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Bethe-Bloch stopping power parameters extracted from recent accurate measurements

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BETHE-ELOPE SCATTERING POWER PARAMETERS EXTRACTED FROM RESIDUAL NUCLEAR MEASUREMENTS

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Bethe-Bloch Stopping Power Parameters Extracted From Recent Accurate Measurements

Director: Leonard E. Porter

Bethe-Bloch theory allows the calculation of stopping power for a given projectile-target material combination, provided the values of the various parameters of the theory are known. Several of the needed parameters cannot be easily determined through analytic calculations. An alternative method for the determination of these parameters is a fit of theory to accurate, experimentally derived stopping power data.

Computer code initiated by Bichsel was modified to allow optimization of parameters in the Bethe-Bloch formula. Experimental data by Anderson et al. provided the standard for the determination of the needed parameters. The mean excitation energy, I, and the z^3-effect parameter, b, were determined for three kinds of projectiles (protons, alpha particles, and lithium nuclei) on each of four target materials (Al, Cu, Ag, and Au). Two sets of shell correction parameters were also examined and compared in the present study.

One of the two sets of shell correction parameters resulted in values of I that were essentially independent of projectile type, and values of b that were essentially independent of both projectile type and target material.

ACKNOWLEDGMENTS

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

CHAPTER I

INTRODUCTION

The ability to predict accurately the amount of energy deposited by a charged particle in its passage through matter is a goal whose attainment would prove useful in many areas of physics.

There are several factors that affect the rate at which a moving charged particle suffers energy loss to an absorbing medium. In a given case, this rate depends upon the type of projectile, its kinetic energy, and various properties of the absorbing medium.

It is convenient to separate the subject at hand into three projectile energy regions, each of which can be treated by a separate theory. Still another theory ensues if the projectile mass is very much less than that of a proton, e.g., that of an electron. The following discussion assumes "massive" projectiles , where massive projectiles are those with a mass equal to or greater than that of a muon.

If the incident particle has a kinetic energy less than approximately 0.5 MeV/amu it is called a low energy projectile. In this low energy region the incident particle interacts primarily with entire atoms of the target material. A complicating factor that is important in this energy range is the capture and loss of electrons by the projectile during its passage through the material. This process leaves the projectile fully ionized along only a portion of its path.

Particles that are significantly relativistic lie in the high energy region. This region begins approximately where the projectile has a kinetic energy equal to its rest mass energy and spans all greater energies. The projectile in this case is stopped primarily by collisions with the nuclei of the target atoms. However interactions of the projectile with the atomic electrons are also important, as are radiative losses (bremsstrahlung).

This thesis deals with the stopping of massive projectiles having kinetic energies intermediate to the high and low energy regions. This energy range is called the medium-energy range.

The dominant mechanism for the loss of kinetic energy by a massive medium-energy charged particle in traversing matter is interactions with the electrons in the absorbing material. Bethe-Bloch theory (1)* allows calculation of the stopping power, S (to be defined later), of a material provided that the values of the parameters of the theory are known.

The path that a massive medium-energy projectile will follow in traversing the target material will deviate little from a straight line if the absorber is sufficiently thin so that the projectile does not lose a large fraction of its initial energy in traversing the material. This feature prevails because the small momentum transfers associated with collisions of the projectile with atomic electrons deflect the projectiles' paths in random directions. The net effect of many such

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*Numbers in parentheses refer to numbered references in the bibliography.

collisions does not, in general, result in a large deflection of the projectile in any one direction.

The partial ionization of the projectile, important in the low energy region, can also be important in the medium energy region if the total charge of the projectile is sufficiently great. This effect can be appreciable even for projectiles with relatively small charges, but in these cases only over the lower portion of the medium-energy region.

The multiplicity of parameters brought about by the many extensions (2,3,4) to the original Bethe-Bloch theory makes their determination difficult at best. Since theoretical determination is not feasible, the alternative is a fit of theory to accurate experimental results. Unfortunately, there has been a dearth of experimental data accurate enough to allow consistent determination of the necessary parameters.

The measurements utilized here are those obtained by Andersen et al. (5). By employing calorimetric techniques the Andersen group was able to claim an accuracy of 0.5% in stopping power, placing these data among the most accurate available. These measurements were done for three kinds of projectiles (protons, alpha particles, and lithium nuclei) on each of four target materials: Al, Cu, Ag, and Au.

The analysis of these data was done with the aid of a computer program based on a code developed by Bichsel (6), Shepard and Porter (7). This FORTRAN program was further modified to allow the determination of a maximum of four parameters simultaneously. The simultaneous determination of these parameters was found to be important

in some cases due to their interdependence.

CHAPTER II

THEORY

The stopping power, S, of a material is defined by

$$S = -\frac{1}{\rho}\frac{dE}{dX} .$$

Here ρ is the density of the target mareriai in g/cm^3, and dE/dX is the average rate of energy loss of the projectile per unit path length. Typical units for stopping power are $MeV\text{-}cm^2/g$. Since a massive projectile loses only a small fraction of its total kinetic energy during each interaction with an atomic electron, the stopping power can be considered to be a continuous function of projectile path length in the absorber. An interaction of the projectile with an atomic nucleus could produce a large energy loss, but these interactions are so rare as to be completely negligible in the medium-energy range.

The first theoretical treatment of stopping power was done by Niels Bohr (8). Bohr used an impact parameter to characterize the transfer of energy from the incident particle to the electrons of the target material. This theory proved unsatisfactory because it had limited accuracy, and only applied over a limited range of projectile energies.

Another major objection to the Bohr theory is that the impact parameter is not an observable quantity. Also, Bohr's theory required that upper and lower limits be imposed on the impact parameter in order that the minimum and maximum energy transfer to an electron be constrained.

Bohr's stopping power theory was later replaced by that of Bethe and Bloch who used a first order quantum mechanical approach characterized by momentum transfer. A treatment of the derivation of the Bethe-Bloch formula can be found in Fano (1). The original form of the Bethe-Bloch formula has been modified in recent years to include terms to correct for recently discovered effects to be discussed later. The current form of the Bethe-Bloch formula is given by

$$S = \frac{4\pi e^4 N_o z^2 Z L}{mc^2 \beta^2 A},$$

where the quantity

$$\frac{4\pi e^4 N_o}{mc^2} = .30708$$

for S given in MeV-cm^2/g. The symbols in the stopping power formula represent the following quantities:

e =electronic charge

N_o=Avogadro's number

mc^2=rest mass of electron

β =v/c=velocity of projectile/velocity of light

z =charge on projectile in units of e

Z =atomic number of target material

A =atomic weight of target material

L =stopping number per target electron

The stopping number per target electron consists of three terms that contain all of the undetermined parameters of the theory and so bears closer examination.

$$L = L_0 + zL_1 + L_2$$

The first term, L_0, and the last term, L_2, come from the original Bethe-Bloch theoretical treatment. The central term was added as a correction to the stopping number after experimental evidence (9) showed that slow negative hyperons lose energy at a slower rate than do positive particles of the same velocity. This phenomenon is called the "Barkas effect" (4) and the correction for this effect was obtained in a theoretical treatment by Ashley, Ritchie, and Brandt (2,3). The form of L_1 is

$$zL_1 = \frac{zF(b/X^{1/2})}{(Z^{1/2}X^{3/2})}$$

where $F(b/X^{1/2})$ is a function derived and tabulated in Refs. 2,3, and b is a free parameter of the theory, henceforth referred to as the projectile-z^3 parameter. The symbol X represents $40.2E/MZ$ where E is the projectile energy in MeV and M is the projectile mass in amu.

The factor L_2 in the expression for L is the Bloch projectile-z^4 correction term, and is considered to be part of the basic theory although it was reintroduced only recently. With the addition of this Bloch term the Bethe formula will reduce, in the limit of high projectile velocities, to the result obtained in the semi-classical treatment of Bohr. The form of the Bloch z^4 term is

$$L_2(y) = \Psi(1) - \mathrm{Re}(\Psi(1+iy))$$

where $y = z\alpha/\beta$ with α being the fine structure constant. The function is the logarithmic derivative of the gamma function. For the purpose of calculational ease this term is approximated to within 0.4% by the function (10)

$$\overline{L_2}(y) = -y^2[1.20206 - y^2(1.042 - 0.3549y^2 + 0.3434y^4)] .$$

It was this form that was used for the calculations in the present study.

L_0 comes from the original treatment by Bethe and is given by

$$L_0=f(\beta)-\ln(I)-\sum_i C_i/Z-\delta/2,$$

where

$$f(\beta)=\ln(2mc^2\beta^2/(1-\beta^2))-\beta^2.$$

These symbols represent the following quantities:

I =mean excitation energy

δ =high velocity density effect correction

C_i=shell correction for the i^{th} shell,

where i=1 refers to the K-shell electrons, i=2 refers to the L-shell electrons, etc. The high energy density effect correction applies only in the case of high energy incident particles and is negligible in the present study.

Charge state corrections are included in some cases to correct for the partial ionization of the projectile. These corrections have the form

$$\overline{z} = z(1-\exp(-k\beta/\alpha z^{1/2}))$$

where α is the fine structure constant, and k is the adjustable parameter for the correction.

The mean excitation energy is the principal adjustable parameter of Bethe-Bloch theory, and must be determined through fits to experimental data. It represents a logarithmic mean over all of the possible excitation energies, E_i, of the target atom above the ground state,

weighted according to oscillator strengths (1), f_i, of bound electrons and is defined by

$$\ln(I)=\sum f_i \ln(E_i) \ .$$

The oscillator strengths are not easily obtained for most materials so I is determined from a fit of theory to experimental data.

The other principal adjustable parameters of the original theory are those found in the C_i, the shell corrections. These corrections are used to correct for the null (or partial) participation of inner shell electrons in stopping the projectile. Experimental evidence has shown this correction to the theory to be more important than was originally thougnt. Without these shell corrections Bethe-Bloch theory rests on the assumption that the speed of the incident particle is much higher than that of atomic electrons in their bound states. Through use of the Bohr atomic model for an order-of-magnitude calculation, this can be shown to be an erroneous assumption. For example, the velocity of a K-shell electron in Cu (Z=29) is, according to the Bohr model,

$$v = Ze^2/\hbar = 0.2c$$

where c represents the velocity of light. If the projectile is a 10 MeV proton, then its velocity is given by

$$E = mv^2/2 \quad v = 0.02c \ .$$

Thus the velocity of a K-shell electron is approximately an order of magnitude larger than that of the incident proton. The shell corrections become more important at low and high projectile energies.

Numerical values of these shell corrections have been calculated theoretically for K- and L-shell electrons by Walske (11,12). An approximation to the shell correction factors for the M- and N-shells have been constructed through the use of "horizontal" and "vertical" scaling factors applied to the Walske L-shell correction. The shell corrections are given by

$$\sum_i C_i = B_1 C_K(\beta^2) + B_2 C_L(\beta^2) + V_M C_L(H_L\beta^2) + V_N C_L(H_N\beta^2)$$

Here C_K and C_L are the K- and L-shell correction factors calculated by Walske. The two multiplicative strength terms B_1 and B_2 are set to 1.0 for the present study. In the case of Al the M-shell correction was applied to the L-shell electrons, in which case H_M and V_M were re-labeled H_L and V_L, respectively.

CHAPTER III

METHOD

The quality of fit of Bethe-Bloch theory to the experimental data is represented by the root mean square relative deviation, σ, defined by

$$\sigma^2 = \sum (S_{exp}(E_i) - S_{th}(E_i))^2 / (\Delta S_{exp}(E_i) N)^2$$

where $S_{exp}(E_i)$ represents the experimentally determined stopping power for the i^{th} energy, E_i, and $S_{th}(E_i)$ is the corresponding theoretical stopping power. Here, N represents the number of data points for a particular projectile-target combination, and $\Delta S_{exp}(E_i)$ is the stated uncertainty in the experimental data. A minimum value of σ corresponds to a combination of Bethe-Bloch parameters that provide the best overall fit of the theory to the data.

The computer program used in these fits of theory to data was automated in order to provide a means of obtaining a minimum in σ by trying various different combinations of numerical values for the free parameters of the theory. These values correspond to points in a "rectangular" grid having a dimensionality equal to the number of free parameters. Upon determination of the point in this grid that provided the smallest σ, a new finer grid can be constructed around this point and the process repeated until the desired accuracy is obtained. Several versions of this program were developed in order to accommodate one, two, three, or four free parameters.

Another feature of the computer program allows determination of the accuracy to which the free parameters are determined, i.e., the degree to which the values of the parameters are affected by the errors

inherent in the experimental data. In order to accomplish this determination of accuracy, Gaussian random perturbations of the experimental data are introduced. These perturbations have a standard deviation equal to the stated accuracy of the experimental data. The free parameters were then again determined using the same procedure applied to the original data. This process was then repeated a minimum of ten times for each set of data to allow a sample of free parameter values large enough to obtain a meaningful standard deviation for each parameter. The standard deviations thus obtained represent uncertainties in the determination of the free parameters arising from random uncertainties in the experimental data.

It was initially hoped that the computer program could be further extended in order to determine all of the experimentally derived parameters simultaneously for a given set of experimental data. Even if the large amount of computer time involved were not the limiting factor, another consideration would currently limit the usefulness of such an approach. This consideration stems from the accuracy of avaliable experimental data. During the course of the determination of values for the free parameters of the theory, the necessity for extremely accurate measurements became obvious. Since some of the adjustable parameters represent small corrections to the total stopping power, the cited random experimental errors in the data allowed a large number of different combinations of the parameters to yield comparable fits in some cases.

The fact that comparable fits can be obtained for several different combinations of values of the free parameters in many cases does not altogether prevent their determination. The assumed dependence or independence of each of the free parameters on target material, projectile energy , and projectile type can aid in the determination of the correct combination of parameter values. Another tool is the comparison of determined values to corresponding values obtained in other studies. This approach can provide useful constraints on some parameters or can at least provide a starting point for a multiparameter search. A third tool is the use of suggested values for some of the parameters based upon the empirically discovered systematics or theoretical calculations of other researchers.

The value of the mean excitation energy, I, should be independent of projectile energy and type, i.e., a constant for a given target material. The limited accuracy of avaliable data and/or the inaccurate determination of other parameters of the theory have yielded many different values of I for a given target material through other studies. One might expect the actual value of I for a material to lie somewhere in the range of determined values previously obtained. Constraints thus imposed may be useful in the determination of other parameters of the theory.

If the theoretical treatment (2,3) of the z^3-effect provides an accurate predictive model, then several conclusions can be drawn about the numerical value of the z^3-effect parameter, b. In the formalism of Ashley, Ritchie, and Brandt it is indicated that b should be essentially

constant for all target materials. A previous study (13) has failed to show this to be the case, possibly due to the limited accuracy of the data used.

The form of b is given in (3) as

$$b = \eta \chi Z^{1/6}$$

where χ is an adjustable parameter of the theory and

$$\eta = \eta_o (1 + 6.02 Z^{-1.19})$$

for Z greater than 12 as is the case in the present study. η_o is a constant of order unity. The product $\eta Z^{1/6}$ has such a weak dependence on Z that b is essentially independent of Z. For the targets used in the present study the values of b are:

$b = 1.94 \chi$ for Al (Z=13)

$b = 1.97 \chi$ for Cu (Z=29)

$b = 2.02 \chi$ for Ag (Z=47)

$b = 2.14 \chi$ for Au (Z=79)

The value of b should vary no more than 0.17χ over the range of target materials examined. This corresponds for a typical value of b=1.4 for a single target element with a variation in b of no more than ± 0.12 due to the target elements tested. This variation is small enough that it probably could not be an observable effect because of the limited accuracy of the data and other factors previously noted.

The data herein analyzed are useful in that they provide a selection of projectiles as well as a selection of target materials. This feature allows the investigation of possible z dependence of b.

Initially it was suggested that this investigation be extended to include higher z projectiles in an effort better to examine this dependence, but time constraints prohibited the further study.

The scaling parameters H_M, V_M, H_N, V_N, H_L, V_L used for the shell corrections have a complicated dependence on the projectile velocity through a dependence on f_i, the oscillator strengths. These oscillator strengths also depend upon the target material, thus indicating a similar dependence for the shell corrections.

Initially a four-parameter fit was attempted for both Al and Cu involving H_M and V_M, (or, in the case of Al, H_L and V_L) b and I. In neither case was a clear minimum in o observed. Indeed it was observed that if the values of I and b were left unconstrained in this search, comparable fits for a large number of combinations of H_M and V_M could be obtained. It was concluded that the limited accuracy of the data did not support a search of these four parameters simultaneously.

The shell correction parameters were then fixed at values obtained in previous studies and a search of I and b for a minimum in o was instituted. These shell correction parameters will herein be called the Bichsel-Hiiler parameters. Alternate values for these shell correction parameters based upon a suggestion by Bichsel (10) were also tried. This alternate set of shell correction parameters will be herein referred to as the Bichsel parameters.

CHAPTER IV

RESULTS

The first analysis uses the Bichsel -Hiiler shell corrections. This set of parameters were held fixed for each projectile-target combination while a two-dimensional search was done for a best fit to the experimental data, with I and b used as the free parameters of the search. The Bichsel-Hiiler shell corrections are given in tabular form below.

BICHSEL-HILLER PARAMETERS

TARGET	H_L	V_L	H_M	V_M	H_N	V_N
Al	1.4	0.7	--	--	--	--
Cu	1.0	1.0	5.0	1.0	--	--
Ag	1.0	1.0	3.0	0.4	2.5	1.0
Au	1.0	1.0	2.3	1.6	9.8	2.7

In the case of Li projectiles, a charge state correction , with k=0.95 , was used in the theoretical calculations. This value for k is given in ref. (14) as a recommended value for all targets examined here.

The results of this analysis are given in table 1. In most cases a good fit was obtained as is indicated by the small values of σ. The exceptions to this generalization are the cases of protons and alpha particles on Ag, and protons on Au.

A major implication of these results is an apparent strong dependence of b on projectile z, which can be seen graphically in fig. 1.* With the possible exception of alpha particles on Ag, b can be seen to be a clearly decreasing function of z.

Another feature is the inconstancy of I for differing projectiles on a given target. This dependence of I on projectile z is shown in fig. 2 for each target element. The projectile dependence of I appears to have a dependence on Z also, as can be seen by comparing the plots for different targets in fig. 2.

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*Dotted lines in the figures are intended to guide the eye only.

TABLE 1

ALUMINUM

PROJECTILE	I	ΔI	b	Δb	σ
Proton	170.2	0.8	2.26	0.21	0.27
Alpha	166.3	1.1	1.10	0.06	0.11
Li	163.8	1.5	0.77	0.04	0.36

COPPER

PROJECTILE	I	ΔI	b	Δb	σ
Proton	329.6	2.5	3.29	1.02	0.22
Alpha	325.8	3.9	1.49	0.17	0.41
Li	339.2	7.4	1.06	0.07	0.18

SILVER

PROJECTILE	I	ΔI	b	Δb	σ
Proton	525.7	1.2	5.22	0.21	3.14
Alpha	488.2	0.7	5.52	0.47	1.60
Li	525.6	1.5	1.37	0.02	0.94

GOLD

PROJECTILE	I	ΔI	b	Δb	σ
Proton	839.2	1.6	4.10	0.05	3.44
Alpha	809.0	10.0	2.20	0.25	0.73
Li	845.8	11.5	1.39	0.05	0.49

I-b SEARCH USING DICKSEL-MILLER SHELL CORRECTIONS
(I is expressed in units of eV.)

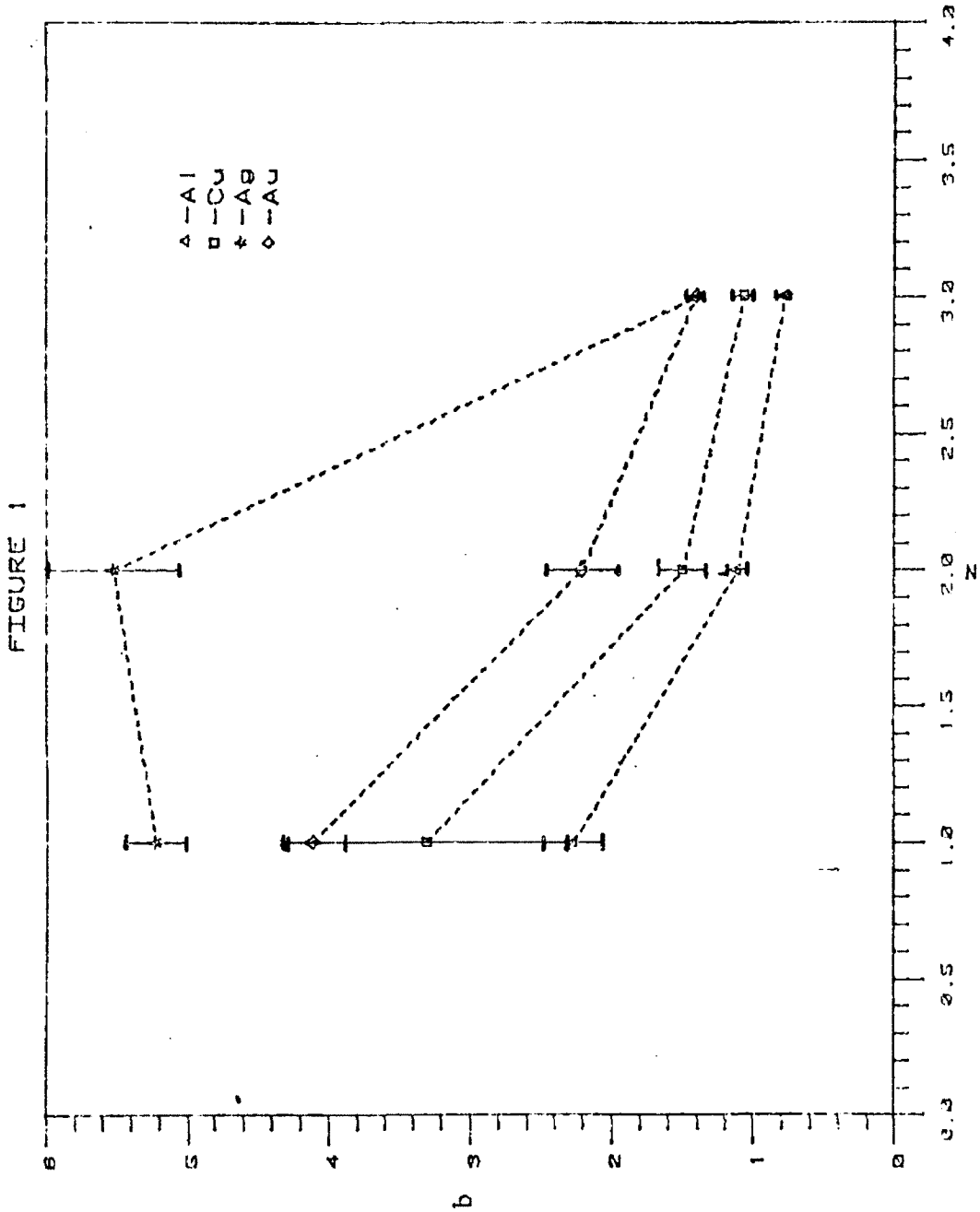
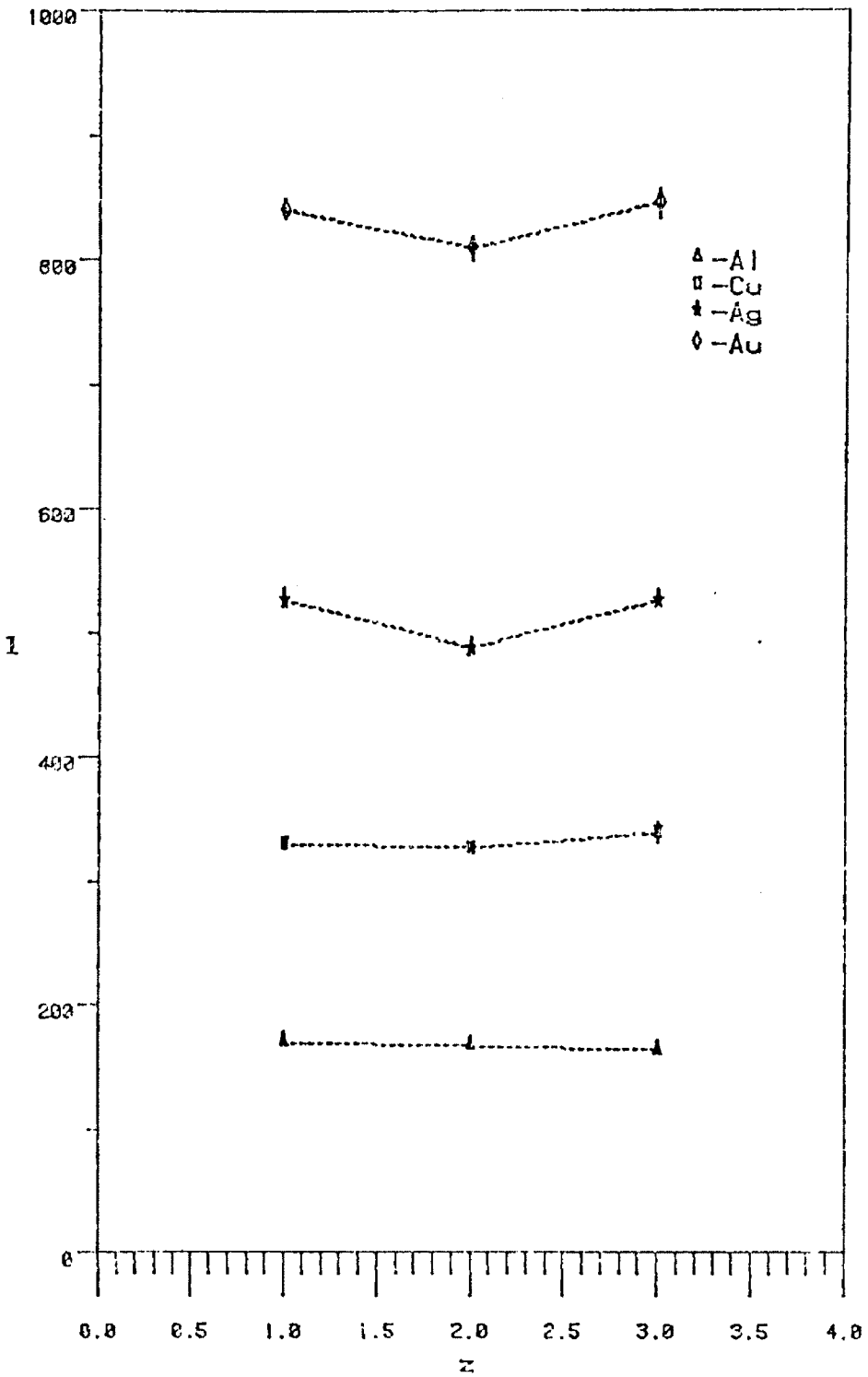


FIGURE 1

b vs. projectile z

FIGURE 2



I vs. projectile z

(I is expressed in units of eV.)

The values for b obtained in the foregoing analysis varied from a maximum of 5.52 ± 0.47 to a minimum of 0.77 ± 0.04. This variation is unacceptable if one subscribes to the supposition that b should be a constant for all target Z and all projectile z.

The second analysis of the experimental data is similar to the previous analysis in all respects except that whenever b exceeded the limit b=2.0 or went below b=1.0, it was fixed at the limit violated and a one-parameter search was continued with I as the remaining free parameter. Thus values for I were obtained for the case of b confined to a closed interval. Although this approach might seem somewhat artificial, it was done in order to ascertain whether more consistent values for I might be obtained if b were constrained to be closer to a single value.

The results of this analysis are shown in table 2. The values of I thus obtained still exhibit a dependence on projectile z and target Z similar to that in the previous analysis, as can be seen by comparing the plots for differing target elements in fig. 3. There is a distinct dependence of I on z similar to the previously observed I-z dependence.

TABLE 2

ALUMINUM

PROJECTILE	I	ΔI	b	Δb	σ
Proton	171.4	0.4	2.00	0.00	0.40
Alpha	166.3	1.1	1.10	0.06	0.11
Li	155.9	0.9	1.00	0.00	1.38

COPPER

PROJECTILE	I	ΔI	b	Δb	σ
Proton	338.3	1.0	2.00	0.00	0.81
Alpha	325.8	3.9	1.49	0.17	0.41
Li	339.2	7.4	1.06	0.07	0.18

Silver

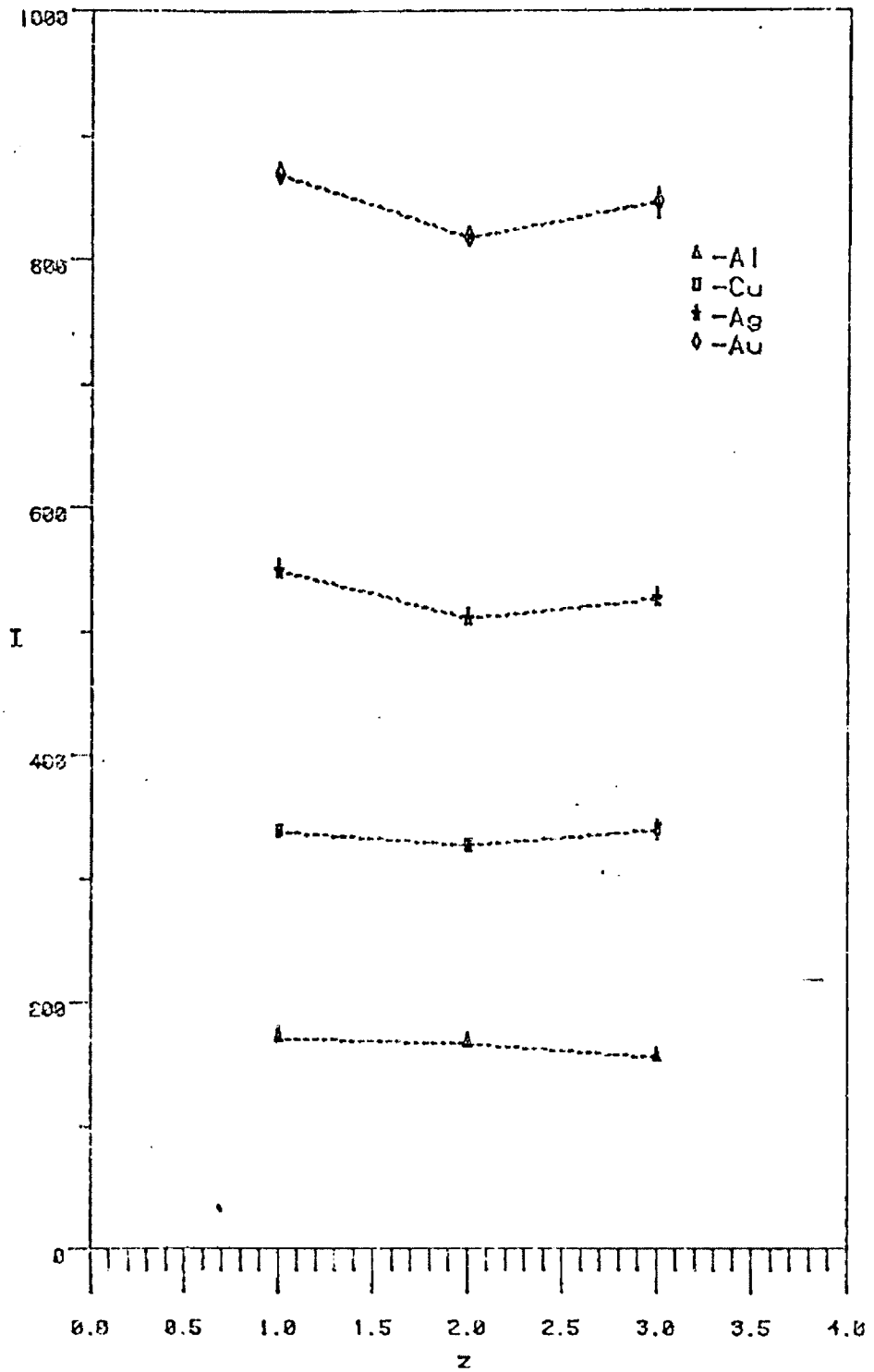
PROJECTILE	I	ΔI	b	Δb	σ
Proton	548.3	0.6	2.00	0.00	4.07
Alpha	510.7	1.5	2.00	0.00	2.07
Li	525.6	1.5	1.37	0.02	0.94

GOLD

PROJECTILE	I	ΔI	b	Δb	σ
Proton	869.1	1.7	2.00	0.00	4.13
Alpha	817.1	1.4	2.00	0.00	0.76
Li	845.8	11.5	1.39	0.05	0.49

CONSTRAINED I-b SEARCH USING BICHSEL-HILLER SHELL CORRECTIONS
(I is expressed in units of eV.)

FIGURE 3



I vs. projectile z

(I is expressed in units of eV.)

The same Miller-Eichsel shell corrections and charge state corrections were also applied in another study (13) that analyzed the same experimental data. In this other analysis b was fixed at b=1.8 and a multiplicative strength term, ξ, was applied to the z^3-effect correction term, L_1. The search for a best fit was done with ξ and I as the free parameters for the cases with ξ unconstrained, and then for ξ constrained to the closed interval [1,2]. This procedure of using ξ as a free parameter and fixing b=1.8 follows a suggestion by Lindhard (4), wherein he suggested that ξ be a constant with a value somewhat less than two. The results of the search with I and ξ as the unconstrained free parameters are shown in table 3. Good fits to the data were obtained in all cases with the exception of Ag as a target material, where reasonable fits were achieved.

There is a definite dependence of ξ on projectile z exhibited in this case that can be clearly seen in fig. 4. This trend is discussed further in ref. 15.

The dependence of I on z can be seen in fig. 5. A comparison of this plot to fig. 2 illustrates that the choice of b=1.8 and the use of the ξ factor have a noticeable effect on the projectile dependence of I.

Additional results for ξ constrained to the closed interval [1,2], and ξ fixed at =1.45 are shown in tables 4 and 5, respectively.

TABLE 3

ALUMINUM

PROJECTILE	I	ΔI	ξ	Δξ	σ
Proton	169.2	0.9	0.58	0.08	0.22
Alpha	170.5	1.4	1.67	0.05	0.23
Li	180.3	2.4	2.58	0.08	0.06

COPPER

PROJECTILE	I	ΔI	ξ	Δξ	σ
Proton	327.2	2.2	0.14	0.20	0.19
Alpha	330.0	5.5	1.30	0.12	0.42
Li	385.1	5.6	2.50	0.10	0.06

SILVER

PROJECTILE	I	ΔI	ξ	Δξ	σ
Proton	462.8	1.7	-2.20	0.08	1.15
Alpha	572.8	4.9	1.95	0.06	1.18
Li	475.0	5.1	0.30	0.08	1.53

GOLD

PROJECTILE	I	ΔI	Δξ	ξ	σ
Proton	712.4	2.2	-3.02	0.06	0.82
Alpha	793.5	7.0	0.62	0.07	0.69
Li	898.4	3.6	1.90	0.00	0.88

I- ξ SEARCH USING BICHSEL-MILLER SHELL CORRECTIONS
(I is expressed in units of eV.)

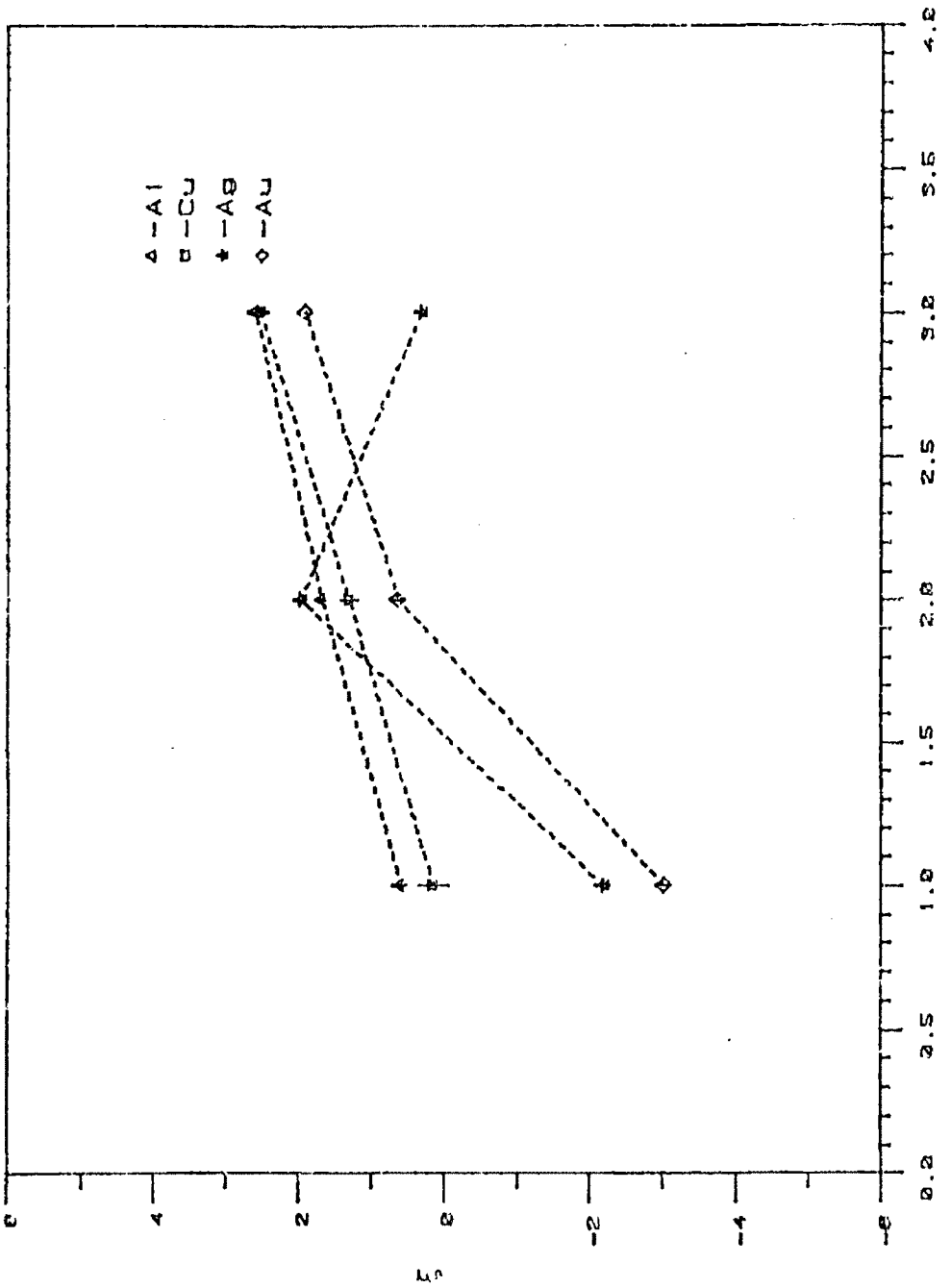
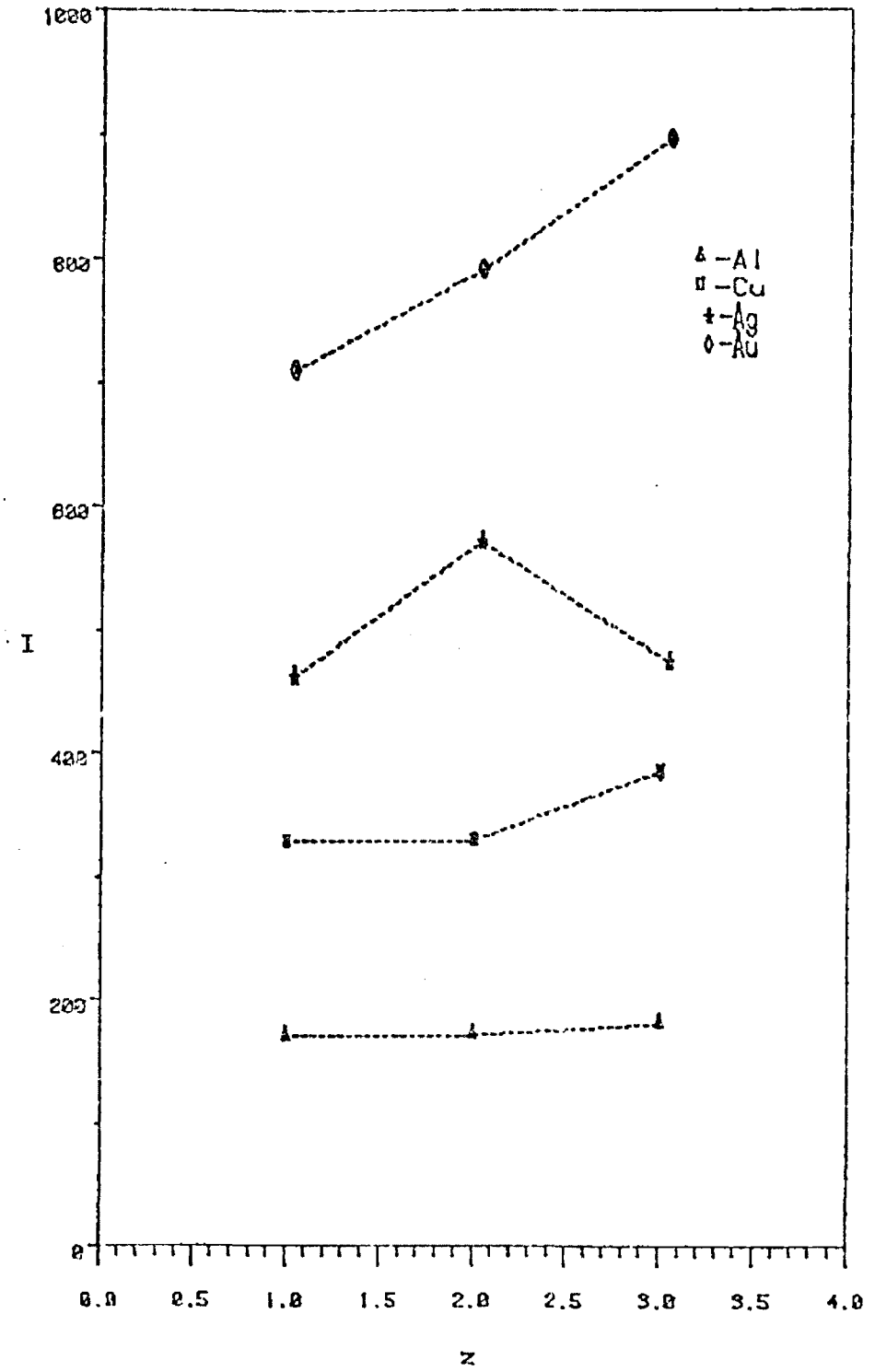


FIGURE 4

ξ vs. projectile z

FIGURE 5



I vs. projectile z

(I is expressed in units of eV.)

TABLE 4

ALUMINUM

PROJECTILE	I	ΔI	ξ	Δξ	σ
Proton	171.4	0.7	1.00	0.00	0.65
Alpha	170.5	1.4	1.67	0.05	0.23
Li	165.2	0.5	2.00	0.00	1.37

COPPER

PROJECTILE	I	ΔI	ξ	Δξ	σ
Proton	341.1	0.7	1.00	0.00	1.11
Alpha	330.0	5.5	1.30	0.12	0.42
Li	352.1	1.5	2.00	0.00	0.76

SILVER

PROJECTILE	I	ΔI	ξ	Δξ	σ
Proton	555.1	0.8	1.00	0.00	4.52
Alpha	572.8	4.9	1.95	0.06	1.18
Li	479.5	1.1	1.00	0.00	2.15

GOLD

PROJECTILE	I	ΔI	ξ	Δξ	σ
Proton	879.9	1.7	1.00	0.00	4.55
Alpha	828.4	2.2	1.00	0.00	0.87
Li	898.4	3.6	1.90	0.00	0.88

CONSTRAINED I-ξ SEARCH USING BICHSEL-NIILER SHELL CORRECTIONS
(I is expressed in units of eV.)

TABLE 5

ALUMINUM

PROJECTILE	I	ΔI	ξ	Δξ	σ
Proton	176.4	0.6	1.45	0.00	1.23
Alpha	166.7	0.6	1.45	0.00	0.82
Li	151.6	0.6	1.45	0.00	2.80

COPPER

PROJECTILE	I	ΔI	ξ	Δξ	σ
Proton	348.7	1.0	1.45	0.00	1.60
Alpha	335.3	1.4	1.45	0.00	1.38
Li	319.0	1.8	1.45	0.00	1.65

SILVER

PROJECTILE	I	ΔI	ξ	Δξ	σ
Proton	569.6	1.4	1.45	0.00	5.15
Alpha	546.9	1.2	1.45	0.00	3.33
Li	521.7	1.5	1.45	0.00	1.51

GOLD

PROJECTILE	I	ΔI	ξ	Δξ	σ
Proton	901.0	2.0	1.45	0.00	4.99
Alpha	871.6	1.9	1.45	0.00	1.29
Li	832.8	2.6	1.45	0.00	1.23

CONSTRAINED I-ξ SEARCH USING BICHSEL-NIILER SHELL CORRECTIONS
(I is expressed in units of eV.)

Yet another analysis of the same data was done using the Bichsel shell corrections below. Unfortunately, no Bichsel shell correction parameters compatible with the utilized computer code were avaliable for Au at the time this research was done.

BICHSEL PARAMETERS

TARGET	H_L	V_L	H_M	V_M	H_N	V_N
Al	1.0	1.0	12.0	0.375	--	--
Cu	1.0	1.0	6.17	2.25	--	--
Ag	1.0	1.0	4.0	2.25	18.5	2.25

Again, a charge state correction of k=0.95 is applied in the case of Li projectiles.

The application of these Bichsel parameters differs slightly from the application of the Bichsel-Miller parameters in that no z^3 correction is applied to the shell corrections above within the computer program.

A search for a minimum in σ was again instituted with the shell corrections held constant throughout and I and b used as free parameters. The multiplicative strength factor ξ is fixed at 1.0 for this portion of the analysis.

The results of this research are given in table 6. With the exception of Li on Al, the values of b all lie in the range $1.24\pm.02$ to $1.40\pm.01$. For the case of Li on Al, a three-parameter search was done in addition to the two-parameter search. The charge state parameter k was used for a free parameter in addition to I and b for this search, in an effort to give a possible explanation for the inconsistent result

mentioned above. The results of this search are included in table 6. An appreciably improved fit is obtained for a value of k=1.24 in the case of Li on Al. It has been found that there is a dependence of the charge state parameter on target Z, and the value of k for Al derived in this search is consistent with the findings of Porter (16).

TABLE 6

ALUMINUM

PROJECTILE	I	ΔI	b	Δ b	σ	
Proton	166.5	1.0	1.37	0.06	0.20	
Alpha	165.2	0.9	1.36	0.03	0.25	
Li	162.7	2.2	0.98	0.04	0.41	
*Li	161.2	---	1.37	----	0.29	(k=1.24)

COPPER

PROJECTILE	I	Δ I	b	Δ b	σ
Proton	329.0	1.4	1.30	0.04	0.23
Alpha	329.0	2.7	1.40	0.01	0.42
Li	329.0	5.9	1.26	0.05	0.76

SILVER

PROJECTILE	I	Δ I	b	Δ b	σ
Proton	483.2	2.2	1.24	0.02	0.89
Alpha	480.0	1.7	1.40	0.01	1.07
Li	467.1	10.4	1.35	0.04	1.51

I-b SEARCH USING BICHSEL SHELL CORRECTIONS
(I is expressed in units of eV.)

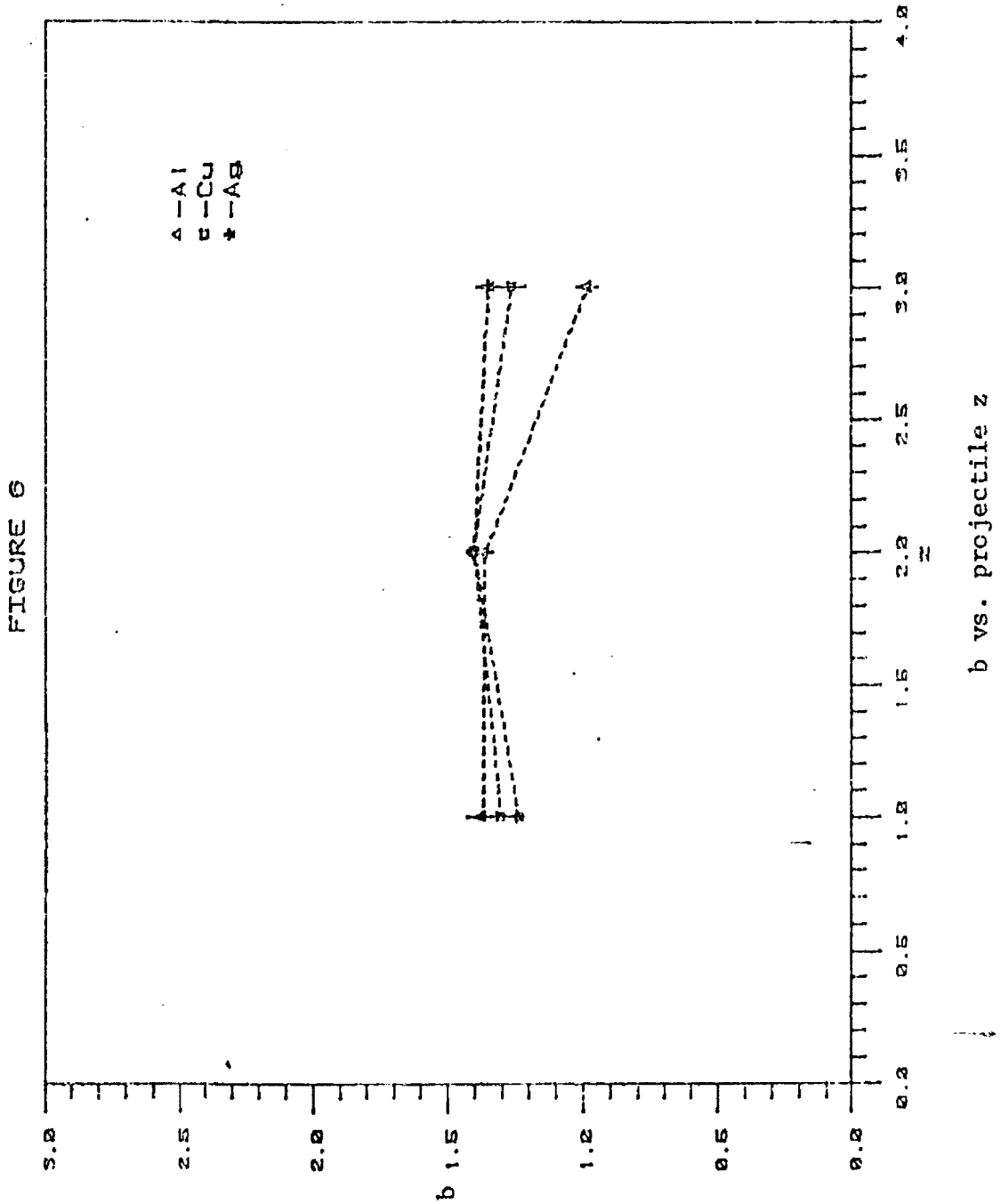
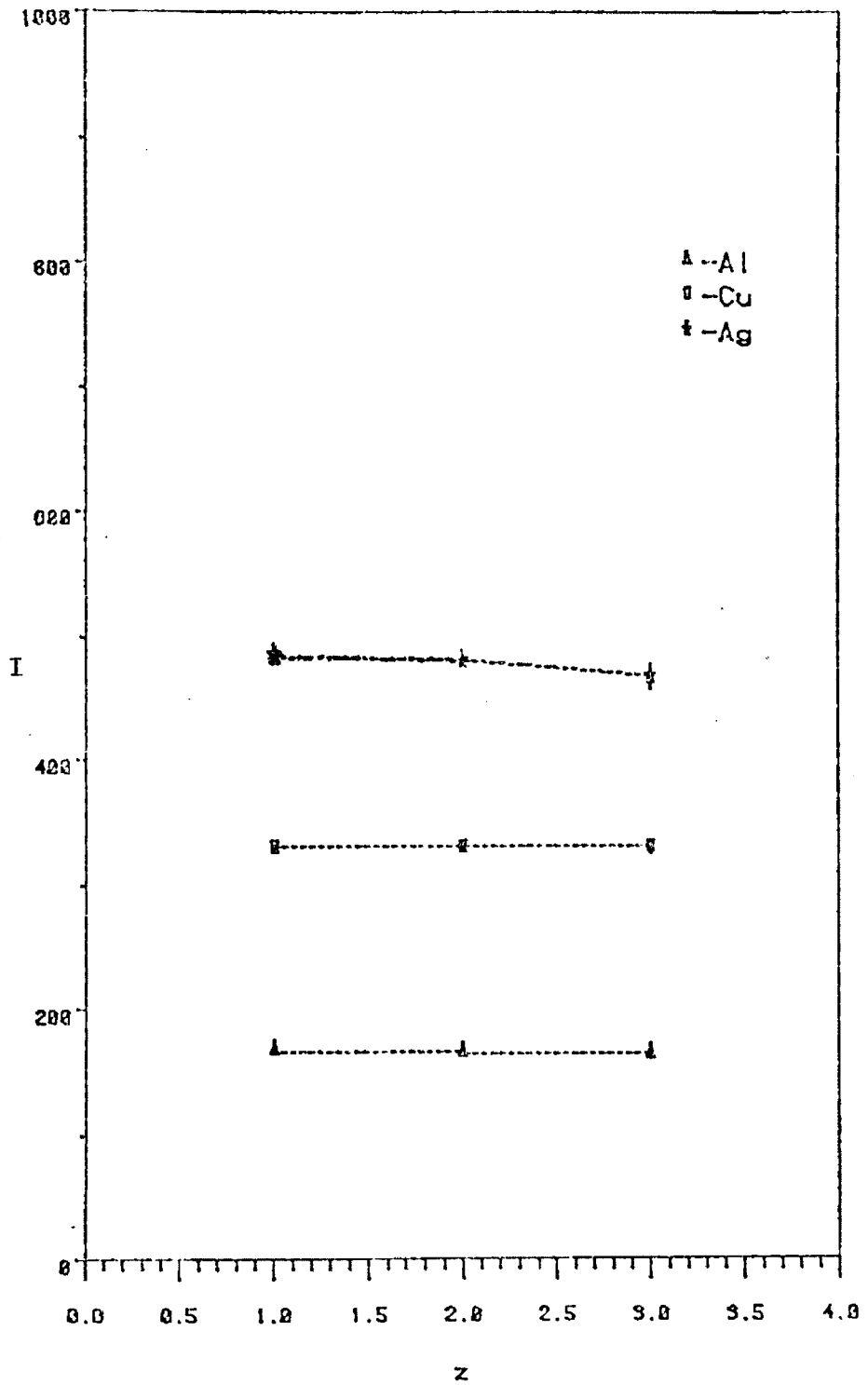


FIGURE 6

b vs. projectile z

FIGURE 7



I vs. projectile z
(I is expressed in units of eV.)

CHAPTER V

CONCLUSIONS

Analyses similar to the present one that have been recently done can provide a basis for comparison. In one such paper (13), experimental data by Ward et al. (17) and Nakata (18) were analyzed using the Bichsel-Miller shell corrections. Al, Ag, and Au were among the elemental targets, but the only projectile type was alpha particles.

In the study by Porter and Bryan (13), a search with b and I as the free parameters was done. However, b was constrained to lie in the interval [0.8,2.0] throughout the study. The lack of a minimum in σ with b thus constrained led the researchers to try another approach, but the values of b and I from the abandoned line of investigation can be compared to results obtained in the present study.

A minimum in σ was achieved for Al (13). The I and b values in this case were found to be I=177eV and b=1.0. These values can be compared to $b=1.1\pm.06$ and $I=(166.3\pm1.1)$eV (see table 2) from the present study. The values for b compare quite favorably in this case but the values for I differ by 6%. Even this magnitude of error in I may not preclude consistency betwen the two studies since the error in the experimental data analyzed in (13) is less than 4%. This large an experimental error may translate into large uncertainties in the determination of the parameters.

Analysis of Ag by Porter and Bryan (13) yielded no minimum in σ for the search. The values of I and b obtained at the limit of b=2.0 can be compared with the results shown in table 2. A value for I of (510.7±1.5)eV from the present study compares with a value of I=546eV (13). The value for I (13) is 7% above the value derived in this study, but in light of the previous observation above concerning the uncertainties in the determination of parameters in ref.13, and considering a comment (19) to the effect that the data analyzed by Porter and Bryan (13) has lower stopping power values than other studies would indicate, this result is not disturbing. This tendency toward lower S would yield I values that are higher than expected (since ln(I) is a negative term in L). This observation, coupled with the error in the determination of the parameters due to experimental error in ref. 13, prevent the conclusion that the present study is inconsistent with the findings of Porter and Bryan (13) in this value for I. The findings of the two studies are consistent in that ref. 13 indicates that a minimum in σ should be expected for Ag when b is somewhat greater than 2.0. The unconstrained I-b search in the present study found the corresponding minimum to lie at b=(5.52±.47eV) (see table 1).

The b-value obtained for Au by Porter and Bryan (13) is b=1.3. This appears to be inconsistent with b=2.20±.25 found here (see table 1). The value I=871eV also does not compare favorably with I=(809±10.0eV) from table 1.

In an additional study (20) a ξ-I search was done with the same data used in ref. 13. This search suggested a best fit ξ-values of $\xi>2$ for Al, $\xi<1$ for Ag and $\xi<1$ for Au. These findings are consistent with the present study with the exception of $\xi=1.95\pm.06$ obtained here for Ag.

Still another source is avaliable for comparison. The extensive compilation of stopping power and range tables by Andersen and Ziegler (21) give I values of I=162eV for Al, I=322eV for Cu, I=466eV for Ag, and I=755eV for Au, with protons as the projectiles. Bichsel-Niiler shell corrections yield I values that differ from these values by 5% for Al, 2% for Cu, 13% for Ag, and 11% for Au (see table 1). Bichsel parameters give values for I that differ from the Andersen and Ziegler values by 3% for Al, 2% for Cu , and 6% for Ag (see table 6). Since all of the I values here are larger than the corresponding I values in ref. 18, it would seem to indicate systematic errors in the case of the data used here (5) or in the data used in the calculation of I from ref. 18.

It is unfortunate that additional comparisons to other studies are not possible, due to the recent inclusion of the z^4 term.

The great success of the Bichsel shell corrections in enabling a fit to the experimental data that is nearly independent of both Z and z is a strong argument for the use of these corrections over the previously used Bichsel-Niiler shell corrections for the calculation of stopping power. More work needs to be done in this area to establish whether or not better fits to other data are achieved with the new shell corrections.

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