Values in effective mathematics lessons in Sweden: what do they tell us?

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Abstract: This study aims to examine values in effective mathematics lessons in Sweden from the perspectives of students in different groups and their teachers. By using methods with lesson observations, student focus group interviews and teacher interviews, it shows that instructional explanation and classroom atmosphere with quietness are shared-values of students and their teachers. The findings propose some crucial issues which related to how mathematics teaching could be adjusted to different students’ learning conditions and whether it needs more instructional explanation in mathematics teaching in Sweden.

Keywords: value, effective teaching and learning, mathematics teaching, explanation, Sweden

1. Introduction

The effective teaching and learning of school mathematics is one of the major objectives of mathematics education. Researchers have drawn on an extensive range of intellectual resources to address how effective mathematic teaching and learning can be implemented, which are demonstrated in several dominant approaches in the history of mathematics.

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education research. The cognitive approach has a long tradition, until 1990s, it was still prominent with the typical vocabularies of *mental schemes*, *misconceptions*, and *cognitive conflicts* (Sfard, Forman & Kieran, 2001). And the subsequent language seems to be given way to the affective approach with *beliefs*, *attitudes* and *emotions* (Zan, Brown, Evans & Hannula, 2006). Both cognitive and affective approach deepens our understanding of how we can better facilitate mathematics teaching and learning. However, the insights from cognitive perspective and the knowledge we have gained from the affective perspective appear to lead to inconclusive attributes to what constitute effective mathematics teaching and learning (Lerman, 2001; Cai, 2007).

With the increasing interest in the socio-cultural nature of mathematics education, researchers are becoming increasingly aware of the significant role that the socio-cultural milieu plays in the teaching and learning of mathematics (De Corte & Verschaffel, 2007). And there is a growing agreement that there may not be a universal trait for effective mathematics teaching and learning within different cultural contexts, therefore, it is expected through a focus on the socio-cultural construct of values to further contribute to our sifting, clarifying and understanding of the different grains of constituents of effective mathematics education (Bishop, Seah & Chin, 2003; Seah, 2007).

This present study is part of a large scale collaborative project (NOTE 1 HERE) which adopts the socio-cultural perspective, to investigate how effective mathematics teaching and learning might be facilitated through an understanding of what teachers and students value in different social contexts. This paper reports the preliminary findings from Sweden, namely, what are valued in effective mathematics lessons from the Swedish students and their teachers’ perspectives.
2. Theoretical background

There are two major issues in this paper. One is about effective mathematics teaching and learning, and the other is about value related to mathematics education. This section offers a general overview of various studies related to these two issues and acts as a point of departure for this study.

2.1 Effective mathematics teaching and learning

During the last decades, the body of research on effective mathematics teaching and learning has grown exponential. These studies focus on characteristics on different levels in mathematics education or some specific kind of intervention in the mathematics educational process. Common to these studies is their main interest in what works in mathematics education. Empirically these studies search for the relationship between these relevant characteristics at the different levels or the intervention, and some dependent variables that often labeled as “effectiveness criteria” or “output measures” (Muir, 2008). Many of these studies focus on the effects on solely one effectiveness criterion. However, despite the broad range of research, the very notion of “effectiveness” remains to be an elusive concept (Seah, 2007). Recently Slavin and his colleagues draw a similar conclusion from a series of comprehensive synthesis on the research on the achievement outcomes of different types of approaches to improving elementary and secondary mathematics, that is, the most striking finding from the review is that the evidence supports various instructional process strategies (Slavin & Lake, 2008; Slavin, Lake & Groff, 2009).

Research into the actions of teachers and their interactions with students is particularly useful as observed features of effective teaching practice. An influential study
in this regard was conducted with 90 primary school teachers and more than 2,000 students in the United Kingdom in the late 1990s by Askew, Brown, Rhodes, Johnson, and Wiliam (1997), which is widely cited in the literature (Muir, 2008). The findings from that study seem interesting that relatively high achievement gains were not necessarily related to specific teaching styles, but were associated with teachers who had ‘connectionist’ orientations (as opposed to ‘transmission’ or ‘discovery’ orientations), focused on students’ mathematical learning (rather than on provision of pleasant classroom experiences), provided a challenging curriculum (rather than a comforting experience), and held high expectations of initially low-attaining students (Seah, 2007).

Many of the features listed above reflect the value of meaningful and constructive classroom interactions between teacher and students, and perhaps also amongst students. Meanwhile, the various international comparative studies generally arrive at similar conclusions that effective teaching is more about responding to and valuing the socio-cultural aspect of the learning environment than it is about adopting particular teaching methods. For example, Hollingsworth, Lokan and McCrae’s (2003) analysis of the Third International Mathematics and Science Survey (TIMSS) 1999 Video Study data revealed that successful teaching of mathematics in schools has not been found to be associated with any one teaching method, and in fact, it was evident that amongst the high achieving countries a variety of teaching methods had been employed. Based on insights and suggestions from the previous studies, instead of defining the effectiveness in our study, we assume that the effective mathematics teaching and learning is reflective of a group of values identified.

2.2 Values related to mathematics education
Important theorists in a variety of fields have emphasized the importance of people’s value priorities in understanding and predicting attitudinal and behavioral decisions (Rohan, 2000). In mathematics education, Bishop (1998) claimed that dealing with issues of democracy in mathematics education clearly requires engaging with values. In light of the findings from TIMSS and the debates on mathematics curriculum standards in the UK, Macnab (2000) argued that standards of attainment in school mathematics are closely connected to belief systems regarding value and purpose; those systems do not always collectively offer a credible and coherent vision for mathematics education which can be effectively implemented in school classrooms; and that this coherence of vision is what to a large extent characterizes the higher performing TIMSS countries.

Meanwhile, researcher argued that it is difficult to identify values. For this, some concepts such as “good” and “bad” are necessary (Swadener & Soedjadi, 1988). According to Fraenkel, a value is a concept or an idea that was considered as by someone in life, and values are ideas about the worth of thinking, they are concepts, abstractions (1977; Cited in Dede, 2006). Jablonka and Keitel (2006) see values as the principles, standards and qualities explicitly or implicitly considered worthwhile or desirable by the participants of a distinct social practice. In fact, mathematics education is designated as social practices where the teaching and learning of mathematics actually occur, and it is deeply rooted in its particular culture. In this sense, cultural values in mathematics education cannot be removed from the environment with which the values are held. Therefore, in our study, what the Swedish students and their teachers value in effective mathematics lessons are what we look for.

Values related to mathematics education operate at different levels. Bishop (1998)
3. Research aim

Before the year 1994, the Swedish schools was centralized and governed in detail on the national level. After two decades of decentralization and deregulation, the Swedish school system is now a goal-based system with a high degree of local responsibility. At the present, one of the most closely followed educational issues in Sweden is centred on pupils who find it difficult to reach educational attainment objectives. There is especial focus on students who do not manage to achieve the pass levels in the three core subjects (Swedish, English and mathematics) and who, for this reason, do not satisfy the requirements for admission to upper-secondary education. Mathematics, one of the three core subjects, stands out and presents the principal concern. For approximately 13% of year nine students in Sweden, mathematics has been the main obstacle to future studies (Skolverket, 2007). This is a major worry not only for students, parents and teachers, but also for politicians and decision-makers. In the election-debates in Sweden in 2006, attention was frequently focused on education-related issues and, in particular, the problem presented by the large number of students who do not manage to satisfy the requirements for admission to upper-secondary education (Sjöberg & Nyroos, 2009). According to the declaration of human rights with the intention “the school for everyone”
in the Swedish school system, teachers have an obligation to teach each student in accordance with his/her own prerequisites and previous knowledge in order to promote further learning. An underlying idea is that each human being is teachable even despite any type of categorized disability, and this assumes that the teaching is adjusted to the individual’s conditions for learning. However, this aim still remains to be fulfilled (Eriksson, 2008). This study aims to make up for this gap, through focusing on an understanding of values of students in different groups and their teachers as well as the values differences they negotiate. Students in the regular group have the regular mathematics teaching and learning, and students in the special group have special needs in mathematics teaching and learning. It is designed to answer the following research questions: What are valued in effective mathematics lessons by students in the regular group? What are valued in effective mathematics lessons by students in the special group? What are valued in effective mathematics lessons by their teachers?

4. Methodology

4.1 Participants

Two mathematics teachers and their students respectively in grade 7 and 8, in the same school, participated in the present study. The school was relatively large and had high prestige, located in a city in the northern part of Sweden. Both teachers are female and experienced, with teaching years of 12 and 24, respectively. The grade 7 class was a regular group and the grade 8 class was a special group. These two classes had been continually observed for one year by one of the authors.

4.2 Data collection

Data collection included lesson observations, student focus group interviews and teacher
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interviews. The data collected as a result of the lesson observation sessions represent valuing in process, and the lessons observed provided a basis for teachers and students to reflect on and discuss. Students were encouraged to take notes of the moments when they feel that they are learning mathematics particularly well in the observed lesson. After each of the observations, interview sessions were held with the student focus group and followed by a session with each of the teacher participants. Structured interview questions were respectively based on students’ recall of the moments of effectiveness and drawn upon each teacher’s reflective thoughts relating to the lessons observed.

Interview questions for the students included the general ones like “What should an effective mathematics lesson look like?”, and the specific ones like “Why do you think this represents a moment of effectiveness?” Interview questions for the teachers included the general ones like “In your opinion, what should an effective mathematics lesson look like?”, and the specific ones like “Here are some episodes mentioned by your students, which are regarded as moments of effectiveness. Have a look through. Which episodes surprise you? Would you like to explain why?” The interviewer tried to be careful not to lead the students and teachers to any answer but to encourage them to express their thoughts freely. The student interview lasted about 30-40 minutes and the teacher interviews about 50-60 minutes. The interviews were performed in Swedish, audio-recorded and transcribed into English. Six students in three different levels in the regular group were chosen by their teacher according to their achievements and genders to participate in the interview session, while all seven students in the special group were interviewed. All students had the permission from their parents.

The data collection was guided by the ethical rules formulated by the Swedish
Research Council, concerning information, consent to participate, scientific use of information and confidentiality (Swedish Research Council, 2006).

### 4.3 Data analysis

The data was analyzed in several phases. Firstly, every sentence which appeared to be valued was recorded. Secondly, different words and expressions connected to the different values were recorded. Thirdly, values were named. For example, when the interviewer asked “Was there any one of you that got an experience that ‘I’m learning mathematics particularly well’ during this lesson?”, one student said that “I understood it better now when she had her lecture”. We recognized that the teacher’s instructional explanation was valued by the student, then, taking the references from the cross-checking with experiences from other participating regions (or countries), the name was given so as to clarify what it means. Here, it was identified as valuing “explanation”.

Furthermore, in the transcripts, different students had the same value, but expressed it in a different way. For example, students said that “Perhaps that I got to hear how others thought about these problems. And I also answered a question myself. I feel that I already knew it, but it’s still good to hear it more when she explains something in front of the whole class”. We analyzed it as the same value “explanation”.

By using this way, values were identified, labeled and categorized. To describe the value priorities within different students, the frequencies of the values was calculated in the student interview transcripts. Two researchers analyzed the data independently. Disagreements concerning the analysis were negotiated until joint agreement was established. Validity of research findings was enhanced through triangulation of data sources.
5. Results

5.1 Values in effective mathematics lessons: students’ perspective

A total of 36 counts of 7 different values from 25 descriptions were identified as values in effective mathematics lessons, which represented the perspective from students in the regular group. And a total of 24 counts of 6 different values from 22 descriptions were identified as values in effective mathematics lessons, which represented the perspective from students in the special group. Table 1 summarizes the results.

Table 1: Values in effective mathematics lessons: students’ perspective

<table>
<thead>
<tr>
<th>Students from regular group</th>
<th>Students from special group</th>
</tr>
</thead>
<tbody>
<tr>
<td>36 counts of 7 values from 25 descriptions</td>
<td>24 counts of 6 values from 22 descriptions</td>
</tr>
<tr>
<td>personalized help [ME]</td>
<td>explanation [ME]</td>
</tr>
<tr>
<td>explanation [ME]</td>
<td>independence [ME]</td>
</tr>
<tr>
<td>quietness [IO]</td>
<td>relaxation [IO]</td>
</tr>
<tr>
<td>collaboration [ME]</td>
<td>quietness [IO]</td>
</tr>
<tr>
<td>sharing [ME]</td>
<td>fun [ME]</td>
</tr>
<tr>
<td>strictness [P]</td>
<td>personalized help [ME]</td>
</tr>
<tr>
<td>concentration [IO]</td>
<td></td>
</tr>
</tbody>
</table>

ME=mathematics educational value
IO=institutional/organizational value
P= (teacher) personality value

Amongst the values identified from students in the regular group, there are 4 mathematics educational values, 2 institutional values, and 1 (teacher) personality value. The values include personalized help, explanation, quietness, collaboration, sharing, strictness, and concentration. This may allow us to depict what an effective mathematics
lesson in Sweden might look like from the perspective of students in the regular group. With reference to the three most cited values, an effective mathematics lesson is likely to be one which contains instructional *explanation* presented by the teacher, and keeps the classroom atmosphere of *quietness*. And the effective mathematics lesson would mostly likely embrace *personalized help*. Of course, this is not to suggest that all these values operated in any one effective mathematics lesson.

Amongst the values identified from students in the special group, there are 4 mathematics educational values, 2 institutional values. The values include *explanation*, *independence*, *relaxation*, *quietness*, *fun*, and *personalized help*. This allows us to depict what an effective mathematics lesson in Sweden might look like from the perspective of students in the special group. With reference to the three most cited values, an effective mathematics lessons is likely to be one in which students have the *independence* on their work, and the classroom atmosphere of *relaxation* would be remained. And the effective mathematics lesson would mostly likely contain instructional *explanation* presented by the teacher.

### 5.2 Values in effective mathematics lessons: teachers’ perspective

A total of 8 different values from 10 descriptions were identified as values in effective mathematics lessons, which represented the perspective from the teacher who teaches the regular group. And a total of 6 values from 13 descriptions were identified as values in effective mathematics lessons, which represented the perspective from the teacher who teaches the special group. Table 2 summarizes the results.
Table 2: Values in effective mathematics lessons: teachers’ perspective

<table>
<thead>
<tr>
<th>Teacher from regular group</th>
<th>Teacher from special group</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 values from 10 descriptions</td>
<td>6 values from 13 descriptions</td>
</tr>
<tr>
<td>explanation [ME]</td>
<td>interests[ME]</td>
</tr>
<tr>
<td>whole-class interaction [ME]</td>
<td>communication [ME]</td>
</tr>
<tr>
<td>quietness [IO]</td>
<td>visualization [ME]</td>
</tr>
<tr>
<td>communication [ME]</td>
<td>quietness [IO]</td>
</tr>
<tr>
<td>group work [IO]</td>
<td>explanation [ME]</td>
</tr>
<tr>
<td>experiment [ME]</td>
<td>authenticity [ME]</td>
</tr>
<tr>
<td>hands-on [ME]</td>
<td></td>
</tr>
<tr>
<td>outdoor learning[ME]</td>
<td></td>
</tr>
</tbody>
</table>

ME=mathematics educational value  
IO=institutional/organizational value

These values are listed according to the order of appearance in the interviews, and the order of the appearance of which might potentially reflect the value priorities. With reference to the first three cited values of the teacher who teaches the regular group, an effective mathematics lessons is mostly likely to be one which contain instructional explanation. And it would capitalize on whole-class interaction, while also keeping the classroom atmosphere of quietness. Likewise, from the perspective of the teacher who teaches the special group, an effective mathematics lessons is mostly likely to be one in which students’ interests would be aroused. And it would embrace communication, while also capitalizing on visualization to eliminate the potential learning difficulties.

6. Conclusion and discussion
6.1 Differences and similarities of values of students in the different groups

The results show that there are differences and similarities of values of students in the regular group and special group.

In general, the values of students in the special group are more divergent, which can be concluded from table 1. 7 values identified from 36 counts compared to 6 values identified from 24 counts, which may suggest that values are more shared by students in the regular group than in the special group. Next, there are specific value differences, for example, collaboration, sharing, strictness, and concentration in the regular group are not mentioned in the special group, while relaxation, independence, and fun in the special group are not evident in the regular group. Among those different values in the two groups, there are two pairs of opposite values, collaboration and sharing as opposed to independence, concentration as opposed to independence. These striking differences of values of students in different groups may reflect their different learning experience and learning requirements, hidden behind these important differences may also reflect something about the origins and role of values for effective mathematics learning for different individuals. This finding is crucial, in the sense that, it gives more evidences for that, to meet the human rights in teaching, we have to abandon the idea of using the same teaching methods for all students of the same age (Eriksson, 2008). And it reminds us that we should be more sensitive to different students’ prerequisites and learning needs, to facilitate the implementation of mathematics teaching adjusted to students in different groups. As the findings show, for students in different groups, it might need to consider opposite methods. This situation is also urgent for the mathematics education in special groups or in special schools, where the main requirements are to arrange teaching and
learning with regard to those students’ specific conditions.

There are three common values of students in both groups, that is, personalized help, explanation, and quietness. Especially, the value explanation are highly valued by both groups, listed respectively as the second one in the regular group and the first one in the special group. This may reflect that teacher’s instructional explanation is very important to facilitate effective mathematics learning for students in both groups. This finding is supported by some related studies, for example, to examine some of the classroom processes that may be responsible for the stellar mathematical performance among Asian children compared to U.S. children, Perry (2000) studied the differences in the frequency and type of mathematical explanations during lessons observed in 80 U.S., 40 Chinese, and 40 Japanese 1st- and 5th-grade classrooms, and found that explanations occurred more frequently in the Japanese and Chinese classrooms than in U.S. classrooms.

6.2 The shared-values of students and teachers

The results suggest that there are some important shared-values of students and their teachers. For the students in the regular group and their teacher, there are the shared-values of quietness and explanation. For the students in the special group and their teacher, there are the same shared-values of quietness and explanation. These shared-values identified may contribute to our understanding how effective mathematics teaching and learning might work.

However, according to the different sources related to this finding, it seems there is a paradoxical situation. On the one hand, explanations seem to be a large and natural part of our cognitive lives, not necessary to mention it in the classroom teaching (Wittwer & Renkl, 2008), thus, it is understandable that clear and detailed explanation are a
shared-value of students in different group and their teachers in effective mathematics lessons. But on the other hand, according to a longitudinal classroom observation study conducted by one of the authors, preliminary findings show that the pattern of mathematics teaching in Sweden seems not based on explanation, on the contrary, it is mainly based on students’ individual work, their work on mathematics textbooks (Peng, 2010). Logically, these two opposite observations cannot be seen to be simultaneously valid. One interpretation could be explained from the limitation of the methodology in this study. Since the qualitative methods adopted in this study, might not lead to the generalization to the pattern of mathematic teaching in the whole Sweden. Another interpretation could be that, indeed there lacks of enough instructional explanation in mathematics lessons or good explanations which should be adapted to the learner’s knowledge prerequisites, and should focus on concepts and principles, and should be integrated into the learner’s ongoing cognitive activities as research suggested (Wittwer & Renkl, 2008). To understand which interpretation is more reasonable, it’s expected to have further investigation on this important issue.

6.3 Differences and similarities of values of the teachers

There are three common values of the two teachers who taught students in different group, namely, explanation, quietness, and communication. Communication are valued by both teachers, which is consistent with the point of view in growing number of researchers have been interested in studying the learning of mathematics as a collective enterprise in socio-cultural contexts, rather than as a process occurring only within an individual mind (Inaqaki, Hatano & Morita, 1998). Thus, it is understandable that teachers value that in effective mathematics lessons students’ mathematical ideas develop through their
communicative practices or dialogical interactions with other members of the classroom.

There are value differences between the two teachers. For example, whole-class interaction, group work, experiment, hands-on and outdoor learning by the teacher who taught the regular group are not mentioned by the teacher who taught the special group, while interests, visualization, and authenticity valued by the teacher who taught the special group are not evident in the values of the teacher who taught the regular group. Among the different values of both teachers, there is only one institutional value group work, and the dominance of mathematics educational values of both teachers reflects the nature of values of showing variety of form in mathematics education. However, are the different mathematics educational values of teachers a testimony to the role that the teacher plays in making different professional choices to facilitate effective mathematics learning according to different students’ learning needs? The answer for this question needs to be confirmed by including more samples.

7. Concluding comments

This study had sought to examine values in effective mathematics lessons in Sweden from the perspectives of students and their teachers. The preliminary findings reveal that both the teachers and the students share some commonalities in what they both value in the shaping of effective mathematics lesson. These include instructional explanation and the classroom atmosphere with quietness. It helps us understand better how the values attribute the effectiveness in mathematics teaching and learning in Sweden. More importantly, the findings in this study propose some crucial issues which related to how mathematics teaching could be adjusted to different students’ learning conditions and
whether it need more instructional explanation in mathematics teaching in Sweden. It is
expected those issues will inform school districts of improved mathematics teaching
practices.

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preparation.


The project is called *The Third Wave: Regional Study of Values in Effective Mathematics Education*, which involves 12 places across 4 continents, designed to investigate how effective mathematics teaching and learning might be facilitated through an understanding of values of teachers and students across different nations/culture, and how they negotiate value differences to optimize their interactions, leading by Dr. Wee Tiong Seah, Monash University.