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An investigation of the factors affecting middle school mathematics students' ability to solve unfamiliar problems involving familiar mathematics concepts

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An Investigation of the Factors Affecting Middle School Mathematics Students' Ability to Solve Unfamiliar Problems Involving Familiar Mathematics Concepts

by

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Abstract

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An Investigation of the Factors Affecting Middle School Mathematics Students' Ability to Solve Unfamiliar Problems Involving Familiar Mathematics Concepts

Director: Libby Krussel LK

Once a student understands a particular concept in the classroom, how does this understanding affect the solving of similar problems? What reasons prevent the student from solving similar problems? How can students who feel that they understand the concepts involved solve similar problems more successfully? And how can teachers help students to be more able to solve similar problems?

There are several factors examined in this study, such as teacher's ability, teacher's attitudes, teacher's beliefs, teaching styles, relationship between teacher and students, students' attitudes, students' motivation, the way students understand, and the way students learn.

This study found a correlation between students' ability to explain an understood concept, and their ability to solve problems involving similar concepts. However, if students felt that their teachers used a teacher-centered approach well, or took too much responsibility in teaching, then those students tended to rely too much on their teachers. Also, if students relied too much on their teacher, then those students tended to be less creative. Moreover, teacher ability is an indirect factor in both helping students understand concepts more deeply and preventing students from thinking by themselves. However, the influence - helping or inhibiting – that teacher will have depends on how students feel about their teaching styles and how much responsibility the teachers take.

Acknowledgments

First of all, I would like to thank my parents and my school in Taiwan for supporting me to come here.

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Chapter 1

Introduction

I. Motivation for study

During my three years of teaching, a problem kept occurring which probably happens all the time for teachers – many students felt that they could understand the principles taught and the accompanying problems, but they couldn't solve similar problems. Wagner (1981) also found this problem when she asked ninth graders to solve a linear equation. In her study, she simply changed the name of the variable and asked those who had solved it correctly the first time to solve the new problem. With their previous work in front of them, the majority of students re-solved the equation from the start, re-doing all of the calculations they had done just minutes before. Wagner concluded that the students did not have the Piagetian notion of conservation, since they looked at each problem presented to them as a separate entity without any reference to what they had done previously. The motivation for this research was an interest in what students' understanding actually *represented* to them, and how it related to the task of the problem. An additional issue was to understand what factors were involved in new concept learning and practical problem solving.

II. Categories to be considered in this paper which affect a student's achievement

Since the interaction in a class is primarily between students and teacher, this study divides the learning that occurs in the class into two main factors, or aspects, which impact a student's achievement: student learning and a teacher's pedagogical methods.

In considering factors affecting students' learning it is important to note that the students' learning strategies directly affect the learning process. The student him/herself is the main person who decides what kind of, and how much, information he/she wants to process. From this viewpoint we can surmise that a student's motivation for learning mathematics, and attitude about learning mathematics might be factors affecting a student's achievement. The student's motivation and attitude can affect the extent to which a student invests him/her self into mathematics. How a student learns a concept in mathematics, either by memorizing, or with deeper understanding, will affect how the student understands a concept and also how the student connects the concept with problem solving. If a student learns mathematics through a deeper understanding of the concept, then the student may be better able to connect to some other already processed mathematics. Therefore, the way that a student learns mathematics could affect how the student integrates concepts. In addition, if there are two students with different math abilities but having the same teacher, the information they grasp about a concept and the depth of understanding of the concept they get may be different. Since two students with different mathematics abilities may understand concepts differently, it would seem that the ability of a student in mathematics could also be a factor in how a student learns the concepts of mathematics. Anxiety about mathematics could also be considered a factor in preventing a student from thinking or solving a mathematics problem efficiently. Even if a student understands a mathematics concept very well, he/she may not be a successful problem solver, because of this anxiety.

In the author's teaching experience, there are some factors, things that teachers do or do not do, that also could affect students' ability to learn concepts. First among all of these factors is the mathematics ability of the teacher. How much of a concept a teacher can convey to students depends on how much mathematical knowledge the teacher has, how deep the cognitive knowledge of the teacher is, and how clearly the teacher explains the concept. The beliefs of the teacher is a second factor that influences a student to learn mathematics. A person's behavior and attitude are affected by that person's beliefs. If two teachers have the same ability but different beliefs, then the two teachers may utilize different teaching styles or attitudes, or emphasize different points of a particular concept. Therefore, students may focus on different aspects because their teachers have different beliefs. A third factor is a teacher's teaching style and attitude toward teaching, since these factors could directly or indirectly affect a student's emotions toward learning mathematics, too. That could in turn influence a student's motivation, so that teaching style and attitude could also affect a student's achievement in mathematics. One more factor is the relationship between a teacher and a student, since some students may study harder because they like the teacher.

A pilot project was first carried out in Taiwan. In that research the author collected information about how a student learns and understands new concepts and how student problem solving was affected. From interviewing some students about how they solved problems, the author realized these aforementioned factors could also affect a student's academic progress.

Although there are many factors which could affect students' overall learning of concepts, there are only certain factors in which a teacher could actually make changes or encourage his/her students, so this paper will only consider those possible factors. Those categories of factors considered in this study are: ability of the teacher, attitude of the teacher, teaching style, beliefs of the teacher, relationship of teacher and student, attitude of the student, motivation of the student, the ways students understand a new concept and the ways students learn a new concept. The following figure was designed for this study to show the relationships between the factors considered. (See *Figure 1*.)

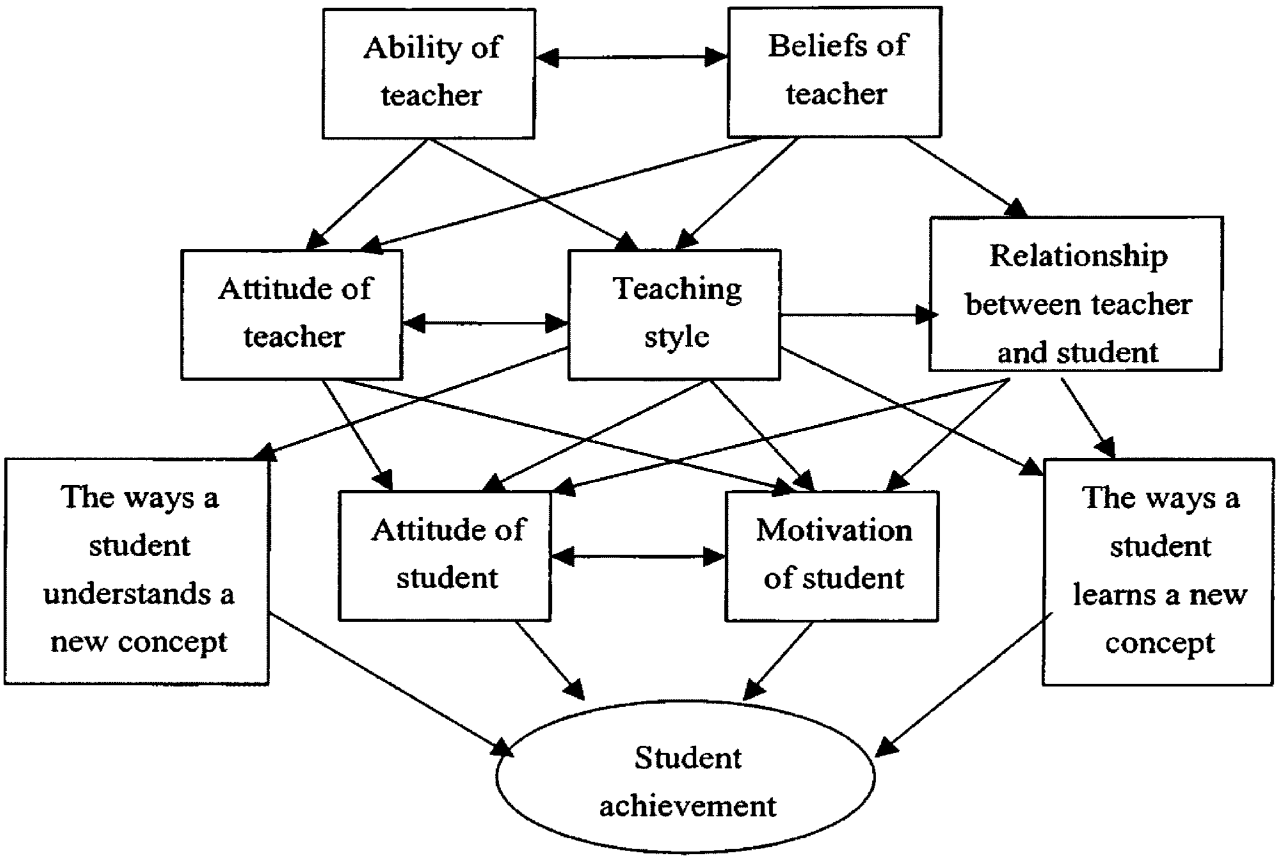

Figure 1. Factors involving students and teachers

III. The purpose and questions of the study

The purpose of this study was to discover why students are able to understand a new concept yet they cannot solve a related problem using that new concept. The pertinent questions, based on the review of the literature and the findings of the pilot study follow:

1. Would teacher ability be a factor that prevented students from solving a different kind of problem involving the concept which they felt that they understood, and if it was a factor, then to what extent would it be?

2. Would teacher attitude be a factor that prevented students from solving a different kind of problem involving the concept which they felt that they understood, and if it was a factor, then to what extent would it be?

3. Would teaching style be a factor that prevented students from solving a different kind of problem involving the concept which they felt that they understood, and if it was a factor, then to what extent would it be?

4. Would teacher beliefs be a factor that prevented students from solving a different kind of problem involving the concept which they felt that they understood, and if it was a factor, then to what extent would it be?

5. Would the relationship between teacher and students be a factor that prevented students from solving a different kind of problem involving the concept which they felt that they understood, and if was is a factor, then to what extent would it be?

6. Would student attitude be a factor that prevented students from solving a different kind of problem involving the concept which they felt that they understood, and if it was a factor, then to what extent would it be?

7. Would <u>student motivation</u> be a factor that prevented students from solving a different kind of problem involving the concept which they felt that they understood, and if it was a factor, then to what extent would it be?
8. Would <u>the ways a student understand a new concept</u> be a factor that prevented students from solving a different kind of problem involving the concept which they felt that they understood, and if it was a factor, then to what extent would it be?
9. Would <u>the ways a student learning a new concept</u> be a factor that prevented students from solving a different kind of problem involving the concept which they felt that they understood, and if it was a factor, then to what extent would it be?

IV. Limitations of the study

This study only considered some of the many factors that could affect students' achievement. There are many other factors that could affect students' achievement, such as gender difference, socioeconomic status, home background or parental expectations for academic performance. The results in this study only considered those factors listed in section II above.

Other limitations in this study concerned the Evaluation Sheet and Problem Sheet. This study focused on the students' point of view, so the Evaluation Sheet was only answered by students. Therefore, all factors which were considered in this study were determined by the students' point of view. Since the study was to discover what factors might affect students' problem solving ability after they felt that they understood a new concept, rather than to discover how deeply students understood the concept, the problems on the Problem Sheet were not so difficult that they challenged their understanding of the concept.

Chapter 2

Literature review

Student attitude

Reynolds & Walberg (1992) showed that students' previous attitudes had the most powerful influence on subsequent attitudes, although there are some other direct and indirect effects, such as motivation and home environment. Rech & Stevens (1996) also agree that students' attitude can be a predictor of mathematics achievement of students. They showed there was a positive correlation between a student's attitude and mathematics achievement, "Attitude was identified as a predictor of mathematics achievement, and educators should be aware of the important role that attitude plays and the need to take action to improve negative attitudes" (p. 348). The work of Bestgen, Reys, Rybolt, & Wyatt (1980) showed that a student with a better attitude will have a greater probability of higher achievement. The National Council of Teachers of Mathematics (1989, 1991) proposed that a student's attitude toward mathematics is one of the critical components affecting achievement in mathematics.

Student motivation

Welberg (1981, 1986) attempted to specify the nine chief productive factors in school learning suggested by several research studies. Among them is motivation. Middleton & Spanias (1999) also stated "Motivations help guide children's activity; they provide a structure for evaluating the outcomes of activity; and they help determine whether or not children will engage in future mathematical activity" (p. 67). Wolleat, Pedro, Becker, & Fennema (1980), Stipek (1998), and Sideridis & Padeliadu (2001) found a similar result, that students with higher motivation in units are more focused on the units, and reach higher achievement in math and other subjects. Singh, Granville, & Dika (2002) believed that motivation and students' attitude toward mathematics are related to academic success, hence their research was to examine the effects of motivation, attitude, and academic time on academic achievement in mathematics and science. Their research showed that students' mathematics attitude and motivation directly affected students' achievement in mathematics and science. Schiefele (1996) investigated the role of interest in learning with texts. He proposed that academic interests are a significant predictor of academic achievement in school. His result showed that the more students are interested in subjects, the longer the students study the texts, then the better their achievement in those subjects.

Student leaning and understanding

The pilot study research in Taiwan revealed information about how students learn and understand a new concept and how that affected the ways in which students solve problems. It was noted that the preceding factors also could affect students' achievement. Mapolelo (1999) suggested that mathematics teaching is to engage children in constructing concepts in such a way that the students can really understand mathematics, since many students learn to do mathematics problems without thinking about the meaning of the problem. Moreover, Bransford, Brown, & Cocking (1999) maintained that students of all ages have a large knowledge base on which to build, including ideas developed in prior school learning and those acquired through everyday experience. Also, Schoenfeld (1988) found that if students could connect new knowledge to prior knowledge in a meaningful way, then they could more easily remember and apply the knowledge. Moreover, the learning principles of the National Council of Teacher of Mathematics (2000) suggests that a major goal of school mathematics is for students to learn mathematics with understanding.

Teacher ability

McDiarmid, Ball, & Anderson (1989) worked on the relationship between how the teachers themselves understand the subject, and how they give instructions to their students:

> Recent research highlights the critical influence of teachers' subject matter understanding on their pedagogical orientations and decisions... Teachers' capacity to pose questions, evaluate their pupils' understanding and make curricular choices all depend on how they themselves understand the subject matter. (pp. 195-196).

Smith & Cotton (1980) reported about the effect of lesson vagueness and discontinuity on students, showing that lesson discontinuity and teacher vagueness[1] both affect students' achievement. Also, Parker (1984) stated that when teachers improved their interactive decision making (i.e. the decisions that teachers make during their instruction), then the teachers more appropriately selected and rejected instructional alternatives which better supported students' achievement. The National Council of Teachers of Mathematics (2000) maintains that effective teaching is based on a teacher's deep understanding of the mathematics they are teaching and their ability to draw on that knowledge with flexibility in their teaching tasks.

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1. Lesson discontinuity and teacher vagueness are considered within the category of teacher ability.

Teacher attitude

There is much research indicating that teacher attitudes and behaviors have a great deal of influence on student attitudes. Quilter and Harper (1988) interviewed a group of students who had negative attitudes toward mathematics and they found that teachers' attitudes were the most pertinent variable affecting learning. When they interviewed those students who had negative attitudes toward mathematics about why they disliked mathematics, the answer was that such attitudes frequently resulted from interaction with teacher's negative attitudes, such as arrogance and assuming background knowledge or "belittling" a lack of ability. Moreover, the National Council of Teachers of Mathematics (1989, 1991) suggested that teachers should develop and maintain positive attitudes and high expectations for all students, including low achievers, in mathematics. Nisbet (1991) maintained that teachers need to have a positive attitude toward mathematics in order to teach mathematics well. Sherman (1999) stated in the conclusion of her research that teachers with positive attitudes could have a powerful influence on improving students' attitudes.

Teaching style

Among the many different types of teaching methods are traditional teaching, anchored instruction, and teaching by using cooperative learning or constructivist learning techniques with their students. Research on teaching styles by Stipek (1998) showed that students in different types of teaching environments exhibited differences in performance, motivation, and behavior. Shyu (2000) provided evidence that different teaching styles could change students' attitudes toward mathematics. Her study showed that students receiving anchored instruction, a technology-based program designed to motivate students and help them learn to think and reason about complex problems in mathematics learning, enjoyed mathematics more and felt that mathematics was more valuable. Thus, the students felt more positive about, interested in, and less anxious toward, mathematics. Vaughan (2002) showed that cooperative learning had a positive effect on attitude and academic achievement levels for students of color. Bauch (1984) maintained that teachers with different beliefs would present different classroom behaviors. Bauch investigated the different characteristics of instructional beliefs and classroom behaviors between teachers as "controllers" and "relators" by using a "Teacher Beliefs Inventory". Controllers were defined as those who got higher scores in the teacher control construct (such as the teacher is in charge of classroom activities, keeping order in most classrooms, keeping students busy in learning, and providing good discipline in the classroom). Relators were those who got higher scores in the student participation construct (such as students are allowed to participate in the choice of activities, students can gauge their own progress, and students feel free to move around the room while class is in session). She frequently found that the controllers placed more emphasis on listening and writing reports, and that they considered the basic skills to be the most important goal of schooling. Moreover, controllers were more influenced by curriculum guides in planning for teaching, and therefore students who were in the controllers' classes felt that they had less freedom to question, to think, or to choose their behaviors in the class. Relator teachers were more likely to use open-ended questioning in addressing students than were their counterparts, and they also used more student-directed activities, student projects, and classroom discussions than did controller teachers.

Teacher beliefs

There is a considerable body of research in the area of teacher beliefs. Fischbein and Ajzen (1975) and Rokeach (1968) stated that all beliefs are predispositions to action, and beliefs underlie attitude formation. As we saw from Shyu and Bauch earlier, teaching style can affect students' learning. However, both Thompson (1984) and Cohen (1990) believe that teachers' beliefs can affect teaching practice. Thompson said: "the observed consistency between the teachers' professed conceptions of mathematics and the manner in which they typically presented the content strongly suggests that the teachers' views, beliefs, and preferences about mathematics do influence their instructional practice" (p. 125). Researching the factors contributing to a teacher's teaching style, Borko and Niles (1982) pointed out that the approaches a teacher used to facilitate student learning – content-centered or student-centered – were directly affected by the teacher's beliefs about student learning. In their study, they found a significant difference between experienced teachers and student teachers. Over a period of time, the experienced teachers placed more confidence in their curricula and had stronger content-centered beliefs. The student teachers had stronger student-oriented beliefs because of their college course work. The research showed that these student-oriented beliefs were not easily changed.

Relationship between teacher and student

Personal experience shows that a good relationship between mathematics teacher and students will improve students' motivation to learn mathematics. Quilter & Harper (1988) found that those students who have a negative attitude towards mathematics exhibit more dislike of mathematics teachers than those students who have a positive attitude in mathematics. Walker & McCoy (1997) also mentioned that most successful students in their study felt welcomed in their mathematics class because they had a strong relationship with their teacher.

As we have seen above, if a student has a better attitude, then that student has a greater probability of higher achievement. Also, motivation can help students do better in their activities. A good learning method, such as learning to connect concepts, is another factor affecting students reaching higher achievement. When we consider the influence of teachers on students, teachers' attitude and relationship with students can have an impact on the students' attitudes toward learning. Moreover, teachers' different beliefs and/or different abilities will result in different types of instruction which will in turn affect students' learning.

Many of the factors considered in this study are defined in different ways in different research studies. In fact, when reviewing the literature there was not always unanimity in defining terms, even in parallel studies, so the following definitions are those that will be used in this study.

Chapter 3

Definitions

This chapter contains the definition of each term that was considered to be a factor, followed by the four questions that were used to measure that term by asking students. Each question in the last two terms will be considered as a separate factor.

Definitions of terms and questions:

Ability of teacher

The ability of a teacher to correct students' misunderstandings and to guide a student towards accepting new concepts.

1. I felt the teacher explained the new concept clearly in this unit.
2. I felt the teacher knew how to guide us to learn this unit effectively.
3. I felt the teacher understood students' problems in learning this unit.
4. I felt the teacher presented the material in a confident and organized manner.

Attitude of teacher

The type of attitude conveyed by the teacher toward the students and the material presented.

1. I felt the teacher enjoyed teaching us this unit.

2. I felt the teacher cared about what students learned in this unit.

3. I felt the teacher liked the students in this class.

4. I felt the teacher worked hard to achieve the unit's learning objectives.

Teaching style

The way in which the teacher presents material – in a teacher-centered way or in a student-centered way.

1. The teacher usually wanted us to solve problems the way that he/she taught us.

2. We worked in groups in this unit.

3. We usually got an answer through discussion with each other.

4. The teacher gave us enough time to finish the problems on our own.

Beliefs of teacher

The extent to which a teacher's beliefs affect a student's perceptions of mathematics learning and relevance.

1. If I did a good job in this unit, the teacher was pleased.

2. I felt the teacher was more interested in the process than the answer.

3. I felt the teacher gave us the confidence we needed to solve the problems in this unit by ourselves.

4. I felt the teacher helped me relate the lesson to real-life situations.

Relationship of teacher and student

The extent of emotional distance between a teacher and a student.

1. I felt the teacher was like one of my friends.
2. I felt free to ask the teacher questions at any time.
3. I liked to talk with the teacher after class.
4. I felt that the teacher empathized with my problems in the class.

Attitude of student

The way the student feels about the material presented and class participation.

1. I paid attention throughout this unit.
2. I enjoyed participating in this unit.
3. I would like to learn more about this unit.
4. I worked hard out of class in this unit.

Motivation of student

The extent to which a student gets involved with a new concept.

1. I felt this unit was interesting to me.

2. I enjoy mathematics.

3. I feel I would use the concepts in this unit in my daily life.

4. I feel it is important for me to get high grades in class.

Understanding of student about a concept

Hiebert & Lefevre (1986) define the terms "procedural" and "conceptual" knowledge. The next four terms are refinements of the preceding terms: Following, transference, explanation, and connection. Each is explored by one of the following items.

1. I could follow how the teacher solved problems in this unit.

2. I could apply a problem-solving method to a similar problem.

3. I could explain the concept taught in this unit to a fellow student.

4. I will be able to apply and broaden the concepts that I learned in this unit to different problems.

Learning of student about a concept

Skemp (1987) associated two structures with the learning of a concept. One is called surface structure, which is characterized by the use of a symbol system. The other one is called deep structure, which is characterized by the use and understanding of underlying mathematical ideas. There are four dimensions that were measured for student learning using Skemp's definition: memorization, relationships, connecting a solution process with a problem type, and connecting different processes.

1. I can do these kinds of problems by memorizing what my teacher said.
2. I tried to find patterns among similar problems.
3. I usually know the reasons why the teacher used a specific way to solve a problem.
4. I looked for alternate ways of solving problems or connecting concepts.

Chapter 4
Methods and Measurement

This chapter describes the methods of research and measurement used in this study.

I. Methods

Subjects

This study was conducted in four public middle school classrooms in Montana, one in Missoula County and three in Ravalli County, and involved 70 students, (25 students were from S1 school and 65 students were from S2 school) enrolled in seventh-grade or eighth-grade mathematics classes, and their teachers. Both teachers had taught for several years. One had about three years experience in teaching of factoring polynomials, which is the unit in this study, and the other teacher had not taught this unit before.

Procedure

The researcher first discussed the concept of factoring polynomials with the teachers to make sure that factoring was a new concept for the students, and to review different ways of presenting this concept to the students. The teachers then taught their students this new concept and solved some problems involving the concept. When the teachers felt their students understood this new math concept, the students were given an Evaluation Sheet (see Appendix A). After the students filled out the

Evaluation Sheet, they were given a Problem Sheet (see Appendix B) and they were asked to solve some problems involving the new concept. After all the students had solved the problems, a sample of students, from those who felt they understood the new concept but could not solve the Problem Sheet correctly, was interviewed to ascertain how they attempted to solve those problems.

Valid Data

On the Evaluation Sheet, question 25 and question 29 were similar questions but were asked in different ways, so if the answers to these two questions were too different, (for example, both the answers are "agree") then the survey was not counted in this study. There were 64 valid data out of a total of 70. Since the object was to study students who felt that they understood the unit on factoring polynomials, this study considered only those students who agreed with the question "I understood what the teacher taught in this unit" which was 49 out of a total of 64 students.

Scoring the Evaluation Sheet

In this study, there were six response categories on the Likert-type scale used on the Evaluation Sheet. They ranged from "strongly agree" to "strongly disagree" without a neutral category option. There was a total of 37 statements for students to respond to. All the answers but # 9 on the Evaluation Sheet were scored in this way: Strongly Disagree-1 point, Disagree-2 points, Partly Disagree-3 points, Partly Agree-4 points, Agree-5 points, and Strongly Agree-6 points. Since question # 9 was asked in the opposite way from the other questions in this category, the scoring was from 6 to 1 instead of the regular scoring (1 to 6). There was a total of nine categories evaluated in this study, each containing four questions. The categories were: ability of teacher, attitude of teacher, teaching style, beliefs of teacher, relationship between teacher and student, attitude of student, motivation of student, understanding of student about a concept, and learning of student about a concept. Because the four questions in each of the first seven categories were closely related, a composite score was computed for each category. This represented the average of the scores in the four questions related to that category. However, each question in the last two categories represented different ways of understanding and learning, so each of the four questions in the last two categories were scored as separate factors. The questions in category eight are designated as U1 through U4 (#30 to #33 in the Evaluation Sheet), and those in category nine as L1 through L4 (# 34 to #37 in the Evaluation Sheet). This gave a total of fifteen factors.

Scoring the Problem Sheet

Whenever students could not solve a problem completely, it was assumed that something was wrong or unclear. Hence, in this study, the Problem Sheet was scored in this way: the outcome of a question is 0 if the answer to the question is either right or the solving procedure was correct but there was a calculation error; otherwise the outcome of the question is 1. In other words, a student outcome on a particular question of 0 means the student comprehends the question; a student outcome on a question of 1 means the student does not adequately comprehend the question.

II. Measurement Model

Logistic Regression

Hosmer & Lemeshow (2000) maintained:

> Regression methods have become an integral component of any data analysis concerned with describing the relationship between a response variable and one or more explanatory variables. It is often the case that the outcome variable is discrete, taking on two or more possible values... What distinguishes a logistic regression model from a linear regression model is that the outcome variable in logistic regression is binary or dichotomous. This difference between logistic and linear regression is reflected both in the choice of a parametric model and in

> the assumptions made. Once this difference is accounted for, the methods employed in an analysis using logistic regression follow the same general principles used in linear regression. Thus, the techniques used in linear regression analysis will motivate the approach to logistic regression. (p. 1).

Since the outcome of a question is either 0 or 1, and each student is an independent entity, we can use this model in this study.

Clinical Interviews

The clinical interview comprised open-ended interviews and think-aloud problem solving protocols. Clement (2000) noted that these techniques have played key roles in seminal studies in science and mathematics education. They also said:

> People have many interesting knowledge structures and reasoning processes that are not the same as academic ones – they have alternative conceptions and use non-formal reasoning and learning processes. Mapping this "hidden world" of indigenous thinking is crucial for the success of instructional design… In some exploratory varieties of clinical interviewing, the investigator can also react responsively to data as they are collected by asking new questions in order to clarify and extend the investigation. Even where the detection of academic

knowledge is sought, clinical interviews can give more information on depth of conceptual understanding because oral and graphical explanations can be collected, and clarifications can be sought where appropriate. (pp. 547-548).

Clinical interviewing was used in this study to discover how students think when they solve problems.

Chapter 5

Results and Analysis

I. Results from computer analysis

The logistic regression mathematical method

Let x be a data vector for a randomly selected experimental unit and let y be the value of a binary outcome variable so that y=1 if x comes from population 1 and y=0 if x comes from population 2. Let $p(y=1|\,x)$ equal the probability that y=1 given the observed data vector x. The form of the logistic regression model is

$p(y=1|\,x) = \dfrac{\exp(\beta_0 + \beta_1 x)}{1+\exp(\beta_0 + \beta_1 x)}$ (where β_0 is constant and β_1 is the coefficient of x)

When we study logistic regression it is customary to consider the logit transformation, a transformation performed on $p(y=1|\,x)$. The logit transformation is the log of the odds that y=1 versus y=0 and defined by

$$g(x)=log\{\, p(y=1|\,x)/[1- p(y=1|\,x)]\} \quad \text{Note that } g(x)= \beta_0 + \beta_1$$

Results from the logistic regression analysis

The reader is advised to keep in mind that all of the data used in this analysis was colleted from student self-reporting. For example, the data concerning teacher ability, or that concerning student attitude, rely solely on the students' own perceptions.

The statistical package used in this study was logistic regression methods in SPSS using the Backward: Wald model. The 15 covariates (variables) were the factors given by the first seven categories plus U1 through U4 and L1 through L4. These were described in detail in Chapter 4. Each of the four problems was used in turn as the individual dependent variable, thus the regression was run 4 times. Moreover, the significant p-value was chosen to be any value less than 0.1. Since we only needed the coefficient B, (which is used by the computer to represent either β_0 or β_1) of the variables to analyze the results, the following tables (Table 1 to Table 4) show only the factors with the B-coefficient and p-value for each problem (see Appendix C for all the data for these results).

Table 1

Significant Factors Affecting the Results of Problem 1 (N=49)

Variable	B	p	Exp(B)
Teaching style	2.727	0.008	15.281
U3	-2.146	0.003	0.117
Constant	-2.378	0.256	0.093

The factors affecting students who understood this unit but were unable to solve problem 1 are teaching style and U3 (# 32 in the Evaluation Sheet). The logistic regression result is that the coefficient of teaching style is 2.727 and p-value=.008; the coefficient of U3 is –2.146 and p-value =.003. Hence, the log of the odds ratio of $P1(1)$ and $P1(0)$ is

$$\log(\frac{P1(1)}{P1(0)}) = -2.378 + 2.727 \times Teaching + (-2.146) \times U3$$

where $P1(1)$ represents the probability of problem 1 being solved incorrectly, and $P1(0)$ represents the probability of problem 1 being solved correctly.

This is interpreted to mean that when the Likert scale rating, by a student, for teaching style increases by 1 point, that student is $e^{2.727} = 15.281$ times more likely to be unable to solve problem 1. That is, if we assume that two students' ratings are identical in all other factors, then a one point difference in students' evaluation of their teachers' teaching style indicates that the student with the higher rating would be 15.281 times more likely to be unable to solve problem 1 than the student with the lower rating. This is the result of the effect of <u>teaching style</u>.

When the rating, by a student, on question U3 increases by 1 point, then that student is $e^{-2.146} = 0.117$ times more likely to be unable to solve problem 1; in other words, that student is $\frac{1}{e^{-2.146}} = 8.547$ times more likely to be able to solve problem 1. That is, if we assume that two students' ratings are identical in all other factors, then a one point difference in students' self-evaluation of their ability to explain a unit to a fellow student indicates that the student with the higher rating would be 8.547 times more likely to be able to solve problem 1 than the student with the lower rating. This is the result of the effect of the ability of students to explain a unit to a fellow student.

Table 2

Significant Factors Affecting the Results of Problem 2 (N=49)

Variable	B	p	Exp(B)
Teaching attitude	2.663	0.040	14.334
U3	-2.187	0.011	0.112
Constant	5.481	0.107	240.024

The factors affecting students who understood this unit but were unable to solve problem 2 are teacher's attitude and U3. The logistic regression result is that the coefficient of teacher's attitude is 2.663 and p-value =.040; the coefficient of U3 is –2.187 and p-value =.011. Therefore, the log of the odds ratio of $P2(1)$ and $P2(0)$ is

$$\log\frac{P2(1)}{P2(0)} = 5.481 + 2.663 \times T.attitude + (-2.187) \times U3$$

where $P2(1)$ represents the probability of problem 2 being solved incorrectly, and $P2(0)$ represents the probability of problem 2 being solved correctly.

This is interpreted to mean that when the Likert scale rating, by a student, for teacher's attitude increases by 1 point, then that student is $e^{2.663} = 14.334$ times more likely to be unable to solve problem 2. That is, if we assume that two students' ratings are identical in all other factors, then a one point difference in students' evaluation of their teachers' attitude indicates that the student with the higher rating would be 14.334 more likely to be unable to solve problem 2 than the student with the lower rating. This is the result of the affect of teacher's attitude.

When the rating, by a student, on question U3 increases by 1 point, then that student is $e^{-2.187} = 0.112$ times more likely to be unable to solve problem 2; in other words, that student is 8.929 times more likely to be able to solve problem 2. That is, if we assume that two students' ratings are identical in all other factors, then a one point difference in students' self-evaluation of their ability to explain a unit to a fellow student indicates that the student with the higher rating would be 8.929 times more likely to be able to solve problem 2 than the student with the lower rating. This is the result of the effect of the ability of students to explain a unit to a fellow student.

Table 3

Significant Factors Affecting the Results of Problem 3 (N=49)

Variable	B	p	Exp(B)
Student attitude	-1.001	0.083	0.368
U1	2.848	0.007	17.256
U3	-1.813	0.02	0.163
Constant	1.526	0.50	4.598

The factors affecting students who understood this unit but were unable to solve problem 3: student's attitude, U1 (# 30 in the Evaluation Sheet), and U3. The logistic regression result is that the coefficient of student's attitude is -1.001 and p-value =.083; the coefficient of U1 is 2.848 and p-value =.007; the coefficient of U3 is -1.813 and p-value =.020. Hence, the log of the odds ratio of $P3(1)$ and $P3(0)$ is

$$\log \frac{P3(1)}{P3(0)} = 1.526 + (-1.001) \times S.attitude + 2.848 \times U1 + (-1.813) \times U3$$

where $P3(1)$ represents the probability of problem 3 being solved incorrectly, and $P3(0)$ represents the probability of problem 3 being solved correctly.

This is interpreted to mean that when the Likert scale rating, by a student, for student's attitude increase by 1 point, then that student is $e^{-1.001} = .368$ times more likely to be unable to solve problem 3; in other words, that student is 2.717 times more likely to be able to solve problem 3. That is, if we assume that two students' ratings are identical in all other factors, then a one point difference in students' self-evaluation of their attitude indicates that the student with the higher rating would be 2.717 times more likely to be able to solve problem 3 than the student with the lower rating. This is the result of the effect of <u>the student's attitude</u>.

When the rating, by a student, on question U1 increases by 1 point, then that student is $e^{2.848} = 17.256$ times more likely to be unable to solve problem 3. That is, if we assume that two students' ratings are identical in all other factors, then a one point difference in students' self-evaluation of the extent of their mimicking the procedure that a teacher taught indicates that the student with the higher rating would be 17.256 times more likely to be unable to solve problem 3 than the student with the lower rating. This is the result of the effect of <u>the degree of mimicking what a teacher taught</u>.

When the rating, by a student, on question U3 increases by 1 point, then that student is $e^{-1.813} = 0.163$ times more likely to be unable to solve problem 3; in other words, a student is 6.135 times more likely to be able to solve problem 3. That is, if we assume that two students' ratings are identical in all other factors, then a one point difference in students' self-evaluation of their ability to explain a unit to a fellow student indicates that the student with the higher rating would be 6.135 times more likely to be able to solve problem 3 than the student with the lower rating. This is the result of the effect of the ability of students to explain a unit to a fellow student.

Table 4

Significant Factors Affecting the Results of Problem 4 (N=49)

Variable	B	p	Exp(B)
Teaching style	1.209	0.044	3.352
U3	-0.772	0.019	0.462
L2	-0.565	0.051	0.568
Constant	1.172	0.554	3.228

The factors affecting students who understood this unit but were unable to solve problem 4 are: teaching style, U3, and L2 (# 35 in the Evaluation Sheet). The logistic regression result is that the coefficient of teaching style is 1.209 and p-value =.044; the coefficient of U3 is –.772 and p-value =.019; and the coefficient of L2 is -.565 and p-value =.05. Hence, the log of the odds ratio of $P4(1)$ and $P4(0)$ is

$$\log\frac{P4(1)}{P4(0)} = 1.172 + .302 \times Teaching + (-.772) \times U3 + (-.565) \times L2$$

where $P4(1)$ represents the probability of problem 4 being solved incorrectly, and $P4(0)$ represents the probability of problem 4 being solved correctly.

This is interpreted to mean that when the Likert scale rating, by a student, for teaching style increases by 1 point, then that student is $e^{1.209} = 3.352$ times more likely to be unable to solve problem 4. That is, if we assume that two students' ratings are identical in all other factors, then a one point difference in students' evaluation of their teachers' teaching style indicates that the student with the higher rating would be 3.352 times more likely to be unable to solve problem 4 than the student with the lower rating. This is the result of the effect of <u>teaching style</u>.

When the rating, by a student, on question U3 increases by 1 point, then that student is $e^{-.772} = 0.462$ times more likely to be unable to solve problem 4; in other words, that student is 2.164 times more likely to be able to solve problem 4. That is, if we assume that two students' ratings are identical in all other factors, then a one point difference in students' self-evaluation of their ability to explain a unit to a fellow student indicates that the student with the higher rating would be 2.164 times more likely to be able to solve problem 4 than the student with the lower rating. This is the result of the effect of <u>the ability of students to explain a unit to a fellow student</u>.

When the rating, by a student, on question L2 increases by 1 point, then that student is $e^{-.565} = 0.568$ times more likely to be unable to solve problem 4; in other words, that student is 1.761 times more likely to be able to solve problem 4. That is, if we assume that two students' ratings are identical in all other factors, then a one point difference in students' self-evaluation of their learning through finding patterns indicates that the student with the higher rating would be 1.761 times as likely to be able to solve problem 4 as the student with the lower rating. This is the result of student learning through finding patterns.

The results in *Figure 2* are from the Evaluation Sheet and Problem Sheet presented in two different school systems. This figure shows the five significant factors versus the average points for each of the questions pertaining to that factor, averaged again over the number of students in each of S1 and S2 schools.

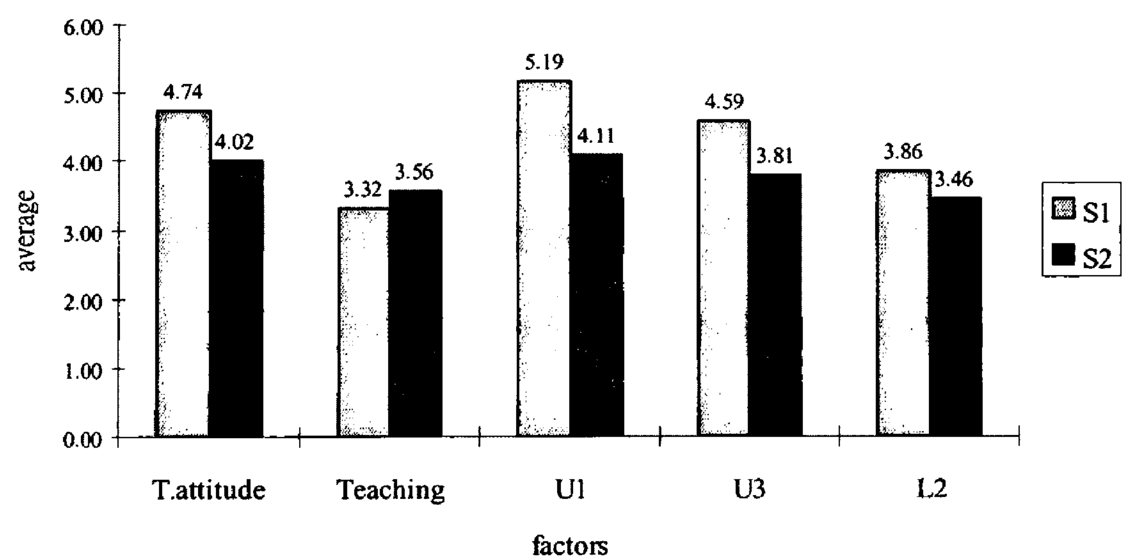

Figure 2. Students' responses to significant factors, by school

In *Figure 2*, T.attitude shows that the S1 students felt that their teacher had a better attitude or was more involved in teaching this unit than the S2 students felt about their teacher. Teaching shows that the S1 students felt their teacher taught in a more teacher-centered way than the S2 students felt their teacher did. U1 shows that the S1 students mimicked the procedure that their teacher taught better than the S2 students mimicked the procedure that their teacher taught. U3 shows that the S1 students were better able to explain the concept in this study than the S2 students, and L2 shows that the S1 students learned by seeking patterns more than the S2 students did.

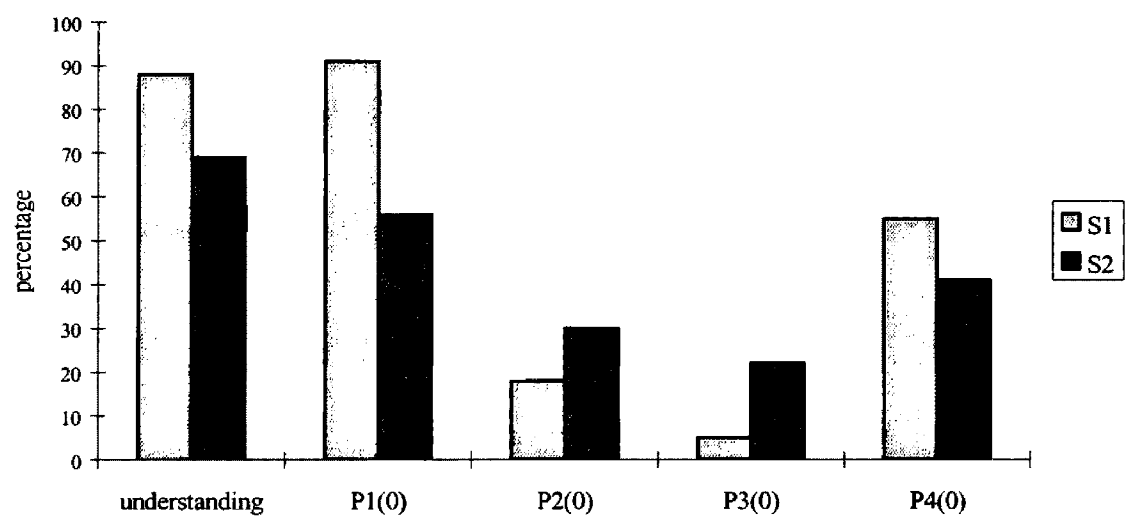

Figure 3. Percentage of students' understanding and the solving problem correctly, by school

Figure 3 shows percentage of students' understanding and the solving problem correctly, by school. There were 25 out of 25 students in S1 and 39 out of 45 students in S2. The number of S1 students understanding the concept is 22, which represents 88%, and the number of S2 students understanding the concept is 27, which represents 69%. The number of S1 students responding correctly to problem 1, is 20, which represents 91%, and the number of S2 students responding correctly in problem 1, is 15, which represents 56%. The number of S1 students responding correctly in problem 2, is 4, which represents 18%, and the number of S2 students responding correctly in problem 2, is 8, which represents 30%. The number of S1 students responding correctly in problem 3, is 1, which represents 5%, and the number of S2 students responding correctly in problem 3, is 6, which represents 22%. The number of S1 students responding correctly in problem 4, is 12, which represents 55%, and the number of S2 students responding correctly in problem 4, is 11, which represents 41%.

II. Results of Problem Sheet and interview

The methods that S1 teacher taught in her class were the formula[1], the FOIL[2], and the box method[3]. The S2 teacher used three different methods in her classes: one class was taught using FOIL and the formula, another class was taught using FOIL and both the formula and the box method, and the other class was taught using FOIL, the formula and algebra tiles.

1. Formula: $x^2 + (m+n)x + mn = (x+m)(x+n)$

2. FOIL: $(x+3)(x-2) = x \cdot x - 2x + 3x - 6 = x^2 + x - 6$

3. The box method: $x^2 + x - 6 = (x+3)(x-2)$

	x	3
x	x^2	$3x$
-2	$-2x$	-6

Generally, most students in S1 used the methods which their teacher taught in this study; either formula, FOIL, or box, to solve problems. Students in S2 used the formula that their teacher taught in this study, or other methods, such as working backwards, to solve problems. All students in S1 and S2 who solved problem 1 successfully used the ways their teacher taught in the class. Most of the students in S1 tried to solve problem 2 and used either FOIL or box to solve it; however, all students in S1 who solved problem 2 successfully used the box method. When students used FOIL to solve problem 2, then most of them were frustrated when they encountered a similar form like $-15 = 9x - 3xa - 5a$ (see *Figure 4* and *Figure 5*). Since students saw that $9x$, $-3xa$, and $-5a$ were all different terms, they did not know how one term could be equal to three terms. However, there were many students who used the box method to solve problem 2 who could not solve it either.

$$\cancel{3x^2} - 4y - 15 = 3x^2 + 5x - 3xa - 5a$$
$$-\cancel{3x^2} \qquad\qquad -3x^2$$
$$-\cancel{4x} - 15 = 5x - 3xa - 5a$$
$$+\cancel{4x} \qquad\qquad +4x$$
$$0 = 9x - 3xa - 5a + 15$$

Figure 4. An example of a student's solving procedure of problem 2 from S1-1

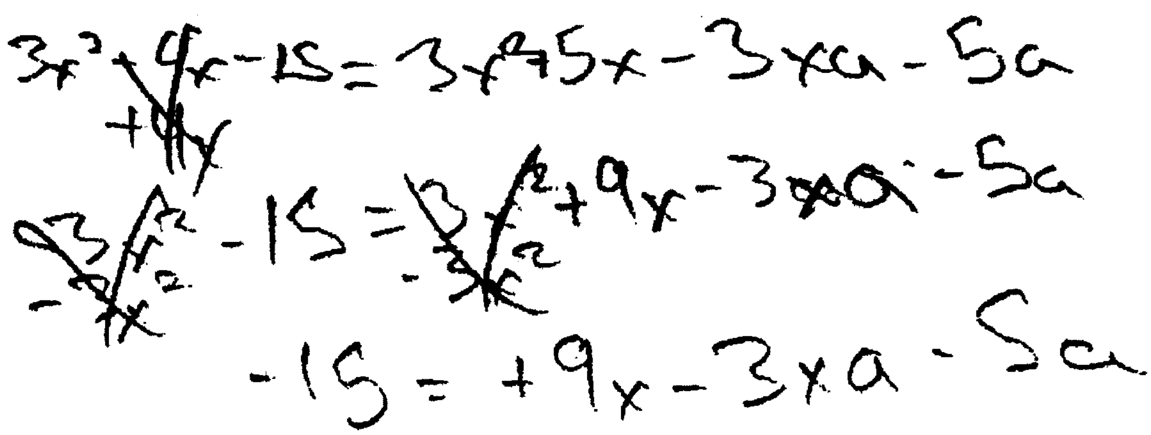

Figure 5. An example of a student's solving procedure of problem 2 from S1-2

Several excerpts from conversations between the researcher and students resulting from asking students questions about the boxes that they set up, and the equation on problem 2, follow:

R: Do you agree that when you add these together [terms were in box], they will equal this equation [pointing to $3x^2+4x-15$]? (See *Figure 6.*)
S5: I don't know, because there is an "*a*" in there.

R: Does $-15=-5a$?
S5: Oh, "*a*" had to equal to 3.

R: If I didn't show you that, $-15=-5a$, could you solve this problem by yourself?
S5: It's really hard. I am sure once I fill out the box…, I was just dumb.

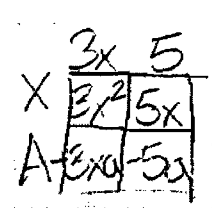

Figure 6. An example of a student's solving procedure of problem 2 from S1-3

Several students in S2 tried to solve problem 2 and used either the formula, multiplied out the constant term, or worked backwards, and most of them solved problem 2 successfully; however, other students in S2 did not know how to start to solve problem 2. There were also some students from both S1 and S2 who could not solve problem 2 because they did not have the necessary algebra skills.

Many students seemed not to understand the meaning of problem 3, so they had no idea how to use the formula or FOIL to solve it. Some other students, who used the box method to try to solve problem 3, could not seem to connect the box with the equation. Thus, there were many students who could not solve problem 3. However, those students who did solve problem 3 successfully worked backwards to solve the problem. In problem 4, some students could not solve this problem because they had forgotten that area equals length times width; however, if the students had known this formula, then most of them could have solved this problem. Yet, there was different thinking between S1 and S2 students. Some students in S1 solved problem 4 with the formula, but they seemed not to pay attention to the equation, so they listed all possible factors and then tried to find out which number could fit in the polynomial (see *Figure 7*). However, all students in S2 who solved problem 4 seemed to know the equation well, so they were able to choose the appropriate factors immediately (see *Figure 8*).

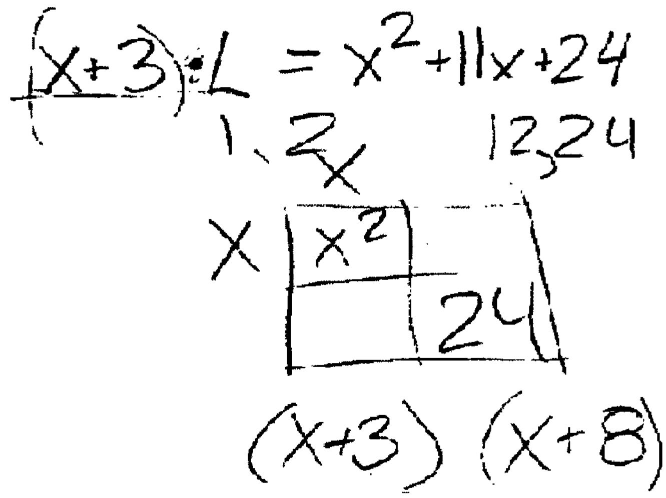

Figure 7. An example of a student's solving procedure of problem 4 from S1

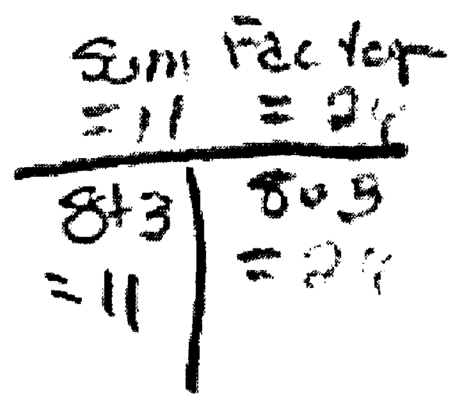

Figure 8. An example of a student's solving procedure of problem 4 from S2

Chapter 6

Discussion and Conclusion

I. Discussion

We cannot know how well a student understands concepts solely from considering the student's own opinion; however, the extent to which the student can explain the underlying concepts in his/her own words is a strong indicator of the student's ability to solve problems. The importance of this is supported by the data, which shows that the extent to which a student can explain a concept correlates significantly with his/her ability to solve problems involving that concept. Moreover, learning by seeking patterns helps students in solving similar problems that are taught in the class, such as problem 4, and the better the learning attitude a student has the better probability that the student can solve non-similar problems. However, the more a student feels that he/she understands concepts just by mimicking how his/her teacher solves problems in the class, the lower the probability that the student can solve non-similar problems taught in the class.

One disappointing result of this study showed that when students felt their teacher used a teacher-centered approach, this approach appeared to be successful in helping students solve problems, such as problem 1 and problem 4. However, problem 1 and problem 4 are similar to problems taught in the class. According to *Figure 3*, the percentage of S1 students who obtained the correct response for problems 2 and 3 is lower than the percentage of S2 students. This is possibly a result of students experiencing a less teacher-centered approach by teacher S2, so a higher percentage of students in the S2 class used many different problem solving techniques, not just what they had been shown in class. Nevertheless, when looking at the unsuccessful problem solving procedures of S1 students, it appeared that they got "stuck" when trying to apply a model that was used previously in class. In addition, some S1 students' Problem Sheets showed an attempt to re-use the original example's strategies on new problems, thus becoming stuck on the new problems. Hence, it seems that if students felt that the approaches used by their teacher were more teacher-centered approaches, then these approaches would not be helpful to students in solving non-similar problems.

Metacognition is the process by which students consciously choose problem solving strategies. There are two important parts to metacognition. One is cognitive knowledge, which is concerned with a person's knowledge of cognitive abilities, processes, and resources in relation to specific task performance. The other is regulation of cognition which leads one to monitor the understanding of one's task, and to regulate strategy usage. (Garofalo & Lester, 1985) A practical observation from the author's classroom is that the greater a student's ability in these two areas, the greater the probability that a student can solve problems successfully. Consequently, an appropriate goal for a teacher is to improve students' knowledge of how to make good strategy choices. The results of this study showed that teacher-centered approaches supply students with many strategies, but they do not help students greatly in making good choices among those strategies. Other support for the apparent weakness of a teacher-centered approach is that the students in S1 who could not solve problem 2 using the FOIL or the box method were unsuccessful in solving it because this teaching approach produced students who simply followed the teacher's instructions, without learning to think creatively. The result was that students were too focused on the mechanics of the method to understand the meaning of the problem, or to think in different non-routine ways. However, the S2 students who were able to solve problem 2 devised their own strategies, because the teacher had not provided them with a rigid, algorithmic strategy for solving such problems. Also when some S1 students solved problem 4, they did this by listing all the factors of the constant term to check, because this was one of the ways that they learned to factor polynomials. If the students had understood the relationship between the two sides of the equation, they would not have found it necessary to check *all* factors, they would have simply picked out the *appropriate* ones. Finally, additional support for this argument comes from the fact that some students reported, during their interview, that they remembered what methods the teacher used in the class too well to think in different ways. It is interesting to observe that a teacher can be so trusted by her students that they "blindly" follow principles, without really analyzing the material taught. One of the results of the Logistic Regression for problem 2 is that teacher attitude from students' self-reporting (i.e. students felt that their teacher took too much responsibility for their learning, or students felt that they could rely heavily on their teacher's teaching) is a significant negative factor affecting students in solving this problem. Hence, when students feel that their teacher uses a teacher-centered approach or when students feel that their teacher takes too much responsibility for telling them everything while teaching, then these students learn to rely more on the teacher's teaching rather than on their own problem solving ability, even if they could explain concepts well.

Although teacher ability (from students' self-reporting) was not a significant factor in this study, from the author's observation of classes and the students' responses on the Evaluation Sheet, the author found teacher ability from students' perceptions was an indirect influence on some significant factors, such as teachers' teaching style (from students' self-reporting), and teachers' attitude (from students' self-reporting). In fact, the way the S1 teacher taught was less teacher-centered than the S2 teacher, but because the S1 teacher has taught this topic about three years and the S2 teacher was teaching this topic for the first time, the S1 teacher knew more about how to deliver her idea to her students than the S2 teacher. As a result the teacher-centered approach appeared stronger to the S1 students than it actually was, while the teacher-centered approach appear weaker to the S2 students than it actually was. Moreover, teacher ability (from students' self-reporting) may be another reason influencing how much students rely on their teachers. S1 students could explain the concept better than S2 students, and also S1 students followed their teacher better than S2 students. (See *Figure 2*). Therefore, the teachers' ability (from students' self-reporting) is an indirect factor in either helping students understand concepts more deeply or preventing students from thinking by themselves. However, the influence, – helping or inhibiting – teachers' ability will have, relates to their teaching styles and attitudes (i.e. how much responsibility the teachers take).

II. Conclusion

A teacher-centered teaching style (from students' self-reporting) is a better way for students to learn to solve traditional problems. But a teacher-centered teaching style is not helpful in improving the ability of students to choose good strategies. Therefore, this can inhibit students from expanding their problem solving abilities.

Students learn mathematics through the experiences that their teachers provide, through students' understanding of mathematics, and through their ability to use mathematics. All are shaped by the teaching they encounter in school. Moreover, students learn more and learn better when they can take control of their learning by defining their goals and monitoring their progress. (NCTM, 2000) Therefore, improving students' decision-making abilities in choosing problem solving strategies is an important goal for teachers. In addition, it is better to give non-routine problems to students, since routine problems will not stimulate the students' thinking when they try to solve them. Also, routine problems could encourage teachers to use teacher-centered approaches more often to teach their students.

Whenever a student can explain a concept in greater detail, then the student is better able to solve related problems. Since students learn mathematics through their teachers, NCTM (2000) states that teachers need to understand and be committed to their students as learners of mathematics. They also need background knowledge about the challenges students are likely to encounter in learning these ideas, about how the ideas can be represented to teach them most effectively, and about how students' understanding can be assessed. As a result, teachers would be better able to teach their students to understand concepts more deeply. Teachers, however, need to be careful about taking too much responsibility when presenting a unit in order to avoid students only learning what they are taught. If students rely too much on their teachers, these students will be less effective in solving non-similar problems. Moreover, teachers' ability (from students' self-reporting) could make a teacher-centered approach work better. Therefore, teachers should use their ability to *guide* their students' learning concepts, instead of *telling* their students how to solve problems.

Students' good attitudes, such as paying attention in the class, participating activities, or work hard on mathematics can also help in solving problems. Hence, teachers should encourage their students to improve their attitude in these ways.

III. Suggestions

The purpose of this study was to discover the factors affecting the problem solving ability of students who understand a new mathematical concept. This study suggests that students who perceive that their teachers use a teacher-centered teaching style, or take too much responsibility for their students' learning, or students who feel that they can rely heavily on their teachers, will be inhibited from solving problems. However, the extent to which students can explain a unit and students' attitude can help the students to solve problems. But the results only show the extent to which each factor affects students' problem solving abilities; therefore, further qualitative research is needed that focuses on those factors showing a significant effect in this study, such as how teaching style affects students' learning, and how students rely on their teachers' teaching. Since it was not the object of this study to specify the nature of the interactions between teachers' ability, teaching style, and the responsibility teachers take in their teaching affects students' learning, this may be another issue for researchers to study in the future.

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Appendix A

Evaluation Sheet

There is no right or wrong answer for the questions below. Please circle that, which most represents your feeling or situation. SD- Strongly Disagree; D– Disagree; PD- Partly Disagree; PA- Partly Agree; A- Agree; SA- Strongly Agree.

Example: I like to learn mathematics. SD-D-PD-PA-A-SA

A. I understand what the teacher taught in this unit. ____ Agree; ____ Disagree

(Please choose one of the answers)

teacher ability	1. I felt the teacher explained the new concepts clearly in this unit.	SD-D-PD-PA-A-SA
	2. I felt the teacher knew how to guide us to learn this unit effectively.	SD-D-PD-PA-A-SA
	3. I felt the teacher understood our problems in learning this unit.	SD-D-PD-PA-A-SA
	4. I felt the teacher presented the material in a confident and organized manner.	SD-D-PD-PA-A-SA
teacher attitude	5. I felt the teacher enjoyed teaching us this unit.	SD-D-PD-PA-A-SA
	6. I felt the teacher cared about what students learning in this unit.	SD-D-PD-PA-A-SA
	7. I felt the teacher liked the students in this class.	SD-D-PD-PA-A-SA
	8. I felt the teacher worked hard to achieve the unit's learning objectives.	SD-D-PD-PA-A-SA
teaching style	9. The teacher usually wanted us to solve problems in the manner that he/she taught us.	SD-D-PD-PA-A-SA
	10. We worked in groups in this unit.	SD-D-PD-PA-A-SA
	11. We usually arrived at an answer through discussion with each other.	SD-D-PD-PA-A-SA
	12. The teacher gave us enough time to finish the problems on our own.	SD-D-PD-PA-A-SA
teacher beliefs	13. If I did a good job in this unit, the teacher was pleased.	SD-D-PD-PA-A-SA
	14. I felt the teacher was more interested in the process than the answer.	SD-D-PD-PA-A-SA
	15. I felt the teacher gave us confidence to solve the problems in this unit by ourselves.	SD-D-PD-PA-A-SA

	16. I felt the teacher helped me relate this lesson to real-life situations.	SD-D-PD-PA-A-SA
relationship between teacher and students	17. I feet the teacher is like one of my friends.	SD-D-PD-PA-A-SA
	18. I felt free to ask the teacher questions at any time.	SD-D-PD-PA-A-SA
	19. I like to talk with the teacher after class.	SD-D-PD-PA-A-SA
	20. I felt that the teacher empathized with my problems in the class.	SD-D-PD-PA-A-SA
student attitude	21. I paid attention throughout in this unit.	SD-D-PD-PA-A-SA
	22. I enjoyed participating in this unit.	SD-D-PD-PA-A-SA
	23. I would like to learn more about this unit.	SD-D-PD-PA-A-SA
	24. I worked hard out of class in this unit.	SD-D-PD-PA-A-SA
student motivation	25. I felt this unit was interesting to me.	SD-D-PD-PA-A-SA
	26. I enjoy mathematics.	SD-D-PD-PA-A-SA
	27. I feel I can use the concepts in this unit in my daily life.	SD-D-PD-PA-A-SA
	28. I feel it is important for me to get high grade in class.	SD-D-PD-PA-A-SA
	29. I didn't enjoy this unit.	SD-D-PD-PA-A-SA
U1	30. I could follow how the teacher solved problem in this unit.	SD-D-PD-PA-A-SA
U2	31. I could apply a problem-solving method to a similar problem.	SD-D-PD-PA-A-SA
U3	32. I could explain the concept taught in this unit to a fellow student.	SD-D-PD-PA-A-SA
U4	33. I will be able to apply and broaden the concepts that I learned in this unit to different problems.	SD-D-PD-PA-A-SA
L1	34. I can do these kinds of problems by memorizing what my teacher said.	SD-D-PD-PA-A-SA
L2	35. I tried to find patterns among similar problems.	SD-D-PD-PA-A-SA
L3	36. I usually know the reasons why my teacher uses a specific way to solve a problem.	SD-D-PD-PA-A-SA
L4	37. I looked for alternate ways of solving problems or connecting concepts.	SD-D-PD-PA-A-SA

Appendix B
Problem Sheet

Problem 1

Factor the polynomial $x^2 + 7x - 60$

Problem 2

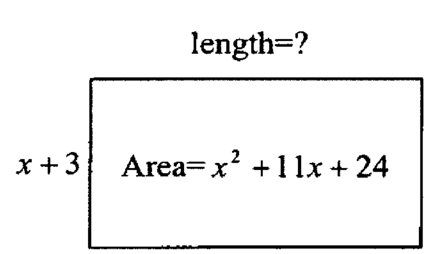
If $3x^2 - 4x - 15 = (x - a)(3x + 5)$, then what is the value of a?

Problem 3

If $x^2 + ax - 2 = (x - b)(x + 1)$, then what are the values of a, b?

Problem 4

There is a rectangle whose area is given by $x^2 + 11x + 24$. If the length of one side of the rectangle is $x + 3$, then what is the length of other side of the rectangle?



Appendix C

Significant variables in the results of logistic regression

Problems	Variables	B	S.E.	Wald	df	Sig.	Exp(B)
Problem 1	Teaching style	2.727	1.020	7.148	1	.008	15.281
	U3	-2.146	.721	8.873	1	.003	.117
	Constant	.-2.378	2.092	1.292	1	.256	.093
Problem 2	Teacher attitude	2.663	1.294	4.237	1	.040	14.334
	U3	-2.187	.862	6.441	1	.011	.112
	Constant	5.481	3.399	2.600	1	.107	240.024
Problem 3	Student attitude	-1.001	.557	3.010	1	.083	.368
	U1	-2.848	1.050	7.358	1	.007	17.256
	U3	-1.813	.777	5.450	1	.020	.163
	Constant	1.526	2.261	.455	1	.500	4.598
Problem 4	Teaching style	.302	.150	4.406	1	.044	1.353
	U3	-.772	.330	5.470	1	.019	.462
	L2	-.565	.289	3.816	1	.051	.568
	Constant	1.172	1.979	.351	1	.554	3.228