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# Problem Posing in Consumer Mathematics Classes: Not Just for Future Mathematicians

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# Problem Posing in Consumer Mathematics Classes: Not Just for Future Mathematicians

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**Abstract**

Problem posing is recognized as a key component of mathematics (Ellerton, 2013). However, in many curricula, problem solving often dominates over problem posing (Stoyanova, 2003). This focus on problem solving exists despite research that shows that problem posing improves students' problem-solving skills, attitudes, confidence, understanding of concepts, and mathematical thinking (Singer, Ellerton, & Cai, 2013); reinforces basic mathematical skills, increases motivation, responsibility, and thinking flexibility (Ponte & Henriques, 2013); and is useful for teachers to assess students' cognitive processes, identify misconceptions, and modify instruction (Ponte & Henriques, 2013). Further, problem posing can play a large part in student motivation (McLeod, 1992). The potential for problem posing as a motivational tool in nonuniversity track mathematics classes has not received much attention. This case study examines a program based on problem posing, in six grade 11 consumer mathematics classes, over a 3-year period. The program was very successful across a number of dimensions, including engagement, motivation, self-efficacy, and achievement. This paper also examines models of problem posing, and suggests modifications to enhance their efficacy.

**Key Words:** problem posing, problem solving, mathematics, motivation

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# Introduction

Ellerton (2013) points out, “To state that problem posing is as fundamental to mathematics as problem solving should be to state the obvious--after all, one cannot solve a problem unless first, a problem has been posed." (p. 87) However, the importance of problem posing is much greater. Freire (2007) says "problem-posing education involves a constant unveiling of reality" (p. 72), and sees problem posing as a fundamental cornerstone of education. Similarly, Nicolaou and Xistouri (2011) among others, identify problem posing as lying "at the heart of mathematical activity" (p. 612). The importance of problem posing is identified in various national curricula including (a) The United States, in both the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000), and the *Common Core Standards* (Flick & Kuchey, 2010); (b) China (Silver, 2013; Van Harpen & Sriraman, 2013); (c) Australia (Silver, 2013); and (d) Ontario (Ontario Ministry of Education, 2005, 2007). The journal *Educational Studies in Mathematics* (January, 2013) recognized the importance of problem posing by devoting a special issue exclusively to the topic.

Problem posing can be either a goal of mathematics education or an instructional strategy (Stoyanova, 2005). However, in many curricula problem solving often dominates over problem posing (Stoyanova, 2003). This is despite research showing that problem posing (a) improves students' problem-solving skills, attitudes, and confidence, understanding of concepts, and mathematical thinking (Singer et al., 2013); (b) reinforces basic mathematical skills, increases motivation, responsibility, and thinking flexibility (Ponte & Henriques, 2013); and (c) is useful for teachers to assess students' cognitive processes, identify misconceptions, and modify instruction (Ponte & Henriques, 2013). There is also extensive research relating problem posing to creativity. Voica and Singer (2013) point out that there are correlations between problem solving, problem posing, and creativity. Among the multiple descriptions of creativity are problem finding, problem solving, and problem posing, and problem posing is frequently used in assessing creativity (Bonotto, 2013; Pelczer & Rodriguez, 2011).

Much of the research in problem posing has focused on elementary students (Barlow & Cates, 2006; Bonotto, 2013; Cai et al., 2013; Kerekes, Diglio, & King, 2009; Nicolaou & Xistouri, 2011; Singer & Voica, 2013; Stoyanova, 2003; Voica & Singer, 2012, 2013); college and university students (Koichu & Kontorovich, 2013; Ponte & Henriques, 2013); gifted high school students (Singer et al., 2013; Stoyanova, 2003; Van Harpen & Sriraman, 2014; Voica & Singer, 2013); teachers and teacher education (Ellerton, 2013; Shriki & Lavy, 2012; Singer et al., 2013; Stoyanova, 2005; Ticha & Hospesova, 2013). There are relatively few references to problem posing in nonacademic streams even though

> Preparing students for life is seen by some educators as an ultimate goal of school education. Those students who will not become professional scientists will need to be able to apply mathematics in everyday life situations. It is therefore important that students' experiences in mathematics classrooms help them to become competent users of mathematics by being able to pose, analyze and solve real world problems. (Stoyanova, 2003, p. 33)

The current case study examines an instructional strategy with a focus on problem posing in six grade 11 consumer mathematics classes over a 3-year period.

## Problem Posing Models

"Problem posing refers to both the generation of new problems and the reformulation of given problems" (Singer, 1994, as cited in Pelczer & Rodriguez, 2011). Problem posing and problem solving are interlinked. Silver (1994) identifies three temporal periods when problem posing can occur while engaging in problem solving: before problem solving (pre-solution), during (within-solution), and after (post-solution), as cited in Singer et al., 2013. Stoyanova (2005) identifies three general strategies for problem posing that are related to problem solving. These are reformulation (rearranging numerical information, adding irrelevant structure, replacing mathematical operations, adding real life context); reconstruction (changing the order of numerical information or operations, changing numerical information, regrouping, using equivalent forms); and imitation (changing the problem goal, looking at information through a different lens, such as ratio instead of division). Brown and Walter, in their book *The Art of Problem Posing* (1990) describe a "What If Not?" strategy for posing new problems based on already-solved problems, by systematically varying problem conditions or goals. This strategy involves identification of the problem's attributes, suggesting alternatives to each attribute, and posing new problems based on the alternatives (Brown & Walter, 1990; Stoyanova, 2003). Tsobota (1987) identified six successful types of problem posing: based on an algorithm, a text, a figure or table, a mathematics topic, an answer, or an already-formulated problem (as cited in Leung, 2013). Christou et al.(2005) identify cognitive processes that interact during problem posing. These processes, all involving quantitative information, are: editing, selecting, comprehending, organizing, and translating from one form to another.

Schoenfeld (1992) identified five dimensions to problem solving: knowledge base, problem-solving strategies, monitoring and control, beliefs and affects, and practices. Based on these dimensions, Kontorovich, Koichu, Leikin, and Berman (2012) have developed a problem posing framework. This framework is shown in Figure 1. They cite Stoyanova's (1998) three levels of problem posing tasks: structured (based on an existing problem), semistructured (based on a story or set of conditions), and free (no constraints). Kontorovich et al. state that their framework addresses all three levels, couched in a small group setting. The category *Individual Considerations of Aptness* encompasses aptness for these participants: the problem poser themselves, potential evaluators, potential problem solvers, group members (Kontorovich et al., 2012).

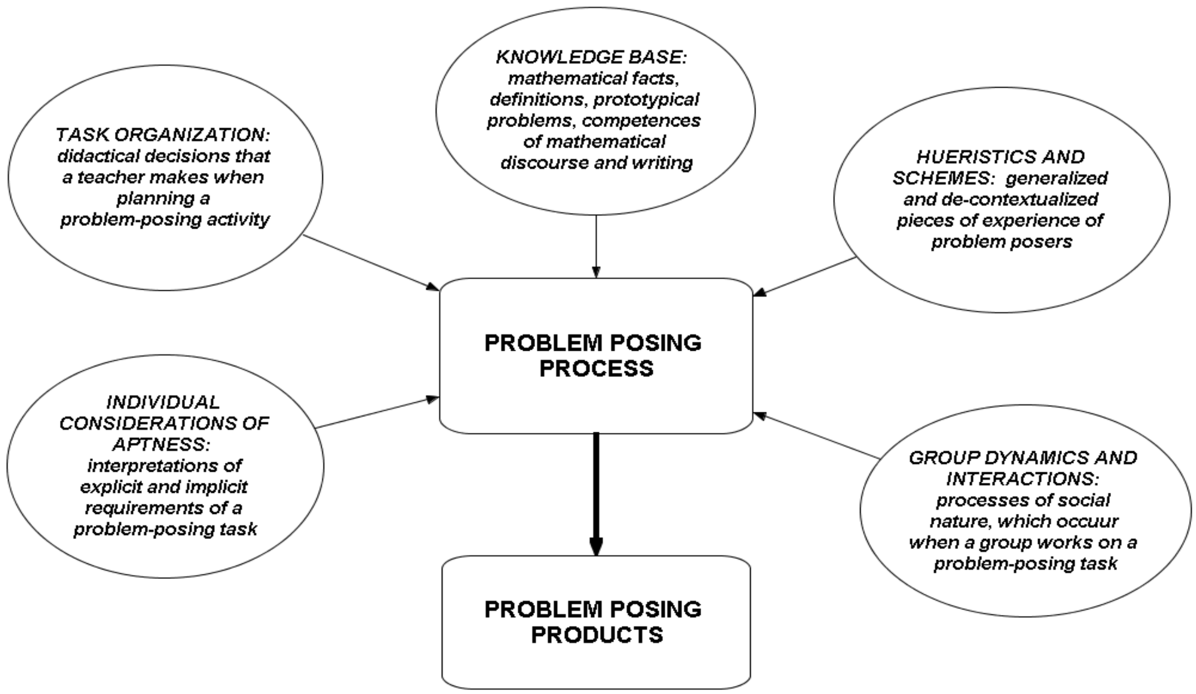


Figure 1. A PROBLEM POSING FRAMEWORK

Kontorovich, I., Koichu, B., Leikin, R., & Berman, A. An exploratory framework for handling the complexity of mathematical problem posing in small groups. The Journal of Mathematical Behavior 31, p. 152

While the categories of *Heuristics and Schemes*, and *Individual Consideration of Aptness* contain some elements of metacognition, this framework is limited across two of the problem-solving dimensions identified by Schoenfeld. The monitoring and control dimension (metacognition) is not explicitly addressed. Metacognition has been identified as an important factor in mathematical problem solving (Yunis & Ali, 2008) and therefore also in problem posing. The problem posing framework would be stronger with metacognition included as an explicit category. The second important omission is beliefs and affects. This is a dimension that was explicitly addressed in the case study described in this paper, which focused on nonacademic students who exhibited negative attitudes to mathematics in general, problem solving in particular, and who had very low self-efficacy beliefs about mathematics tasks. The case study illustrates how problem posing in the context of real life situations can have positive and reciprocal effects on student beliefs and affects. The inclusion of these two categories in the problem-posing framework would enhance its effectiveness.

## Case Study

The case examined here is a holistic single case study (Yin, 2009) with the unit of analysis six grade 11 consumer mathematics classes. The case examines diachronic covariation (Gerring, 2007) of student motivation, achievement, and self-efficacy, with different students over three school years.

The research questions were:

1. How can a program of problem posing using real-world artifacts be enacted in nonuniversity track classes of secondary school mathematics?
2. What is the effect on student motivation, achievement, and self-efficacy of such a program?

Propositions:

- A program emphasizing problem posing, based on real world artifacts, will have a positive impact on student motivation.
- The program will have a positive impact on student achievement.
- The program will have a positive impact on student self-efficacy.
- There will be interrelated positive effects among all three variables of interest, particularly the interrelationship between motivation and self-efficacy.

The case study utilized semistructured self-interviews. As the participant-observer in this case I conducted self-interviews consisting of three 1-hour sessions. To do this I first developed questions on the major facets of the program. I then responded in writing to each of these questions. The third of these self-interviews was for clarification and elaboration, after investigating other sources of information. This process was supplemented by artifacts consisting of student journals, sample teacher-posed problems, and sample student-posed problems, as well as a final exam.

**Background**

This case occurred in a secondary school of 1,800 students, located in a city with a population of 500,000 in Ontario, Canada. The city and the school were culturally and ethnically diverse, with significant minority groups of South Asian, Indian, Caribbean Blacks, Somali Blacks, Vietnamese, with Caucasians being a slight majority. The study examined two grade 11 Consumer Mathematics classes per year for 3 years, with an average enrolment of 25 students per class. The students in these classes were of mixed ethnicity. They shared negative attitudes towards school in general and mathematics in particular. These students had low self-efficacy, poor work habits, and completed little or no homework in their previous mathematics classes. They were often disengaged in class and expressed the opinion that the mathematics they had been exposed to in previous years had nothing to do with their lives. They viewed mathematics as a game in which they did not know or understand the rules, and that mathematics was questions in a textbook, disembodied and unconnected to themselves as persons, and unconnected to anything else in their lives, not even other subjects in school. The teacher of these classes had 25 years of experience in secondary school mathematics teaching across a variety of levels, in several secondary schools located in two cities in Ontario. The classes all occurred in a relatively isolated portable classroom located near the edge of the school property. Other sections of the same course were taught by several different teachers, all located inside the school building. These teachers utilized the traditional "banking model" (Freire, 2007), or instrumental method of instruction.

**Gradual Release Instructional Program**

It was clear that traditional instruction had been unsuccessful with these students. While they had accumulated two previous mathematics credits, there was little or no evidence of concept understanding, and no ownership of their mathematics learning. Further, student motivation and engagement were very low. For these reasons the teacher introduced a radically different program, focusing on problem posing and problem solving, using a gradual release model, whereby over time students assumed greater responsibility for formulating their own learning.

**Resources**

This program relied on real world artifacts to foster problem posing (Bonotto, 2013). Since these students had had very negative experiences with mathematics textbooks, there were no assigned textbooks in the course. The major resource was a Toronto newspaper. The teacher obtained class subscriptions enabling the classes to receive multiple copies each day. This resource was supplemented by advertising flyers, brochures, guest speakers, and library resources.

Figure 2 illustrates the structure and the various components of the program. Each component was selected based on student disengagement as the lens.

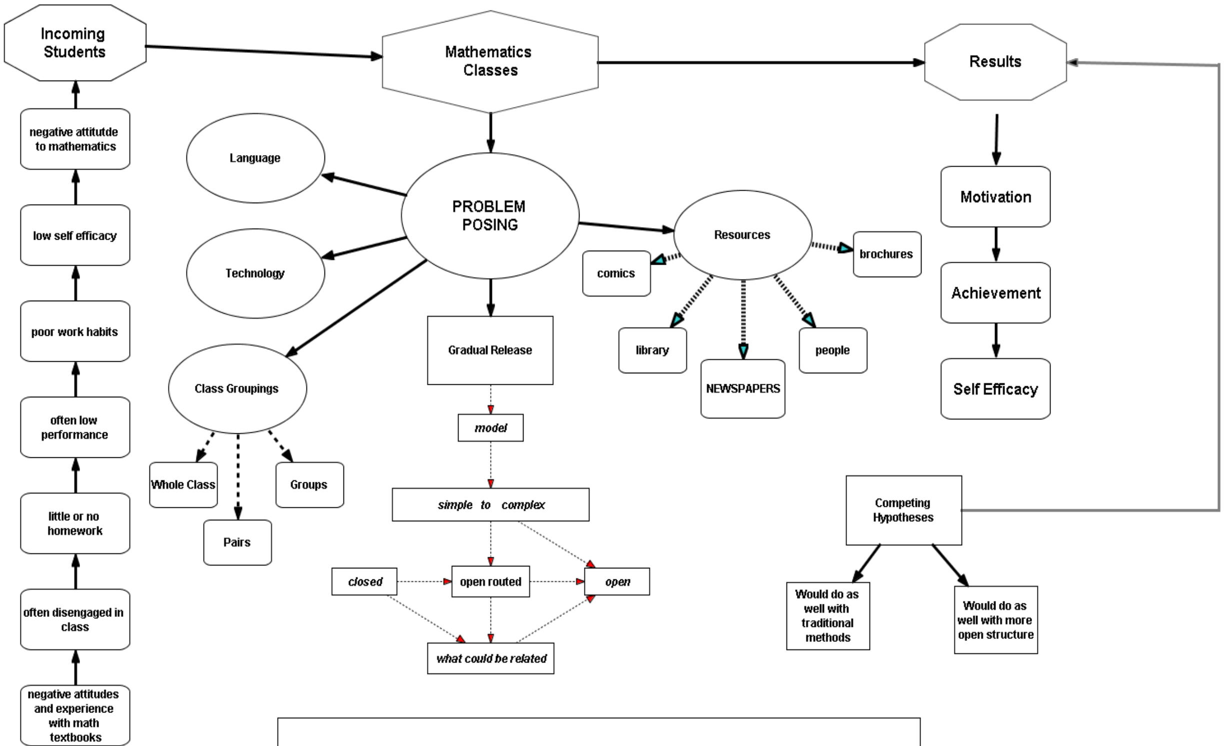


**Figure 2. PROBLEM POSING LOGIC MODEL**

## Structure of the Program

The vignettes that follow provide a more detailed description of some topics and activities. Initially the teacher provided problems, all based on newspaper articles or advertising flyers. At first the problems were straightforward, closed questions (Vignette #1). As student competency and confidence increased the program moved toward student-posed problems. The class then created more complex problems, moving from closed problems to open routed problems, having multiple solution paths to a single answer (Small, 2009), and then to open problems (Vignette #3). As the course progressed the teacher was required to engage in *just in time teaching* (Irvine, 2015). Since there was no restriction on the problems posed, and the mathematics content required to solve them, the teacher had to be prepared to adapt his lessons to address the content as needed.

**Language**

The teacher was very careful about the phraseology used in class. Since "problems," especially "word problems" had a very negative connotation for these students, the class engaged in "missing information situations." If students proposed a problem that involved mathematics not usually taught in Grade 11, they were always encouraged to explore the topic and the teacher structured activities to build their knowledge. They were never told "That's something you'll learn in grade 12" or "That's too difficult for this class." Instead the teacher helped direct their learning, using guiding questions to move student learning forward.

**Technology**

Technology such as scientific and graphing calculators was always available in the classroom and viewed as tools to both increase student self-efficacy by removing arithmetic stumbling blocks and also to allow students to tackle more difficult problems (Vignette #2). Access to computer spreadsheets was available in the library, along with print resources. When the teacher learned that many of these students had never voluntarily been in the library he arranged for the librarian to offer a workshop on how to research, how to use the print and computer resources, and how to avoid plagiarism by citing sources. This workshop was done during math class time and was driven by the students' need to find information to use in their problem posing and solving (Vignette #3).

**Groupings and Instructional Strategies**

The class used a mixture of flexible groupings, consisting of whole class, pairs, and groups. Most activities were done in self-selected groups, with students naturally forming a math talk learning community (Hufferd-Ackles, Fuson, & Sherin, 2004). Problem carousels in student pairs were a main feature. These carousels began with the students solving teacher-posed problems based on newspaper articles and progressed to student problem-posing activities, again based on newspaper articles. There was also frequent use of instructional strategies like jigsaw, stay and stray, and placemat, as well as activities that promoted student critical thinking and evaluation, such as judging the best or most interesting carousel problem posed by a student pair or group. These judgments had to be supported by reasoned arguments without invoking personal attributes.

**Assessment**

Traditional tests were preceded by jigsaw activities in which student groups posed questions and passed them on to other groups who generated solutions. The test then consisted of a mix of similar (sometimes identical) problems. As student competence and confidence increased tests became more open, based on the current day's newspaper. Individual students or pairs were asked to pick a newspaper section or article and generate a specified number of problems based on these resources. Some tests contained more structure, whereby students could be asked to generate three questions based on one newspaper article, with the questions involving three different mathematics topics that had been studied or, given a newspaper article, generate one question, then two related "what if?" questions, where the constraints were changed. In either case the expectation was that students would also provide solutions to their problems. The final exam in the course was similar to a class test, with a mixture of teacher-generated problems to be solved, and student problem-posing tasks. All these problems and tasks were based on newspaper articles. While cognitive process in problem posing were not specifically an intended target of the program, it was noted that when completing these assessment tasks students demonstrated all the levels of problem posing identified by Christou et al. (2005), i.e., editing, selecting, comprehending, organizing, and translating.

Both work portfolios and showcase portfolios were also used extensively. The work portfolios were used to keep students' work organized. These portfolios might be individual, pair, or group, and group membership could change based on flexible groupings, depending on the task. The showcase portfolios were part of student evaluation. For these portfolios students chose their best work according to criteria provided by the teacher, and included written justification for their choices as well as reflections on their learning. The showcase portfolios were assessed using rubrics. Initially the rubrics were teacher-generated. As the semester progressed the rubrics became co-created, and subsequently student-generated. Journals were also a major feature. Weekly entries consisted of two portions. The first portion was always "This week in math." In this section students identified their best work for the week, asked clarifying questions, and identified areas of difficulty. The second part of the weekly journal involved a sentence stem that focused on metacognitive or affective attributes. Examples of sentence stems include "My study plan for the test next week is ...", "The activity I liked best this week was...because...", "I feel confident that I understand... because..." This portion of the journal could also include a Likert scale on attitudes towards mathematics, self-efficacy, or interest.

## Vignettes

### Vignette #1: Advertising Claims

Initially teacher generated problems used flyers advertising sales. By concealing one or more of sale price, regular price, discount, or percent discount, students were challenged to discover what information was hidden. A major thrill for students was finding an error in the advertised information. When this happened the class wrote letters to the advertiser pointing out the error and asking for a correction in the flyer. This technique was also used for markup on cost, margin on selling price, percent change, investment interest rates or amounts, and other published numerical information. Very quickly the students began posing their own "hidden information" problems. They were also asked to engage in critical thinking. For example, "A flyer advertises 25% off, but the actual discount is 27%. Why might the store use 25% in their advertisements?" or " A flyer advertises 25% off, but the actual discount is 23%. Is this OK? Justify your decision."

**Vignette #2: Depreciation**

Students determined depreciation of cars using advertisements for new and used cars. They then formulated and graphed linear models. Based on real life data indicating that depreciation was not linear students reformulated their models as exponentials. Then using their knowledge of compound interest they determined how long they would have to save to be able to buy the used car of their choice by finding the point of intersection of two exponential functions using technology. Cunningham (2004) discusses a similar activity with college students. He indicated that his students found finding the intersection of a linear function and an exponential function using technology to be very challenging. This difficulty was not encountered with the Grade 11 students. After some preliminary work on compound interest and exponentials the students who had posed this problem solved it quite easily, and their classmates were also motivated to solve the problem since it was very much real world to students of this age, whose main goals in real life often involved purchasing a car.

**Vignette #3: Prisoners**

Based on an article about the number of federal prisoners in Canada, groups of students posed and investigated a variety of problems on topics such as number of prisoners per capita, by province, by gender, by age, by ethnicity, over time, by sentence length. Some groups investigated salaries of court and jail personnel as well as lawyers. This topic was close to home for some students since some of them had been involved in the youth criminal justice system or had parents or relatives who were incarcerated.

**Vignette #4: Guest Speakers**

Guest speakers were invited when they had information that would be of use to the class. Prior to having a guest speaker the students generated questions they wanted answered. These were given to the speaker beforehand to ensure that the speaker's information was useful and relevant to the students. A student was always selected to introduce the guest, and another student was tasked with thanking the guest. The class always composed a letter thanking the guest. After the guest speaker, students posed and solved problems based on the information they had obtained.

**Vignette #5: Role Play**

Some situations lent themselves to student role play activities. For example, to apply for a car loan, students researched occupations and salaries, created a budget, completed a loan application, and role played a meeting with a bank loan officer. Other role play topics included investments, surveys and polls, and accommodation decisions such as to rent or buy.

**Vignette #6: Comics**

Newspaper comics were an excellent source of problem posing resources. A placemat activity involved each student group having a comic in the centre of the placemat. Each student generated at least one problem based on the information in the comic. The group then determined, with reasons, which problem to share with the class in a carousel activity. A second placemat activity with comics used a comic with blank speech bubbles. The task was to outline a strategy for a specified mathematical situation, such as computing percent increase, determining commission earnings, or constructing a personal net worth statement. Each student proposed statements for the strategy and the group edited for clarity and completeness. The best edited version was written in the blank speech bubbles and was posted in the classroom.

## Discussion

### Validity

To confirm construct validity data were triangulated using semistructured self-interviews and artifacts consisting of student journals, teacher-posed problems, student-posed problems, and a final examination. The author, as the participant-observer, read a number of drafts of this paper to ensure accuracy. Internal validity was addressed using the logic model shown in Figure 2. External validity is supported by the research cited in this paper. Reliability was addressed through the use of case study protocol (Yin, 2009).

This is an ex post facto study of a program that was instituted a number of years ago. As a participant in the study, there is the risk that both the program and its outcomes are viewed through rose coloured glasses and that remembered observations are biased towards positive outcomes. Triangulation involving artifacts was used to minimize this bias. However, the artifacts themselves consisted of activities and journals that could also involve positive bias through the sampling and archival process.

### Case Results

All propositions were supported by the evidence, although the proposition concerning the interrelated effects of motivation, achievement, and self-efficacy, could not be verified with the evidence available. There was anecdotal evidence of significantly increased student motivation and engagement. There was also increased student ownership of their learning. One student remarked "We do a lot of problems in math class, but they're *our* problems." Attendance records showed decreased rates of absenteeism across all six classes relative to previous consumer mathematics classes. A comparison with other consumer mathematics classes taught by other teachers was not possible.

Schoenfeld (1992) and Lee (2012) have cited research that traditional "story problems" are often seen as difficult due to the stereotypical and unrealistic nature of the problems, and are actually demotivating for students since they have no relevance to real life. In this case study problems were always based on artifacts from the real world. The prominent role of newspaper articles as the basis for problem posing and problem solving meant that the real world connection was both obvious and emphasized. There is an interesting anecdote concerning student engagement. Over the 3-year period a number of students began going to the school library before math class to read the newspaper before their classmates. The librarian complained that frequently articles had been cut out of the library copy of the newspaper. The students were posing problems based on these articles before they came to class, engaging in mathematical problem posing on their own time. As the teacher, I made an arrangement with the library that every day we would take one of the class copies of the newspaper to the library to replace the copy that my students had defaced. I was excited that these students, many of whom had never voluntarily entered the library during their high school years, were actively engaged in problem posing and problem solving. This was an indicator of the program's impact on student motivation and engagement.

Journal entries indicated increased student self-efficacy. This is consistent with research by Cai et al. (2013) which indicates that problem-posing activities enhance self-efficacy. The gradual release structure of the program supported Silver's (1994) three levels of problem posing (before, during, and after problem solving). Initially most student-posed problems were similar to teacher-posed problems. As the students gained confidence the problems posed by individual students and groups became both more complex and more open. By the end of the course most students were posing open-routed or open-ended problems a large portion of the time. As the teacher, my involvement moved through the three levels outlined by Leung (2013): teacher as helper, teacher as junior partner, and teacher as collaborator. Thus, my function changed from active problem formulation to facilitator as students took on more responsibility for the problem-posing and problem-solving activities.

**Self -Determination Theory**

The structure of this problem-posing program addressed the three needs articulated by self-determination theory (Deci, Koestner, & Ryan, 2001; Stipek, 2002). These needs are autonomy, affiliation, and competence. Autonomy was a cornerstone of the program. Under the gradual release structure students gained increasing control over their learning. From initially posing problems based on teacher-generated problems the students moved to posing open-routed and open-ended problems across a variety of mathematical content areas, based on newspaper articles selected by the students, without teacher restrictions. The majority of activities occurred with students working in pairs or flexible groups thus addressing the need for affiliation. Competence was gained progressively as students moved from posing and solving relatively low level problems to posing multifaceted open problems, sometimes involving mathematics content significantly beyond the usual Grade 11 consumer mathematics curriculum.

**Active Learning Framework**

Ellerton (2013) has proposed an *Active Learning Framework* (Figure 3) that models the problem posing process in schools. While there are positive aspects to this model there are also significant limitations. First, the initial four blocks of Classroom Actions are very similar to the traditional instrumental teaching approach, with the exception that in block three students locate examples rather than teachers directing students to specific textbook questions. A second concern is block five, where students are limited to posing problems similar to the model

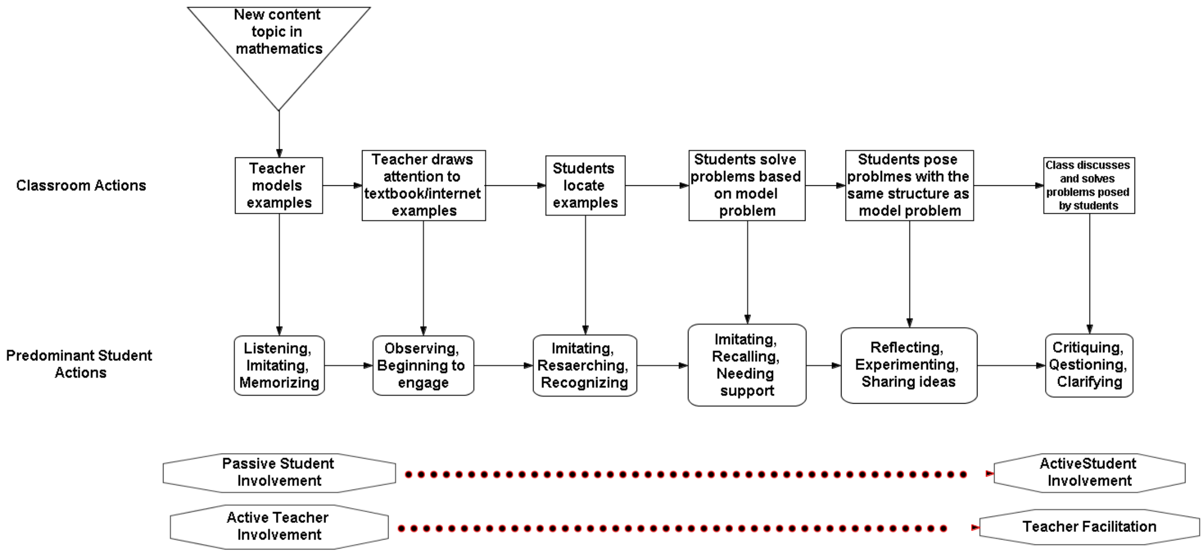


Figure 3. Active Learning Framework

Ellerton, N.(2013). Engaging pre-service middle-school teacher-educaiton students in mathematical problem posing: development of an active learning framework. Educational Studies in Mathematics 83.

problem. While a good start, modeling the initial actions described in the case study, the Active Learning Framework does not go far enough after this step. Because the driver of this framework is specific content topics there is insufficient flexibility for students to pose problems outside the limits of the content topic. There needs to be the freedom to use additional real world generators as the basis for student problem posing without the constraints of specified mathematics content. I propose a "Level 2" framework where the initiator is a real world encounter, such as a newspaper article; students pose problems; the teacher provides supporting content under a "just in time teaching" paradigm (Irvine, 2015); students solve student-posed problems; students generate additional problems by varying the constraints of the solved problem, and support each other in "thinking mathematically" (Mason, Burton, & Stacey, 2010).

Ellerton's framework represents a good first step. However it does not go far enough to truly engage students in problem posing based on their real world.

**Alternative Hypotheses**

The logic model (Figure 2) proposes two alternative hypotheses. The first, that students would do as well with traditional methods, is not supported by the evidence. These students had been subjected to traditional mathematics teaching methods in the past. The outcomes were generally mediocre performance, negative attitudes, low self-efficacy, poor work habits, and lack of engagement. The students in the case study demonstrated increased motivation and self-efficacy, reduced absenteeism, increased engagement, and reduced failure rates. The second alternative, that these students would do as well with a more open structure, is an open question. While I, as a teacher with extensive classroom experience, believe that the gradual release format was optimal, no research was conducted with a more open classroom format.

**Limitations**

While there was no identified control group, those sections of the course taught by other teachers, using the instrumental instructional technique, could be considered as "involuntary" control groups. Anecdotal evidence indicates that these classes featured similar patterns of attendance, achievement, and engagement as those classes taught in the past using the same strategies. Teachers commented about students' lack of interest in the mathematical content, their general lack of understanding and retention, and other indicators of disengagement, such as high numbers of lates, behaviour issues, and lack of homework completion. However, the lack of an identified control group does limit the causal implications of this study. A second limitation is the sample size. While this study involved approximately 150 students, with very positive results, the program would need replication to support the validity and reliability of the conclusions. This process will be made more difficult because of the generally restrictive structure of current curricula. For example, in Ontario an emphasis on overall and specific content expectations mitigates against the "just in time teaching" that allows problem posing across wide areas of content. Finally, the teacher-observer was responsible for selecting the questions to be addressed in the self-interviews. Bias may have been created through the selection or omission of questions that were addressed in the interviews.

## Future Research

Research needs to be undertaken to first verify Ellerton's (2013) Active Learning Framework and then extend it to broaden and deepen the model, relaxing the restrictions described earlier. Replication of the program described in the case study, on a more limited basis due to curriculum restrictions, is also necessary. Conducting similar studies at other grades and levels would assist in confirming the effectiveness of a similar program. While a focus on high achieving students and the impact of problem posing is understandable, a significant portion of the student population is being ignored by that focus. The case study illustrates how this segment of students is being underserved (and often ill served) and deserves the attention required to improve and enhance their education.

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