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A modified program for the teaching of arithmetic in the primary department of the Billings public schools

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A MODIFIED PROGRAM FOR THE TEACHING OF ARITHMETIC IN THE PRIMARY DEPARTMENT OF THE BILLINGS PUBLIC SCHOOLS

by

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FOREWORD

This study was undertaken because it has long been felt by a number of Billings teachers that the primary arithmetic program could be so designed as to provide the same type of flexibility required to meet individual differences in rates of growth and development provided by the primary reading program now functioning in the Billings Public Schools.

During the winter of 1949 a number of Billings teachers participated in an in-service training course on arithmetic, under the direction of Dr. Charles D. Dean. The writer was a member of that group, and following the exchange of ideas concerning the teaching of arithmetic at all levels active work on this paper was started. References in the field were consulted and an effort made to organize the material to fit the needs of primary children in the Billings Schools. The material was used experimentally in one school during the past year.

The writer gratefully acknowledges the assistance and encouragement of many people, especially staff members of the School of Education of Montana State University and administrators and primary teachers of the Billings Public Schools.

L. M. M.

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CHAPTER I

A BASIC GROWTH CONCEPT

If education is conceived as the continuous reconstruction of experience, then an educational program must be built on what is known about human and child development.

Foremost educational writers[1] of today agree on the importance of the implications of child growth for those dealing with children. Goals and practices of education at any growth level should be in keeping with the individual's capacities and potentialities at that level.

I. GENERAL NATURE

Normal children are essentially similar in their sequence of growth, but they are not alike in the way in which they pass through this sequence. Kilpatrick puts it this way, always with emphasis on education as the process of leading the young most effectively to learn and live the "good life":

> No two children and no two classes will ever show exactly the same aggregate achievement or status either in knowledge or skill or attitude or personality maturity. General averages will, however, be maintained wherever conditions remain sufficiently stable; and those pupils who vary from these averages will be kept in the teacher's mind to receive such attention as they seem to need.[2]

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[1]Arthur T. Jersild, Child Psychology (New York: Prentice-Hall, Inc., 1947).

[2]William Heard Kilpatrick, Modern Education: Its Proper Work (New York: Hinds, Hayden & Eldredge, Inc., 1949), p. 21.

It is essential that the teacher understand and take into account the growth rate of each child in order that his chances of continuous growth during school life may be at a maximum. Some children grow physically at a rapid rate; others develop very slowly. Mental development is just as variable. Growth is always continuous but need not be steady. Sometimes there will be a week or more at a time when a child seems to stand still in his development. At times he may even seem to go backwards in that he temporarily may do some things less well than he did before. In this connection, Huggett and Millard give a word of caution:

> The teacher who follows the developmental pattern of the child as a guide for the introduction of instructional material does not become alarmed because certain pupils progress at slower rates than others. Pressure should not be used to drive pupils toward unnaturally high standards.[3]

Lee and Lee describe the close relationships among the physical, mental, and emotional aspects of growth:

> Just as a child grows and develops physically in height and weight and muscle coordination; just as he grows and develops mentally in memory and reasoning and ingenuity; just so does he grow and develop emotionally in his feeling toward himself, toward other people, and toward his place in the world. The physical, mental and emotional do not represent three separate and clearly distinguished categories to which all information about the child can be assigned. Far from it! Nearly every fact about the child is a result of the interaction of these phases and has been affected by them to a certain extent.[4]

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[3]Albert J. Huggett and Cecil V. Millard, _Growth and Learning in the Elementary School_ (Boston: D. C. Heath and Company, 1946), p. 44.

[4]J. Murray Lee and Dorris May Lee, _The Child and His Curriculum_ (New York: Appleton-Century-Crofts, 1950), p. 64.

The outcome of normal development is increasing maturity. At any phase of his growth, a child may be regarded as being mature for his age group, but immature in subject matter areas, or vice versa. This places a great responsibility upon educators to design the educational program so as to help the child at any stage in his career to reach his potential level of maturity. In doing this, the teacher must be aware not only of what children are like but also of the relation between learning and growth. Jersild says:

> In connection with many changes in behavior which take place in the course of development, it is impossible to tell precisely how much of the change is due to growth and how much to learning. The effects of the two factors interact and are closely interwoven. But findings from many fields indicate that many of the changes in what a child is able to do and strives to do come about by a process of ripening or inner growth. Regardless of the opportunities for learning which a child may have, this process of ripening requires time, varying with different performances.[5]

II. IMPLICATIONS FOR CURRICULUM

The school does not and cannot _produce_ growth.[6] It merely stimulates and guides the normal growth process. It does this through an organized body of experiences known as the curriculum.

The content of any educational program represents a

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[5]Arthur T. Jersild, _et. al._, _Child Development and Curriculum_ (New York: Bureau of Publications, Teachers College, Columbia University, 1946), p. 9.

[6]John J. DeBoer, _Design for Elementary Education_ (New York: Hinds, Hayden and Eldredge, Inc., 1945), p. 10.

choice among many alternatives.[7] The school curriculum, no matter how liberal it may be, has to be selective. Choices are necessary. In order to be sound ones, they must be in line with the principles of child growth and development.

Strickland says:

> Planning experiences for young children in the modern school requires knowledge of the general characteristics of children of each age level and knowledge of the specific characteristics and needs of each child in the group for which one is making plans. The level of maturity which children have reached, their background of experiences, and their interests must all be taken into account.[8]

Sequence in learning experiences is stressed by Swenson:

> What a child or a group of children should be taught at any particular time is not so much a matter of teaching a certain piece of subject matter when the children are a certain age or in a certain grade. It is much more a matter of planning subject matter and teaching method so that each learning experience follows naturally upon preceding learning experiences and results.[9]

The continuous growth process appears in many fields. In regard to science, for example, we read:

> Instruction in science during the year should provide a continuity of experience which will help the children through the extension of environment to grow continuously. Continuity of experience implies that the science

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[7] Jersild, *et. al.*, *op. cit.*

[8] Ruth Strickland, "Organizing a Program for the Primary Grades," in DeBoer, *op. cit.*, p. 52.

[9] Esther J. Swenson, "Arithmetic for Preschool and Primary-Grade Children," *Fiftieth Yearbook of the National Society for the Study of Education*, Part II (Chicago: The University of Chicago Press, 1951), p. 60.

> program should be developed in terms of meanings and understandings which children have and that it should lead on to the building of larger and fuller meanings. Such meanings are developed as children solve problems of social value which are significant and challenging to them.[10]

III. IMPLICATIONS FOR THE ARITHMETIC CURRICULUM

The traditional arithmetic program, with its major emphasis on the acquisition of skill in computational procedures and on the mastery of subject matter largely abstract in nature, was not in harmony with recognized principles of child development. Therefore, it is not surprising that the results have been unsatisfactory.

Administrators, supervisors, and teachers, according to one authority,[11] cannot dismiss lightly criticisms of the arithmetic program of the American elementary school which have come from people in the profession who have spent much time and effort in studying the psychological foundations of the teaching of arithmetic, the problems involved in methods of teaching, and problems of the place of arithmetic in the curriculum.

Huggett and Millard indict the traditional program in

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[10]"Science Education in American Schools," in _Forty-Sixth Yearbook of the National Society for the Study of Education_, Part I (Chicago: The University of Chicago Press, 1947), p. 71.

[11]G. Max Wingo, "The Organization and Administration of the Arithmetic Program in the Elementary School," G. T. Buswell, editor, _Arithmetic 1948_ (Chicago: The University of Chicago Press, 1948), p. 69.

this manner:

> The amount of time normally spent on the teaching of arithmetic should result in a much better product than the one well known to most of us. Six or even eight years of persistent drill, home study, threats from both teachers and parents should allow us to expect something more than inability on the part of most children to handle simple numbers in computations and to solve most of the arithmetical problems faced in daily living. In most schools arithmetic is not learned, let alone taught, as a functional tool, but as a device for labelling children according to their ability or inability to handle arithmetical abstractions.[12]

For a more complete summarization of the contrasts between traditional and modern arithmetic programs or procedures than it is possible to include here, the reader is referred to Brueckner and Grossnickle.[13]

Morton suggests that there are three criteria which aid in determining the role which arithmetic should play in the curriculum:

> (1) The logical criterion, which has reference to the structure and organization of arithmetic as a science.
>
> (2) The social criterion, which indicates a concern with the usefulness of arithmetic in life's affairs.
>
> (3) The psychological criterion, which is concerned with how children learn.[14]

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[12]Huggett and Millard, *op. cit.*, p. 181.

[13]Leo J. Brueckner and Foster E. Grossnickle, *How to Make Arithmetic Meaningful* (Chicago: The John C. Winston Company, 1947), pp. 20-21.

[14]Robert Lee Morton, "The Place of Arithmetic in Various Types of Elementary-School Curriculums," G. T. Buswell and Maurice L. Hartung, editors, *Arithmetic 1949* (Chicago: The University of Chicago Press, 1949), p. 1.

Probably the most tenable view is that these fundamental criteria must operate together, even though to plan and put into operation a program which is simultaneously based upon the implications of these three criteria is not an easy task.

The least neglected of the three criteria is, undoubtedly, the logical criterion, because there is, in arithmetic, a sequential pattern which is quite commonly observed. Unless certain topics precede others, the efforts of the learner are doomed to failure. For example, children do not add until after they can count; measuring with a yard stick would have little or no meaning if the child did not have some understanding of foot, inches in a foot, feet in a yard, and inches in a yard.

Occasionally cases arise in social situations where there is an apparent need for arithmetic skills which the pupils do not have. In such cases, logical criteria should be followed. Arithmetic sequence must be observed regardless of the demands of the social criteria.

In many instances the logical and psychological criteria appear to be the same. It is true that there is often a close relationship between the two. For example, the basic addition facts without zero must be learned before higher-decade addition is begun. Logically, the basic facts come first, because they necessarily precede the higher-decade facts in a program which develops the subject of addition systematically. However, since no violence is done to the systematic organization of the subject by postponing the teaching of zero facts, they should be taught at a later date when they can be used in realistic situations. This plan for timing the teaching of the zero facts is based upon the psychological criterion.

The social criterion indicates the usefulness of the arithmetic program in life-activities of children and adults. This criterion justifies the inclusion of arithmetic in the curriculum and should be used when grade placement of topics is considered. For example, when the need arises, the child should learn that there are sixty seconds in a minute and sixty minutes in an hour. Long before that, however, he will have been telling time in a fashion suitable to his needs.

The selection of topics to be taught also is determined by the social criterion. Until recently, nearly all children learned the multiplication tables to 12 x 12. Today, this requirement seems to have been reduced to 9 x 9. However, some authorities advocate need for combinations such as 12 x 12 for a gross and combinations involving 15 for social usage. Since there are bound to be differences in opinion as to what should be included in the curriculum, the social criterion is extremely useful.

Also, the problem solving program in arithmetic depends largely on the social criterion. The problems should be real, or at least realistic. The data and conditions described should be true to life, and the problems, if possible, should arise from the experiences of the children themselves.

Presumably a curriculum represents an organization of experience which will facilitate learning. According to the psychological criterion, new learning should be based upon related old learning. An illustration is found in teaching higher-decade addition. A pupil who is confronted with the situation 6 + 9 + 8 + 4 in the form of a column of numbers must add 8 to 15 in the second step. In learning to add 8 to 15, the pupil should see 15 + 8 as related to 5 + 8. He knows that 5 and 8 are 13. He concludes that the sum of 15 and 8 must be similar to 13 but in the twenties. All higher-decade addition facts may be seen as extensions of the related facts into the higher decades.

Also, according to the psychological criterion, children should build understandings gradually, one step at a time. As an example, consider the teaching of carrying in addition. An adequately built program will first guarantee that the pupil is given a background of experience which will enable him to understand why he carries and what carrying is. He understands ones, tens, and hundreds before he begins to carry. Also, before he begins to carry, he learns to use his recently acquired addition facts in adding two- and three-place numbers without carrying.

Children should be led to discover new truths for themselves. In most learning situations, telling is not teaching. Pupils can be led to discover sums and differences in the primary addition and subtraction facts by counting groups.

Vocabulary in arithmetic is another area in which the psychological criterion should be used. No words should be taught that have no use in the vocabulary of the children or tend to confuse them.

All three of these fundamental criteria must operate simultaneously, in spite of the fact that at certain times and for good reason one seems to take precedence over the others. Arithmetic is by nature, logical. Curriculum must be socially useful. The teacher must be concerned with how children learn.

Buckingham insists that arithmetic must be meaningful to even the youngest children:

> If we accept the current philosophy as to the value of meaningful arithmetic, we must be prepared to begin that type of arithmetic when children enter school. We must select and co-ordinate such meanings as are likely to favor insight. We must grade these meanings according to the maturity of the child. We must build our arithmetic from the beginning with the <u>intention</u> of teaching it with meaning.[15]

One of the most important factors in the learning process--the teacher--needs to be mentioned here, even if only briefly.

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[15]B. R. Buckingham, "The Social Point of View in Arithmetic," <u>Fiftieth Yearbook of the National Society for the Study of Education</u>, Part II (Chicago: The University of Chicago Press, 1951), pp. 280-281.

> ... The teacher of arithmetic must have an understanding of child development; a genuine love for children; an understanding of mathematics; a genuine liking for the subject.[16]

Although the writer agrees with Williams[17] that curriculum development is a dynamic on-going process and that the newer practices in the teaching of arithmetic are neither fixed nor final, some phases presumably are sufficiently stable so that one may with reasonable confidence suggest suitable learning experiences in primary arithmetic. The principles of child growth and their implications for curriculum presented briefly above form the basis of succeeding chapters.

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[16] Catharine Williams, _Teaching of Arithmetic in the Elementary School_ (Danville, Illinois: Interstate Printers and Publishers, 1950), p. 17.

[17] _Ibid._, p. 12.

CHAPTER II

OBJECTIVES AND GUIDING PRINCIPLES OF A SOUND PRIMARY ARITHMETIC PROGRAM

The major objectives of the modern arithmetic program, according to Brueckner and Grossnickle, are:

> (1) To develop in the learner the ability to perform the various number operations skilfully and with understanding, and
>
> (2) To provide a rich variety of experiences which will assure the ability of the pupil to apply quantitative procedures effectively in social situations in life outside the school.[18]

Numbers become significant and meaningful to a child through contact with the quantitative aspects of daily living. Formal aspects of arithmetic should be developed as the pupil has sufficient experience of the type which builds and interprets meaningful number concepts.

As sufficient number understanding is established, a planned program for building the fundamental arithmetic skills is essential. The teacher will make this systematic instruction most effective by capitalizing on every opportunity to apply learning to the problems that pupils meet in their everyday lives.

I. OBJECTIVES

An excellent statement of objectives in arithmetic for

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[18]Brueckner and Grossnickle, *op. cit.*, p. 1.

primary years is to be found in the Cincinnati course of study.

> Instruction in arithmetic should endeavor to:
>
> Develop an awareness of the usefulness of numbers in everyday life
>
> Develop the ability to meet number situations which arise in connection with classroom activities
>
> Stimulate a permanent interest in number processes
>
> Promote self-reliance in discovering quantitative relationships in natural situations
>
> Develop concepts of time in connection with the activities in the home and school
>
> Provide and use every available opportunity for developing understanding and appreciation of money values
>
> Provide a variety of activities in which simple measurement will be needed
>
> Develop a meaningful number vocabulary and rich background of number concepts through school activities
>
> Encourage and provide opportunities for writing numbers in classroom situations
>
> Develop in children fundamental personal and social habits, such as economy, fair play, sharing, good judgment, and co-operation.[19]

II. GUIDING PRINCIPLES

Guiding principles for a sound primary arithmetic program upon which there is considerable agreement among leaders in the field and which served as a basis for the proposed

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[19] The Primary Manual, Althea Beery, editor, Curriculum Bulletin 95 (Cincinnati: Cincinnati Public Schools, 1947), p. 480.

course of study in Chapter III are as follows:

(1) The first step in effective arithmetic teaching is to stimulate an interest in numbers by providing the children with many and varied useful quantitative experiences which will lead the children to identify these experiences with their appropriate symbols. Both planned and incidental number experiences are necessary. The children should acquire number ideas through discovery from opportunities offered them.

(2) An arithmetical vocabulary for each child must be built through a wide variety of his own actual experiences and used in many practical situations to insure his understanding.

(3) Each new process should begin in the environment, since the social need for arithmetic processes is essential. Every subject matter area provides untold opportunity for supplementary arithmetic experiences that will not only "fix" the principles but will develop further insight, accuracy, and speed.

(4) The age or grade level at which any given child can effectively attempt any arithmetic process or phase depends on how successfully his personal experiences have established his underlying insight into number relationships.

(5) The teacher should test, if possible, and explore the child's background of number experiences and should discover his individual capacity for understanding of new processes.

(6) Both individual and group instruction on all levels is necessary, due to the wide range of arithmetic readiness and ability. Instruction must fit the individual needs and levels of understandings of the children in order to challenge their continuous effort.

(7) In order that all may have a common background as far as possible, it is necessary to set up a minimum program of "normal" achievement for each year.

(8) Devices are justified when they stimulate the pupil's interest or increase his participation in the correcting, proving, or improving of arithmetic understandings.

(9) In order to help the children learn more easily, the mathematical processes should be arranged in logical steps or levels according to the amount or difficulty of reasoning required for understandings.

(10) Understandings, developed through many varied experiences, must precede practice if learning is to be effective.

(11) Frequent evaluation of individual and group learning and progress is essential to effective teaching.

(12) No teacher of arithmetic can hope to lead a student to a level of functional competence higher than that which the teacher himself has attained.

(13) The transition from the concreteness of real experiences to the abstractness of numbers written on blackboard and paper must be gradual.

(14) Problem solving techniques which involve critical thinking should be begun in the primary years.*

*These principles were derived from the writer's extensive teaching experience, authenticated by authorities in the field, some of whom are:

Ibid.
Brueckner and Grossnickle, op. cit.
Williams, op. cit.
Swenson, op. cit.
Lee and Lee, op. cit.

CHAPTER III

PRIMARY COURSE OF STUDY

Content of learning experiences in arithmetic and some suggested procedures are included in this chapter. They are classified in ten categories, with separate headings under each for the first, second, and third years of school.

READINESS

Readiness is the process of getting a child more nearly ready for the next learning experience. This process is important, recurring, and highly specific. Also, it is an individual matter. As Swenson so aptly observes:

> Only to the uninitiated can readiness appear to be a simple matter of reaching some mythical, magical point preceding which the learner is clearly not ready and following which he is clearly and unequivocally ready to learn.[20]

And again:

> Learning readiness is always readiness for learning particular content or learning in a particular area . . . . With young children the question of "readiness for what?" needs particular attention for the simple reason that adults who guide the learning of children are inclined to assume that children know "what" they are working into even when they have little or no notion about it.[21]

Assessing children's readiness for number experiences and instruction is not easy, but children supply an abundance

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[20]Swenson, op. cit., p. 54.

[21]Ibid., p. 55.

of clues if teachers understand children well enough and are observant enough to note the clues.

In-school number situations that arise naturally with young children include money, measurement, time, objects, school subjects, and distance, according to studies reported by Koenker[22], and the alert teacher will detect and make use of them.

In out-of-school experiences of first-grade children, the most frequent number operations were found[23] to be addition, counting, subtraction, fractions, reading Arabic numerals, and measuring. These arithmetic processes occurred in transactions in stores, games involving counting, reading Roman numerals on clock, finding pages in a book, dividing with playmates, depositing money in toy banks, playing store, running errands, measuring in sewing, buying, and selling, and acting as newsboy.

Many children entering their first year of school have well-developed understandings of number concepts and can profit from activities involving quantitative thinking. The teacher should discover what each child knows and understands, through observation, personal interviews, study of the arith-

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[22]Robert H. Koenker, "Arithmetic Readiness for the Primary Grades," G. T. Buswell and Maurice Hartung, editors, Arithmetic 1949 (Chicago: The University of Chicago Press, 1949), p. 28.

[23]*Ibid*., p. 29.

metic sections of standardized tests, and information from the kindergarten teacher, if any is available.

Progress of an individual child depends upon his intelligence and the situations which have enabled him to obtain number experiences. By the time the first year in school is reached there are wide variations in abilities of children to deal with various phases of number. The following list of number skills and knowledge might be used by the teacher as a check:

> Counting by ones.
> Counting by tens.
> Telling how many objects are in a group.
> Recognizing coins.
> Recognizing circles and squares.
> Knowing about pints and quarts.
> Knowing age, birthday, address, parents' name and phone number.[24]

Brueckner and Grossnickle have a very good individual test[25] for readiness in arithmetic. A teacher may construct her own test for measuring the number concepts she is planning for her group. A record should be kept of the findings on the first test so that a retest will indicate progress made. In chapter V there is a list of standardized arithmetic tests that might be used.

At the beginning of each and every year it is necessary for the teacher to check to see whether material presented in

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[24]Louise Pedigo, editor, Wyoming Course of Study for Primary Arithmetic (Cheyenne, Wyoming: State Department of Education, 1949), p. 18.

[25]Brueckner and Grossnickle, op. cit., p. 56.

the first year has meaning and to proceed from there. Many times some essential learning has been forgotten, and a readiness period for individuals or groups is needed. This should precede any new work, as arithmetic learning progresses upon understanding and is unsuccessful without it.

CONTENT AND SUGGESTED PROCEDURES

I. BASIC NUMBER UNDERSTANDINGS

A. Learning Experiences

1. First Year

a. Counting--1's to 100, 2's to 50, 10's to 100, 5's to 100.

b. Reading--1's to 100, and such numbers as pages in books, as far as needed.

c. Writing--1's to 100, 2's to 50, 10's to 100, 5's to 100.

d. Using ordinals--first through sixth, for understanding and general use in social situations, as need arises.

2. Second Year

a. Counting--1's to 200, to establish sequence; additional understanding in counting and using 5's and 10's to 100, and 2's to 100.

b. Reading--1's to 500.

c. Writing--1's to 300; 5's, 10's, and 2's to 100.

d. Using ordinals--review first through sixth; continue through tenth.

e. Introduction of concept of place value.

3. Third Year

   a. Counting--review. Habits should be pretty well established by now.

   b. Reading--1's to 1000.

   c. Writing--all numbers to 1000.

   d. Using ordinals--eleventh, twelfth, and so on to twentieth.

   e. Place value taught objectively: thousands, hundreds, tens, ones.

B. Suggested procedures

Counting

"The process of counting," says Spitzer[26], "is so fundamental and important to the child's understanding of and progress in arithmetic that it warrants much more careful consideration than is usually given to it."

Complete counting, according to Brueckner and Grossnickle[27], includes: rote counting, enumeration or rational counting, identification, reproduction, comparison, and grouping. Some sources add serial counting to this group.

Rote. Saying the number names in their order or sequence is rote counting. It is done in a mechanical manner with little thought of meaning, but it is a basic ability in all counting. The teacher may use number rhymes to help establish the number names in proper order.

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[26]Herbert F. Spitzer, *The Teaching of Arithmetic* (Chicago: Houghton Mifflin Company, 1948), p. 67.

[27]Brueckner and Grossnickle, *op. cit.*, pp. 170-171.

<u>Rational</u>. Associating the number names with the corresponding number of objects is called rational counting or enumeration. Pointing to each picture or object is necessary. This ability comes slowly to some children, as they are inclined to skip over objects giving no number name, or to say two or three number names when pointing to one object. The child should learn that the last number name is the total of the objects counted.

The teacher should be certain that the child does not merely say number names mechanically without meaning in this type of counting. Simple tests can measure whether the child is doing rote counting or really engaging in rational counting. There are many uses of counting rationally, chiefly social.

<u>Identification</u>. This consists of a pupil being able to name the number of white marbles in a group of black and white; or being able to answer, "In which group are there four black marbles?"

<u>Reproduction</u>. This consists of the pupil being able to select a given number of black or white marbles and knowing how many he has selected; or being able to answer, "Pick out three black marbles from the pile."

<u>Comparison</u>. This consists in having the child determine whether there are more marbles in one color than the other; or being able to answer, "Of which kind of marble are there more (or less)?" "How many more?"

Serial. This type of counting involves an understanding of the order of our number system. A child who can place numbered objects in their correct numerical order or who can locate a number in a series has the idea of number.

Two types of designation are involved: the cardinal and the ordinal. The cardinal designation is 1, 2, 3, 4, 5, etc. It is concerned with a higher level of counting than rote counting because the child must be aware of the sequence of number. The ordinal designation is first, second, third, fourth, fifth, etc.

Grouping. This is the final stage of counting. The pupil may be able to see from arrangement of the marbles that there are four in one group and then continue counting by ones to get the total.

Making crude comparisons and counting to determine the exact number in a group are prerequisite to making exact comparisons of two or more groups. The child strengthens his knowledge by describing various groups: a group of six is seen as three more than three; a group of four is seen as three less than seven. Practice should be given in counting groups, matching groups one to one, and in counting the excess of one over the other. Both oral and written language are used to describe and compare groups.

Up to this time the child has counted to determine the number in a concrete group. In order to provide for progression in thinking, it is necessary to lead the child to recognize the number in a group without counting. Cards may be used, suggesting various arrangements of number groups. These are exposed without giving the child time to count the number in the group.

These patterns will serve as meaningful symbols and may be posted about the room after they have been taught. There are many natural patterns in the classroom that may be used to strengthen these concepts, such as: panes of glass, panels in the door or above the blackboard, or the arrangement of the seats.

II. VOCABULARY

Size, form, time, measure, number, speed, place, money concepts, and other words involving number concepts are typical terms to be extended with children's experiences and then used naturally by the children. As Strickland observes:

> Many experiences are necessary before he (the child) can understand all the common words of English which deal with measurement and number. The full significance and meaning of quantitative and mathematical terms may develop very slowly. Some children require more time and many more experiences than others. Insight cannot be passed from one individual to another; each must arrive at it through his own experience.[28]

Each year the teacher should provide review experiences for the vocabulary used in the previous year, and extend it as the maturity of the children permit, before using additional words. The vocabulary should follow the basic and supplemen-

[28]Strickland, _op. cit._, p. 63.

tary readers in use.

A. Suggested Words

1. First Year

above, after, afternoon, again, all, another, any, around, as many as, before, below, beside, big, both, bottom, buy, by, calendar, cent, circle, clock, cost, curve, day, dime, dollar, down, dozen, draw, early, enough for, eight, evening, every, fast, far, few, first, five, four, fourth, from, half, heavy, high, hour, how many, how much, in, inside, large, last, late, less, less than, little, long, low, many, middle, morning, money, more, more than, much, narrow, near, new, next, nickle, night, nine, none, noon, nothing, now, on, once, old, one, over, pay, penny, pint, place, price, quart, quarter, recess, round, second, sell, seven, several, short, slow, small, side, six, some, spend, square, straight, tall, ten, than, then, thick, thin, third, three, too big, too little, top, two, under, up, very, week, whole, wide.

2. Second Year

about, across, ahead, a long time, almost, altogether, always, among, check, column, corner, deep, each, end, even, examples, feet, foot, forth, front, full, gone, great, hard, heavy, high, inch, last year, least, light, lot, minute, month, most, never, none, number left, often, oldest, only, outside, paid, past, plenty, pound, prompt, quick, slowly, soon, suddenly, swift, tiny, this year, through, today, together, tomorrow, tonight, twice as many, underneath, upside, until, weight, while, yard, year, yesterday, youngest.

3. Third Year

addition, altogether, buying, carry, center, change, chart, comparison, complete, count, different, distribute, divide, divided by, division, each, enough, entire, equal to, example, excursion, explain, find, figure, group, length, less, liquid, longer, money, multiply, necessary, need, number, order, paid, parts of, plan, price, prove, reach, receive, reverse, sale, selling, sizes, space, spend, supply, tall, thickness, times, value, withdrew.

B. Suggested Procedures

Children should have many opportunities to hear and use the above vocabulary in meaningful situations. In general, the words are in their meaning vocabulary, not their reading vocabulary. Comparative and superlative degrees of adjectives are not listed often. Young children use these terms indiscriminately. The teacher will use the correct form, and the children will gradually acquire these forms through use.

It is helpful to use objects, pictures, actions, and any experiences possible to make the words meaningful. Dramatizations and manipulations of objects will help the child in forming a clearer picture of the word. Story plays, counting, and rhyming games will help here, too.

III. ADDITION

A. Suggested Content

1. First Year

Develop understanding and meaning of addition through concrete experiences.

Give many experiences in separating, comparing, and combining groups. Begin with simplest forms and continue through combinations to ten.

Recognition of groups of 2, 3, 4, and 5, without counting, using objects, pictures, or semi-concrete materials.

2. Second Year

Recognition of groups of ten.

Introduce 64 basic combinations, sums to ten.

Establish meaning through all possible groupings. This will form background for abstract presentation of addition facts in second semester.

Terms: add, sign + and - should be taught through use.

3. Third Year

Complete the introduction of 100 addition facts.

Addends of two or three digits without carrying, including zero facts.

Adding by endings, as:

```
24    28
 2     5
──    ──
```

Column addition: two addition facts, sums exceeding ten, as:

```
 5
 3
 6
──
```

Column addition, with a sum greater than ten for the first two facts, plus adding by endings:

```
 8
 7
 2
──
```

Column addition, with a sum greater than twenty:

```
 5
 8
 9
──
```

Double column of three addends, with and without carrying.

Adding dollars and cents, with and without carrying, using dollar sign and decimal point.

B. Suggested Procedure

1. First Year

Use objects in teaching group recognition. Use picture cards as flash cards. Follow this with semi-concrete materials using groups of lines, circles, and so on. The children need much help to develop meaningful concepts of numbers. It is necessary to give them opportunity to understand that number is a language that describes things that actually happen. The children will learn many combinations as they work in groups of things, but no attempt should be made to teach abstract combinations at this level. This is verified in the following quotation from Cole:

> Children's concepts of number grow slowly. The first concept they develop, some time before they have reached

school age, is the difference between "one" and "more than one" or "many." Later on, they add such definite concepts as "two," or "five," or "ten" and general ideas for such terms as "equal" or "add." These later concepts do not develop until the child is about seven years old. First and second grade children do not yet have the concepts needed for learning addition and subtraction, but they have the necessary capacity to learn the simplest forms of these processes if the teacher first develops their concepts by means of concrete materials and if she teaches at first in terms of objects, not abstractions. As soon as the children can count objects correctly, they can begin to add by means of counting, even though they may not yet be ready for generalizing.[29]

2. Second Year

Many opportunities should be given the children to work with concrete materials during the first few weeks. The experiences should include grouping, rearrangement of sub-groups, one-to-one relationships prior to learning more mature methods. When the children are ready, the teacher begins instruction with recall as the outcome. She should teach both forms of the same addition combination, as well as the related subtraction facts. All children will not arrive at the maturity level for combinations at the same time. It may be necessary for some to use less mature methods much of the year.

For purposes of recall, some sort of drill is necessary. None should be used until the children really understand the number relationships involved in addition combinations. Drill with abstract number combinations seems to

[29]Luella Cole, The Elementary School Subjects, (New York: Rinehart and Company, Inc., 1946), pp. 333-334.

create and fix low-level response. A child who is counting to solve combinations continues to count. Drill merely teaches him to count more rapidly. A child who is using grouping and counting together to get answers continues to use this method. Drill only helps him become more expert. Drill does not lead to ability to think in groups. This comes only with complete understanding. It is the task of every teacher to evaluate her teaching to the point where she knows whether it is advisable to use drill for purposes of recall, without further fixing immature habits of addition.

When drill is thus defined in terms of its functions, it becomes clear that the time for drill cannot be specified in an all-or-none way, in terms either of pupil age or of pupil grade. Instead, it must be specified with respect to each separate fact and skill and it must be used then in the last stages of learning of each child. This means that there is still a place for drill in primary arithmetic but only after skills and facts have first been invested with meaning and given significance through vital experiences.

3. Third Year

Use objects, pictures, and semi-concrete and abstract presentation of the addition facts, sums eleven to eighteen. The hundred addition facts should be mastered during this year. The child learns combinations best as he discovers group relations and makes generalizations in handling objects and as he uses number symbols to tell about the things he himself actually does.

Both in review and new work, as she teaches the combinations, the teacher will help the children in the following ways:

1. To understand the usefulness of number in solving problems.

2. To see the relation between addition and subtraction.

3. To recognize the relation between the two addition combinations and between the two subtraction combinations of a family.

4. To discover and generalize other number relations such as adding and subtracting one.

5. To understand and use the terms sum, plus, remainder, difference, minus.[30]

Carrying in addition should be taught with concrete materials. Toothpicks or bundles of cards of ten may be used to represent tens and single ones used for the ones' column. In the example, 36 cards and 45 cards, the steps are as follows:

In adding the ones' column, 6 cards and 5 cards make 11 cards, or 11 ones.

Show that 11 ones can be made into one bundle of 10, with 1 single card remaining in the ones' column.

The bundle of 10 can then be carried to the tens' column and added to the other tens, making 8 tens.

With the 8 tens and the 1 in ones' column, the sum is, therefore, 81.

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[30]Mrs. Leslie Beatty, _Guide to the Teaching of Arithmetic in Kindergarten and Grades One and Two_, Bulletin XVIII, No. 8 (Sacramento: California State Department of Education, 1949), p. 35.

This concrete method should be used enough times to insure complete understanding of the carrying process. With the addition combinations, teach the related subtraction combinations.

IV. SUBTRACTION

A. Suggested Content

1. <u>First Year</u>

Build subtraction understanding through the use of objects, pictures, games, and semi-concrete materials. Begin with simplest forms and build understanding through ten. Terms "more than," "less than," "how many left," and "how many more" will be needed.

2. <u>Second Year</u>

Subtraction facts with minuends not exceeding ten.

Subtraction should be taught with addition, using objects, pictures, semi-concrete materials, and abstract numbers.

Use signs: - and = .

3. <u>Third Year</u>

Complete introduction of 100 subtraction facts.

Subtraction of two-digit numbers, with no borrowing.

Subtraction of zero facts and double zero, with borrowing.

Subtraction of two- and three-place numbers, with and without borrowing.

Subtraction of dollars and cents, with and without borrowing.

B. Suggested Procedures

At all age levels it is necessary to utilize varied concrete experiences to establish the four meanings of subtraction: more than, less than, how many left, how many more.

The nature of the task is set forth in Stern:

> In mathematics, addition and subtraction are expressions of relationship that exist between any two numbers. Every number equals itself only; it is different from any other number. This difference may be expressed by stating what has to be added to the smaller one or taken away from the larger one to make them equal. Thus addition and subtraction are interconnected, and at the same time opposite in effect.[31]

1. First Year

Build subtraction concept through games, objects, and other materials. They will experience subtraction along with addition of concrete materials after their skill in counting is developed.

2. Second Year

Subtraction and addition facts are to be taught together at all times. Much use of objects, pictures, semi-concrete materials is still necessary in introducing combinations.

3. Third Year

In addition to the work of the preceding years, borrowing is new. Use concrete illustrations, as in the teaching of carrying in addition. Show the children that

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[31]Catherine Stern, Children Discover Arithmetic (New York: Harper & Brothers, 1949), p. 80.

since the minuend is not large enough, we must open a bundle of tens to put with the ones we already have.

V. MULTIPLICATION

A. Suggested Content

1. First and Second Years

No formal multiplication work is included in these years. However, a readiness for and understanding of it is begun when children see that three twos are six or that three threes are nine. This will grow through experiences as needed and depending on the maturity of the children. Many will not acquire any understandings of multiplication, due to immature methods of counting and grouping.

2. Third Year

Introduce multiplication facts through fives.

Multiply two-digit multiplicand by a one-digit multiplier, with no carrying.

Use the zero in multiplication.

B. Suggested Procedures

1. First and Second Years

The children gain understandings in multiplication and division from working with groups. They see that groups can be taken apart and put together. Taking apart into equal groups gives the idea of multiplication. This knowledge may be used in oral language. Dramatization, verbal descriptions, and pictures illustrating solutions of problems involving multiplication and division, help to give meaning to the process.

2. Third Year

Through the use of concrete materials, the teacher should review the facts learned but not mastered in the two preceding years. The children are now ready to organize this information into sequence and hence the tables. The children will already be familiar with all the facts about twos from their study of groups to eighteen. All relationships involved in the table should be discovered. It is important for the children to see that any known fact may be used to discover unknown facts by the addition of one or more groups.

Knowledge of all relationships within the table and between tables will aid children in learning the multiplication and division combinations. Eventually the children should know the combinations apart from these related ideas. Memorization of facts before understanding has been established should be discouraged. Problem solving and drill as practice in thinking should be used at every level in progressing from concrete to abstract stages.

Multiplying two-place numbers will be new in this year's work. The teacher should review the fact that tens are added and subtracted like ones. The children know how to multiply ones and should very readily understand that tens are multiplied like ones. The concrete materials of sticks or cards in bundles of tens should be used. Three times thirty should be illustrated by:

Adding three thirties

Multiplying three tens, getting 9 tens, or 90

Setting the problem down as $\begin{array}{r} 30 \\ \underline{x\ 3} \end{array}$

In the same way thirty-two, or three tens and two ones, can be multiplied by three. The child thinks:

Three 2's are 6, and writes the 6 in the ones' column.

Three times 3 tens are 9 tens, and writes this 9 in the tens' column.

Practice in multiplying at this level should be provided in preparation for development of more complex steps in multiplication.

From this step, the children are led to see that carrying in multiplication is just like carrying in addition. Drill or practice in thinking in the concrete, semi-concrete, and abstract stages, must be provided. Solving problems will afford many opportunities for this practice in thinking. Extension of number ideas previously developed should be continued and basic skills maintained.

VI. DIVISION

A. Suggested Content

1. <u>First and Second Years</u>

No formal division.

In social situations it is often necessary to divide things, and this should be done, as a basis for teaching the process with understanding.

2. <u>Third Year</u>

Introduce exact division through fives. This should be presented and taught with multiplication. Be sure to use the long-division form, in accordance with modern texts, to avoid reteaching.

B. Suggested Procedures

1. <u>First and Second Year</u>

As already suggested, some experience with and understanding of the idea of division takes place, with the use of concrete materials. Use of division in passing materials is common practice. Many other forms of division occur in the modern primary room, and the teacher will make use of such situations.

2. <u>Third Year</u>

The first of this year the teacher will use concrete materials so that the children may see that we can find out how many equal groups there are in a number and discover that the size of one group equals that of another. She will develop the containing idea (four "contains" two and <u>not</u> two "goes into" four), through the use of successive subtractions, done with concrete materials, and the "sharing" idea of division, with partition done with concrete materials.

Concrete materials should be used again to teach the division of tens. After many experiences with bundles of tens and ones, the children should arrive at the generalization that tens are divided like ones.

During this year, the teacher should include only examples and problems of exact division. Children may need to solve problems arising from their social experiences involving uneven division. The teacher should aid the children in their solutions but should not expect them to use the written form involving uneven division in this year's work. Difficulty arises in writing zero in the quotient to hold the place of ones. The importance of zero as a place holder is further demonstrated in the proof. The children should be given opportunity to suggest ways to verify their answers. These suggestions should include returning to the concrete, in subtracting, adding, and multiplying.

VII. FRACTIONS

A. Suggested Content

1. First Year

Introduce the concept of a whole and a half of an object.

More mature children can grasp one-fourth.

Use words only.

2. Second Year

Review one-half and one-fourth of objects.

Use words and symbols both: $\frac{1}{2}$, $\frac{1}{4}$.

3. Third Year

Teach understanding of: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{3}$.

Introduce understanding of:
$\frac{1}{2}$ of a group, $\frac{1}{3}$ of a group, $\frac{1}{4}$ of a group, as $\frac{1}{3}$ of 12 apples, $\frac{1}{4}$ of 16 pencils.

Stress the symbolism of fractions as used.

B. Suggested Procedures

The study of fractions in the first three years of school is actually a readiness program. It consists of concept building to establish meanings needed later in addition, subtraction, multiplication, and division of fractions. Fractions have different meanings in different situations, and the teacher should develop each of these meanings individually.

A fraction may mean a part of a group. Children will work first with $\frac{1}{2}$. Six children may arrange themselves into two equal groups. The children might arrange ten books into two equal piles.

A fraction may equal part of a whole. Children entering first year frequently are familiar with the term one-half, although they do not always recognize the fact that one-half implies two equal parts. This concept may be developed by tearing a sheet of paper in exactly two halves.

A fraction may be part of a unit of measure. Units of measure frequently are divided into fractional parts which in themselves are units of measure. Thus: one fourth of a dollar is a quarter; one third of a yard is a foot. While the children deal with these measures as units, they also should develop the concept of fractional amounts involved.

Fractions may be used to express comparisons. Ruth's pencil might be only half as long as Betty's. A foot ruler is only one third as long as a yard stick.

Williams reminds teachers:

> The fraction concept does not develop all at once. It must be built up gradually. Through many experiences children must be guided so that they get not only the idea of division and the equality of the parts involved in the division but also the fact that the sum of the parts makes up the whole or unit. They need discussion and experimentation to make meaningful the indications of size of parts and number of parts.[32]

VIII. MONEY VALUES

A. Suggested Content

1. <u>First Year</u>

Develop the concept of penny or cent, nickel, dime, quarter, cents in a nickel, and cents in a dime.

2. <u>Second Year</u>

Concepts of: half-dollar, nickels in a quarter, dimes in a half-dollar, cents and dimes in a dollar.

Use symbols: ¢ and $.

3. <u>Third Year</u>

Develop concepts: dimes and quarters in a dollar; reading and writing of dollars and cents, to five digits.

Place value of money. Use of money problems in all fundamental operations previously taught.

B. <u>Suggested Procedures</u>

1. <u>First Year</u>

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[32] Catharine Williams, <u>op</u>. <u>cit</u>., p. 53.

When children bring money to school for some purpose, discuss the coin, its value, and its importance. Count the milk money. Set up natural experiences using money.

2. Second Year

Show relationship of money to counting by fives and tens. Use real money for development. Toy money may be used successfully for practice. Continue concepts started during preceding year.

3. Third Year

Expand use of money in real situations. Take a trip and collect bus money. Wise use of expending allowances might be used. Use money in all previous operations taught.

IX. MEASUREMENTS

Almost all of the child's daily living activities involve measurement. Using these experiences with varied appropriate illustrative materials should insure functional pupil understanding.

A. Suggested Content

1. First Year

<u>Time</u> -- to begin school, to go home, for lunch, for recess.

Hour, half-hour, today, tomorrow, yesterday.

Realization that days and months have names.

Changes of season, names of days of week.

<u>Linear</u> -- long lines, short lines, edges of big box, edges of little box, inch, foot, yard.

Liquid -- half-pint of milk, pint, quart, cup.

Quantitative -- teaspoon, tablespoon, dozen, pound.

Geometric figure -- triangle, square, circle, star.

Temperature -- hot, cold, warm, cold enough to wear snow suits, etc., colder than yesterday.

2. Second Year

Time -- hour, half-hour, minutes.
Names of the days and months in order.
Use of calendar.
Reading hours and half hours on clock.
Year.

Linear -- inch, foot, feet, yard.
1/2 and 1/4 inch on ruler.
12 inches in foot.

Liquid -- cups in pint, half-pint, quart
1/2 quart
2 pints in a quart.

Quantitative -- dozen, half-dozen, pound, 1/2 and 1/4 pound.

Geometric figure -- triangle, square, circle, snowflake star.

Temperature -- freezing, zero, above and below zero.
Use real thermometer.
Discuss radio reports on temperature.

3. Third Year

Time -- minutes, 60 minutes in an hour.
Reading exact time on clock.
Calendar to show weeks in a month.

Linear -- inch, foot, feet, yard, feet in yard, inches in yard.

Liquid -- cups in a quart, quarts in a gallon, gallon, half-gallon.

Quantitative -- half-dozen, 1/4 dozen, ounces in a pound, 1/4 of 16 oz., 1/2 of 16 oz.

Geometric figure -- continue concepts already used.
Add rectangle.

Temperature -- read both indoor and outdoor thermometer.
Keep temperature chart.

B. Suggested Procedures

1. First Year

The teacher will find many ways to use all forms of measurement if she is on the lookout for them. Time to do special things, such as go to music or physical education, is quickly learned. The calendar is used to mark special days or birthdays. As the daily calendar takes on meaning, the children tear off a sheet each day. This points out the end of the month also, and it tends to establish the orderliness of the time concept.

Many projects require measurement. It need not be too exact, but the children should do it. They may measure a given space in which to put a frieze. Ours happens to be "two yard sticks and 21 inches long." How they knew it was 21 inches is a mystery to the teacher; perhaps someone had read inches on a yard stick at home.

The children should actually measure liquids to establish the needed facts. This needs to be reviewed often. They readily remember that their morning milk contains a half-pint, though.

Geometric figures are used in many ways and take on more and more meaning as the year progresses. A triangle is played in the rhythm band; we make a snowman from circles. Many objects handled daily have a given geometric shape, and they should be referred to by their proper name.

2. Second Year

The children should construct clock faces with movable hands. One good way is to make them from paper plates. The time to do the many different things throughout the day becomes very important. It is easy to remember these times on the clock.

The children should continue to use linear and liquid measure in actual experiences. These will become increasingly difficult as the children become more mature and are able to make closer measures. Their terminology also becomes more mature.

The geometric figures are used in making greeting cards, snowflakes, booklets, record sheets, and many other things.

Temperature takes on more meaning when the children can read a thermometer and compare their findings with those on the weather report. It also takes on meaning in connection with health study and the proper clothing to wear.

3. Third Year

By this time, most measurements have a general meaning for the children and they are ready to apply specific learnings to things close at hand. These should be extended in all areas possible and combined with science, art, and health.

Pupils should be able to keep height and weight charts and note changes from month to month. A record of actual readings on the thermometer is of value at this level. Both an indoor and outdoor thermometer should be used. Some records may be represented graphically.

X. PROBLEM SOLVING

Many arithmetic problem-solving techniques can be extended to show children how to solve problems growing out of their day-by-day living together. They can begin to develop critical thinking in experience situations. As Williams points out:

> In a democracy the school has a definite obligation to foster in children the attitudes and methods of critical thinking. The teacher who accepts this obligation will guide pupils in such a way as to develop their powers to formulate problems, to make plans for the solution of these problems, to find and select data bearing upon the problems, to organize and interpret the data, to make inferences and draw conclusions from data, and to apply the conclusions or solutions to events.[33]

A. Suggested Content

1. First Year

Use many oral problems on every-day occurences arising in and out of school. Make many number stories about objects and pictures in the classroom.

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[33] Catharine Williams, op. cit., p. 21.

2. <u>Second Year</u>

Solve simple one-step problems around meaningful situations arising from anything being studied. Use in problems the addition and subtraction facts learned at this level.

3. <u>Third Year</u>

Solve one-step problems growing out of every-day experiences. Use in problems all four fundamental processes, including carrying in addition and borrowing in subtraction. Use exact division facts only, through fives.

B. Suggested Procedures

1. <u>First Year</u>

Since arithmetic is taught in all curriculum areas here, problems should be those which in most part arise from actual experiences. Many problems can be made from stories about books, articles with which the children work and play, and familiar objects and pictures. Games provide another means of using problem solving. Children will enter these story problems with enthusiasm.

2. <u>Second Year</u>

Many problems may be used without numbers, just to describe the situation. These should come from real life problems. Each new process may be introduced through a practical problem. Dramatization of story problems to illustrate combinations being taught is very effective. Children can learn to illustrate their own story problems.

3. Third Year

The children should learn to find out the following things about their story problems:

What does the problem tell?

What does the problem ask you to find?

What would be a reasonable estimate?

Which process will you use in finding the answer?

Children of this age are capable of doing critical thinking at a more advanced level and should have much practice in using problem-solving techniques.

CHAPTER IV

LEARNING AIDS

A volume could be written concerning teaching aids and effective ways of using them to clarify mathematical processes and principles, to give the story of man's struggles to develop number, to teach the instruments of measurement, and to help children to understand the role mathematics plays in helping man to gain control over his environment. However, space limitations prohibit doing more than offering a few suggestions here.

In the broadest sense, learning aids in arithmetic include all those activities which tend to enrich the child's awareness of the quantitative side of social living. Thus, the teacher needs to be alert to the number possibilities in classroom experiences, the games children play, the books children read, the parties, picnics, and trips children plan, the money children spend, and the many home activities in which children engage--such as eating, setting the table, cooking, sleeping, dressing, going to the store, buying and sharing candy, going on automobile trips, and dialing radio stations and telephone numbers. Each of these can contribute to the child's growth in mathematical understanding.

I. LABORATORY EQUIPMENT

When children are encouraged to think and act, to discover and to experiment for themselves, there is definite need for laboratory equipment. It is advisable for each teacher to gradually build up her own equipment from year to year. A partial list of materials suitable for an arithmetic laboratory would include:

Balls, jumping rope, bean bags, marbles, small toys, small clothespins, blocks, toothpicks, colored beads, abacus, measuring cups, bottles and jars (for pint and quart), scale, calendars, tape measure, yardstick, rulers, toy money, paper pie plates, circles, half circles, rectangles of colored paper or oak tag, counters (including such things as colored sticks, buttons, beans, spools, checkers, small blocks, milk bottle caps), a large clock with movable dial, number cards showing numerals and dots, toy telephones, number charts, thermometer, and flash cards.

II. GAMES AND SONGS

SOME GAMES THAT MAY BE PURCHASED

Bingo	Parchesi
Contack	Pick-up Sticks
Dominoes	Racing with Numbers
Hot Numbers	Rook
Jacks	Seven Up
Lotto	Target and Darts
Marble Games	Ten Pins
Number Authors	Tiddly Winks

SOME GAMES FOR PRACTICE AND DRILL

Ring Toss. A large board may be made with nails or pegs spaced 4 or 5 inches apart. These may be numbered from 1 to 9. Each player may throw 2 jar rubbers. His score will be the sum of the numbers he rings.

Guess the Number. The teacher may say, "I am thinking of a number smaller than ten and more than five." The child who guesses the right number will be "it." He may say, "I am thinking of a number that is more than ten and less than fifteen," or may use other numbers. For lower grades, this may be changed to, "I am thinking of two numbers the sum of which is 7." Have the one who is "it" whisper the numbers to the teacher first.

Bean Bag. Construct a solid frame of plywood so that it fits securely on the floor; then saw three or four holes of various sizes in the front of frame. Mark the value of each hole below it. Have the children take turns in throwing bean bags into the holes. A score should be kept on the blackboard.

Basketball. Balls are thrown into a wastebasket or other container. Decide ahead the number of points each ball will count. The player getting the most points wins. Children should take turns acting as scorekeeper.

Number Relay. Form children into two teams with nine on each team. Pass out cards numbered from 1 to 9, to each team. Have each team in a corner of the room opposite the blackboard. At a signal from the teacher they will race to see how quickly each team can line up their numbers in the right order on the chalk rail.

Fish Pond. Make a fishing pole by tying a magnet to a dowel stick. Cut out nine cardboard fish and attach a paper clip to each. On one side of each put a number. Place them face down. Each child is allowed two turns to fish.

Guess! Guess! Use a box containing ten marbles. One child takes a few marbles in each hand and says: "I have seven (or some other number of) marbles. Guess how many I have in each hand." The child who guesses correctly takes his place.

Spin the Plate. The group decides to base the game on a certain number; e.g., 10. One child in the center spins the plate, at the same time calling out, "John, 4." John must run out to the center, catch the plate before it stops spinning and call out, "4 and 6 are 10."

Upset the Number Basket. Use two sets of number cards, numbered 1 to 9. Form children into circle, each holding his number. One who is "it" will say a combination such as 2 and 3. The two children who have the answer (Number 5) will exchange places, and the one who is "it" will try to get one of the vacated places. The one left in the center will give the next combination such as 4 + 5 = . Then the two nines will change places.

Follow Me. The teacher will say something like this: "Follow me. I go to the store with a half dollar. I buy a loaf of bread for 20¢ and an ice cream cone for 5¢. How much change should I get?"

Fox and Geese. The fox stands in the center of a circle. He calls on a goose by name, and announces a combination of numbers. If the sum or difference is not given correctly, the goose is caught and joins the fox. Then another goose is named, and another combination called. A goose who has been caught may escape to the circle by giving any answer before the goose called upon can give it, in which case the goose called upon is caught.

SOME NUMBER SONGS

Ten Little Indians	The American Singer,
Seven Steps	The American Singer
Five Little Girls	Music Hour II
This Old Man	New Music Horizons

Here We Go 'Round the Mulberry Bush, used as a marching song to the following words:

"Here we go walking two by two, two by two, two by two.
Here we go walking two by two, so early in the morning."

III. BOOKS

It is advisable that each room library contain some books for helping children build number concepts. The following are some inexpensive ones that are suited to primary children:

Title	Publisher
A Penny for Candy	Children's Press, Chicago
Animated Numbers	Garden City Publishing Co., New York
Animals and Little Ones	Hampton Publishing Co., New York
Arithmetic Can Be Fun	Lippincott, Philadelphia
Around the Clock	Grossett and Dunlap, N. Y.
Around the Week	Algonquin Publishing Co., New York
Bobbie Had a Nickel	John Martin's House, Inc. Kenosha, Wisconsin
Chicken Little Count to Ten	Children's Press, Chicago
Children's Picture Cook Book	Wm. R. Scott, Inc., N. Y.
Count in Color	Merrill Co., Columbus
Counting Rhymes	Simon and Schuster, N. Y.
Count to Ten With the Little Red Hen	Artists and Writers Guild, Poughkeepsie, N. Y.
Easy Tricks with Numbers	Pelham, New York
Fascinating Figure Puzzles	Burroughs, New York
Five Chinese Brothers	Coward-McCann, Inc., N. Y.
Finger Play Book	Grossett & Dunlap, N. Y.
Five Little Bears	Rand McNally, Chicago
Four Little Bunnies	" " "
Four Little Kittens	" " "
Fun With Numbers	Row Peterson, Evanston
Gresha and His Clay Pig	Lippincott, Philadelphia

How Far?	David McKay Co., Philadelphia
I Had a Penny	Viking Press, N. Y.
Into the Ark	Franklin Watts, Inc., N. Y.
Junior Bank Book and Picture Puzzle	Platt and Murk Company, New York
Jumbo's Tons	Samuel Gabriel and Sons, New York
Little and Big	Abingdon-Cokesbury Press, New York
Little Wooden Farmer	Macmillan, New York
Millions of Cats	Coward-McCatt, Inc., N. Y.
Money, an Elementary Science Reader	Albert Whitman and Co., Chicago
Mother Goose Tells Time	Citadel Press, N. Y.
Number Rhymes from One to Ten	Merrill Co., Columbus
Number Ten	Children's Press, Chicago
On the Airways	Row Peterson, Evanston
One is the Engine	Wilcox and Follett, Chicago
Puppy Dog Number Book	Saalfield Publishing Co., Akron
Seven Diving Ducks	David McKay Co., Philadel-phia
Tall Enough Tommy	Children's Press, Chicago
Telltime the Rabbit	Thomas Y. Crowell Company, New York
Timothy-Tick-Tock	Ziff Davis Publishing Co., Chicago
The Animals of Farmer Jones	Simon Schuster, N. Y.
The 500 Hats of Bartholomew Cubbins	Vanguard Press, N. Y.

The Numbers Book	Lothrop, Lee, Shepard Co., New York
The Story about Ping	Viking Press, N. Y.
The Taxi that Hurried	Simon & Schuster, N. Y.
The Poky Little Puppy	" "
This Little Piggy and Other Counting Rhymes	" "

IV. FILM-STRIPS

Encyclopedia Britannica Films, Inc., 20 North Wacker Drive, Chicago 6, Illinois. A set of film-strips and teacher's manual have been prepared by John R. Clark, Teachers College, Columbia University, and by Caroline H. Clark, Child Education Foundation. These film-strips deal with number concepts.

Counting to Five
Counting to Ten
Reading Numbers to 10
Writing Numbers to 10
Counting by 10's to 20
Counting by 10's to 50
Counting by 10's to 80
Counting by 10's to 100
Counting from 10 to 15
Counting from 15 to 20
Counting from 20 to 40

Eye Gate House, Inc., 330 West 42nd St., New York 18, N. Y.

"Work and Play With Numbers"

Arithmetical Concepts
We Learn Numbers--Part I
We Learn Numbers--Part II
Time and Money
Addition and Subtraction Concepts
Work and Play with Numbers--5 and 6
" " " " " 7 and 8
" " " " " 8 and 9
" " " " " 9 and 10

Popular Science Publishing Company, Inc., N. Y.

The "Teaching Guide" and film-strip scripts were written by Dr. Foster E. Grossnickle, State Teachers College, Jersey City, New Jersey.

The Two in Division
Zero, Place Holder
What Numbers Mean
Compound Subtraction
A Number Family in Addition
Primary Arithmetic
The Three

CHAPTER V

EVALUATION

Evaluation of a child's progress in the use and understanding of numbers is essential to the arithmetic program. Huggett and Millard explain the word evaluation in this way:

> The meaning of the word "evaluation" is much more comprehensive than "measurement" and "testing." Most standardized tests attempt to appraise amount of information, whereas, the goal of evaluation is to determine such development as social attitudes, character traits, and abilities to use information.[34]

Evaluation must be a continuous process. Only the teacher who consistently knows how far the child has progressed can suit his instruction to the child's maturation level. Evaluation should be in terms of teaching aims and goals. For younger children, observational techniques are just as important as pencil-and-paper tests. Performance tests, too, are very enlightening.

The way a child meets each successive situation provides opportunity for evaluation. Actual situations in which arithmetic is used afford excellent tests for determining what the child's next experience should be. Such tests have the advantage that the child does not know he is being tested and therefore no emotional factor is involved. Then, too, the response is real, not artificial, and the "tests" are worked out by the teacher, who best knows the child.

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[34]Huggett and Millard, _op_. _cit_., p. 281.

The teacher should keep a program record for each child. The child and the teacher together should work out a record in the form of a folder. The child should keep his written work, with a memo as to date, number of correct responses, and his rating. The teacher's record should include cumulative materials: test data, summaries of anecdotal records, reports to parents, summaries of parent conferences, and other pertinent guidance materials. For, only if the teacher studies and teaches the whole child, will that child achieve the greatest growth in the field of arithmetic that his maturity and intelligence permit.

An effective evaluation program requires close co-operation between principal and teacher. Together they must make sure that:

1. The uses of the evaluation instruments are carefully planned.

2. The measurement techniques selected will furnish information which can be used to improve the learning of the children.

3. Tests are given at suitable times and wisely interpreted.

4. An adequate program of follow-up work is carried on.

Teachers should construct many of their own tests. Lee and Lee point out that the results should be used as follows:

1. In determining to what extent the understandings of a given unit of work have been attained.

2. In determining what changes in attitudes, beliefs, and appreciations have taken place.

3. To show pupils where they need to place more emphasis in their work.

4. To evaluate strengths and weaknesses of instruction.

5. To guide in the future selection of work units and types of activities.

6. To discover weaknesses in skills for which standardized tests are not available.[35]

Too often teachers have hoped for certain outcomes but have not actually checked to see whether or not the desired outcomes were being attained. The purpose of evaluation is not to see if the child has failed or passed but to determine whether individuals or groups have acquired certain understandings and made other kinds of growth.

FIRST YEAR

Evaluation should occur at any time throughout the day, as the children engage in activities that involve number. For example, most of the children bouncing the ball may be observed to count correctly. On the other hand, certain children may indicate the need for some assistance in learning to count. The teacher who analyzes difficulties and records the progress of individuals, keeping in mind the goals to be achieved, is able to build an effective program best suited to the needs of all.

Through observation and individual conferences, the teacher should check on such items as the following, in order

[35]Lee and Lee, op. cit., p. 681.

to measure attitudes and habits:

> Do my children like arithmetic?
>
> Do they feel secure in their ability to deal with number situations?
>
> Do they bring to class outside experiences that happened away from school?
>
> Do the children try to help each other?
>
> Are my children careful in their planning?
>
> Do they show evidence of thinking through an arithmetic situation?[36]

Or the teacher may want to evaluate social understanding and social skills. Questions such as these could be used:

> Are my children able to count milk bottles, paper, and other materials usually distributed in the classroom?
>
> Can they find out how many children are present or absent?
>
> Can they check the number of books in our library?
>
> Can they find page 43 in the reading book?
>
> Do they recognize coins? Do they know the value of these coins and relationships among them?
>
> Can they read the calendar? Do they realize the significance of dates?
>
> Do they use a quantitative vocabulary effectively in social situations?[37]

These methods and many others will suggest themselves as the teacher is on the look-out for informal as well as formal ways and means of evaluating the group and the individuals in it.

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[36] Arithmetic (Philadelphia Public Elementary Schools, 1947), p. 40.

[37] Ibid, pp. 40-41.

Suggested Test for Children Entering First Grade

1. How far can you count? Go ahead; count as far as you can. (The number after he stops is his score.)

2. Can you count by tens? Try it.

3. Hand me five blocks. (If correct, ask for 6, 7, 8, 9, 10. There should be at least 20 on the table. If incorrect response is given, go down from 5.)

4. Count these blocks out loud, touching each one as you count. (20)

5. Place a certain number of objects, and ask, "How many are there here?" (Begin with 5, and go up if he succeeds; down if he fails.)

6. Verbal problems:
   You have 2 cookies and mother gave you two more. How many have you now? (etc.) (If the child can give the correct answer, continue. If not, let him again use objects. Credit is given for both the invisible and the visible test.)

Combinations used:			
	2 + 2	1 + 7	2 + 8
	8 + 1	3 + 1	2 + 6
	6 + 1	2 + 4	7 + 3
			4 + 6

SECOND YEAR

Children who developed desirable attitudes and habits in the first year are well on their way toward success. There may be some children, however, who, due to a variety of factors, may have a negative mind set. The teacher in the second year should discover the attitudes and habits they possess. After she knows her children she should attempt, through new approaches and sympathetic understanding, to eliminate undesirable habits and attitudes and build strong foundations.

Observation of children in action, and conversations with children, should disclose answers to such questions as:

> Are my children interested in meeting number situations in other areas?
>
> Do my children like arithmetic? If they do not, why not?
>
> Can my children work independently?
>
> Do my children feel free enough to ask for help when they need it?[38]

Social understandings and skills to be evaluated in any classroom will vary with the experiences the children have had at home and in school. Through observation and conversation, the teacher should be able to answer such questions as these:

> Do my children recognize the relationship between a quart and a pint?
>
> Are they using a ruler as an instrument of measure?
>
> Do they know that short periods of time are measured in hours and long ones in years?
>
> Do my children know that certain things are sold by the pound? By the dozen? By the yard?
>
> Can they keep score in games?
>
> Can my children compare different sizes and shapes and amounts?
>
> Do they recognize the importance of the use of number by man?
>
> Can my children apply number ideas and relationships to social situations?[39]

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[38] *Ibid.*, p. 70.

[39] *Ibid.*

Many other ideas will occur to the teacher as work with the group develops and she concerns herself with the progress of each individual.

There are several types of teacher-made, paper-and-pencil tests that are useful in certain ways. (See Appendix.)

THIRD YEAR

The teachers in the first and second years presumably have been conscious of the need for establishing desirable attitudes and habits in arithmetic. The teacher in the third year should determine which need to be strengthened or extended. Constant watchfulness should be the keynote. That what has been taught will necessarily be permanent should never be taken for granted. The child is growing, and as he grows new factors may affect his development. In some cases, factors relating to home conditions, past achievement, bodily changes, etc. may be so strong that they tend to negate the developing of habits and attitudes that are desirable and useful.

To aid in planning the development of desirable understandings, the teacher will need to evaluate continuously. Through observation and conversation the teacher can readily obtain answers to such questions as:

Do my children know why and how man developed the number system?

Can my children make purchases effectively:

Do they recognize the importance of time in our relationships with people?

Do my children know the difference between A.M. and P.M.?

Do they recognize the importance of the thermometer?[40]

The teacher in the third year should evaluate the child's arithmetic understandings. She should determine where the child is in order to proceed to next steps. For this purpose the teacher may use paper-and-pencil tests to supplement other evaluation methods. (See Appendix.)

The third-year teacher should also check on the computational skill of the children resulting from their experiences during earlier years. It will be her task in this year to aid the children in establishing mastery of the 100 addition and subtraction facts, in addition to some multiplication and division, and addition and subtraction of tens. Paper-and-pencil tests may be used to determine computational skill.

FORMS OF EVALUATION

Personal observation. Daily observation of the pupils at work, coupled with specific questioning, is a most fruitful mode of appraisal and diagnosis of difficulty. But this should not be casual to the extent that it is not planned, and it should be done with a high sense of responsibility. All teachers must realize that help given a pupil is most timely when he realizes a difficulty.

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[40] *Ibid.*, p. 94.

This type of evaluation can be specific and fully as valuable as a written test. Difficulties ranging from such concepts as the size of an inch and the kinds of things measured in pints, to the ability to borrow in subtraction and to know what to do in attacking a problem, are easily apparent to the teacher who is constantly working with pupils.

The wise teacher will also note the opportunities to use arithmetic and the ways in which her pupils do use it in situations outside the regular arithmetic class. Anecdotal records and reports of pupils supply an additional opportunity for appraisal and diagnosis. Remedial measures and self-directed helps should be immediately available to the teacher. Often the best help is what which merely redirects the pupil's thinking by a simple question instead of a full explanation.

Interviews. One of the most effective diagnostic procedures is to have individual interviews with children, in which they "think aloud" as they work an example or solve a problem. By following the steps carefully the teacher is able to discover the exact point at which the child has difficulty or fails to understand a process. Help can then be given on the spot.

Pupil self-appraisal. Pupils who are learning through meaning and understanding are frequently able to discover their own difficulties and to appraise their own work. Modern methods of learning are a powerful aid to self-help. For example, the pupil who has learned addition combinations by counting real objects or by associating such combinations as $8 + 9$ with the double combination of $8 + 8$ has a means of checking himself and arriving at a correct answer. A teacher who has established good rapport with her class will find that the individual pupils are very good at appraising their own strengths and weaknesses.

Oral tests. Oral tests save time and also provide excellent practice in "mental" arithmetic. The procedure usually followed is to have the children number to five on slips of paper. The teacher explains that all computations are to be done without pencil and paper and that the answers are to be placed in order on the slip of paper. She then gives the example or problem to be solved mentally, varying her questions so as to include several aspects of arithmetic. For the third year she might ask,

> About how long is this window sill?
>
> Add $5 + 1 + 3$.
>
> If I had a dime and bought a top for four cents, how much change should I receive?
>
> Write the number that shows three hundreds, no tens, and seven ones.
>
> Sally had 8 cookies. She gave half to Billy. How many cookies did Billy receive?

When they have finished, the children discuss their answers and offer demonstrations of proof. No formal record need be kept of the scores, although the teacher should be alert to individual differences in proficiency.

The teacher's appraisal of her pupils through frequent use of oral or short informal tests variously given is probably a more valid measure of their real understanding and achievement than the results of a longer and more formal written test regardless of how well it may have been "standardized."

Formal testing. Formal testing serves both for appraisal and for diagnosis. But to do this well, the tests must be carefully constructed. This includes attention to selection of content in terms of its importance both in the sequence of arithmetic and for functional usefulness; to range of difficulty of content; to method of presentation of materials; to pupil expectation both in methods of work and in achievement; and to the teacher's mode of marking the test. Certainly, if the test is to be used for a "mark," the content of the test should be a fair reflection of the kind of arithmetic that the school expects. To devote much attention to the development of meanings and understandings and then measure only in terms of computations and "typical" written problems is grossly unfair. Making good comprehensive tests is not easy; this requires a great deal of understanding and appreciation on the part of the teacher.

Standardized tests. Standardized tests--tests for which national or state-wide norms have been established--are useful for comparing local class groups with national standards. In interpreting such tests, however, it must be remembered that this comparison is made on the basis of the content of the specific test used and the sampling of the population on which the norms were based. The validity for a test in terms of any specific class or school is limited to the content of that test and the degree to which it reflects the comprehensive aims of the school.

Any test should be carefully studied before it is chosen. This requires the attention of a competent teacher or supervisor. Content should be educationally important for the pupils for whom the test is intended. Sharing of test results with pupils or parents should be very carefully done, if at all.

Some Standardized Arithmetic Tests

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CHAPTER VI

SUMMARY AND CONCLUDING STATEMENT

The program for the teaching of primary arithmetic presented above is based on sound principles of child growth and of the way arithmetical understandings and skills are acquired. Expectations, learning experiences, and methods of teaching and testing are suggested in sufficient detail, it is hoped, so as to be usable by both experienced and inexperienced teachers, with children of varying backgrounds. The discussion for all three years is organized under each content and procedure heading in order to encourage all primary teachers to get the total primary program in mind.

Characteristics of this program may be briefly outlined as follows:

(1) It is based on logical, social, and psychological criteria.

(2) It is systematic but not formal.

(3) A minimum program of "normal" achievement is indicated for each year.

(4) Primary arithmetic is broadly conceived to include number understandings, vocabulary, addition, subtraction, multiplication, division, fractions, money values, measurements, and problem solving.

(5) The program takes into account the level of maturity which children have reached, their background of ex-

periences, and their interests.

(6) The developmental pattern of the child is used as a guide for the introduction of instructional material.

(7) The shift from concrete to abstract use of number is made very gradually.

(8) Children's number needs and experiences--both in and out of school--are recognized and utilized.

(9) Continuous growth is provided for to the extent that each number experience starts where the child is.

(10) Fundamental processes are not introduced or practiced until a broad and thorough understanding of number has been established.

(11) Larger and fuller meanings are developed in terms of meanings and understandings which the children have.

(12) Problems are significant and challenging.

(13) Diagnosis of needs and evaluation of progress is continuous.

(14) Children need never experience a sense of failure.

(15) Pressure is not used to drive pupils toward unnaturally high standards.

(16) The teacher does not become alarmed because certain pupils progress at slower rates than others.

(17) Film-strips and other learning aids are to be utilized when they stimulate the pupil's interest or increase his participation in the correcting, proving, or improving of arithmetic understandings.

(18) It is assumed that the teacher of arithmetic must have an understanding of child development, a genuine love for children, an understanding of mathematics, and a genuine liking for the subject.

(19) Although much individualized attention is expected, this program may be used with large classes by dividing the children into sub-groups at various times for specific purposes.

CONCLUDING STATEMENT

The logical outcome of such a program in primary arithmetic should be happier and more successful experience with number for children during their primary years plus a better foundation for later work than is usually acquired.

It is recommended that the Billings Public School system modify its present program in primary arithmetic by substituting the curriculum and procedures suggested here.

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APPENDIX

TEACHER-MADE EVALUATION FOR END OF SECOND YEAR

Write the number that comes after the numbers in each group:

6	7	8	______
11	12	13	______
18	19	20	______
62	63	64	______
17	18	19	______
44	45	46	______
37	38	39	______
20	21	22	______
97	98	99	______
57	58	59	______

****

Write the number that belongs in each space:

Before			Between		
______	11	12	9	______	11
______	21	22	27	______	29
______	96	97	68	______	70
______	45	46	80	______	82
______	35	36	52	______	54
______	30	31	98	______	100
______	73	74	11	______	12
______	80	81	49	______	50

Write Yes or No on the line after each sentence.

1. 5 is more than 6. ________
2. 11 comes after 10. ________
3. 5 pennies are more than a nickel. ________
4. 20 is larger than 12. ________
5. 80 is less than 60. ________
6. A dime is the same as 10 pennies. ________
7. 2 is half of 4. ________
8. Six cookies and two more are 8 cookies. ________
9. A dime will buy more than a nickel. ________
10. We measure with a ruler. ________

****

Match the words with the numbers.

four	9
six	4
two	3
five	6
one	7
eight	7
nine	5
seven	2
three	10
ten	1

Add:

4 +1	2 +7	3 +6	4 +4	2 +3	8 +1
4 +2	7 +3	6 +2	3 +3	5 +2	6 +3
1 +7	5 +4	1 +2	2 +8	3 +5	6 +1
1 +4	1 +9	5 +5	2 +1	3 +4	1 +1
3 +7	2 +4	9 +1	3 +2	5 +1	4 +6
7 +2	3 +1	2 +6	1 +8	4 +3	7 +1
6 +4	2 +5	1 +3	4 +5	8 +2	1 +6
5 +3	2 +2	1 +5			

Subtract:

8 -1	10 -4	7 -5	4 -3	9 -5	10 -2
7 -6	4 -1	6 -2	10 -1	5 -2	6 -1
9 -9	10 -6	8 -3	9 -8	7 -3	8 -6
10 -9	8 -5	7 -1	10 -5	3 -1	7 -4
2 -1	10 -7	4 -2	8 -2	6 -3	7 -2
9 -3	8 -7	9 -4	3 -2	10 -8	5 -4
9 -7	6 -5	5 -1	9 -6	8 -4	5 -3
9 -1	6 -4	10 -3			

TEACHER-MADE EVALUATION FOR THIRD YEAR
END OF FIRST SIX WEEKS

Write Yes or No on the line after each sentence.

1. A yardstick is longer than a ruler. ______
2. A quarter will buy more than a dime. ______
3. I would use the scales to see how hot the room is. ______
4. You say 2 4 6 8 when you count by twos. ______
5. A week is longer than a day. ______
6. 5 10 15 20 is the way you count by 10's. ______
7. We use scales to tell us how much we weigh. ______
8. 13 means one ten and three ones. ______
9. A dime is larger than a quarter. ______
10. Ten pennies will buy more than a dime. ______

****

Draw a line from the word in the first row to the number in the second row.

twelve	16	three	14
nine	20	fourteen	8
twenty	9	eight	7
ten	10	eighteen	3
sixteen	12	seven	18

Put a ring around the answer you think is right.

1. 80 is larger than 92 78 83 89.
2. A nickel is more than a dime a penny a dollar a quarter.
3. 70 comes after 67 69 65 68 in counting.
4. We measure the time of day in months years hours weeks.
5. A dime will buy as much as five pennies ten pennies eight pennies six pennies.
6. There are 5 7 6 4 days in a week.
7. Eight boys and four boys are 10 11 12 13 boys.
8. There are 9 10 8 7 pennies in a dime.
9. We measure heat with a thermometer scale stove book.
10. 9 take away 3 is 5 6 7 8.

****

Write the correct number in the space:

Before	After	Between
_____10	79_____	76_______78
_____97	20_____	29_______31
_____24	66_____	34_______36
_____32	19_____	58_______60
_____51	72_____	62_______64
_____69	39_____	41_______43
_____71	99_____	9_______11

Add or subtract

8 +1	2 +9	5 +2	5 +0	11 -3	6 -3
0 +7	5 +3	9 -7	10 -4	9 -4	2 -1
4 +4	6 +5	1 +8	5 -2	6 -2	5 +6
11 -2	10 -7	9 -8	7 +3	5 +4	9 -2

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1 +7	4 +8	9 -5	4 +5	12 -6	2 +1
8 +4	0 +5	6 -6	8 -1	7 -5	10 -2
6 +4	10 -8	12 -4	9 +3	5 -5	2 +5
10 -5	3 +9	3 -2	7 -0	9 -3	4 +6

Add or subtract

3 +2	4 +2	12 -8	3 +5	8 -5	7 -4
1 +4	7 +2	2 +4	4 -2	5 -3	11 -6
5 +1	6 +0	5 -0	8 -4	7 -2	8 +2
7 -1	6 -4	5 +5	6 +3	2 +2	10 -6

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9 +1	2 +0	11 -9	3 +3	10 -9	8 -3
12 -9	2 +3	2 +7	9 -1	6 +6	2 +8
1 +5	3 +6	7 -3	2 -2	5 -1	4 +3
6 -1	9 +2	12 -3	3 +4	3 +8	1 +1

TEACHER-MADE EVALUATION FOR THIRD YEAR
END OF SECOND SIX WEEKS

Write Yes or No on the line after each sentence.

1. 25 means two tens and five ones. ______
2. I would use a ruler to measure the school grounds. ______
3. We add when we want to find out how many are left. ______
4. Both together means to add the numbers. ______
5. Six is the same as a half dozen. ______
6. I could divide six cookies evenly between three children. ______
7. We subtract when we want to find out how many there are altogether. ______
8. We buy eggs by the pound. ______
9. Thirty minutes is one-half of an hour. ______
10. Altogether means we add the numbers in the problem. ______

****

Draw a line from the thing in the first row to how it is measured in the second row.

milk	hours
cheese	quart
dress goods	yard
time	pound
eggs	yard

Put the right number in each blank space:

296 means ________ hundreds, ________ tens, ________ ones.

864 means ________ hundreds, ________ tens, ________ ones.

96 means ________ hundreds, ________ tens, ________ ones.

536 means ________ hundreds, ________ tens, ________ ones.

8 means ________ hundreds, ________ tens, ________ ones.

****

Put a ring around the answer you think is right.

1. A ruler is 10 8 9 12 inches long.
2. A half dozen eggs is 4 6 7 5 .
3. Four cookies and three cookies are 7 5 6 4 cookies.
4. If I had 8 pennies and lost 3 , I would have 7 4 5 3.
5. If I wanted to know how much I weighed, I would use a ruler scale thermometer book.

****

Write the correct number in the space:

Before			After			Between		
___	11	12	98	99	___	89	______	91
___	91	92	64	65	___	72	______	74
___	76	77	43	44	___	69	______	71
___	48	49	58	59	___	20	______	22
___	20	21	26	27	___	49	______	51
___	55	56	40	41	___	81	______	83
___	32	33	79	80	___	32	______	34
___	61	62	68	69	___	78	______	80

Add or subtract

83 +24	114 -92	15 +93	106 -52	118 -25	94 +34
25 +94	108 -63	43 +95	115 -54	129 -65	75 +41
107 -44	67 +42	127 -93	82 +36	109 -24	128 -54
52 +52	137 -85	41 +67	53 +84	72 +53	104 -91
136 -54	16 +93	68 +50	119 -63	114 -40	52 +65
51 +71	123 -72	43 +83	125 -85	138 -92	91 +46
136 -46	70 +62	26 +81	119 -78	136 -63	133 -70
84 +43	30 +39	107 -82	118 -85	82 +51	118 -36
23 +80	61 +50	128 -41	61 +73	64 +61	134 -71

TEACHER-MADE EVALUATION FOR THIRD YEAR
END OF THIRD SIX WEEKS

Write <u>Yes</u> or <u>No</u> on the line after each sentence.

1. Several means more than two. ________
2. We use a thermometer to see how much we weigh. ________
3. If I divided nine cookies between three children each would have three cookies. ________
4. You would multiply numbers that are the same to get the answer more quickly. ________
5. If an apple was divided evenly between two children, each would have 1/3. ________
6. If you had 24 cookies, 3 children, you would divide to see how many each would get. ________
7. There are 12 inches in a foot. ________
8. 267 means two hundreds, six tens, and two ones. ________
9. A zero is used in writing numbers when you want to show there is nothing in one of the number columns. ________
10. A gallon is smaller than a quart. ________

****

Circle the answer that you think is nearest right.

1. Six is 1/3 1/4 1/5 1/2 of a dozen.
2. A quarter is 10 50 25 30 cents.
3. A dime will buy as much as a penny a nickel two pennies two nickels.

Draw a line from the things in the first row to how each is measured in the second row.

eggs	yard
a trip	dozen
cream	mile
dress goods	pint

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gasoline	quart
a day	gallon
milk	pound
cheese	hours

****

Write the correct number in each blank:

Before	After	Between
______ 111	318 ______	439 ______ 441
______ 672	209 ______	204 ______ 206
______ 990	107 ______	937 ______ 939
______ 836	199 ______	320 ______ 322
______ 421	111 ______	581 ______ 583
______ 500	863 ______	110 ______ 112
______ 735	749 ______	649 ______ 651
______ 220	530 ______	989 ______ 991
______ 329	409 ______	428 ______ 430
______ 468	899 ______	860 ______ 862

Do what the sign tells you to do.

53 23 +43	14 50 +63	20 47 +51	35 4 +10	82 4 +21	5 40 + 3
834 -213	615 +233	747 -443	572 +425	943 -330	426 +463
33 +47	65 +28	30 -17	42 -16	47 +37	91 -44
463 -227	571 -347	654 -337	832 -417	129 - 54	138 - 55
8 3 +4	3 7 +8	6 2 +5	9 0 +4	8 3 +5	7 4 +3
5 2 +5	9 3 +3	1 7 +5	2 6 +4	5 1 +6	9 3 +7
143 - 73	114 - 82	135 - 92		17 x 2	49 x 2
23 x 2	41 x 2	22 x 2	30 x 2	26 x 2	35 x 2
2) 14	2) 10	2) 16	2) 8	2) 2	2) 12