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A two-dimensional model of the earth's atmosphere with application to stratospheric debris transport

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A TWO-DIMENSIONAL MODEL OF THE EARTH'S ATMOSPHERE

WITH APPLICATION TO STRATOSPHERIC DEBRIS TRANSPORT

by

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ABSTRACT

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A Two-Dimensional Model of the Earth's Atmosphere with Application to Stratospheric Debris Transport (67pp.)

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A two-dimensional transport model of the atmosphere from 0 to 50 km is developed. The transport includes advection by the mean meridional circulation and a unique method of modeling the diffusion by large scale eddies. This method utilizes the assumption that the large scale eddies can be characterized by random fluctuations in the velocity field. The mean winds and the magnitude of the random fluctuations vary with latitude, height and season. A Gaussian distribution is assumed for the random fluctuations at each grid point.

Observed mean winds are used below fifteen kilometers. Above this level the mean winds are computed by solving the thermodynamic and continuity equations. The horizontal random velocity fluctuations are made proportional to the observed root mean square of the meridional wind variance from seasonal averages. The vertical random velocity fluctuations are parameterized in terms of the static stability.

The model is capable of simulating the transport of tracers from both low and high latitude sources. As a test of the model, the behavior of Tungsten-185 from low latitude U.S. tests and Zirconium-95 from the high latitude Chinese tests were simulated. The overall distribution of the tracer, rate of spread and the rate of decrease of the maximum concentration agree qualitatively with observations. Discrepancies in the low latitude simulation are discussed in terms of the method used in deriving the characteristic time of the random fluctuations in the equatorial region.

This work demonstrates that diffusion due to eddy fluxes can be modeled under the assumption that the diffusion is due to random processes. This work also shows that the parameterization of diffusion coefficients typical of existing two-dimensional transport models can be circumvented.

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Chapter 1

INTRODUCTION

The possibility that human activity may be affecting the ozone layer of the stratosphere has placed an importance on understanding atmospheric transport processes. There is also the possibility of a nuclear event or an explosive volcano in which case it is advantageous to know that general areas would be affected by the long term fallout of debris originally introduced in the stratosphere. For these reasons, and also for scientific interest in obtaining a more complete picture of our atmospheric environment, the study of atmospheric motions and transport properties are pursued. But the atmosphere forms a very complex system and an accurate determination of the relationship between the atmosphere and an introduced substance requires a knowledge of atmospheric dynamics, photochemical reactions, and many interactions between radiative properties and motions of the atmosphere. The problem is further complicated by the inability to obtain data at high altitudes. Therefore, numerical modeling and computational techniques are necessary in order to relate the complex physical laws of the atmosphere with accessible data.

Because of the amount of data, computational time and space required to accurately solve the physical equations in three dimensions, it is convenient to consider the two-dimensional problem. In two-dimensional modeling all the variables are averaged over longitude and time. Although these models cannot accurately describe the details of atmospheric motions they address themselves to the significant climatic averages. In the process of averaging the governing equations, most of the feedbacks between interdependent quantities are removed. The only one left is that between the radiative properties of the constituents and the mean motions. This feedback relationship is important in studying chemicals such as O_3, H_2O, and CO_2 where the radiative properties of the chemicals have a significant effect on the motions and hence on their distributions. But in considering chemicals without radiative properties such as most nuclear debris, it can be assumed that the dynamical changes will be small perturbations.

The advantages of simplifying the calculations by averaging over time and longitude are offset by a different problem. When the governing equations are averaged, there are correlation terms between the deviations of the motions and the deviations of the constituent concentration from their means. These correlations are generally non-zero and produce a net flux (the eddy flux) of the tracer. Since there is no quantifiable data for the correlation terms, they must be parameterized in terms of mean atmospheric quantities. In working out the equations, most two-dimensional models express the rate of change of a constituent as being dependent on two terms; the advection due to the mean motions and transport due to the correlation term. So the correlation term actually represents the effect of turbulent eddies and must be modeled accordingly. By analogy with the microscopic processes of molecular diffusion, the diffusion due to the eddy fluxes is made proportional to the concentration gradient. The proportionality constant between the correlation term representing the eddy fluxes and the concentration gradient is a second order tensor which is shown to be symmetrical. The parameterization of eddy fluxes is thus accomplished by evaluating or estimating the diffusion coefficients.

The plan of this work was to circumvent the necessity of these diffusion coefficients. By reexamining the formalism used in obtaining the averaged governing equation for the tracer concentration, it was found that simple statistical techniques could be used to simulate eddy diffusion. This method required the assumption that the eddy fluxes are due completely to random fluctuations of the velocity field, which is not completely true. This assumption nonetheless provides one with a useable approximation to eddy transport.

In modeling random processes, the random fluctuations are considered to be related to mean quantities of the atmosphere that are observed or easily calculated. In this way, the necessity of parameter manipulation typical of conventional two-dimensional models employing diffusion coefficients is eliminated. Although in this work the diffusion coefficients of a model developed at NCAR, the National Center for Atmospheric Research (Louis, 1974), were used to derive the time scale over which the random fluctuations occur, this model demonstrates the feasibility of using random processes in two-dimensional modeling to explain eddy transport using only observed mean quantities.

As a test of the model, two simulations of previous nuclear detonations were performed: the distribution of ^{95}Zr from the Chinese test of December 27, 1968 (40° N latitude) and the ^{185}W distribution from the HARDTACK test series of the summer of 1958 (11° N latitude). The results were found to be in general agreement with observations. Some major discrepancies occurred as the simulations were carried out over long periods of time, but these are attributed to the method used in estimating the time scales of the fluctuations. A more comprehensive treatment of the outline developed by this work should eliminate the disparities that have been encountered.

Chapter 2

A DISCUSSION OF ATMOSPHERIC PROCESSES AND STRATOSPHERIC DEBRIS TRANSPORT

The introduction of nuclear debris into the atmosphere has led to two general fields of investigation: the use of radioactivity to observe atmospheric processes, and the investigation of atmospheric processes to predict the distribution of atmospheric debris. The use of radioactive tracers in the atmosphere was intensively investigated by HASP (High Altitude Sampling Program) and other investigators from 1958-1960 following numerous nuclear weapons tests by the U.S., Britain, and the U.S.S.R. (Friend, *et al.*, 1961). Subsequent tests in the early 1960's by the U.S. and U.S.S.R. and more recent French and Chinese tests have contributed much in the way of observed atmospheric motions, especially in the stratosphere (List and Telegadas, 1969; Seitz, *et al.*, 1968; Reiter, 1974).

This presentation is concerned with the use of known atmospheric parameters to predict the distribution of debris by means of computer modeling. The meterological information necessary to model the distribution of radioactive debris is generally unavailable. Specifically, one needs to know:

1) Distribution of vertical motions throughout the atmosphere

2) The complete horizontal wind field (zonal and meridional)

3) The rate of turbulent mass exchange (eddy diffusion or Austausch) in the horizontal and vertical directions.

There is data for the horizontal winds up to 15 km but the time and distance intervals over which the data was taken makes it useful only for longitude and/or time averages. Above 15 km not enough data has been taken to make valid statistical computations, and vertical wind data is virtually nonexistent. The fluctuations of the winds are large compared to the mean winds, the vertical mean winds in particular being practically undetectable (of the order of mm/sec.). Crude calculations of the mean winds where data is unavailable are possible using the heat balance and continuity equations. The procedure involves the evaluation of temperature changes due to adiabatic and diabatic heating, and of vertical and horizontal heat fluxes due to mean motions and turbulent eddy exchange. Sufficient data is available from rocket, satellite and rawinsonde observations for horizontal components but observations are imcomplete as to what extent the vertical eddy heat flux contributes to the total heating rate.

The quantification of turbulent eddy exchange is the main stumbling block in transportation modeling and also the most essential feature of a workable model. The distinction between turbulent exchange and exchange due to mean motions seems to be mainly a problem of semantics. In viewing the atmosphere as a total system the division of atmospheric transport into categories of mean motion and turbulent eddy fluexes is a heuristic over-simplification. Early stratospheric models considering only mean motions showed limited applications, other models considering turbulent diffusion to be the only transport mechanism have shown good agreement with atmospheric inventories, while still more recent models where mean winds and diffusion play a more equal role in transport have also shown good agreement with observation. Turbulence occurs over a wide range of spatial and temporal scales from the smallest wind gusts on the planet surface to cyclones and anticyclones of daily weather patterns to large scale eddies which extend to the order of thousands of kilometers. It is still uncertain which scales of turbulence contribute what portion of the total mixing. The incomplete knowledge of transport processes is compounded by the inability to obtain pertinent data.

Information regarding the initial conditions of a detonated bomb is also necessary for debris transport modeling. In particular:

1) The vertical distribution of the debris cloud after the establishment of thermal equilibrium
2) Particle size distribution in the stabilized cloud
3) Distribution of radioactivity as a function of particle size
4) Meteorological conditions at the time of injection.

The first three are functions of bomb yield, fusion/fission ratio, amount of fractionation and coalescence of debris particles, tropopause height, and other detonation related factors. The fourth is a function of time, place, and 'how the wind blows' at the time of injection. These problems and how they are dealt with will be explained in conjunction with individual simulations.

Structure

A brief picture of the system under consideration may aid in understanding characteristics of the atmosphere that are significant in tracer propagation and deposition. Figure 1 illustrates typical associated temperatures over a longitude average. The lowest layer of the atmosphere, the troposphere, is characterized by a decreasing temperature with height or what is called a positive lapse rate. It extends from the earth's surface up to altitudes that vary with season and latitude. The extent of the troposphere in the tropics is approximately 15 km while at the poles it decreases to 9 km or less. The summer pole tropopause extends higher than that at the winter pole since the summer pole is warmer and will exhibit a positive lapse rate over a greater height. Above the troposphere lies the stratosphere with a zero or positive vertical temperature gradient up to about 48 km. The term lower stratosphere applies to the lower 21 km of the stratosphere or the isothermal layer, the upper stratosphere refers to the remaining portion. At the upper level of the stratosphere the temperature begins to decrease again up to about 81 km in the region known as the

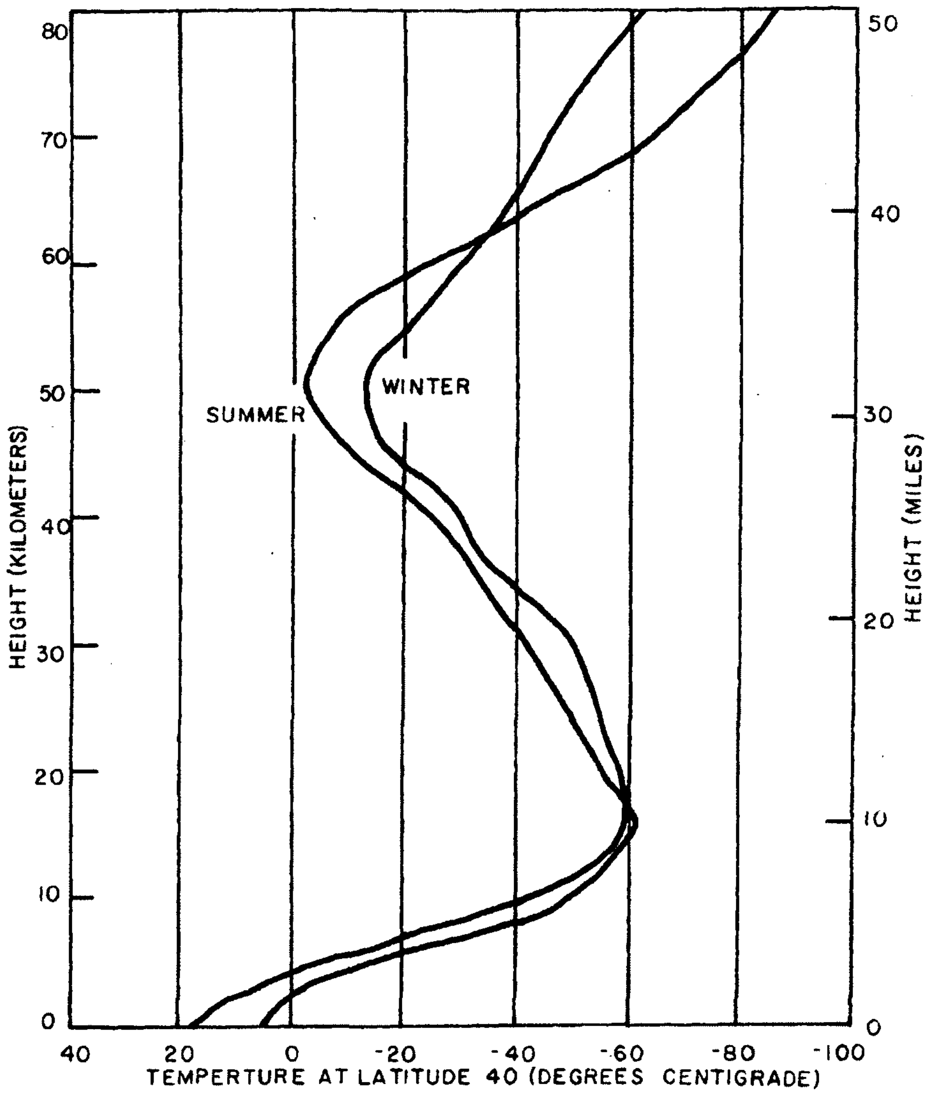


Figure 1. Taken from Newell (1964, pg. 68).

mesosphere. The region between the stratosphere and mesosphere where the temperature gradient changes from positive to negative is termed the stratopause, and the fact that its mean pressure is .1% of sea level pressure indicates that 99.9% of the earth's atmosphere lies below this layer. The definition of the stratosphere-troposphere boundary (tropopause) is that altitude where the temperature gradient changes in the lower 17 km of the atmosphere. This is the important region as far as stratospheric long term fallout is concerned since it represents the boundary for which less mixed stratospheric air enters the well mixed troposphere. Since the mean pressure of the tropopause is about 25% that of sea level, 75% of the atmosphere is in the troposphere. Another positive temperature gradient above the mesophere occurs. This region is referred to as the thermosphere and extends to undefined limits. The outer edge of the atmosphere, called the exosphere, begins at about 600 km above earth and is defined roughly as the layer at which inter-atomic and inter-molecular collisions become negligible. In considering nuclear explosions that have occurred so far, we need only consider the lower 50 km of the atmosphere. The mean cloud height of a 20 megaton explosion is estimated to be about 35 km.

Tropopause Structure

The transition from troposphere to stratosphere is sometimes so abrupt that the tropopause can be identified as a surface with a positive lapse rate below (temperature decrease with height) and an inversion (temperature increase with height) above. Just as often, the transition is gradual and the boundary is undefined for a couple of kilometers, or a number of apparent tropopauses may exist in a longitude average. The tropopause is not a continuous surface, but rather a number of overlapping surfaces, as illustrated in Figure 2. The tropical tropopause is nearly

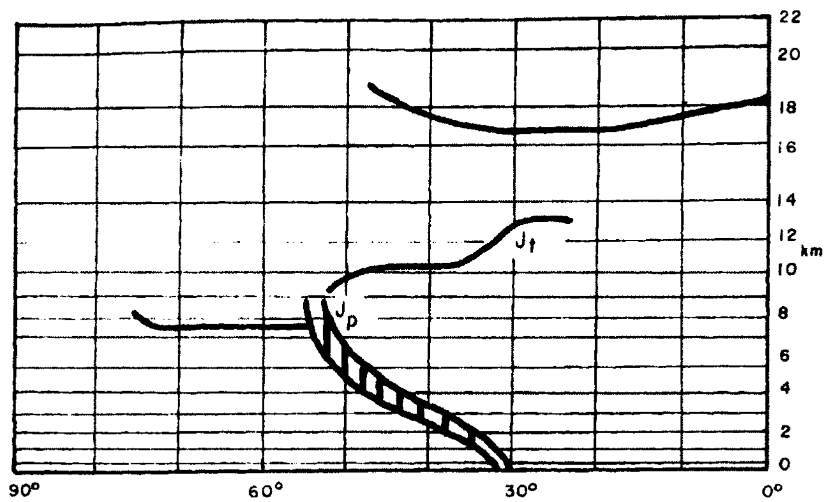


Figure 2. Taken from Palmen (1969, pg. 92). J_p denotes location of polar jet stream, J_t denotes location of tropical jet stream. The dashed lines represent an area of high baroclinicity. The solid lines represent the tropopause boundary in winter northern hemisphere.

horizontal at about 15 km above earth with a horizontal extent from about 35 degrees S. to 35 degrees N., being higher in summer than in winter. A less distinct subtropical tropopause exists at about 11 km with a horizontal extent from 30 degrees to 50 degrees in each hemisphere. The polar tropopause, on the other hand, slopes from the poles upward to about 8 km at 50 degrees latitude for the winter pole. The polar tropopauses are more variable than the tropical and subtropical, varying not only seasonally but with day-to-day air mass movements. When the tropopauses overlap there is a very important region, as far as stratospheric debris is concerned, termed the tropopause gap. The tropopause gap is a transition zone between stratospheric air on the poleward side and tropospheric air on the equatorial side. Particularly in winter, narrow bands of strong winds, the jet streams, are found meandering in wavelike currents around each gap. Turbulence is often severe in the jet stream regions and throughout each gap as a consequence of the large wind shears created by the jet streams. The latitude of each gap varies with season, being closer to the poles in the summer when the jets are weak, and closer to the equator in the winter when they are strongest. However, there are large variations from day to day associated with migratory pressure systems. As a result, the stratosphere acts like a leaky reservoir of stratospheric debris. The leakage is effective in certain places at different times of the year.

Energetics

A concise but thorough account of atmosphere energetics is given by Newell (1964). The vertical change in the temperature gradient can be explained by the radiation of the sun, reradiation by the earth and emission and absorption of energy by ozone, carbon dioxide, and water vapor. The atmosphere absorbs approximately 19% of the sun's radiation, another 34% is reflected into space, and 47% is absorbed by the ground. The portion of energy absorbed by the earth is divided between heating the air, evaporating water, and reradiation. The heating of air and reradiation from earth produces the negative temperature gradient from the ground to 15 km region. At 50 km absorption by ozone is a maximum, decreasing vertically in both directions, thus accounting for the rise in temperature to that region and the decline above. One must also consider the change of temperature with latitude at various heights, as seen in Figure 3. In the 0 to 10 km region, the temperature decreases from equator to pole. In the 15 to 20 km region the temperature increases from equator to pole, exactly contrary to intuition (the coldest temperature below the mesosphere is 18 km above the equator). In the 25-50 km region the temperature increases from equator to summer pole and decreases from equator to winter pole, as would be expected under consideration of the heating unbalance between poles. But from 50-80 km the temperature increases from summer pole to winter pole. It has been estimated that at ground level the radiation arriving at the poles is 120 calories/ cm^2-day (annual average) with 390 calories/cm^2-day reradiated, giving a net deficit. At the equator there is 580 cal/cm^2-day incident and 500 cal/cm^2-day reradiated or a net surplus. The poles would reach a colder equilibrium, and the equatorial regions warmer, were it not for the net poleward flux of heat. This poleward migration of air also transfers momentum, which is the cause of the predominant westerly winds at mid-latitudes and of the jet streams. The method of transporting this energy is due partly to ocean currents, but mostly to the eddy fluxes of cyclones and anticyclones of everyday weather patterns. So the region from 0 to 10 km acts as a heat engine, taking thermal energy and converting a small part of it to kinetic energy of

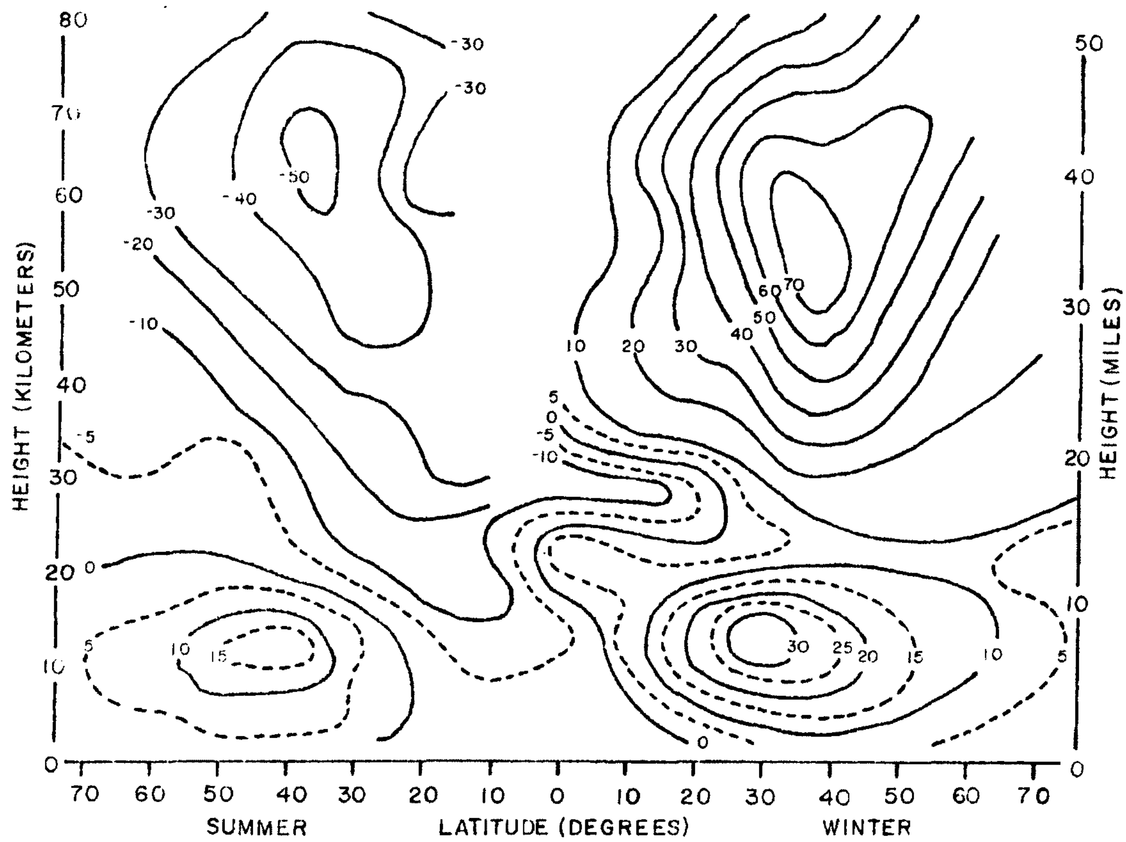


Figure 3. Taken from Newell (1964, pg. 64). Temperature °C.

non-random flow. In the region from 15 to 20 km, ozone concentration and temperature studies have shown that there is a downward flux of poleward moving air from the equator and an upward flux of equatorward moving air from the poles in this region, which represents creation of potential energy against opposing buoyant forces. The forcing of air parcels at angles greater than angles of lines of constant potential temperature (temperature the air would have at ground level) has been conjectured to be due to leaking of kinetic energy from the lower atmosphere, and since there is much less air in the overlying region, only a small percentage of the total kinetic energy in the lower region is necessary to accomplish the forcing. Above the 20 km region up to the 50 km region the summer pole is 20°C warmer than the winter, creating kinetic energy through unbalanced heating rates. The pole-to-pole flux of heat energy transfers momentum and accounts for the westerly winds in the winter hemisphere and for the easterly winds in the summer hemisphere at this region. This is because the angular momentum of an air parcel is conserved in moving from one latitude to another. Above 50 km, a pole-to-pole refrigerator operating in the same way as the equator-to-pole counterpart at lower altitudes is responsible for the observed counter-gradient of temperature. So the picture of the atmosphere is two coupled heat-engine, refrigerator cycles, one from 0 to 20 km acting from equator to pole, the other from 20 to 80 km acting from pole to pole. The vertical interactions between the two separate coupled systems account for the spring maximum of ozone and radioactive transport into the troposphere. In the Northern Hemisphere on December 21st, in the 30 km region the air is moving poleward and ascending from the circulation induced by the warmer south pole, while the air underneath it in the refrigerator cycle of the lower system is moving northward and downward, creating a dead space as far as intermixing goes. But as the radiation balance between poles starts to shift, the vertical motions of the upper level heat-engine begin to change and eventually come into phase with the downward motions of the lower layer, thus forcing any stratospheric debris into mid-latitudes in early spring.

Static Stability

A quantity of importance in explaining the stability and instability of different regions of the atmosphere is the static stability. Here it is advantageous to consider the atmosphere as adiabatic in order to simplify the calculations. Atmospheric motions are generally divided into diabatic and adiabatic categories, depending on their origin.

In considering an air mass in equilibrium, let a small air particle at level z_o, pressure p_o, density ρ_o and entropy s_o be displaced to level z_1, p_1, ρ_1, and s_1. Assume the displacement does not affect the pressure field so the particle arrives at z_1 with p_1, ρ', s_o. The vertical buoyancy force per unit mass becomes

$$F = -g\frac{(\rho'-\rho_1)}{\rho'}$$

The density can be considered a function of entropy and pressure in an unsaturated atmosphere, so

$$d\rho = \gamma dp - \mu ds, \text{ where}$$

$$\gamma = \frac{C_v}{C_p R T} \qquad \mu = \frac{\rho}{C_p}$$

C_v = specific heat at constant volume

C_p = specific heat at constant pressure

$\bar{R}$ = ideal gas constant (per gram) from $\rho = \frac{p}{\bar{R}T}$

$\bar{R} = 2.87 \times 10^6$ cm^2/sec^2-°K)

T = temperature in °K

So,

$$\rho'-\rho_1 = -\mu(s_o-s_1) = \mu \frac{ds}{dz}(z_1-z_o)$$

$F = -\frac{g\mu}{\rho} \cdot \frac{ds}{dz} \cdot (z_1-z_o)$ where F is positive upwards. The particle is restored to equilibrium (or stable) if $\frac{ds}{dz} > 0$ and the particle is moved away from equilibrium (or unstable) if $\frac{ds}{dz} < 0$.

This treatment is only a demonstration of a more rigorous proof which arrives at the same result by considering the extrema (and the signs of the extrema) of the energy stored (kinetic and potential energy) of a virtually displaced particle (Eliassen and Kleinschmidt, 1957).

The entropy/unit mass (s) is defined by

$Tds = de + pd\alpha$, where

$\alpha = \frac{1}{\rho}$, e = internal energy/unit mass.

For dry air $e = C_vT$ + a constant. So,

$$Tds = CpdT - \frac{\bar{R}Tdp}{p} \quad \text{or} \quad ds = \frac{CpdT}{T} - \bar{R}\frac{dp}{p}.$$

From the hydrostatic equation,

$$\frac{dp}{dz} = -\rho g, \text{ hence } \frac{ds}{dz} = \frac{Cp}{T}\frac{dT}{dz} + \frac{\bar{R}\rho g}{p}. \quad \text{So,}$$

$$F/(z_1-z_o) = \frac{g}{T}\left(\frac{dT}{dz} + \frac{g}{Cp}\right); \quad \frac{dT}{dz} > -\frac{g}{Cp} \Longrightarrow \text{static stability.}$$

Physically this means that if the temperature gradient is negative and larger than the dry adiabatic lapse rate (g/Cp) the air is unstable and turbulent mixing will occur.

The stratosphere is considered stable because the temperature gradient is always positive. The instability of the troposphere can be attributed to its negative temperature gradient, while the tropopause can be considered a transition region.

Characteristics and Removal Mechanisms of Stratospheric Radioactive Debris

A prediction of the deposition of long term fallout requires the specification of the following quantities: the initial injection, transport processes throughout the atmosphere, stratospheric-tropospheric exchange processes, and deposition mechanisms. In a nuclear event, the radioactive debris injected into the stratosphere is carried with the fireball to a height where thermal equilibrium is reached (Figure 4). The height of the radioactive cloud therefore depends on the yield and type of burst and also on the tropopause height. The type and amount of radioactive debris will also depend on the fission/fusion ratio of the reaction, the higher the ratio the more heavy nuclei particles are injected. A 10 MT ground burst in the tropics would inject anywhere from 25% to 50% of its total yield of ^{90}Sr into the stratosphere. The particles that remained in the troposphere will be rained out or will settle, the troposphere having a residence time (time for which only 1/e of the original amount

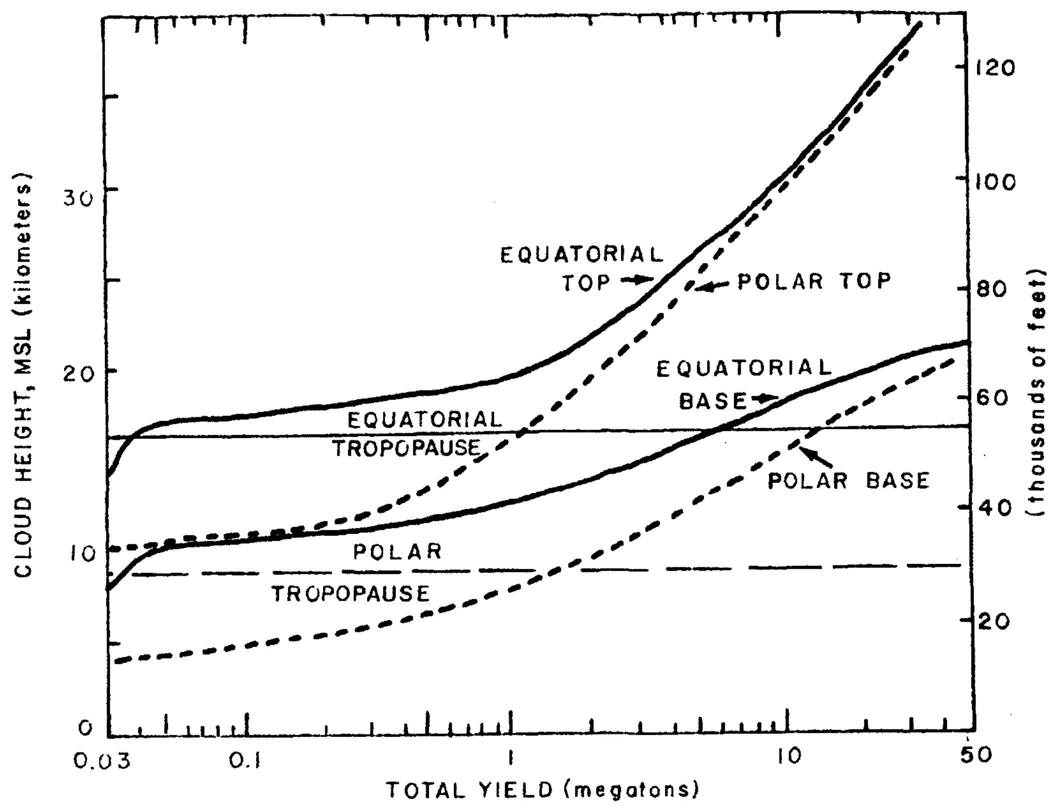


Figure 4. Taken from Peterson (1970, pg. 360). Height of cloud top and base as a function of bomb yield for equitorial and polar injections.

is present) of 20 to 40 days. The rest of the material remains in the stratosphere with residence times of a few months to a few years depending on location and season of injection. A striking feature of past stratospheric inventories is that, after enough time, the concentration maximum lies along certain defined angles regardless of the height and latitude of injection (Figure 5, Machta, et al., 1970). These angles of maximum concentration nearly coincide with the isentropic surfaces of the lower stratosphere (lines of constant potential temperature

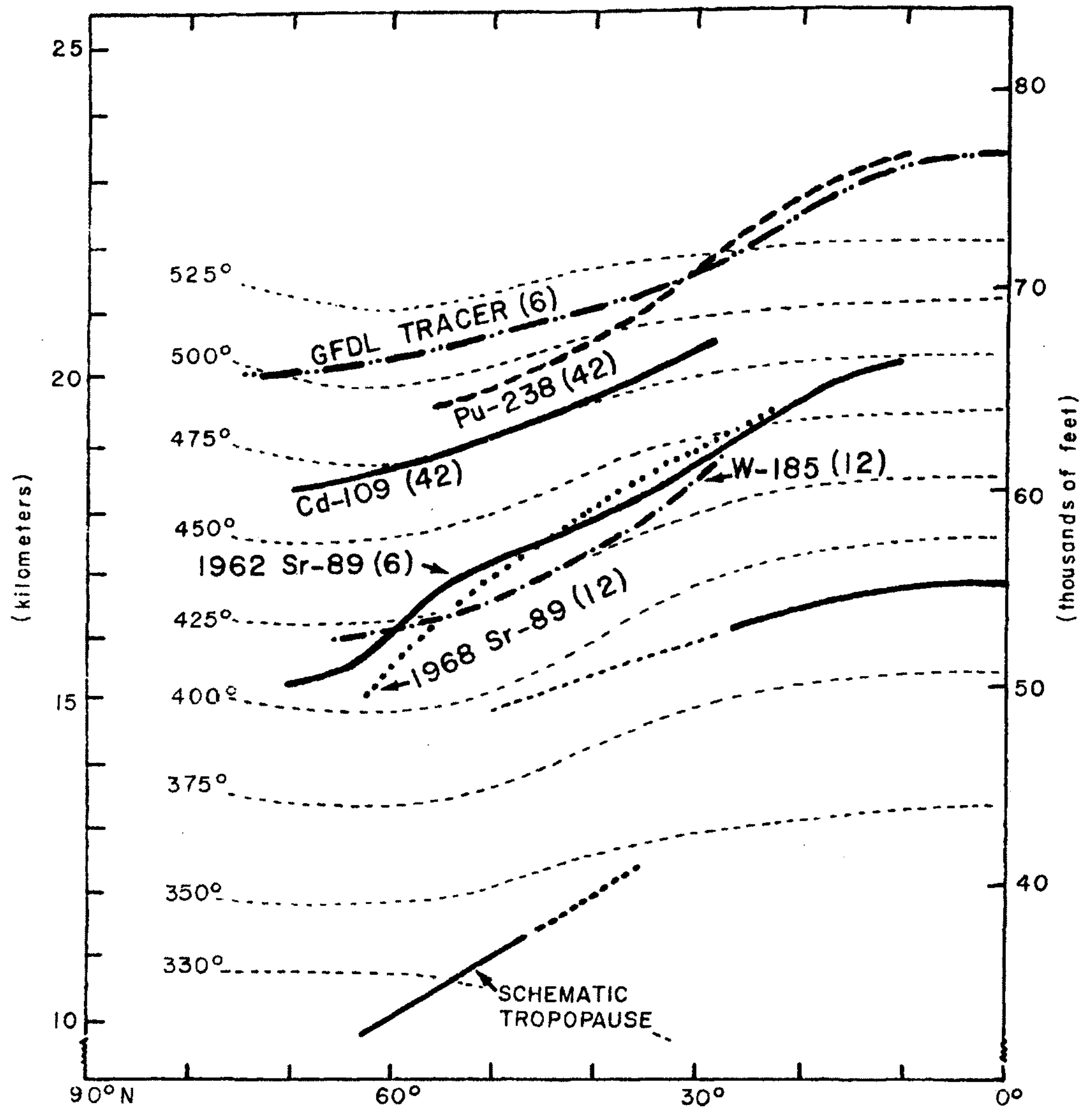


Figure 5. Observed levels of maximum concentration and mean annual isentropes. Numbers in parentheses are months from injection into the stratosphere to observation. Taken from Matcha, et al., (1970, pg. 2288).

Tracer	Latitude of Injection	Date (and height) of Injection
1962 Sr-89	75°N	September 1 - November 4
1968 Sr-89	40°N	June 17, 1967
W-185	11°N	August 21, 1958
Pb-238	11°S	April, 1964 (about 55 km satellite burnup)
Cd-109	17°N	July 9, 1962 (about 400 km satellite burnup)
GFDL tracer	11°N	January 21, 1968 (about 20 km)

are lines of constant entropy). This approximation is often utilized in models to simulate the angle of turbulent diffusion exemplified by Figure 6. But evidence more closely supports the notion that these levels lie along lines of constant potential vorticity $[(\nabla x \vec{v}) \cdot \alpha \nabla \Theta]$, a quasi-conserved (adiabatically conserved) quantity.

The exact mechanisms behind the transfer from stratosphere to troposphere are not completely understood, and not quantifiable in terms of what process contributes what percent to the total exchange since any conjectured 'process' is a simplification taken from scattered observations of complex atmospheric phenomena. Evidence strongly suggests that the tropopause gap is a major center for exchange, and some outstanding general features of debris deposition can be attributed to gap phenomena. Examples are the spring and mid-latitude maxima illustrated in Figures 6 and 7. Besides horizontal and vertical eddy diffusion being greater at this region, there is a high frequency of baroclinic storms in the region, especially during the spring. The tropopause gap moves north and south with the migration of polar fronts. The gap is not a well defined hole, but a weak spot in the tropopause, fluctuating in intensity at a given location. To further complicate matters the higher latitude tropopause has vertical motion, ascending in the spring and descending in the fall, which brings the gap up to a level near the highest concentration. The forcing down of polar air from the spring

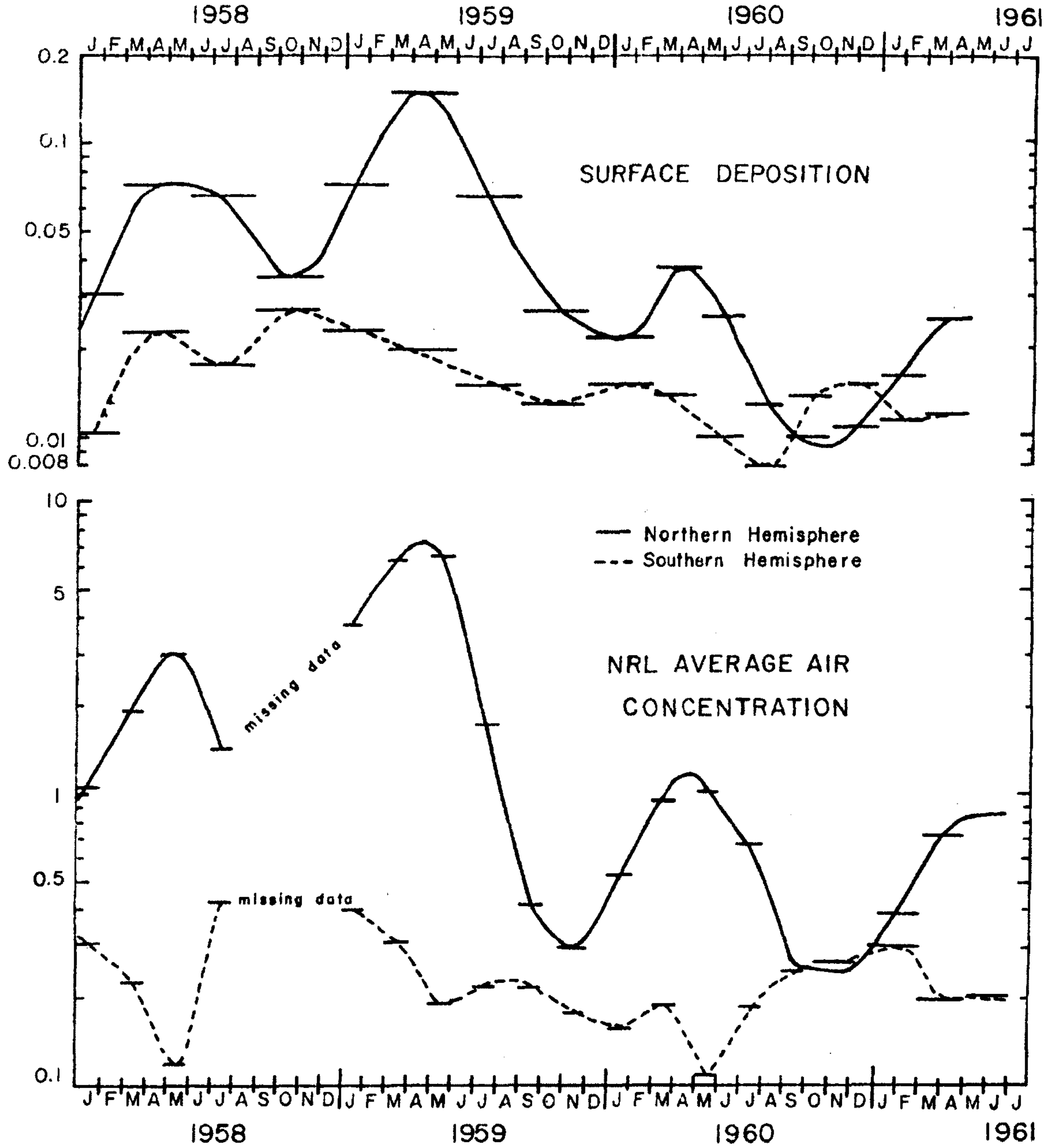


Figure 6. Taken from Matcha, et al., (1962, pg. 168).

breakup of the polar vortex and the relation of the gap to migration patterns of the jet stream intensity have been conjectured to be responsible when and where the gap is effective. Other portions of the tropopause facilitate leakage through high level

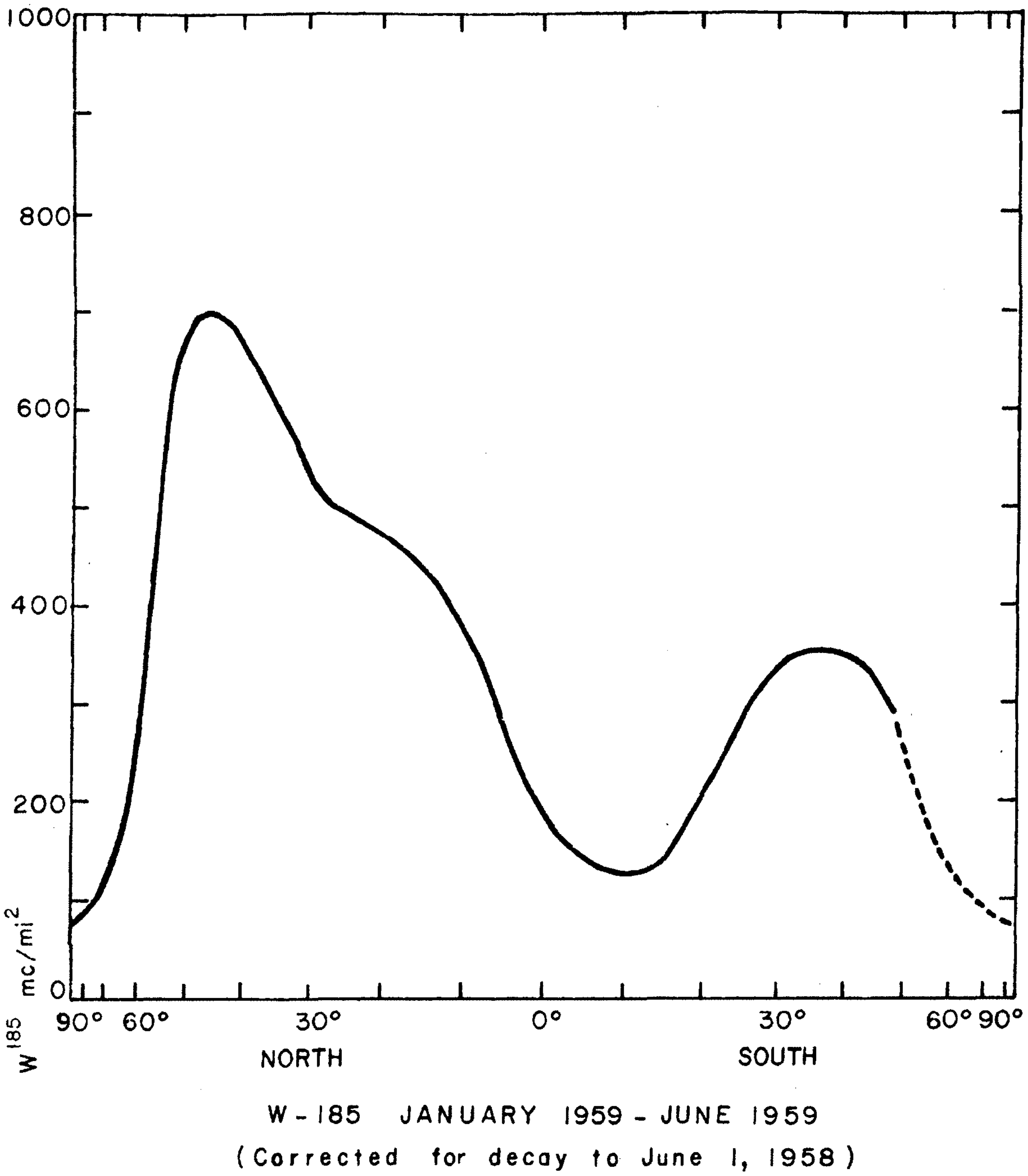


Figure 7. Taken from Matcha, et al., (1962, pg. 160).

troughs and upper cloud lows of high altitude weather systems. Short time injections of high level air due to folding of the tropopause has been demonstrated to occur continuously throughout the tropopause area. A stratospheric extrusion may contain approximately 10^{12} metric tons of air, being replaced by a near equivalent amount of tropospheric air, with about 5 extrusions active around a hemisphere at any time. Penetration from large convective storms will mix debris with clouds, bringing down higher concentrations of radioactivity in the form of rainout. Recent studies (Reiter, R., 1975) have shown a well-marked correlation between solar flare activity and large scale instusions of stratospheric air. All of these factors tend to make accurate modeling more dependent on empirical parameterization, and in most cases, all the exchange phenomena are simply lumped into 'eddy diffusion.'

Dry fallout comprises anywhere from 10% to 30% of the total fallout, depending on precipitation rates and the latitude under consideration. The mechanisms by which rain scavenges nuclear debris are not understood. It should be kept in mind that the fallout associated with ^{90}Sr and ^{137}Cs actually refers to aerosol particles of diameter less than .2 microns (or .02 microns at altitudes above 30 km), while the term 'fallout' is also associated with gaseous radioactive material such as ^{14}C which is unaffected by precipitation scavenging. There is no convincing evidence that the fallout particles themselves act as condensation nuclei in the formation of droplets, but to some extent the particles in the upper troposphere attach themselves to sulfur compound particles which do act as condensation nuclei. The correlation

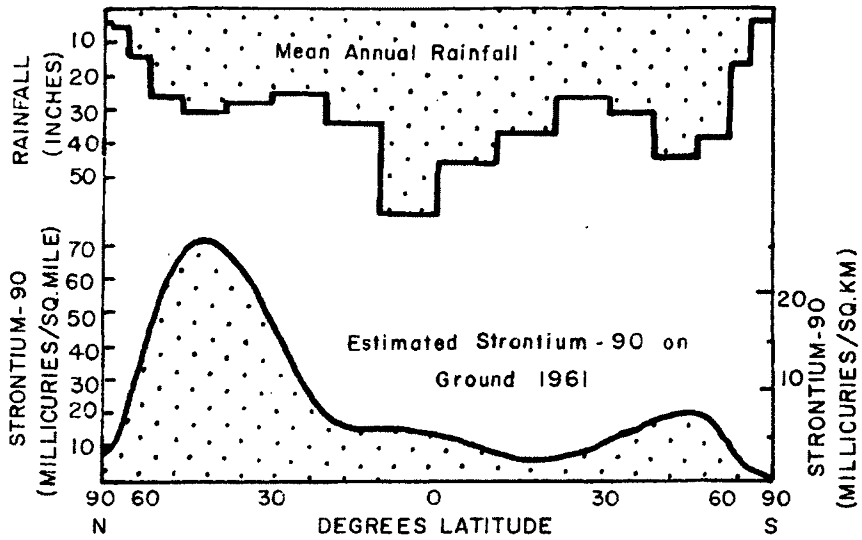


Figure 8. Taken from Glasstone (1962, pg. 287).

of deposition to high level convective storms indicates that water vapor and precipitation simply wash the debris from the sky once the material has mixed with the unstable air. In some regions the deposition rate is proportional to rainout while in others, there are large deviations from linearity (see Figure 8). Completely empirical but thorough models using only interpolation from previous tests and observations yield moderately successful predictions (Peterson, 1971). Deposition models rely heavily on empirical and semiempirical parameterization. Many models divide the atmosphere into a number of boxes and use observed residence times for each region to approximate diffusion coefficients. Settling velocities for particles are sometimes included, and it is generally assumed that all fallout is linearly dependent on the precipitation rate with removal factors varying with height. The most noteworthy model is one by Davidson, et al., (1966, 1968), which includes a 4 box model with 2 gap regions, and assumes diffusion, rainout and settling to be the determining factors. After sufficient parameter manipulation the model was able to simulate time and latitude maxima and minima for polar and equatorial injections.

Chapter 3

A DISCUSSION OF MODELING

The importance of an accurate description of atmospheric transport goes beyond scientific curiosity to problems of immediate concern. Stratospheric pollutants and their effects on the ozone layer have prompted the SST controversy, and have led to many attempts toward an accurate description of relevant processes. Since only a limited amount of data and knowledge exists concerning the photochemistry, radioactive properties and dynamics of the atmosphere, numerical modeling is essential. Although it was generally concluded in the early 1970's that the immediate effects of stratospheric fallout from the nuclear tests conducted so far were not significant in terms of human health, the comparison between observed radioactive distributions and those predicted by a model provides a good check of the modeler's intuition. In considering long term fallout, there are completely empirical models (Peterson, 1970) that interpolate and extrapolate data produced from previous nuclear tests to estimate deposition given the latitude and yield of the explosion. It is more advantageous in terms of descriptive processes to model the atmosphere and then to study the dispersal of an introduced tracer.

Atmospheric modeling on local and global scales has a long history. The first effort was made in 1911 by the British mathematical physicist L. F. Richardson, who did hand calculations of finite difference hydrodynamic equations that gave pressure differences over Europe 60 times greater than had ever been observed. His errors lay in the size of his time step, and in meteorological noise due to errors in the initial conditions. It was not until the late 1940's that, under the direction of John von Neumann, the Maniac computer at the Institute of Advanced Studies at Princeton was used to give a reasonably accurate description of an incompressible, two-dimensional flow over the Northern Hemisphere. The mid-1950's brought further attempts to predict weather patterns by the Navy, Air Force, and Weather Bureau which were the beginnings of recent three-dimensional global circulation models (GCM's). More recent GCM's have been developed by several research groups (e.g., Manabe et al., 1970; Smagorinsky et al., 1965; Kasahara and Washington, 1971; Kasahara and Sasamori, 1974). These models solve simultaneously the continuity, momentum and energy equations in order to simulate the evolution of the pressure, temperature, and wind fields in the atmosphere. Tracers can be included in these models, and their transport can be studied. Photochemical reactions could also be included.

All GCM's must obey the physical laws of the atmosphere which can be written as

1) $\rho \frac{D\vec{v}}{Dt} = -\rho\nabla\phi_g - 2\rho\vec{r}\times\vec{v} - \nabla p + \vec{F}$ Equation of Motion (momentum equation)

2) $\frac{dp}{dz} + \rho g = 0$ Hydrostatic Balance Approximation

3) $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$ Equation of Continuity of Air

4) $\frac{\partial q}{\partial t} + \nabla \cdot (q\vec{v}) = 0$ Equation of Continuity of Substance q'

5) $p = \rho \bar{R} T$ Ideal Gas Law

and

6) $\frac{\partial (\rho T)}{\partial t} + \nabla \cdot (\rho T \vec{v}) = \rho S_T$ The Conservation of Energy or Thermodynamic Energy Equation.

ρ = density

T = temperature

$\vec{v}$ = velocity

S = source or sink of constituent q'

ϕg = geopotential

S_T = sources of thermal energy

$\vec{\Omega}$ = angular velocity of earth

p = pressure

F = frictional force terms

q = density of constituent q'

g = 980 cm/sec^2

$\bar{R} = 2.87 \times 10^6$ cm^2/sec^2-°K.

The set of equations is a closed set if $\vec{F}$ and S_T plus all the source terms for interactive substances q' in the atmosphere are known. That is, for one constituent (q'), there are six equations and six unknowns, for two constituents there are seven equations and seven unknowns ... etc. When finite difference techniques are used, all subgrid scale processes of importance, such as the vertical flux of horizontal momentum and subgrid turbulent eddy diffusion, must be parameterized in the friction and source terms. According to the needs of the model, such processes as the release of latent heat by cumulus convection (the principal heat source in tropospheric circulation), the effects of clouds, ozone and ocean-atmosphere interactions must all be parameterized in the heating rate and friction terms of the governing equations. The errors inherent in GCM's arise from the parameterization of subgrid processes, the specification of the initial state and the approximation of nonlinear systems of differential equations by finite difference equations. All the variables appear simultaneously in several equations in such a way that it is not possible to separate them. The variables are related to each other by feedback phenomena and it is not possible to modify one variable without affecting the others. For example, a change in temperature will modify the pressure and wind patterns. These modifications will affect the transport of sensible heat, thus changing the temperature again. Another feedback loop exists between the continuity equation for constituents with radiative or photochemical properties and the heat balance and continuity of air equations. If the distribution of a radiatively active constituent (such as ozone) changes, the thermal structure of the atmosphere changes, the wind pattern changes, and consequently the constituent distribution is itself modified.

Tracers are introduced into the model through the constituent continuity equation and the distribution is simulated, but the typical three-dimensional GCM has a very large vertical grid space which does not allow accurate simulation of vertical processes such as tropopause folding in the region of the tropopause. Three-dimensional computer simulations take a large amount of storage space and time. An average model may have 100 km grid points horizontally and 15 grid points vertically with about 2×10^3 arithmetic and logical operations per grid point at each time step. For a 5 minute time step (typical for long range forecasting) and a computing ratio of 20 to 1 (1 day's computing forecasts 20 days ahead) 10^8 operations/sec would be required. This rate is near the upper limit of presently existing computers. Therefore there are distinct disadvantages to three-dimensional modeling in describing debris transport, and until a new generation of computers exists to overcome these disadvantages the most promising approach toward a more workable description of atmospheric transport seems to be two-dimensional modeling.

Two-Dimensional Modeling

In conventional two-dimensional models all the equations are averaged over time (month, season, or year) and/or longitude. These models cannot describe all the atmospheric motion, but address themselves to the more important climatic averages. By averaging the equations most of the feedback effects between different quantities are removed. The only effect left is the one between the radiative properties and the mean motions. For radioactive tracers the radiative properties are negligible and the dynamical changes can be assumed to be small perturbations.

With this simplification the transport of a tracer can be described by a single equation, the averaged tracer continuity equation. The advantages of two-dimensional averaged modeling are obvious. The amount of computer time and storage space necessary makes such modeling feasible for existing computers, even for the University of Montana Decsystem-10. There are inherent disadvantages with conventional two-dimensional models. The main disadvantage is the parameterization of correlation terms between the deviation (from zonal and time averages) of the winds and the tracer concentration that arise in averaging the equations. These correlation terms are generally nonzero and produce a net flux (the eddy flus) of the tracer. In almost all two-dimensional transport models the spreading of introduced material is accomplished by these eddy fluxes which represent the effect of turbulent eddy diffusion. There is no measurable quantity that can be assigned to eddy diffusion, so it must be approximated in terms of mean atmospheric quantities. By analogy with the microscopic processes of molecular diffusion, the macroscopic diffusion produced by the large-scale eddies is parameterized by assuming that the eddy flux is proportional to the gradient of the mean concentration of the tracer. Since the mixing processes in the atmosphere are not isotropic (along vertical and horizontal axes), the proportionality factor (or diffusion coefficient) must be a second order tensor. Modeling under the assumption that the mixing processes are due to mean motions and eddy diffusion has met with varied success.

The first two-dimensional models originated in the late fifties and were a consequence of two different fields of research. The detonations by the U.S. and U.S.S.R. of 1958 and 1959 were extensively studied. Numerical models appeared to predict the evolution of radioactive materials from these sources and from others purposely introduced for transport investigation, such as ^{185}W tracer isotopes. At the same time it was realized that the peculiar distribution of ozone in the stratosphere could not be explained by photochemical processes alone and that atmospheric motions were a definite influence, thus adventing the use of two-dimensional models in ozone studies.

The work by Prabakhara (1963) was an early attempt to explain the ozone distribution. This study combined a mean circulation due to Mergatroyd and Singleton (1961) and a Fickian diffusion with no off-diagonal terms in the diffusion coefficient. The magnitudes of the diffusion coefficients were based on the ^{185}W distribution from the Hardtack tests as observed by Feeley and Spar (1960). The circulation of Mergatroyd and Singleton was inaccurate in that they neglected the effect of eddy heat transport in their calculations of the mean winds. Thus Prabakhara made diffusion the main mechanism for the poleward flux of ozone in the lower equatorial stratosphere. The omission of the eddy transport made it necessary to reduce the winds by 80% to describe mid- and high-latitude distributions.

At the same time studies at MIT (Oort, 1964) showed an equatorial flux of heat in the lower stratosphere which is unexplainable in terms of eddy transport by Fickian diffusion. As mentioned in the previous chapter (Energetics section) Newell (1964) showed that this process was possible if parcels of air moved from equator to pole along trajectories with angles greater than that of mean isentropic surfaces. Hence the off-diagonal terms of the diffusion coefficient must be non-zero.

Reed and German (1965) tried to devise a way to determine the diffusion coefficients from observed mean quantities without arbitrary parameter adjustments typical of existing two-dimensional models, but they were unable to obtain an analytical solution without setting the diffusion coefficients at constant values throughout the atmosphere. This model and a more successful one by Davidson, et al., (1966) have only diffusion as the means of transport, but unlike Reed and German, Davidson varied the values of diffusion coefficients over specific regions and allowed for a transition zone at the tropopause. In experimenting with the principal axis of diffusion Davidson, et al., observed that the ^{185}W concentration pattern and rainout were quantitatively well produced when the principal diffusion axis was parallel to lines of constant potential vorticity. Injections of ^{90}Sr in the polar regions were qualitatively reproduced.

Gudisken, et al., (1968) used the varying diffusion coefficients of Reed and German (divided by a factor of 8 horizontally and 2 vertically) and a circulation based on the horizontal wind observations of Tewerles (1963). The ^{185}W distribution was reproduced quite well by this model but it could not account for mid-latitude maxima actually observed. More recently, there are many other two-dimensional models that may or may not utilize diffusion coefficients. Stone (1974) uses results from baroclinic wave theory to calculate eddy heat fluxes. A tropospheric model by McCracken (1971) solves the complete set of governing equations but in two dimensions, avoiding seasonal averages.

The NCAR Model

It is illustrative to elaborate on one particular two-dimensional model that is relatively successful and is actually the basis for much of this study. This model was developed by Jean-Francois Louis (1974) for the National Center for Atmospheric Research at Boulder, Colorado. Essentially, the averaged equation for the continuity of the tracer concentration is solved numerically with 5 degree latitude grid space from pole to pole and 1 km grid space from ground level to 50 km, using seasonally averaged wind, density, and diffusion calculations. To obtain the tracer concentration continuity equation, the equations of continuity for air and for the tracer are averaged over longitude and time. For any quantity b, a function of space and time,

$$\bar{b} = \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} b \, dt \, d\phi$$

ϕ = longitude
t = time
T = one season (90 days)

Then,

$b = \bar{b} + b'$, where b' is the deviation of b from $\bar{b}$. Clearly $\bar{b'} = 0$. Difficulties arise in taking the average of products. For any quantities a and b, functions of time and space,

$$\frac{1}{2\pi} \int_{0}^{2\pi} \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} (ab)\, dt\, d\phi =$$

$$\frac{1}{2\pi} \int_{0}^{2\pi} \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} (\bar{a} + a')(\bar{b} + b')\, dt\, d\phi .$$

When t equals the midpoint of the season and a and b are constant over the season,

$$\overline{ab} = \bar{a}\bar{b} + \overline{a'b'}.$$

$\overline{a'b'}$ is called the correlation between the deviations of a and b. The equation of continuity for air becomes

$$\frac{\partial \bar{\rho}}{\partial t} = -\nabla \cdot (\bar{\rho}\bar{\vec{v}}) \qquad 1)$$

using the common assumption that all density correlations are negligible. The density of the tracer, $\rho_t = x\rho$ and the continuity equation for the tracer $\frac{\partial \rho t}{\partial t} = \frac{\partial \rho x}{\partial t} = -\nabla \cdot \rho v x + \rho S$ (where S is the source term), when averaged becomes,

$$\frac{\partial \bar{\rho}\bar{x}}{\partial t} = -\nabla \cdot (\bar{\rho}\bar{\vec{v}}\,\bar{x}) - \nabla \cdot (\bar{\rho}\overline{\vec{v}'x'}) + \bar{\rho}\bar{S} . \qquad 2)$$

Multiplying equation 1) on both sides by $\bar{x}$, subtracting this result from equation 2) and dividing this result by $^{-}$ gives

$$\frac{\partial \bar{x}}{\partial t} = - \bar{\vec{v}} \cdot \nabla \bar{x} - \frac{1}{\bar{\rho}} \nabla \cdot (\overline{\bar{\rho} v' x'}) + \bar{S} \; . \qquad 3)$$

The first term on the right hand side of 3) represents the effect of advection by the mean winds. The second term involving the correlation between the deviations of wind and concentration represents the effect of diffusion by eddies. Since there is not data for this correlation, the diffusion term must be approximated in terms of mean atmospheric quantities. The correlation term could be considered as an unknown and continuity equations for the two components of $\overline{\vec{v}'x'}$ could be formalized but these equations involve triple correlations $\overline{v'v'x'}$, $\overline{v'w'x'}$, and $\overline{w'w'x'}$ where v' is the horizontal component of $\vec{v}'$ and w' is the vertical component of $\vec{v}'$. This veature introduces the problem of closure. The technique generally employed is to use a first order closure method and make $\overline{\vec{v}'x'}$ proportional to $\nabla\bar{x}$. Then $\overline{\vec{v}'x'} = -K \cdot \nabla \bar{x}$ where K is the second order diffusion coefficient tensor. Equation 3) then becomes

$$\frac{d\bar{x}}{dt} = -\bar{\vec{v}} \cdot \nabla \bar{x} + \frac{1}{\bar{\rho}} \nabla \cdot (\bar{\rho}\; K \cdot \nabla \bar{x}) + \bar{S} \quad . \qquad 4)$$

The time-dependent equation for $\bar{x}$ is solved using a semi-implicit alternating direction scheme once $\bar{\rho}$, $\bar{v}$, $\bar{w}$, and K have been chosen. $\bar{\rho}$ was determined from satellite and rocket data of mean temperature distributions and the 500 millibar pressure height. Below 15 km

$\bar{v}$ and $\bar{w}$ can be estimated from observations, but above 15 km $\bar{v}$ and $\bar{w}$ were determined simultaneously from the averaged heat balance equation and the equation of continuity for air. The heating rates were given by Kuhn (1969) and the horizontal eddy heat fluxes were taken from rocket and rawinsonde observations. Although the vertical eddy heat fluxes are unknown they were assumed to be negligible (see justification on pg.46).

First attempts at evaluating the diffusion coefficients followed somewhat the formalism set by Reed and German (1965). A simplified version of a more formal approach shows how these coefficients can be approximated. By analogy with molecular processes, the net flux due to turbulence (correlation term) is assumed to be proportional to the gradient of the mean tracer mixing ratio. Since atmospheric turbulence is nonisotropic the proportionality constant must be a second order tensor, i.e.,

$$\begin{pmatrix} \overline{v'x'} \\ \overline{w'x'} \end{pmatrix} = \begin{pmatrix} Kyy & Kyz \\ Kzy & Kzz \end{pmatrix} \begin{pmatrix} \frac{\partial \bar{x}}{\partial y} \\ \frac{\partial \bar{x}}{\partial z} \end{pmatrix} \qquad 5)$$

It was argued that if a certain parcel of air carries with it the characteristics of the environment a distance $\vec{\lambda}$ then $x' = \vec{\lambda x} \cdot \begin{pmatrix} \frac{\partial x}{\partial y} \\ \frac{\partial x}{\partial z} \end{pmatrix}$. By further assuming

$$\vec{\lambda} = \vec{v}'\tau \qquad 6)$$

where τ is the characteristic time of the mixing processes, equation 5) can be written as

$$\begin{pmatrix} \overline{v'x'} \\ \overline{w'x'} \end{pmatrix} = \begin{pmatrix} \overline{v'v'} & \overline{v'w'} \\ \overline{w'v'} & \overline{w'w'} \end{pmatrix} \tau \begin{pmatrix} \frac{\partial x}{\partial y} \\ \frac{\partial x}{\partial z} \end{pmatrix} \qquad 7)$$

The assumption of equation 6) that $\vec{\lambda}$ and $\vec{v}$ are in the same direction may not be true and is currently under investigation at NCAR. Equation 7) states the diffusion coefficient tensor is the Reynolds stress tensor multiplied by a characteristic time τ. Equation 7) also shows that the tensor is symmetrical, which means a rotation of the original coordinate axis by a unique angle would make the tensor diagonal. Only $\overline{v'v'}$ can be estimated from observations; the other two terms must be parameterized. $\overline{w'w'}$ was made proportional to the static stability, and once the principal diffusion axis was determined, $\overline{v'w'}$ was found by a coordinate change of the tensor.

After several attempts this approach was abandoned because of the inability to negate the effect of the mean vertical circulation in the tropical troposphere. A larger downward flux of tracer from the 1968 Chinese test was predicted than was actually observed. The coefficients were then determined in certain regions by subtracting the mean circulation from observed ozone fluxes, and were parameterized or linearly approximated in other regions. This second model worked very well in simulating high latitude and equatorial injections when the mean winds and diffusion coefficients were divided by two - a procedure consistent with the limits of accuracy for the computed winds above 15 km.

Modeling with Random Processes

The time dependent solution of equation 4) is actually an approximation. To obtain equation 2) it was assumed in averaging the term $x\vec{v}$ that

$$\frac{1}{T}\int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \frac{1}{2\pi}\int_{0}^{2\pi} \overline{x\vec{v}}'\, d\phi\, dt = 0 \quad .$$

But in the time dependent approach, this term will not necessarily be zero. Also, the lengthy arguments and calculations involved in obtaining the diffusion coefficients led me to search for a more direct way of modeling eddy transport in terms of physical processes.

In obtaining equation 3) the continuity equations for the tracer and air were first averaged over longitude and time, and then combined by subtraction to give a continuity equation for the tracer concentration $\bar{x}$. By first averaging the continuity equation for the tracer and the continuity equation for air over longitude (to make the equations over two dimensions),

$$\frac{\partial\rho}{\partial t} = -\nabla\cdot(\rho\vec{v}) = -\frac{\partial(\rho v)}{r\partial\phi}\quad\frac{\partial(\rho w)}{z} \qquad 8)$$

and

$$\frac{\partial(\rho x)}{t} = -\nabla\cdot(\rho\vec{v}x) = -\frac{\partial(\rho x v)}{r\partial\phi}\quad\frac{\partial(\rho x w)}{z} \quad . \qquad 9)$$

Then multiplying 8) by x and subtracting this product from 9), one obtains

$$\frac{\partial x}{\partial t} = - v \frac{\partial x}{r\partial\phi} - w \frac{\partial x}{\partial z} = -\vec{v}\cdot\nabla x \qquad 10)$$

which is just the two-dimensional Liouville equation for x. With such a simplification, all the transport properties must be modeled in the velocity term. It is convenient to consider the velocity as consisting of two parts: a mean circulation and a random deviation from the mean circulation that represents a random eddy transport, or

$\vec{v} = \bar{\vec{v}} + \vec{v}_{RAN}$, where $\bar{\vec{v}}$ = mean velocity, $\vec{v}_{RAN}$ = random velocity.

Within a time period characteristic of the mixing processes (of the order of a day) a number of random wind events will effectively diffuse the tracer. For the eddy transport to be modeled as completely random, the average over the characteristic time interval of the random winds must be zero. There is some physical reason for believing the diffusion process to be partly nonrandom (that portion due to standing eddies). Then the average of the random winds would not necessarily be zero, and a predetermined random wind average corresponding to observed fluxes would need to be incorporated. But transport mechanisms such as clear air turbulence and transient eddies on all scales suggest that random modeling may be useful to some extent. The random portion of the velocity may be parameterized in terms of atmospheric quantities quite easily, which is a major advantage over diffusion modeling. However, a distinct disadvantage and probably the biggest drawback to the method is the lack of a diffusion term

($\nabla^2 x$) in the governing equation. A more complete formalism could possibly combine random processes and diffusion processes to better simulate a distinction between transient and standing eddies.

For the purpose of this work, the random winds formed a Gaussian distribution and averaged to zero over a long time. The horizontal component of the random winds was set proportional to the meridonal wind variance, the vertical portion being proportional to the static stability. To simulate random diffusion off the regular axis the random winds were assumed to originate in a different coordinate system rotated through an angle from the regular coordinate axis. For convenience only, the angle was chosen to be proportional to the off-diagonal term of the diffusion tensor given in the NCAR model. The mean circulation below 15 km was teken from observations as given by NCAR and above 15 km the winds were recomputed in the same fashion as in the NCAR model. All the data used were seasonally averaged, which is inappropriate for a precise time dependent solution, but the main features of the mean transport are identifiable.

Chapter 4

THE MODEL

As mentioned in the previous chapter, the modeling of debris transport is done by numerically solving equation 10) for the mass mixing ratio, x. With the partitioning of the velocity into mean and random components, equation 10) becomes

$$\frac{dx}{dt} = \frac{(\bar{v} + v_{RAN})}{r}\frac{dx}{d\phi} + (\bar{w} + w_{RAN})\frac{dx}{dz} \,. \qquad 11)$$

For a given time step, x is determined for several random wind possibilities. The average x determined by these events is used in the calculation of the next time step. The density of the tracer, ρ_t , is related to x by $\rho_t = x\bar{\rho}$. So once the amount of the initial injection is known in terms of the density of the tracer, it is necessary to know $\bar{\rho}$ in order to solve equation 11) for x. Therefore, there are five quantities that must be known in order to solve 11); $\bar{\rho}$, $\bar{v}$, $\bar{w}$, v_{RAN}, and w_{RAN}. The following section describes how these quantities were obtained.

Determination of Density

Once a temperature field over the region considered has been chosen, by knowing the height for a given pressure, the density can be found over the entire vertical extent. Using the hydrostatic equation, and the equation of state,

$$\frac{dp}{dz} = -\bar{\rho}g \qquad \text{and} \qquad \bar{p} = \bar{\rho}\,\bar{R}\,\bar{T} \quad ,$$

then $\frac{d\bar{p}}{\bar{p}} = -\frac{g\,dz}{R\,\bar{T}}$ and $p(h) = 500(\text{mb})\exp\left(-\int_{h(500\text{mb})}^{h} \frac{g\,dh}{R\,\bar{T}}\right)$

where h(500mb) is the height of the 500 mb pressure and p(h) is the pressure at height h. With this pressure field, $\bar{\rho}$ is determined by the equation of state. The temperature field used is the one adopted by J. F. Louis (1974) and the pressure height was taken from the NCAR data bank. The temperature field is based on rocket and satellite observations (SCR, 1972). These observations were first hand smoothed by Louis in order to ensure that the important characteristics of the actual soundings were preserved, and then a computational data smoothing technique was used. When using the temperature field to determine the zonal winds by the geostrophic equation,

$$f\bar{u} + \frac{\bar{u}^2 \tan\phi}{r} = \frac{1}{\bar{\rho}}\frac{\partial\bar{p}}{\partial\phi}$$

$\bar{\rho}$ = mean density
ϕ = degrees latitude
$f = 2\omega\sin\phi$
ω = earth's angular frequency
r = earth radius
$\bar{p}$ = mean pressure
$\bar{u}$ = zonal (east-west) wind component

it was found that the zonal winds were in good agreement with observed winds given by Newell, et al., (1969) and Groves (1971), except in two regions: the southern polar stratosphere and the equatorial region. Around the equator, the Coriolis parameter is small, and $\bar{u}$ depends on the ratio of two small quantities. Therefore, small errors in the temperature field produce large errors in the zonal winds. The temperature in these areas was adjusted to minimize the discrepancies between computed and observed zonal winds and temperature, while keeping the two fields related to each other through the geostrophic equation. The adopted temperature field is estimated to be within $\pm$ 5° K error in the southern stratosphere.

The Mean Winds

The longitude and season averaged heat balance equation (see pg. 28) becomes

$$\frac{\Delta \overline{T}}{\Delta t} + \frac{\overline{v}\partial \overline{T}}{r\partial \phi} + \overline{w}\left(\frac{\partial \overline{T}}{\partial z} + \Gamma\right) + \frac{\partial(\overline{v'T'}\cos\phi)}{r\cos\phi\,\partial\phi} + \frac{\partial(\overline{w'T'})}{\partial z} = \overline{\Phi} \quad 12)$$

where $\overline{T}$ = mean temperature
Γ = adiabatic lapse rate
r = earth radius
$\overline{\Phi}$ = mean diabatic heating rate
ϕ = latitude angle
$\overline{v'T'}$ = mean horizontal eddy heat flux
$\overline{w'T'}$ = mean vertical eddy heat flux.

It was assumed that $\frac{\partial \overline{\rho}}{\partial t}$ is negligible since it is at least two orders of magnitude smaller than the other terms. $\frac{\Delta \overline{T}}{\Delta t}$ is the seasonal trend, and the next two terms represent the heat advection due to the mean circulation. The last two terms on the left hand side represent the horizontal and vertical eddy diffusion of heat. Equation 12) and the continuity equation

$$\frac{\partial(\overline{\rho}\,\overline{v}\cos\phi)}{r\cos\phi\,\partial\phi} + \frac{\partial(\overline{\rho}\,\overline{w})}{\partial z} = 0 \quad 13)$$

form a set of equations that can be used to solve for $\overline{v}$ and $\overline{w}$. Below 15 km $\overline{v}$ and $\overline{w}$ can be determined from rocket and rawinsonde observations. Above 15 km too little data exists to make valid statistical computations, so equations 12) and 13) are solved simultaneously for $\overline{v}$ and $\overline{w}$ in this region. The observed circulation as given by Newell, et al., (1969) was used as a lower boundary condition for the computation above 15 km.

With the temperature field given and the density already calculated, the quantities necessary in solving equation 12) are $\overline{w'T'}$, $\overline{v'T'}$, and Φ. Not enough observations are available to estimate the vertical eddy heat flux $\overline{w'T'}$. Results from three-dimensional general circulation models (Kasahara and Sasamori, 1974) suggest that this term contributes very little to the heat balace in the stratosphere. Therefore, the term was neglected in the calculations. The horizontal eddy heat flux used was given by NCAR. In the troposphere and lower stratosphere the data published by Newell, et al., (1960) was used; these data were derived from ten years of rawinsonde observations. In the upper stratosphere, $\overline{v'T'}$ consisits of weekly analyses of the 4, 2, and .4 mb levels published by ESSA (U.S. Department of Commerce, 1964 to 1967). These data are based on rocket observations convering only the western half of the northern hemisphere.. These data were considered representative of both halves of both hemispheres. The horizontal eddy heat flux term is of the order of 1° K/ day, which is a fraction of the diabatic heating rate, so amy discrepancies should not be significant.

The original heating rate used by J. F. Louis (1974) was determined from ultraviolet absorption by O_2 and O_3 as given by Park (1972), and the cooling due to CO_2 and H_2O was calculated by a program developed by Kuhn (Kuhn and London, 1969) at the University of Michigan. The cooling due to O_3 above 30 km was given by Kuhn and London (1969) and below 30 km by Dopplick (1970). These rates are combined to give the net diabatic heating rate.

After several trials at different iteration schemes, it was decided to use the same iteration technique that was used by J. F. Louis (1974) for the NCAR model. A convergent solution of $\bar{v}$ and $\bar{w}$ independent of a reasonable initial guess is reached if a contrived $\bar{w}$ is first substituted into equation 13), which is then integrated to give a $\bar{v}$. This $\bar{v}$ is substituted back into equation 13) and the process repeated. For lack of a better upper boundary condition the top boundary given in the NCAR data was used. Since $\bar{v} = 0$ at the poles, $\bar{w} = 0$ at the ground and the equation of continuity must be satisfied over any region, it follows that $\int_{90°S}^{90°N} \bar{\rho}\,\bar{w}\,d\phi = 0$ for any given height. But this integral was not 0, so a corrected vertical velocity $\bar{w}^*$ was introduced after $\bar{w}$ had been determined from equation 12),

$$\bar{w}^* = \bar{w} - C\cos\phi \text{ where } C = \frac{\int_{90°S}^{90°N} \bar{\rho}\,\bar{w}\,d\phi}{\int_{90°S}^{90°N} \bar{\rho}\cos\phi\,d\phi}\,. \qquad 14)$$

The integral of $\bar{\rho}\,\bar{w}^*$ then vanished. Central differences were used which provided for a smooth transition at the 15 km boundary. Figures 9 and 10 show the vertical and meridional winds for the winter calculation. There are slight discrepancies between my computation and that given by NCAR. This result can be accounted for by the fact that the diabatic heating rate used in the present study is different than the one used in the NCAR calculations. When J. F. Louis did the calculations, it was found that the constant C of equation 14) did not converge to 0 as the winds converged, thus the final winds did not correspond to the original heating rate. The

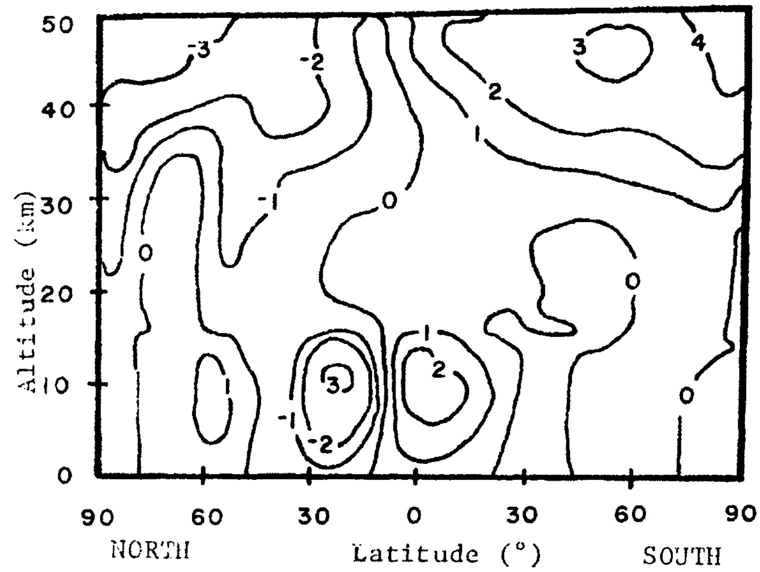


Figure 9). Vertical winds (mm/sec), winter

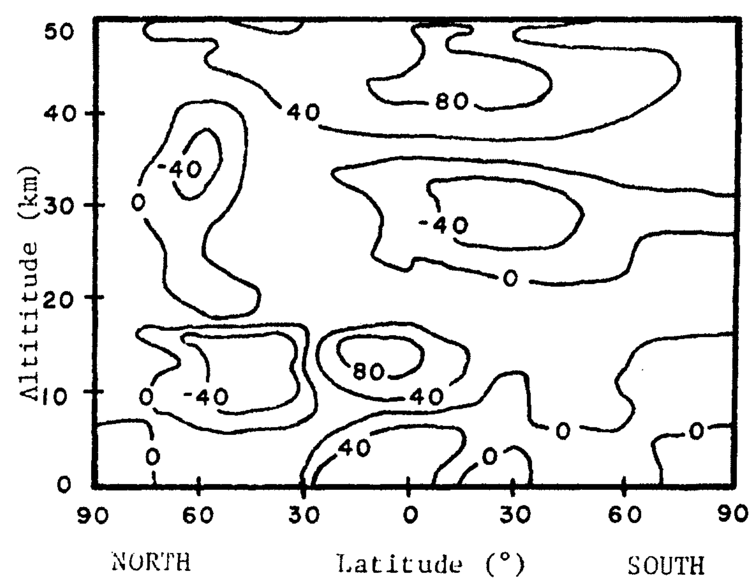


Figure 10). Meridional winds (cm/sec), winter

heating rate used herein corresponds to the heating rate that would be observed if the correction factor of equation 14) were added. Because of this fact, it was decided to use the NCAR mean circulation in the calculations. The errors associated with this circulation are estimated to be as large as 50% due to the many uncertainties involved in averaging the data that was observed below 15 km, and the data used in the calculations above 15 km. The random winds used to simulate spreading are at least one order of magnitude larger than the mean winds, so any errors in the mean circulation should not have an important over-all effect.

The Random Winds

To determine the random winds, at each point a Gaussian distribution centered at zero for each component was assumed. The probability density P of randomly choosing v and w is,

$P(v) = A_v \exp(-v^2/2\sigma_v^2)$ and $P(w) = A_w \exp(-w^2/2\sigma_w^2)$, respectively, where σ_v and σ_w are the standard deviations and A_v and A_w are normalization constants for the respective winds. In order that

$\int_{-\infty}^{\infty} A_v \exp(-v^2/2\sigma_v^2)\,dv = 1$, $A_v = 1/\sigma_v\sqrt{2\pi}$. Similarly,

$A_w = 1/\sigma_w\sqrt{2\pi}$.

The expectation values become

$$\int_{-\infty}^{\infty} vA_v \exp(-v^2/2\ {}_v{}^2)\,dv = \sqrt{\frac{2}{\pi}}\,\sigma_v \qquad \text{and} \qquad \int_{-\infty}^{\infty} wA_w \exp(-w^2/2\ {}_w{}^2)\,dw = \sqrt{\frac{2}{\pi}}\,\sigma_w.$$

Horizontally, there is data that can be used to approximate the expectation value. These data were furnished by NCAR as the meridional wind variance, or the deviation of the horizontal wind from its seasonal average. These data, like the horizontal heat flux data, were published by Newell, et al., (1969) for altitudes below 30 km and by ESSA (U.S. Department of Commerce, 1964 to 1967) above 30 km. The actual observations were of $\overline{v'v'}$, where v' is the meridional wind variance, so $\overline{v}'$ is always positive. As a first guess, it was assumed this variance should be equal to the horizontal random wind expectation value, so $\sigma_V = \sqrt{\frac{\pi}{2}}\,\overline{v}'$.

Vertically, there is no data for an analogous $\overline{w}'$, so a $\overline{w}'$ was estimated from the static stability. Baroclinic theory shows that the rate of growth of baroclinic disturbances depends on the static stability ($\frac{g}{T}\left(\frac{dT}{dz} + \frac{g}{C_p}\right)$). The vertical motions, especially at the scale of thermal convection, are critically dependent on the static stability. It is well known that tracers diffuse much faster in the troposphere than in the stratosphere, and the main physical difference between these two regions is the static stability. Therefore the random vertical component was related to the static stability through an exponential function,

$\overline{w}' = C_2 \exp(-C_3\Sigma)$ where $\Sigma = \frac{g}{T}\left(\frac{dT}{dz} + \frac{g}{C_p}\right)$. Then, $\sigma_w = \sqrt{\frac{\pi}{2}}\,\overline{w}'$, where C_2 and C_3 are adjustable parameters.

It was mentioned in the previous section that in order to simulate random diffusion along axes other than vertical-horizontal, it is necessary to assume that the random winds originate in a coordinate system rotated by an angle α from the regular. It was shown in the first chapter, Figure 5, that a common characteristic of all stratospheric debris, regardless of height or latitude of injection, is the slope of the maximum concentration trajectory. The maximum concentration and therefore the diffusion slopes up toward the equator and down toward the poles. The angle at which this diffusion occurs is about the same as the angle of the mean isentropic surfaces. Eady (1949) computed that the maximum rate of growth for baroclinic disturbances occurs when the angle of the disturbance is half the angle of the mean isentropic surfaces. This angle corresponds to a maximum transfer of potential energy into kinetic energy. Further calculations by Green (1970) show that the angle decreases to 0 at the ground, near the tropopause, and throughout the middle troposphere. It was pointed out in chapter one that the investigations of Newell (1964) and Oort (1964) have shown that in the lower stratosphere parcels of air must have trajectories greater than the angle of the mean isentropic surfaces to account for the transfer of heat from cold equatorial regions to warmer polar regions. The data used for α was furnished by the NCAR program in the following way. The diffusion coefficient tensor of equation 5)

$$K = \begin{pmatrix} K_{yy} & K_{yz} \\ K_{zy} & K_{zz} \end{pmatrix} \qquad \text{where } K_{yz} = K_{zy}$$

can be transformed into a diagonal tensor

$$K' = \begin{pmatrix} K_{11} & 0 \\ 0 & K_{22} \end{pmatrix}$$

by imposing a coordinate transformation

$$\lambda = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}.$$

Then

$$K' = \lambda^{-1} K \lambda$$

or

$$K_{yy} = \cos^2\alpha K_{11} + \sin^2\alpha K_{22}$$
$$K_{zz} = \cos^2\alpha K_{22} + \sin^2\alpha K_{11}$$
$$K_{yz} = K_{zy} = \sin\alpha\,\cos\alpha\,(K_{11} - K_{22})\ .$$

The angle α is of the order of 10^{-3} radians and $K_{22} << K_{11}$. Therefore,

$$K_{yy} = K_{11} \qquad K_{yz} = \alpha\,K_{11} \qquad K_{zz} = K_{22} + \alpha^2 K_{11}\ , \text{ or } \quad \alpha = \frac{K_{yz}}{K_{yy}}\ .$$

In the NCAR data α was actually given by Green (1970) and used to calculate K_{yz} in every region but the lower stratosphere and equatorial troposphere, where the observed distribution of ozone was used to determine the diffusion coefficients. So the angles obtained by dividing the coefficients are actually the angles given by Green (1970) except in the lower stratosphere and equatorial regions. In these regions the angles are also consistent with observations. The random velocities are now related to $\overline{v}'$ and $\overline{w}'$ by the coordinate rotation $\underset{\sim}{\lambda}$,

$$v_{RAN} = \overline{v}'\cos\alpha - \overline{w}'\sin\alpha$$
$$w_{RAN} = \overline{v}'\sin\alpha + \overline{w}'\cos\alpha\ .$$

Tests, Results, and Further Revisions

In order to establish the parameters for the vertical random winds, the Gunung Agung volcanic eruption of March 17, 1963 (11° south latitude) was simulated and good agreement was obtained with predictions by Cadle, Kiang and Louis (1976) when the constants relating the static stability and vertical random winds were set at

$$C_2 = .473 \quad \text{and} \quad C_3 = 2 \times 10^6\ \text{sec}^2/\text{gm}\ .$$

The major discrepancy was in the rate of spread of the maximum concentration. After 60 days, the model showed the maximum concentration to be twice as much as that predicted by the NCAR model, althogh the rest of the volcanic ash cloud spread in agreement.

When the same set of data was used to simulate the December 27, 1968 Chinese 3 megaton detonation (45° N latitude) it was found that the model spread 7 or 8 times faster than was observed. This reult led me to believe there was an important process missing in the model, and it turned out the time scales of the random winds had been neglected. In different parts of the atmosphere turbulent diffusion is accomplished in different ways. The mid-latitudes are characterized by a high frequency of baroclinic storms and represent a barrier between two distinct regions. The motions of the high latitudes can be characterized by large scale waves, such as Rossby waves (1942), that are present because of the large Coriolis effect. The equatorial region, where the Coriolis force is small, can also be characterized by large-scale waves but of a completely different nature (Kelvin-Helmholtz waves, studied by Lindzen, 1967, and others). Therefore, the time scales of these different processes are different, and must be modeled accordingly. In considering a point in the atmosphere imagine taking hundreds of horizontal wind measurements in a given time period t. Then, if the distribution of these winds is Gaussian, the normalized distribution function would look like

$$P(v) = \frac{1}{\sigma_v\sqrt{2\pi}} \exp(-v^2/2\sigma_v^2)$$

where σ_v is the standard deviation as taken from the wind measurements. The expectation value is given by

$$\int_{-\infty}^{\infty} (v/\sigma_v 2\pi) \exp(-v^2/2\sigma_v^2)\, dv = \sigma_v\sqrt{\frac{2}{\pi}} \ .$$

A characteristic distance of spread for this sample would be $d = \sigma_v \Delta t$. This distance can be thought of as the distance over which a parcel of air retains certain characteristics of its environment. But this distance can also be thought of in terms of the deviation of the mean meridional velocity component $\overline{v'}$. If t is a time characteristic of the mixing processes, then $\overline{v'}t$ represents an analogous characteristic distance, and the two distance scales could be related by

$\sigma \Delta t = D\overline{v'}t$ where D is a proportionality constant.

In the discussion of diffusion, it was shown that the diffusion coefficient tensor is related to the Reynolds stress tensor by

$$\begin{pmatrix} K_{yy} & K_{yz} \\ K_{zy} & K_{zz} \end{pmatrix} = \begin{pmatrix} \overline{v'v'} & \overline{v'w'} \\ \overline{w'v'} & \overline{w'w'} \end{pmatrix} \tau , \qquad 15)$$

so $K_{yy} = \overline{v'v'}\ \tau$.

I simply divided the horizontal diffusion tensor coefficient by $\overline{v'v'}$ to get an estimate of τ or t. Then

$$\sigma_v = \frac{Dt\overline{v'}}{\Delta t} \quad . \qquad 16)$$

The proportionality constant D represents a free constant that tunes the rate of spread of this model to the quantities K_{yy} and $\overline{v'}$ of the NCAR model. Best agreement was found with D = 9.0 .

The Chinese Test:

During the period 1967-1972, a number of nuclear bombs were detonated by the French and Chinese. The distributions of the debris clouds were observed for several months after each test, and documented by Telegadas (1974). ^{95}Zr from the December 27, 1968 detonation of a 3 megaton device was chosen as the first check of the model. It was estimated that 4.6 megacuries of ^{95}Zr were introduced into the stratosphere.

Although the location of the Chinese explosions was 40° N latitude (90° E longitude, Lop Nor, China), the location of the zonally-averaged center of mass of the stratospheric debris cloud was estimated to be between 60° N and 65° N latitudes a few days after the injection. The initial vertical distribution of the cloud follows that predicted by Peterson (1970) with the maximum at 18 km. The latitude of injection was chosen to be 60° N.

Figures 11 a), b), and c) show the computed distributions various dates after injection with comparison to Telegadas' (1974) observations. The spreading appears to be a little slow, and the computed distribution lacks the large concentration observed in the southern hemisphere.

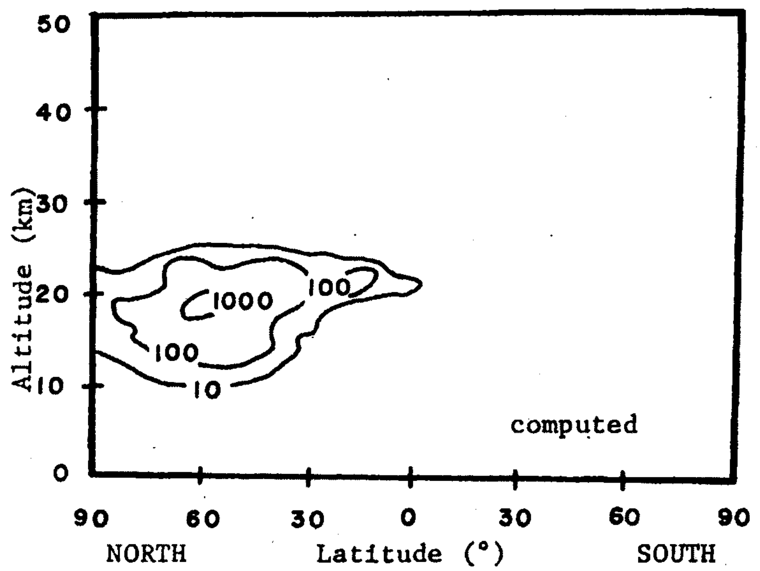
It should be noted that the numerical technique used in solving equation 11) had to be changed 1.5 months after the injection. The large concentrations in the southern hemisphere were calculated using a 3 point central difference, semi-implicit, alternating direction scheme. This method involves the solution of a set of linear equations by a matrix inversion technique, hence all points in the atmosphere are interrelated and the distribution in one region of the atmosphere has an effect on the distribution in another region. But this technique proved to be unstable after about 3 months of simulation. It actually predicted the distribution as becoming stationary and then contracting. This was thought to be due to the small number of random samples considered for each days calculation.

For each day 15 random velocity fields were taken, which is a small number statistically, so there exists a definite probability that the maximum concentration could actually increase. When this happens, it affects the distribution throughout the entire grid space. Eventually, the distribution becomes unstable and begins to converge.

To correct this result, all further calculations employed a completely explicit difference scheme. In doing so, the governing equation 11) is solved locally for each grid point and the distribution in one region doesn't affect the distribution in another region. Although the maximum concentration may still increase, it will not make the entire distribution unstable, and on the average the tracer disperses as the figures illustrate.

The HARDTACK Test Series:

During the project HARDTACK in 1958, the United States detonated several bombs in the south Pacific around 11° N latitude and introduced a previously undetected radioisotope into the stratosphere, ^{185}W. From the amount of ^{185}W observed to have been deposited on earth in the following years, it is estimated that about 95 megacuries (decay corrected to August 15, 1958) were initially injected. The date of the injection was taken to be August 15, 1958 and the latitude of injection was taken to be 10° N. Again, the initial vertical distribution and location of the maximum concentration were taken from predictions of Peterson (1970). Figures 12 a), and b) show the computed distributions 3, and 10 months after injection, compared to that given by Friend, et. al., (1961). The 3 month computed distribution is in fair agreement with observation except at heights below 10 km. The computation predicts a rapid drop of debris around 15° latitude in each hemisphere that is not observed. This disagreement can be attributed to the way in which the time scale of the random fluctuations was calculated in this region. Equation 16) shows the relationship between the standard deviation of the random wind probability distribution (which is a measure of the horizontal rate of spread) and the characteristic time of the fluctuations. Equation 15) explains how this characteristic time was obtained, by dividing the horizontal eddy diffusion coefficient, K_{yy}, by the observed data for $\overline{v'v'}$. In the equatorial region Louis (1974) obtained his diffusion coefficient by canceling the effect of his mean circulation from observed ozone fluxes. As a result, the horizontal diffusion coefficient is a maximum in this region. In comparison, Gudisken (1968) set his diffusion coefficients equal to zero up to 14 km in the equatorial region. Since the vertical random fluctuations are directly related to the horizontal fluctuations by the diffusion angle, the large characteristic time calculated from the diffusion coefficients makes the vertical spreading much larger than observed. After 10 months of calculation, this disparity is further emphasized. The model also predicts a tendency towars dispersal in the northern hemisphere, while observations show a near equal distribution in both hemispheres. The location of the maximum concentration and the slope of diffusion are qualitatively reproduced.



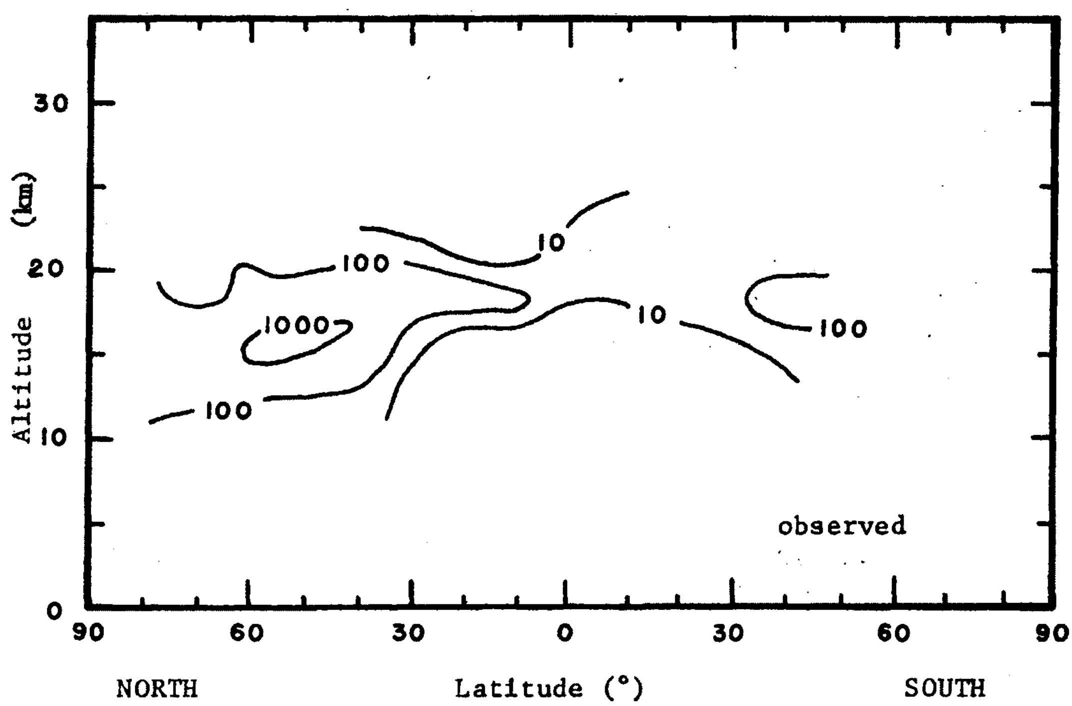
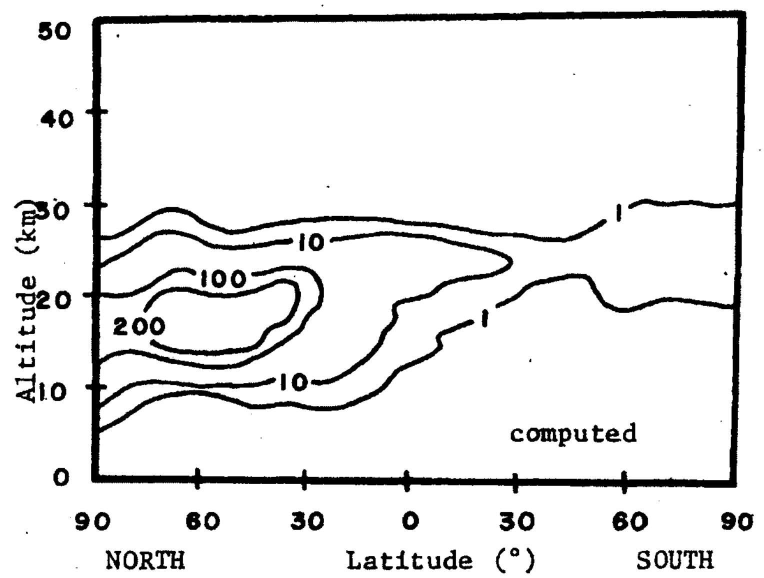


Figure 11 a). Distribution of ^{95}Zr (piCi/SCM)

February 1969, approximately 1.5 months after the 12/27/68 Chinese test.



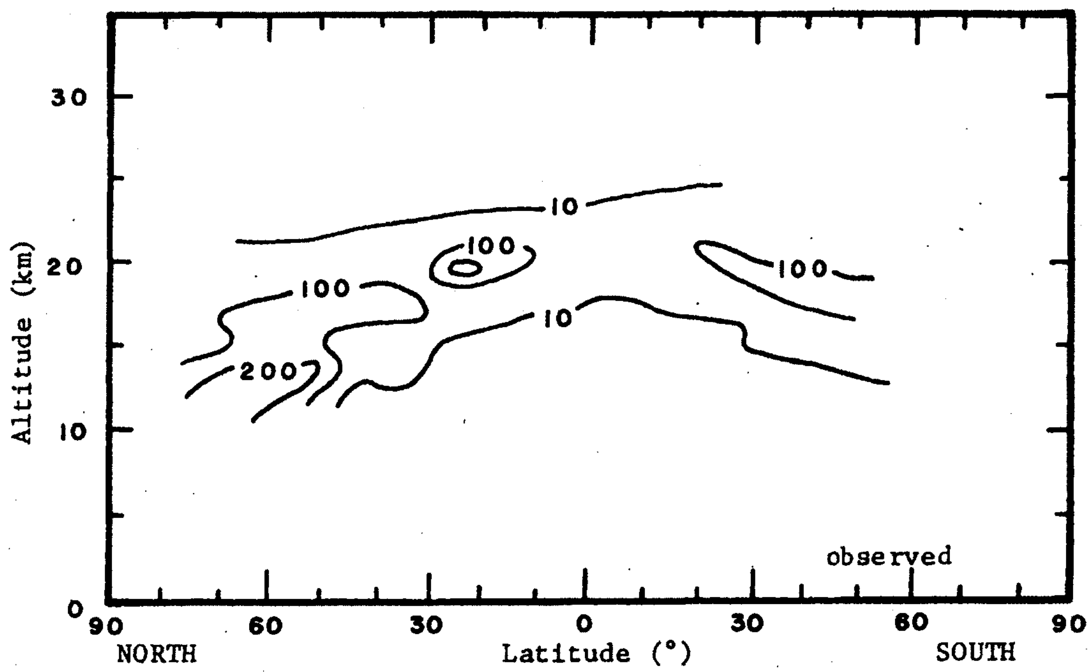


Figure 11 b). Distribution of ^{95}Zr (piCi/SCM)

May 1969, approximately 4.5 months after the 12/27/68 Chinese test.

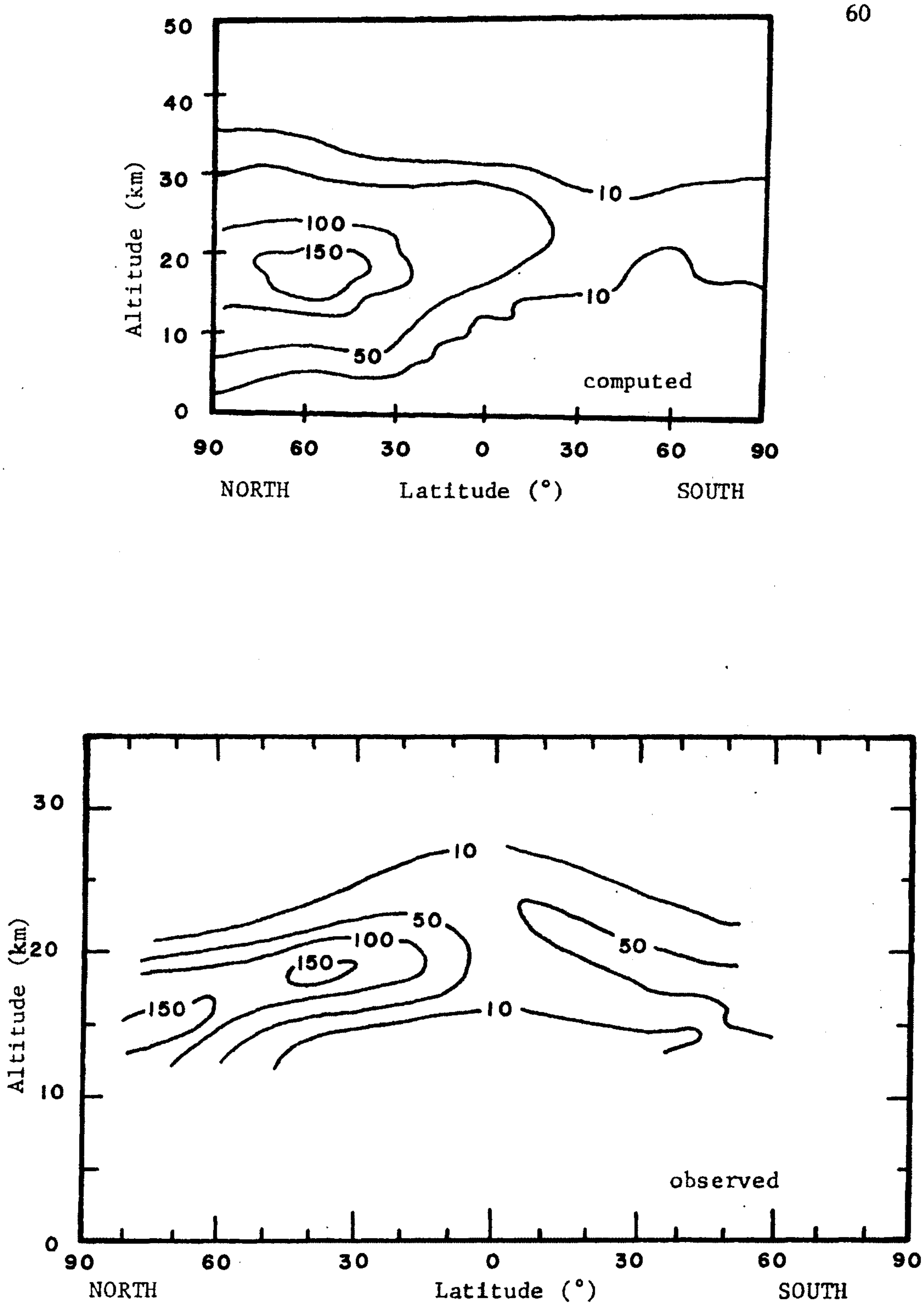
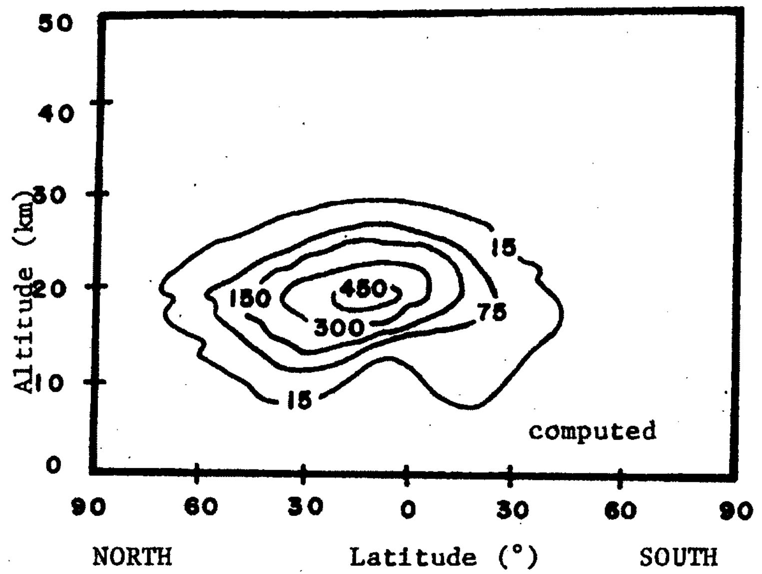


Figure 11 c). Distribution of ^{95}Zr (piCi/SCM)

July 1969, approximately 6.5 months after the 12/27/68 Chinese test.



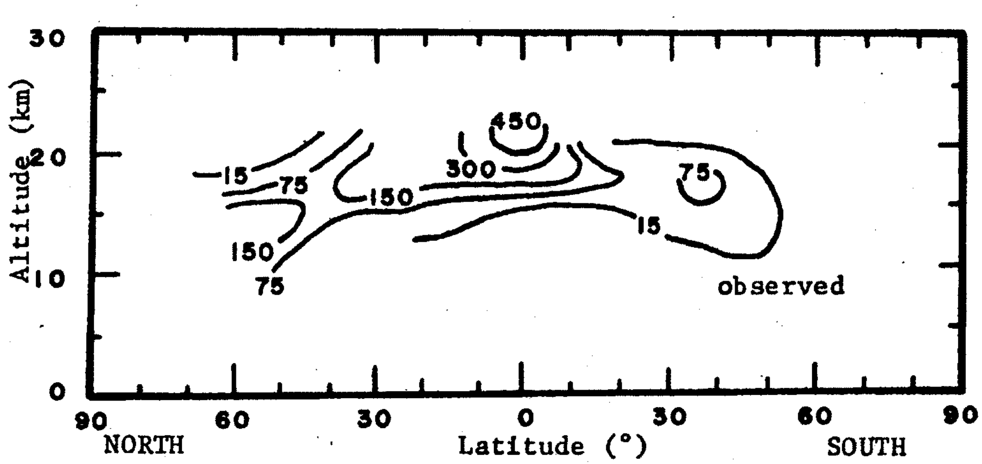
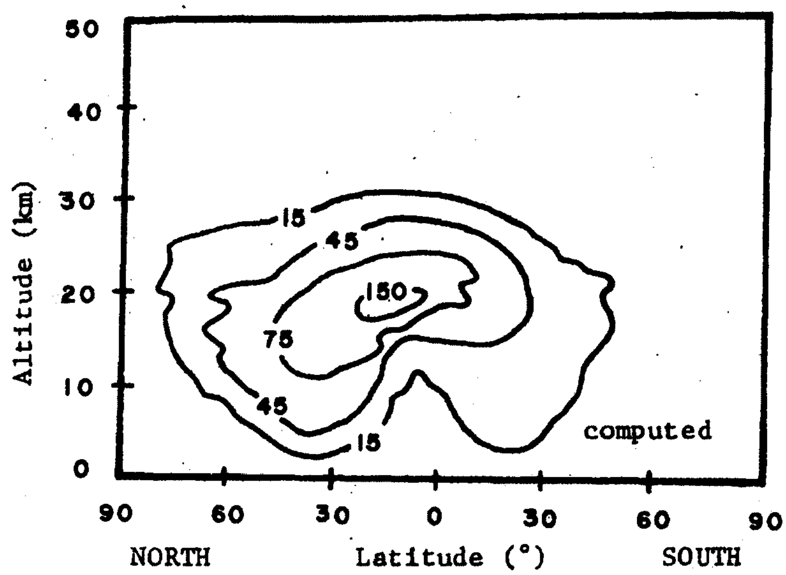


Figure 12 a). Distribution of ^{185}W (piCi/SCM)

December, 1958, approximately 3 month after the 8/15/58 HARDTACK test.



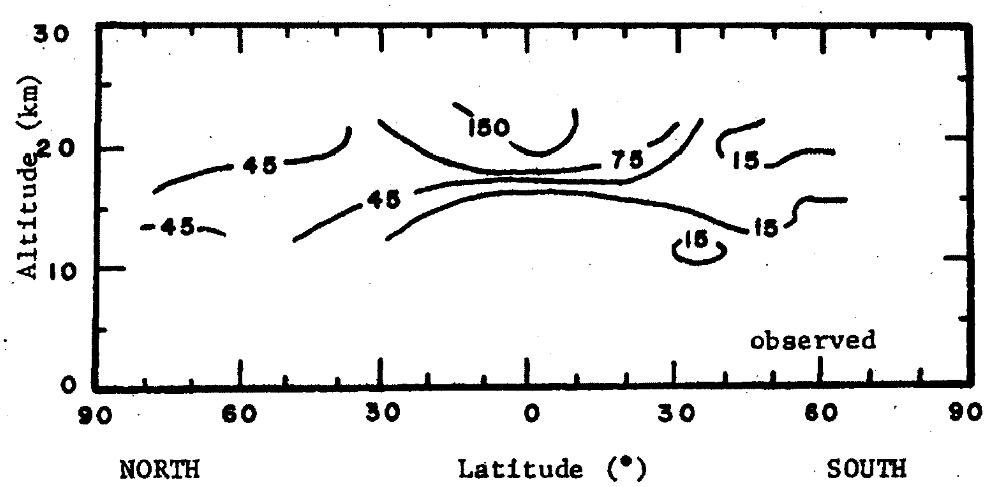


Figure 12 b). Distribution of ^{185}W (piCi/SCM)

June 1959, approximately 10 months after the 8/15/58 HARDTACK test.

CONCLUSION

In this work a two-dimensional model of the atmosphere has been developed. It simulates the distribution of introduced tracers by assuming transport is due to advection by a mean circulation and by turbulent eddies. A unique method of modeling the effect of eddies is introduced by assuming that the turbulent eddy fluxes are due to random fluctuations in the velocity field. The main advantage to this approach is that the diffusion due to the random fluctuations can be parameterized from observed data, while similar two-dimensional models require emprical or semi-empirical evaluations of diffusion coefficients.

The model was tested with independent sets of data by simulating the radioactive debris from the mid-latitude Chinese numclear explosion of Dec. 1968, and the equatorial U.S. explosions of Aug. 1958. Calculations were carried out to 6 months, and 10 months after injection, respectively. The simulation of both tracers was qualitatively reproduced, despite discrepancies in some of the data. This result supports the assumption that eddy transport can be characterized by random processes.

It is hoped that the modeling of random processes as set forth in this work could provide a starting point for a more complete description of debris transport without the necessity of arbitrary parameterization typical of existing two-dimensional models.

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