Aerial inventory of Ponderosa pine stands

Winsor Fernette

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AERIAL INVENTORY OF PONDEROSA PINE STANDS

by

WINSOR FERNETTE

B. S. Montana State University, 1949

Presented in partial fulfillment of
the requirements for the degree of

Master of Science in Forestry

MONTANA STATE UNIVERSITY

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[Signatures]

Chairman, Board of Examiners

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CHAPTER I

OBJECT OF STUDY AND REVIEW OF LITERATURE

Introduction

Aerial photographs have been used by foresters for more than twenty years. During this period of time they have proven to be a useful and almost indispensable tool in all phases of the profession.

The uses of aerial photographs in forestry are many and varied. Planimetric and topographical maps may be readily prepared from them. By viewing a stereo-pair of aerial photographs under a stereoscope the forester obtains a perspective three-dimensional picture of the forest and surrounding terrain. With this instrument and a few simple photographic measuring devices he may map a timber stand according to forest type, site, height and density classes, productive and non-productive areas, etc., and determine the size of each. The logging engineer has found that aerial photographs are useful in planning the preliminary surveys of logging roads. From them he obtains a view of the terrain ahead and can plan his road so that such "pitfalls" as rock outcrops are avoided. Aerial photographs have also proven extremely useful in fire control, and insect-damage surveys.

It has only been in the last few years, however, that
the forester has come to realize the full potential of this common tool. Schools of forestry, for instance, have only recently recognized the need to educate foresters in the intelligent use of aerial photographs. Many of them, as a result, have added courses in forest photogrammetry to their regular curriculum. The U. S. Forest Service has also taken steps to educate its personnel in forest photo interpretation. This progressive attitude toward educational needs, plus the rapid technical advancement in cameras, films, and photo-interpretation equipment should result in a more intense application of aerial photographs in forestry; especially in the infant field of aerial inventories.

In the last few years the use of aerial photographs for making volume estimations of timber stands has received wide spread attention in this country. However, this use of aerial photographs is not a recent development. The German Forester Zeiger (22) experimented with aerial cruises back in 1928. The Dominion Forest Service of Canada has been experimenting with aerial cruising and mensurational techniques since 1929 (16). In the United States it has only been since the last war, when the type of photography and interpretation devices necessary for this purpose became economically available. Since then rapid advancement has been made in this field. The Central States Forest Experimental Station and the Pacific Northwest Forest and Range Experimental Station of the U. S. Forest Service have conducted extensive experiments on different
methods of aerial inventory. (18)(25)(26) Many of these experiments have met with notable success; especially in the Pacific Northwest, where the character of some timber stands is ideal for the application of aerial inventories. Other Forest Service regions, such as the Lake States and Northeast, as well as several private agencies have conducted less publicized experiments in this field.

In the Northern Rocky Mountain Region (Region One, U.S. Forest Service) there are three prominent forest types whose stand characteristics and composition make them readily adaptable to aerial inventory techniques. They are: (1) Douglas fir (Pseudotsuga taxifolia (Lamark) Britt.) and western larch (Larix occidentalis Nutt.); (2) lodgepole pine (Pinus contorta Loud.), and (3) ponderosa pine (Pinus ponderosa Laws.).

The first type, Douglas fir and larch, usually appears as a pure type in western Montana. However, in many instances this type will contain a heterogeneous mixture of ponderosa pine and/or lodgepole pine. On medium-scaled aerial photographs, Douglas fir is readily distinguishable from larch. Here ponderosa pine is present in the type the problem of identification becomes increasingly difficult. Douglas fir can only be distinguished from ponderosa pine on very large-scaled aerial photographs. On the other hand, larch is easily distinguishable from either species. In the case of lodgepole pine no problem of identification occurs. Both Douglas fir and larch are distinguishable
from this species.

The outstanding characteristic of the Douglas fir-larch type which makes it readily adaptable to aerial inventories is its two-storied formation. Since larch is a more intolerant species than Douglas fir it greatly outstrips the latter in height growth. Also, larch does not form an unbroken canopy over its shorter associate, but occurs widely scattered throughout the stand. Consequently, there are, theoretically, two different timber types, each composed of a single species. On aerial photographs these intermingled species are readily distinguishable.

The Douglas fir understory of this type usually has a high stand homogeneity, with sufficient openings in the crown canopy to permit height measurements. This characteristic, along with the single-species composition, is ideal for the application of the stand-aerial volume table method of aerial inventory.¹ The larch overstory, on the other hand, presents a more complex problem. The crown characteristics of this species, for example, are not readily adaptable to photographic measurements. The thin, feathery, and sharply pointed crown does not

¹ The terms: "stand-aerial volume table" and "tree-aerial volume table" are borrowed from Spurr(7). The former table estimates volume by measurements of the trees occurring on a unit of area. In the latter table, volume is estimated on the basis of individual tree measurements.
readily resolve on the photograph. Consequently, there will be a significant difference between the actual crown diameter and total height of the tree, and the photographic crown diameter and total height. This discrepancy could be corrected by computing a correction factor based on actual field and photographic measurements of the same trees. For estimating volume of this species on aerial photographs, the tree-aerial volume table method would, in all probability, give the most accurate values.

The second forest type, lodgepole pine, is readily adaptable to aerial inventory techniques because of the high homogeneity of the stand. Pure stands of this species are of uniform height and density; the latter usually being very high. These characteristics are ideal for the application of the stand-aerial volume table method. On aerial photographs lodgepole pine stands are readily identifiable and delineated.

The last type, ponderosa pine, has none of the stand characteristics exhibited by the above two types. In Western Montana this species occurs mostly in pure stands of widely scattered trees on south and west exposures. Where site conditions are favorable, Douglas fir and larch are common associates. Another associate, to a lesser degree, is lodgepole pine.

The open nature of the ponderosa pine type and the relative large size of the individual trees which compose it make this type readily applicable to aerial volume estimations by
the tree-aerial volume table method. For example, in a pure stand of mature ponderosa pine practically every tree may be measured on aerial photographs for total height and crown diameter. There will be small clumps of trees present in every stand of this species which will require an estimation of average height and average crown diameter. However, these will form only a minor part of the total stand.

Where ponderosa pine is associated with Douglas fir of commercial size a problem of identification is introduced which could nullify any potential advantage gained by aerial inventory methods over conventional field methods. As mentioned in a preceding section, it is difficult, if not impossible to distinguish between mature trees of these two species on medium-scaled photographs. In the case of large-scaled photography these two species may be separated by the application of such factors as site, crown characteristics, and photographic tone of the tree crowns. However, in order to correctly apply such factors, the photo interpreter must have an intimate knowledge, gained by field observations, of the stand, or a similar stand, of which he is making an aerial cruise.

Ponderosa pine is one of the more important commercial conifers found in the western part of Montana. In this region there are approximately 2.2 million acres of timberland classified as Ponderosa pine type. Sawtimber stands of this species have an estimated volume of 9.4 billion feet, b.m. Production
of ponderosa pine lumber in western Montana is 215 million feet, b. m., annually.

Many of the more valuable stands of mature ponderosa pine in western Montana are located in remote inaccessible areas. One of the more important of these is the Bitterroot National Forest. To reach such an area, for cruising purposes, would require a long difficult journey by horse, or on foot. Practically all supplies would have to be packed in. In such a case the initial cost, before any actual cruising is done, is quite large. Because of these factors the possibilities of applying aerial inventories to remote stands of ponderosa pine in this region should be closely examined.

Object of Study and how Study was Made

Because of its economic status in western Montana and high potential adaptability to aerial inventory techniques, ponderosa pine was selected over the above mentioned types, lodgepole pine and Douglas fir-larch, as the species to be studied in this problem.

Objects of this study were to determine: (1) what correlation exists, and to what degree, between diameter, breast high (b.b.h.), crown diameter, total tree height, and tree volume of ponderosa pine; (2) what statistical method is best for preparing tree-aerial volume tables to estimate the volume of this species; (3) that problems are involved in
species identification on aerial photographs; (4) what is the influence of site quality on tree form and volume; (5) how site quality can be predicted on aerial photographs; and (6) what photogrammetry factors are involved in aerial inventories.

In order to initiate the above objectives an area of Ponderosa pine on the Bitterroot National Forest having good photographic coverage was selected. The problem of applying aerial inventories to this area was facilitated by dividing the study into the following phases: (1) statistical, (2) aerial cruise, (3) ground cruise, and (4) comparison and summary.

The first phase involved the gathering of the basic field data and preparation of aerial volume tables. Data concerning the relationship between the different tree variables, d.b.h., crown diameter, total tree height, merchantable height, and tree volume were obtained by measurements on randomly selected plots. These data were segregated according to height, crown diameter, and site classes. Three different statistical methods were used to prepare the tree-aerial volume tables. In two of these methods, the coefficient and 3-variable solution for d.b.h., d.b.h. was directly correlated with crown diameter and total height, while in the third method, 3-variable solution for volume, tree volume was correlated with these two variables and d.b.h. was ignored. Site was segregated into three general classifications and tree-aerial volume tables based on the coefficient method were prepared from data collected.
on each site classification. Aerial volume tables were also prepared from data collected on all three site classifications by each of the three statistical methods. The latter tables are referred to as the "all sites" aerial volume tables.

In the second phase a series of aerial cruises were made of parts, and of the whole experimental area. The area was segregated according to general site and density classes to form small blocks of timber having a high stand homogeniety. Different intensities of aerial cruises were made of four blocks for the purpose of comparing the different types of volume tables under all possible conditions of site, density, type, exposure, and slope. In every cruise each tree measurement was classified according to site. The corresponding site volume table was used to estimate volume. Also, the volume of each tree measured was estimated by each of the three "all sites" volume tables. The volume of the area was then determined for each type of volume table. A final aerial cruise, consisting of a 20 percent representative\(^1\) random sample, was made of the entire area. In this cruise, like the smaller ones, each measurement was classified according to site. A total volume value was then computed for each type of aerial volume table used in the survey.

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\(^1\) By a "representative" random sample is meant that the location of each plot taken in the sample is determined by random selection.
In the third phase of the problem a 50 percent systematic random field sample, using one-fifth acre circular plots, was made of the experimental area. Aerial photographs were used to plan and control all phases of the field cruise. The boundary of the area was first delineated on the photographs and then transferred to the field. Direction and spacing of the cruise lines, and the spacing of plots on each line, were first determined on this medium. Type and density classes were determined and delineated on the aerial photographs before the field cruise. The size of the entire study area and of each site and density class was determined from a planimetric map prepared from the aerial photographs by means of a Kail Radial Line Plotter.

In planning this cruise on the aerial photographs no consideration was given the individual site and density classes. The cruise and computed cruise intensity were for the entire area. However, in making the actual field cruise the plots were segregated according to what class they fell in. In some instances it was necessary to divide a plot between two classes. By this segregation it was possible to determine the volume of each site and density class. A statistical analysis was made of the field cruise to determine the limits of accuracy; both for the individual classes and for the entire area.

2 By a "systematic" random sample is meant that only the location of the first cruise line, and first plot in each line, is determined by random selection.
In the final phase the results of the aerial and field cruises were compared and summarized.

**Review of Literature**

In the past much has been written on the use of aerial photographs in forestry. However, most articles on this subject are of a general scope and contain very little material of a specific or technical nature. This has been especially true of articles concerning aerial inventories. The large majority of these articles appear in three professional magazines: the *Journal of Forestry*, *Photogrammetric Engineering*, and the *Forestry Chronicle*; the latter being a Canadian publication.

There are, however, a few publications which deal specifically with the technical problems and statistical methods involved in aerial forest inventories. For example, Nash (16) mentions an investigation conducted by the Dominion Forest Service of Canada in which three different statistical methods were used to prepare tree-aerial volume tables. This investigation was for the purpose of testing the validity of a new formula technique, developed by that organization, for predicting d.b.h. and tree volume from the known tree variables of crown diameter and total treat height. These two variables being the only tree measurements which are directly measurable on aerial photographs. The other tree variables, d.b.h. and volume, must be directly correlated with them. In one of the
statistical methods discussed by Nash, no assumptions were made as to probable relationship between the variables. This method was used as the criterion with which the results of the formula method are to be compared. The third method made use of the diameter-height relationship in order to determine the superiority, if any, of the formula. The resulting volume tables were compared and it was concluded that the formula table would give slightly conservative values in volume estimations.

The type of volume table discussed by Nash is nothing more than a conventional volume table in which d.b.h. is estimated from crown diameter and total height. Other authors have questioned the statistical validity of this method. Spurr (7), for instance, states that this method is statistically unsound. He concludes that such volume tables are open to question because d.b.h. is first introduced as a dependent variable and then is changed to an independent variable in the final volume table. This method, according to Spurr, is only justified when the variables are perfectly correlated. Otherwise the method is unsound and should only be used for volume estimations when nothing better is available.

Spurr also discusses several methods for preparing tree-aerial volume tables which he considers to have a sound statistical basis. Spurr refers to these two techniques as the "direct method of computation" and the "combined-variable method." In the former method d.b.h. is ignored and tree volume is directly
plotted over crown diameter by total height classes. Volume being expressed in cubic feet in this case. This method requires more work than the indirect d.b.h. volume table approach and, even though the standard error is much larger, it has a sound statistical basis.

The "combined-variable method" is, according to Spurr, the simplest means of preparing a tree-aerial volume table in terms of total height and crown diameter. In this method the crown diameter is squared and then weighted by the height. The data are then sorted by $G^2H$ classes, and the average tree volume for each class is obtained. Volume was then plotted over $G^2H$. Large standard errors are obtained by this method. Spurr cites standard errors of 56 percent for red spruce, 47 percent for black spruce, and 45 percent for balsam fir.

Other investigators have reported the results of experiments on the stand-aerial volume table method of forest inventory. For instance, Pope (18), discusses the results of an aerial inventory experiment in which three types of aerial volume tables were used to determine the volumes of Douglas fir stands in the Pacific Northwest. One table was for estimating the volumes of individual trees from total height and crown diameter; the other two were of the stand-aerial volume type and employed total height, crown diameter, and crown closure as the basis for estimating stand volumes.
Aerial photographs, whose Representative Fraction (RF) scales ranged from 1:23,000 to 1:12,000, were used in this investigation.

The types of Douglas fir stands sampled ranged from young stands growing on good sites to mature stands growing on poor sites. The difference between gross volumes estimated on aerial photographs, and the true gross volumes ranged from 10 percent height, in the case of the younger stands, to 25 to 45 percent low for the older stands.

Pope concluded that the individual tree approach was less reliable than the stand volume approach. He also stated that future work in this field will be concentrated on the stand-aerial volume method because of its greater reliability and that data necessary for its preparation are easier to obtain than for the individual tree method.

Another investigation on the use of stand-aerial volume tables is reported by Moessner and Jensen. (26) The volume tables used in this investigation employ total tree height, crown diameter, and crown closure as the basis for estimating volume. (25)

Aerial photographs used in this investigation were regular panchromatic, 1:20,000 scale Department of Agriculture prints.

Three test areas, ranging from 40 acres to several thous-
and acres, were selected. Aerial cruises of these areas were made by three photo interpreters. The true volume of each area was determined by one or more field cruises. The mean variation between aerial and field cruises for the 40-acre tract was 3.5 percent; for the 640-acre tract, 2.3 percent; and for the several thousand acre tract, 6 percent. According to Moessner and Jensen, the results of this investigation indicate that reliable gross volumes can be obtained by aerial inventories without field work, regardless of the size of the tract.

In concluding the authors point out that there are definite limitations to this method of aerial inventory. For example, there is no possible way to obtain data on the individual tree directly from aerial photographs. This pertains to diameter, merchantable length, growth, and cull factor. Also, the volume tables are limited to average situations and cannot reflect volume extremes caused by local conditions.
CHAPTER II

FIELD TECHNIQUES

To initiate the study it was first necessary to: (1) select an area of ponderosa pine which met certain specification, and (2) collect field data on the selected area for preparation of the tree-aerial volume tables.

Selection of Study Area

Because of the limited scope of this study it was necessary to establish certain specifications in selecting an area of ponderosa pine for the problem. These specifications were to assure the selection of an area which would present all, or most, of the problems of a large-scaled aerial inventory of this species. Specifications required of the selected area were: (1) it had a virgin stand of mature ponderosa pine, (2) there was a wide range of site qualities, (3) on some sites Douglas fir and/or larch were associates, (4) the boundaries were well defined, (5) the size was approximately 100 acres, and (6) it had recent medium-scaled aerial photographic coverage. Well-defined boundaries and the limited size of the area were for the purpose of controlling and facilitating the field and aerial cruises. Recent medium-scaled photography means photographs taken in the last ten years at a minimum scale of 1:15,040. This scale was considered the smallest practical scale for aerial inventory work; especially where each indivi—
dual tree must be measured.

After considering many potential locations, an area was finally selected on the Bitterroot National Forest in the center of Sec. 11, T.I.N., R. 19E., M.F.M., which conformed with most of the specifications listed above. (Figure 1) This area is part of Piquette Creek Experimental Forest; an area set aside by the U. S. Forest Service for experimental work in ponderosa pine. The spur drainage which forms the north and south boundaries of this area is without a name. Thus, for the purpose of reference and identification this drainagewill be called Benchmark Creek; and the upper forks, the North, Middle, and South Forks of Benchmark Creek.

In selecting this area for the study, aerial photographic coverage was the controlling factor. The Piquette Creek Experimental Forest was photographed in the summer of 1946 on pan-chromatic film at a datum scale of 1:8,000. These photographs were systematically viewed under a stereoscope until an area was located which appeared to have all the stand and site characteristics required for this study. Subsequent ground examination confirmed the accuracy of the photographic selection.

In practically every respect the area selected conforms with the required specifications. A virgin and mature ponderosa pine stand, 300 years in age, grows on the area. There is a wide range in site qualities. Site indices on the exposed ridges
Legend:
- control point
- section corner
- section line
- boundary of plot

Figure I.
Aerial photograph of Piquette Creek-Benchmark Creek Area
Figure 2.

Photographs illustrating the different sites found on the Piquette Creek-Benchmark Creek Area.
Figure 2.
are 30 to 40; those for sites in canyon bottoms, 80 to 90. Exposures range from southeast to northwest. On the latter, Douglas fir of commercial size is an associate. The area boundaries are readily definable on both the photographs and the ground, and the size is approximately 93 acres. Photography, as stated above, is excellent. The scale of the photographs, however, is not 1:8,000 for this area. Since it is located below datum the scale will be smaller. In this case the average scale for the area is 1:13,280, or one inch equals 1,107 feet.

Collecting Tree-Aerial Volume Table Data

Data for the preparation of tree-aerial volume tables were collected on the study area during the fall of 1951.

To initiate the collection of data a 15 percent representative random sample, using one-quarter acre plots, was made of the study area. Random selection was obtained by placing a plastic dot grid over a photograph of the area and assigning a number to each dot falling on the area. The number of each dot was then written on a slip of paper and placed in a box. Numbers were drawn and the corresponding dot for each number was determined. The position of each selected dot was recorded on the photograph by means of a needle. Each dot selected represented the center of a plot. A total of 48 plots were selected in this manner.
In the field the center of each plot marked on the photograph was located on the ground. This transfer was sometimes difficult, but in practically all cases the center of each plot was located on the ground within a plus or minus ten feet of the true, or photo, location.

On each plot every tree of commercial size (1½ inches d.b.h. and over) was measured for: (1) d.b.h., (2) total visible height, (3) visible crown diameter, and (4) merchantable height. Where Douglas fir of commercial size occurred on the plot, the same measurements were made of this species. Each plot was also classified according to site quality. All diameter measurements were made with a steel diameter tape and all height measurements with a percent abney and 50-foot metallic tape.

By total visible height is meant the height of a tree to a point where the crown narrows to three feet in diameter. (6) The theory for this procedure is that nothing less than three feet in diameter will resolve on photographs of 1:20,000 scale. What this factor is for photographs of 1:13,000 scale is not definitely known. Because this factor was an unknown quantity at the time the volume table data was collected, the above assumption was followed in making total height measurements.

Measurements of total visible height are not important in the case of ponderosa pine. Because of the rounded-top
crown characteristic of mature trees of this species, total visible height and actual total height are usually the same. Only in young, thrifty trees is the difference great enough to require correction for visible height. However, in Douglas fir there is a significant difference between these two measurements of height. This species has a conical crown with a long tapering apex. In many instances, visible height was more than 20 feet lower than total tree height. This crown characteristic of Douglas fir will have a significant influence in the preparation of tree-aerial volume tables for this species. If this correction factor is not taken into consideration, volumes of individual trees will be greatly over-estimated.

Visible crown diameter is a term used by forest photogrammetrists to denote that part, or the diameter of that part, of the crown which resolves on the aerial photograph. In this way it is similar to total visible height. Of the four tree measurements made in the field, crown diameter was the most difficult measurement to make. In the case of ponderosa pine the crown often consisted of nothing more than a few branches; either scattered about the tree bole, or concentrated on one side. Many of the branches were too small to resolve on the photographs. These conditions made it difficult to estimate what portion of the crown could be classed as "visible." By applying the 3-foot diameter rule it was possible to arrive at a fairly accurate visible crown diameter value. Some of the
trees for which the visible crown diameter measurement was six feet, had an actual crown diameter of 10, 12, and even 14 feet.

The problem of estimating visible crown diameter for Douglas fir was not as difficult. Usually visible and actual crown diameters were the same. Exceptions were over-mature trees of this species. In this case, visible crown diameter was determined by the same method applied to ponderosa pine.

Once the visible crown diameter was determined it was transferred to the ground by means of a plumb line. Stakes were driven at these transfer points, and the distances from the center of the tree to the stakes were measured with a metallic tape.

Many investigators in the field of aerial inventories have advanced theories on how crown diameter measurements should be made. (6)(15) Some suggest a four-way average, others suggest measurements in an east-west direction, while still others are advocates of only a north-south direction of measurement. In large surveys, where a large number of measurements, under varying forest conditions, are involved the four-way average would be the more accurate; it would also require more work than the other methods. Where the aerial inventory is of a small area, such as in this study, crown diameter should be measured at approximately right-angles to the radial
displacement line of the aerial photograph. The reason for this is that when the tree images being measured are off-center of the photograph, only the side of the tree which is perpendicular to the radial displacement line can be measured. Therefore, a similar measurement should be made in the field. In this study the plane perpendicular to the radial displacement line was easily determined in the field; being approximately a north-south direction. All field measurements of crown diameter were made in this direction.

**Site Classification**

If it is common silvicultural knowledge that a tree growing on a poor site will not only be shorter, but will also have a larger crown diameter than a tree of the same species and age growing on an adjacent site of higher quality.

Site quality has considerable influence on the tree form of ponderosa pine. For example, compare two typical trees of the same age growing on the study area: one growing on a bottomland site having a site index value of 80, the other growing on an exposed ridge site having a site index value of 40. Dimensions of these two trees are as follows:

---

1 Figure 2 may be correlated with Figure 1 by comparing the Roman numerals under each photograph shown in Figure 2 with the corresponding Roman numerals in Figure 1. The individual trees shown in photographs IV and V of Figure 2 are circled in Figure 1. Photographs I and IV illustrate slope sites; photograph II a northwest exposure slope site; photograph III a canyon site; and photographs V and VI, exposed sites.
Because of this wide variation in tree form exhibited on sites of different qualities it was thought best to separate the field measurements into general site classes, and prepare the aerial volume tables according to these segregated data. These general site classes are:

1. canyon site - defined as the area from the bottom of a canyon to approximately one-quarter of the way up the slope;

2. exposed site - defined as the area from the top of the ridge to approximately one-quarter of the way down the slope; and

3. slope site - defined as the area in between the above extremes.

The average site index value for canyon sites is 80; for slope sites, 60; and for exposed sites, 40.

In some cases the above limits did not adequately classify a site and it was necessary to make adjustments. This was

<table>
<thead>
<tr>
<th>Site 80</th>
<th>Site 40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>320 years</td>
</tr>
<tr>
<td>D.B.H.</td>
<td>38 inches</td>
</tr>
<tr>
<td>Total Height</td>
<td>125 feet</td>
</tr>
<tr>
<td>Merchantable Height</td>
<td>7 logs (112')</td>
</tr>
<tr>
<td>Crown Diameter</td>
<td>16 feet</td>
</tr>
<tr>
<td>Volume</td>
<td>3,010 feet, B.M.</td>
</tr>
</tbody>
</table>
usually done in accordance with the vegetation growing on the site. If the trees appeared to belong in the exposed site category, although they were growing on a slope site as defined in (3) above, the measurements of these trees were listed under the exposed site classification. Measurements made of trees growing on spur-ridge tops were usually listed under the slope-site classification.
CHAPTER III

OFFICE TECHNIQUES

Statistical solutions used in this study for preparing the tree-aerial volume tables are: (1) coefficient, (2) 3-variable solution for d.b.h., and (3) 3-variable solution for volume. The coefficient solution was the only solution used to prepare the individual site volume tables. A method for estimating site quality on aerial photographs was also developed by computing the average total height-crown diameter ratio for each of the three general site classifications discussed in the preceding chapter.

Coefficient Solution

The coefficient solution of estimating d.b.h. from known values of crown diameter and total height was developed by the German Forester Zeiger, (22) This investigator established a relationship between crown diameter and d.b.h. by the formula:

\[
\frac{k}{d} = E
\]

where

\[ k = \text{crown diameter} \]
\[ d = \text{d.b.h.} \]
\[ E = \text{a coefficient} \]

-30-
Once the value of the coefficient "E" for any species and height class is established, d.b.h. of any tree of the same species and height class could be determined by measuring the crown diameter and applying the coefficient "k" according to the formula

\[ d = \frac{k}{E} \]

In managed forests of Germany the trees are usually of fairly uniform age and height. For this condition, the above formula was especially adaptable to aerial inventory work. Only one photographic measurement is required - the diameter of the crown.

When the Dominion Forest Service of Canada began experimenting with this formula they found that it was not applicable, in its original form, to the natural-grown forests of this country. (22) Unlike the intensive managed forests of Germany, trees composing Canadian forests grow under varying degrees of competition and suppression, and in stands of varying density. Canadian foresters reasoned that the degree of competition under which a tree has grown throughout its life will be reflected in the proportion of height growth over stem diameter growth. It was suggested, therefore, that the competition ratio, \( \frac{h}{d} \) be incorporated into the original German formula; which then became
\[
\frac{kh}{d^2} = E
\]

According to Nash (16), when plotted over total height, \( \frac{kh}{d^2} \) becomes a curve which has a rapidly decreasing slope with increasing height.

Once the average value of the coefficient \( E \) is established, d.b.h. of any tree can be determined by measuring the crown diameter and total height and applying the coefficient according to the formula

\[
d = \sqrt[3]{\frac{kh}{E}}
\]

The advantage of the coefficient solution in preparing tree-aerial volume tables is that the amount of data necessary to establish a regression is considerably less than that required by the other statistical solutions.

In this study the coefficient solution was employed to prepare four ponderosa pine, and one Douglas fir tree-aerial volume tables. The ponderosa pine volume tables were segregated according to the three general site classifications outlined in the preceding chapter. Each site volume table was prepared from data obtained from the particular site it represents. For example, the exposed sites tree-aerial volume table was prepared from data obtained in the field from exposed sites only. A fourth ponderosa pine volume table was
prepared from data obtained from all three sites, and is termed the "all sites" tree-aerial volume table.

The coefficient volume tables for ponderosa pine, and the graphs used in their preparation, are found on pages 16 to 23 of the appendix. The coefficient volume table and graph for Douglas fir are found on pages 24 and 25 of the appendix.

Only the coefficient solution was used to prepare the individual site tree-aerial volume tables. The reason for this is that there wasn't sufficient data for each site classification to apply the other statistical solutions.

Three-Variable Solution for D.B.H.

In this method the original data were listed according to 2-foot crown diameter classes, and 10-foot total height classes. The average d.b.h. for each crown diameter class was determined, and d.b.h. was then plotted over crown diameter by total height classes. For both ponderosa pine and Douglas fir the graphic solution indicated that the least squares method would be the most suitable statistical solution.

Unlike the coefficient solution, no assumptions are made in the 3-variable solution for d.b.h. concerning probable
relationships between the tree variables. Consequently, an unbiased result is obtained. Another statistical advantage of the 3-variable solution for d.b.h. is that the relationship between the different tree variables is readily computed. For example, the correlations between d.b.h. and crown diameter, d.b.h. and total height, and crown diameter and total height are shown in the statistical analysis. (5)

One tree-aerial volume table, for all site classifications, was prepared for each species by the 3-variable solution for d.b.h. No attempt was made to segregate the data by site classes in the case of ponderosa pine. This is because the amount of data available was not sufficient to prepare a volume table for each individual site class by this solution. The tree-aerial volume table, prepared by the 3-variable solution for d.b.h., for ponderosa pine is found on pages 28 and 29 of the appendix, and for Douglas fir, pages 30 to 37 of the appendix.

**Statistical Comparison Between Coefficient Solution and Three-Variable Solution for D.B.H.**

Results of the statistical analysis of ponderosa pine data by these two solutions are found in Table I. In Table II the results of the statistical analysis of Douglas fir are listed.

A complete analysis of the relationship between the
TABLE I
PONDEROSA PINE

STATISTICAL COMPARISON OF COEFFICIENT SOLUTIONS
AND THREE-VARIABLE SOLUTION FOR D.B.H.

<table>
<thead>
<tr>
<th>Site</th>
<th>Three-Variable</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Std. error</td>
<td>Std. error</td>
</tr>
<tr>
<td></td>
<td>(percent)</td>
<td>Correlation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>index</td>
</tr>
<tr>
<td>All sites</td>
<td>3.8&quot;</td>
<td>14.9%</td>
</tr>
<tr>
<td></td>
<td>74.5%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canyon sites</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope sites</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exposed sites</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* 688 measurements
TABLE II

DOUGLAS FIR

STATISTICAL COMPARISON OF COEFFICIENT SOLUTION AND THREE-VARIABLE SOLUTION FOR D.M.R.

<table>
<thead>
<tr>
<th>Solution</th>
<th>Std. error of estimate</th>
<th>Std. error (percent)</th>
<th>Correlation index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>1.8&quot;</td>
<td>9.3%</td>
<td>80%</td>
</tr>
<tr>
<td>3-Variable</td>
<td>1.0&quot;</td>
<td>5.0%</td>
<td>85%</td>
</tr>
</tbody>
</table>

* 343 measurements
different tree variables is provided in the 3-variable solution for d.b.h. For example, the correlation coefficient for total height and crown diameter of ponderosa pine is 0.25; for d.b.h. and total height, 0.67; and for d.b.h. and crown diameter, 0.71. The multiple correlation coefficient is 0.862. This means that 45 percent of the variance in d.b.h. is associated with total height, and 50 percent of the variance in d.b.h. is associated with crown diameter. The variance in d.b.h. associated with both total height and crown diameter is 70 percent.

Volume Curves

The two statistical methods discussed above are only for the purpose of estimating d.b.h. from crown diameter and total height. Once d.b.h. is known it is still necessary to estimate tree volume. Conventional volume tables cannot be used because, in this case, volume is estimated on the basis of merchantable height; a measurement unobtainable on aerial photographs. In order to obtain a volume value it is necessary to correlate this variable with the estimated d.b.h. and the measured total tree height.

In this study graphs were prepared in which volume is the dependent variable, and d.b.h. and total tree height the independent variables. The data being segregated according to total height classes. Once d.b.h. is known, the correspond-
ing volume can be obtained by scaling the appropriate height-class curve. These volume graphs for ponderosa pine are found on pages 39 to 48 of the appendix, and for Douglas fir, pages 49 to 54 of the appendix.

Three-Variable Solution for Volume

This method differs from the coefficient solution and 3-variable solution for d.b.h. in that d.b.h. is eliminated entirely. Volume is directly plotted over crown diameter by total height classes. In this case volume is expressed in board feet values, not in cubic feet, as has been the practice in previous experiments by other investigators. (7)

By estimating volume directly from crown diameter and total height the necessity of preparing volume curves is eliminated. However, other statistical complications arise. What statistical solution is applicable to these data is sometimes difficult to determine. The curve forms do no follow any general trend. For example, the individual height class curves for ponderosa pine were of three types. The 55-foot, 65-foot and 75-foot height class curves exhibited a parabolic form; the 125-foot height class curve was hyperbolic in character; and the remaining curves were linear correlations.

With this complex distribution of curve forms it was first considered best to solve each height class curve individually and compute a weighted-average standard error of estimate.
This procedure was followed and a weighted standard error computed. The value of this measure was much smaller than that which would have been obtained by solving, in entirety, the complete set of data. However, the statistical validity of this method is questionable. Since the original data were obtained by random selection, the distribution of the measurements in these data should correspond with the distribution of measurements in a random sample aerial cruise. Consequently, in order to obtain a reliable measure of dispersion, the standard error of estimate must be computed from the entire set of data. Because of this factor, solving each height class data individually was discarded.

To determine what statistical method would provide the most accurate analysis, the entire set of ponderosa pine data were plotted on cross-section paper. The resulting graphic solution indicated that the trend of regression was either linear or hyperbolic. Because curve form was definitely linear in the heavily weighted lower crown diameter classes, and only in the highest crown diameter classes did it tend to become slightly hyperbolic, the least squares method was first used to solve the data. Also, most of the individual height class curves exhibited a linear, or nearly so, correlation. Examination of the results of this method indicated that the analysis was sufficient, and no attempt was made to further analyze the data by the semi-logarithmic method. The reason for this is
apparent if Figure 30 in the appendix is examined. In the lower crown diameter and height classes, the resulting volumes are minus quantities. The semi-logarithmic method would have increased these minus quantities.

Statistical results obtained in the analysis of the entire set of ponderosa pine data by the least-squares method are: (1) standard error of estimate, 32.2 feet, B.M., or expressed as a percentage of the mean, 28.7 percent; and (2) coefficient of correlation, 87.4 percent. Further discussion of this statistical analysis may be found on pages 55 to 56 of the appendix.

The procedure followed in analyzing the Douglas fir data by the 3-variable solution for volume is similar to that of ponderosa pine. Volume was plotted over crown diameter by total height classes. The data of each height class were solved individually and a weighted-average standard error of estimate computed. As in ponderosa pine, the height class curves did not conform to any one general pattern. Graphs of these individual height class curves, and the different statistical methods used to solve each height class data, are found on pages 66 to 71 of the appendix. No attempt was made to prepare a volume table based on the individual curve solution.

Two statistical methods were used to analyze the Douglas
fir data in entirety. A graphic solution of the data indicated that either the least squares or logarithmic methods were applicable. Since the trend of regression could not be identified with any certainty on the graph, it was necessary to analyze the data by both methods. The results of the two methods were compared, and the logarithmic method was found to have the smallest standard error of estimate. Graphical illustration of these two statistical methods may be found on pages 64 and 65 of the appendix. The tree-aerial volume tables prepared from these graphs on pages 62 and 63 of the appendix.

The statistical results of least squares and logarithmic solutions are found in Table III. A comparison between a numerical analysis, such as the least squares method, and a logarithmic analysis is difficult to make. The only statistical measure which can be directly compared is the standard error of estimate, expressed as a percentage of the mean.

Estimation of Site Quality on Aerial Photographs from the Total Height Crown Diameter Ratio

The conventional method of indicating site quality is in terms of site index; or productive site value. This is expressed as the height, in feet, of average-diameter dominants and codominant trees at the age of 100 years. If the
### Table III
**Douglas Fir**

**Statistical Results of Least Squares and Logarithmic Solutions**

<table>
<thead>
<tr>
<th>Statistical Method</th>
<th>Statistical Measure</th>
<th>Std. Error of Estimate</th>
<th>Std. Error of Est. (Percent)</th>
<th>Correlation Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logarithmic</td>
<td>0.62 log. units</td>
<td>26%</td>
<td>0.98 log. units</td>
<td></td>
</tr>
<tr>
<td>Least squares</td>
<td>11.4 feet, b. m.</td>
<td>34%</td>
<td></td>
<td>85%</td>
</tr>
</tbody>
</table>

* 344 measurements
dominant trees on an area are 100 feet in height at 100 years of age, the area is a site index 100. This method, however, cannot be used to indicate site quality on aerial photographs. It is a function of tree age, and age cannot be measured on this medium.

Losee (13)(14) describes a method for classifying sites on aerial photographs in terms of topography and tree cover. According to this investigator:

"The topographical situation of a specific area, interpreted according to geology and soils, permits an evaluation of site which is sufficient for all practical purposes. Should a finer delineation of site be necessary, it may be accomplished by considering the site requirements of the vegetation present."

This method is, if used for precise site mapping, somewhat complicated. It would require on the part of the photo interpreter, a broad knowledge of pedological, ecological, and geomorphic principles, and the ability to apply them to what he observes on the aerial photograph.

It is also possible to determine site quality on aerial photographs by computing the ratio of total tree height to

---

1 In order to facilitate the study of site interpretation on aerial photographs, especially in regards to geology and soils, the articles by Belcher (3) and Hittle (10) are excellent references. Although these authors are primarily concerned with the engineering significance of soil patterns as interpreted from aerial photographs, their data may be readily applied to the interpretation of forest site quality.
crown diameter. This method was developed by the German Forester Zeiger (22), and recently investigated in this country by Spurr. (6) The basic reasoning behind this method is common silvicultural knowledge. A tree growing on a poor site will not only be shorter, but will also have a larger crown diameter than a tree of the same species and age growing on an adjacent site of higher quality. Consequently, the tree growing on the poorer site will have a lower total height-crown diameter ratio than the tree growing on the adjacent site of higher quality.

To determine the validity of this method in regards to this study, the total height-crown diameter ratio was computed for every mature tree in the original ponderosa pine data. The average ratio was determined for each site classification, and then plotted over the average site index value. Figure 4 graphically illustrates the correlation between total height-crown diameter ratio and site index value.

Although this analysis is of limited scope, it does indicate the possibilities of estimating the productive value of ponderosa pine sites on aerial photographs.
Predicting site quality by the total height-crown diameter ratio.
CHAPTER IV

AERIAL INVENTORY

Before an aerial inventory can be made of a ponderosa pine stand, certain photographic procedures and problems have to be considered and solved. The most important of these are: (1) photogrammetric factors, (2) type and density classification, (3) preparation of base map from aerial photographs, (4) area determination, (5) species identification, and (6) estimation of cull factor.

Once the above factors had been solved, it was then possible to make the aerial cruises of the experimental area. In this study, ten different aerial cruises, of varying intensities and systems, were made of sections I, II, V, and VI, and of the study area in entirety.

Photogrammetric Factors

Photogrammetric factors which must be known before any tree height or crown diameter measurements can be made are photographic scale and absolute parallax base. Both of these factors are a function of the flying height of the camera station above the ground level. Consequently, if an aerial inventory is to be made of an area having a large elevation differential, it will be necessary to compute new scale and absolute parallax base values for a given change in elevation. Photographic measuring devices, like the
parallax wedge, have constants which are based on scale and absolute parallax base. When the values of these two factors are corrected for elevation changes, the constant must also be corrected.

The scale of the aerial photographs used in this study was determined by establishing a baseline, 2,660 feet in length, in the bottom of Fiquette Creek canyon. In Figures 1 and 3 this baseline is designated by the letter "A", and considered the datum plane of the photograph. Another baseline was established on the ridge near the upper boundary of the experimental area. Its length is 1,200 feet. In Figures 1 and 3, this baseline is designated by the letter "B". The purpose of baseline B is to serve as a check on baseline A. Actually, neither of these baselines was measured in the field. Their lengths were scaled from a G.L.O. base map having an allowable error of 1:5000. However, the author did establish a baseline, 460 feet in length, near the top of the ridge.

The photographic scale in the datum was determined from the baseline to be 1:14,760, or 1 inch equals 1,230 feet. The scale at baseline B, 1:12,000, or 1 inch equals 1,000 feet. For a given camera focal length of 0.6875 feet, the difference in elevation of the two baselines, computed on a scale basis, should be 189.3 feet. From a topographic map of the area the difference in elevation was found to be 1,800 feet; an error of 98 feet. The scale at the baseline
measured in the field by the author is 112,120. This baseline was located about two hundred vertical feet down from
the ridge top.

The absolute parallax base is defined as the distance between camera stations of two successive, or stereo-pair,
photographs, expressed in units of photographic distance. The actual value of this distance remains constant. It is
only its photographic measurement which varies. For example, consider the two baselines, A and B, in the above paragraph.
The actual distance between camera stations of the two aerial used in this study is 3,300 feet. The absolute parallax base
for baseline A would be 2.68 inches, and for baseline B, 3.30 inches; a difference of 0.63 inches. If this discrepancy
was not corrected from A to B, a large error in height measurements at B would result.

The usual practice is to substitute the average distance between the principal and conjugate principal point of a
stereo-pair of photographs for absolute parallax base. In this example the distance between principal and conjugate
principal points is 2.96 inches. If this value were substituted for absolute parallax at either A or B, a significant error
would occur in height measurement at both points. For a 100-
foot tree, the error at point A would be a minus 18 percent,
and at point B, a plus 14 percent.
The elevation differential of the study area is approximately 1,200 feet. In order to obtain precise measurements of crown diameter and total height, scale and absolute parallax base values were corrected every 200 vertical feet. Whether or not this precision was necessary is difficult to determine because other limiting factors present cannot be closely evaluated. The methods and formulae used in this study to determine scale and absolute parallax base are found on pages 1 to 7 of the appendix.

The parallax measuring devices used in this study to measure tree height are the Abrams Academy Height Finder, designated the HF-2 (12), and the Harvard parallax wedge. (6) Because of its faster operation and greater accuracy, the HF-2 was used to make all height measurements in the aerial cruise. The only use made of the parallax wedge was in checking some of the HF-2 measurements.

Before either the HF-2 or parallax wedge can be used to measure tree heights, a constant for each must be computed. This constant is based on HF scale and absolute parallax base values. In the case of the HF-2, the constant is expressed as so many vertical feet of height per one-hundredth of a millimeter of parallax difference, or differential parallax.

1 Differential parallax is defined as the difference between the absolute parallax of the base of a photographic image, and the absolute parallax of the top of the same image. This quantity is usually very small, being measured in thousandths of an inch, or hundredths of a millimeter.
For the parallax wedge the constant is expressed as so many feet of vertical height per one-thousandth of an inch of parallax difference. The constant for the HF-2 may be determined by means of a circular slide rule, called a photogrammetric computer. (12) To solve for the constant with this device, the absolute parallax base and the flying height of the camera station above the base of the object being measured must be known. The constant for the parallax wedge may be found in a published table of constants, or else computed by the parallax formula. The solution of this formula is found on pages 7 to 9 of the appendix.

Another method was used in this study to determine tree heights which is not based on parallax measurements. In this method length of shadow is used to determine height. If the altitude of the sun at time of photography is known, height can be determined by application of basic trigonometric functions. Data needed in order to determine the sun's altitude are (1) time of photography, (2) date of photography, and (3) longitude and latitude of photography. The declination of the sun and the equation of time on the date of photography may be obtained from a solar ephemeris. Solution for the sun's altitude is found on pages 11 to 14 of the appendix.

This method of determining tree height was used in this study because in some instances the tree was not visible, but its shadow was.
The determination of tree height by the shadow method was complicated by the fact that in every case the shadow was cast on slopes of 30 to 60 percent. Consequently, before any tree height could be computed, the gradient of the slope on which the shadow fell had to be known. The only way in which slope could be determined was by measuring it with the HF-2.

Crown diameter measurements were made in this study by means of the crown diameter dot scale (24). This device was developed by the Central States Forest Experimental Station, U. S. Forest Service, and consists of a series of dots printed on a plastic transparency. Each successive dot is 0.0025 of an inch larger than its predecessor. To determine crown diameter the scale was placed on the aerial photograph and viewed under a stereoscope. The dots were compared with the tree crown, and the dot selected which appeared to be of the same dimension as the crown. Size of dot selected was noted and the crown diameter was determined by converting dot value to photographic scale units. Another device, the crown diameter wedge, was used as a check and for measuring shadow lengths.

**Type, Density, and Site Classification**

*Figure 5 illustrates the type and density classification used to segregate the study area into blocks of timber having*
a high stand homogeneity. If the map in this figure is compared with the aerial photograph shown in Figure 1 the classification system is more apparent. For the purpose of identification and reference each type and density class is designated by a Roman numeral.

There are only two distinct types found on the study area. One consists mainly of ponderosa pine, and the other is a heterogeneous mixture of ponderosa pine and Douglas fir; the former being the predominant species. The ponderosa pine-Douglas fir type occurs on northwest slopes and in canyon bottoms. The pure ponderosa pine type occurs on south and west exposures. Sections III and V in Figure 5, are examples of the former type, and the other sections, of the latter type.

The pure ponderosa pine type has been segregated on the basis of density, and to a lesser extent, site. However, some of the sections include two, and sometimes three, of the general site classifications. For example, section I includes all three site classes. Section II, canyon and slope sites; while section VI is classified as part slope site, and part exposed site. Only the slope site classification occurs on section IV; while section VII is classified as exposed site.

The basis of this classification system is, more or less, arbitrary. For all practical purposes the pure ponderosa pine type on this area could be listed under one density class, as
the whole area has a stand density of ten percent or less. However, from ground observations it was apparent that some segregation of the timber was necessary if a high stand homogeneity was to be achieved. Because the demarcation lines were obscure no attempt was made to determine the general boundaries of each class in the field. All typing was done from aerial photographs.

In some cases the boundaries were obvious. For example, the ridge and canyon boundaries of sections III and V could be readily identified on both the ground and aerial photographs. The determination of the east or upper, boundaries of these two sections was more difficult; especially for section V. To determine these boundaries required a close correlation between field and phot observations. Once sections III and V were delineated, the boundaries of sections IV, VI, and VII were readily identified.

On the aerial photograph shown in Figure 3, sections I and II appear to be of the same density and site class. However, there is a wide variation in timber characteristics exhibited by these two sections. For example, the average individual tree volume for section I is 1,068 feet, b.m., and for section II, 1,246 feet, b.m. Although section I has larger mature trees than section II, it also has a greater proportion of pole size timber. This accounts for the smaller average volume value. Another reason for separating these
two sections, is that section I has well defined boundaries. This factor facilitates both aerial and field cruises.

**Preparation of Base Map from Aerial Photographs**

The planimetric maps of the study area shown in Figures 3 and 5 were prepared from aerial photographs #0-1-55 and #0-1-56 of the Piquette Creek aerial survey. Ground control for preparing the maps was obtained from a G.L.O. base map having an allowable error of 1:5,000. The scale of the base map is 1:12,000. Control points on the base map were transferred to the aerial photographs during the printing process.

These aerial photographs were placed on a Kail Radial Line plotter and the desired scale obtained by adjusting the linkage of the instrument until the control points on the photograph corresponded with the same control points on the G.L.O. base map. Terrain features, such as ridges and creeks, and area and type boundaries were then transferred from the aerial photographs to the base map at a scale of 1:12,000, or 1 inch equals 1,000 feet.

**Area Determination**

The area of each type and density class, or section, and of the whole area was determined by means of a planimeter. Each measurement was repeated five times and an average value
TABLE IV
AREA OF EACH TYPE AND DENSITY CLASS

| Section | Area
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Planimeter</td>
</tr>
<tr>
<td></td>
<td>acres</td>
</tr>
<tr>
<td>I</td>
<td>18.47</td>
</tr>
<tr>
<td>II</td>
<td>13.03</td>
</tr>
<tr>
<td>III</td>
<td>2.73</td>
</tr>
<tr>
<td>IV</td>
<td>6.86</td>
</tr>
<tr>
<td>V</td>
<td>10.74</td>
</tr>
<tr>
<td>VI</td>
<td>20.01</td>
</tr>
<tr>
<td>VII</td>
<td>12.63</td>
</tr>
<tr>
<td>Total</td>
<td>93.37</td>
</tr>
</tbody>
</table>
computed. Area of each class was also determined by a plastic dot grid. As in the case of the planimeter, five measurements were made of each class, with the grid being moved to a new position each time. The results of these measurements are found in Table IV.

**Species Identification**

Species identification is one of the major factors preventing a large-scaled application of aerial inventories in the Northern Rocky Mountain Region. Timber types found in this region are, for the most part, heterogeneous, with as many as six different coniferous species and one deciduous species composing a single type (White Pine Type, *Pinus monticola*, D. Don.). Most of the coniferous species found in such a type have similar crown characteristics which makes identification difficult, if not impossible, on medium-scaled aerial photographs. Another factor complicating species identification is the wide variation in site found in mountainous terrain. In a given general forest type a large number of "micro-climates" will exist. These areas will not only vary in composition, but density, form class, tree height, and crown characteristics.

Since the relative timber values of the individual tree species are so diversified, it is important that the true volume of each be known. For example, the present stump-
the value of white pine is about 4 dollars per thousand board feet. Its photographic indiscernible associates, headlock (Juniperus heterophylla (Linn.) var.) and grand fir (Abies grandis Lind.), sell for about 2 dollars per thousand board feet. In such a case an accurate volume estimation must be made of each species.

A similar difference in value exists between ponderosa pine and douglas fir. Consequently, when making an inventory of a timber stand composed of these two species it is imperative that the true volume of each be known. Since ponderosa pine and douglas fir have similar tree and crown characteristics, these two species are difficult to distinguish from each other, even on large-scaled photography.

In this study it was found that ponderosa pine could be distinguished from douglas fir by the application of the following factors: (1) site, (2) photographic tone of the crown, and (3) crown characteristics.

The value of site in distinguishing ponderosa pine from douglas fir on aerial photographs is indirect. Its only use is in indicating the most likely areas where these two species will be associated; not in helping distinguish one individual tree from another. In eastern montana, it is common silvicultural knowledge that douglas fir seldom occurs on dry
south and west exposures which ponderosa pine favors. However, in a general ponderosa pine type, Douglas fir can always be expected to associate with this species wherever soil moisture conditions are favorable. Such places as canyon bottoms, ridge tops between north and south exposures, and northwest exposures will usually have a mixture of Douglas fir and ponderosa pine.

On the study area Douglas fir was an associate of ponderosa pine on sites which follow the above general pattern. In sections III and V (Figure 5), which are northwest exposures, Douglas fir of commercial size was generally distributed over the area. Large clumps of Douglas fir reproduction are found in section III. In sections I and II, Douglas fir is found scattered along the canyon bottom.

The use of photographic tone, or degree of color (black in this case), is of doubtful value as an aid in distinguishing ponderosa pine from Douglas fir on aerial photographs. From ground observation a difference in foliage color between these two species is apparent, if not obvious. The foliage of Douglas fir is a dull blue-green, while that of ponderosa pine is a dark yellowish green. Perhaps, more important than foliage color in influencing photographic tone is the density of crown. Because of its denser crown, Douglas fir should photograph in darker tones than ponderosa pine, even if the foliage of both species were the same color. The practical
application of this factor in species identification is limited because, as far as the author is concerned, the degree of tonal difference is not great enough for positive identification. This conclusion is based on observing trees of known species on high quality panchromatic photographs of 1:12,000 scale.

The most important factor in distinguishing ponderosa pine from Douglas fir on aerial photographs is crown characteristics. In a previous paragraph it was stated that both species have similar crown types. This is only true in the case of mature trees. The greatest difference in crown characteristics occurs in young trees and, to a lesser degree, in overmature trees. Although young trees of both species tend to have conical crowns, that of Douglas fir is much more prominent. However, a limiting factor which must be considered in the application of this characteristic is the scale of photography. If the scale is not sufficiently large enough to resolve the top portion of the crown this identifying characteristic will have been lost. In this study young Douglas fir trees could be readily identified by their conical crowns. Over-mature Douglas fir trees are readily identified by their star-shaped crowns which have a fuzzy appearance around the edges. If trees of this species and age class are growing in the open, the crowns will appear to be growing the full length of the bole. Crowns of mature and over-mature ponderosa pine trees appear as a blob on top of a thin stick.
In section V of the study area most Douglas fir trees could be identified. In section III only a small proportion of the trees of this species could be distinguished from ponderosa pine. In this section ability to distinguish between species was impeded by the stand density and dark background. Identification of Douglas fir in sections I and II was not difficult. Due to the relatively large size of the individual trees and small confined area of occurrence, this species could readily be distinguished from ponderosa pine. In section VI there are a few widely scattered young-mature Douglas fir trees. These trees are approximately the same size, and have similar crown characteristics as the surrounding ponderosa pine. The writer knows the approximate locations of these trees, yet is unable to identify them after a detailed stereoscopic examination of the known areas.

**Estimation of Cull Factor**

No attempt was made in this study to determine cull factor. Both the ground and aerial cruises were only concerned with gross per-acre volumes. However, a few observations were made of this factor which may have practical application in aerial inventories of ponderosa pine stands.

Most of the mature ponderosa pine trees on the study area have "cat faces", or old fire scars. Ground observation of these scars indicated that between one-half, and all of
the butt log of trees having a fire scar would have to be culled. A prominent characteristic of these trees is that they had a very pronounced flare at the base.

In a subsequent stereoscopic examination of the aerial photographs many of these trees could be identified by the flare at the base. By the application of an average cull factor, determined by ground measurements, the net volume of trees with this defect could be estimated.

Another factor noticed in connection with defect in ponderosa pine is that a diseased tree of this species will tend to have a larger and denser crown, and be shorter than its surrounding associates.

Aerial Cruises

The aerial cruises of the study area were designed to test the validity of the different tree-aerial volume table solutions discussed in Chapter III. To initiate the aerial cruises, four sections were selected on the study area which have different combinations of site classifications. The sections selected are I, II, V, and VI (Figure 5). Aerial cruises of varying intensities were applied to each section and the volume determined, individually, by each volume table solution. All measurements of height and crown diameter were classified according to the three general site classifications discussed in Chapter II. Topographic location, exposure, and total height-crown diameter ratio were the basis for
classifying photographic site. The Abrams 40-2 and crown diameter dot scale were used to determine height and crown diameter values in all of the aerial cruises.

A final aerial cruise was made of the entire study area. As in the small highly controlled cruises, all three volume table solutions were used to estimate total volume. These results were compared with those of the sections to determine which type of cruise, general or highly controlled, produced the most accurate results.

Section I was selected as the first test area. This section has well defined boundaries, a range in site, covering all three classifications, and variable tree characteristics. Three 100 percent, and two 50 percent aerial cruises were made to determine the volume of ponderosa pine on this section. The purpose of three 100 percent cruises was to determine the degree of consistency in classifying site and the relative accuracy of height and crown diameter measurements.

The first 100 percent cruise was made a year prior to the other cruises. At that time, the author had used the instruments for measuring photographic height and crown diameter only a short time and was not fully aware of the errors involved. For example, there is a tendency to over estimate the crown diameter because of the proximity of the crown
shadow. This error was subsequently identified and corrected.

The second aerial cruise made of section I was a 50 percent representative random sample,\(^1\) using one-half acre square plots. A 100 percent aerial cruise was made in conjunction with this sample by measuring the residual trees on the area. This was for the purpose of determining whether or not the 50 percent sample was representative of the entire area. Another 50 percent sample was made of section I, using a systematic random selection,\(^2\) instead of a representative random selection as the basis for determining the location of each plot. These two types of inventory systems were compared, and the one giving the most representative coverage determined. A 100 percent cruise was also made in conjunction with the 50 percent systematic sample by measuring the residual trees on the area.

All three volume table solutions were used to estimate, individually, the volume of this section for each of the above cruises. Results and comparison of the different aerial cruises are found in Table V.

---

1 By representative random sample is meant that the location of every plot was determined by random selection.

2 By systematic random sample is meant that only the first cruise line, and the first plot in each line are located by random selection.
TABLE V
PONDEROSA PINE

RESULTS OF THE AERIAL CRUISES OF SECTION I

<table>
<thead>
<tr>
<th>Aerial cruise</th>
<th>No. of measurements</th>
<th>No. of plots</th>
<th>Volume table solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>3-Variable for volume</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>per-acre volume, ft. c. b.m.</td>
</tr>
<tr>
<td>100A</td>
<td>274</td>
<td>-</td>
<td>19,182 ± 0</td>
</tr>
<tr>
<td>100A</td>
<td>269</td>
<td>-</td>
<td>15,514 ± 0</td>
</tr>
<tr>
<td>100A</td>
<td>266</td>
<td>-</td>
<td>15,513 ± 0</td>
</tr>
<tr>
<td>50A</td>
<td>119</td>
<td>19</td>
<td>14,732 ± 1656</td>
</tr>
<tr>
<td>50A</td>
<td>137</td>
<td>19</td>
<td>15,254 ± 1480</td>
</tr>
</tbody>
</table>
In order to determine the accuracy with which Douglas fir could be distinguished from ponderosa pine on aerial photographs, a series of aerial cruises were made of sections II and V. On both of these sections, Douglas fir of commercial size forms a significant component of the stand. However, the occurrence of this species differs considerably in each of the two sections. For example, in section II Douglas fir is confined to sites along the canyon bottom; while in section V, this species occurs generally throughout the entire area. Consequently, because of the limited site range of this species on section II, the problem of species identification should be easier than that of section V. Another factor to consider in comparing these two sections is the relative size of the individual trees. The average Douglas fir tree occurring on section II has a much larger d.b.h. and crown diameter, and a greater height than the average tree of this species found on section V. The density of the stand where Douglas fir occurs in section II is also much higher than the stand density of section V.

The aerial cruises of section II consisted of a 100 percent sample and a 50 percent representative random sample in which one-half acre square plots were used. The purpose of these two cruise intensities was to determine if the random sample was representative of the entire area. The 100 percent
cruise was used as control in this case to determine the accuracy of the 50 percent sample.

The volume of ponderosa pine on section II was determined, individually, by each of the three volume table solutions. Douglas fir volume was determined in an identical manner. Results and comparison of these aerial cruises are found in Table VI for ponderosa pine, and in Table VII for Douglas fir.

Section V differs from section II in that the exposure of the former is northwest, while that of the latter is southern. Consequently, the soil moisture content will be higher on section V. This will tend to increase site quality and provide favorable growing conditions for Douglas fir. Because of its exposure, most of this section is classified as a canyon site. Only a small portion of the area near the ridge top is classified as a slope site. Unlike section II, where Douglas fir is limited to a small area, the occurrence of this species in section V is general throughout the area.

The identification of Douglas fir on aerial photographs was relatively easy in the canyon bottom of the North Fork of Benchmark Creek. However, where this species was associated with ponderosa pine on the slope, the problem of identification became increasingly difficult. On this area Douglas fir had similar crown characteristics as that of ponderosa
TABLE VI
FOND GACA PINE

RESULTS OF THE AERIAL CRUISES OF SECTION II

<table>
<thead>
<tr>
<th>Aerial Cruise</th>
<th>No. of measurements</th>
<th>No. of plots</th>
<th>3-Variable for Volume</th>
<th>3-Variable for d.b.h.</th>
<th>Coefficient &quot;all sites&quot;</th>
<th>Coefficient &quot;ind. sites&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100%</td>
<td>218</td>
<td>-</td>
<td>17,522 ± 0</td>
<td>16,982 ± 0</td>
<td>17,588 ± 0</td>
<td>17,304 ± 0</td>
</tr>
<tr>
<td>50%</td>
<td>116</td>
<td>13</td>
<td>21,302 ± 2045</td>
<td>20,462 ± 1842</td>
<td>20,746 ± 1846</td>
<td>20,636 ± 1857</td>
</tr>
</tbody>
</table>
TABLE VII

DOUGLAS FIR

RESULTS OF THE AERIAL CRUISES OF SECTION II

<table>
<thead>
<tr>
<th>Aerial cruise</th>
<th>No. of measurements</th>
<th>No. of plots</th>
<th>Volume table solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>3-Variable for volume</td>
</tr>
<tr>
<td>100%</td>
<td>20</td>
<td>-</td>
<td>482 ± 0</td>
</tr>
<tr>
<td>20%</td>
<td>5</td>
<td>13</td>
<td>270 ± 127</td>
</tr>
</tbody>
</table>
pine, was approximately of equal average height, and did not conform to any general pattern of distribution. It was also on the slope area that Douglas fir trees having the greatest volume content occurred. Most of the trees found in the canyon bottom were pole size and contributed little towards the total Douglas fir volume of this section.

Only one aerial cruise was made of section V. This cruise consisted of a 50 percent representative random sample, using one-half acre square plots. The 100 percent control cruise was abandon in this case because the purpose it was supposed to serve was of doubtful value. Number of trees and inability to determine which trees along the boundaries fell in the area were also contributing factors in abandoning the 100 percent sample.

The volume of ponderosa pine on section V was determined, individually, by each of the three volume table solutions, Douglas fir volume was determined in an identical manner. Results and comparison of these aerial cruises are found in Table VIII for ponderosa pine, and Table IX for Douglas fir.

In order to determine the effect an area having only slope and exposed site classifications would have on the accuracy of volume estimation by the different volume table solutions, an aerial cruise was made of section VI. This section is divided about equally between slope and exposed site clas-
### TABLE VIII

**CONIFEROSA PINE**

**RESULTS OF A FIFTY PERCENT AERIAL CRUISE OF SECTION V**

<table>
<thead>
<tr>
<th>Aerial cruise measurements</th>
<th>No. of plots</th>
<th>Volume table solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>3-Variable for volume</td>
</tr>
<tr>
<td>50%</td>
<td>90</td>
<td>22,711 ± 2952</td>
</tr>
</tbody>
</table>

per-acre volume, feet, b.m.
TABLE IX

DOUGLAS FIR

RESULTS OF A FIFTY PERCENT AERIAL CRUISE OF SECTION V

<table>
<thead>
<tr>
<th>Aerial cruise</th>
<th>No. of measurements</th>
<th>No. of plots</th>
<th>Volume table solution</th>
<th>Coefficient per-acre volume, feet, b.m.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>3-Variable</td>
<td>3-Variable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>for volume</td>
<td>for d.b.h.</td>
</tr>
<tr>
<td>50%</td>
<td>31</td>
<td>11</td>
<td>1658 ± 481</td>
<td>1609 ± 434</td>
</tr>
</tbody>
</table>
ifications and only ponderosa pine occurs on the area. The main problem in cruising this section was in determining the demarcation line between these two site classifications. The transition from one site to the other is so gradual that identification is difficult. This factor is readily illustrated if the aerial photograph shown in Figure I is examined. The final site classification of this area was based on topographic location and the application of the height-crown diameter ratio graph shown in Figure 4.

A 50 percent representative random sample cruise, using one-half acre square plots was made of section VI. The per acre volume of ponderosa pine on this section was determined, individually, by each of the three volume table solutions. Results of this aerial cruise are found in Table X.

The final aerial cruise was a 20 percent representative random sample, using one-half acre square plots, of the entire study area. The purpose of applying a 20 percent intensity in this cruise, instead of the 50 percent intensity as used in the other aerial cruises, was to determine a practical cruise intensity for use in aerial inventories of the ponderosa pine type. A cruise of 50 percent intensity, either field or aerial, is too laborious and time consuming to be of any practical value. Also, the increase in accuracy is not worth the additional work. For example, the standard error of a 50 percent cruise of the ponderosa pine stand surveyed in this
### TABLE X

**Ponderosa Pine**

RESULTS OF A FIFTY PERCENT AERIAL CRUISE OF SECTION VI

<table>
<thead>
<tr>
<th>Aerial measurements</th>
<th>No. of plots</th>
<th>No. of plots</th>
<th>Volume table solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>S-Variable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>for volume</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>per-acre volume, feet, b.m.</td>
</tr>
<tr>
<td>50'</td>
<td>69</td>
<td>18</td>
<td>20,300 ± 1627</td>
</tr>
</tbody>
</table>
study is 4.6 percent. The standard error of a 20 percent
cruise of the same stand is 6.9 percent. This difference is
insignificant if other limiting factors present in inventory
work are considered.

A representative random sample was obtained of the
study area by placing a dot grid on an aerial photograph of
the area. Each dot falling on the area was assigned a number.
These numbers were written on a slip of paper and placed in
a box. In order to obtain a truly representative selection,
each section was weighted according to the timber volume found
on that particular section. For example, section VII was
assigned the weight of one. This means that the number of
each dot falling on this area was only recorded once. Sections
I, II, V, and VI were assigned a weight of three; and sections
III and IV, a weight of four. A dot falling on either section
III or section IV had four times the chance of being selected
than a dot falling on section VII. These numbers were then
drawn at random and the position of each dot corresponding
to the drawn number was permanently recorded on the aerial
photograph by means of a needle. The drawn number was re-
placed in the box each time. This procedure was repeated
until the required number of plots were selected. In this
cruise a total of 38 one-half acre plots were needed to obtain
the desired 20 percent cruise intensity.

As in the preceding sectional aerial cruises, all three
volume table solutions were used to estimate, individually, the volumes of both ponderosa pine and Douglas fir. Results of this aerial cruise are found in Table XI for ponderosa pine, and Table XII for Douglas fir.

**Comparison and Summary of Aerial Cruises**

An examination of the tabulated results of the aerial cruises reveals that there is no orderly sequence in volume estimation by the various volume table solutions. In one cruise the coefficient "all sites" solution may estimate the highest volume value; while in another cruise, the 3-variable solution for volume will give the highest estimate. This is as it should be. Each of the aerial cruises discussed above was made of a section having different site combinations and tree characteristics than the other sections. Trees growing on different sites and exposures will exhibit variation in height and crown diameter, which in turn will be reflected in the final volume values obtained by each volume table solution. These solutions were prepared by different statistical systems and, as a result, the volume weight assigned to a certain height and crown diameter class will differ in each case. However, if the aerial sample is truly representative of the entire population the end results should be the same for each volume table solution.

As stated in a previous paragraph, the purpose of making a highly controlled aerial cruise of only one section
TABLE XI
PONDIROSA PINE
RESULTS OF A THIRTY PERCENT AERIAL CRUISE OF ALL SECTIONS

<table>
<thead>
<tr>
<th>Aerial cruise measurements</th>
<th>No. of plots</th>
<th>No. of plots</th>
<th>Volume table solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>3-Variable</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>for volume</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>per-acre volume, feet, b.m.</td>
</tr>
<tr>
<td>20%</td>
<td>283</td>
<td>38</td>
<td>18,709 ± 1740</td>
</tr>
</tbody>
</table>
TABLE XII

DOUGLAS FIR

RESULTS OF A TWENTY PERCENT AERIAL CRUISE OF ALL SECTIONS

<table>
<thead>
<tr>
<th>Aerial cruise measurements</th>
<th>No. of plots</th>
<th>Volume table solution</th>
<th>Coefficient for d.b.h.</th>
<th>&quot;all sites&quot; per-acre volume, feet, b.m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>105</td>
<td>16</td>
<td>38</td>
<td>349 ± 146</td>
<td>258 ± 116</td>
</tr>
</tbody>
</table>
at a time was to test the aerial volume table solutions under varying conditions of site, exposure, and density. For example, in section I, 20 percent of the area is classified as "canyon" site; 20 percent "exposed" site; and 60 percent "slope" site. This section provided a representative segment of the entire study area, although not exactly in the same proportions. In section II, 20 percent of the area is classified as "canyon" site, and the remainder, "slope" site. The exact opposite is found in section V, where 80 percent of the area is classified as "canyon" site, and the remaining 20 percent as "slope" site. Section VI is about equally divided between slope and exposed site classifications. In order to determine the reason why one aerial volume table will estimate the volume at a particular area differently from that of another volume table, a direct comparison was made of the various volume table solutions. The coefficient "all sites" solution was used as the basis for the comparison. Each of the other solutions was compared with the base, and the feet, B.M. and percent differences noted. Results of this comparison are found in Table XIII. Actually the value of a comparison, like that shown in Table XIII, is misleading. The figures listed in this table have as their basis a comparison of each and every height and crown diameter class volume value. However, the difference between these various values will vary from one height class
### TABLE XIII

**FORDROSA MINE**

**COMPARISON OF THE VARIOUS ALKALI VOLUME TABLE SOLUTIONS, USING THE COEFFICIENT "ALL SITES" TABLE AS THE BASIS FOR COMPARISON**

<table>
<thead>
<tr>
<th>Volume table solution</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Feet, B.M.</td>
</tr>
<tr>
<td>Coefficient &quot;all sites&quot; solution</td>
<td>0</td>
</tr>
<tr>
<td>Coefficient &quot;canyon sites&quot; solution</td>
<td>+5730</td>
</tr>
<tr>
<td>Coefficient &quot;slope sites&quot; solution</td>
<td>+3563</td>
</tr>
<tr>
<td>Coefficient &quot;exposed sites&quot; solution</td>
<td>+14,380</td>
</tr>
<tr>
<td>3-Variable solution for d.b.h.</td>
<td>+8120</td>
</tr>
<tr>
<td>3-Variable solution for volume</td>
<td>-10,572</td>
</tr>
</tbody>
</table>

* Coefficient "canyon sites" table over estimates coefficient "all sites table by 3.4 percent.
to the next. For example, further analysis of the comparison of the coefficient "all sites" table and 3-variable solution for volume table reveals that in the 55-foot to 85-foot height classes, the latter solution over estimates the former, while in the 115-foot to 145-foot height classes, the reverse is true to a far greater degree.

Therefore, it does not necessarily mean that the coefficient "all sites" table will over estimate the 3-variable solution for volume table by four percent when both are applied to a specific aerial cruise. What the final volume results of each table will be depends on the distribution of measurements in the aerial cruise. If most of the measurements should fall within the 55-foot to 85-foot height-class limits, the 3-variable solution for volume table will over estimate the coefficient "all sites" table. And if the majority of measurements fall within the 115-foot to 145-foot height-class limits, the reverse will be true.

Both of these tables, however, have as their statistical basis the compensating error principle; it is only in the manner in which this principle is applied which differs. An excellent example of the compensating error factor, in regards to these two volume table solutions, is illustrated in Table VI. In the 50 percent random sample aerial cruise of section II, a total of 115 trees were measured. Of this
total figure, 67 trees fell within the 55-foot to 85-foot height-class limits, where the 3-variable solution for volume over estimates the coefficient "all sites" solution; and 48 measurements fell within the 95-foot to 145-foot height-class limits. The proportion of measurements falling within each group was sufficient to balance the total volume figures.

What is trying to be brought out in the above paragraphs is that the principal factor in this study is the compensating error. It could even be said that an aerial inventory is a "comedy of compensating errors." However, without its influence an accurate volume estimate on aerial photographs would be impossible. The author believes that this factor is excellently illustrated in the above table summarizing the various aerial cruises of the study area. Each of the sections cruised presented a wide range of different site and timber conditions. The statistical solutions used to prepare the volume tables from which the volumes of these sections were estimated bore no mathematical similarity. Yet, the results of each solution, for a given aerial cruise, vary, on the average, only 630 feet, b.m. per acre from the highest to the lowest estimate. Only the sequence from high to low volume estimates of the solutions changed. Over the limits of accuracy computed for each random sample aerial cruise were identical for each volume table solution within a given cruise. For example, the limits of accuracy computed for each
volume table solution used to estimate volume in the 20 percent random sample aerial cruise of the study area are: (1) for the 3-variable solution for volume, 9.3 percent; (2) 3-variable solution for d.b.h., 9.5 percent; (3) coefficient "all sites" solution, 9.3 percent; and (4) coefficient "individual sites" solution, 9.4 percent. This example again illustrates the importance of the compensating error, because considerable variation was exhibited in the estimation of a single plot's volume by the various solutions.

In view of the above results it is the opinion of the author that the main limiting factor in this study is not the aerial volume table, but rather the ability of the photo interpreter to accurately measure tree height and crown diameter on aerial photographs.
CHAPTER V

FIELD INVENTORY

A field survey of the Benchmark Creek study area was
necessary to establish the total volumes of both ponderosa
pine and Douglas fir. The results of this survey were con-
sidered the true volume values and used as the basis on which
the accuracy of each aerial cruise was judged. In order to
initiate the field survey it was divided into the following
phases: (1) office preparation, and (2) field measurements.

Office Preparation

To obtain an adequate control cruise certain statis-
tical factors had to be considered before the cruise was made.
The most important of these factors was what should be the
allowable standard error. Because of other limiting factors
present in the actual cruise, i.e., estimating d.b.h., mer-
chantable height, and plot diameter, the author felt that a
too precise standard error would be a waste of time and effort.
It was finally decided that a standard error of five percent
was sufficiently precise for all practical purposes.

To determine the cruise intensity necessary to obtain
a standard error of 5 percent, a statistical analysis was
made of the plots used in the collection of the aerial volume
table data. Although this sample is limited in scope, it is
a random selection and representative of the area. The analysis of this sample indicated that a cruise intensity of 35 percent was necessary in order to obtain a standard error of 5 percent. A margin of error was introduced by increasing the cruise intensity to 50 percent. To achieve this intensity, 233 one-fifth acre plots, randomly distributed over the area, were necessary.

With this many plots a representative random sample is too difficult to control in the field. It requires that the center of each plot be located on the aerial photographs by random selection and then transferred to the ground when making the actual field cruise. Much time is consumed in the transfer process, and the photographic and ground locations do not always correspond with each other. However, if only a few plots were to be sampled, this method would suffice. Instead of the representative random sample, a systematic random sample was used. This method of sampling timber is similar to the regulation "line-plot" field cruises, where there is a fixed, or constant, interval between the cruise lines, and between the individual plots on each line. In the systematic type of cruise the interval between cruise lines, and the individual plots on each line is determined by the number of plots needed, and the size of the area to be cruised. In order that each plot be located by chance, the first line, and the first plot in each line, are randomized. All other
cruise lines and plots are a fixed distance apart. Although the statistical validity of this method is questionable, there is still no control over the location of each plot. Besides being easier to control in the field, this method provides a more thorough coverage of the area being cruised.

The spacing of the cruise lines, and the individual plots, was first planned on the planimetric map found in Figure 5. The starting point selected was the forks of benchmark Creek, and the direction of the cruise lines was perpendicular to the spur ridges, or a north-south bearing. Once the spacing had been determined, the position of the first line, and the first plot in each line, was obtained by random selection. Each plot was then located on the planimetric map to determine if the correct number of plots were obtained. In order to expedite the field cruise, the location of each plot was transferred from the planimetric map to the aerial photograph found in Figure 1.

**Field Measurements**

The actual field cruise of the study area was similar to that of a regulation line-plot cruise. The only difference being, other than the random selection of the first cruise line, that the line-plot cruise is controlled by G.L.O. surveyed section lines, while this cruise was controlled entirely by aerial photographs. The location of each cruise
line was first determined on the photograph and then transferred to its corresponding field location. The location of the first plot in each line was also determined in a similar manner. After a cruise line and the first plot had been established in the field, each successive plot on that particular line was located by staff compass and "stick" pacing. This procedure was for the purpose of saving time. The transfer of photographic location to its corresponding field location is sometimes a tedious and time-consuming job; especially in areas having an obscure ground cover similar to that found in section III (Figure 1). Also, this transfer is not always as accurate as it should be. Because of similar topographic and timber characteristics an appreciable error is possible in transferring photographic location to its corresponding field location. The usual practice was to check the compass and pacing location of the last plot in each line against its photographic location and adjust for any error present. The accuracy of this method was found to be sufficient for all practical purposes. In this cruise the number of field plots was only one less than the original number of plots called for.

Once the location of a plot and its boundary were determined, every tree of commercial size falling within the plot area was measured for d.b.h. and merchantable height.
Merchantable height in this case being the height to a 2½-inch top. At least one tree on the plot was measured by tape and percent abney to accurately determine its merchantable height. This tree was strategically located and was used as control from which the merchantable heights of other trees occurring on the area were estimated. The d.b.h. of every tree of commercial size on the plot was measured by a steel diameter tape.

Each plot cruised in the field was listed according to the section on which it occurred. Occasionally it was necessary to divide a plot between two of the sections. This was only done on a 50 percent basis; that is, one-half of the plot was considered to fall on one section, and one-half on the other section.

The volume of each plot was estimated by conventional ponderosa pine and Douglas fir volume tables (M-1067-R1). Average plot volume, by species, for each section was then determined. To obtain the average volume per acre for the section, the average plot volume of that section was multiplied by five (one-fifth acre plots). This procedure was followed in determining the per-acre volume of each section. The total volume of the study area was determined by finding the average volume for all plots and multiplying this figure by the plot size proportional factor.

A statistical analysis was made of the plot volumes
by sections, and for the whole study area, to determine the limits of accuracy of this field cruise. The results of this analysis are found in Table XIV.

Since a 50 percent random sample cannot possibly provide a true estimate of volume on the study area, it was necessary to compute limits of accuracy for predicting true volume. The mean per-acre volume of the study area, as established by the field cruise, is 18,992 feet, b.m. (Table XIV). The standard error of 232 cruise plots was computed to be 4.6 percent. Therefore, the true per-acre volume of the area should fall, 2 out of 3 times, within a plus or minus 4.6 percent of the mean volume, or 18,113 feet, b.m. to 19,866 feet, b.m. If the mean per-acre volume established by an aerial cruise of the same area was to fall within these limits, it could be considered a valid volume estimate, even though it did not coincide with the mean volume established by the field cruise.
### TABLE XIV

RESULTS OF A FIFTY PERCENT SYSTEMATIC RANDOM FIELD CENSUS OF THE BROOMMARK CREEK STUDY AREA

<table>
<thead>
<tr>
<th>Section</th>
<th>Area (acres)</th>
<th>Mean per-acre volume Ponderosa Pine (feet, b.m.)</th>
<th>Douglas Fir (feet, b.m.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>15.47</td>
<td>18,684 ± 5939</td>
<td>-</td>
</tr>
<tr>
<td>II</td>
<td>13.03</td>
<td>20,770 ± 1329</td>
<td>475 ± 297</td>
</tr>
<tr>
<td>III</td>
<td>9.73</td>
<td>25,585 ± 2354</td>
<td>2,200 ± 581</td>
</tr>
<tr>
<td>IV</td>
<td>8.86</td>
<td>25,825 ± 2466</td>
<td>90 ± 90</td>
</tr>
<tr>
<td>V</td>
<td>10.74</td>
<td>21,615 ± 2507</td>
<td>2,120 ± 522</td>
</tr>
<tr>
<td>VI</td>
<td>20.01</td>
<td>18,090 ± 2334</td>
<td>295 ± 176</td>
</tr>
<tr>
<td>VII</td>
<td>12.53</td>
<td>7,054 ± 14II</td>
<td>-</td>
</tr>
<tr>
<td>all sections</td>
<td>93.37</td>
<td>18,992 ± 874</td>
<td>6II ± 11E</td>
</tr>
</tbody>
</table>
CHAPTER VI

COMPARISON OF FIELD AND AERIAL INVENTORIES

To determine the accuracy of volume estimations on aerial photographs a comparison was made of each aerial cruise and its corresponding field cruise. The true volume of any section cruised on aerial photographs was considered to be that volume falling within the limits of accuracy established by the statistical analysis of the field cruise. If the per-acre volume obtained by an aerial cruise fell within the established limits of accuracy it was considered to be a valid estimation of volume. In the case of a random sample aerial cruise it was also possible to compute limits of accuracy. Consequently, there exists two overlapping standard errors when a random sample aerial cruise is compared with its corresponding field cruise. Where two such standard errors exist, a wide margin of error is allowed in estimating the true volume of an area. For example, it is possible that the lower limits of the aerial cruise mean volume may fall within the upper limits of the field cruise mean volume.

Such a wide range in allowable error is necessary when estimating timber volumes because there are many variable factors which will influence the accuracy of a timber volume estimation; both in the field and on aerial photographs. For example, the accuracy of a field or aerial cruise will be influenced by such factors as: (1) site, (2) density, (3) expo-
ure, (4) species or type, (5) measurements of height, d.b.h., and crown diameter, (6) determination of plot boundaries, and (7) representative coverage of the sample even though a 100 percent cruise was to be made of an area, some of these factors would still greatly influence the accuracy of the volume estimation.

Although it was not considered practical to make a 100 percent field estimate of the study area, another type of cruise could have been employed which may have proved to be more accurate than the 50 percent sample. In this type of cruise the average tree volume is determined by field measurements, and the number of stems on the area are determined from aerial photographs. The average volume is then multiplied by the number of stems to obtain the total volume of the area. The major limitation to this method is that the correct number of stems on an area is difficult to determine on aerial photographs, unless the boundaries are well defined and the stand is open, or scattered. On the study area only section I was applicable to this method of volume estimation.

The comparison of aerial and field inventories in this study is segregated according to section. A final aerial cruise, made of the entire experimental area, was compared with the combined field volumes of all sections. Areas in which comparisons were made are (1) Section I, (2) Section II,
Comparison Between Field and Aerial Cruises of Section I

Three 100 percent aerial cruises and two 50 percent random sample aerial cruises were made of section I. To determine the accuracy of these aerial samples they were compared with the mean volume value established by a 50 percent systematic field sample of the section. This comparison of aerial and field cruises is further segregated according to the different volume table solutions. The results of these comparisons are found in Table XV for the 3-variable solution for volume, Table XVI for the 3-variable solution for d.b.h., Table XVII for the coefficient "all sites" solution, and Table XVIII for the coefficient "individual sites" solution.

Examination of Tables XV through XVIII reveals a significant discrepancy between the results of the aerial cruises and the field cruise of section I. This discrepancy may be due to any number of factors. For example, the main source of error probably lies in the ability of the author to correctly measure tree heights and crown diameters on aerial photographs; especially the latter. Because of the proximity of the tree shadows, crown diameters are difficult to measure on the aerial photographs used in this study. The side of the crown against the tree shadow is obscure, and in the majority of cases required the edge of the crown to be estimated. Since the tree-
### TABLE XV

**COMPARISON OF THE AERIAL AND FIELD CRUISE ESTIMATES OF POPLAR TREE VOLUME FOR SECTION I**

**THREE-VARIABLE SOLUTION FOR VOLUME**

<table>
<thead>
<tr>
<th>Type of cruise</th>
<th>Mean per-acre volume</th>
<th>Difference from field cruise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Feet, B.M.</td>
<td>Feet, B.M.</td>
</tr>
<tr>
<td>50% field cruise</td>
<td>18.624 ± 1.999</td>
<td>0</td>
</tr>
<tr>
<td>100% aerial cruise</td>
<td>19.182 ± 0</td>
<td>+4.98</td>
</tr>
<tr>
<td>100% aerial cruise</td>
<td>18.514 ± 0</td>
<td>-2.76</td>
</tr>
<tr>
<td>100% aerial cruise</td>
<td>18.513 ± 0</td>
<td>-2.71</td>
</tr>
<tr>
<td>50% aerial cruise</td>
<td>14.732 ± 1.856</td>
<td>-3.952</td>
</tr>
<tr>
<td>50% aerial cruise</td>
<td>15.254 ± 1.480</td>
<td>-3.430</td>
</tr>
</tbody>
</table>
TABLE XVI

COMPARISON OF THE AERIAL AND FIELD CRUISE ESTIMATES
OF TONDuftIPA FINE VOLUME FOR SECTION I

THREE-VARIABLE SOLUTION FOR D. B. H.

<table>
<thead>
<tr>
<th>Type of cruise</th>
<th>Mean per-acre volume</th>
<th>Difference from field cruise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>feet, b.m.</td>
<td>Feet, B.M.</td>
</tr>
<tr>
<td>50% field cruise</td>
<td>18,564 ± 1999</td>
<td>0</td>
</tr>
<tr>
<td>100% aerial cruise</td>
<td>20,000 ± 0</td>
<td>+1316</td>
</tr>
<tr>
<td>100% aerial cruise</td>
<td>15,968 ± 0</td>
<td>-3216</td>
</tr>
<tr>
<td>100% aerial cruise</td>
<td>15,046 ± 0</td>
<td>-3638</td>
</tr>
<tr>
<td>50% aerial cruise</td>
<td>14,832 ± 1864</td>
<td>-3362</td>
</tr>
<tr>
<td>50% aerial cruise</td>
<td>14,652 ± 1443</td>
<td>-4652</td>
</tr>
</tbody>
</table>
### TABLE VII

**COMPARISON OF THE AERIAL AND FIELD CRUISE ESTIMATES OF COMBINE HAYE VOLUME FOR SECTION I**

**COEFFICIENT "ALL SITES" SOLUTION**

<table>
<thead>
<tr>
<th>Type of cruise</th>
<th>Mean per-acre volume</th>
<th>Difference from field cruise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>feet, b.m.</td>
<td>Feet, B.M.</td>
</tr>
<tr>
<td>50% field cruise</td>
<td>18,684 ± 1999</td>
<td>0</td>
</tr>
<tr>
<td>100% aerial cruise</td>
<td>20,126 ± 0</td>
<td>+1451</td>
</tr>
<tr>
<td>150% aerial cruise</td>
<td>15,826 ± 0</td>
<td>-2656</td>
</tr>
<tr>
<td>166% aerial cruise</td>
<td>15,713 ± 0</td>
<td>-2671</td>
</tr>
<tr>
<td>50% aerial cruise</td>
<td>14,988 ± 1814</td>
<td>-3666</td>
</tr>
<tr>
<td>50% aerial cruise</td>
<td>16,362 ± 1469</td>
<td>-3382</td>
</tr>
</tbody>
</table>
### TABLE VIII

Comparison of the Aerial and Field Cruise Estimates of Coniferous Line Volume for Section I

Coefficient "Individual Sites" Solution

<table>
<thead>
<tr>
<th>Type of cruise</th>
<th>Mean per-acre volume (feet, b.m.)</th>
<th>Difference from field cruise (feet, B.M.)</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>50% field cruise</td>
<td>16,684 ± 1999</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100% aerial cruise</td>
<td>20,758 ± 0</td>
<td>+2074</td>
<td>+11.1</td>
</tr>
<tr>
<td>200% aerial cruise</td>
<td>16,145 ± 0</td>
<td>-1529</td>
<td>-15.5</td>
</tr>
<tr>
<td>400% aerial cruise</td>
<td>15,611 ± 0</td>
<td>-2033</td>
<td>-16.2</td>
</tr>
<tr>
<td>80% aerial cruise</td>
<td>15,358 ± 1947</td>
<td>-2252</td>
<td>-17.9</td>
</tr>
<tr>
<td>90% aerial cruise</td>
<td>15,336 ± 1976</td>
<td>-2268</td>
<td>-17.6</td>
</tr>
</tbody>
</table>
aerial volume tables used in this study are very sensitive to
crown diameter changes, an error of one crown diameter class
in only a small proportion of the total number of measurements
is sufficient to cause the discrepancy between the aerial and
field cruises listed in the above tables. This error tends
to be cumulative rather than compensating.

Another possible source of error is in the representa-
tive coverage of the random field cruise. This factor, instead
of the one mentioned in the above paragraph, could be the
major cause of error. Usually a 50 percent sample will, if
representative, give a fairly accurate estimate of volume.
However, in cruising the ponderosa pine type the variability
in the stand may require a higher cruise intensity in order
to obtain an accurate coverage. To determine the relative
accuracy of the 50 percent field sample of section I, another
type of cruise, using a combination of field and aerial tech-
niques was used. This type of cruise employs the average
individual tree volume, determined by field measurements,
and the number of stems occurring on the area, determined by
a photographic count.

On section I, the average individual tree volume, deter-
mined from the 50 percent field cruise, is 1068 feet, b.m.
The average number of stems on the area, determined in conjunc-
tion with the 100 percent aerial cruises (Table V) is 270.
The total volume of the area is then, 270 times 1,068, or 288,360 feet, b.m. On a per-acre basis the volume value is 15,612 feet, b.m. Comparing this figure with the mean per-acre volume obtained by the 50 percent field sample, a difference of 3,072 feet, b.m. per acre is noted. This difference is considered significant and the source of error should be determined.

An examination of the 50 percent field cruise data reveals that 151 measurements were made in this sample. Interpreted in terms of area this means that there are 17.5 trees of commercial size per acre, or 323 trees occurring on the entire section. However, from three photo counts the average total number of trees on section I is found to be 270; a discrepancy of 53 trees. If the average individual tree volume is 1,068 feet, b.m., this error is 53 times 1,068, or 56,604 feet, b.m. On a per acre basis the volume value would be 3,064 feet, b.m. Therefore, if this figure can be assumed correct the adjusted volume of section I is 15,620 feet, b.m. per acre. However, since this is a theoretical correction factor it would be best to modify the above figure and assume that the true volume of the section falls near the lower volume limits established by the statistical analysis of the field sample, or 16,684 feet, b.m.

If this adjusted volume be considered the true volume of section I, the results of the aerial cruises listed in
Tables XV through XVIII are sufficiently accurate for all practical purposes. The average mean volume difference is about 1,000 feet, b.m. per acre, or 18,470 feet, b.m. for the entire area. On the other hand if the computed limits of accuracy of the 50 percent random sample aerial cruises are considered, the volume results of these cruises are the true volume of the area. This is due to the overlapping of the aerial cruises' upper limit and the field cruise's lower limit. For example, the upper limit of the systematic random sample aerial cruise listed in Table XV is 15,254 feet, b.m. plus 1,480 feet, b.m., or 16,734 feet, b.m. The lower limit of the 50 percent field cruise is 13,684 feet, b.m. minus 1,999 feet, b.m., or 11,685 feet, b.m. Thus, the upper limit of the aerial cruise falls 49 feet, b.m. within the lower limit of the field sample.

A more feasible method of comparing volume results of aerial and field cruises is to determine the standard error of the difference. By this method it can be determined if the difference between the two volume values is significant or accidental. Only the results listed in Table XV are used in this determination because results of the other methods exhibit similar variation.

In a 100 percent cruise the mean volume value and the standard deviation may be computed, but not the standard error of the mean. Consequently, the standard error of the difference
between the 100 percent field sample mean volume listed in Table XV cannot be computed. Instead a direct comparison will have to be made between the mean volume values of these cruises.

The standard error of the difference for the mean volume and standard error of the 50 percent representative random sample aerial cruise, and the mean volume and standard error of the 50 percent field sample, expressed as a normal deviate, is ±1.44 standard deviations. The area under the normal curve frequency between a plus and minus 1.44 standard deviations is 0.92432 minus 0.07568, or 0.84964. In the two tails beyond these limits is 1.00000 minus 0.84964, or 0.15036. This means that there are about 18 chances in 100 that the difference is an accident of sampling. This may also be stated: that one chance in six can a greater error be expected than the one shown above.

The standard error of the difference in this case indicates that the difference between the aerial and field cruise means volume values is not due to an accident in sampling. Therefore, only two other factors remain which could cause the difference in mean volumes. The first one is the tree-aerial volume tables; and the second factor is the accuracy of the individual measurements. There is no doubt that part of the difference will be due to the volume tables. However, errors caused by this source tend to be compensating
(Chapter IV), and will have only a slight influence on the final results. The main error, then, is in the individual measurements of height and crown diameter on aerial photographs. These errors, unlike those caused by the volume tables, tend to be accumulative and will be responsible for the greater part of the difference between the volume results of aerial and field cruises.

The standard error of the difference for the 50 percent systematic random sample aerial cruise and the 50 percent field cruise, expressed as a normal deviate, is ±1.37 standard deviations. From the normal frequency curve it is found that the difference is an accident in sampling in only one chance in five. Therefore, it is concluded that the cause of the difference between the results of the two cruises in this case is the same as that for the representative random sample.

The analysis of the aerial and field cruises of section I to determine the cause of the difference between these two types of cruises appears to be contradictory. It was first pointed out that a large discrepancy exists in the total number of trees of commercial size predicted for the section by each cruise. In most cases attempting to prove that the total number of trees predicted for an area by a particular cruise indicates the degree of representative coverage is a fallacy. However, application of this method to section I is sounder than it appears. On this section the stand is of even density.
and trees composing the stand are, more or less, of uniform size. There are a few small areas, mostly in the upper ranges, in the stand which vary in tree size and density. Therefore, any random sample of section I should predict the correct number of commercial-sized trees occurring on this section within reasonable limits.

It is readily apparent, however, that no random sample cruise, either field or aerial, predicted the correct number of trees occurring on section I. For example, the field cruise predicted 323 trees of commercial size; the 50 percent representative random aerial cruise, 231; and the 50 percent systematic random aerial cruise, 266. The number of trees measured in the three 100 percent aerial cruises are 274, 268, and 269 respectively. The average of these three measurements, 270, is assumed to be the true number of trees occurring on section I.

The standard error of the difference determined for these various cruises indicates there is no significant difference between the aerial and field random samples. If this statistical measure is assumed correct, then the error does not lie in the sampling methods, but in either the aerial volume tables, individual measurements of height and crown diameter, measurement of plot boundaries, or in a combination of these factors. This is a direct contradiction of what has been said in the above paragraphs. The direct analysis
indicates there is a discrepancy in sampling, while the statistical analysis repudiates this theory. Because of the many limiting factors present in this study the author accepts the results of the statistical analysis and assumes that the difference in volume values is not due to sampling.

**Comparison of Aerial and Field Cruises of Section II**

Two aerial cruises were made of section II. The first cruise was a 100 percent inventory of the area, and the second cruise consisted of a 50 percent representative random sample, using one-half acre square plots. These cruises are compared with a 50 percent systematic random field sample of the area to determine their relative accuracy. Results of this comparison of aerial and field cruises are found in Table XIX for 3-variable solution for volume, Table XX for 3-variable solution for d.b.h., Table XXI for coefficient "all sites" solution, and Table XXII for coefficient "individual sites" solution.

A comparison was also made between the aerial and field cruises of section II in regards to Douglas fir volume. Results of this comparison are found in Table XXIII for the 3-variable solution for volume, Table XXIV for the 3-variable solution for d.b.h., and Table XXV for the coefficient "all sites" solution.
### Table XIX

COMPARISON OF THE AERIAL AND FIELD CRUISE ESTIMATES OF MCDONALD PINE VOLUME FOR SECTION II

THREE-VARIABLE SOLUTION FOR VOLUME

<table>
<thead>
<tr>
<th>Type of cruise</th>
<th>Mean per-acre volume</th>
<th>Difference from field cruise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>feet, b. a.</td>
<td>Feet, B.M.</td>
</tr>
<tr>
<td>50% field cruise</td>
<td>20,770 ± 1323</td>
<td>0</td>
</tr>
<tr>
<td>700% aerial cruise</td>
<td>17,523 ± 0</td>
<td>-3245</td>
</tr>
<tr>
<td>50% aerial cruise</td>
<td>21,302 ± 2015</td>
<td>+532</td>
</tr>
<tr>
<td>Type of cruise</td>
<td>Mean per-acre volume</td>
<td>Difference from field cruise</td>
</tr>
<tr>
<td>------------------------</td>
<td>----------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td></td>
<td>Feet, B.M.</td>
<td>Feet, B.M.</td>
</tr>
<tr>
<td>50% field cruise</td>
<td>20,770 ± 1329</td>
<td>0</td>
</tr>
<tr>
<td>100% serial cruise</td>
<td>16,982 ± 0</td>
<td>-3788</td>
</tr>
<tr>
<td>50% serial cruise</td>
<td>20,462 ± 1842</td>
<td>-508</td>
</tr>
</tbody>
</table>
TABLE XXI

COMPARISON OF THE AERIAL AND FIELD CRUISE ESTIMATES OF PSEUDOSASA PINS VOLUME FOR SECTION II

COEFFICIENT "ALL SITES" SOLUTION

<table>
<thead>
<tr>
<th>Type of cruise</th>
<th>Mean per-acre volume</th>
<th>Difference from field cruise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>feet, b.m.</td>
<td>Feet, B.M.</td>
</tr>
<tr>
<td>50% field cruise</td>
<td>20,770 ± 1329</td>
<td>0</td>
</tr>
<tr>
<td>100% aerial cruise</td>
<td>17,566 ± 0</td>
<td>-3162</td>
</tr>
<tr>
<td>50% aerial cruise</td>
<td>20,740 ± 1646</td>
<td>-50</td>
</tr>
</tbody>
</table>
### Table XXII

**Comparison of the Aerial and Field Cruise Estimates of Coniferous Pine Volume for Section II**

Coefficient "Individual "Citen" Solution

<table>
<thead>
<tr>
<th>Type of Cruise</th>
<th>Mean Per-Acre Volume</th>
<th>Difference from Field Cruise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Feet, B.M.</td>
<td>Percent</td>
</tr>
<tr>
<td>50% field cruise</td>
<td>20,770 ± 1329</td>
<td>0</td>
</tr>
<tr>
<td>100% aerial cruise</td>
<td>17,204 ± 0</td>
<td>-3586</td>
</tr>
<tr>
<td>50% aerial cruise</td>
<td>20,836 ± 1857</td>
<td>-134</td>
</tr>
<tr>
<td>Type of cruise</td>
<td>Mean per-acre volume</td>
<td>Difference from field cruise</td>
</tr>
<tr>
<td>------------------------</td>
<td>----------------------</td>
<td>------------------------------</td>
</tr>
<tr>
<td></td>
<td>feet, b.m.</td>
<td>Feet, b.m.</td>
</tr>
<tr>
<td>50% field cruise</td>
<td>475 ± 297</td>
<td>0</td>
</tr>
<tr>
<td>100% aerial cruise</td>
<td>482 ± 0</td>
<td>± 7</td>
</tr>
<tr>
<td>50% aerial cruise</td>
<td>276 ± 127</td>
<td>-206</td>
</tr>
<tr>
<td>Type of cruise</td>
<td>Mean per-acre volume</td>
<td>Difference from field cruise</td>
</tr>
<tr>
<td>-----------------------</td>
<td>----------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>50% field cruise</td>
<td>472 ± 297</td>
<td>0</td>
</tr>
<tr>
<td>100% aerial cruise</td>
<td>461 ± 9</td>
<td>-14</td>
</tr>
<tr>
<td>50% aerial cruise</td>
<td>250 ± 117</td>
<td>-225</td>
</tr>
</tbody>
</table>
TABLE XXV

COMPARISON OF THE AERIAL AND FIELD CRUISE ESTIMATES
OF DOUGLAS FIR VOLUME FOR SECTION II

CONFIDENT "ALL SITES" SOLUTION

<table>
<thead>
<tr>
<th>Type of cruise</th>
<th>Mean per-acre volume</th>
<th>Difference from field cruise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Feet, b.m.</td>
<td>Feet, B.M.</td>
</tr>
<tr>
<td>50% field cruise</td>
<td>475 ± 297</td>
<td>0</td>
</tr>
<tr>
<td>100% aerial cruise</td>
<td>505 ± 0</td>
<td>+50</td>
</tr>
<tr>
<td>50% aerial cruise</td>
<td>270 ± 132</td>
<td>-203</td>
</tr>
</tbody>
</table>
One of the first items to be examined in the above comparison is the validity of the field cruise of section II. Although the primary interest of this study is to determine the relative accuracy of aerial inventories, it still remains that the field cruise, since it is used as the basis to predict this degree of accuracy, be scrutinized for possible error. Because the volume of a particular area has been determined by a field cruise of 50 percent intensity does not necessarily indicate that the results of this cruise represents the true volume value for the area.

The field cruise of section II is difficult to evaluate. A check cruise, similar to that made of section I, is not applicable in this case because an accurate count of the number of trees on this section is impossible to obtain from the aerial photographs. The stand density along the canyon bottom prohibits an accurate delineation of the section boundary and, consequently, which trees fall within the area cannot possibly be determined with any degree of accuracy.

An examination of both field and aerial cruises of section II reveals a glaring discrepancy in the estimation, by the different cruises, of the total number of commercial-sized trees occurring on the area. For example, the total number of trees on section II as predicted by the 50 percent field cruise is 208; by the 100 percent aerial cruise, 218; and by the 50 percent aerial cruise, 237. Since the average
individual tree volume, determined from field measurements, is 1,301 feet, b.m., this is an important factor. If the tree count result of the 100 percent aerial cruise is considered as the basis, then, theoretically, the 50 percent field cruise is 13,010 feet, b.m. low, and the 50 percent aerial cruise is 24,720 feet, b.m. high. However, if Tables XIX through XXII are examined this sequence of volume difference does not hold true. The field cruise gives the highest volume estimates, while the 50 percent aerial cruise is a close second. The 100 percent aerial cruise averages 15 percent low in all examples.

If the 50 percent field cruise of section II is further analyzed a discrepancy in the representative coverage is noted. For example, it was found that 29 percent of the cruise plots fell within the canyon site classification, or along the canyon bottom. This area comprises only 25 percent of the total acreage of section II, but contains 31 percent of the total volume of ponderosa pine. On a cruise intensity basis, this would mean that on the canyon site, 62 percent of the area was sampled, while on the slope site, only 49 percent was sampled. Since a greater proportion of the cruise plots fell on the area having the highest per unit volume the total volume of the section estimated by the 50 percent field cruise will tend to be higher than the true volume of the section. However, this difference should be small because
the slope site area has four times the weight as the canyon site area. Like in section I, the true volume of section II should fall between the mean volume and the lower limit, or 20,770 feet, b.m. to 19,441 feet, b.m.

The standard error of the difference for the field and aerial cruises of section II, expressed as a normal deviate, is ± 0.20 standard deviations. The area under the normal frequency curve between a ± 0.20 deviations is 0.57926 minus 0.42074, or 0.15852. In the two tails beyond these limits is 1.00000 minus 0.15852, or 0.84148.

This is the reverse of what was found in the analysis of the field and aerial cruises of section I. The standard error of the difference for the aerial and field cruises of section II indicates that in five chances out of six the difference between the mean volumes of these two cruises is an accident of sampling. This may also be stated: that there is only one chance in six that another volume estimate of the same area will fall within the indicated limits.

Therefore, it can be concluded from this analysis that the difference between the mean volumes of the field cruise and 50 percent representative random sample aerial cruise is due to an accident of sampling.

A comparison of the Douglas fir volume values obtained by aerial and field cruises of section II would be insignificant. The limiting factor in such a comparison is the accur-
asy in which trees of this species can be recognized on aerial photographs. If only a small percentage can be identified with any certainty, and a calculated guess must be made of the others, it may be assumed that any difference in mean volumes of aerial and field cruises is due to this factor, and not errors of sampling, etc.

**Comparison of Aerial and Field Cruises of Section V**

Only one aerial cruise was made of section V. This cruise consisted of a 50 percent representative random sample, using one-half acre square plots. A comparison was made of the mean volume values of ponderosa pine obtained by the aerial and field cruises of this section. The results of this comparison are found in Table XXVI for all volume table solutions. A comparison of Douglas fir-mean volume values obtained by the aerial and field cruises is found in Table XXVII for all volume table solutions.

A detailed examination of each type of cruise made of section V reveals no apparent discrepancies. Both appear to give a good representative coverage of the area. Because of its systematic basis the field cruise sampled all parts of section V equally. However, each plot in the aerial cruise was located by random selection. If the aerial cruise did not give a representative coverage of the area a discrepancy in mean volume results between the aerial and field cruises would
<table>
<thead>
<tr>
<th>Type of cruise</th>
<th>Mean per-acre volume</th>
<th>Difference from field cruise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>feet, b.m.</td>
<td>feet, b.m.</td>
</tr>
<tr>
<td>50% field cruise</td>
<td>21,615 ± 2307</td>
<td>0</td>
</tr>
<tr>
<td>50% aerial cruise</td>
<td>22,711 ± 2852</td>
<td>+1096</td>
</tr>
<tr>
<td>3-var. for volume</td>
<td>22,387 ± 3131</td>
<td>+752</td>
</tr>
<tr>
<td>50% aerial cruise</td>
<td>22,196 ± 2688</td>
<td>+581</td>
</tr>
<tr>
<td>10% aerial cruise</td>
<td>22,452 ± 2953</td>
<td>+837</td>
</tr>
</tbody>
</table>

TABLE XXVI

COMPARISON OF THE AERIAL AND FIELD CRUISE ESTIMATES OF PONDEROSA PINE VOLUME FOR SECTION V
## Table XXVII

Comparison of the April and Field Cruise Estimates of Douglas Fir Volume for Section V

<table>
<thead>
<tr>
<th>Type of Cruise</th>
<th>Mean Per-Acre Volume</th>
<th>Difference from Field Cruise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Feet, B.M.</td>
<td>Feet, B.M.</td>
</tr>
<tr>
<td>50% Field Cruise</td>
<td>2720 ± 522</td>
<td>0</td>
</tr>
<tr>
<td>50% Serial Cruise 3-var. for Volume</td>
<td>1858 ± 401</td>
<td>-162</td>
</tr>
<tr>
<td>50% Serial Cruise 3-var. for d.b.h.</td>
<td>1609 ± 4.4</td>
<td>-511</td>
</tr>
<tr>
<td>50% Serial Cruise Coeff. &quot;all sites&quot;</td>
<td>1665 ± 403</td>
<td>-455</td>
</tr>
</tbody>
</table>
be readily apparent, regardless of other possible errors present. The timber stand of section V exhibits a high degree of variation, and if a large proportion of sample plots should happen to fall in one particular area it would be reflected in the volume results of the aerial cruise. For example, it has been found from an analysis of the field cruise that 60 percent of the total ponderosa pine volume is concentrated on 45 percent of the total acreage of section V. This concentration occurs in the upper ranges of the section. In the middle ranges of the section, 20 percent of the area contains less than 10 percent of the volume; and in the lower ranges, 30 percent of the volume is on 35 percent of the area. If the greater proportion of sample plots of the aerial cruise fell on the middle ranges of the section the volume value would be much less than that of the field cruise; if most of the plots were concentrated in the upper ranges of the section the volume value of the aerial cruise would be much higher than that of the field cruise.

Another method of analyzing the aerial and field cruises of section V is to determine the total number of trees, predicted by each cruise, for the entire section. In this case the aerial cruise predicted 187 trees of commercial size for the area, while the field cruise predicted 232 trees. If the average individual tree volume, determined by field measurements, is 1,000 feet, b.m., the difference in volume values
of the two cruises should be 45 times 1,000, or 45,000 feet, b.m. On a per acre basis the volume value is 4,180 feet, b.m. However, because of the wide variation in stand density and individual tree size present in section V this figure cannot be considered absolutely correct.

This analysis does indicate the possibility that the aerial cruise sampled too heavily in the low-volume areas of the section, and was compensated for by an over estimation in the measurements of photographic height and crown diameter. Another possible error indicated by this discrepancy is in the determination of aerial plot boundaries. However, there is no tangible basis from which the actual presence of these possible errors, or their degree of influence, can be predicted.

The feasible method of comparing two cruises having characteristics similar to those of section V is to determine the statistical measure of the difference between the two cruises. However, the statistical solution, in this case, only indicates the errors of sampling; it indicates nothing of the errors incurred in sampling.

The standard error of the difference for the field cruise and aerial cruise, whose volume value was obtained by the 3-variable solution for volume, expressed as a normal deviate, is ± 0.30 standard deviations. The area under the
normal frequency curve between ± 0.30 standard deviations is 0.61791 minus 0.38209, or 0.23582. In the two tails beyond these limits is 1.00000 minus 0.23582, or 0.76418. This means that three chances in four the difference is an accident of sampling.

A different value is obtained for the standard error of the difference if the field cruise is compared with the aerial cruise where volume value was obtained by the 3-variable solution for d.b.h. In this example the difference is ± 0.20 standard deviations. The area falling within these limits is 0.15852; and in the area beyond these limits, 0.84148. This means that six chances in seven the difference is an accident of sampling.

In the first comparison between aerial and field cruises, the standard error of the difference indicates that three chances in four another volume estimate of the same area will fall outside the limits shown. In the second comparison there are seven chances in eight that another volume estimate of the same area will fall outside the limits. This would mean that the difference between the aerial and field cruises in the first comparison would have the greater significance.

It may be concluded from the above analysis that there is no significant difference between the aerial and field
cruises of section V. What difference there is present is due to an accident in sampling.

**Comparison of Aerial and Field Cruises of Section VI**

The aerial inventory of section VI consisted of a 50 percent representative random sample, using one-half acre square plots. This cruise was compared with a 50 percent systematic random field sample to determine its relative accuracy. Results of this comparison are found in Table XXVIII for all aerial volume table solutions.

An examination of the field cruise of section VI reveals that 30 percent of the area was classified as exposed site, and 70 percent as slope site. The average per acre volume of the exposed site classification was 12,140 feet, b.m., and the average individual tree volume was 1,215 feet, b.m. For the slope site classification the per acre volume was 19,880 feet, b.m., and the average individual tree volume, 1,125 feet, b.m. The average individual tree volume for both classifications was 1,140 feet, b.m., and the total number of commercial-sized trees predicted for the section was 319.

The analysis of the aerial cruise, using the 3-variable solution for volume as the basis, reveals a significant difference between the aerial and field cruises of section VI. For example, 44 percent of the total area is classified by the
<table>
<thead>
<tr>
<th>Type of cruise</th>
<th>Mean per-acre volume</th>
<th>Difference from field cruise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>feet, b.m.</td>
<td>Feet, b.m.</td>
</tr>
<tr>
<td>50% field cruise</td>
<td>18,090 ± 2306</td>
<td>0</td>
</tr>
<tr>
<td>50% serial cruise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-var. for volume</td>
<td>20,300 ± 1897</td>
<td>+2210</td>
</tr>
<tr>
<td>50% serial cruise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-var. for d.b.h.</td>
<td>20,766 ± 1994</td>
<td>+3076</td>
</tr>
<tr>
<td>50% serial cruise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>coeff. &quot;all sites&quot;</td>
<td>21,348 ± 2071</td>
<td>+3258</td>
</tr>
<tr>
<td>50% serial cruise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>coeff. &quot;ind. sites&quot;</td>
<td>20,466 ± 1844</td>
<td>+2336</td>
</tr>
</tbody>
</table>
aerial cruise as exposed site, while 56 percent is classified as slope site. The average per-acre volume of the exposed site classification is 17,050 feet, b.m., and the average individual tree volume is 1,115 feet, b.m. For the slope site classification the average per-acre volume is 23,884 feet, b.m., and the average individual tree volume is 1,300 feet, b.m. The average individual tree volume for both classifications is 1,225 feet, b.m., and the total number of trees predicted for the entire area is 340.

If the above figures are compared it is readily apparent that there is a large discrepancy in representative coverage of the two types of cruises. For example, the most prominent difference between these cruises is in the percentage of total area classified as exposed site. If it can be assumed that methods of classifying site are the same for each cruise, then the above figures indicate that the exposed site was sampled heavier in the aerial cruise than in the field cruise. Because this is a lower per-acre volume area than the slope site area, it follows that the aerial cruise should have a smaller mean-volume value than that of the field cruise. However, examination of the results listed in Table XXVIII does not substantiate this theory.

Another source of error, then, must account for the discrepancy between the aerial and field cruises. If the
total number of commercial-sized trees predicted for the section by each cruise are examined it is found that the field cruise predicts 319, while 340 trees of commercial size are predicted by the aerial cruise; a difference of 21 trees. If the average individual tree volume, determined from field measurements, is 1,140 feet, b.m., the total volume difference should be 21 times 1,140, or 23,940 feet, b.m., and the per acre volume value, 1,200 feet, b.m. This would account for approximately half of the existing difference between the aerial and field cruises. However, there is still 1,200 feet, b.m. per acre to be accounted for.

Therefore, from the above comparison it may be concluded that part of the error in volume results of the aerial cruise is due to sampling, and part is due to an over estimation in either measurements of photographic height and crown diameter, determination of aerial plot boundaries, or a combination of these factors.

The above comparison between the two types of cruises of section VI is obtained by a direct analysis of each cruise data. This method indicates, but does not prove, whether or not the difference between these two cruises is significant, or merely an accident of sampling. In order to obtain a more definite measure of the discrepancy between the aerial and field cruises, a statistical comparison is made.
The standard error of the difference for the field and aerial (3-variable solution for volume) cruises, expressed as a normal deviate, is ± 0.56 standard deviations. The area under the normal frequency curve falling within these limits is 0.42396; and in the tails beyond these limits is 0.57604. This means that there is about one chance out of two that the difference is merely an accident of sampling. This may also be stated: there is about one chance in two that another volume estimate of the same area will fall within the indicated limits.

The conclusion reached by the statistical comparison is similar to that reached by the direct comparison: that it is a fifty-fifty proposition on whether or not the difference between these two cruises is significant, or an accident of sampling.

Comparison of the Aerial and Field Cruises of the Benchmark Study Area

The final aerial cruise made in this study consisted of a 20 percent representative random sample of the entire Benchmark Creek study area. Although the purpose of this cruise was to secure the total volume of the entire area, consideration was given each section by weighting it according to its per unit volume. For example, section VII was given a weight of one, while section III was weighted four times.
The volume results of this cruise were compared with the results of a 50 percent systematic field cruise of the entire area to determine if an aerial cruise of 20 percent intensity would give accurate volume estimates. Results of this comparison are found in Table XXIX for ponderosa pine, and Table XXX for Douglas fir.

An analysis of both aerial and field cruises of the study area is shown in Table XXXI. The purpose of this analysis is to illustrate where, and what degree, the aerial cruise varies from the field cruise. If the table is examined, the most prominent discrepancies are between the areas of each site classification, and the average individual tree volumes. The variation between these factors indicates a significant difference in the representative coverage of the two cruises. Actually, this is as it should be. Because the field cruise has a systematic basis, all areas will be sampled equally. The aerial cruise, on the other hand, was weighted according to per unit volume. Since the canyon sites contain the greatest per acre volume they were weighted the heaviest, and should also have been sampled the heaviest. The slope sites were given the second heaviest weight; and the exposed sites the least weight. However, this system of weighting only holds true in the exposed and slope site categories. In the canyon site classification, where the aerial cruise should have given the higher area estimate, the reverse is
<table>
<thead>
<tr>
<th>Type of cruise</th>
<th>Mean per-acre volume</th>
<th>Difference from field cruise</th>
</tr>
</thead>
<tbody>
<tr>
<td>50% field cruise</td>
<td>18,592 ± 674</td>
<td>0</td>
</tr>
<tr>
<td>20% serial cruise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-var. for volume</td>
<td>18,709 ± 1740</td>
<td>-283</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1.5</td>
</tr>
<tr>
<td>20% serial cruise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-var. for d.b.h.</td>
<td>18,493 ± 1757</td>
<td>-499</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-2.6</td>
</tr>
<tr>
<td>20% serial cruise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>coeff. &quot;all sites&quot;</td>
<td>18,591 ± 1736</td>
<td>-301</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1.6</td>
</tr>
<tr>
<td>20% serial cruise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>coeff. &quot;ind. sites&quot;</td>
<td>18,275 ± 1716</td>
<td>-717</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-3.8</td>
</tr>
</tbody>
</table>
### Table XXX

Comparison of the Aerial and Field Cruise Estimates of Douglas H. V. Volume for the Benchmark Creek Study Area

<table>
<thead>
<tr>
<th>Type of Cruise</th>
<th>Mean per-acre volume</th>
<th>Difference from field cruise</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>feet, b.m.</td>
<td>Feet, b.m.</td>
<td>percent</td>
</tr>
<tr>
<td>50' field cruise</td>
<td>611 ± 118</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20' serial cruise</td>
<td>349 ± 146</td>
<td>-822</td>
<td>-23.9</td>
</tr>
<tr>
<td>20' serial cruise</td>
<td></td>
<td>-192</td>
<td>-17.3</td>
</tr>
<tr>
<td>20' serial cruise</td>
<td></td>
<td>-149</td>
<td>-16.1</td>
</tr>
<tr>
<td>20' serial cruise</td>
<td></td>
<td>-45</td>
<td>-8.7</td>
</tr>
</tbody>
</table>
TABLE XXXI

DIRECT ANALYSIS OF THE AERIAL AND FIELD CENSUSES
OF THE BENFORD CREEK STUDY AREA

<table>
<thead>
<tr>
<th>Comparison factor</th>
<th>Canyon site</th>
<th>Slope site</th>
<th>Exposed site</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Field</td>
<td>Aerial</td>
<td>Field</td>
</tr>
<tr>
<td>Area (acres)</td>
<td>25</td>
<td>22.4</td>
<td>44</td>
</tr>
<tr>
<td>Volume per acre</td>
<td>25,030</td>
<td>29,664</td>
<td>21,065</td>
</tr>
<tr>
<td>Total volume</td>
<td>645,875</td>
<td>642,074</td>
<td>926,860</td>
</tr>
<tr>
<td>Number of trees</td>
<td>368</td>
<td>340</td>
<td>526</td>
</tr>
<tr>
<td>Average tree volume</td>
<td>1106</td>
<td>1888</td>
<td>1768</td>
</tr>
</tbody>
</table>
true. Also, according to the weighting system, the aerial cruise should have given a much lower estimate of area for exposed sites than that shown.

What can be surmised by the direct analysis approach in this case is difficult to evaluate because of the weighted system used in the aerial cruise. However, it may be concluded that the difference between the results of these two cruises is an accident of sampling.

This conclusion is supported by a statistical comparison between the aerial and field cruises of the study area. The standard error of the difference, expressed as a normal deviate, for these two cruises is ± 0.15 standard deviations. The area under the normal frequency curve between these two limits is 0.11910. In the tails beyond these two limits is 0.88090. This means that eight chances in nine the difference is due to an error of sampling.

Comparison of Aerial and Field Measurements of Individual Trees

A final test of the validity of the tree-aerial volume tables, the accuracy of photographic height and crown diameter measurements, and the accuracy in determining plot boundaries was made by comparing the aerial and field measurements of three one-half acre plots. These plots were first
located on the aerial photographs and each tree of merchantable size occurring on these plots was measured for height and crown diameter, and the volume determined. The photograph on which these plots were located was then taken into the field and the aerial location of each plot was transferred to its corresponding ground location. Field measurements were made of each tree as to d.b.h., merchantable height, total height, crown diameter, and volume. Plot boundaries were also measured in the field. The results of the field sample were then compared with those of the aerial sample to determine the accuracy of the photographic measurements.

In Table XXXII are found the results of the comparison between aerial and field measurements for plot no. one; in Table XXXIII for plot no. two; and in Table XXXIV for plot no. three.

Each of the three plots selected represented a site classification. The first plot was located in the upper ranges of section VI and was classified as an exposed site. The second plot was also located on section VI and represented a slope site. The last plot was located in the canyon bottom of the North Fork of Benchmark Creek and was classified as a canyon site. The first two plots contain only ponderosa pine, while Douglas fir is the only species occurring on the third plot.
### TABLE XXXII

**Comparison of the Aerial and Field Measurements of the First Act**

<table>
<thead>
<tr>
<th>Tree number</th>
<th>Total height</th>
<th>Crown diameter</th>
<th>Volume</th>
<th>Th. CD ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Aerial Field</td>
<td>Aerial Field</td>
<td>aerial Field</td>
<td>feet, b.m.</td>
</tr>
<tr>
<td>1</td>
<td>72</td>
<td>77</td>
<td>22</td>
<td>28</td>
</tr>
<tr>
<td>2</td>
<td>62</td>
<td>66</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>65</td>
<td>65</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>67</td>
<td>65</td>
<td>18</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>67</td>
<td>65</td>
<td>22</td>
<td>18</td>
</tr>
<tr>
<td>6</td>
<td>83</td>
<td>84</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>7</td>
<td>92</td>
<td>92</td>
<td>24</td>
<td>26</td>
</tr>
</tbody>
</table>
Examination of the comparison between aerial and field measurements found in Table XXXII reveals a significant discrepancy in estimate of volume by the two methods. The reason for this lies in the aerial volume table used as the basis for estimating volume. The 3-variable solution for volume method used in this problem will greatly overestimate volume in the lower height classes. Since the heights of all trees occurring on this plot fell in the lower height classes, the estimated, or aerial, volume will be larger than the true volume.

Comparing the height measurements of trees occurring on the first plot it is found that in only two cases is the difference between aerial and field measurements greater than five feet. In the case of tree #2 the error is due to the inability of the observer to determine the exact photographic location on the slope of the tree base. This problem was encountered in the height measurement of every tree occurring on the plot. The trees all had large dense crowns growing the full length of the boles; and in no case could the tree base be observed on the aerial photographs. Since the slope on which this plot fell is 55 percent, the estimated location of the tree base would not have to vary much from the true location in order to cause an appreciable error in height measurement.

The error incurred in the case of tree #4 is due to
two trees being close together. In observing these two trees on the aerial photographs it appears that the base of the upper tree, or #4, was at least 15 to 20 feet up the slope from the base of the lower tree, and the height measurement was made according to this assumption. However, subsequent field examination revealed that these trees possessed a common base.

The field measurements of crown diameter listed in Table XXXII are for the actual dimensions of the tree crowns along a north-south line. This procedure for measuring tree crowns was for the purpose of determining the loss of crown width on aerial photographs due to resolution. However, due to other limiting factors present, such as shadows and adjacent crowns, it was not possible to determine the relationship between photographic crown diameter and true crown diameter with any degree of certainty.

These measurements do, however, indicate a general trend in the relationship between aerial and field values of crown diameter if the average of each is considered. The average of the photo measurements is 20 feet, and for the field measurements, 21 feet. On this basis the difference between actual crown diameter and visible crown diameter may be considered as one crown diameter class, or 2 feet. If this may be assumed correct, then the correction applied for visible crown diameter in preparing the aerial volume tables is
from two to four feet too high.

In the last column of Table XXXII is found the total height/crown diameter ratio. This ratio was computed from the aerial measurements of these two variables. To determine the site index of this plot, the average ratio was computed and the site index value, corresponding with this average ratio, was scaled from the graph illustrated in Figure 4. In this example, the average ratio was 3.6, and the corresponding site index value 40.

The comparison between aerial and field measurements of trees occurring on the second plot is similar to that of the first plot. However, there are a few significant differences. For example, the aerial and field estimates of plot volume were approximately the same. The reason for this is the influence volume table. If the volume figures listed in Table XXXIII are examined this factor and its effect on total plot volume is readily apparent. Another problem which occurred in aerial measurements of trees found on this plot, is where the crown image was indistinct on aerial photographs. In this case the crown of tree #14 was almost invisible. Consequently, it was necessary to determine the height and crown diameter values of this tree by measuring its shadow. Subsequent field examination of this tree revealed that its crown was 40 feet in diameter along a northeast-southwest line, and only about 8 feet in diameter along a line perpendicular to
<table>
<thead>
<tr>
<th>Tree number</th>
<th>Total height</th>
<th>Crown diameter</th>
<th>Volume</th>
<th>Tht./Db ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>117</td>
<td>18</td>
<td>2254</td>
<td>6.5</td>
</tr>
<tr>
<td>2</td>
<td>126</td>
<td>20</td>
<td>2780</td>
<td>6.5</td>
</tr>
<tr>
<td>3</td>
<td>101</td>
<td>16</td>
<td>1768</td>
<td>6.3</td>
</tr>
<tr>
<td>4</td>
<td>122</td>
<td>30</td>
<td>3654</td>
<td>4.1</td>
</tr>
<tr>
<td>5</td>
<td>127</td>
<td>32</td>
<td>3841</td>
<td>3.7</td>
</tr>
<tr>
<td>6</td>
<td>69</td>
<td>20</td>
<td>1046</td>
<td>3.5</td>
</tr>
</tbody>
</table>
the above. To obtain the field measurement listed in the table for this tree, the crown was projected onto a north-south line, and its dimensions determined on that basis.

If the total height/crown diameter ratio column in Table XXXIII is examined it is found that the average ratio for all trees occurring on this plot is 5.1. Referring to the graph in Figure 4, the corresponding site index value for the above figure is 50. However, the actual site index value for this plot is 70.

A comparison of aerial and field measurements of trees occurring on the third plot is found in Table XXXIV. Because the Douglas fir trees found on this plot are small and bunched together it was difficult to determine the position of the tree bases, and to discern the individual tree crowns. Consequently, in many instances the tree heights were over estimated. The only aerial measurements made of trees occurring on this plot which may be considered correct are those for tree #1 and #2. These trees were isolated and it was possible to obtain accurate measurements of both crown diameter and total height. If the aerial and field measurements listed in Table XXXIV for these two trees are examined it is found that the aerial measurements are five feet less than the field measurements. This difference is, in all probability, due to the failure of the sharply-pointed crown tips to resolve on
<table>
<thead>
<tr>
<th>Tree number</th>
<th>Total height</th>
<th>Crown diameter</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Aerial Field</td>
<td>Aerial Field</td>
<td>Aerial Field</td>
</tr>
<tr>
<td></td>
<td>feet</td>
<td>feet</td>
<td>feet, b.m.</td>
</tr>
<tr>
<td>1</td>
<td>66</td>
<td>71</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>71</td>
<td>71</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>69</td>
<td>67</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>72</td>
<td>68</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>65</td>
<td>60</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>73</td>
<td>71</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>61</td>
<td>64</td>
<td>14</td>
</tr>
<tr>
<td>8</td>
<td>71</td>
<td>70</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>71</td>
<td>76</td>
<td>14</td>
</tr>
<tr>
<td>10</td>
<td>53</td>
<td>45</td>
<td>12</td>
</tr>
<tr>
<td>11</td>
<td>77</td>
<td>65</td>
<td>16</td>
</tr>
<tr>
<td>12</td>
<td>77</td>
<td>71</td>
<td>16</td>
</tr>
<tr>
<td>13</td>
<td>63</td>
<td>65</td>
<td>14</td>
</tr>
</tbody>
</table>
The results of the comparison of aerial and field measurements of individual trees indicate:

1. that the limiting factors in aerial height measurements (considering photographs of 1:12,000 scale and a competent photo interpreter) are percent of slope, size of crown, density of crown, contrasting background, sharpness of tree image, and whether or not the tree base is visible;

2. that where the tree base is visible and the tree image sharply defined, height measurements on aerial photographs of 1:12,000 scale, using the academy height finder (HF-2), can be consistently classified into 5-foot height classes;

3. that where the tree base is not visible, and other limiting factors are present, height measurements on aerial photographs of 1:12,000 scale, using the HF-2, can be classified into 10-foot height classes;

4. that visible crown diameter on aerial photographs of 1:12,000 scale, is from two to four feet less than actual crown diameter;

5. that the visible crown diameter value used in the preparation of the tree-aerial volume tables is from one to two crown diameter classes low; and

6. that the prediction of site index value from an average ratio between the photographic measurements of total height and crown diameter is not accurate.

Summary

The comparison of the various aerial and field cruises illustrated in this chapter does not indicate any general trend in the aerial inventory of ponderosa pine stands. For
example, in four of the aerial cruises the volume estimates were low in comparison with the corresponding field cruise estimates, while in three of the aerial cruises the estimates of volume were higher. In only two aerial cruises did the volume estimates approach the field estimates. However, in most examples the aerial estimates were accurate enough for all practical purposes.

But aerial volume table solution gives the most reliable estimates is another factor which is difficult to evaluate. However, from the preceding comparison the best solution for estimating volume on a combination of all three site classifications is the 3-variable solution for volume. Where the area consists mostly of one site classification, the coefficient "individual site" volume table solution will give the most accurate estimates. Usually the difference between the various aerial volume table solutions for volume estimation of a particular area, is so small as to be negligible.

Another factor in the comparison of aerial and field cruises which has little influence on the final results, but is worth mentioning at this point, is the statistical difference between systematic and representative random samples. In most instances, a systematic random sample will give a more thorough coverage of the area being cruised. Because of this factor the mean per-acre volume and the standard deviation obtained by a systematic random sample will be more
reliable than the same values obtained by a representative random sample. However, in the case of the standard error of the mean, the opposite is true. The representative random sample will yield a more reliable measure of this value than will the systematic random sample. The reason for this lies, mainly, in the basic principles of statistics. This type of mathematics is based on the laws of chance and, theoretically, does not apply to a mechanical system, such as a systematic sample, for determining the location of sample plots.

An illustration of the statistical difference between systematic and representative random samples is found in Table V. The first 50 percent serial cruise of section listed in this table is a representative random sample. The second 50 percent serial cruise is a systematic random sample. The standard error for the representative sample is 1836 feet, b.m., and the standard error for the systematic sample is 1480 feet, b.m.; a difference of 356 feet, b.m. Whether or not this difference is due entirely to the system of sampling, or to some other factor, cannot be determined. However, because of the known characteristics of each type of sample, it is fairly reasonable to assume that the difference in the values of the standard error is the result of the sampling systems employed.
CHAPTER VII

CONCLUSIONS

The comparisons made in the preceding chapter between the various aerial and field cruises of the Benchmark Creek study area indicate that it is possible to obtain an accurate volume estimate of ponderosa pine stands from aerial photographs. However, the regional acceptance of this inventory technique depends on such factors as: (1) minimum degree of accuracy acceptable, (2) species identification, (3) photographic estimation of stem quality, (4) scale, type, and quality of aerial photography, and (5) preparation of the necessary tree-aerial volume tables.

Minimum Degree of Accuracy Acceptable

The degree of accuracy could be the controlling factor in the acceptance of aerial inventory surveys. However, this factor, as applied in the Northern Rocky Mountain Region, is only a relative value. For conventional line-plot cruises are prepared for the purpose of obtaining a certain degree of accuracy. For example, a certain area is being considered for sale and the total volume on the area must be known before bids can be accepted. Cruisers are sent out and a 10 percent line-plot cruise is made. The total volume is then computed, and this figure is considered the true volume of the area. The limits of accuracy of this cruise might be one percent.
or it might be 100 percent; no attempt is made to find out. The logs cut on the sale will be scaled, and the operator pays for the timber on this basis. Because of the method employed the accuracy of the cruise is not important. All that is necessary is a fair estimate of the volume by species so that both buyer and seller are acquainted with the possibilities of the sale.

Therefore, the total volume value obtained by a conventional line-plot cruise of a certain area is accepted regardless of whether or not it is within 10 percent, 20 percent, or even 100 percent of the true volume. Sometimes the error involved reaches considerable proportions. The author has knowledge of some conventional field cruise whose estimates ranged from 100 percent to 300 percent low of the final volume cut.

If the above is true of conventional line-plot cruises, a similar margin of error might be applied to aerial cruises. However, common sense dictates that a 10 percent aerial cruise is not as accurate as a 10 percent field cruise. Therefore, the most probable solution to this problem is to increase the intensity of the aerial cruise. Since there is no way of predicting from this study the relationship between accuracy and cruise intensities, a hypothetical "rule at thumb" is discussed which could be of practical application.
A "rule of thumb" for the relationship between aerial and field cruise intensities will largely depend upon the ease with which a particular timber stand is interpreted on aerial photographs. For example, if the stand is scattered and composed of large trees, the "rule of thumb" in this case might be that a 20 percent aerial cruise will give the same accuracy as a 10 percent field cruise. If the stand under consideration is of medium density and it is difficult to discern some of the individual trees on aerial photographs, then the relationship between cruise intensities might be 30 percent to 10 percent. In an extreme case the aerial cruise will have to be of a 50 percent cruise intensity to achieve the same accuracy as a 10 percent field cruise. If an aerial cruise intensity of more than 50 percent is necessary, then it would probably be best to cruise the area in the field.

What the relationship between aerial and field cruise intensities for a given stand of timber should be, will depend upon the ability of the photo interpreter to recognize the limiting factors present in regards to the aerial cruise. These will include scale and quality of photography, photo interpretation devices available, and stand characteristics as to density, species composition, and size of individual trees.

Species Identification

On some areas species identification will be the limit-
ing factor in the application of aerial inventories to ponderosa pine stands in Western Montana. However, this species usually occurs in pure open stands and any associate will usually form only an insignificant portion of the total stand. The ponderosa pine stand used in this study is a typical example. This stand contained nearly two million feet, b.m., of ponderosa pine to only about fifty thousand feet, b.m., of Douglas fir. Where Douglas fir is an associate of ponderosa pine it is confined to moist sites, and these are readily identified on aerial photographs. However, segregation of these two species where they do occur in association is difficult. In this study, it was found that only 50 percent of Douglas fir occurring on the area could be identified with certainty on aerial photographs. Consequently, where Douglas fir is a commercially important component of ponderosa pine stands, aerial inventories are not applicable.

Species identification, however, is not an insurmountable obstacle in aerial inventories of the ponderosa pine type. With the larger-scaled and higher-quality panchromatic photography, along with improved photo-interpretation devices and techniques, the problem of segregating Douglas fir from ponderosa pine could readily be solved. Another possible solution is color photography. Color transparencies viewed through filters which only emit those light waves reflected by Douglas fir crowns, might be a feasible answer to this problem. These
items are expensive, and whether or not there is justification in their application is a future consideration.

**Photographic Estimation of Stem Quality**

The percentage of cull in ponderosa pine stands cannot be accurately evaluated in aerial photographs at the present time. Since no attempt was made in this study to correlate the cull factor with height, crown diameter, site, or a combination of these variables, a prediction as to the possibilities of estimating the percentage of cull on aerial photographs cannot be made in this conclusion.

At the present time, estimation of cull in aerial inventories of the ponderosa pine type must be obtained from either a given cull and breakage percentage, or from a limited field sample. The cull and breakage factor for ponderosa pine has been established by the U. S. Forest Service in this region from measurements made of this species in past logging operations. It will run, on an average, between ten and twenty-five percent of the gross stand volume. Factors which influence the application of cull and breakage deductions are: age of stand, presence and intensity of fire scars, site, exposure, terrain characteristics, etc.

In this study, the stand is in the thrifty mature stage, with fire scars the only apparent defect. These occur
on about forty percent of the mature stems, and will cull one-quarter of the butt log.

The terrain is steep on the study area, but there are few rock outcrops, or other similar obstacles, which would hinder logging.

From these conditions it is estimated that the cull factor will run about twelve percent, and the breakage, three percent; a total deduction of fifteen percent of the gross volume. The net volume of the aerial cruise (3-variable solution for volume method) made of the Benchmark Creek study area is then 1,746,859 feet, b.m. minus the fifteen percent deduction, or 1,484,830 feet, b.m.

The other method of estimating cull deduction requires that a small percentage of the plots taken in an aerial cruise be checked in the field. This check cruise should not have to consist of more than ten percent of the total number of plots taken in the aerial cruise, and they should be selected at random so an unbiased correction factor is obtained. The field sample will not only serve as a check on the cull factor, but also on species identification and photographic measurements of height, crown diameter, volume, and site.

**Photography**

Aerial photographs which are to be used for aerial
inventory purposes should meet certain specifications as to: (1) photographic scale, (2) type of photography, and (3) quality of photography.

The selection of a suitable photographic scale for aerial inventory of the ponderosa pine type requires that four factors be considered. These are: (1) cost of photography, (2) accuracy desired in inventory work, (3) stand characteristics, and (4) multiple uses, or objective of photography. The scale finally selected will usually be a compromise between two or more of these factors. For example, if the funds available for a certain project are limited, but the vegetative characteristics require medium to large-scaled photography in order to accomplish the original objectives, a compromise scale of 1:13,840 might be decided upon. However, in another, similar project the characteristics of the vegetation might only require a scale of 1:20,000 to accomplish the same objective.

Since large-scaled photography is usually too expensive for one project, it could be justified by considering its application to other forest activities. For example, large-scaled photography would not only facilitate engineering, but also game and watershed management, recreation, fire control, and forest management.

It is the opinion of the author that the best compromise scale for aerial photographs, on which are to be made
inventories of ponderosa pine stands, is either 1:12,000, or 1:15,840. Which of these scales is selected depends on the characteristics of the stand under consideration. For example, if the stand is similar to that found in section I of the study area (Figure 1), the latter scale would be selected. If the stand was similar to section III of the study area, the former scale would be considered best.

The type of photography selected for a particular set of forest conditions is an important factor to consider in aerial inventory work. Different types of photography will resolve different forest values. Therefore, before a certain area is photographed, it must be determined which film-filter combination will yield the best results in accordance with the character of vegetation and objective of photography.

At the present time there are three aerial films, and four filter combinations commonly used for forest aerial photography. These are: (1) panchromatic film and minus-blue filter, (2) panchromatic film and green filter, (3) infrared film and red filter, (4) infrared film and minus-blue filter, and (5) Aero-color, or Kodochrome film. (6)(11)(20)

In the Northern Rocky Mountain Region panchromatic minus-blue photography has the widest application. This type of photography is considered best for the many different
forestry purposes aerial photographs are used for in this region. For example, the two principal uses of aerial photographs are forest management and topographic mapping. If forest management alone is considered, there are many instances where infrared photography would give better results in the identification of species and delineation of forest types. However, this type of photography is of little value for mapping purposes because shadows register in true blacks and, consequently, obscures all ground detail falling within the shadow area. Panchromatic minus-blue photography is a compromise between these two factors. This type of photography may be used for both mapping and forest management purposes. Although distribution of forest types is not as apparent on panchromatic photographs, more can be learned about the individual tree, and ground features are not obscured by shadows.

The use of color photography for aerial inventory purposes in this region is an unexplored potential. However, experimental insect-damage surveys using aerial color film indicates that this type of photography might provide a solution to the problem of species identification on aerial photographs. At the present time this type of photography is too expensive for practical application to any, or combination of any, forest activities.
The type of photography which is best suited for aerial inventories of ponderosa pine stands cannot be accurately determined because only panchromatic photographs were available for study. However, the photographic characteristics of each type of photography is known, and a prediction can be made as to which one will best resolve the desired forest values.

The main problem in aerial inventories of most ponderosa pine stands will be species identification. Therefore, the type of photography is needed which will facilitate the segregation of the different species without eliminating other necessary photographic characteristics. There will be some cases, i.e., pure ponderosa pine stands, where the problem of species identification does not exist.

In this analysis color photography is eliminated on the basis of prohibitive cost. However, if cost were not a factor, this type of photography would, in all probability, be best for aerial inventory purposes.

By eliminating color photography, the choice is now between infrared and panchromatic photography. Which of these types would be best for aerial inventories of ponderosa pine stands is difficult to accurately predict. The solution of this problem probably lies in the characteristics of the ponderosa pine stand under consideration. If it contains Douglas fir of commercial value, infrared minus-blue photography would probably be best in this case because of the greater tonal contrast it produces between the different
species. However, if the stand under consideration is composed only of ponderosa pine, then panchromatic minus-blue photography is superior to any type of infrared photography. This type of photography will yield more information concerning the individual tree. In aerial inventories whose basis is individual tree measurements, this is an important factor.

These assumptions as to the type of photography which would be best for a particular set of conditions is theoretical. They cannot be accepted as definite conclusions because they have as their basis experimental aerial surveys made in other regions of different forest types. What type of photography is best for a certain timber type is this region cannot only be established by intensive photographic tests of that type.

The quality of aerial photographs used for aerial inventory purposes will greatly affect the accuracy of volume estimations. For example, if there is an excessive amount of "tip and tilt" present, accurate measurements of tree heights cannot be made with parallax instruments. Accuracy of height measurement is also affected by the amount of parallax present in the aerial photographs. If there is too little, stereovision is hampered by the lack of displacement, and accurate height measurements are impossible. If there is too much parallax, difficulty in fusing the tops of tall objects under a stereoscope is experienced.
Another important factor in photographic quality is resolution. This is a function of scale, image movement, film-filter combination, and contrasting backgrounds. If resolution is poor, the tree images are blurred and, consequently, species identification and height measurements are not accurate.

Preparation of Necessary Tree-Aerial Volume Tables

In order to prepare ponderosa pine tree-aerial volume tables for regional use would involve collecting thousands of random measurements throughout the region. The cost of such a project is enormous and would have to be balanced against the potential advantages of aerial inventories. This factor, plus the additional cost of necessary photo-interpretation equipment and the lack of competent photo interpreters, would require a thorough study of the problem before such a project is initiated.

If such a project was initiated, the methods used to prepare tree-aerial volume tables on a regional scale would be similar to those employed in this study. However, only the 3-variable solution for volume method is recommended for such a project. The coefficient and 3-variable solution for d.b.h. methods are not statistically sound, and results of this study indicate that more reliable volume estimates can be obtained by the recommended method.
Since the site quality has a pronounced influence on the tree form of ponderosa pine, the aerial volume tables should be segregated according to broad site classifications. This will require more work to prepare the volume tables, but the resulting increase in accuracy of aerial volume estimations would make it worthwhile. Each site classification data should contain enough measurements so that the 3-variable solution for volume is applicable. The same data used in the preparation of the volume tables may also be used to prepare site quality curves based on the ratio between total height and crown diameter. These two solutions of the same data will complement each other. In an aerial cruise the tree measurements are first solved for the average total height/crown diameter ratio and site quality index determined. The volume table prepared for the computed index is then used to estimate volume.

Field techniques used in collecting the necessary aerial volume table data will differ slightly from those used in this study. For example, crown diameter measurements should be a four-way average instead of in a north-south direction. The tree volume should also be determined by falling and bucking each tree and scaling out the volume of each 16-foot log, instead of estimating the volume from d.b.h. and merchantable height. This factor should increase the accuracy of estimating volume from crown diameter and total height.
Another important factor to consider is the correction of actual total height and crown diameter to visible total height and visible crown diameter. Since a wide variety of photographic scales will be used in regional aerial surveys, the best method would be to prepare the aerial volume tables on the basis of actual total height and crown diameter. A percentage correction for visible height and crown diameter could then be computed for broad classifications of photographic scale. The correction factors could easily be computed by making precise measurements of a few trees on aerial photographs of various scales, and comparing these with their corresponding field measurements. The computation of correction factors is complicated by the variation exhibited in height and crown diameter of trees growing on sites of different qualities. For example, due to the denser and larger crowns of trees growing on poor sites, the percent reduction for "visible" measurements would be less for these trees, than for trees growing on sites of higher quality.

Summary

If aerial cruises of ponderosa pine stands in western Montana are considered for practical application, certain procedures, or rules, on how they are to be conducted will have to be established. This would involve stand characteristics as to density and size, intensity of cruise, size of
aerial plots, estimation of stem quality, species identification, and other factors which will influence the accuracy of an aerial cruise. For example, if the stand under consideration is open and composed of mature ponderosa pine trees, the procedure might be: (1) that the cruise intensity will be twenty percent, (2) size of aerial plots will be one acre, and (3) at least ten percent of the total number of plots taken in the aerial cruise will be checked in the field as to species, cull percentage, and accuracy of volume estimations.

In establishing a certain set of procedures, all of the above mentioned factors would have to be evaluated by further study of the problems involved in aerial inventories of the ponderosa pine type. There is no accurate way to predict from this limited study what the procedure should be for an aerial inventory of a certain type of ponderosa pine stand.
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APPENDIX
PHOTOGRAMMETRY FACTORS INVOLVED
IN AERIAL INVENTORIES

The principal photogrammetry factors which must be known before tree height and crown diameter measurements can be made on aerial photographs are: (1) photographic scale, (2) absolute parallax base, and (3) constant for parallax wedge, or academy height finder (HF-2).

Photographic Scale

In this study photographic scale was determined by the following procedure:

(1) If the aerial photograph shown in Figure 1 is examined, two baselines, designated A and B are seen. Baseline A is located in the canyon bottom of Piquette Creek, and baseline B is located on the ridge top, near the upper boundary of the study area. Although one baseline would have been sufficient, two were used for check purposes.

(2) Baseline A is two inches long on the aerial photographs, while baseline B is one and two-tenths inches in length. These lengths were arbitrarily chosen. Neither baseline has definite terminals. Using a G. L. O. base sheet of 1:12,000 scale as control, these baselines were transferred from the aerial photographs to the base sheet by means of a radial
line plotter. The length of baseline A on the base sheet is 2.46 inches, and baseline B, 1.20 inches (Figure 2). If the scale of the base sheet is 1:12,000, then the scale of the photograph at baseline A is 1:14,760, and at baseline B, 1:12,000.

(3) The principal formula for computing scale of aerial photographs is

\[ RF = \frac{f}{H} \quad (1) \]

where

- \( RF \) = a representative fraction
- \( f \) = focal length of camera (feet)
- \( H \) = flying height of camera station

If the flying height and focal length are given, then the RF scale of any aerial photograph may be computed by the above formula. However, in actual practice the value of the flying height is seldom given. On the other hand, focal length is usually always known. Consequently, when aerial photographs are to be used for aerial inventory purposes of an area where there is a large difference in elevation, it will be necessary to establish a baseline in the field similar to the ones used in this study. From this baseline the scale of the aerial photograph at that point may be determined, and if the focal length is known, the flying height of the camera station
above the baseline may be computed by reversing the above formula and solving for H. The scale for any given point on the area covered by the aerial photograph may then be obtained by scaling, from a topographic map of the area, the difference in elevation between the baseline and given point, and substituting in the formula the new value for flying height. For example, the given focal length in this study is 8.25 inches. The flying height of the camera station above baseline A is

\[
\frac{1}{4,750} = \frac{6875}{H}
\]

or

\[H = 10,148 \text{ feet}\]

From a topographic map of the area the difference in elevation between baseline A and baseline B was found to be 1,800 feet. Substituting this value in the formula, the scale at baseline B is

\[
RF = \frac{6875}{10,148 - 1800}
\]

or

\[RF = \frac{1}{12,143}\]

However, the scale determined from the baseline
at that point is 1:12,000. The error then is 1:143 of RF scale, or 98 feet in elevation. The actual difference in elevation between baseline A and baseline B is 1,898 feet.

**Absolute Parallax Base**

The absolute parallax base is nothing more than the distance between successive camera stations, converted to units of photographic scale. Since photographic scale will vary according to flying height of the camera station, successive aerial photographs of an area having a large difference in elevation, will have many different values of absolute parallax base. To illustrate what absolute parallax base is, and how it is computed, the stereo-pair of aerial photographs used in this study are taken as an example.

In order to determine the absolute parallax base of aerial photographs, #0-1-22 and #0-1-23, Piquette Creek survey, the following procedure was used:

1. The distance between the principal point and conjugate principal point of photograph #0-1-22 was

---

1 The principal point is the optical center of an aerial photograph. If there is no tip or tilt present, it will be the actual center of the area photographed. The conjugate principal point of a photograph is the principal point of its adjacent photograph. Aerial photographs are usually taken with a 60 percent overlap, which makes possible this projection of principal points.
The difference in elevation, as scaled from a topographic map, between baseline A and the principal point projected on the ground is 800 feet, and between the baseline and the conjugate principal point, 1,100 feet. The scale at the principal point is

\[ RF = \frac{.6875}{10,143 - 800} \]

or

\[ RF = \frac{1}{13,597} \]

and at the conjugate principal point

\[ RF = \frac{.6875}{10,143 - 1100} \]

or

\[ RF = \frac{1}{13,151} \]

The average scale for the principal and conjugate principal points is

\[ RF = \frac{1}{13,379} \]

or

1 inch = 1,115 feet

If the distance on the aerial photograph between
the principal and conjugate principal points is 3.05 inches, then by applying the average scale found in (3) the actual distance between successive camera stations is 3.05 times 1,115, or 3,400 feet.

(5) The distance between the principal and conjugate principal points of photograph #0-1-23 was then measured and found to be 2.37 inches. Since the scale at baseline A is the same for both photographs, the actual distance between the same successive camera stations is 2.87 times 1,115, or 3,200 feet.

(6) Thus, there is an error of 200 feet between the measurements. This is due to photographic distortion. To compensate for this error an average value is computed. In this case the average distance between successive camera stations is 3,300 feet.

(7) The distance between successive camera stations established, it is now possible to determine the absolute parallax base of any point on the photograph by determining the flying height of the camera station above the point and applying this value according to the formula

\[ P = \frac{Bf}{h} \quad (2) \]

where

\[ P = \text{absolute parallax base} \]
\[ B = \text{distance between successive camera stations (feet)} \]
\[ f = \text{focal length (inches)} \]
\[ H = \text{flying height (feet)} \]

(8) For example, the absolute parallax base for baseline A is

\[ P = \frac{3,300 \times 8.25}{10,019} \]

or

\[ P = 2.68 \text{ inches} \]

and for baseline B

\[ P = \frac{3,300 \times 8.25}{10,118-1.398} \]

or

\[ P = 3.30 \text{ inches} \]

(9) The actual value of \( P \) does not change, it is only the photographic value which changes. The influence of absolute parallax base on the accuracy of height measurements will be illustrated in the following section.

Constant for Parallax Wedge

The constant for the parallax wedge is usually expressed as so many feet of height per one-thousandth of an inch of
parallax difference at 1:1,000 scale. It may also be expressed in terms of the flying height of the camera station above object being measured.

The formulae used to determine the constant for the parallax wedge are

\[ h = \frac{f(dp)}{H(p + dp)} \quad (3) \]

and

\[ h = \frac{H(dp)}{p + dp} \quad (4) \]

where

- \( h_0 \) = height in feet
- \( f \) = focal length in feet
- \( dp \) = parallax difference
- \( p \) = absolute parallax base
- \( H \) = flying height

In actual practice, \( dp \) in the numerator of the above formulae is ignored. Its influence on the final results is insignificant.

For illustration purposes, the computation of the parallax wedge constants for height measurements at baselines A and B are used as examples.

The constant for baseline A is

\[ h = \frac{0.6675}{1,000} \left( \frac{0.001}{1.00} \right) \]

or
\( h = 0.256 \text{ feet} \)

Since the scale at baseline A is 1:14,760, the above value would have to be multiplied by 14.76 to obtain the constant for that particular scale. The constant would then be 3.78 feet. If formula (4) was used, the constant is

\[
\frac{10.123(\cdot001)}{2.68+\cdot001}
\]

or

\[ h = 3.78 \text{ feet} \]

which checks with the results of formula (3). If a tree was measured at that point and found to have a parallax difference of 0.030 of an inch, the height of the tree would be 30 times 3.78, or 113.4 feet.

For baseline B, the constant is

\[
\frac{8250(\cdot001)}{3.30+\cdot001}
\]

or

\[ h = 2.50 \text{ feet} \]

The height of a tree at this point having a parallax difference of 0.030 of an inch, would be 75 feet in height.

The above difference between height measurements of trees having the same parallax difference illustrates the error which would result if the absolute parallax base and scale were not corrected for changes in elevation.
Constant for Academy Height Finder

The constant for the academy height finder is determined by the same formulae used to compute the parallax wedge constant. However, this instrument measures parallax difference in 0.01 millimeters. Therefore, it is necessary to convert these units before substituting in the formulae. For example, if 0.01 millimeters equals 0.000394 inches, then the constant for baseline A is

\[ h_o = \frac{10.118(0.000394)}{2.68+0.000394} \]

or

\[ h_o = 1.49 \text{ feet} \]

and the constant for baseline B is

\[ h_o = \frac{2250(0.000394)}{3.30+0.000394} \]

or

\[ h_o = 0.99 \text{ feet} \]

A much simpler method for determining the constant for the height finder is by means of a photogrammetric computer. This is nothing more than a circular slight rule graduated to units of scale, flying height, and absolute parallax base. Using this instrument, the constant for baseline A is 1.50, and baseline B, 0.99.
DETERMINATION OF SUN'S ALTITUDE ON DATE AND TIME OF PHOTOGRAPHY

In order to determine heights of trees on aerial photographs from shadow measurements, the sun's altitude on the date and time of photography must be known. Once the sun's altitude is known, the height of any tree may be determined by the application of simple trigonometric functions.

In this study the altitude of the sun on the date and time photographs #0-1-22 and #0-1-23, Piquette Creek survey, were taken was determined as follows:

(1) In order to determine the sun's altitude the date, time, longitude, and latitude of the photography must be known. In this example, the date of photography was August 10, 1946; the time, 1308; the longitude, 114°11'; and the latitude, 45°52'.

(2) The next step is to determine the sun's declination and the equation of time on the above date and apply the hourly correction factor for the time. These may be found in a solar ephemeris. For August 10, 1946, at 1308 hours, the sun's declination is north 15°47', and the equation of time is 5 mins. 14 secs. The latter value means that on this date true solar time is 5 mins. 14 secs. behind mean solar time.

(3) Determination of the hour angle is the next step in the solution of this problem. By hour angle is
meant the angle formed at the north celestial pole by the projection of the meridian of longitude passing through the area of photography, and the meridian of the sun at the instance of photography. To solve for the hour angle it is first necessary to determine the angular distance between the time meridian of the region and the meridian of the sun at the instance of photography. In this region the time meridian is the 100 degrees of longitude. The instance the mean sun passes this meridian it is 1200 hours throughout the time belt. However, on this date the true sun sees five minutes slow. Consequently, the sun did not pass the time meridian until 1205 hours. Therefore, the time differential is 1205 minus 1200, or 5 hr. 03 mins.. If one hour of time equals 15 degrees of longitude, the angular distance between the time meridian and the meridian of the sun at the instance of photography is 15 degrees, 47 mins.. The longitude of the sun is then 100 degrees plus 15 degrees, 47 mins., or 120 degrees, 47 mins.. Subtracting the longitude of photography from this figure, a value of 6 degrees, 32 mins. is obtained.

(4) All of the values necessary to solve for the sun's altitude are now known. These values are substituted in the formula

\( \sin h = \cos d \cos l \cos t \sin d \sin l \)
-13-

where

\[ h = \text{sun's altitude} \]
\[ d = \text{sun's declination} \]
\[ L = \text{Latitude} \]
\[ t = \text{hour angle} \]

In the above formula the latter part is added if the sun is north of the equator, and subtracted if south of equator.

(5) Solving formula (5) the altitude of the sun was found to be 59°34'.

(6) The shadow factor, or height per one thousandth of an inch of shadow length at a given scale, may now be determined. This factor is a function of photographic scale, percent of slope upon which the shadow is cast, and whether the slope is positive or negative in regards to the direction of the shadow. For example, at baseline A the shadow factor for level ground is 2.09, or 0.001 of an inch of shadow length equals 2.09 feet of tree height. If the shadow of a tree at this point was 0.050 of an inch in length, the height of the tree would be 50 times 2.09, or 104.5 feet. For a positive slope of 45 percent the shadow factor would be 2.42, and for a negative slope of the same value, 1.40. A shadow length of 0.050 of an inch on the positive slope would indicate a tree height of 121 feet;
and on the negative slope, a height of 70 feet.

(7) Slope has a very pronounced influence on the accuracy of height measurements by the shadow method. Where the terrain is steep and mountainous the application of this method, except on a limited scale is not practical. Its use necessitates the determination of slope percentage on aerial photographs by either the parallax wedge, or the academy height finder. The reason it was used in this study was because there were some instances in which the shadow of the tree could be discerned on the aerial photographs, but not the tree casting the shadow. This includes trees having a very sparse crown, and those obscured by other trees.

COEFFICIENT METHOD OF PREPARING TREE-AERIAL VOLUME TABLES

In this method the coefficient $\frac{kh}{d^2}$ was computed for every tree measured in the field. These values were grouped according to 10-foot height classes and the average coefficient, and average height determined. The average coefficient value was then plotted over height on cross-section paper. Figure 1 illustrates the coefficient curve for ponderosa pine all sites data; Figure 2 for canyon sites; Figure 3 for slope sites; Figure 4 for exposed sites; and Figure 5 for Douglas fir data. Because of the wide variation between the individual averages, it was difficult to determine the trend of the regression.
It was finally decided that the correlations were linear in nature.

The standard error of estimate was computed for each set of data by applying the estimated coefficient values to the measured height and crown diameter and obtaining d.b.h. from

\[ d.b.h = \frac{kh}{2} \]

The difference between the estimated value of d.b.h. and the original value was then determined for each measurement. This difference was squared, and the sum of the squared differences obtained substituting the value of the sum of the squared differences in the formula

\[ S = \sqrt{\frac{s(d)^2}{N-1}} \]

and solving for S to obtain the standard error of estimate.
TABLE I

Ponderosa Pine

Tree-Aerial Volume Table for All Sites.*

<table>
<thead>
<tr>
<th>Visible Crown Diameter (feet)</th>
<th>TOTAL VISIBLE HEIGHT</th>
<th>55</th>
<th>65</th>
<th>75</th>
<th>85</th>
<th>95</th>
<th>105</th>
<th>115</th>
<th>125</th>
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*Based on the coefficient method.
TABLE II
PONDEROSA PINE

TREE-AERIAL VOLUME TABLE FOR CANYON SITES*

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*Based on the coefficient method.
### TABLE III

PONDEROSA PINE

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*Based on the coefficient method.*
**TABLE IV**  
FONDEROSA PINE

**TREE-AERIAL VOLUME TABLE FOR EXPOSED SITES**  

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*Based on coefficient method.*
Figure I.
Figure 3.
Figure 4.
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*Based on the coefficient method.*
Figure 5.
THREE-VARIABLE SOLUTION FOR D.B.H. METHOD OF PREPARING TREE-AERIAL VOLUME TABLES

In this method the data were listed according to total height classes and solved by the least squares method of statistical analysis. The resulting d.b.h.-crown diameter curves are found in Figure 6, for Ponderosa Pine, and in Figure 7, for Douglas Fir.

Equation for the curve of the Ponderosa Pine data is:

$$\text{D.B.H.} = 0.18 \text{ total height} \ 0.81 \text{ crown diameter} - 0.74$$

and the equation for Douglas Fir is:

$$\text{D.B.H.} = 0.16 \text{ total height} \ 0.48 \text{ crown diameter} \ 0.66$$

An examination of the analysis of Ponderosa Pine data reveals that the correlation coefficient for total height and d.b.h. is 0.67; for crown diameter and d.b.h., 0.71; for total height and crown diameter, 0.28; and for total height, crown diameter, and d.b.h., 0.36. These results indicate that 45 percent of the variance in d.b.h. is associated with total height, 50 percent is associated with crown diameter, and 74 percent is associated with the combination of total height and crown diameter.

If these results are compared with those of Nash (7) considerable difference is noted. According to this investi-
The correlation between height and d.b.h. for eastern white pine is 80 percent, and nothing was mentioned concerning the correlation between crown diameter and d.b.h. Apparently the correlation between these two variables was insignificant. The opposite is concluded in this study: there is a stronger correlation between crown diameter and d.b.h. of Ponderosa Pine than there is between total height and d.b.h.

Similar results were obtained in the analysis of Douglas Fir data. The correlation coefficient for crown diameter and d.b.h. is 0.80; for total height and d.b.h., 0.76; for crown diameter and total height, 0.33; and for total height, crown diameter, and d.b.h., 0.96. These results indicate that 64 percent of the variance in d.b.h. is associated with crown diameter; 57 percent is associated with total height; and 91 percent is associated with the combination of total height and crown diameter.

Besides the statistical analysis made of the entire set of Douglas Fir data, another analysis was made of individual height class data by the same statistical method. This analysis serves no useful purpose other than to illustrate the influence height class curves have on each other. This influence is illustrated in Figures 8 through 13.
TABLE VI

PONDEROSA PINE

TOTAL VISIBLE VOLUME FOR ALL SITES*

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*Based on the 3-variable solution for d.b.h.*
Figure 6.


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*Based on the 3-variable solution for d.b.h.*
Figure 7.
Douglas Fir
50-59' ht. class

Figure 8.
Figure 9.
Douglas Fir
70-79' ht. class

Figure 10.
Figure II.
Douglas Fir
90-99' ht. class

Figure I2.
Figure 13.
The estimation of d.b.h. from total height and crown diameter is only the first phase in determining tree volume by the coefficient and 3-variable solution for d.b.h. methods. The next phase is to determine the volume from the estimated d.b.h. and measured total height. Conventional volume tables cannot be used because volume is correlated with merchantable height; a value not measurable on aerial photographs. Therefore, it is necessary to prepare intensity curves in which total height is substituted for merchantable height as the independent variable.

In Figures 14 through 23 are found the volume curves for Ponderosa Pine. These curves were prepared by obtaining the average volume value for each d.b.h. and total height class. Volume was then plotted over crown diameter by height classes.

Volume curves were prepared in a similar manner from the original Douglas Fir data. These curves are found in Figures 24 through 29. Separate volume curves were also prepared for each Ponderosa Pine site volume table.

The 3-variable solution for volume method differs from the other two methods in that d.b.h. is eliminated entirely.
Figure 14.

Ponderosa Pine
50-59' ht. class
Figure 15.

Ponderosa Pine

60-69' ht. class

Volume-Scribner Dec. c

DBH-inches

Figure 15.
Figure 16.
Ponderosa Pine
80-89' ht. class

Figure 17.
Figure 18.

Ponderosa Pine
90-99' ht. class

Volume-ScribnerDec.C

16 20 24 28 32 36 40 44 48

DBH-inches
Figure 19.

Ponderosa Pine
100-109' ht. class

Volume-Scribner Dec. C

DBH-inches

Figure 19.
Figure 20.

Ponderosa Pine
III 0-II 9' ht. class
Figure 21.

Ponderosa Pine
120-129' ht. class
Ponderosa Pine
130-139' ht. class

Figure 22.
Douglas Fir
50-59' ht. class

Figure 24.
Douglas Fir
60-69' ht. class

Figure 25.
Douglas Fir
70-79' ht. class

Figure 26.
Douglas Fir
80-89' ht. class

Figure 27.
Douglas Fir
90-99' ht. class

Figure 28.
Instead tree volume is directly correlated with crown diameter and total height. This method eliminates the necessity of preparing volume curves and, although it has a much higher standard error of estimate in comparison to the two d.b.h. methods, there is no question as to the statistical validity of this method. In Figure 30 are illustrated the intensity curves prepared by this method for estimating the volume of Ponderosa Pine from crown diameter and height measurements. Figures 31 and 32 illustrate intensity curves for estimating the volume of Douglas Fir. In the former figure the logarithmic method of statistical analysis was used to solve the data, and in the latter the least squares method.

The procedure used in the solution of this method is as follows:

1. The data were listed according to 10-foot height, and 2-foot crown diameter classes.
2. The average volume value for each height and crown diameter was determined.
3. This average volume value was then plotted over crown diameter by height classes.
4. From the graphical solution of the Ponderosa Pine data, the statistical method which would best fit the data was selected. Two methods, least squares and semi-logarithmic, were considered, with the former finally being chosen.
(5) The graphical solution of the Douglas Fir data indicated that both the logarithmic and least squares methods were possible solutions. Consequently, it was necessary to solve the data by both methods and compare the resulting standard errors of estimate.

(6) Equation of curve for Ponderosa Pine data is

\[ \text{Vol.} = 9.34 \text{ crown diameter} \times 2.79 \text{ total height} - 263.6 \]

and the curve equations for Douglas Fir data are

\[ \log \text{Vol.} = 1.31 \log \text{crown diameter} \times 2.54 \log \text{height} - 3.69 \]

and

\[ \text{Vol.} = 38.6 \text{ crown diameter} \times 10.5 \text{ total height} - 1056.6. \]

(7) Examination of the least squares analysis of Ponderosa Pine data reveals that 50 percent of the variance in volume is associated with crown diameter, 64 percent is associated with the combination of crown diameter and total height. In the 3-variable solution for d.b.h. it was found that the correlation between d.b.h. and crown diameter was greater than the correlation between d.b.h. and total height. In this analysis the correlation between volume and total height is greater than between volume and crown diameter. Therefore, from these analyses it can be
concluded that crown diameter has the greatest influence on d.b.h., while total height will have the greatest influence on tree volume. Another interesting comparison between these two methods concerns the correlation coefficients for total height and crown diameter. In the 3-variable solution for d.b.h. the correlation coefficient for these two variables is 0.28, and the 3-variable solution for volume, 0.31. This would mean that the relationship between the two independent variables remains the same, regardless of whether the dependent variable is d.b.h. or volume.

(8) Examination of the logarithmic analysis of Douglas Fir data reveals that the correlation coefficient for volume and crown diameter is 0.8226 logarithmic units; for volume and total height, 0.8288 logarithmic units; and for volume and the combination of crown diameter and total height, 0.983 logarithmic units. The standard error of estimate, expressed as a percentage of the mean, is 26 percent.

(9) In the least squares analysis of this species it was found that 71 percent of the variance in volume is associated with crown diameter, 52 percent is associated with total height, and 85 percent is associated with the combination of crown diameter and total height. The standard error of estimate, expressed as a percentage of the mean is 34 percent.
(10) If the two analyses used to solve the Douglas Fir data are compared on the basis of the standard error of estimate, expressed as a percentage of the mean, the logarithmic solution is the more accurate.

A separate analysis was also made of each Douglas Fir height class data. Since the individual height class curves did not conform to any general pattern, more than one statistical method was necessary in solving the series. In Figures 33 through 38 are illustrated the different methods used to solve each height class data.

To illustrate how these different statistical methods were applied, Figure 34 is taken as an example. In this height class the average volume values, when plotted graphically, did not exhibit any definite trend. Because of the wide variation between the individual average points, it was impossible to even select the statistical method which might provide the best fit. Consequently, in order to obtain the most accurate solution, it was necessary to apply the least squares, logarithmic, and semi-logarithmic methods to the data. The results of these methods were compared on the basis of the standard error of estimate, expressed as a percentage of the mean, and the one with the smallest value was selected.

The curve equation obtained by the analysis of the height class data by the least squares method is
Vol. = 21.2 crown diameter - 98.8

and the standard error of estimate, expressed as a percentage of the mean is 19 percent. For the logarithmic method the curve equation is

\[
\text{Log. Vol.} = 1.69 \log \text{crown diameter} + 0.34035
\]

and the standard error of estimate, 17 percent. For the semi-logarithmic method the curve equation is

\[
\text{Vol.} = 601.1 \log \text{crown diameter} - 489.9
\]

and the standard error of estimate, 12 percent.

A comparison of these results indicates that the most accurate solution or best fit, is provided by the semi-logarithmic method of statistical analysis.
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*Based on the 3-variable solution for volume.
Figure 30.

Ponderosa Pine

Crown Diameter-feet

Volume-Scribner Dec.C

100 200 300 400 500

85' 95' 105' 115' 125' 135' 145'

55' ht. class
### TABLE IX

**DOUGLAS FIR**

**TREE-AERIAL VOLUME TABLE**

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*Base on 3-variable solution for volume - logarithmic analysis.*
TABLE X

DOUGLAS FIR

TREET-AERIAL VOLUME TABLE

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Douglas Fir

Figure 31.
Figure 32.

Douglas Fir

Volume-Scribner Dec. C

Crown Diameter-feet

55' ht. class

65'

75'

85'

95'

105'
Douglas Fir
50-59' ht. class

Logarithmic
Least Squares
Semi-logarithmic

Volume-Scribner Dec.C

Crown Diameter-feet
Figure 33.
Volume-Scribner Dec.C

Crown Diameter-feet

20 40 60 80 100

8

12

16

20

24

28

32

36

Douglas Fir

60-69, ht. class

Figure 34.
Douglas Fir
70-79' ht. class

Figure 35.
Douglas Fir
80-89' ht. class

Volume-Scribner Dec.C

Crown Diameter-feet

Figure 36.
Douglas Fir
90-99' ht. class

Figure 37.
Douglas Fir
I00-I09' ht. class

Figure 38

Crown Diameter-feet

Volume-Scribner Dec.C

Least Squares

Logarithmic