The Mathematics Enthusiast

Volume 14
Number 1 *Numbers 1, 2, & 3*

Article 8

1-2017

Linking pre-service teachers' questioning and students' strategies in solving contextual problems: A case study in Indonesia and the Netherlands

Rahmah Johar

Sitti Maesuri Patahuddin

Wanty Widjaja

Linking pre-service teachers' questioning and students' strategies in solving contextual problems: A case study in Indonesia and the Netherlands

Rahmah Johar
Syiah Kuala University, Indonesia

Sitti Maesuri Patahuddin
University of Canberra, Australia

Wanty Widjaja[1]
Deakin University, Australia

Abstract: This study examined the relationship between teachers' questioning techniques and students' strategies in solving contextual mathematical problems. This case study was undertaken with one pre-service teacher (and 22 Year 4 students) from Indonesia and one pre-service teacher (and 25 Year 4 students) in the Netherlands. Both pre-service teachers assigned the same problems to their students and these problems were novel for the students in both countries. The lessons were observed by the first author and video recorded for data analysis. Qualitative data analysis was undertaken through within-case and cross-case analysis. The findings suggest that the contextual problems, the way pre-service teachers prompt students' thinking, and the curriculum context were highly influential in the way the students solved the problems.

Key words: questioning, contextual problem, and students' strategies, fractions, Realistic Mathematics Education

Introduction

There is now a widespread body of knowledge that identifies the potential and challenges of using contextual problems to advance children's learning of mathematics. Some studies have demonstrated that context plays a significant role as a starting point of learning for students because it allows students to start with informal strategies and offers opportunities for students to solve the problems at different levels of formality (Freudenthal, 1983, 1991; Gravemeijer & Doorman, 1999; van den Heuvel-Panhuizen, 1996, 2000; van den Heuvel-Panhuizen & Wijers, 2005). This was also reported in earlier studies from Indonesia

[1] w.widjaja@deakin.edu.au

The Mathematics Enthusiast, **ISSN 1551-3440, vol. 14, nos1, 2&3**, pp.101 – 128

where the use of contextual problems supported students to develop mathematical understandings (Dolk, Widjaja, Zonneveld, & Fauzan, 2010; Sembiring, Hadi, & Dolk, 2008; Widjaja, Dolk, & Fauzan, 2010). However, existing studies have also revealed the challenges of incorporating contextual problems into students' learning for two main reasons. Firstly, students bring different experiences and prior knowledge to the classroom. This may influence the students' interpretations of the contextual problem and lead them to solve the problem in an inappropriate way (Boaler, 1993; Carraher & Schliemann, 2002; Widjaja, 2008, 2013; Wijaya, van den Heuvel-Panhuizen & Doorman, 2015). Secondly, the way students solve contextual problems depends on how the students have been taught (van den Heuvel-Panhuizen, 2005).

Since 2000 a movement called '*Pendidikan Matematika Realistik Indonesia' (PMRI)* was adapted and implemented in Indonesia. This was inspired by Realistic Mathematics Education (RME) which was developed in the Netherlands (Freudenthal, 1983, 1991; Sembiring, Hadi, & Dolk, 2008; Sembiring, Hoogland, & Dolk, 2010). PMRI advocates the teaching of mathematics which moves away from teaching based on symbolic manipulation. Studies incorporating the use of contextual problems in Indonesian classrooms confirm this assertion (Widjaja, Dolk, & Fauzan, 2010; Dolk, Widjaja, Zonneveld, & Fauzan, 2010). Nevertheless, the bottom up approach in the PMRI movement implies that these changes are not mandated. As a result the PMRI movement has not been implemented nationally and is not reflected in Indonesian national curriculum. To date there are still limited opportunities for students to engage in making sense of mathematics through contexts in Indonesian textbooks (Fauzan, Plomp & Gravemeijer, 2013; Wijaya, 2015; Zulkardi, 2002). Similarly, solving contextual problems in multiple ways is another approach which is not common in Indonesia. Many concepts in mathematics are introduced without context, nor do they enable students to explore multiple meanings of the concept. For instance, fractions are only introduced as parts of a whole. Fractions as quotients and ratios are excluded. This is not in line with the RME principles where fractions are introduced through various real-life contexts such as fair sharing, measurements and money so as to enable students to understand the relationship between all the concepts involving fractions. According to RME principles, students are expected to learn fractions in informal ways using their "own investigations " before moving on to more abstract and formal mathematics (van den Heuvel-Panhuizen & Wijers, 2005).

This case study is based on teaching experiments conducted by volunteer Indonesian and Dutch pre-service teachers using contextual problems on fractions with Grade 4 children in Indonesia and the Netherlands. The study aims to examine how pre-service teachers' questioning influence students' reasoning in solving contextual problems with fractions. Whilst a number of cross-national studies on the topic of fractions have been documented (Alajmi, 2012; Cai & Wang, 2006; Moseley, Okamoto, & Ishida, 2007; Mosvold, 2008; Son & Senk, 2010; Yim, 2010), to date there is no study where pre-service teachers examine their own questioning as a way to engage students when solving contextual problems about fractions. Hence, the following research questions were examined:

1. What strategies do students use when solving contextual problems involving fractions?
2. How do pre-service teachers' questioning promote students' strategies when using contextual problems involving fractions?

Literature Review

Contextual Problems

Much research has been conducted on contextual problems in mathematics (e.g., Chapman, 2006; Gravemeijer & Doorman, 1999; van den Heuvel-Panhuizen & Drijvers, 2014). A considerable quantity of this research was based on the Realistic Mathematics Education (RME) that was established in The Netherlands, which has now spread across many countries, including Indonesia. Lubienski (2000) pointed out that when solving mathematical contextual problems, some students "approached the problems in a way that caused them to miss the intended mathematical points" (p. 454). Relatedly, Kastberg, D'Ambrosio, McDermott, and Saada (2005) identified one difficulty in incorporating contextual problems as national assessment items (e.g., NEAP) was due to the difficulties in scoring the items. This is because contextual problems required the integration of mathematical and practical knowledge. They argued that "the interpretations of students' solutions should be based on the assumptions that each student makes about the problem context and his or her ability to integrate personal, mathematical and other content knowledge to craft a reasonable solution" (p. 11). Hence, further research is required to fully understand how contextual problems are utilized in the classroom.

Designing a contextual problem is a good step to find out whether the students are interested in the task and therefore will be challenged to solve it. Gravemeijer (2010) states with RME teachers play a vital role in choosing and structuring the problem as well as eliciting and stimulating students' thinking during a whole class discussion. Gravemeijer (2010) asserts that RME tries to relate mathematics to students' experiences. It has to be understood that the term, 'real' in 'realistic' is the sense of being meaningful for the students. Therefore, the selection of realistic problems is very important. In RME, van den Heuvel-Panhuizen (1996) offered two requirements of the problems, that is, the problems must be meaningful and informative to students to re-invent mathematics in their own way. In order for a problem to be meaningful, it should be obvious to the students why an answer to the contextual problem is needed. Meaningful also relates to a subject matter, it covers the entire breadth and depth of the mathematical area. Another requirement is informative, which means that the problem must be as clear as possible to the students. Furthermore students must have the opportunity to give their own answers in their own words so they can explore and establish different strategies at different levels of sophistication.

Conceptions of fractions

In general, conceptions of fractions involve five sub-constructs: part-whole, quotient, ratio, measure, and operator (Behr, Khoury, Harel, Post, & Lesh, 1997; Pantziara & Philippou, 2012). Several studies documented that the part-whole sub construct was heavily emphasized in textbooks and in the teaching and learning of fractions in countries such as United States, Australia and Indonesia (Charalambous & Pitta-Pantazi, 2005; Gould, 2005, 2011; Masitoch, Mukaromah, Abidin & Julaeha, 2009; Thompson & Saldanha, 2003; Watanabe, 2007). Charalambous and Pitta-Pantazi (2007) found that this lack of attention to the other four sub-constructs affects students' conceptual understanding of fractions and is realised in student performance "the differences in students' performance on the five interpretations of fractions mirror the imbalance in emphasis placed on them during instruction" (p. 309). The consequences of placing an imbalanced emphasis on the part-whole conception are apparent in the student's treatment of fractions as two different whole numbers: numerators and denominators when comparing two fractions (Gould, 2005; Olive & Vomvoridi, 2006; Steinle & Price, 2008) and their limited understanding of fractions as rational numbers (Lamon, 2007, 2012). This difficulty was also found among pre-service teachers when solving problems that required other conceptions of fractions beyond the part-whole notion (Browning et al., 2014; Lo & Grant, 2012; Olanoff, Lo, & Tobias, 2014).

More attention to other conceptions of fractions is found in the Japanese textbooks. Fractions are introduced using the measurement contexts of length, capacity, volume and mass where the whole is one measurement unit (Alajmi, 2012; Watanabe, 2007). A familiar context such as measuring the length of a student's arm using a metre ruler presents a meaningful context where a fraction is needed and this allows students to develop a notion of fraction as quantities. The part-whole notion of fractions is mainly used to establish the idea of unit fractions for instance ½ as one half of one metre. A concerted effort to help students in establishing links between the context of measuring length and the number line model is depicted consistently throughout the textbooks. While number lines are also frequently used in the U.S textbooks, Watanabe (2007) argued that the limited exposure to the part-whole meaning of fractions in the U.S textbooks might hinder students to develop understandings of fractions as numbers.

There is a clear focus in introducing different conceptions of fractions grounded in contextual problems that could be considered real for students in the Dutch educational system (Bruin-Muurling, 2010; Keijzer, 2003; Keijzer & Terwel, 2001; Streefland, 1991). For instance, fractions are closely linked with length measurement using linear models such as bars and number lines (Keijzer & Terwel, 2001). Other meanings of fractions such as part-whole, ratio and operator are also represented in the textbooks. However, detailed analysis of fractions in four primary and two secondary textbook series by Bruin-Muurling (2010) revealed that while fractions and fraction multiplication are firmly grounded in contexts, students' various informal strategies are often followed by teaching specific procedures. They contend that this approach does not support the development of more advanced understanding of fractions such as "the dual conceptualization of a fraction as a number and as an operation" (p. 38). Gravemeijer et al. (2016) called for shifting the effort from "task propensity – the tendency to think of mathematics education in terms of individual tasks that have to be mastered by students" – to inclusion of more advanced conceptual understandings as key instructional goals (p. 26).

Student reasoning of fractions and choice of fractions representations

Prior studies have underscored the significance of choice of representations including words, models, contexts, and diagrams, and alternation between representations in developing students' and pre-service teachers' conceptual understanding of fractions (Charalambous & Pitta-Pantazi, 2007; Harvey, 2012; Olive & Vomvoridi, 2006; Tobias, 2013). The use of area models such as circles representing pizzas or cakes for teaching fractions situated in the context of fair sharing situation is quite universal despite the fact that creating equal parts of area in circles is not always easy. Mistakes in representing a fraction such as a third by dividing the whole circle into three unequal parts horizontally or vertically are often observed. Gould (2005, 2011) argues that these mistakes reflect students' limited understanding of the area concept and the requirement to divide the whole into three equal parts of area. Furthermore, students need to shift away from additive to multiplicative thinking by comparing the relative size of one third of the circle to the whole. Gould is critical of the use of tasks in primary schools that focuses on shading pre-partitioned shapes because these tasks tend to lead students in associating fractions by "double count" – count the number of parts shaded (numerator), count the total number of parts (denominator) and shade the number of given parts and record the first count over the second count rather than thinking about the relative size area of the parts (p. 69).

Teacher's representations and procedures are critical in establishing the multiplicative relation between a unit fraction and the whole. Olive and Vomvoridi (2006) found that the teacher's initial way of representing "a line segment of indefinite length and then segment this line with the appropriate number of tick marks for the fractional quantity that was to be represented (e.g., for 11/9 she would make 11 tic marks)" led students to see the whole as growing (p. 29). Gould (2011) observed a similar trend among students in Australia and posited a "growing whole" phenomenon could be explained by thinking of fractions additively or thinking of fractions using discrete models or a collection of objects whereby one third is associated with one object out of 3 objects and one sixth is associated with one objects out of 6 objects. In comparing fractions and finding equivalent fractions, the important assumption of using the same whole needs to be realized but this assumption is often implicit. Chinappan (2005) calls for teachers to engage students in reasoning and examining their representations of fractions to validate their choice of an area model to represent part-whole relations of fractions.

Division by fractions is well known to be challenging for both students and adults. Bulgar (2003) conducted a yearlong teaching experiment using a problem called "Holiday Bows" with Year 4 students who had not received formal instruction on division of fractions. Students were given various ribbons, pre-cut to the indicated sizes along with a meter rule, string and scissors to determine "how many bows could be made from each length of ribbon?" (p. 322). It was found that four different strategies emerged with each of the four strategies involving counting. As the problem was explored using a measurement context of dividing metric units of length, naturally the students' strategies involved conversion of metric units. While some students were not able to come up with the exact answer using a fraction, a reasonable estimate was provided. Keijzer and Terwel's (2002) used a context of "the fraction-lift" to divide fractions by two, three and more and noticed that one student

(Audrey) struggled with the problem of dividing ¼ by three as this problem could not be solved using her preferred method of repeated halving. The fraction-lift context offered Audrey opportunities to build on her understanding of equivalent fractions as “fractions living on the same floor of the fraction-building”. This knowledge was also consolidated with “fractions at the same position on the number line” (p. 69). A similar result was reported by Yim (2010) who found nine high achieving students devised various pictorial strategies to solve division by fractions in ‘the Invert Cartesian Product’ (ICP) context. These studies highlight the complexities involved in division of fractions and indicate the potential of using contexts and appropriate representations to help students make sense of division of fractions.

Teachers’ Questioning

Teachers’ questioning plays an important role in shaping classroom interaction (Akkus & Hand, 2010; Funahashi & Hino, 2014). Questioning helps students make connections between prior knowledge and new mathematics content (Heinze & Erhard, 2006; Koizumi, 2013), understand the problem (Warner, Schorr, Arias, & Sanchez, 2013; Weiland, Hudson, Amador, 2013), and develop their mathematical reasoning (Moyer & Milewicz, 2002). Funahashi and Hino (2014) highlight the teachers’ role in eliciting, elaborating and extending students’ thinking during classroom interaction to learn new mathematical content guided by the lesson objectives. Through questioning, teachers can gather insights about the procedure students use (Koizumi, 2013), clarify the students’ understandings (Weiland, Hudson, & Amador, 2014), promote students’ interactions (Martino & Maher, 1999; Mason, 2000), and scaffold their thinking to develop powerful mathematical ideas (Warner et al., 2013).

There are many types of questions teachers use, such as factual, probing, and guiding questions (Sahin & Kulm, 2008); check-listing, instructing, and follow up questions (Moyer & Milewicz, 2002); reproductive, closed, open, evaluative, and rhetorical questions (Heinze & Erhard, 2006). Some researchers have examined the most frequently used questions by teachers and found that teachers ask more factual questions than critical question and have some difficulties in posing productive questions (Sahin & Kulm, 2008). Teachers are found to be asking questions which require yes or no responses more frequently than the higher order questions which require students to explain or justify their thinking (Kawanaka & Stigler, 1999; Weiland, Hudson & Amador, 2014).

International comparative studies such as TIMMS 1995 video study and other studies have examined teachers’ questioning in mathematics lessons. Akkus and Hand (2010) found that the types of questions teachers’ ask impact on the interaction among students during problem solving. Kawanaka, Stigler, and Hibert (1999) compared teachers' questionings from Germany, Japan, and the USA using the TIMSS 1995 Video Study data. They found that German teachers often asked questions that require short responses, American teachers often asked only yes or no questions, and Japanese teachers requested descriptions or explanations of a mathematical object. Perry, VanderStoep and Yu (1993) revealed Japanese teachers' questions covered conceptual knowledge.

There is limited research about pre-service teachers' questioning. Weiland, Hudson, and Amador (2014) examined two pre-service teachers' questioning during a clinical interview. This study found that pre-service teachers' questioning tends to lead students to solve problems in a certain way rather than allowing students to solve the problem their own way. Pre-service teachers also missed a number of opportunities to ask follow-up probing questions, which may have elicited a deeper level of thinking from the students. Yet, there is no study comparing pre-service teachers' questioning in different countries solving the same contextual problems. As Stigler and Hiebert (1999) pointed out, teaching is a cultural activity and it is important to take into consideration the cultural assumptions underpinning teaching and learning. This study aims to extend our understanding in this area by examining how pre-service teachers' from Indonesia and the Netherlands use questioning to influence the strategies students use when solving the same contextual problems which involve fractions.

Methods

Research settings

This study was conducted in 2011 within an exchange program between Katholieke Scholenstichting Utrecht in the Netherlands and Syiah Kuala University in Indonesia, following the tsunami disaster in Aceh. In this exchange program, a group of primary teachers and primary pre-service teachers visited Aceh to teach in several schools, including the *PMRI* pilot schools. The first author worked closely with this group in designing lessons, discussing teaching approaches, as well as reflecting upon the enacted lessons. At the same time, the first author supervised pre-service teachers at Syiah Kuala University during their teaching practicum in primary schools in Aceh. Since both cohorts of pre-service teachers were in the last year of their study, and both were familiar with RME teaching principles, the opportunity arose for the researchers to gain insights about pre-service teachers' questioning in mathematics lessons in Aceh and in the Netherlands. This also allowed us to expand our understanding of mathematics teaching in primary schools through a cross-cultural study.

Participants

The participants of this study included the two pre-service teachers, one from Aceh and one from Utrecht and their students. Both pre-service teachers were enrolled in a four-year teacher education program and were in semester eight of their study. However, their pathways of teaching experiences were different. The Dutch pre-service teacher had already completed relatively longer teaching practicums than the Indonesian pre-service teacher. The Dutch pre-service teacher in this study had already completed relatively longer teaching practicums than the Indonesian pre-service teacher. A personal communication from Fokke Munk (29 May 2016) reports the Dutch pre-service teacher had begun her teaching experiences in schools since Semester 2 (1 day a week in Semester 2, 3, and 4; 2 days a week in Semester 5, 6, and 7; and 3 days a week in Semester 8) while the Aceh pre-service teacher started her teaching experiences in Semester 7 which was for 4 months with 6 days a week. The lessons reported in this paper were carried out with fourth grade students, consisting of 22 male students in an Aceh school and a mixture of 11 male and 14 female students in the Netherlands school.

Contextual mathematics problem

Two problems were given to the students, one problem for each lesson (see Figure 1). Both problems were novel for the students since they were substantially different from what they had learnt.

Problem for Day 1:
Mr. Ahmad has 15 kilograms of meat that he will give to 20 people. How many kilograms of meat does each person receive?
Problem for Day 2:
Mother has 25 kilograms of rice. She cooks 3/4 of a kilogram every day. How many days will it take for the rice to run out?

Figure 1. Contextual problems given to the students

Prior to the lessons, the first author and both pre-service teachers agreed to pose both problems to their students. They also discussed the importance of encouraging students to find their own solutions. After their first lesson, the first author discussed the strategies the students' used with each pre-service teacher and how they could be improved in solving the second problem in the second lesson.

Curriculum Context

Prior to the lessons, the fourth grade students in Indonesia had learnt (1) whole number operations including standard algorithms in addition, subtraction, multiplication, and division (2) fractions as part of a whole, fractions as a quotient, equivalent fractions, addition, and subtraction of fractions. The Dutch students had learned addition, subtraction, multiplication, and division of whole number. However, they had not learned the formal strategies for whole number operations. Division problems involving large numbers were very new for these students, however they were familiar with dividing special numbers, for example 10÷2, 100÷2, 50÷2, 500÷2, and 1000÷4. Another difference was, the Dutch students were familiar with different strategies in solving whole number operations, without using standard algorithms. They were also familiar with different strategies in solving fraction problems that involved fractions as part of a whole and as a ratio (van den Heuvel-Panhuizen & Wijers, 2005).

A notable difference between the Indonesian and Dutch curriculum was in its structure. This is reflected in the number of mathematics topics that were organised and displayed in the textbooks. The Indonesian textbook content is presented in chapters and each chapter contains one particular topic such as fractions. Meanwhile the Dutch textbooks cover many topics and diverse contexts in one unit or lesson. For example, in lesson 2 grade 4 students learn fractions using multiple contexts such as fractions as part of cake, part of volume with liters and even multiplatication of whole numbers in the same lesson.

Data Collection

Data were collected using a video recorder, a digital camera, field notes, and through discussions with the pre-service teachers. The data focused on the interactions between student and student as well as student and the pre-service teacher when solving contextual problems in the classroom. To record the strategies the students' used, work samples and posters were collected. Dialogue between the pre-service teacher and their students was transcribed to allow the researchers to examine the questioning the pre-service teachers' used to support their students while working out the contextual problems. The first author observed all the lessons and took field notes.

Data analysis

The four lessons were videoed and transcribed so as to capture teachers' language and actions to facilitate students' interaction. Analysis was conducted in two stages, namely within-case analysis and cross-case analysis (Merriam, 1998a, 1998b). To do a within-case analysis, the first and second researcher re-read the transcript and the field notes of each case a number of times so as to gain deeper insights of each case. Questions asked in analysing each case included: What strategies did the students use to solve the two contextual problems? What questions were posed by the pre-service teachers? How did these questions affect the strategies the student's used when solving the problems? Both researchers applied constant comparison for the cross-case analysis. As Merriam (1988a, 1988b) suggests a cross-case analysis provides the opportunity to compare each case to determine the similarities and differences (Merriam, 1988a, 1988b). This analysis involved many discussions between the two researchers about interpreting the data. The results of the data analysis were then further discussed with the third researcher, particularly to address the different interpretations that emerged during the initial data analysis. Finally, the most telling or representative extracts were selected for reporting.

Results

This section presents the interactions that took place in both lessons. The focus is on examining the strategies the students' used to solve the problems on Day 1 and Day 2 and how the teachers shaped the students' strategies with their questions or prompts. Strategies the students use are related to various fraction sub-constructs and different representations. The questions the teachers' posed are indicated by statements followed by a question mark and prompts are an affirmative to elicit students' ideas. Questions or prompts that are considered influential on students' strategies are presented in bold fonts.

Overall, this study suggests that the Indonesian students utilised one sub-construct of fractions, i.e., the part-whole construct whereas the Dutch students applied various sub-constructs of fractions including part-whole, ratio, quotient, and operator. Table 1 summarised students' strategies in solving both problems.

Table 1. *Summary of students' strategies in solving the contextual problems*

Contextual problem	Indonesian students' strategies	Dutch students' strategies
Day 1	1. Division of whole number: (none) 2. Fraction Part of a whole	1. Division of whole number a. Ratio [5]

	a. 1 unit represent 1 kilogram [17] b. 1 unit represent 15 kilogram consist of 15 small units [5]	b. Quotient 1) Splitting [2] 2) division with skip 0, e.g: 15000÷20 = 1500÷2 [16] 2. Fraction as Operator [2]
Day 2	Fraction as part of a whole a. 1 unit represent 1 kilogram b. 1 unit represent 25 kilogram consist of 25 small units	Fraction as: a. Part of a whole: 1 unit represent 1 kilogram [3] b. Part of a whole then ratio [5] c. Operator then ratio [4] d. Operator then quotient: repeated subtraction [1]

Note: [ ] mean the number of students

The following section illustrates the conversations between the pre-service teachers and their students. Two different groups from each class are selected to represent students' strategies and teachers' questions. Group A and Group B are the Indonesian class and Group C and Group D are the Netherlands class. To ensure the participants' anonymity, all participants were coded. SI is used for Indonesian students and SD for Dutch students.

The First Lesson in the Indonesian School

The pre-service teacher began the lesson through a demonstration of distributing 1 kilogram of sugar equally into 5 plastic bags and introduced the problem: "*This sugar is distributed into these five small plastic bags. How many kilograms of sugar is in each small plastic bag?"*. Some students answered 1/5 of a kilogram. Other students seemed to agree with it. The teacher then directed the students to a new problem written on the board. She read the problem, "*Mr. Ahmad has fifteen kilograms of meat that he will give to twenty people. How many kilograms of meat does each person receive?*" and asked the students to solve this problem in their groups.

She continually monitored the progress of each group, encouraged them to move on, asked for clarifications, and prompted students' thinking. Eventually all groups represented their answers pictorially using an area model, but with different processes. Two groups' answers are presented to illustrate the links between teachers' questions and students' strategies. Both of these groups utilised the same sub-construct of fractions, i.e., the part-whole but the whole was 'defined' and represented differently at the beginning. Group A considered each kilogram of meat as the whole and used halving to create 20 halves to be divided equally among 20 people. Then the group partitioned the remaining 5 kilograms into 20 quarters and divided these equally again among 20 people whereas Group B considered 15 kilograms as the whole and attempted to use paper folding to represent a division of the whole by 20.

Group A:

T : How do you solve this problem? [T asks Group A].
SI4 : Using division.
T : **What is the answer? You may use any method, such as drawing or counting. It's up to you**.
SI4 : [Talks to his group] How is fifteen divided by twenty? Fifteen times what equals twenty? Five times three is fifteen, it is less than twenty.
SI3 : Should we draw a picture? [He proposes an idea to his friends then asks his teacher] What kind of picture should we make for the meat, Miss?
T : **It's up to you. You can draw rectangles, circles, or any other shapes**.

One of student (SI1) draws 15 rectangles to represent 15 pieces of meat, each of 10 rectangles is divided into two parts] as Figure 2.

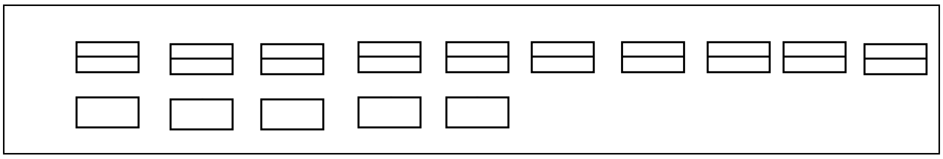


Figure 2. The first step the student uses to share 15 kg of meat into 20 parts

T : You have fifteen pieces of meat. Each is divided into two, so how many [pieces] in total?
SI4 : There will be some leftover meat.
T : How many [parts] of the meat will be left?
SI4 : Five.
T : Five what?
SI4 : Five kilograms.
T : So, what are you going to do with the 5 leftover [meat]?
SI3 : Divide it again.
T : How many parts do you want to divide each meat into?
SI3 : Divide it into four [for each] as in Figure 3.

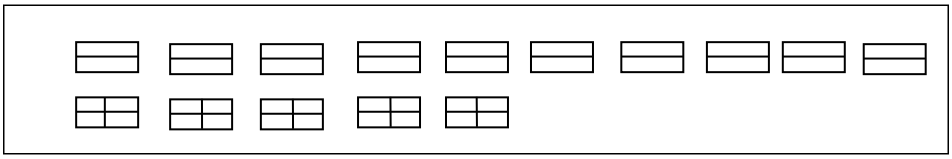


Figure 3. The second step the student uses to share 15 kg of meat into 20 parts.

T : Why do you want to divide it into four for each?
SI3 : To get enough meat for twenty people.

Group A eventually solved the task by representing 15 kilogram of meat by 15 rectangles. The first 10 rectangles were divided into 2 [equal] parts and the remaining 5 shapes were divided into 4 [equal] parts. They also presented how they found 3/4 kilogram of meat by combining the ½ and ¼ parts (See Figure 4).

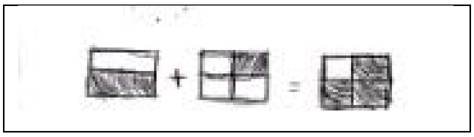


Figure 4. Student’s strategy to combine ½ and ¼ to get 3/4.

Group B

Similar to Group A, Group B’s first attempt at solving the problem was through division, namely 15÷20 and 20÷15. Finally, SI9 concluded, “we cannot divide 15 by 20”. This is reasonable since the students have yet to learn decimal numbers. One student (SI6) began folding an A4 paper. They e attempted to fold it into 20 parts. SI9 responded by trying to measure each part using their ruler and expressed his disappointment because each section was a different size. SI9 looked frustrated and emphasized, “You can’t find fifteen divided by twenty” causing SI6 to laugh at him.

The teacher came around to each group to ask questions that required students to re-check the strategy they had used.

T : Come on, try it first. Fifteen kilograms are shared among twenty people.
SI9 : You can’t (he talked to his friends).
T : **How about fifteen slices of cake, shared among twenty children, is that possible?** Now just try it first, you can definitely do it. Which one have you drawn? Or is it the one you have folded?
SI10 : Fifteen divided by twenty.
SI9 : Twenty divided by fifteen.
T : **Fifteen divided by twenty, can you do that? How do you even do that?**
SI9 : Can we write it Miss? ...
T : **Yes, you can write anything. You can draw circles or other shapes, it’s up to you all.**

From the group discussion, the teacher came and encouraged the students to solve the problem and prompted them to draw any shape. SI9 drew a rectangle, then divided it into 20 parts whilst it should have been divided it into 15 parts. The teacher noticed this and scaffolded the students through a simpler problem as documented in the dialogue below.

T : Come on, you can do this!
SI9 : Here Miss… The people will get one each.
T : **Okay, let’s see. For example, say I have two mangoes, then I share it with 4 people, how many parts does each person get?**
SI10 : A half.
T : Yes, that’s right. Now, how about fifteen kilograms of meat shared amongst twenty people?
SI9 : One and a half.
T : Come on, try it first.
SI9 : So fifteen pieces of meat (Draws a large rectangle and divide it into 15, indicating the division by vertical lines).
T : Yes, now try dividing it.
SI9 : If you divide it by a half, then there’ll be too many.
[SI9 divides each part into halves]
T : Now what do you do with the left-overs?
SI9 : Do we divide it again Miss?

T : Yes, that's absolutely right.
SI9 : Divide it again into two, that's a quarter Miss.
(S9 completes his drawing as Figure 5a)
T : How much does each child get? ...
SI9 : (Draws two rectangles and finds that the result is three quarters as in Figure 5b)
Here Miss…[shows Figure 5b] Three quarters.

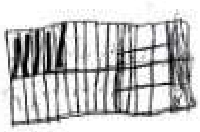


Figure 5a. Group B shared 15 kg of meat with 20 people

Figure 5b. Strategy of Group B to combine ½ and ¼ to get 3/4.

Group B considered fifteen kilograms as the whole and partitioned the whole into fifteen parts as a sub-unit. The teacher's example prompted the group to consider halving ten partitions that resulted 20 equal parts then the group divided the five partitions left into four parts in creating 20 equal parts. It appears that the teacher focused on the notion of a fraction as part of a whole throughout the conversation with students.

The First Lesson in The Netherlands School

The pre-service teacher began the lesson by reminding the students about the importance of explaining their strategies. She said:

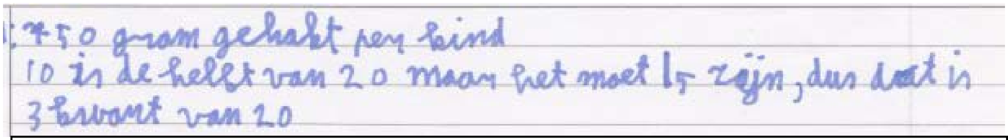
> "*It's not about the right answer, but it's about how you can write down your strategy. How did you find the answer? Maybe you can draw it or write it down in another way. That's the most important thing. I is all about dividing*".

Then she poses a question "Do you know what is division?". A student answered, "Yes, for example two children have 6 sweets and they have to divide it between the two of them". The teacher said "*.....you can divide a lot of things, you can divide sweets, football cards, but now it's about dividing weights."* The teacher then reminded the students about the connection between kilograms and grams.*"So it's about kilograms and grams. Do you know about that?* **Do you know how many grams [there are] in one kilogram?".** One student answered "a thousand". Then the teacher wrote the problem on the white board.*"And now I'm going to ask you, can you calculate how many kilograms or how many grams each person gets? So you have fifteen kilograms of meat and you are going to divide it among twenty people.* Within less than 2 minutes all students in the classroom finished writing their answers on the paper, but most of them only showed their final answer - 750 grams. The teacher prompted the students to write and discuss their strategies. Two different solutions from Group C and D were presented to illustrate the students' reasoning in solving the problem.

Group C

Initially only SD1 (see Figure 6) and SD2 (see Figure 7) had written strategies.

Figure 6. SD1's strategy to divide 15,000 to 20



Translation: 750 gram of meat for each child
10 is a half of 20 but this shoud be 15, so it must be
3 quarters of 20

Figure 7. SD2's strategy to divide 15,000 into 20

After the teacher asked the students to explain their reasoning to each member of their group, they discussed it as per the transcript below:

SD1: See, I have 750, 1500 is 10, so 750 is 20.
SD2: Yes. Well, I thought first 15 you can't divide into 10, it is the half of 20, and 15 not so, it must be ¾.

SD1 converted kilograms to grams and used ratio. They concluded that 1500 grams is for 10 people so 750 grams would be for 20 people. Meanwhile, SD2 worked with fractions as an operator. They thought that 10 is ½ of 20 so 15 is ¾ of 20. It means ¾ kilogram is 750 grams.

Group D

Similar to group C, students in group D only wrote their final answer "750 grams". They then explained their answers within their group. The teacher then asked, SD3 to explain their answer to the rest of the class.

SD3 : [*writes 15000÷20*] and then I thought I'm just going to try…if I just ignore the 0….just like this to skip the 0 (ignore *one 0 off 15000 and 20*) and I just do like I have to divide this to 2…is 750. [He means 15000÷20 = 1500÷2]
T : O…so then you just ignore the 0?
SD3 : Yes.

The majority of these students solved the problem by converting 15 kilograms into 15,000 grams and then used division of whole numbers. Some students used the 'ignore the zeros strategy' to divide the number, 1,500÷2 = 750 grams. The other students used the 'splitting strategy' by splitting 15,000 into 10,000+5,000 then 10,000÷20=500 and 5,000÷20=250 to reach the final answer of 500+250=750 grams. One student used the sub-construct of fractions as an operator. They divided the numerator and denominator of 15/20 by the same number as shown in his work 15÷5 and 20÷5 and obtained the answer ¾. Then, they reasoned that there were 750 grams in ¾ kilogram since ¼ kilogram = ¼ of 1,000 grams = 250 grams. Thus ¾ kilogram = 3×250 grams = 750 grams.

The Second Lesson in the Indonesian School

The pre-service teacher started the second lesson by showing a picture of a large bag of rice weighing 25 kilograms. She then read the problem aloud "Mother has twenty five kilograms of rice. Every day, she cooks three quarter kilogram". Next she prompted her students to predict what the question could possibly be and invited the students to write their prediction on the whiteboard. SI1 wrote: "How many days [does it take] mother [to cook all] the rice?" SI9 wrote on the whiteboard "How many days [does it take for] the rice, [to] run out?"

T : [has] ¾ reached 1 kilogram yet?
SIs : Not yet, we need ¼ more to reach 1 kilogram.
T : That's right, so, if mother cooks 1 kilogram of rice, how many days [will it be till the] rice runs out?
SIs : 25 days (chorus).
T : That's right. What about [if] mother cooks ¾ kilogram per day? How many days till the rice runs out? **Try to solve it, write anything. You can also draw pictures**.

In the second lesson, both groups (A and B) in Indonesia used a similar process to the first lesson for solving the problem. Group A represented one kilogram of rice using one rectangle. They drew 25 rectangles, divided each of them into four parts, then took away ¾ kilogram from each rectangle then took away the remaining ¼ kilogram and combined it to the other ¼ parts. Group A had 33 groups of ¾ kilogram portions and concluded that mother could use 25 kilograms of rice for 33 days with ¼ kilogram of rice leftover (see Figure 8a). Group B represented 25 kilograms of rice by drawing one large rectangle and dividing it into 25 columns. Each of thesecolumn was further divided into four [equal] parts. They then coloured one quarter of each column resulting in 25 little shaded boxes which represented 25 quarters. They then reasoned it took 25 days to cook all the non-shaded boxes and 8 days to cook the blue boxes with ¼ of each box remains uncooked (see Figure 8b). They also concluded that 25 kilograms of rice was enough for 33 days with ¼ kilogram of rice leftover. While monitoring the students' as they were working in their groups, the Indonesian pre-service teacher asked clarifying questions such as: "How far have you gone in reaching the solution? What happened to the remainder? Did you delete it?"

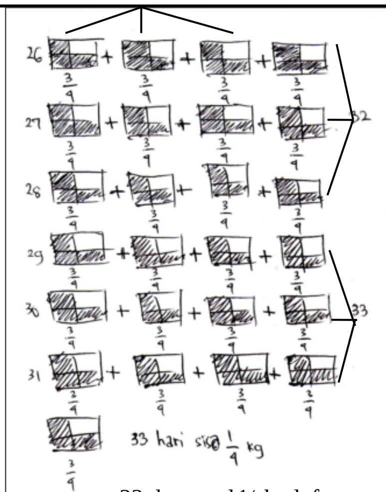


Figure 8a. Strategy of group A

Figure 8b. Strategy of group B

The Second Lesson in the Netherlands School

Before introducing the second problem, the pre-service teacher reviewed two strategies from the previous lesson on division and simplifying fractions (see number 1 and 2 on the whiteboard in Figure 9). She reminded the students that in simplifying fractions they could use a ratio table. She introduced a new strategy by drawing some circles to represent 1/2 kilogram portions of meat on the whiteboard then combined it with ¼ portions to show ¾ was the amount of meat for each person (see number 3 on Figure 9). The next day the teacher presented Day 2's problem "Mother has 25 kilograms of rice. She cooks 3/4 of a kilogram every day. How many days will it take for her rice run out?" and encouraged her students to write down their strategies.

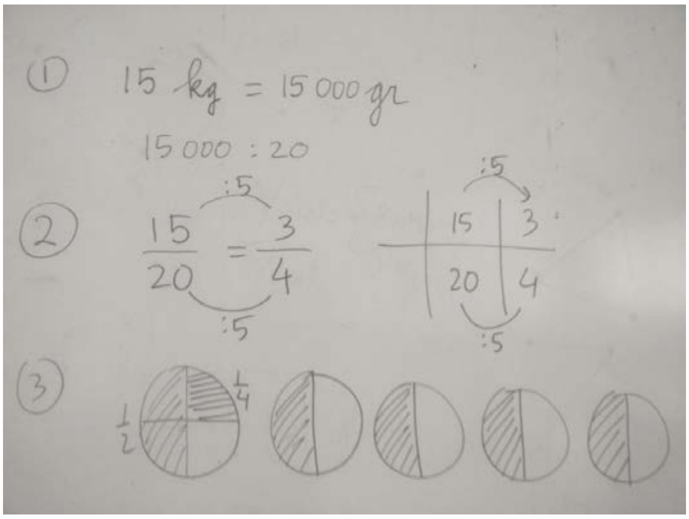


Figure 9. The teacher reviewed the students' strategies from the previous lesson

Group C

The students in group C had not found the solution to the problem as yet. They discussed in their group the dialogue below:

T : How are you going to solve this? **Do you prefer to draw it?**
SD4 : [Draw one circle)
T : Okay…then we are going to draw 25 circles, that means 25 kilos. … I'll come back to you.
SD4 : (draws 25 circles)
SD1 : It is just to add…to add….Look you have 25 kilos….so each day is 750, and do you know what you have to do then? You have 250 grams left every day.
T : This one first (*point the first circle*)
SD4 : So then you have this one and then you have one day (points to the shaded area which is ¾ of the circle)
T : Yes exactly, **so then you have 1 day, 2 days, 3 days, but then [what happens to the] little pieces that are left, you have 4 days** And so you can now continue.
SD4 : SD1,, 4 days you take…eh….you use 3 kilo's in 4 days.

It appeared that SD4 and SD1 understood the problem even though they did not record their strategy precisely. SD1 raised his hand to explain his thinking in front of the class.

SD1 : (*Draws 4 circles and divides the first two circles into 4 parts).*
I thought, if this was 750 [grams], then this is 1500 [grams] [points at the picture]
T : Yes, very good, you coloured ¾ [of the circle] of the next kilo.
SD1 : Well …this is 750, then this is 1500, then this is 2250 and then this is 3000. (Refers to the four circles as shown at Figure 10)

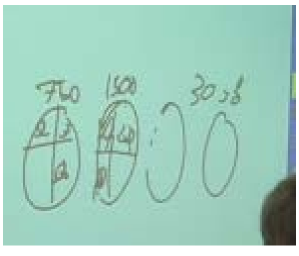


Figure 10. SD1 drawn 4 circles as a representation of 4 kilograms

T : 3000. And then, how did you continue?
SD1 : I added.
T : You went to draw circles and continued?
SD1 : Well, actually I counted in my head.
T : Yes?
SD1 : And then I…then then…then it was 10 times….ehm…10 days is ehm…o I can't go on very well.
T : Well I think you explained it and I appreciate your early strategy. Thank you very much!

The teacher then ended the presentation without asking any more questions. She moved on to ask another group to explain their strategies.

Group D

The students in group D solved the problem successfully. They wrote the final answer 33 days and ¼ kilogram of rice left. They obtained their solution without any help from the teacher. A representative of the group explained their strategies in front of the classroom.

SD5 : Ehm…I drew like this…. I coloured this one and this one and this one (points at the quarter parts one by one, Figure 11a) And then …I thought 25 kilos and I went to add ¾ all the time but it didn't really fit so I had 33 days and a quarter.
T : Yes, I understand your strategy but I think that for most of us it is a good idea if you explain some more steps. Can you write above this circle…how many kilos is this circle?

Since the pre-service teacher encouraged SD5 to further clarify their answer, the student then shaded the circles and added the labels "1 day" for the first three quarters, "2 days" after shading the second three quarters, and "3 days" after shading the third three quarters as shown in Figure 11b. They then mentally added the days to conclude that 25 kilograms of rice will run out within 33 days with a leftover of ¼ kilogram.

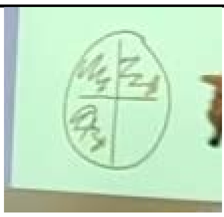


Figure 11a. The first step of SD5

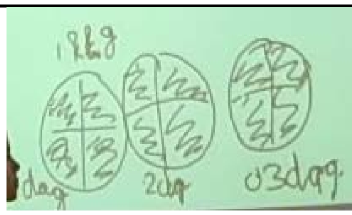


Figure 11b. The second step of SD5

A ratio sub-construct was evident in the strategy used by one student. This student used doubling to work out that 4 days equalled 3 kilograms, 8 days equalled 6 kilograms, 16 days equalled 12 kilograms and 32 days equalled 24 kilograms of rice. Interestingly, this student recorded a final answer of 34 and ¼ days (Figure 12a). Three of the 25 students used part-whole interpretations of fractions which is evident in their representations of the 25 circles or 25 rectangles that represent the 25 kilograms of rice. This strategy was similar to the strategy used by the Indonesian students who divided each of the rectangles into four equal parts then took away ¾ kilogram from each rectangle. They took away the remaining ¼ of the whole rectangle and combined it with two other ¼ parts from the next two rectangles. They found 3 kilograms of rice were enough for 4 days. Continuing with this process, they concluded that 25 kilograms of rice was needed for 33 days and they had ¼ kilogram rice left (Figure 12b). Only one student used the 'repeated subtraction strategy' in solving the problem, and they converted kilograms into grams and did not use any fraction construct at all.

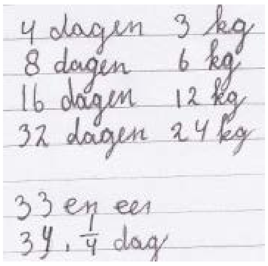


Figure 12a. Student used ratio

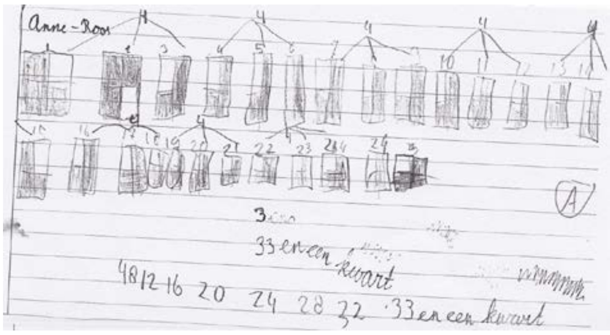


Figure 12b. Students used part-whole interpretation of fractions

Discussion

This study shows that contextual problems and teacher questioning enabled and influenced how students engaged with the different sub-constructs of fractions. These differences were evident through various strategies and representations displayed by students in both countries. The Dutch students utilised different operations (e.g., division and repeated subtraction) and various fractions sub-constructs. While the Indonesian students used solely part-whole concepts with different representations of the part-whole concept (e.g., 15 kilograms of meat were represented by 15 rectangles or by one big rectangle). This finding is consistent with previous studies particularly in the context where RME ideas were implemented (Dolk, Widjaja, Zonneveld, & Fauzan, 2010; Gravemeijer & Doorman, 1999; Sembiring, Hadi, & Dolk, 2008; van den Heuvel-Panhuizen, 1996, 2000; Widjaja, Dolk, & Fauzan, 2010). As argued by Johnson (2002) a contextual problem should enable students to make a link between the mathematics explored in the problem and the context. Similarly, van den Heuvel-Panhuizen (1996) and van den Heuvel-Panhuizen & Wijers (2005) advocates the use contextual problems to develop students' mathematical understanding and their capacity to solve the problems differently.

When reviewing the questions the pre-service teacher used the Dutch pre-service teacher posed fewer questions than the Indonesian pre-service teacher however, the Dutch students displayed a broader repertoire of strategies than the Indonesian students. This suggests that the pre-service teachers' questions were not the only factor that influence the students' strategies but other aspects such as: the local context of the curriculum, textbooks, and classroom culture all play an influential part. We concur with Watanabe (2007) who points out that curriculum is a critical factor that influences the teaching and learning mathematics.

In the Netherlands one mathematics lesson encompasses several topics such as addition, multiplication, fractions, and measurement (Kolovou van den Heuvel-Panhuizen & Bakker, 2009; TAL Team, 2001) as required by the curriculum. The Dutch students displayed their competence in solving the problems in multiple ways by using splitting, halving, doubling or a ratio table (van den Heuvel-Panhuizen, 2000). It may be the case that solving a contextual problem with different approaches has become a part of Dutch classroom culture. The findings of this study confirm a previous study by Gravemeijer, Bruin-Muurling, Kramer and van Stiphout (2016) where Dutch students have shown their flexibility in approaching mathematical problems.

For the Indonesian case, it appears that the pre-service teacher's questions and the curriculum simultaneously impact upon students' strategies in solving contextual problems. The questions and prompts used by the Indonesian pre-service teacher supported students to apply different strategies even though the main discussion seemed to be limited to one fraction sub-construct: the part-whole notion of fractions with the area model as the main representation. However, this study offered a new insight where Indonesian students modelled different strategies to solve the problem. This indicates that the Indonesian students also performed flexible mathematical thinking within the constraint of the part-whole construct.

The curriculum and textbooks appear to be influential factors of mathematics teaching and learning. In Indonesia, teaching is based around units of work. Students were aware they were working on a unit about fractions, so they assumed that the task needed to be solved using fractions. Hence, even though the Indonesian pre-service teacher posed questions to elicit multiple strategies, the students still solved the problem using fractions.

This study also suggests the impact of the adaptation of RME in Indonesia (Sembiring et al., 2010; Dolk et al., 2010; Widjaja, Dolk, & Fauzan, 2010). Since the adapting Realistic Mathematics Education (RME) in Indonesia in 2000, the use of contextual problems have been incorporated as starting points in mathematics lessons to engage students when solving problems at various levels (van den Heuvel-Panhuizen & Wijers, 2005; Widjaja, Dolk, & Fauzan, 2010). However, this study suggests that adapting an idea from one country to another requires a holistic approach such as considering the curriculum context and the textbook structure.

The way the pre-service teachers introduced the lesson and the questions they posed to the students helped to build a link with the students' prior knowledge. This was highly influential in the way the students then approached the problem. Even though the tasks were designed to elicit student knowledge on fractions, the Dutch pre-service teacher initially led her students to the relationship of kilograms and grams and as such the task did not encourage the students to work with fractions. Even after providing a solution strategy involving fractions in the second lesson, the Dutch students still focused on translating the problem into division of whole numbers using conversion to solve the problem.

Both pre-service teachers helped their students to make connections between prior knowledge and new mathematics content (Heinze & Erhard, 2006; Koizumi, 2013), so as to understand the problem (Warner, Schorr, Arias, & Sanchez, 2013; Weiland, Hudson & Amador, 2013), and develop the students' mathematical reasoning through questioning (Moyer & Milewicz, 2002). While the Indonesian pre-service teacher's questions led her students to think solely about the part of a whole construct to solve the problems, she offered area models when the students struggled to solve it. The Dutch pre-service teacher led her students to think about conversion to solve the problems. This means, both pre-service teachers led their students to start solving each problem in particular ways. This result highlights the importance of improving pre-service teachers' questioning skills during their teacher education training as pointed out by Weiland, Hudson, and Amador (2014).

Concluding comments

Both pre-service teachers agreed that the problems were related to fractions. It was evident that the Indonesian pre-service teacher followed this up by positioning the teaching of these lessons around the time the unit of work on fractions was being taught. She did not think outside the topic of fractions. Her questions provided space for her students to solve the problems their own way. This was evident in the different strategies the students' used when considering the whole in the problem. The enacted lessons in the Netherlands were quite different. The decimal increments of the metric system allowed the Dutch pre-service teacher and her students to convert the unit into grams and consequently solve the problem using division of whole numbers rather than fractions.

We argue that these pedagogical differences were somewhat influenced by the opportunity of the Dutch students to utilize different approaches to solving the task, whereas the Indonesian students were limited to one process. Nevertheless, the Dutch classroom was more open to student engagement and exploration. Further research is needed on how to foster pre-service teachers' capacity to support students in solving problems involving division of fractions. Contextual problems of this nature are difficult to generate, so sound content knowledge is required in order to support students through the process. The intent of this paper was not to generalize the characteristics of Indonesian and Dutch pre-service teachers, rather to highlight the subtleties and nuisances of contextually-based problems that involved fractions. To some degree, the capacity to convert units into decimals elevates some of the challenges presented in this paper. However, student conversations and engagement with the contextual features of the task help a great deal.

References

Akkus, R. & Hand, B. (2010) Examining teachers' struggles as they attempt to implement dialogical interaction as part of promoting mathematical reasoning within their classrooms. *International Journal of Science and Mathematics Education*, 9, 975-998

Alajmi, A. H. (2012). How do elementary textbooks address fractions? A review of mathematics textbooks in the USA, Japan, and Kuwait. *Educational Studies in Mathematics, 79*(2), 239-261.

Armanto, D. (2002). *Teaching multiplication and division realistically in Indonesian primary Schools: A prototype of local instructional theory*. Enschede: PrintPartners Ipskamp.

Behr, M., Khoury, H. A., Harel, G., Post, T., & Lesh, R. (1997). Conceptual unit analysis of preservice elementary school teachers' strategies on a rational-number-as-operator task. *Journal for Research in Mathematics Education, 28*(1), 48-69.

Boaler, J. (1993). Encouraging the transfer of 'school' mathematics to the 'real world' through the integration of process and content, context and culture. *Educational Studies in Mathematics, 25*(4), 341-373.

Browning, C., Thanheiser, E., Edson, A. J., Kimani, P. M., Olanoff, D., Tobias, J. M., & Whitacre, I. (2014). Prospective Elementary Mathematics Teacher Content Knowledge: Introduction. *The Mathematics Enthusiast, 11*(2), 203-215.

Bruin-Muurling, G. (2010). *The development of proficiency in the fraction domain. Affordances and constraints in the curriculum.* (PhD), Eindhoven University of Technology, Eindhoven. Retrieved from http://alexandria.tue.nl/extra2/692951.pdf

Bulgar, S. (2003). Children's sense-making of division of fractions. *Journal of Mathematical Behavior, 22*, 319-334.

Cai, J., & Wang, T. (2006). U.S. and Chinese teachers' conceptions and constructions of representations: a case of teaching ratio concept. *International Journal of Science and Mathematics Education, 4*(1), 145-186.

Carraher, D., & Schliemann, A. D. (2002). Is everyday mathematics truly relevant to mathematics education. *Journal for Research in Mathematics Education Monograph, 11*, 131-153.

Chapman, O. (2006). Classroom practices for context of mathematics word problems. *Educational Studies in Mathematics*, *62*, 211-230. DOI: 10.1007/s10649-006-7834-1.

Charalambous, Y. C., & Pitta-Pantazi, D. (2005). Revisiting a theoretical model on fractions: Implications of teaching and research. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29 Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 233-240). Melbourne: PME.

Charalambous, Y. C., & Pitta-Pantazi, D. (2007). Drawing on a theoretical model to study students' understanding of fractions. *Educational Studies in Mathematics, 64*, 293-316.

Chinappan, M. (2000). Preservice teachers' understanding and representation of fractions in a java bars environment. *Mathematics Education Research Journal*, *12*(3), 234-253.

Chinappan, M. (2005). Children's mappings of part-whole construct of fractions *Proceedings of xxx MERGA* 241-248.

Chinnapan, M., & Forrester, T. (2014). Generating procedural and conceptual knowledge of fractions by pre-service teachers. *Mathematics Education Research Journal, 26(4)*, 871-896.

Dolk, M., Widjaja, W., Zonneveld, E., & Fauzan, A. (2010). Examining teacher's role in relation to their beliefs and expectations about students' thinking in design research. In Sembiring, R. K., Hoogland, K. & Dolk, M. (Eds.), *A decade of PMRI in Indonesia* (pp. 175-187). Utrecht: APS International.

Fauzan, A., Plomp, T., & Gravemeijer, K. (2013). The development of an rme-based geometry course for Indonesian primary schools. In T. Plomp and N. Niveen (Eds), *Educational design research–Part B: Illustrative cases*, 159-178. Enschade, the Netherlands: SLO

Freudenthal, H. (1983). *Didactical phenomenology of mathematical structures*. Dordrecht: D. Reidel Publishing Company.

Freudenthal, H. (1991). *Revisiting mathematics education, China lectures*. Dordrecht: Kluwer Academic Publishers.

Funahashi, Y., & Hino, K. (2014). The teacher's role in guiding children's mathematical ideas toward meeting lesson objectives.*ZDM Mathematics Education, 46*, 423-436.

Gould, P. (2005). Really broken numbers *Australian Primary Mathematics Classroom, 10*(3), 4-10.

Gould, P. (2011). Developing an understanding of the size of fractions *The Australian Association of Mathematics Teacher* (pp. 63-70).

Gravemeijer, K. (2010). Realistic mathematics education theory as a guideline for problem-centered, interactive mathematics education. In Sembiring, R. K., Hoogland, K. & Dolk, M. (Eds.), *A decade of PMRI in Indonesia* (pp. 175-187). Utrecht: APS International.

Gravemeijer, K., & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational Studies in Mathematics, 39*, 111-129.

Harvey, R. (2012). Stretching student teachers' understanding of fractions. *Mathematics Education Research Journal*.

Heinze, A & Erhard, M. (2006). How much time do students have to think about teacher questions? An investigation of the quick succession of teacher questions and student responses in the German mathematics classroom. *ZDM Mathematics Education, 38*(5), 388-398.

Johnson, E., B. (2002*). Contextual teaching and learning*: *what it is and why it's here to stay*. Corwin Press: USA.

Kastberg, S. E., D'Ambrosio, B., McDermott, G., & Saada, N. (2005). Context matters in assessing students' mathematical power. *For the Learning of Mathematics, 25*(2), 10-15.

Kawanaka, T., & Stigler, J. W. (1999). Teachers' use of questions in eighth-grade mathematics classrooms in Germany, Japan, and the United States. *Mathematical Thinking and Learning*, *1*(4), 255-278.

Keijzer, R. (2003). *Teaching formal mathematics in primary education- fraction learning as mathematising process*. Utrecht: Freudenthal Institute.

Keijzer, R., & Terwel, J. (2001). Audrey's acquisition of fractions: a case study into the learning of formal mathematics. *Educational Studies in Mathematics, 47*(1), 57-73.

Koizumi, Y. (2013). Similarities and differences in teachers' questioning in German and Japanese mathematics classrooms. *ZDM Mathematics Education*, 45, 47-49.

Kolovou, A., van den Heuvel-Panhuizen, M. & Bakker, A. (2009). Non-routine problem solving tasks in primary school mathematics textbooks – a needle in a haystack.*Mediterranean Journal for Research in Mathematics Education, 8* (2), 31-69.

Lamon, S. J. (2007). Rational numbers and proportional reasoning: toward a theoretical framework for research. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 629-667). Charlotte, NC: Information Age.

Lamon, S. J. (2012). *Teaching fractions and ratios for understanding: Essential knowledge and instructional strategies for teachers*. New York: Routledge.

Lo, J.-J., & Grant, T. J. (2012). Prospective elementary teachers' conceptions of fractional units. In T. Y. Tso (Ed.), *Proceedings of the 36th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 169-176). Taipei, Taiwan: PME.

Lubienski, S. T. (2000). Problem Solving as a Means Toward Mathematics for All: An Exploratory Look Through a Class Lens. *Journal for Research in Mathematics Education, 31*(4), 454-482.

Martino, A. M., & Maher, C. A. (1999). Teacher questioning to promote justification and generalization in mathematics: What research practice has taught us. *The Journal of Mathematical Behavior*, *18*(1), 53-78.

Masitoch, N., Mukaromah, S., Abidin, Z., & Julaeha, S. (2009). *Gemar matematika untuk SD dan MI Kelas III*. Jakarta: Pusat Perbukuan Departemen Pendidikan Nasional.

Mason, J. (2000). Asking mathematical questions mathematically. *International journal of mathematical Education in Science and Technology*, *31*(1), 97-111.

Merriam, S. B. (1988a). *Case study research in education: a qualitative approach*. San Francisco: Jossey-Bass.

_______(1988b). *Qualitative research and case study applications in education*. San Francisco: Jossey-Bass Publishers.

Moseley, B. J., Okamoto, Y., & Ishida, J. (2007). Comparing US and Japanese elementary school teachers' facility for linking rational number representations. *International Journal of Science and Mathematics Education, 5(1)*, 165-185.

Mosvold, R. (2008). Real-life connections in Japan and the Netherlands: National teaching patterns and cultural beliefs. *Journal for Mathematics Teaching and Learning*. Retrieved from http://www.cimt.plymouth.ac.uk/journal/mosvold.pdf

Moyer, P. S. & Milewicz, E. (2002). Learning to question: categories of questioning used by preservice teachers during diagnostic mathematics interviews. *Journal of Mathematics Teacher Education*, 5, 293-315

Olanoff, D., Lo, J.-J., & Tobias, J. M. (2014). Mathematical content knowledge for teaching elementary mathematics: a focus on fractions. *The Mathematics Enthusiast, 11*(2), 267-310.

Olive, J., & Vomvoridi, E. (2006). Making sense of instruction on fractions when a student lacks necessary fractional schemes: The case of Tim. *Journal of Mathematical Behavior, 25*(1), 18-45.

Pantziara, M., & Philippou, G. (2012). Levels of students' "conception" of fractions. *Educational Studies in Mathematics, 79*, 61-83.

Perry, M., VanderStoep, S. W., & Yu, S. L. (1993). Asking questions in first-grade mathematics classes: Potential influences on mathematical thought. *Journal of Educational Psychology*, *85*(1), 31.

Sahin, A., & Kulm, G. (2008). Sixth grade mathematics teachers’ intentions and use of probing, guiding, and factual questions. *Journal of mathematics teacher education*, *11*(3), 221-241.

Sembiring, R. K., Hadi, S., & Dolk, M. (2008). Reforming mathematics learning in Indonesian classrooms through RME. *ZDM Mathematics Education, 40*, 927-939.

Sembiring, R. K., Hoogland, K., & Dolk, M. (2010). *A decade of PMRI in Indonesia*. Utrecht: APS International.

Son, J.-W., & Senk, S. L. (2010). How reform curricula in the USA dan Korea present multiplication and division of fractions. *Educational Studies in Mathematics, 74(2),* 117-142.

Steinle, V., & Price, B. (2008). *What does three-quarters look like? Students' representations of three-quarters.* Paper presented at the the 31st Annual Conference of the Mathematics Education Research Group of Australasia, Brisbane.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: The Free Press.

Streefland, L. (1991). *Fractions in realistic mathematics education*. Dordrecht: Kluwer Academic Publishers.

TAL Team. (2001) *Children learn mathematics: a learning-teaching trajectory with intermediate attainment targets for calculation with whole numbers in primary school.* In van den Heuvel-Panhuizen, M. (Edt.). Freudenthal Institute Utrecht University: the Netherlands.

Thompson, P. W., & Saldanha, L. A. (2003). Fractions and multiplicative reasoning In J. Kilpatrick, G. Martin, & D. Schifter (Eds.), *Research companion to the principles and standards for school mathematics* (pp. 95-114).

Tobias, J. M. (2013). Prospective elementary teachers' development of fraction language for defining the whole. *Journal of Mathematics Teacher Education, 16*(2), 85-103.

van den Heuvel-Panhuizen, M. (1996). *Assessment and realistic mathematics education*. Utrecht, The Netherlands: CD-ß Press/Freudenthal Institute, Utrecht University.

_______ . (2000). *Mathematics education in the Netherlands: A guided tour*. Freudenthal Institute CD-rom for ICME9. Utrecht.

van den Heuvel-Panhuizen, M., & Drijvers, P. (2014). Realistic mathematics education. In S. Lerman (Ed.), *Encylocpedia of Mathematics Education* (pp. 521-525). Dordrecht, The Netherlands: Springer.

Warner, L. B., Schorr, R. Y., Arias, C. C., & Sanchez, L. (2013). A case study of two teachers: teacher questions and student explanations. In Lesh, R., et al. (Eds.), *Modelling students' mathematical modelling competencies, International perspectives on the teaching and learning of mathematical modelling* (pp.512-529). Springer.

Watanabe, T. (2007). Initial treatment of fractions in Japanese textbooks *Focus on Learning Problems in Mathematics, 29*(2), 41-60.

Weiland, I. S., Hudson, R. A., & Amador, J. M. (2014). Preservice formative assessment interviews: The development of competent questioning. *International Journal of Science and Mathematics Education*, *12*(2), 329-352.

Widjaja, W. (2013). The use of contextual problems to support mathematical learning. *Indonesian Journal of Mathematics Education, 4*(2), 151-159.

Widjaja, W., Dolk, M., & Fauzan, A. (2010). The role of contexts and teacher's questioning to enhance students' thinking. *Journal of Science and Mathematics Education in Southeast Asia, 33*(2), 168-186.

Widjaja, W., & Stacey, K. (2009). Growth of pre-service teachers' knowledge and teaching ideas about decimals and fractions: The case of Vivi. In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Proceedings of the 32nd Conference of Mathematics Education Research of Australasia* (Vol. 2, pp. 587-595). Palmerston North: MERGA.

Wijaya, A. (2008). Design research in mathematics education: Indonesian traditional games as means to support second graders learning of linear measurement. (Master thesis), Utrecht University, Utrecht.

Wijaya, A., van den Heuvel-Panhuizen, M., & Doorman, M. (2015). Opportunity-to-leran context-based tasks provided by mathematics textbooks. *Educational Studies in Mathematics, 89*, 41-65.

Yim, J. (2010). Children's strategies for division by fractions in the context of the area of a rectangle. *Educational Studies in Mathematics, 73*, 105-120.

Zulkardi, Z. (2002). *Developing a learning environment on realistic mathematics education for Indonesian student teachers* (Doctoral dissertation, University of Twente, Enschede).

Zulkardi. (2013). *Future challenges and educational responses: Curriculum 2013 and PMRI as an innovation in Indonesian mathematics education*. Paper presented at the EARCOME 6 Phuket.