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Applying ant colony optimization (ACO) metaheuristic to solve forest transportation planning problems with side constraints

Marco A. Contreras

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APPLYING ANT COLONY OPTIMIZATION (ACO) METAHEURISTIC TO
SOLVE FOREST TRANSPORTATION PLANNING PROBLEMS WITH
SIDE CONSTRAINTS

by

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for the degree of
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Approved by:

Chairperson
Dean, Graduate School

Date
Applying Ant Colony Optimization (ACO) Metaheuristic to Solve Forest Transportation Planning Problems with Side Constraints

Chairperson: Dr. Woodam Chung

Timber transportation is one of the most expensive activities in forest operations. Traditionally, forest transportation planning problem (FTPP) goals have been set to find combinations of road development and harvest equipment placement to minimize total harvesting and transportation costs. However, modern transportation problems are not driven only by economics of timber management, but also by multiple uses of roads and their social and ecological impacts. These social and environmental considerations and requirements introduce side constraints into the FTPP, making the problem larger and much more complex. We develop a new problem solving technique using the Ant colony optimization (ACO) metaheuristic, which is able to solve large and complex transportation planning problems with side constraints. A 100-edge hypothetical FTPP was created to test the performance of the ACO metaheuristic. We consider the environmental impact of forest road networks represented by sediment yields as side constraints. Results show that transportation costs increase as the allowable sediment yield is restricted. Four cases analyzed include a cost minimization, two cost minimization with increasing level of sediment constraint, and a sediment minimization problem. The solutions from our algorithm are compared with solutions obtained from a mixed-integer programming (MIP) solver used solve a comparable mathematical programming formulation. For the cost minimization problem the difference between the ACO solution and the optimal MIP is within 1%, and the same solution is found for the sediment minimization problem. The current MIP solver was not able to find a feasible solution for either of the two cost minimization problems with a sediment constraint.

Key words: Forest transportation planning, ant colony optimization metaheuristic, forest road networks.
ACKNOWLEDGEMENT

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PREAMBLE

This thesis is composed of two parts, Part I introduces forest transportation planning problems and the ant colony optimization metaheuristic. Part II is a manuscript prepared for publication. Part I consists of: i) a more detailed introduction to various forest transportation planning problems and the optimization techniques that have been used to solve such problems, ii) the type of forest transportation planning problems addressed in this thesis, and iii) a detailed description of the ant colony optimization metaheuristic.

Part II is in the format of a manuscript for submission to a scientific journal describing the research under a number of subheadings. The abstract at the beginning of this thesis will be submitted as part of the publishable paper.

PART I. FOREST TRANSPORTATION PLANNING PROBLEMS AND THE ANT COLONY OPTIMIZATION METAHEURISTIC.

Introduction

Problems related to forest transportation planning have been an important concern since the beginning of the last century, due to the fact that timber transportation is one of the most expensive activities in forest operations (Greulich 2002). The cost of timber transportation activities may reach 30-40% of the total forest operation costs, and 50-60% of the total manufacturing cost of finished forest products (Neuenschwander 1998).
In general forest transportation planning problems (FTPP) can be divided into; off-road and on-road phases, which are heavily dependent on each other (Heinimann 2001). Off-road activities are related to wood transportation from the stump location to either roadside or to centralized landings. On-road activities refer to wood transportation on ground vehicles to final destinations.

Two different approaches have been applied to solve FTPP: exact algorithms and approximation algorithms. Exact algorithms use mathematical programming techniques, such as Linear Programming (LP), Integer Programming (IP) and Mixed-Integer Programming (MIP). Approximation algorithms, generally called heuristics, consist basically of evaluating a large number of feasible solutions and selecting the best. The most important advantage of exact algorithms is that they provide optimal solutions. However, they are limited to small scale problems. Contrarily, heuristic techniques, although they may not provide optimal solutions, have successfully been applied to solve large scale problems and are relatively easy to formulate compared with exact algorithms (Jones 1991; Weintraub 1994, 1995; Martell et al 1998; Falcao 2001; Olsson 2003).

Since integer and mixed integer models can represent transportation problems in a better way than continuous variable models, due to the discrete nature of FTPP variables such as road building, IP and MIP have received attention in the past years. On the other hand, IP and MIP models are restricted to solve small to medium scale problems due to their relatively large computational complexity (Weintraub 1995; Olsson 2003). Since real world problems are usually large scale problems with thousands of variables, heuristic
techniques have been the focus of a large number of researchers (Zeki 2001; Boyland 2002). In addition, advances in Geographic Information Systems (GIS) have made possible the creation and manipulation of data representing large areas, facilitating the creation of large scale problems. Besides, since some FTPP do not have a formal mathematical formulation derived, exact algorithms cannot be applied (Murray 1998). An example is the problem of building a road network in a forested region that provides access to identified timber sales while minimizing overall road building costs. This problem has been defined by Dean (1997) as the multiple target access problem (MTAP), which have only been solved by heuristic approaches (Murray 1998).

Some approaches combining MIP with heuristic techniques have also been developed (Martell et al 1998; Boyland 2001). Although these approaches intend to capture the advantages of both techniques, they improve the efficiency of exact algorithm while providing only partial optimal solutions, thus making a trade-off between efficiency (given by heuristics) and solution quality (given by exact algorithms).

Most FTPP considering fixed and variable costs are complex optimization problems that to date can often only be solved using heuristic approaches, mainly because of two reasons. First, there is not a formal mathematical formulation that can adequately represent the complexity of the problem, which is heavily dependent on the type of variables and objectives. Second, real world problems often become too large to efficiently solve using exact solution techniques that are currently available. In order to overcome the limitation of exact techniques, several programs using heuristics have been
developed to solve FTPP with fixed and variable transportation costs (Chung and Sessions 2003). Road construction costs for proposed segments in the road network are considered fixed costs, while transportation costs themselves are considered variable costs. NETWORK II (Sessions 1985) and NETWORK 2000 (Chung and Sessions 2003), which use a heuristic approach combined with the shortest path algorithm (Dijkstra 1959), have been widely used for the last twenty years. NETWORK 2000 can solve multi-period, multi-product, multi-origin and -destination transportation planning problems, but it considers only either profit maximization or cost minimization without taking into account other attributes of road links.

Traditionally, FTPP goals have been set to find combinations of road development and harvest equipment placement to minimize total harvesting costs. However, modern FTPP are not driven only by economic of timber management, but also multiple uses of roads and their social and ecological impacts such as recreation, soil erosion, wildlife and fish habitats among others. For that reason, FTPP have evolved from single-objective (only cost minimization) to multi-objective problems (economic, environmental and social aspects). These environmental and social considerations and requirements introduce side constraints to the FTPP, making the problems larger and much more complex.

NETWORK 2001 (Chung and Sessions 2001) was developed to solve multiple objective transportation planning problems by combining a k-shortest path algorithm with a simulated annealing heuristic. NETWORK 2001 provides the function for the users to
modify the objective function that evaluates solution for multiple objectives, but currently
does not allow having side constraints.

Since there is no guarantee for the optimal solution when using these heuristic
approaches to solve large scale problems, testing different heuristic techniques has been a
constant effort for numerous researchers because a very small increment in the solution
quality can be translated into large monetary savings in forest management. Moreover,
heuristics developed to solve a specific problem can be modified relatively easy to solve
other similar problems. Consequently, new heuristics and hybrids of existing heuristics
are continually being developed, and yet many promising algorithms have not been
applied to FTPP with fixed and variable costs with side constraints.

The objective of this study is to develop a new approach using the Ant Colony
Optimization (ACO) metaheuristic to efficiently solve these challenging multi-objective
FTPP with side constraints. The Ant Colony Optimization (ACO) metaheuristic is a
recently developed optimization technique (Dorigo 1999a) which has not been applied to
solving FTPP. Up to date there have been numerous successful applications of ACO
metaheuristic developed to solve a number of different combinatorial optimization
problems (Dorigo 2002, Dorigo 1999a). The ACO metaheuristic approach is promising
for solving FTPP with fixed and variable costs due to the following reasons: i) the
inspiring concept of ACO metaheuristic is based on a transportation principle, and it was
first intended to solve transportation problems that can be modeled through networks, ii)
its effectiveness in finding very good solutions to difficult problems, as introduced in the
literature, and iii) the nature of the FTPP, which allows the problem to be modeled as a network problem.

**Problem Statement**

In this study a new problem solving technique based on the ACO metaheuristic is developed to solve FTPP considering fixed and variable costs with side constraints. The problem under consideration is to find the set of least cost routes from multiple timber sales to the selected destination mills, while considering environmental impacts of forest road networks represented by sediment yields. Like most other transportation problems, this particular problem can be modeled as a network programming problem.

The road network system is represented by a graph $G$, where vertices represent destination points (i.e. mill locations), entry points (i.e. log landing locations) and intersections of road segments, and edges represent the road segments connecting these different points. The graph $G$ has variables associated with each edge. These variables may represent distance, cost or some other edge attributes. Thus, a network $N$ is formed representing the transportation planning problem. For this particular FTPP under consideration, there are three variables associated with every edge; variable cost, fixed cost, and the amount of sediment yield. Variable costs are proportional to the traffic volume. On the other hand, fixed costs are one time costs that occur when the road is used for the first time. Like fixed costs, we assume sediment is produced when roads are in use regardless of the traffic volume. Consequently, this transportation problem
considers not only an economic factor, represented by the fixed and variable cost, but also an environmental factor, represented by the sediment yields to be delivered from the road segment.

Therefore, the problem is to find the set of routes from multiple timber sales to the selected destination mills, which minimizes the total variable and fixed costs subject to the maximum allowable sediment delivered from the road network. In other words, the problem is to find a set of best routes connecting multiple pairs of vertices in a given network while considering the three mentioned variables associated to every edge. Figure 1 illustrates an example of the described transportation problem.

![Diagram of transportation network](image)

Figure 1. Example of the transportation problem with three timber sales and one mill location.

The transportation network may be composed of existing roads and/or proposed roads which are planned to be built. Fixed costs for existing road segments could either be zero or an assigned fixed maintenance cost. In the case of proposed roads, the construction
cost of the road segment plus the fixed maintenance cost will represent the fixed cost associated to the road segment. The fixed cost associated to a road segment can be expressed either in dollars per road segment or in dollars per unit of length. On the other hand, variable cost refers to the hauling cost, which is expressed in dollars per unit of volume per edge (i.e. $ / vol - edge). Although there are several ways to estimate this variable cost, in most cases it is a function of the road length and driving speed (Byrne et al 1960, Moll and Copstead 1996). Since every road segment has different conditions, there exists a different variable cost associated with every edge. Depending on how detailed the calculations of the variable and fixed costs are, a road segment can be divided into sub-segments, which results in adding more vertices and edges to the network that have different variable and/or fixed costs. The sediment yield associated with every edge represents the amount of sediment eroding from that road segment. This sediment amount can be expressed either in tons per edge or in tons per unit of length. The WEPP model (Elliot et al 1999) can be used to estimate average annual sediment yields from each road segment.

In addition to these three variables associated to every edge, it is also required to know the total volume of timber per product to be harvested in each timber sale or harvest unit and delivered to the selected mill locations. In the case of having multiple harvest periods, the harvest year should also be specified.
Ant Colony Optimization Metaheuristic

Inspiring Concept

The Ant Colony Optimization (ACO) is a metaheuristic approach to solve difficult combinatorial optimization problems. Motivated by its success, ACO metaheuristic was proposed as a common framework for existing applications and algorithmic variants. Thus, algorithms which follow the ACO metaheuristic are called ACO algorithms (Dorigo 2002).

ACO algorithms are inspired by the observation of the foraging behavior of real ant colonies, and in particular, the question of how ants find the shortest path between the food source and the nest. When walking, ants deposit on the ground a chemical substance called pheromone, ultimately forming a pheromone trail. An isolated ant moves essentially at random, but an ant that encounters a previously laid pheromone trail can detect it and decide with a high probability to follow it, therefore reinforcing the trail with its own pheromone. This indirect form of communication is called autocatalytic behavior, which is characterized by a positive feedback, where the more ants following a trail, the more attractive that trail becomes for being followed (Dorigo 1999).

Consider the example shown in Figure 2. Ants are walking along a path between the nest and a food source or vice versa (Fig.2a). Suddenly, an obstacle appears cutting off the path. At position B, for the ants walking from the nest N to the food source F, or at
position D for the ants walking from the food source to the nest, both have to decide whether to turn left or right (Fig.2b). Since there is no previously laid pheromone trail around the obstacle, and the choice is influenced by the intensity of pheromone trials left by preceding ants, the first ant reaching point B or D have the same probability of turning right or left. The ants choosing path BCD will arrive at D earlier than the ants choosing path BHD, because it is shorter. Therefore, ants returning from F to D will find a stronger pheromone trail on path DCB, caused by half of all the ants that by random decided to take path DCBN and by the already arrived ones coming via BCD; thus they will prefer in probability path DCB to path DHB. As a consequence, the number of ants per unit of time following path BCD will be higher than the number of ants following BHD. This causes the amount of pheromone on the shorter path to grow faster than on the longer one. Consequently, the final result is that very quickly all ants will choose the shorter path BCD. (Example and explanation taken from Dorigo 1996).

Several experiments have been carried out with laboratory colonies of real ants (Argentine ants – *Iridomyrmex humilis*), where the colony is given access to a food

Figure 2. An example with real ants (Dorigo 1999).
source in an arena linked to the colony’s nest by a bridge with two branches. The experiments include branches of different length, as well as single and multiple bridges (Figure 3). Dorigo (1999b) observed that, after a transitory phase of apparently a few minutes, most ants use the shortest branch. He also observed that the colony’s probability of selecting the shortest branch increases with the difference in length between the two branches.

The ability of ant colonies to find the shortest path can be viewed as a certain kind of distributed optimization mechanism, where each ant contributes to form a solution. This ant’s behavior can be modeled as an artificial multi-agent system applied to the solution of difficult optimization problems.

![Figure 3. Different experimental apparatus for the bridge experiment.](image-url)
ACO Approach

The concept of ACO metaheuristic is to set a colony of artificial ants that cooperate to find good feasible solutions to combinatorial optimization problems. Cooperation is one of the most important components of ACO algorithms. Computational resources are allocated to relatively simple agents – artificial ants. These artificial ants have a double nature. On one hand, they are the abstraction of those behavioral traits of real ants, which seem to control the shortest path finding ability. On the other hand, they are enriched with some capabilities not present in their natural counterpart (Dorigo 1999a).

There are four main ideas taken from real ants that have been incorporated into ACO metaheuristic (Dorigo 1999a, 1999b); the use of:

i) Colony of cooperating individuals. Ant algorithms are composed of a colony of ants which globally cooperate to find “good solutions” to the given problem. Although, each artificial ant is capable of finding a feasible solution, high quality solutions can only emerge as a result of the collective interaction among the entire ant colony.

ii) Pheromone trail and indirect communication. Artificial ants change some numerical information stored in the problem’s stage they visit, just as real ants deposit pheromone on the path they visit on the ground. This numerical information is called artificial pheromone trail. These pheromone trails are communication channels among ants and their main effect is to change the way the environment (problem landscape) is locally perceived by ants as a function of the past history of the whole ant colony.
iii) Shortest path searching and local moves. Artificial ants as real ones have a common purpose: to find the shortest (minimum cost) path connecting the nest (any origin vertex) to the food source (a destination vertex). Similar to real ants, artificial ants move step by step through adjacent states (adjacent vertices in a graph).

iv) Stochastic and myopic state transition policy. Artificial ants, as real ones, move through adjacent states applying a probabilistic decision policy. This policy employs only local information, not utilizing look-ahead to predict future states. Consequently, the artificial ants transition policy is a function of both, the information represented by the problem specifications (terrain conditions for real ants) and the local modifications in the problem states (by pheromone trails) induced by previous ants.

To increase the efficiency and efficacy of the colony, some enriching characteristics have been given to artificial ants, although not corresponding to any capacity of their real counterparts, some of these characteristics are;

i) Artificial ants live in an environment where time is discrete, moving from discrete states to discrete states.

ii) Artificial ants have an internal state, which contains the memory of the ants’ previous actions.

iii) Artificial ants deposit an amount of pheromone proportional to the quality of the solution found.

iv) Artificial ants are not completely blind and can incorporate look-ahead information, local optimization, and backtracking to improve overall system efficiency.
The ACO Metaheuristic

In ACO algorithms, a finite colony of ants concurrently and asynchronously move through adjacent states of the problem (through adjacent vertices in a graph), applying a stochastic transition policy, which considers two parameters called *trail intensity* and *visibility*. Trail intensity refers to the amount of pheromone in the path, which indicates how proficient the move has been in the past, representing *a posteriori* indication of the desirability of the move. Visibility is usually computed as some heuristic value indicating the *a priori* desirability of the move (Maniezzo 2004).

Therefore, ants incrementally build a feasible solution to the optimization problem being solved. Once an ant has found a solution, or during the construction phase, the ant evaluates the solution and deposits pheromone on the connections it used, proportionally to the goodness of the solution.

Ants deposit pheromone in various ways. They can deposit pheromone on a connection (an edge in a graph) directly after the move is made without waiting for the end of the solution. This is called *online step by step pheromone update*. Ants also can deposit pheromone after a solution is built by retracing the same path backwards and updating the pheromone trail of the used connections. This is called *online delayed pheromone update* (Dorigo 2002).
In addition to the ants’ activity that uses an incremental constructive approach, ACO algorithms include two more mechanisms, namely *pheromone trail evaporation* and *daemon activities* (Dorigo 1999b, 2002; Maniezzo 2004). Pheromone trail evaporation refers to the process of decreasing the pheromone intensity on all connections (the entire set of edges E in a graph) over time to avoid unlimited accumulation of pheromone over some components. It is to say, pheromone evaporation avoids a too rapid convergence of the algorithm towards a sub-optimal solution, thus allowing the exploration of other areas of the solution space. Daemon activities can be used to implement centralized actions, which cannot be performed by single ants. Examples include the activation of local optimization procedures (such as 2-opt, 3-opt move or Lin-Kernigham) and the update of global information to decide whether to bias the search process.

Figure 4 shows a description of ACO metaheuristic reported in pseudo-code. Some of the components are optional (daemon actions) and implementation dependent, such as when and how pheromone is deposited (taken from Dorigo 1999a).
Figure 4. The ACO metaheuristic in pseudo-code. Comments are enclosed in braces. All the procedures are the first level of indentation in the statement in parallel and are executed concurrently. The procedure `daemon_actions()` at line 6 is optional and refers to centralized actions executed by the daemon processing global knowledge. The `target_state` (line 19) refers to a complete solution built by the ant. The step-by-step and delayed pheromone updating procedures at lines 24-27 and 30-34 are often mutually exclusive. When both of them are absent the pheromone is deposited by the daemon.

Applications of ACO Algorithm

ACO algorithms, as a consequence of their concurrent and adaptive nature, can be applied to solve numerous problems that can be modeled through graphs. Several
implementations of ACO metaheuristic have been developed to solve a number of different \textit{NP-hard} combinatorial optimization problems. These problems can be classified in two classes: static and dynamic combinatorial optimization. Static problems are those in which the conditions of the problem are given once and do not alter while the problem is being solved. On the other hand, dynamic problems have conditions that change over time such as communication networks.

Most of the ACO algorithms applied to solve static problems are strongly inspired by the first work on ant colony optimization, Ant System (AS) (Dorigo 1991). Many of the successive applications of the original idea are relatively straightforward applications of AS to specific problems (Dorigo 1999a). The first application of an ACO algorithm was developed for solving the traveling salesman problem (TSP), due to the fact that the TSP is one of the most studied \textit{NP-hard} problems and the easiness to adapt the ant colony metaphor. Table 1 shows some of the most important ACO applications for the TSP and other important static combinatorial optimization problems. More detailed description and other ACO applications can be found in Dorigo and Stutzle (2002).
Table 1. Applications of ACO algorithms to static combinatorial optimization problems.

<table>
<thead>
<tr>
<th>Problem name</th>
<th>Authors</th>
<th>Algorithm name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traveling salesman</td>
<td>Dorigo (1991)</td>
<td>AS</td>
</tr>
<tr>
<td></td>
<td>Dorigo &amp; Gambardella (1996a)</td>
<td>ACS &amp; ACS-3-opt</td>
</tr>
<tr>
<td></td>
<td>(1996b)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Stutzle &amp; Hoos (1997a)</td>
<td>MMAS</td>
</tr>
<tr>
<td></td>
<td>Stutzle &amp; Hoos (1997b)</td>
<td></td>
</tr>
<tr>
<td>Scheduling problems</td>
<td>Colomi et al (1994)</td>
<td>AS-JSP</td>
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<tr>
<td></td>
<td>Stutzle (1998)</td>
<td>AS-FSP</td>
</tr>
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<td></td>
<td>Merkle et al (2000)</td>
<td>ACO-RCPS</td>
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<tr>
<td></td>
<td>Dowsland &amp; Thompson (2005)</td>
<td></td>
</tr>
<tr>
<td>Quadratic Assignment</td>
<td>Maniezzo et al (1994)</td>
<td>AS-QAP</td>
</tr>
<tr>
<td></td>
<td>Maniezzo (1999)</td>
<td>ANTS-QAP</td>
</tr>
<tr>
<td></td>
<td>Maniezzo &amp; Colomi (1999)</td>
<td>AS-QAP</td>
</tr>
<tr>
<td></td>
<td>Stutzle &amp; Hoos (2000)</td>
<td>MMAS-QAP</td>
</tr>
<tr>
<td>Sequential ordering</td>
<td>Gambardella &amp; Dorigo (1997)</td>
<td>HAS-SOP</td>
</tr>
<tr>
<td>Graph Coloring</td>
<td>Costa &amp; Hertz (1997)</td>
<td>ANTCOL</td>
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<tr>
<td></td>
<td>Bui (2005)</td>
<td>ABAC</td>
</tr>
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</table>

Most of the research on the application of ACO algorithms to dynamic combinatorial optimization problems has been centered on communication networks, in particular to routing problems. Implementations of ACO algorithms for communication networks are grouped into two categories: a) connection-oriented networks, where data follow a common path selected by a preliminary setup phase, and b) connection-less networks, where data can follow different paths (Dorigo et al 1999a). Connection-oriented networks are modeled through directed graph, where only one direction is considered for each edge. On the other hand, connection-less networks are modeled through graph where both directions are considered for each edge. Table 2 shows some of the main implementations of ACO algorithms for dynamic problems.
Table 2. Applications of ACO algorithms to dynamic combinatorial optimization problems.

<table>
<thead>
<tr>
<th>Problem name</th>
<th>Reference</th>
<th>Algorithm name</th>
</tr>
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<tbody>
<tr>
<td>Connection-oriented network routing</td>
<td>Schoonderwoerd et al. (1996)</td>
<td>ABC</td>
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<td></td>
<td>White et al. (1998)</td>
<td>ASGA</td>
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<tr>
<td></td>
<td>Di Caro &amp; Dorigo (1998a)</td>
<td>AntNet-FS</td>
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<td></td>
<td>Bonabeau et al. (1998)</td>
<td>ABC-smart ants</td>
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<td></td>
<td>Sim &amp; Sum (2002)</td>
<td>MACO</td>
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<td>Walkowiak (2005)</td>
<td>ANB</td>
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<td></td>
<td>Subramanian at al. (1997)</td>
<td>Regular ants</td>
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<td></td>
<td>Heusse et al. (1998)</td>
<td>CAF</td>
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<td></td>
<td>van der Put &amp; Rothkrantz (1999)</td>
<td>ABC-backward</td>
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<tr>
<td></td>
<td>Di Caro (2004)</td>
<td>ACR</td>
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</table>

Based on the introduced ACO metaheuristic, a new ACO algorithm for solving FTPP with side constraints was developed, ACO-FTPP. In order to validate the performance of the algorithm, a 100-edge hypothetical FTPP considering five timber sales and one mill destination was developed. The results of ACO-FTPP were compared with the results of a MIP solver. In the next section, ACO-FTPP is described in detail and the results are presented.

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PART II.

Applying Ant Colony Optimization (ACO) Metaheuristic to Solve Forest Transportation Planning Problems with Side Constraints

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Introduction

Problems related to forest transportation planning have long been an important concern due to the fact that timber transportation is one of the most expensive activities in forest operations (Greulich 2002). Traditionally, the goals of forest transportation planning problems (FTPP) have been set to find combinations of road development and harvest equipment placement to minimize total harvesting and transportation costs. However, modern FTPP are not driven only by the economics of timber management, but also multiple uses of roads and their social and ecological impacts such as recreation, soil erosion, wildlife and fish habitats among others. These environmental and social considerations and requirements introduce side constraints to the FTPP, making the problems larger and much more complex.

Two different approaches have been applied to solve FTPP: exact algorithms such as mixed-integer programming (MIP), and approximation algorithms generally called heuristics (Falcao 2001; Weintraub 1994). The most important advantage of exact algorithms is that they provide optimal solutions. However, they are limited to small scale problems. Contrarily, heuristic techniques, although may not provide optimal solutions, have successfully been applied to solve large scale problems and are relatively easy to formulate compared with exact algorithms (Olsson 2003; Martell et al 1998; Weintraub 1995; Jones 1991). Since real world problems are usually large scale problems with thousands of variables, heuristic techniques have been the focus of a large number of researchers (Boyland 2002; Zeki 2001).
The case of FTPP with fixed and variable costs are complex optimization problems that to date have only been solved efficiently using heuristic approaches. NETWORK II (Sessions 1985) and NETWORK 2000 (Chung and Sessions 2003), which use a heuristic approach combined with the shortest path algorithm (Dijkstra 1959) have been widely used for the last twenty years. NETWORK 2000 can solve multi-period, multi-product, multi-origin and -destination transportation planning problems, but it considers only either profit maximization or cost minimization without taking into account other attributes of road links. NETWORK 2001 (Chung and Sessions 2001) was developed to solve multiple objective transportation planning problems by combining a k-shortest path algorithm with a simulated annealing heuristic. NETWORK 2001 allows users to modify the objective function in order to evaluate solutions considering multiple objectives, but currently does not allow having side constraints.

Since heuristic approaches usually do not guarantee the optimality of solutions, testing different heuristic approaches has been a constant effort of numerous researchers. New heuristics and hybrids of existing heuristics are continually being developed, but only a few algorithms have been applied to FTPP with both fixed and variable costs. One of the promising algorithms that have not been applied to FTPP is the Ant Colony Optimization (ACO) metaheuristic, an optimization technique introduced in 1991 by Dorigo and colleagues (Dorigo 1999a). To date there have been numerous successful applications of ACO metaheuristic developed to solve a number of different combinatorial optimization problems. Currently, some ACO algorithms have provided the best known results for
solving many of the most important combinatorial optimization problems (such as the traveling salesman problem (TSP), quadratic assignment problem (QAP), job-shop scheduling problem (JSP), vehicle routing problem (VRP) among others), while others have matched the results of the best known algorithms (Dorigo 2002; Dorigo1999a).

The ACO metaheuristic approach is promising for solving FTPP with fixed and variable costs due to the following reasons: 

1) the inspiring concept of ACO metaheuristic is based on a transportation principle, and it was first intended to solve transportation problems that can be modeled through networks, 

2) its effectiveness in finding good solutions to difficult problems, as introduced in the literature, and 

3) the nature of the FTPP, which allows the problem to be modeled as a network problem.

In this manuscript, we introduce ACO-FTPP, a specially designed ACO algorithm for solving FTPP with fixed and variable costs while considering total sediment yields from the road network as a side constraint. To validate the performance of the algorithm, we developed a 100-link hypothetical FTPP and compared the results of ACO-FTPP with the results obtained with a mixed-integer programming solver applied to solve a comparable mathematical programming formulation (Weintraub et al 1994). A description of the algorithm and the results from the applications are presented.
Ant Colony Optimization Metaheuristic

The Ant Colony Optimization (ACO) is a metaheuristic approach for solving difficult combinatorial optimization problems. Motivated by its success, ACO metaheuristic was proposed as a common framework for existing applications and algorithmic variants. Thus, algorithms which follow the ACO metaheuristic are called ACO algorithms (Dorigo 2002).

ACO algorithms are inspired by the observation of the foraging behavior of real ant colonies, and in particular, by how ants can find shortest paths between food sources and the nest. When walking, ants deposit a chemical substance on the ground called pheromone, ultimately forming a pheromone trail. While an isolated ant moves essentially at random, an ant that encounters a previously laid pheromone trail can detect it and decide with a high probability to follow it, therefore reinforcing the trail with its own pheromone. This indirect form of communication is called autocatalytic behavior, which is characterized by a positive feedback, where the more ants following a trail, the more attractive that trail becomes for being followed (Dorigo 1999).

The concept of the ACO metaheuristic is to set a colony of artificial ants that cooperate to find good feasible solutions to combinatorial optimization problems. Cooperation is one of the most important components of ACO algorithms. Computational resources are allocated to relatively simple agents – artificial ants. These artificial ants have a double nature. On one hand, they are the abstraction of those behavioral traits of real ants, which
seem to control the shortest path finding ability. On the other hand, they are enriched with some capabilities not present in their natural counterparts (Dorigo 1999a).

There are four main ideas taken from real ants that have been incorporated into ACO metaheuristic (Dorigo 1999a, 1999b); the use of: i) colony of cooperating ants – although each artificial ant is capable of finding a feasible solution, high quality solutions can only emerge as a result of the collective interaction among the entire ant colony, ii) pheromone trail and indirect communication – artificial ants change some numerical information, called artificial pheromone trail, stored in the problem stage they visit, just as real ants deposit pheromone on the path they visit on the ground, iii) shortest path searching and local moves – artificial ants as real ones have a common purpose: to find the shortest path moving step by step through adjacent states, and iv) stochastic and myopic state transition policy – artificial ants move through adjacent states applying a probabilistic decision policy, which is a function of the information represented by the problem specifications (terrain conditions for real ants) and the local modifications in the problem states (by pheromone trails) induced by previous ants.

To increase the efficiency and efficacy of the colony, some enriching characteristics have been given to artificial ants. Some of these characteristics are that artificial ants i) live in an environment where time is discrete, ii) have an internal state, which contains the memory of the ants’ previous actions, iii) deposit an amount of pheromone proportional to the quality of the solution found, and iv) are not completely blind and can incorporate
look-ahead information, local optimization and backtracking to improve overall system efficiency.

In ACO algorithms, a finite colony of ants concurrently and asynchronously moves through adjacent states of the problem, applying a stochastic transition policy that considers two parameters called *trail intensity* and *visibility*. Trail intensity refers to the amount of pheromone in the path, which indicates how proficient the move has been in the past, representing *a posteriori* indication of the desirability of the move. Visibility is usually computed as some heuristic value indicating the *a priori* desirability of the move, such as cost or distance (Maniezzo 2004). Therefore, moving through adjacent steps, ants incrementally build a feasible solution to the optimization problem.

Once an ant has found a solution, it evaluates the solution and deposits pheromone on the connections it used, proportionally to the goodness of the solution. Ants deposit pheromone in various ways. They can deposit pheromone on a connection (an edge in a graph) directly after the move is made without waiting for the end of the solution. This is called *online step by step pheromone update*. Ants also can deposit pheromone after a solution is built by retracing the same path backwards and updating the pheromone trail of the used connections. This is called *online delayed pheromone update* (Dorigo 2002).
The Forest Transportation Planning Problem

The specific FTPP we address in this paper are finding the set of least cost routes from multiple timber sales to selected destination mills, while considering environmental impacts of forest road networks represented by sediment yields. As most of transportation problems, these FTPP can be modeled as network problems. The road system is represented by a graph $G$, where vertices represent destination points (i.e. mill location), entry points (i.e. log landing location), and intersections of road segments, and edges represent the road segments connecting these different points. The graph $G$ has three variables associated with every edge; fixed cost, variable cost, and the amount of sediment.

The transportation network may be composed of existing and/or proposed roads. Fixed cost for an existing road segment could either be zero or a fixed maintenance cost for the road segment. In the case of proposed roads, the construction cost of the road segment plus fixed maintenance cost will represent the fixed cost associated to that specific edge. Fixed cost is a one-time cost which occurs if the road segment is used. Variable cost refers to the hauling cost. Unlike the fixed cost, variable cost is proportional to traffic volumes. Although there are several ways to estimate the unit variable cost ($/vol$-edge), in most cases it is a function of the road length, driving speed, and operating costs (Byrne at al 1960, Moll and Copstead 1996). Since every road segment has different conditions, there will be a different unit variable cost associated with each edge. The sediment associated with each edge represents the amount of sediment eroding from the road.
segment in tons per year per edge. Like fixed cost, we assumed that sediment is produced when roads are open regardless of the traffic volume. The WEPP model can be used to estimate average annual sediment yields from each road segment (Elliot et al 1999). In addition to the three variables related to each edge, it is also required to have the total volume of wood per timber sale to be delivered to the selected mill location.

In this context the problem under consideration becomes a minimization problem where the objective function is set to minimize the combination of fixed and variable costs (Eq. 1) subject to a sediment yield restriction (Eq. 2).

\[
\text{Minimize } \sum_{i=1}^{e} \left[ (\text{var}_i \cdot \text{vol}_i) + (\text{fixed}_i \cdot B_i) \right] \quad [\text{Eq. 1}]
\]

Subject to

\[
\sum_{i=1}^{e} (\text{sediment}_i \cdot B_i) \leq \text{allowable}_\text{sed} \quad [\text{Eq. 2}]
\]

where,

- \( \text{var}_i \): variable cost for edge \( i \) in \$/vol.
- \( \text{fixed}_i \): fixed cost for edge \( i \) in \$.
- \( \text{sediment}_i \): amount of sediment eroding from edge \( i \) in tons.
- \( \text{vol}_i \): total volume transported over edge \( i \)
- \( B_i \): binary variable (1 if edge is used and 0 otherwise)
- \( e \): total number of edges in the network
- \( \text{allowable}_\text{sed} \): maximum allowable sediment in tons.
Methodology

ACO-FTPP algorithm

ACO-FTPP is the specialized ACO algorithm we developed to solve the FTPP described above. ACO-FTPP has a finite number of ants \((m)\) that search for \(r\) shortest paths, one from each timber sale-destination pair, in a network of \(v\) vertices and \(e\) edges. In ACO-FTPP a move is defined as the transition of an ant from one vertex to another. After a certain number of moves, an ant arrives at its destination thus completing a route. Once all ants have completed their routes for one timber sale, a shortest path is found among the \(m\) routes. When all ants finish one timber sale they move to the next timber sale to find \(m\) routes for the sale. An iteration is completed when all timber sales are routed to the destination vertex.

When an ant is located on a given vertex, it has to choose where to go. An ant decides what vertex to visit next, based on a transition probability on each edge calculated by the following equation (Eq. 3).

\[
\rho_j(c) = \frac{(\tau_j)^{\alpha} (\eta_j)^{\beta}}{\sum_{i=1}^{l} (\tau_i)^{\alpha} (\eta_i)^{\beta}} \quad \text{if } j \in N_i \quad [\text{Eq. 3}]
\]

where, \(\rho_j(c)\) indicates the transition probability with which an ant, chooses the edge \(j\) in iteration \(c\); \(l\) is the number of edges in the set \(N_i\) sharing the same origin vertex; \(\alpha\) and \(\beta\)
are the parameters that control the relative importance of the pheromone trail intensity ($\tau_j$) and the visibility ($\eta_j$) values on edge $j$. The visibility value is calculated by adding the reciprocal of the variable cost (unit variable cost on edge $j$ multiplied by the volume ($vol_j$) from origin $s$), the reciprocal of the fixed cost and the reciprocal of the sediment amount associated to edge $j$ (Eq. 4).

$$\eta_j = \left(\frac{\text{var}_j \cdot \text{vol}_j}{\text{fixed}_j} + \frac{1}{\text{sediment}_j}\right)^{-1}$$

[Eq. 4]

Consequently, by combining equations 3 and 4, the resulting transition probability formula for a given edge is determined as follows:

$$\rho_j(c) = \frac{\left(\tau_j\right)^a \cdot \left(\frac{\text{var}_j \cdot \text{vol}_j}{\text{fixed}_j} + \frac{1}{\text{sediment}_j}\right)^{a}}{\sum_{i=1}^{N_i} \left(\tau_i\right)^a \cdot \left(\frac{\text{var}_i \cdot \text{vol}_i}{\text{fixed}_i} + \frac{1}{\text{sediment}_i}\right)^{a}} \quad \text{if } j \in N_i \quad \text{[Eq. 5]}

Based on the transition probability values of all edges in $N_j$, accumulated transition probabilities for each of these edges are computed. Then, a random number between zero and one is selected using a random number generator. If this random number is smaller than the accumulated transition probability of edge $i$ and larger than the accumulated transition probability of edge $i-1$, then edge $i$ is selected.

Starting from a given timber sale and ending on the selected mill destination, an ant incrementally builds a route, moving through adjacent edges according to the transition probability equation (Eq. 5). At the end the best route among the $m$ routes generated by
the $m$ ants is selected as the shortest path. At the end of each iteration the edges forming all shortest paths (one for every sale-destination pair) are identified, the total solution value is computed and the solution feasibility is evaluated. If the current solution is not better than the best found so far or is infeasible, the solution is ignored, the pheromone trail intensities remain the same and another iteration starts. However, if the current solution is better than the best solution found so far, the current solution becomes the new best solution and the pheromone trail intensity of the edges forming all shortest paths is updated. At the same time, pheromone intensity on all edges decreases (evaporates) in order to avoid unlimited accumulation of pheromone. Also pheromone evaporation avoids a too-rapid convergence of the algorithm towards a sub-optimal solution, allowing the exploration of other solution spaces. Pheromone trail intensity is updated using the following equation (Eq. 6):

$$
\tau_i(c+1) = \lambda \times \tau_i(c) + \Delta \tau_i
$$  \quad [\text{Eq. 6}]

where two components are considered; the current pheromone trail intensity on edge $i$ at iteration $c$, indicated by $\tau_i(c)$, multiplied by $0 < \lambda < 1$ which is a coefficient such that $(1 - \lambda)$ represents the pheromone evaporation rate between iteration $c$ and $c + 1$; and $\Delta \tau_i$ which represents the newly added pheromone amount to edge $i$, calculated as follows:

$$
\Delta \tau_i = \sum_{k=1}^{s} \Delta \tau_i^k
$$  \quad [\text{Eq. 7}]

where, $s$ is total number of timber sales, and $\Delta \tau_i^k$ is the quantity of pheromone laid on edge $i$ by the ants in iteration $c$; which is given by:
\[ \Delta \tau_i^t = \begin{cases} Q / L_k & \text{if the ants used edge } i \text{ in the shortest path} \\ 0 & \text{otherwise} \end{cases} \quad [\text{Eq. 8}] \]

where \( Q \) is a constant and \( L_k \) is the total transportation cost over the selected route. The value of \( Q \) has to be chosen so the amount of pheromone added to edge \( i \) by a given ant slightly increases the probability of that edge during the following iterations.

Given the definitions above, ACO-FTPP can be stated as follows (see Figure 5). At iteration 1 an initialization phase takes place in which ants start at a random timber sale location. An initial equal small amount of pheromone \( q \) is set for each edge, and transition probabilities for each edge are computed considering the volume of the chosen timber sale. Thereafter each ant can find a route by moving from edge to edge until the mill destination is reached.

When an ant moves through an edge, the edge is recorded with its from- and to- vertex in the ant’s internal memory. This memory is used to avoid ants returning to a previously visited vertex. When an ant is located at a vertex whose all adjacent vertices have been previously visited, it stops without reaching its destination and a high cost (i.e. $999,999) is assigned to the ant’s route as a penalty. Likewise, if an ant has not found its destination after a maximum number of moves \( \text{Max\_moves} \), the ant stops and a high cost is assigned. For the applications used in this paper, the \( \text{Max\_moves} \) is set to be the number of vertices in the network plus one (\( v + 1 \)).
After each of all ants finds its own route, the least cost route is selected as the *shortest path*, and all ants move to the next randomly chosen sale (origin). The transition probabilities are re-calculated using the current sale volume and ants start moving through adjacent edges until they find the destination mill. When an *shortest path* is complete for this second sale, all ants move to the next sale and so forth. At the end of an *iteration* the objective function and total sediment values are calculated using the best route for each timber sale. The edges forming the $r$ best routes (one per timber sale) are identified and their pheromone trail intensity is updated. This process continues until a stopping criterion is met. We used a maximum number of iterations $I_{\text{max}}$ to stop the process in a reasonable time. If in a given *iteration* the solution found does not satisfy the constraints (the calculated sediment amount is greater than the maximum allowable sediment) or is worse that the best solution found so far, the solution is ignored, pheromone trail intensities remain the same as the previous iteration, and the next *iteration* starts.
Deposit initial pheromone amount on every edge
Locate ants on a randomly selected timber sale
Compute transition probability for every edge
Send out an ant
Move to next edge according to the transition probability
Is destination mill reached?
Yes
Evaluate route's cost
No
Have all ants been sent out?
Yes
Select the minimum cost route
No
Do all timber sales have a selected minimum cost route?
Yes
Compute total solution value and evaluate feasibility
No
Is it a feasible solution?
Yes
Is current solution better than the best solution found so far?
No
Ignore current solution
Yes
Identify edges forming the set of minimum cost routes
Deposit pheromone on identified edges, update pheromone trail intensity for every edge and compute new transition probabilities
Save current solution as best solution found
Is stopping criterion reached?
No
Stop and report best solution
Yes
Locate ants on the next randomly chosen timber sale
Iteration = Iteration + 1

Figure 5. Flowchart of the ACO-FTPP search process
Hypothetical Transportation Problem

To examine the behavior and performance of the algorithm, we applied the ACO-FTPP to a 100-edge hypothetical forest transportation problem (see Figure 6). This 100-edge problem includes five timber sale locations (indicated by the circles on the left) and one destination mill (indicated by the circle on the right). Appendix A contains the variable cost, fixed cost, and sediment amount associated to every edge as well as the volume to be delivered from each timber sale to the selected destination.

![Figure 6. Hypothetical forest transportation problem with 100 edges, five timber sales and one destination mill.](image)

Although this FTPP is a hypothetical example with 100 edges, this problem is represented by 200 edges since we consider both directions on every edge. This hypothetical problem which forms a grid-shaped road network may not often exist in real forest road networks, but we used this example to test the algorithm since it is relatively more difficult to solve. Usually in real transportation problems, most road segments have obvious loaded-truck directions, or there are not many alternative routes from a timber...
sale to a given destination, especially in small scale problems. In addition, there are not many intersections at where four or even more road segments meet in real forest road networks.

An increasing number of road segments leaving an intersection point exponentially increases the number of alternative routes. The degree of a vertex is defined as the number of adjacent edges. In our hypothetical FTPP, the minimum degree is two (i.e. vertex 1 and 40), the maximum is seven (i.e. vertex 14 and 17), and the average degree of the graph representing this hypothetical FTPP is five.

**Results and Discussion**

**Setting Parameters**

ACO-FTPP requires parameters such as $\alpha$, $\beta$, $\lambda$, $q$, $Q$, $m$, and $I_{max}$. The parameters $\alpha$ and $\beta$ control the relative importance of the pheromone trail intensity and visibility, respectively. Pheromone evaporation is controlled by $\lambda$. The constant $q$ is an initial small amount of pheromone deposited on every edge at the first iteration. $Q$ is also a constant related to the additional amount of pheromone deposited by ants on selected edges. Lastly, $m$ indicates the total number of ants and $I_{max}$ is the stopping criteria of the algorithm, which is expressed by a maximum number of iterations.
Since our initial test runs of ACO-FTPP confirmed the findings of previous studies in recognizing that different parameter combinations affect the performance of the ACO (Dorigo 1991), we conducted a search for the best parameter combination. Several parameter combinations among the many we tested could find the same best solution, probably because the 100-edge transportation problem constitutes a relatively small problem. To select one best parameter combination, we considered the number of iterations taken to find the best solution as well as solution quality.

Three of the seven parameters required by ACO-FTPP ($q$, $m$, and $I_{\text{max}}$) do not affect the calculation of the transition probability (Eq. 3-8). Therefore these parameter values were fixed in our trials. Because $q$ is an equal small amount of pheromone deposited at time zero, $\tau_i(0)$, on every edge, it does not affect the ants search (Dorigo 1991). In most ACO algorithm $q$ is set to a small positive constant. For our applications, $q$ was set to 0.001. Similarly, the number of ants $m$ is usually set to be the number of vertices (Dorigo et al 1996). Since our FTPP are complex problems that consider three variables associated with every edge instead of one, to diversify the search in our applications, $m$ was set to be equal to the number of edges ($e$), which is larger than the number of vertices. Based on initial runs the maximum number of iterations ($I_{\text{max}}$) was set to give the algorithm enough time to find the best solution; in our applications $I_{\text{max}}$ was set to 100.

The parameters $Q$, $\alpha$, $\beta$, and $\lambda$, directly affect the calculation of the transition probability (Eq. 3-8), therefore they largely affect the performance of the algorithm. The constant $Q$, related to the quantity of pheromone deposited by ants, has to be chosen so the

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transition probability of an edge from one iteration to the next is slightly increased. Because our initial test runs showed that $Q$ did not have a significant effect on the solution quality, we set $Q$ to 0.001. The remaining parameters ($\alpha$, $\beta$, and $\lambda$) were identified to directly affect the performance of the algorithm, and therefore subject to the search for the best parameter combination.

To test different values of the parameters $\alpha$, $\beta$, and $\lambda$, a range for each parameter was defined and partitioned into ten, fifteen, and ten discrete values respectively. Table 3 shows the range of values and the corresponding discrete values for each parameter. This yields 1,500 different parameter combinations. Considering the values of $m$ and $I_{\text{max}}$, the algorithm took approximately 4 hours to execute all 1500 parameter combinations. The algorithm was implemented in C programming language and run using a 2.66Ghz Pentium(R)4 CPU with 512MB of RAM.

### Table 3. Range of values for the variable parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value Range</th>
<th>Discrete Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$0 &lt; \alpha &lt; 10$</td>
<td>${0.5, 1.5, 2.5, \ldots, 9.5}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$0 &lt; \beta &lt; 15$</td>
<td>${0.5, 1.5, 2.5, \ldots, 14.5}$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$0 &lt; \lambda &lt; 1$</td>
<td>${0.05, 0.15, 0.25, \ldots, 0.95}$</td>
</tr>
</tbody>
</table>

The best parameter combination found after this search was $\alpha = 1.5$, $\beta = 0.5$ and $\lambda = 0.65$. It was noticed from our runs that visibility values ($\eta_i$) become very large compared with pheromone trail intensity values ($\tau_i$) (Eq. 4). Therefore, since $\beta$ and $\alpha$ are the exponents of the visibility and pheromone trail intensity, respectively (Eq. 3), $\beta$ smaller than one decreases the visibility value, and $\alpha$ larger than one increases the trail intensity value.
Consequently, the relative importance of the visibility ($\eta_i$) and the pheromone trail intensity ($\tau_i$) are more homogeneous.

The best value of $\lambda$ found (0.65) may be explained by the fact that the ants need to forget part of the experience gained in the past, represented by the accumulated pheromone amount, to better exploit new incoming pheromone information and to avoid a fast convergence to sub-optimal solutions. Dorigo et al (1996) observed the same behavior in their parameter setting procedure.

**Solutions from the Hypothetical Transportation Problem**

To evaluate the effect of the sediment constraint on solutions, four different cases for the hypothetical example were analyzed. *Case I* is a cost minimization problem without a sediment constraint, *Cases II and III* are cost minimization problems subject to increasing levels of sediment constraints, and *Case IV* is a sediment minimization problem without a cost constraint. While *Cases I and IV* address single goal transportation planning problems, *Cases II and III* address multiple goals with different levels of sediment restriction.

Once *Case I* is solved, the minimum cost solution is obtained and the associated total sediment amount can be calculated. This sediment amount becomes the upper limit for the sediment constraint because any larger sediment constraint values would not affect the minimum cost solution. On the other hand, *Case IV* provides the lower limit for the
sediment constraint since requiring sediment below the limit will result in an infeasible solution. Consequently, Cases II and III are set with sediment constraint values within the range between the upper and lower limits obtained by Cases I and IV, respectively. The level of the sediment restriction is increased from Case II to Case III.

To efficiently guide ants in their search for the shortest path, the transition probability function (Eq. 5) was modified according to the objective function of the problem to be solved. For Case I the transition probability considered only the variable and fixed costs associated with each edge (Eq. 9):

\[
\rho_j(c) = \frac{(\tau_j)^\alpha * \left( (\text{var} \cdot \text{cost}_j * \text{vol}_j)^{-1} + \text{fixed} \cdot \text{cost}_j^{-1} \right)^\beta}{\sum_{i=1}^{I} (\tau_i)^\alpha * \left( (\text{var} \cdot \text{cost}_i * \text{vol}_i)^{-1} + \text{fixed} \cdot \text{cost}_i^{-1} \right)^\beta}, \quad \text{if } j \in N, \quad [\text{Eq. 9}]
\]

Likewise, for Case IV the transition probability considered only the sediment amount associated with each edge (Eq. 10):

\[
\rho_j(c) = \frac{(\tau_j)^\alpha \cdot (\text{sediment}_j)^\beta}{\sum_{i=1}^{I} (\tau_i)^\alpha \cdot (\text{sediment}_i)^\beta}, \quad \text{if } j \in N, \quad [\text{Eq. 10}]
\]

Because there is no guarantee for optimality when using ACO, which is a heuristic approach, we compared our results with a mixed-integer programming solver, MIPIII, which uses a branch-and-bound algorithm to solve mixed-integer programming problems.
MIPIII is the mixed-integer component of the mathematical programming system MPSIII (Ketron 2001).

The results from the 100-edge hypothetical FTPP obtained by ACO-FTPP and MIPIII are presented in Figures 7 through 10. A solution was found for each of the four cases by ACO-FTPP. On the other hand, MIPIII was able to find an optimal solution only for the single goal transportation problems, Case I (cost minimization) and Case IV (sediment minimization).

For Case I the optimal solution found by MIPIII is slightly better than the best solution found by ACO-FTPP (Figure 7a and 7b respectively). The optimal MIP solution found has an objective function value of $\$128,057$ ($\$33.27/vol$) with an associated total sediment amount of 610.96 tons. The best solution found by ACO-FTPP has an objective function value of $\$129,388$ ($\$33.62/vol$) and an associated total sediment amount of 660.46 tons. ACO-FTPP reached an optimality level of 99% since the difference between two solutions is only 1% or $\$1,281$. For Case II the maximum allowable sediment value was set to 550 tons. Based on this sediment constraint the best solution found by ACO-FTPP has a minimum total cost of $\$170,833$ ($\$44.38/vol$) reaching a total sediment amount of 527.70 tons (Figure 8). For Case III, where a maximum allowable sediment was set to 450 tons, ACO-FTPP found the best solution of $\$197,667$ ($\$51.35/vol$) with a related total sediment amount of 440.69 tons (Figure 9). MIPIII failed to find any feasible solution for both Cases II and III. For Case IV ACO-FTPP was able to find the same solution as the optimal one found by MIPIII (Figure 10a and 10b respectively). The
The minimum objective function value is 393.67 tons with an associated total cost of $247,080. Although both algorithms solved this 100-edge transportation problem in a short time, ACO-FTPP solved it much faster than MIPIII. ACO-FTPP took 4 seconds, whereas MIPIII took approximately 1 minute.

**Figure 7.** Case I, cost minimization problem without sediment constraint.  
a) Results from ACO-FPTT, and b) Results from MIPIII.

<table>
<thead>
<tr>
<th></th>
<th>ACO-FPTT</th>
<th>MIPIII</th>
</tr>
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<tbody>
<tr>
<td>Variable cost</td>
<td>80,381 ($20.89/vol)</td>
<td>86,165 ($22.39/vol)</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>49,007 ($12.73/vol)</td>
<td>41,892 ($10.88/vol)</td>
</tr>
<tr>
<td>Total cost</td>
<td>129,388 ($33.62/vol)</td>
<td>128,057 ($33.27/vol)</td>
</tr>
<tr>
<td>Total sediment</td>
<td>660.46 tons</td>
<td>610.96 tons</td>
</tr>
</tbody>
</table>

**Figure 8.** Case II, cost minimization problem subject to a sediment constraint of 550 tons.  
a) Results from ACO-FPTT, and b) Result from MIPIII.

<table>
<thead>
<tr>
<th></th>
<th>ACO-FPTT</th>
<th>MIPIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable cost</td>
<td>109,931 ($28.54/vol)</td>
<td>-------- ($-- /vol)</td>
</tr>
<tr>
<td>Fixed cost</td>
<td>60,962 ($15.84/vol)</td>
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<td>Total cost</td>
<td>170,833 ($44.38/vol)</td>
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<td>Total sediment</td>
<td>527.7 tons</td>
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</table>
Figure 9. Case III, cost minimization problem subject to a sediment constraint of 450 tons. a) Results from ACO-FPTT, and b) Result from MIPIII.

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<th>MIPIII</th>
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<td>Variable cost</td>
<td>$153,006 ($39.75/vol)</td>
<td>$200,450 ($52.09/vol)</td>
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<td>Total cost</td>
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<td>Total sediment</td>
<td>440.49 tons</td>
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Figure 10. Case IV, sediment minimization problem without constraint. a) Results from ACO-FPTT, and b) Result from MIPIII.

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<td>Total sediment</td>
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<td>393.67 tons</td>
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The total cost and total sediment values associated with the best solutions found by ACO-FTPP for the four cases are presented in Figure 11. In Case II, when the total allowable sediment is restricted to 550 tons, (approximately 20% less than the associated total sediment for Case I), the minimum total cost obtained increased by 32% compared to Case I. In Case III, where we further restricted the sediment constraint to 450 tons, (around 47% less than the associated total sediment for Case I), the minimum total cost obtained increased by 53%. When the goal was to minimize total sediment, Case IV, the
optimal solution obtained a minimum total sediment amount of 393 tons, which is approximately 40\% less than the sediment associated with Case I. On the other hand, the total cost associated with Case IV, increased by 91\% from $129,399 to $247,080. This increment of the total cost from Case I to Case IV may be explained by the fact that edges that produce lower sediment amount do not necessarily have low costs.

To have a better understanding of the algorithm’s performance, the best solution found at every iteration for Cases I to IV are shown in Figures 12a through 15a respectively. These figures illustrate the solution improvement until the algorithm reached the best solution found, at iterations 14, 15, 16, and 12 for Cases I through IV, respectively. We also plotted the average transition probabilities for the edges included in the final best solution at the end of each iteration to see the evolution of the transition probabilities affected by pheromone accumulation over time. These transition probabilities for Cases I through IV are shown in Figures 12b through 15b respectively.
From this analysis it is possible to see that after a few iterations, when the ants are exploring different alternative routes, the transition probabilities of the chosen edges rapidly increase, because these edges are more attractive than others, and selected as part of the solution found at every iteration. After the best solution is found, the probabilities of the chosen edges keep slowly increasing until they become close to one. This slowdown phase happens because the increase of the pheromone amount does not proportionally increase transition probability as it approaches the maximum value of one.

Figure 12. Algorithm performance from Case I. a) Solution found at each iteration, and b) average transition probability of all edges forming the final best solution.

Figure 13. Algorithm performance from Case II. a) Solution found at every iteration, and b) average transition probability of all edges forming the final best solution.
Sensitivity Analyses

To evaluate the effects of small parameter changes on the algorithm performance, sensitivity analyses were carried out for $\alpha$, $\beta$, and $\lambda$ using Case I. Several values for each of $\alpha$, $\beta$, and $\lambda$ were tested while others were held constant. The default values for $\alpha$, $\beta$, and $\lambda$ were 1.5, 0.5 and 0.65, respectively (the best parameter combination found previously). Each time only one of the parameters was changed while other parameters remained constant. The tested values for $\alpha$, the relative importance of the pheromone trail intensity, were 0.5, 1.5, 2.5, 3.5, 4.5 and 5.5. Figure 16 shows how the solution quality changes
with the different values of $\alpha$. When $\alpha$ was 0.5, 1.5, and 2.5, the solution found was the same. The solution quality, however, decreased as $\alpha$ became larger than 2.5, the number of iterations taken to reach the solution increases. When $\alpha$ is 1.5, the same quality solution was found quicker than the other values (14 iterations).

![Graph showing the algorithm sensitivity to alpha.](image)

Figure 16. Algorithm sensitivity to alpha.

Different values for $\beta$ were also tested: 0.1, 0.5, 1.5, 2.5, 3.5, 4.5 and 5.5. Figure 17 shows how the solution quality changes with increasing values of $\beta$. The results show that as $\beta$ deviates from 0.5, the solution quality decreases (total cost increases). However, when $\beta = 4.5$ and 5.5, the solution quality improves compared with the two previous values of $\beta$. It seems, the probabilistic nature of the algorithm causes the inconsistent results.
Lastly, we also tested several values for the pheromone evaporation rate \((1 - \lambda)\). The tested values for \(\lambda\) are 0.35, 0.45, 0.55, 0.65, 0.75, 0.85 and 0.95. The best solution found was the same for all these values, a total minimum cost of $129,338. However the number of iterations the algorithm took to reach the solution changes (Figure 18). As \(\lambda\) deviates from the best value found at 0.65, the number of iteration the algorithms increases.
As mentioned above this 100-edge transportation problem is a relatively small problem, therefore the algorithm was able to reach the same solution with different levels of pheromone evaporation rate. However, the value of $\lambda$ affects the algorithm efficiency as shown in Figure 18, this result implies that an incorrect value of $\lambda$ may need more iterations to find a similar quality solution than one carefully selected through initial algorithm trials.

**Conclusions**

In this paper, we introduced a new heuristic approach, the ant colony optimization (ACO) metaheuristic, and developed a specialized algorithm (ACO-FTPP) to solve forest transportation planning problems with fixed and variable costs considering side constraints. The ability to consider these constraints allow us to address various environmental issues in road system management decision making.

A 100-edge hypothetical FTPP was developed to test the performance of our algorithm. ACO-FTPP was able to find a solution for the four cases analyzed; two single goal transportation problems (cost minimization and sediment minimization) and two multiple goal problems (cost minimization subject to an increasing level of sediment restriction). A detailed sensitivity analysis of the most important ACO parameters was conducted to better understand the impact of the parameters on the algorithm performance, and to obtain the best parameter combination for the hypothetical FTPP analyzed.
We compared the results from our ACO-FTPP algorithm with those from a mixed-integer programming (MIP) solver. The current MIP solver was only able to find optimal solutions for the two single goal transportation problems. For the cost minimization problem there was less than a 1% difference between the ACO-FTPP solution and the optimal MIP, and both methods found the same solution for the sediment minimization problem.

Based on the results obtained by ACO-FTPP, we believe our approach is very promising for solving large, real forest transportation problems. Although the hypothetical example used is a relatively small scale problem, it represents a complex problem due to the grid-shaped road network with a large number of road segment leaving each road intersection (an average of five), and the MIP solver could not find an optimal solution for the sediment constrained cases analyzed.

ACO-FTPP can be easily modified to solve more complex transportation problems that consider multiple periods, products, origins and destinations. ACO-FTPP can also solve the problem of mills having a maximum volume capacity by including these mill capacities into the ACO-FTPP formulation as additional constraints.

Further development of the algorithm will need to be done in the following three areas to enhance its performance. First, because the magnitudes of the three variables associated with each edge (fixed cost, variable cost, and sediment amount) are likely to be different,
it would be necessary to evaluate transition probability equations that incorporate these different magnitudes in order to better predict the goodness of a road segment in the solution. Second, local search techniques such as the 2-opt heuristic can also be combined with ACO-FTPP to improve solution quality, although it may likely increase the computing time. The 2-opt heuristic is an exhaustive search of all permutations obtainable by exchanging 2 edges adjacent in solution found at the end of each iteration. Lastly, since the algorithm parameters are heavily dependent on the nature and size of the problem, further evaluation of the robustness of the parameters should be done by applying ACO-FTPP to different problem types and sizes. As shown in the sensitivity analyses the right tuning of parameters can significantly improve the solution quality and efficiency of the algorithm.

References


APENDIX A

INPUT INFORMATION FOR THE 100-EDGE HYPOTHETICAL PROBLEM

a) Costs and sediment data per edge

b) Volume data per timber sale
a) Cost and sediment information per edge

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b) Volume information per timber sale

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