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A DYNAMICAL ANALYSIS OF THE OBSERVED TIME
RESIDUALS FOR ECLIPSING BINARY STARS

By

John H. Doolittle

B.A. Physics, University of Montana, 1973

Presented in partial fulfillment of the requirements of the degree of

Master of Arts

UNIVERSITY OF MONTANA

1976

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John B. Stewart
Dean, Graduate School

Date Sept 30, 1976

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A Dynamical Analysis of the Observed Time Residuals for Eclipsing Binary Stars (174 pp.)

Director: Thomas E. Margrave *RJM*

This thesis presents a method of analysis for determining the masses and dynamics of unseen companion bodies whose presence about an eclipsing binary star may be inferred from the cyclic variation in residuals of the observed times of eclipse. Application to the eclipsing binary system RZ Cassiopeiae indicates that the observed time residuals may be the result of the motion of two unseen companions of spectral types M3-M5. Included are FORTRAN computer codes which predict future eclipses, create and manipulate O-C residual data files, and fit observations with a theoretical O-C residual curve generated by a three-body dynamical interaction model. A tabulation of time residuals is given for fourteen eclipsing binary stars.

ACKNOWLEDGEMENTS

This study has been supported in part by University of Montana Small Research Grant 841-3R, which has allowed the author to make eclipsing variable star observations at the Blue Mountain Observatory. The literature search has been greatly simplified by access to the University of Pennsylvania's Eclipsing Variable Star card catalog, provided through the courtesy of R. S. Koch. Appreciation is also extended to Mrs. E. H. Doolittle for assistance in locating reference material at Wesleyan University, and to H. G. Krogstad for clerical assistance in assembling eclipsing variable star data files.

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CHAPTER I

INTRODUCTION

An effect of companions on eclipsing binary stars

It has long been recognized that the presence of unseen companion bodies in motion about an eclipsing binary pair could result in the observed deviation from predicted times of occurrence for the binary eclipses. The gravitational reaction of the central pair of stars to moving companions would cause it to wander with respect to the center-of-mass (COM) of the multibody system. This wandering motion of the eclipsing binary will cause its distance from the sun to be variable. Since the speed of light is a constant, the amount of time required for light to arrive from an eclipsed binary pair is directly proportional to its distance. The variation of light travel-time with distance is exhibited in the long term variation of the residuals found by subtracting the expected time of eclipse from the observed time of eclipse. The light travel-time effect has the underlying assumption that the period of revolution of the binary pair about each other is constant. Any radial motion toward or away from the sun that the entire multibody system might have will be constant and thus adds a constant amount to the intrinsic period of the eclipsing binary pairs without affecting the amplitude of the period variation.

While the light travel-time effect has often been postulated to explain the historical behavior of certain eclipsing binary systems, no

method of analysis has yet been devised which describes the masses and time-dependent positions and velocities of the possible companions which would account for the observed residuals. A method of determining these parameters by applying a technique of fitting the residual curve using dynamical models is developed in this thesis. The method is then applied to the eclipsing binary system RZ Cassiopeiae.

Abundance of multiple-body systems

Before suggesting the existence of companions to explain the time residuals observed for the light minima of eclipsing binary systems, it is worth considering whether such systems are likely. Choosing as a representative sample of the galaxy's spiral arm population, the 253 stars found within 10 parsecs (32.62 light-years) of the sun, 42.3% are found to be single stars, while the majority (57.7%) are members of multiple star systems. It seems probable that less massive dark companions would be associated with stellar systems in even greater abundance, since the greatest gravitational interaction experienced by any one companion would most likely be due to the central pair, while the other companions would only offer secondary perturbations, and gravitational stability of the system could be expected. B. M. Oliver (1972) lists examples of several stars known to have dark companions, either through astrometric or spectroscopic studies, and concludes "that a more or less continuous spectrum of systems exists between symmetrical binaries at one extreme and single stars with a giant planet, or planets, at the other." Apparently, multiple-body systems are the rule rather than the

exception, and it is quite plausible to hypothesize low-mass companions about the central pair in eclipsing binary systems.

Binary star classification

Binary star systems can be classified in one or more of several categories. Stars which appear on nearly the same line of sight yet are separated by such large distances that they are not bound gravitationally, are known as optical pairs and are of little interest. Visual binary stars are also resolvable but are indeed bound. Their motion about a common barycenter, or two-body center-of-mass, is usually well described by the laws of Kepler and observations can yield a determination of their orbital elements and masses.

For many binary systems only one point of light can be seen due to its particular distance from the earth and spatial separation and the relative luminosities of the component stars. One classification of unresolved pairs is that of the astrometric binary. Although these systems often are relatively close to the earth and the apparent magnitude of one body is beyond detectability, the presence of the dark companion is seen as a wavy displacement of the visible star rather than the linear proper motion expected for a single star when sky photographs that are taken several years apart are compared. An example of an astrometric binary is Barnard's star. Explanations for the observed motions of this star have been offered which postulate the presence of one (Van de Kamp 1962, 1969a), two (Van de Kamp 1969b), or even three (Suffolk & Black 1973) unseen companions whose masses are comparable to that of Jupiter.

Spectroscopic binaries are those whose spectra show a periodic doppler effect. Much information about the projected orbit of one or both stars about the system's COM can be derived by noting the temporal manner in which the spectral lines are shifted towards the red as a star recedes and towards the blue as it approaches the point of observation. Binary stars whose motions are in a plane other than the plane of the sky (i.e. the plane whose normal is the line of sight) will have radial velocities with respect to the earth and might be expected to exhibit such a doppler effect. A maximum doppler effect for any system occurs when the system's orbital plane lies along the line of sight to earth. These systems are particularly interesting because their orientation causes each star to periodically eclipse the other and are thus referred to as eclipsing binaries. Since eclipsing binaries are also spectroscopic systems, information about the relative orbits and masses of the stars can be determined through spectroscopic analysis.

Eclipsing binary star light curves

The variation of the brightness of eclipsing binary stars is conveniently displayed by a graph depicting the system's apparent magnitude versus time. Usually referred to as a light curve (figure 1; Rossati, 1970), it is characterized by constant brightness except during times when eclipses cause a reduction in the amount of light reaching the observer. Since the two stars may be of different absolute magnitudes, alternate eclipses can differ in depth as measured from the near constant brightness seen during times outside of eclipse. The deeper or primary eclipse occurs when the brighter star is eclipsed by the less

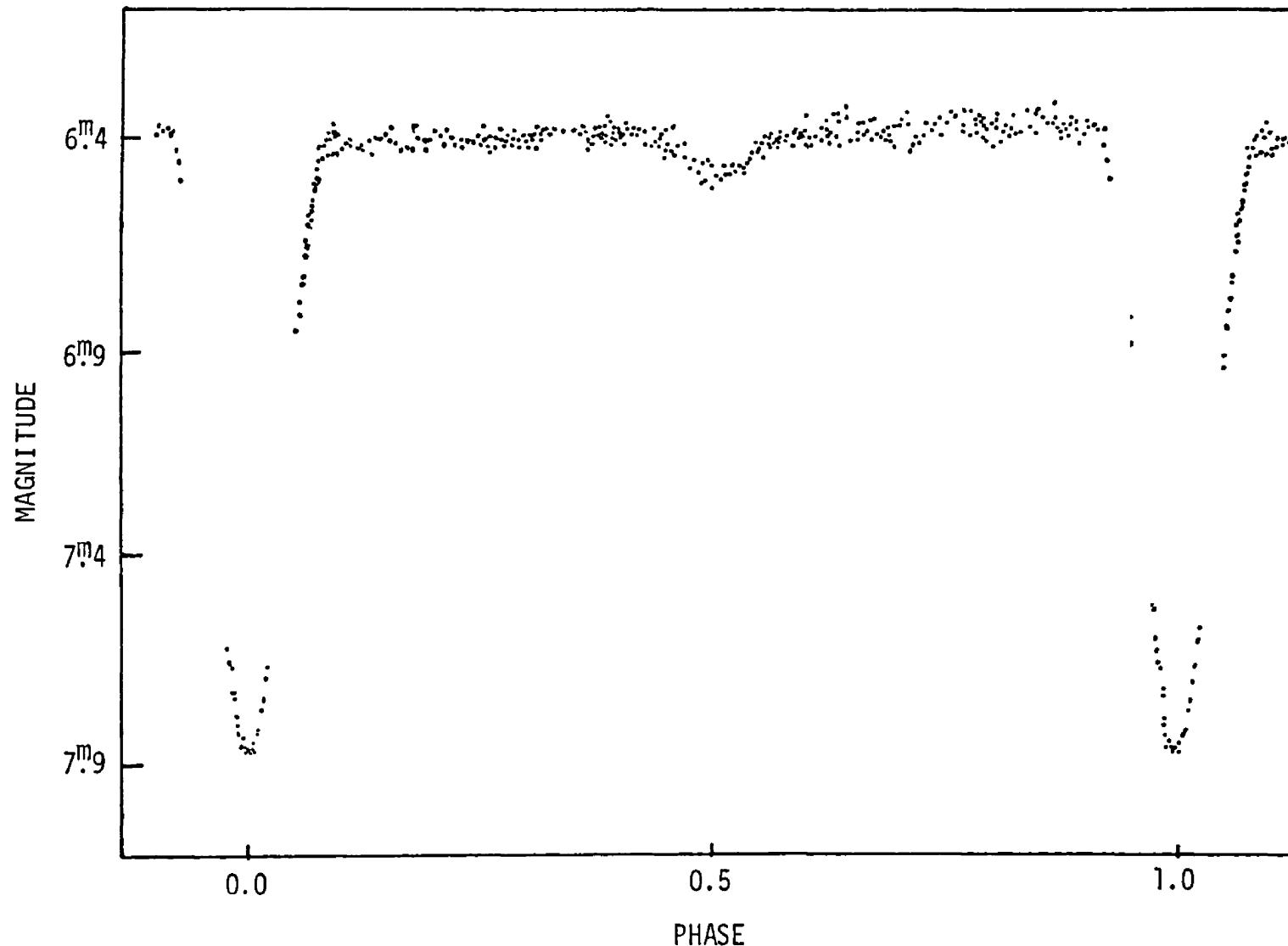


Fig 1. Light curve of RZ Cassiopeiae

bright star. The secondary eclipse occurs approximately one-half period later when it is the less bright star which is being blocked from view by the brighter star. Since the light curve is repetitive, the period can be defined between any two times with the same relative phase. However, by convention the time of zero phase occurs at the midpoint of the primary eclipse.

Predicting eclipses

When the period (P) of an eclipsing binary is well defined, the prediction of times of occurrences of eclipses (T_{calc}) can be made simply by extrapolating from a known time of occurrence (T_{known}) of a past eclipse.

$$T_{\text{calc}} = T_{\text{known}} + E * P \quad (1.1)$$

The epoch (E) is necessarily an integer indicating the number of events which have occurred since the known time.

The linear elements of this expression (i.e. P and T_{known}) are occasionally revised for binary systems which display apparent variations in eclipse period.

The difference between the observed time of an eclipse (O) and the time calculated from the linear elements (C) is referred to as an O-C time residual. A plot of O-C residuals against time, in years (appendix 1), or against epoch number, shows a temporal history of the system's eclipse period. The sign of any particular residual indicates whether that eclipse occurred earlier (negative) or later (positive) than predicted. The slope of the O-C curve shows whether the eclipse period at

that time tends to be shorter (positive) or longer (negative) than the value of the period chosen for the calculations.

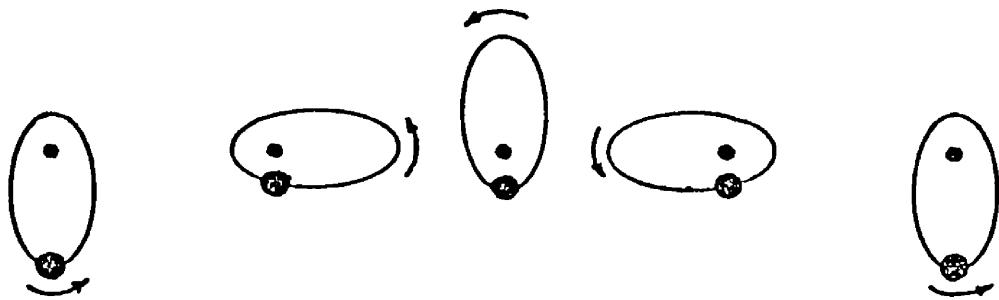
Variations of eclipse period

The period variations exhibited in the O-C residual curves of eclipsing binary stars may be classified as being either continuous or discontinuous.

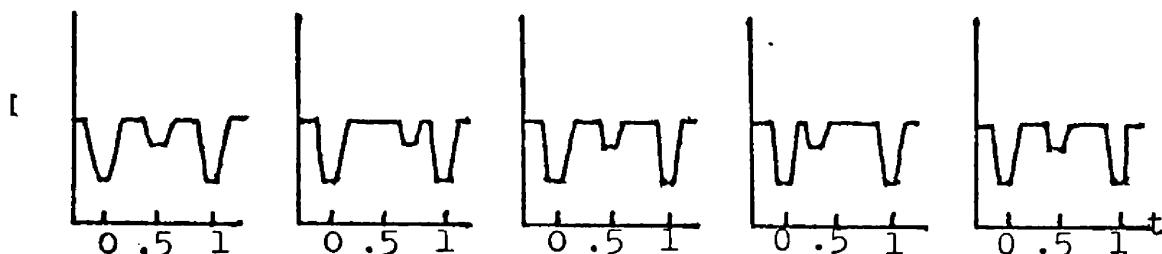
Continuous variations. A possible explanation of a continuous variation which is periodic could be apsidal rotation. If the relative orbit of the eclipsing pair of stars is an ellipse of eccentricity greater than zero, then a rotation of the line of apsides is possible. The effect of the variable orbital velocities associated with elliptical orbits compounded by the rotation of the orbit is to produce a sinusoidal variation in the O-C curve (figure 2). Observational evidence for apsidal rotation is found when a graph of the time residuals for the secondary minima has the same shape as the graph of primary minima residuals but is of the opposite phase (Wood, 1950). In such cases the secondary minima will oscillate about the 0.5 phase point of the light curve with a period corresponding to the period of apsidal rotation.

It is interesting to note that a light travel-time effect also is associated with these systems. The spatial excursion of the point of eclipse, however, is relatively small. Typically this effect has an O-C amplitude of less than one minute during the cycle of apsidal rotation and therefore may be considered to be of minor importance.

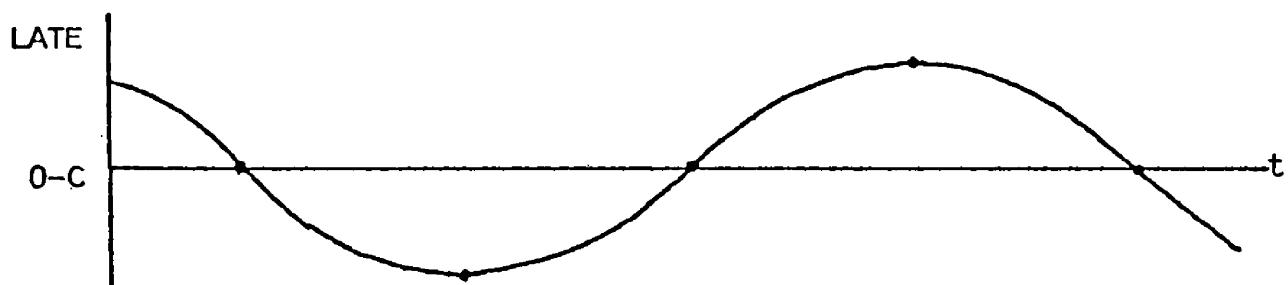
In other cases in which the O-C curve shows continuous variation, the secondary minima residual plot again shows the same form as the



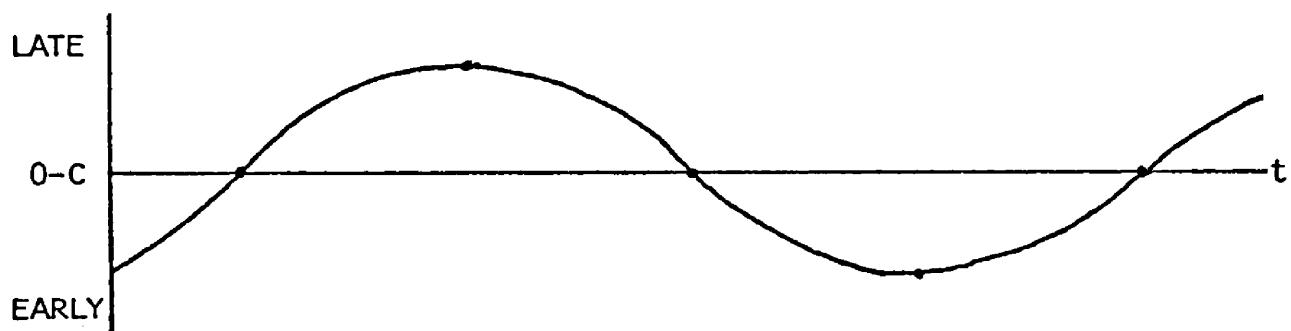
• HOT ● COOL
ROTATION OF APSIS



LIGHT CURVES



PRIMARY RESIDUALS



SECONDARY RESIDUALS

Fig. 2. Evidence of apsidal rotation

primary minima residual plot, but the phases of both curves are the same. In such cases the secondary minima remain fixed halfway between the primary minima, further excluding apsidal rotation as an explanation. The continuous variation exhibited by the O-C curve may then be attributable to the presence of a third body in motion about the COM of the entire system. The displacement of the eclipsing binary stars with respect to the three-body center-of-mass is observed at earth as a light travel-time effect. Translation of the three-body COM relative to our heliocentric coordinate system will cause a difference between the observed eclipse period and the absolute period. However, since this difference is a constant, it is of little consequence in a study of period variation. If there is more than one periodicity in the O-C curve it may be necessary to postulate the presence of additional bodies to satisfactorily explain the nature of the system.

Discontinuous variations. The second category of eclipsing binary O-C curves is characterized by abrupt discontinuities attributed to sudden changes in the eclipse period. The explanation for this is mass ejection by either or both of the binary's components. According to Kepler's Third Law,

$$(m_1 + m_2)P^2 = a^3 \quad (1.2)$$

where m_1 and m_2 are the masses of the components, P is the orbital period of revolution, and a is the average linear separation between the components, a change in period can be caused by either a change in separation or in either body's mass. F. B. Wood (1950) has shown that while the ejection of mass can only provide an increase in period, the

eruptive force of this action working within the gravitational field of the other star can provide either an increase or a decrease in period as a result of a change in the separation, depending on the point on the star's surface at which the thrust acts. He concludes that the greater effect is ascribed to the explosion with less importance given to the effect of mass loss and points out that sometimes the two will reinforce each other, and at other times they will cancel. Quantitatively, he finds that a mass loss of about 10^{-6} solar masses (M_\odot) can cause a period change of one second in a typical system which has a total mass of $2M_\odot$, a period of revolution of about 2 days, and a mean separation of 0.039 A.U.

In cases where the mass loss model is applicable, the primary and secondary O-C residual curves are effected equally and maintain the same phase as they do for systems classified under the unseen companion hypothesis. Therefore, for systems whose residual periodicity is not well defined, there remains some ambiguity as to which of these latter two groups it belongs. It seems likely that perturbing forces of a multiple-body system acting to change the separation between the eclipsing binary components could also cause sudden jump discontinuities in an otherwise periodically varying O-C curve.

Selection criteria

The intent of this study is to investigate eclipsing binary stars whose behavior is explained by the unseen companion hypothesis. Selecting suitable stars for analysis began with a search through the Flower and Cook Observatory Finding List of Eclipsing Variable Stars (1963).

Attention was given only to those entries which are noted as having variable eclipse periods. A further selection criteria imposed was that of observability at latitude 46.8°N from early July until the middle of November, the operating season of the University of Montana Blue Mountain Observatory. This allows for continuing investigation at that site of any stars determined to be of interest. An upper limit to apparent magnitude was set at 10^m5 . Table 1 shows the uneclipsed apparent magnitude (m_v), the depth in magnitude of both the primary (Pri.) and secondary (Sec.) eclipses, and the orbital eccentricity (e) of several candidate stars selected.

A literature search was begun with an initial objective of finding the published values of eccentricity of the relative orbits for each candidate star. Systems with an eccentricity greater than 0.10 were considered to be susceptible to apsidal rotation and were excluded from further consideration under the multiple-body hypothesis. It should be noted, however, that a study of the secondary minima behavior and a comparison of the secondary minima O-C residuals with those of the primary minima might reveal that apsidal rotation is, in fact, not evident. In such a case, a system would warrant further consideration for application of the three-body analysis.

Observed times of minima

In order to determine whether the period variation noted in the Finding List mentioned above is of a continuous or discontinuous nature, it next became necessary to assemble a list of observed times of primary minima to be used in plotting an O-C curve for each candidate star.

TABLE 1
CANDIDATE PROGRAM STARS

Star	m_v	Pri.	Sec.	e	Ref.
KO Aql	8. ^m 5	1. ^m 0	0. ^m 1	0.02	1, 2
RY Aqr	9.0	1.3	0.1	0.00	1, 3
BF Aur	8.5	0.7	0.7		1,
44i Boo	6.5	0.6	0.5	0.00	1, 4
DO Cas	8.5	0.4	0.1	0.13	1, 5
RZ Cas	6.4	1.5	0.1	0.052	1, 4
TW Cas	8.5	0.6	0.1	0.071	1, 6
U Cep	7.0	2.8	0.1	0.47	1, 7
XX Cep	8.5	1.1	0.1	0.14	1, 8
U CrB	7.5	1.2		0.13	1, 4
SW Cyg	9.5	2.6		0.30	1, 4
WW Cyg	10.0	3.8	0.1	0.00	1, 4
TW Dra	7.5	2.3	0.1	0.027	1, 4
TX Her	8.0	0.7	0.4	0.00	1, 4
? Ori A	3.0	0.2		0.016	1, 4
Z Ori	10.0	0.9	0.1	0.23	1, 6
AT Peg	8.5	0.7	0.2	0.024	1, 9
RT Per	10.5	1.4	0.2	0.043	1, 6
U Sge	6.5	3.6		0.035	1, 4
λ Tau	4.0	0.5	0.1	0.055	1, 4
TX UMa	7.1	2.2		0.162	1, 10

References:

1. Flower and Cook Ob., Finding List of Eclipsing Variables.
2. Ap.J., Vol. 102, p. 470.
3. A.J., Vol. 56, No. 1, p. 3.
4. Lick Observatory Bulletin, No. 521.
5. A.J., Vol. 71, No. 1, p. 44.
6. A.J., Vol. 76, No. 6, pp. 547-8.
7. Obs., Vol. 69, p. 203.
8. Var. Stars, Vol. 12, No. 21.
9. P.A.S.P., Vol. 84, p. 432.
10. P.A.S.P., Vol. 52, p. 287.

This task was greatly simplified through access to the University of Pennsylvania's eclipsing variable star card catalog, which is a rather complete list of literature references categorized by star name. More than 200 journal articles were reviewed in varying degrees of thoroughness and a total of about 1400 heliocentric times of minima for 14 stars were compiled chronologically (appendix 1). These lists will remain archived on magnetic tape at the University of Montana where they will be accessible for future related investigations. Included in the lists are two photoelectric minima of RZ Cassiopeiae (Margrave) and one unpublished minima of AT Pegasi from observations made at the Blue Mountain Observatory.

The process of compiling the data lists was accomplished through the execution of the FORTRAN computer program code FILCHG (appendix 2). The information is written onto a disk file to facilitate further data handling. Since the code has been designed with full update capability, the data files need never be considered closed. As more eclipse times become available, either through additional journal research or through observation, these can be added.

The O-C time residuals corresponding to each observed event are determined by the linear elements of eq. (1.1). The set of values for O-C, and thus the shape of the residual curve, will depend on the choice of both a known time of primary eclipse occurrence (T_{known}) and the assumed eclipse period. The values of the periods used to calculate the files found in appendix 1 are those recently quoted in the literature. Although these values closely fit the current observations, it is not correct to suggest that they are the genuine orbital periods of one

binary component about the other, since the fact that the periods vary implies that they may include the effects of spatial motion (i.e. apsidal rotation or perturbations caused by unseen companions).

Plots of the O-C residuals for 14 stars are given in appendix 1. Also included there are a few plots of secondary minima O-C residuals.

Average eclipse period

It was found to be advantageous in this study to accept as a useful value of the eclipse period one which causes the O-C residuals of the primary minima to be symmetrically distributed about the line O-C= 0.0. This step is accomplished by the FORTRAN code AVEPER (appendix 2). Admittedly, this method is susceptible to statistical biasing due to occasional inaccurate data points or intervals of few observations, yet defining an "average period" has a definite advantage in a study which is concerned with the entire recorded history of a binary since published values usually apply only to limited time spans.

Visual Detectability

It is important to consider whether a hypothetical companion will be visually detectable, since if it proves to be theoretically possible to observe a companion but such an observation has not been made, then the validity of the hypothesis will be in doubt. The following analysis is pursued to insure that in any subsequent postulation of unseen companions associated with stellar systems, the hypothetical companions would indeed be non-visible.

In a statistical study on the completeness of binary discovery, W. D. Heintz (1969) adopts the "measure of difficulty" of discovering a visual binary which was previously introduced by E. Öpik in 1924. When the visual binaries discovered over the past decades were plotted in a diagram relating the angular separation (ρ) of the two component stars to the difference in their apparent visual magnitudes (Δm_V), it was found that nearly all are located in a strip bounded by the straight lines $0.22\Delta m_V - \log \rho = 0.47$ and 0.99 . Heintz assumes that three ranges of the discoverability index, defined as $D = 0.22\Delta m_V - \log \rho$, are delimited by $D = 0.5$ and $D = 1.0$. Binary pairs for which $D < 0.5$ have been discovered prior to the past decades and are considered to be "completely discovered", while those with $0.5 < D < 1.0$ are "half discovered", and those with $D > 1.0$ remain "undiscovered". In the present application $D > 1.0$ is considered a criteria for non-detectability.

An eclipsing binary pair appears visually as one point of light. The detectability index (D), as well as intuition, suggests that a dim companion would be seen more easily if the brightness of the visible point of light was reduced to a minimum. Since this occurs at the time of primary eclipse, it is the published value of apparent visual magnitude for the midpoint of the primary eclipse which is used in the calculation of the detectability index.

To determine the apparent visual magnitude of the dark companion it is first necessary to find its fractional luminosity (L/L_\odot) in solar units through the mass-luminosity relation,

$$L/L_\odot = (M/M_\odot)^\alpha \quad (1.3)$$

where M/M_{\odot} is the companions fractional mass, also in solar units, and the exponent α is found to vary with mass as seen in Table 2 (Allen).

TABLE 2
MASS-LUMINOSITY RATIO

$\log M/M_{\odot}$	$\log L/L_{\odot}$	α
-1.0	-2.9	2.9
-0.8	-2.5	3.125
-0.6	-2.0	3.33
-0.4	-1.5	3.75
-0.2	-0.8	4.0
0.0	0.0	4.0
+0.2	+0.8	4.0

Assuming a value for the companion mass and choosing the appropriate value for α , the luminosity is easily obtained. This is used in eq. (1.4) to determine the magnitude that the companion would have if it were located at a standard distance of 10 parsecs and if radiation of all wavelengths was included. This quality is usually referred to as an absolute bolometric magnitude (M_{bol}).

$$M_{bol} = 4.77 - 2.5 \log(L/L_{\odot}) \quad (1.4)$$

The absolute bolometric magnitude of the sun is +4.77.

In order to convert the absolute bolometric magnitude of the companion into an absolute visual magnitude (M_V), it is necessary to add (eq. 1.5) the appropriate bolometric correction (BC).

$$M_V = M_{bol} + BC \quad (1.5)$$

As explained by T. L. Swihart (1968), "the BC is a measure of the ratio

of the total energy radiated by a star to that which it radiates in the visual region of the spectrum". The apparent visual magnitude (m_v) of the companion is calculated according to eq. (1.6),

$$m_v = M_v + 5 \log(r/10) \quad (1.6)$$

which is dependent on the distance (r) in parsecs of the system from earth. Finally, the difference in apparent visual magnitude between the eclipsing binary at primary minimum and the companion is found by subtraction.

Since one parsec is the distance at which one astronomical unit subtends an angle of one second of arc, a knowledge of the distance (r) to the system in parsecs and the spatial separation (d) in A.U.'s between the companion and the visible star, as measured across the line of sight, will give the angular separation in seconds of arc at that time.

$$\rho(") = d(\text{AU})/r(\text{pc}) \quad (1.7)$$

The magnitude difference and angular separation are then used to calculate the index of discoverability.

$$D = 0.22\Delta m_v - \log \rho \quad (1.8)$$

In application, it is easy to determine detectability of a suggested companion by examining its mass and greatest separation from the visible component of the system. A study has been made using the FORTRAN code DETECT (appendix 2) which incorporates the preceding analysis. The results shown in figure 3 show the curve corresponding to $D = 1.0$. In the region to the right of the curve, D is less than 1.0, indicating that such stars would be visible, while stars located in the left-hand region are considered non-detectable because D there is greater than 1.0. Since

DETECTABILITY

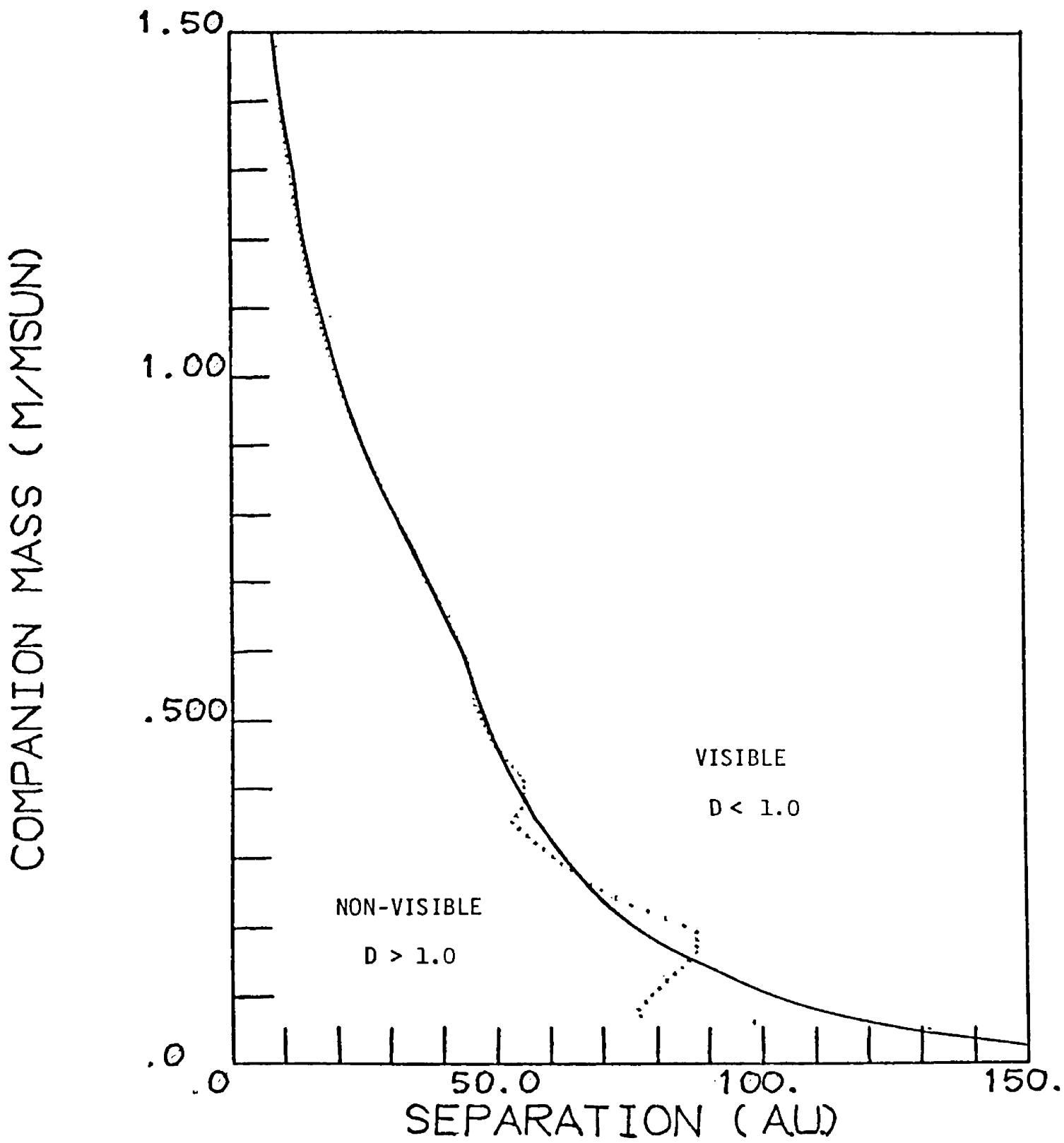


Fig. 3. Detectability of companions

the index D is dependent on distance and apparent visual magnitude, a separate study must be made for each star system considered. Figure 3 is characteristic of RZ Cassiopeiae, whose distance is about 90 pc and mid-eclipse magnitude is about +7^m.88.

CHAPTER II

PHYSICAL ANALYSIS

General procedure

Once it is assumed that one or more unseen companions are causing the observed variation in the eclipse period of a binary, it then becomes desirable to derive the parameters describing the companion(s) and the dynamics of the multi-body system.

A method of trial curve-fitting was used wherein a set of initial parameters is assumed and applied to a physical model which describes the relationships among all the bodies involved. Incremental time steps are then taken to simulate the hypothetical motion of the masses. The displacement of the eclipsing binary pair toward or away from the earth is used to determine the light travel-time effect during the time interval covered by eclipse minima observations. This result is superimposed on the empirically-derived O-C plot and a judgement is made as to how well the theoretical curve fits the actual history of the eclipsing binary. The best-fitting trial then gives the parameters which, based on the assumptions of the model, describe the dynamics of the system.

The models evolved with the specific case of RZ Cassiopeiae in mind. It seems evident from the double periodicity seen in the O-C residual curve of RZ Cassiopeiae (appendix 1) that at least two

companions are necessary to fit the past observations. Therefore, for considerations of spatial displacement, all analysis which follows assumes three coplanar point masses; two which represent the companions, and the third which represents the total mass of the binary pair located at the common barycenter. The simplifying assumption that the binary is reduced to a point seems to be justified in a first-order calculation, since the separation of the binary pair is typically less than 1% of the distance to the closest companion. The direction of revolution of all bodies about the system's center-of-mass is arbitrarily chosen to be counterclockwise with increasing time.

Circular orbits

The simplest model which was considered to explain the behavior of RZ Cassiopeiae is one in which the orbits of the companions are circular. The orbital motion (figure 4) of the closest companion (m_2) about the two-body center-of-mass (COM_1) causes a simultaneous circular motion of the binary. The total binary mass (m_1) is known from published spectroscopic results. Adopting the shorter periodicity exhibited in the O-C curve as the value for P_1 (in years), and deducing a value of the distance (r_1) of the binary from COM_1 from the amplitude of the O-C curve (i.e. $r_1 = \text{O-C half-amplitude in days} \times \text{speed of light in A.U./day}$), then the companion mass (m_2) in solar units, and its distance (r_2) from COM_1 can easily be determined through a simultaneous solution of eq. (2.1), which is the definition of the center of mass, and Kepler's Third Law (2.2).

$$m_1 r_1 = m_2 r_2 \quad (2.1)$$

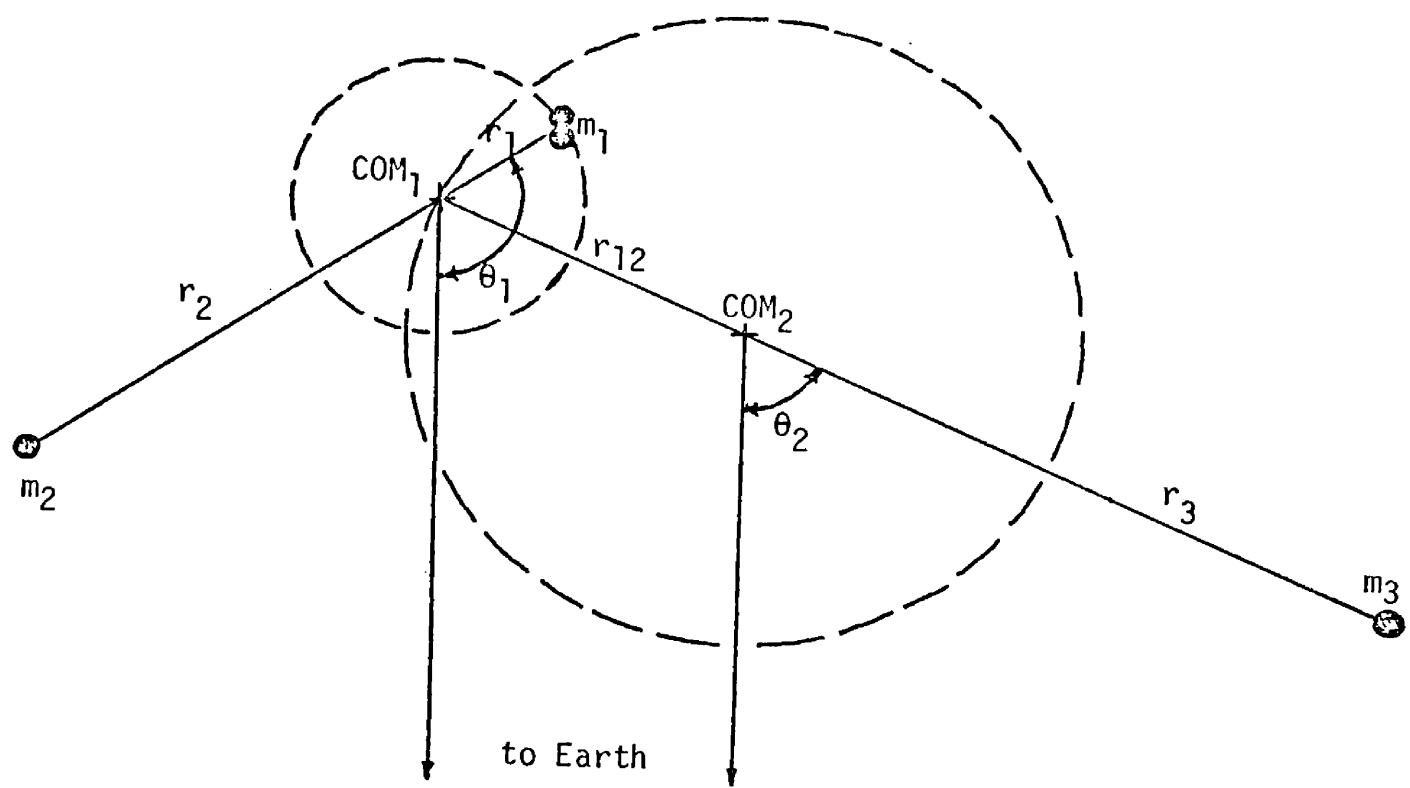


Fig. 4. Epicyclic model

$$(m_1 + m_2)P_1^2 = (r_1 + r_2)^3 \quad (2.2)$$

Because the angular velocity is constant, the angular displacement (θ) at any time (t) may be found by

$$\theta = 2\pi \frac{t}{P_1} + \theta_0 \quad (2.3)$$

where θ_0 gives the initial longitude.

The motion of the outer companion is determined by a similar treatment. The masses m_1 and m_2 are combined and assumed to be located at COM₁. The values of P_2 and r_{12} are again obtained from the O-C curve and are used to determine r_3 and m_3 (see figure 4).

$$(m_1 + m_2)r_{12} = m_3r_3 \quad (2.4)$$

$$(m_1 + m_2 + m_3)P_2^2 = (r_{12} + r_3)^3 \quad (2.5)$$

The epicyclic displacement of the binary produces the expected superposition of sinusoidally-varying O-C values once the factor relating distance to time delay (i.e. the speed of light) is applied.

Through a procedure of making trial fits of the O-C curve by slightly varying the input parameters, a best fit was found for the case of RZ Cassiopeiae. The inadequacy of the model was immediately apparent. Since it is based on circular geometry, this model can only generate the sum of two symmetrical sine waves. The periodic variations seen in the O-C curve of RZ Cassiopeiae (appendix 1) show a definite skewness. Therefore it was decided to pursue the problem further by introducing elliptical orbits.

Elliptical Orbits

The method used to determine the instantaneous position of a body which is constrained to an elliptical orbit is outlined in most texts on celestial mechanics (e.g. Roy, 1965). It is necessary in this analysis to specify the following set of parameters (figure 5) which define the relative orbit and the position of the body at a known time:

T = Period of revolution

a = Semi-major axis

e = Eccentricity

t_0 = Time of perapse passage

ω = Longitude of the perapse

where ω is measured with respect to the line of sight and in the direction of revolution.

The calculations begin by determining the mean angular velocity

$$\eta = 2\pi/T \quad (2.6)$$

This is used in finding the mean anomaly M which corresponds to a specific time t through the relation

$$M = \eta(t - t_0) \quad (2.7)$$

The mean anomaly is inserted into Kepler's Equation,

$$E = M + e \sin E \quad (2.8)$$

which must be solved through a process of converging iteration. The resulting value of the eccentric anomaly (E) is used to locate the body in terms of the radius vector r measured from the focal point of the ellipse and the angular displacement f measured from the perapse, using the standard equations

$$r = a(1 - e \cos E) \quad (2.9)$$

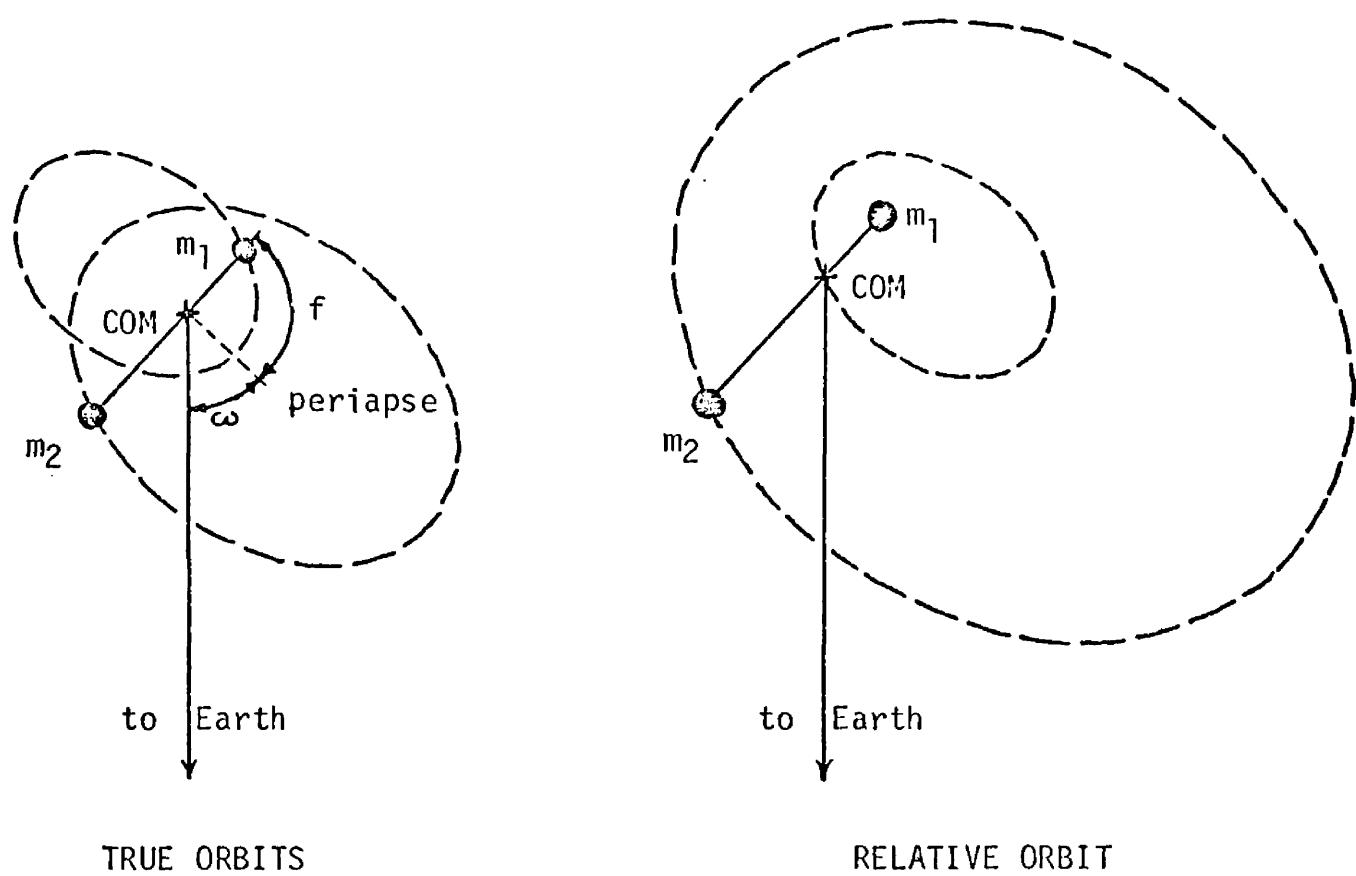


Fig. 5. Elliptical Orbits

$$f = 2 \arctan \left(\sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2} \right) \quad (2.10)$$

The longitude of the periapse is added to refer the angular displacement to the line of sight, whence

$$\Theta = \omega + f \quad (2.11)$$

The focal points of both of the orbital ellipses (figure 5) are located at the two-body center-of-mass. Although the two orbits are proportionate ellipses, the longitude of each periapse differs by 180^0 with respect to the other.

The companion mass and orbital semi-major axis are also found from two-body mechanics. At any time, the center-of-mass expression (eq 2.1) must hold true. At the particular time when the bodies are located at apastron, their distances (r_1 and r_2) from the COM can be related to the semi-major axes of their orbits by the following expressions:

$$r_1 = a_1 (1 + e) \quad r_2 = a_2 (1 + e) \quad (2.12)$$

Inserting these into eq. (2.1), we obtain..

$$m_1 a_1 = m_2 a_2 \quad (2.13)$$

Since the semi-major axis of the relative orbit is the sum of both true orbit semi-major axes, Kepler's Third Law may be expressed here as

$$(m_1 + m_2)P^2 = (a_1 + a_2)^3 \quad (2.14)$$

Solving eq. (2.13) for a_2 and substituting into eq. (2.14)

$$(m_1 + m_2)P^2 = a_1^3 \left(1 + \frac{m_1}{m_2}\right)^3 \quad (2.15)$$

This may be solved for m_2 which, in turn, may be used in eq. (2.13) to find a_2 .

In the calculational procedure, it is easiest to write the quartic expression (eq. 2.15) as

$$g(m_2) = (m_2^4 + m_1 m_2^3)P^2 - a_1^3(m_2 + m_1)^3 \quad (2.16)$$

and then to find the value of m_2 which causes $g(m_2)$ to be equal to zero.

By Descartes' rule of signs, there is, at most, one positive value of m_2 .

At this point the analysis follows the same general sequence as for the circular model. The superposition of ellipses has a greater degree of freedom which allows the skewness displayed in the O-C curve of RZ Cassiopeiae to be fitted more easily.

It was decided to choose as the fundamental plane of the three-body motion that which is defined by the binary itself. From spectroscopy the inclination of this plane (figure 6; X, Y) for RZ Cassiopeiae is known to be about 82° (82.14° , Horak, 1951) with respect to the plane of the sky (Y', Z'). Observations of light travel-time effects are due strictly to the component of the eclipsing binary pair's motion which is directed toward or away from the earth, and therefore the sine of the inclination is used to project the x-value of the binary's position onto the line of sight. According to Chambliss (1976), the value of the total binary mass of RZ Cassiopeiae is 2.4 solar masses ($m_1 = m_a + m_b$, where $m_a = 1.75M_\odot$ and $m_b = 0.61M_\odot$).

The best fit obtained using an elliptical orbit model is shown as a solid curve in figure 7. The rms deviation from the observations is about $\pm 5^{m}15^s$. The orbital parameters which lead to this result are given in table 3. Also included there are the companion masses expressed in solar units. While these masses are great enough to be luminous stars, their luminosities are faint enough such that they are not visually detectable at their greatest separation from the central pair (see figure 3).

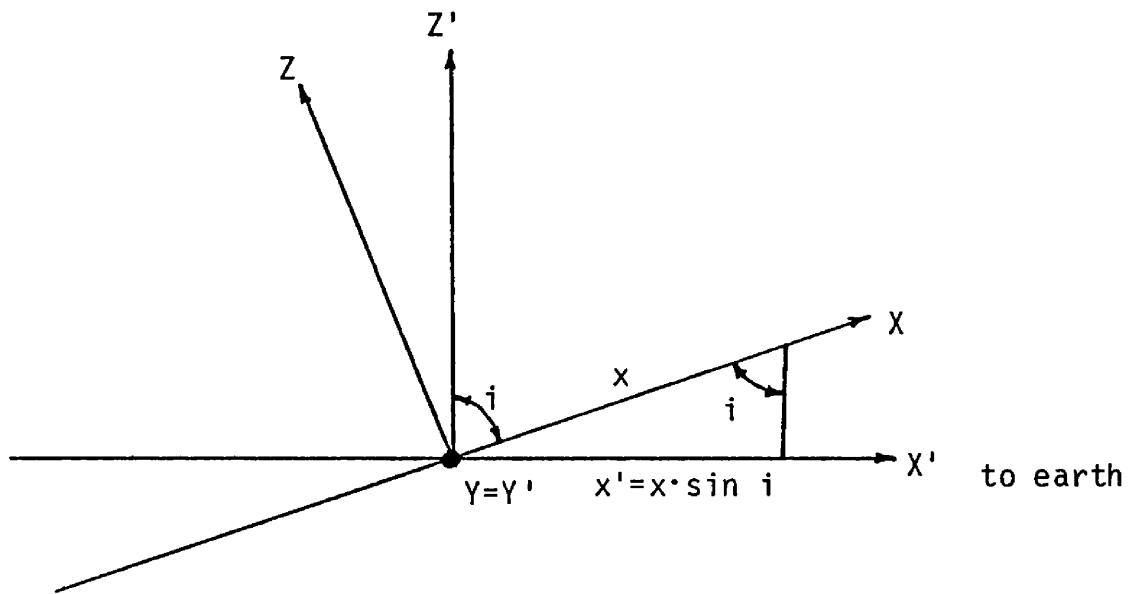


Fig. 6. Inclination of fundamental plane

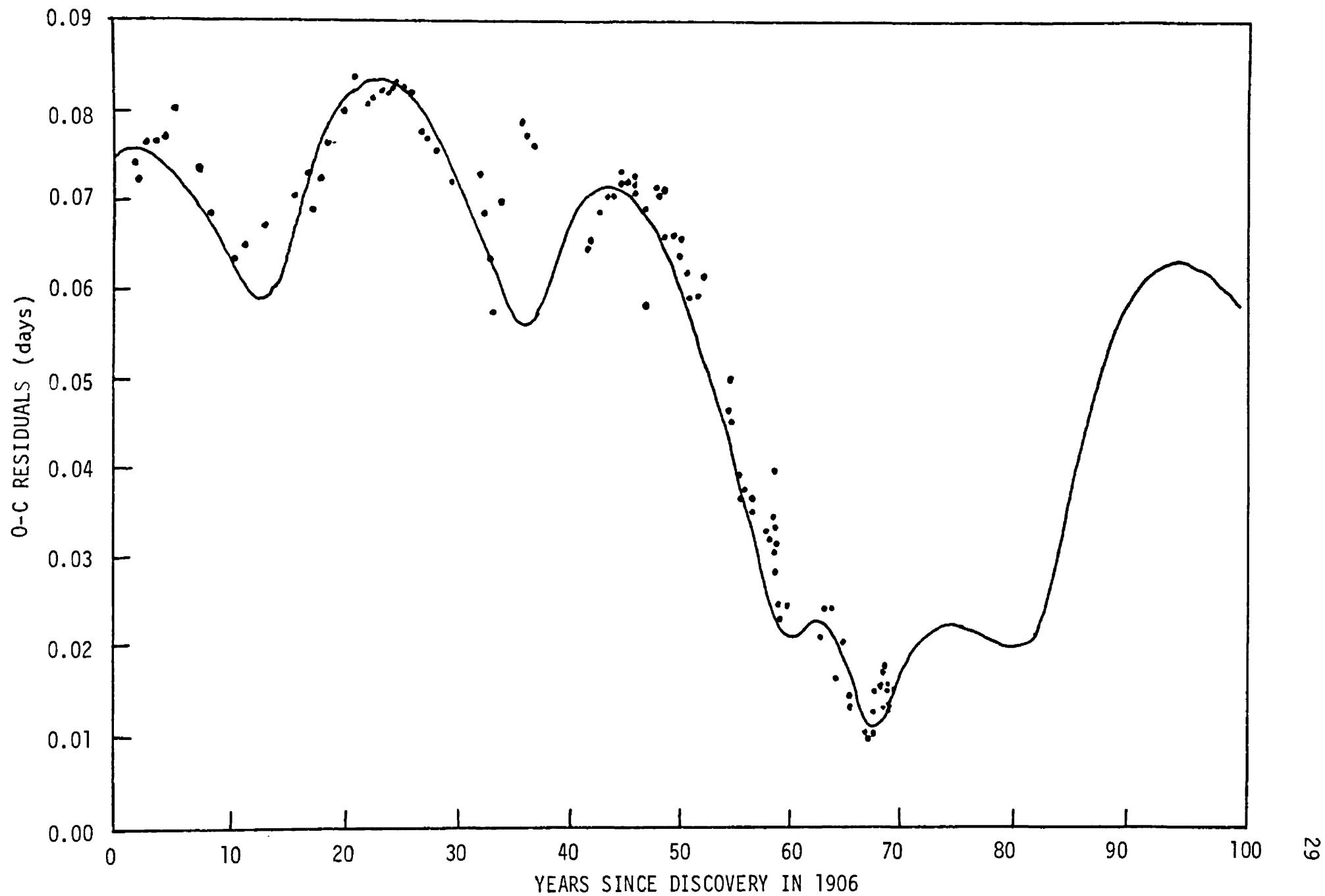


Fig. 7. O-C curve of RZ Cassiopeiae fit by elliptical model

TABLE 3
MASSES AND ORBITAL PARAMETERS OF THE HYPOTHETICAL COMPANIONS

Companion	M/M_{\odot}	a(A.U.)	P(yr)	e	t_0	ω
Outer	0.37	31.25	105	0.85	1973	155°
Inner	0.32	11.29	23	0.4	1920	210°

The closeness of the fit gives encouragement that the presence of faint companions can be postulated to explain the historical patterns of the O-C residuals. Upon closer inspection, however, it is found that the gravitational attraction between the two companions at some time exceeds the attractive force exerted on either by the binary. Therefore, since it is incorrect to neglect the companion interaction, the value of this model is only in suggesting plausibility. A physically realistic configuration of the multi-body system can only be simulated by a model which incorporates all of the gravitational interactions simultaneously.

Mutual interaction model

The solution of the three-body problem may be found by the method of special perturbations. This requires a step-by-step numerical integration of the differential equations of motion and results in position and velocity information which describes the system at any desired time based on some initial configuration.

Reference is made in the following derivation of the equations of motion to figure 8, which shows the coplanar relationship among the three bodies. The origin is chosen at the system's center-of-mass.

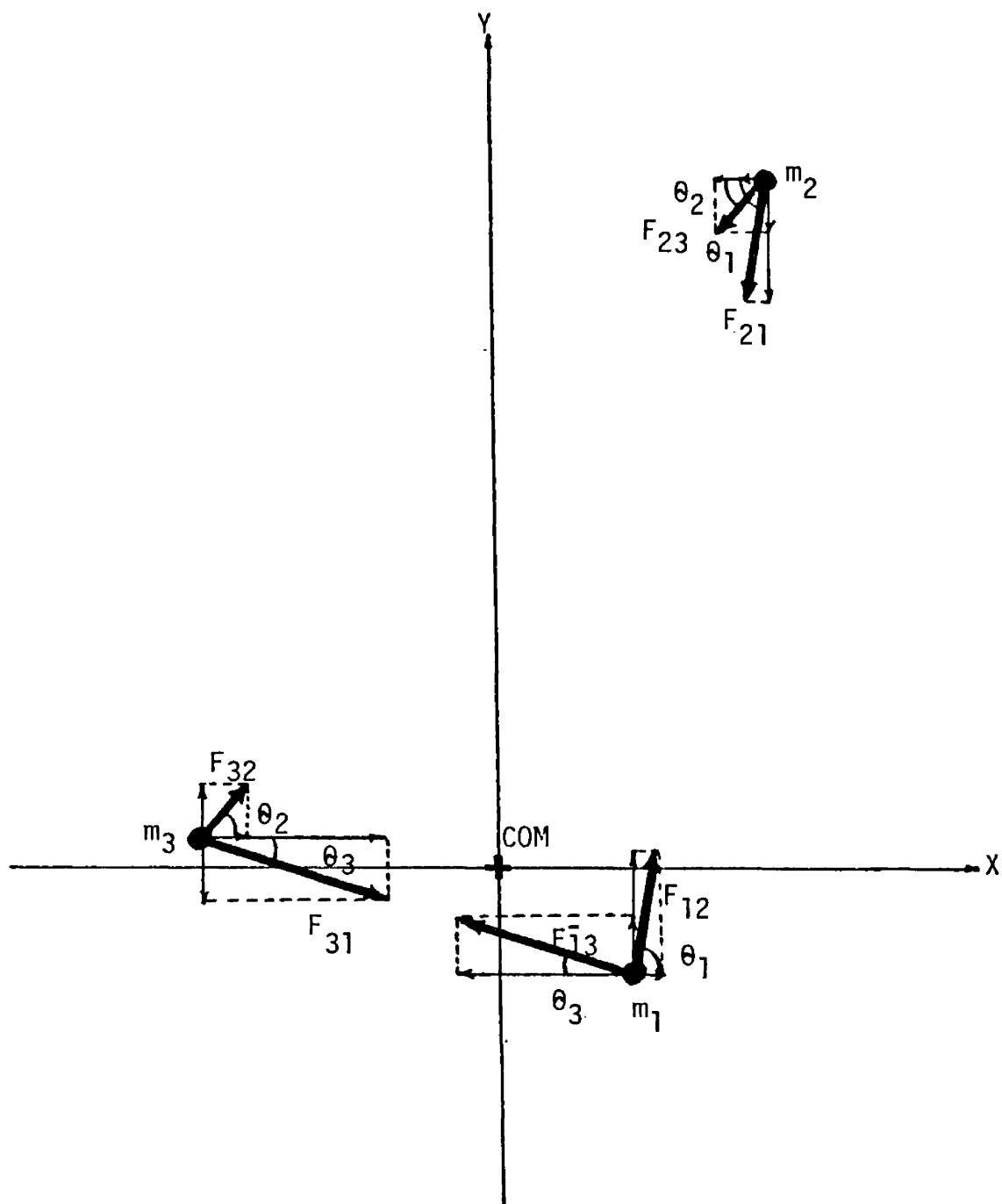


Fig. 8. Three-body interaction forces

The total force \vec{F}_i , acting on each body is the vector sum of the individual gravitational forces \vec{F}_{ij} due to the presence of the other bodies,

$$\vec{F}_i = \sum_{j \neq i} \vec{F}_{ij} \quad (2.17)$$

From Newton's Second Law

$$\vec{F}_i = m_i \vec{a}_i = m_i \frac{d^2 \vec{r}_i}{dt^2} \quad (2.18)$$

Since the position and velocity of one of the bodies is most easily obtained through COM considerations once these quantities are known the the other two, the analysis can proceed by concentrating on masses m_2 and m_3 . Equating eqs. (2.17) and (2.18), and resolving into rectangular coordinates, we obtain

$$m_2 \frac{d^2 x_2}{dt^2} = F_{21} \cos \theta_1 + F_{23} \cos \theta_2 \quad (2.19a)$$

$$m_2 \frac{d^2 y_2}{dt^2} = F_{21} \sin \theta_1 + F_{23} \sin \theta_2 \quad (2.19b)$$

$$m_3 \frac{d^2 x_3}{dt^2} = F_{31} \cos \theta_3 + F_{32} \cos \theta_2 \quad (2.19c)$$

$$m_3 \frac{d^2 y_3}{dt^2} = F_{31} \sin \theta_3 + F_{32} \sin \theta_2 \quad (2.19d)$$

Now, according to Newton's third law and his law of universal gravitation,

$$\vec{F}_{ij} = \frac{G m_i m_j}{r_{ij}^3} \vec{r}_{ij} = \vec{F}_{ji} \quad (2.20)$$

where $r_{ij} = ((x_j - x_i)^2 + (y_j - y_i)^2)^{\frac{1}{2}}$; $\vec{r}_{ij} = \vec{i}(x_j - x_i) + \vec{j}(y_j - y_i)$

Applying this to eqs. (2.19), we obtain

$$\frac{d^2x_2}{dt^2} = -\frac{Gm_1}{r_{12}^2} \cos \theta_1 + \frac{Gm_3}{r_{23}^2} \cos \theta_2 \quad (2.21a)$$

$$\frac{d^2y_2}{dt^2} = -\frac{Gm_1}{r_{12}^2} \sin \theta_1 + \frac{Gm_3}{r_{23}^2} \sin \theta_2 \quad (2.21b)$$

$$\frac{d^2x_3}{dt^2} = -\frac{Gm_1}{r_{13}^2} \cos \theta_3 - \frac{Gm_2}{r_{23}^2} \cos \theta_2 \quad (2.21c)$$

$$\frac{d^2y_3}{dt^2} = -\frac{Gm_1}{r_{13}^2} \sin \theta_3 - \frac{Gm_2}{r_{23}^2} \sin \theta_2 \quad (2.21d)$$

From the geometry shown in figure 8

$$\cos \theta_1 = \frac{(x_2 - x_1)}{r_{12}} \quad \sin \theta_1 = \frac{(y_2 - y_1)}{r_{12}} \quad (2.22)$$

$$\cos \theta_2 = \frac{(x_3 - x_2)}{r_{23}} \quad \sin \theta_2 = \frac{(y_3 - y_2)}{r_{23}}$$

$$\cos \theta_3 = \frac{(x_3 - x_1)}{r_{13}} \quad \sin \theta_3 = \frac{(y_3 - y_1)}{r_{13}}$$

Expressing the fractional masses as follows

$$\gamma_2 = \frac{m_2}{m_1} \quad \gamma_3 = \frac{m_3}{m_1} \quad (2.23)$$

and inserting into eqs. (2.21), we obtain

$$\frac{d^2x_2}{dt^2} = Gm_1 \left(\frac{-(x_2 - x_1)}{r_{12}^3} + \frac{\gamma_3(x_3 - x_2)}{r_{23}^3} \right) \quad (2.24a)$$

$$\frac{d^2y_2}{dt^2} = Gm_1 \left(\frac{-(y_2 - y_1)}{r_{12}^3} + \frac{\gamma_3(y_3 - y_2)}{r_{23}^3} \right) \quad (2.24b)$$

$$\frac{d^2x_3}{dt^2} = Gm_1 \left(\frac{-(x_3 - x_1)}{r_{13}^3} - \frac{\gamma_2(x_3 - x_2)}{r_{23}^3} \right) \quad (2.24c)$$

$$\frac{d^2y_3}{dt^2} = Gm_1 \left(\frac{-(y_3 - y_1)}{r_{13}^3} - \frac{\gamma_2(y_3 - y_2)}{r_{23}^3} \right) \quad (2.24d)$$

Also,

$$\frac{dx_2}{dt} = v_{x_2} \quad \frac{dy_2}{dt} = v_{y_2} \quad (2.25)$$

$$\frac{dx_3}{dt} = v_{x_3} \quad \frac{dy_3}{dt} = v_{y_3}$$

Eqs. (2.24) and (2.25) are the eight equations of motion for masses m_2 and m_3 . Using the Runge-Kutta method of numerical integration, these are solved simultaneously for x_2 , y_2 , x_3 , y_3 , v_{x_2} , v_{y_2} , v_{x_3} , and v_{y_3} at each step in time.

The values of x_1 , y_1 , v_{x_1} , and v_{y_1} at each step are determined through the defining expression for the center-of-mass.

$$0 = m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 \quad (2.26)$$

In rectangular component form

$$0 = m_1 x_1 + m_2 x_2 + m_3 x_3$$

$$0 = m_1 y_1 + m_2 y_2 + m_3 y_3$$

or,

$$x_1 = \frac{-m_2 x_2 - m_3 x_3}{m_1} \quad y_1 = \frac{-m_2 y_2 - m_3 y_3}{m_1}$$

or,

$$x_1 = -\gamma_2 x_2 - \gamma_3 x_3 \quad y_1 = -\gamma_2 y_2 - \gamma_3 y_3 \quad (2.27)$$

Differentiating eqs.(2.27) we find the velocity components of mass m_1 to be

$$v_{x_1} = -\gamma_2 v_{x_2} - \gamma_3 v_{x_3} \quad v_{y_1} = -\gamma_2 v_{y_2} - \gamma_3 v_{y_3} \quad (2.28)$$

To determine the light travel-time effect associated with the eclipsing pair's motion, it is necessary to project x_1 onto the line of sight using the sine of the inclination. The inclination will influence the companion masses and the sizes of the open orbits (figure 6) which are necessary to fit the residual curve. An inclination of 90° causes the plane of motion to coincide with the plane including the line of sight. This will result in the smallest companion masses and spatial excursions from the COM and therefore tend to minimize the interaction between the companions. As the inclination decreases, the spatial excursion must increase to give the same projected effect and the masses must increase to maintain the same periodicity. The result is that the companion interactions become stronger and maintaining stability becomes more difficult.

A simplification of the curve-fitting is made by defining the projection of the initial position of the eclipsing binary pair as the origin of a coordinate system (figure 9) to which the light travel-time effect is referred. This results in an initial light travel-time residual of zero for the model. As the integration continues in the

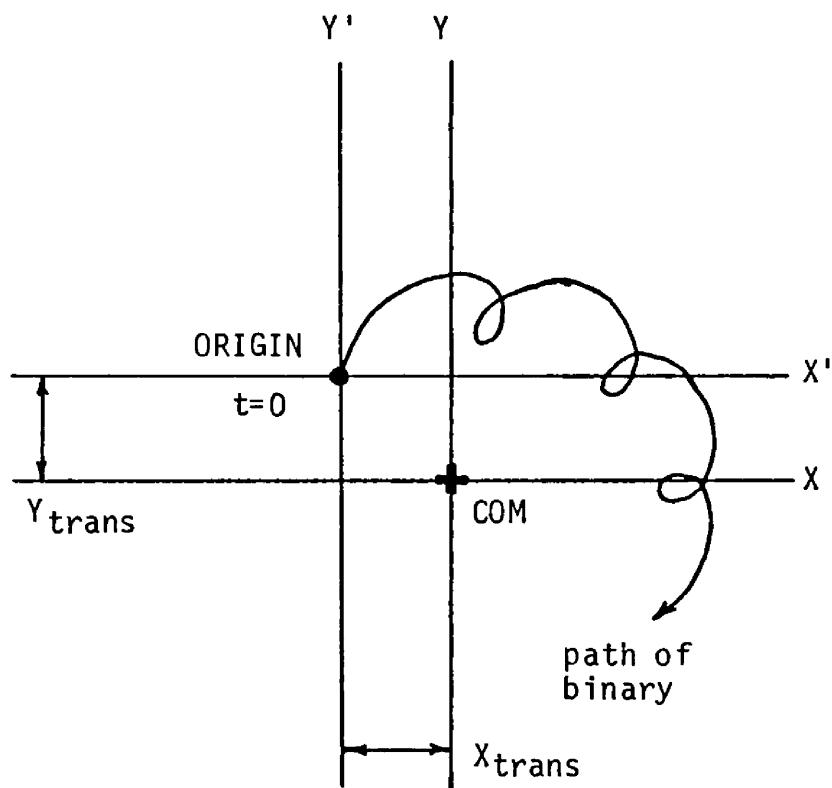


Fig. 9. Coordinate translation

center-of-mass coordinate system, the projected value of the binary pair's position is translated (by subtracting its initial COM coordinates) to determine the light travel-time effect at each step in time. If the integration is assumed to start at the ephemeris date, which is a time when the observed residual is also zero (see eq. 1.1), then the observed and model residual curves will coincide initially.

In the integration process, the time since the start of the calculations (i.e. the ephemeris date, by assumption) is found by adding the successive time steps. The clock may be caused to run into the future or into the past by selecting the time steps to be either positive or negative, respectively.

Units

The scale of physical systems being considered suggests that a convenient system of units to be used in the dynamical calculations is one in which distance is measured in astronomical units (1 A.U. is the mean distance between the earth and the sun), time is measured in years, and mass is measured in solar units (M_{\odot}). This will cause velocity to be measured in astronomical units per year (A.U./yr) and the universal gravitational constant (G) to have units of

$$\frac{(A.U.)^3}{(yr.)^2 M_{\odot}}$$

The O-C residuals are most appropriately measured in units of days (although the conversion into minutes and seconds may offer a better perspective). Since distance is measured in astronomical units, the

light travel-time calculations are made most easily if the speed of light (c) is defined in astronomical units per day (A.U./day).

CHAPTER III

APPLICATION OF MUTUAL-INTERACTION MODEL TO RZ CASSIOPEIAE

The eclipsing binary system RZ Cassiopeiae has a non-eclipsed apparent visual magnitude of +6^m.38 and a depth of primary eclipse of 1^m.5 (Wood, 1950). Therefore a mid-eclipse visual magnitude of +7^m.88 was used as input for the determination of the detectability thresholds for hypothetical companions.

The distance in parsecs from the earth to the binary was found as follows. From the literature the spectral types of the visible components of the system are known (Chambliss, 1976). The corresponding luminosities are found from the Hertzsprung-Russell diagram (Novotny, 1973).

$$\text{A2V} \quad \log(L_a/L_\odot) = +1.2$$

$$\text{G5IV} \quad \log(L_b/L_\odot) = -0.1$$

The total luminosity of the pair is used to determine their absolute visual magnitude (eqs. 1.4, 1.5), assuming as a value for the bolometric correction that which applies to the more luminous star ($BC = -0.10$).

$$M_V = +1.62$$

The difference between the apparent and absolute visual magnitudes then yields a distance of about 90 parsecs through a solution of eq. (1.6).

A data file of about 600 Julian Dates of minima was compiled from the literature for RZ Cassiopeiae. About 128 of these were selected as being a representative set, since it is impractical to repeatedly plot the larger number of points during the curve-fitting sequence and since also a maximum number of 150 is allowed by the subroutine HISTRY.

An average eclipse period (P_{ave}) was determined for the unabridged file by an execution of the code AVEPER. The value obtained after 42 iterations was

$$P_{ave} = 1.19524788 \text{ days}$$

Both the original (RZCAS) and abridged (RZABR) data files are included in appendix 1. The O-C residuals of the abridged file have been calculated through an execution of the code FILCHG, using the average value of the period. Those of the original file have been calculated using a value of the period ($P = 1.19525189$ days) common to much of the current literature. A comparison of the plots for the two files (appendix 1) shows the effect of changing the eclipse period.

The trial procedure of fitting the residual history by a dynamical model required many hours of patient execution of the code NTERAC (appendix 2). The sensitivity of the model caused many unproductive choices to be made of the initial parameters of the multiple-body system. These often resulted in close encounters among the bodies, sometimes causing one or both of the companions to escape* from the system. The

*In an isolated multiple-body system, one body can never completely escape the attraction of the others. The term "escape" is used here to suggest that the component of the velocity of a body which is directed away from the system COM will go to zero and then be directed towards the COM at a time much greater than the duration of this study.

procedure in the trial method became one of trying to maintain stability in the system while continually improving the fit of the model O-C curve to the observations by refining the input.

The orbital parameter input mode was chosen during the initial execution of the code, since approximate values of the binary orbital periods and semi-major axes could be estimated from the O-C curve. A double periodicity is exhibited by the curve and therefore two companions were assumed. The starting values of orbital periods and semi-major axes due to assumed inner and outer companions were estimated to be 25 years, 1.3 A.U., and 100 years, 3.5 A.U., respectively.

The analysis had adjusted the model O-C value to be zero at the initial time through a spatial coordinate translation and therefore the placement of the O-C curve which resulted from the model was found to be sensitive to the input values of the longitudes of periapsides chosen. The initial choice was simplified by referring to figure 10, which exhibits both the horizontal and vertical translations of the O-C curve along the epoch and residual axes, respectively, which occurred when the longitudes were varied.

First consideration in the curve-fitting procedure was given to the case where the inclination was assumed to be 90° , since this case would exhibit the least companion interaction. After 67 trials, a best fit was obtained and is shown in figure 11. The rms deviation between the observed and model residual curves is $\pm 7^{m}39^s$ (i.e. ± 0.00531 day). The center-of-mass system rectangular coordinates of position and velocity for the three bodies at the initial time (i.e. ephemeral J.D. = 2442340) are given in table 4. Also listed are the masses of the companions

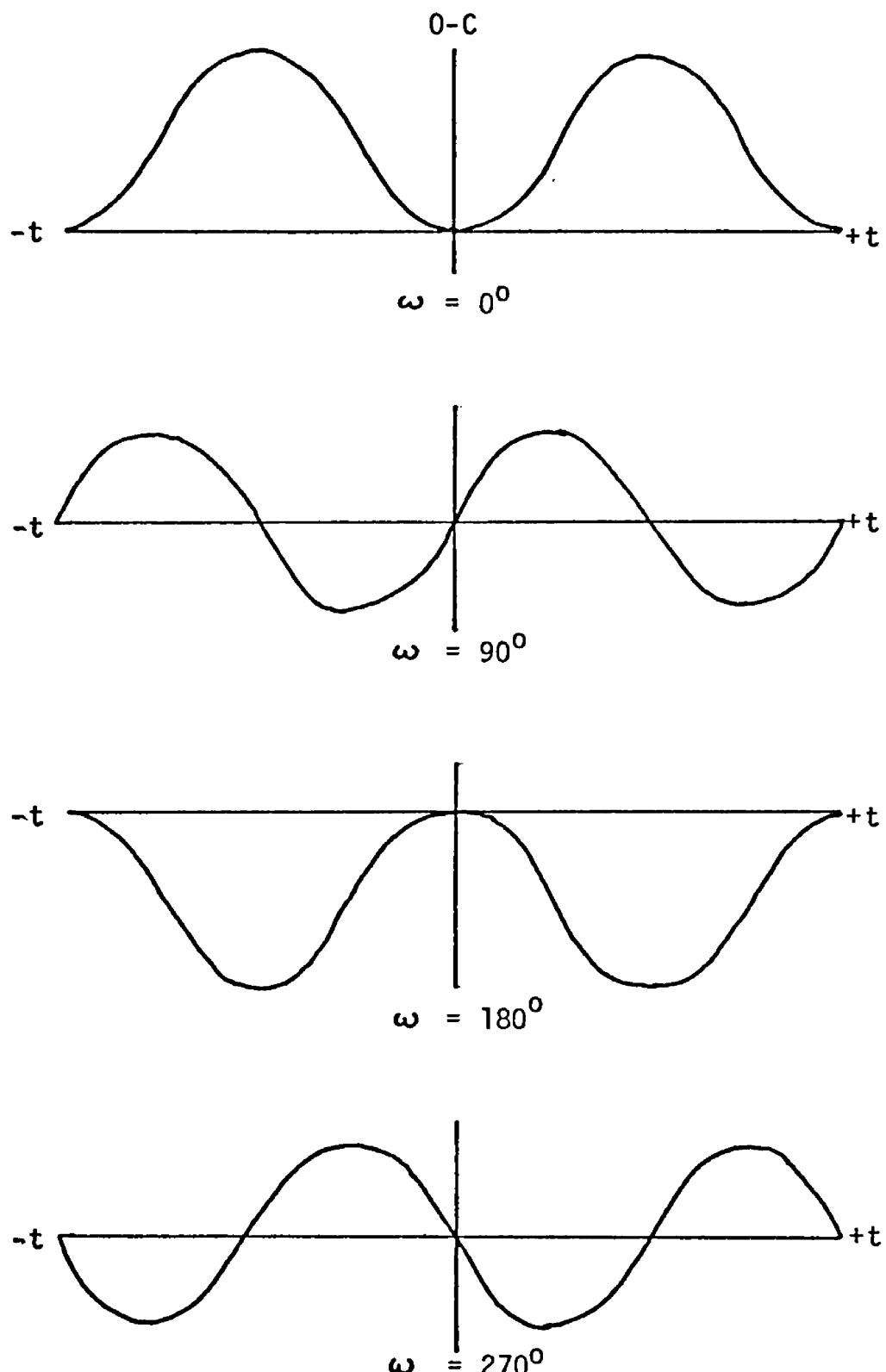


Fig. 10. Effect of the longitude of periapse on the placement of the O-C curve

MODEL # 67: O-C RESIDUALS

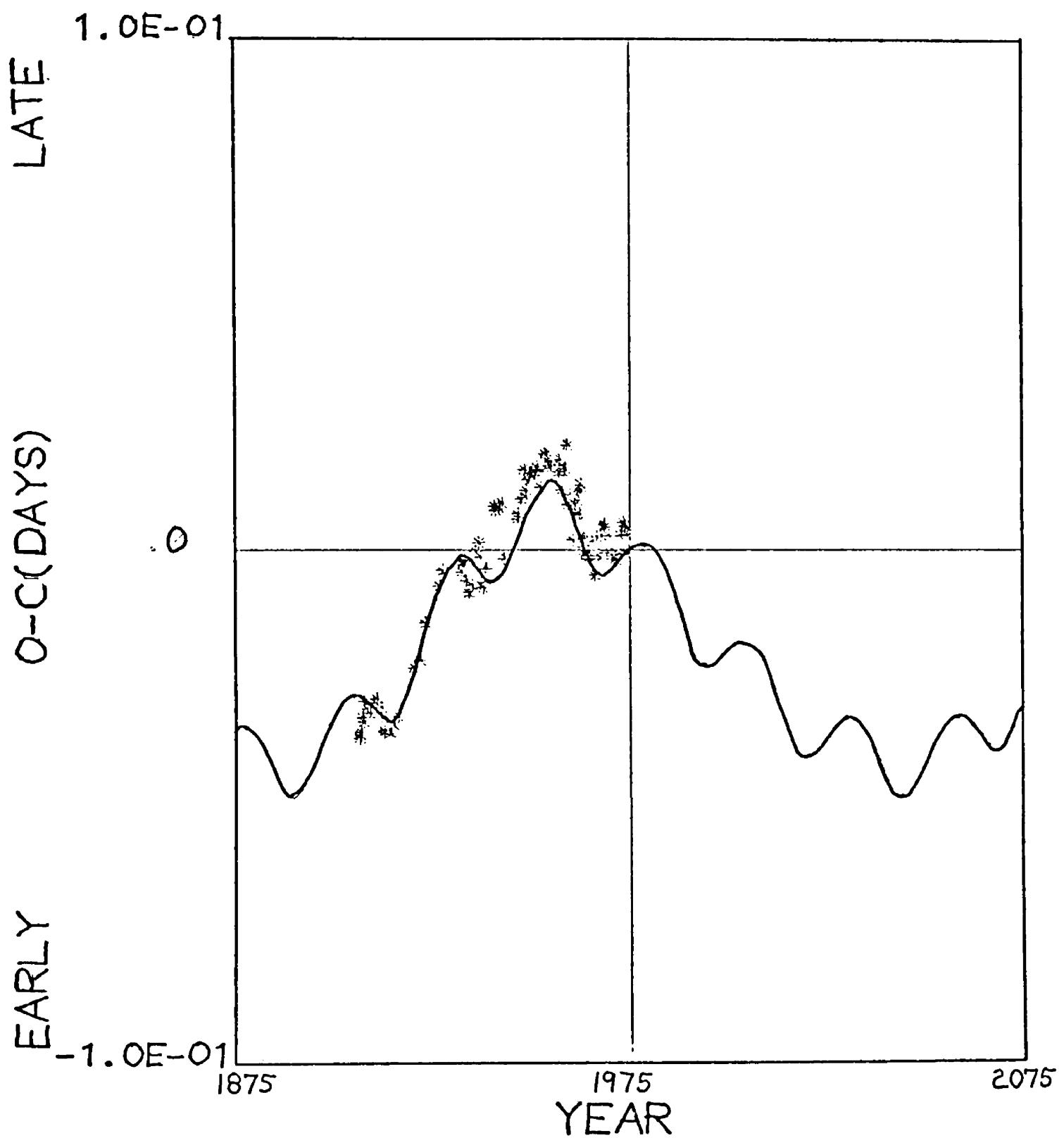


Fig 11. O-C curve fit by interaction model 67

TABLE 4
COMPANION MASSES AND INITIAL RECTANGULAR PARAMETERS

$m_y = +7^m.88$ $d = 90 \text{ pc}$	MODEL 67			MODEL 127		
	BINARY PAIR	INNER COMPANION	OUTER COMPANION	BINARY PAIR	INNER COMPANION	OUTER COMPANION
M/M_\odot	2.4	0.263	0.288	2.4	0.287	0.330
X (AU)	-2.83234	14.86341	-41.00514	-2.64463	15.30136	-38.64449
Y (AU)	-2.46345	-9.12520	17.86768	-2.32152	11.26606	-28.63106
VX (AU/YR)		1.30292	-0.35300		-0.68683	0.52601
VY (AU/YR)		2.00634	-1.21160		2.19624	-1.02252

(M/M_\oplus) in solar units. The equivalent orbital parameters are not presented because the motion of the companions relative to the binary exhibits continuous variation of the orbital parameters, and therefore the instantaneous values may not be representative of the mean orbits.

In the spatial plots of the binary and companion motions (appendix 3) the positive x-axis is directed towards earth. Model 67 has been allowed to run for 6300 years into the past and 2000 years into the future and shows no sign of becoming unstable. Although an extended investigation gives insight into the stability of a system, it should be noted that round-off error becomes significant after such a long integration period, and that the model becomes less valid with increasing time.

The second model considered was constrained to a plane of motion whose inclination was $82^\circ 14'$ with respect to the plane of the sky. This also is the inclination of the fundamental plane of the central binary pair's revolution about each other, and thus it has the aesthetic appeal of coplanarity which might be attributed to a common origin for all four bodies.

After an additional 60 trials (i.e. model 127), a best fit for this inclined orientation was found as is shown in figure 12. The resulting rms deviation between the curves is $\pm 9''30''$ (i.e. ± 0.0066 day). Again the companion masses and rectangular position and velocity components corresponding to the initial time are listed in table 4. A comparison of the two models shows that a change in inclination of only 8° requires the companion masses to increase by 10-15% to maintain a fit of the residual curve and therefore increases the likelihood of strong interactions between them.

MODEL #127: O-C RESIDUALS

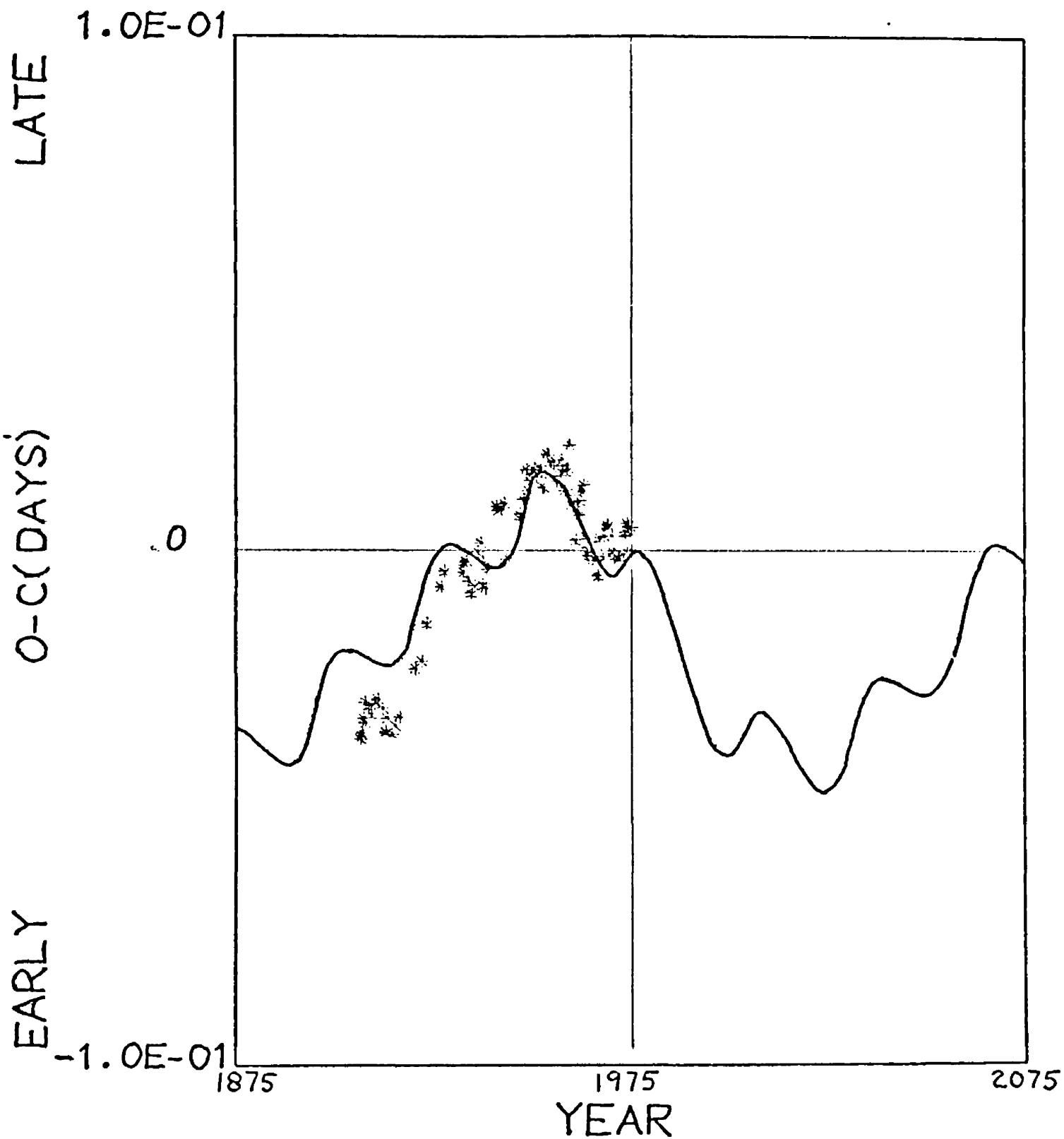


Fig. 12. O-C curve fit by interaction model 127

Model 127 has been run for about 3,000 years as seen in the spatial plots in appendix 3. It appears to remain stable for at least 800 years into the future and for more than 300 years into the past. However at about year -350 a close interaction occurred, causing the outer companion to be thrown far out into space. It exceeded the detectability threshold at about year -382 and remained visible until about year -534. The system then seemed to be well-behaved until about year -1180 when another close encounter between the companions caused the two to capture each other. The newly-formed pair revolved about a common barycenter while continuing to orbit the eclipsing binary. At the end of one orbit, the central pair exerted a large enough perturbation on the companion pair to again cause the outer companion to be thrown far from the COM, becoming visible again at about year -1334. This rather wild motion continued as is seen in the spatial plots.

Although model 127 gave an adequate fit to the observed O-C residual curve, it is unlikely that such a system could maintain stability for a time span approaching the age of the main sequence A2 component of the eclipsing binary (2 billion years maximum). On the other hand, the model's behavior suggests that relatively short-lived gravitational encounters between stars which are migrating through space can occur and result in the dynamical effect which is observed in the O-C residuals.

If the investigation is restricted to systems which fit the observed behavior yet remain stable over a long period of time, it then appears that the possible companion masses would be smaller than those found in model 127 and that the inclination of the companion orbital plane would be greater than 82° . Since the masses would be at least as

great as those of model 67 to account for the amplitude of the residual curve, these two models may be regarded as bracketing possible companion masses for the eclipsing binary system RZ Cassiopeiae. Apparently the companion bodies are red dwarf stars of spectral type M3-M5.

CHAPTER IV

DISCUSSION

Radial velocity residuals

Additional credibility would be given to a hypothetical companion model if it explained the observed radial velocity residuals while fitting the photometric O-C curve. The binary orbital elements are determined through a trial procedure of fitting the spectroscopic radial velocity curves which have been observed over several consecutive revolutions. Historically these orbital elements have been the subject of continual revision. If instead of revising the elements, an investigator were to assume some average set, he might notice that the long term residuals between the observed and model radial velocity curves have a periodic behavior which could be explained by a spatial motion of the binary pair, caused through interaction with companions.

Figure 13 shows a typical radial velocity residual curve associated with an interaction model (#67) which also fits the observed light travel-time residuals of an eclipsing binary. The amplitude of this curve is about 4 km/sec, which is twice as large as that used by A. H. Batten and E. L. van Dessel (1975) to support their hypothesis of a third body orbiting about the spectroscopic binary 70 Ophiuchi.

A suggested further procedure, then, in the analysis of eclipsing binary stars with periodic O-C curves and extensive spectroscopic

MODEL # 67: RADIAL VELOCITY RESIDUAL

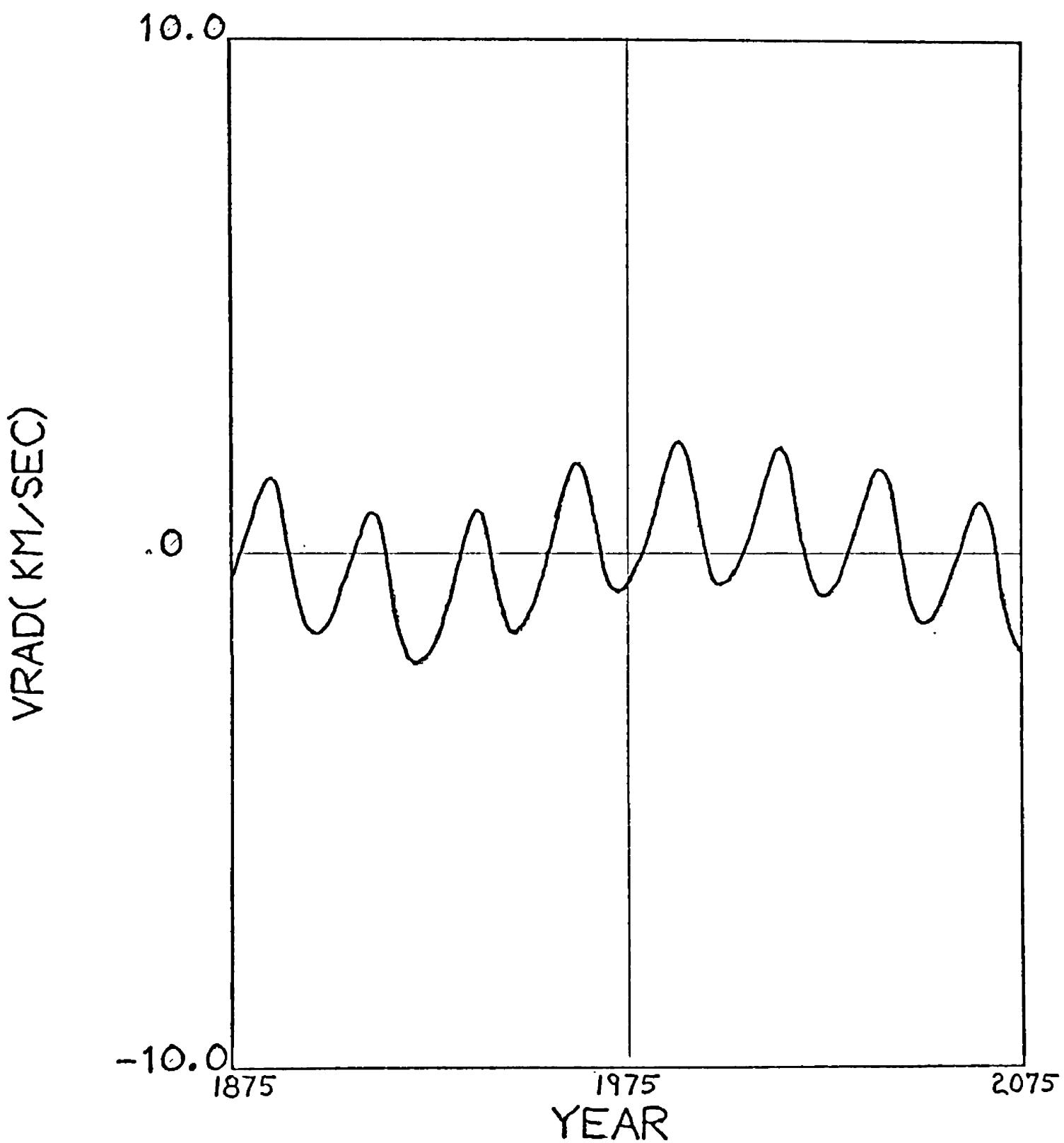


Fig. 13

observations, is to assemble from the literature a data file containing radial velocities and corresponding times. Assuming a set of binary orbital elements, the residuals of the observed radial velocities could be determined for each date. These values then could be read into the interaction model code NTERAC and displayed on the plot (figure 13) containing the radial velocity residuals of the model. The most probable model would be one which simultaneously gives a best fit to both the eclipse minima residuals and radial velocity residuals.

Reduced O-C residuals

Figure 14 shows the difference between the observed O-C residual values and the corresponding values which result from the hypothetical interaction of a binary with companions. The reduced residual curve offers a convenient way of estimating whether or not there may exist undetected periodicities which might be attributable to additional unseen companions. A good fit of the O-C curve should produce a reduced residual curve which shows observational scatter or noise which is equally distributed about the abscissa.

Osculating orbital elements

How far in time a model can be used to represent the interactions between bodies is limited by the real time required to make the necessary calculations. An interesting further project which could be undertaken would be to write a subroutine to determine the osculating orbital elements (ref. Danby, 1962) at specific intervals during the dynamic calculations of the code NTERAC. It would then be possible to

MODEL # 67: REDUCED O-C RESIDUALS

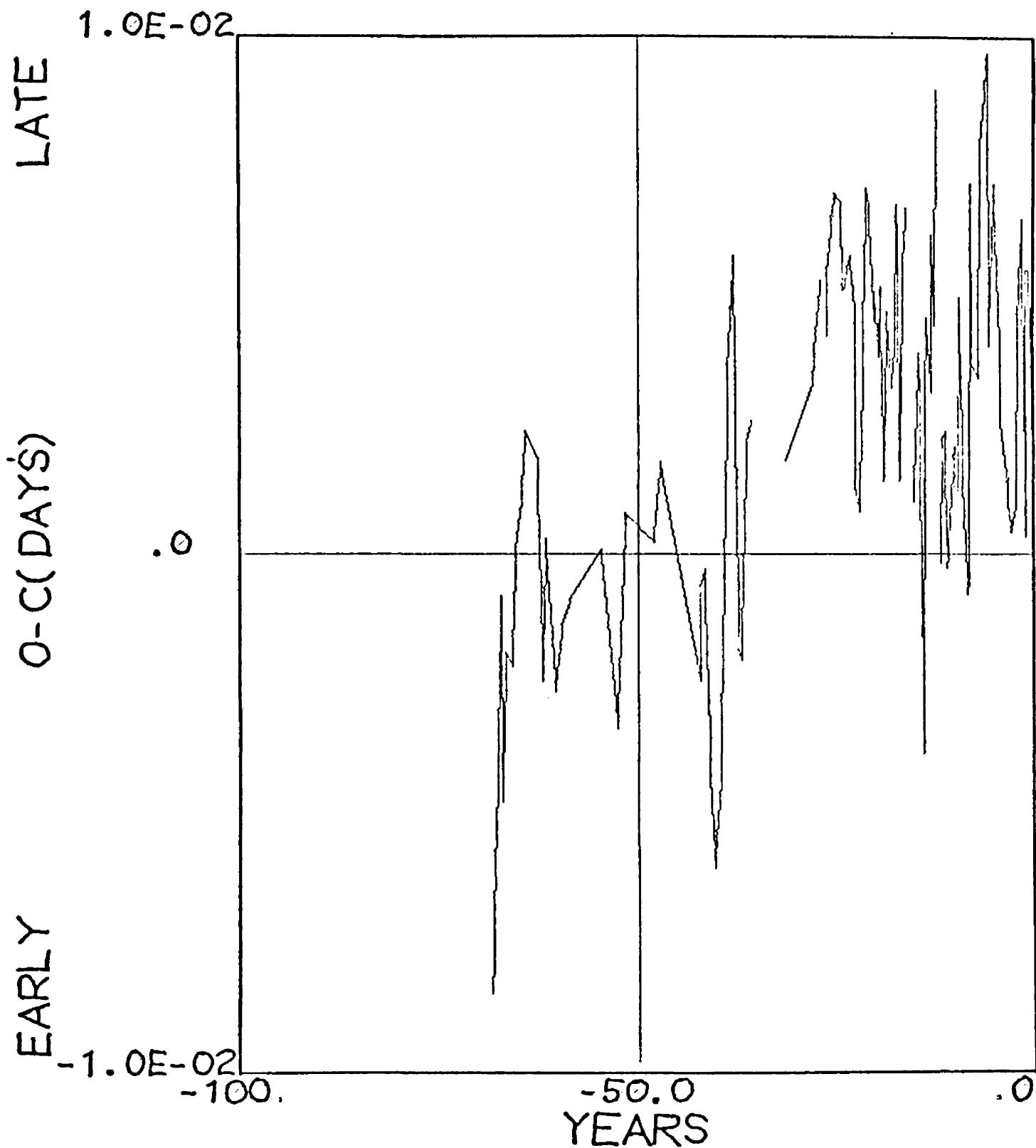


Fig. 14

execute the model for a time period long enough to show patterns of variation in the orbital elements of the companions. A variation-of-parameters technique could then be used to make the large calculational steps necessary to reconstruct the motions of the system at a time several million or even billions of years ago. This method has been applied to the orbit of Pluto by Bensen and Williams (1971), who used time steps of 500 years.

Error control

Round-off error is cumulative as the number of calculation steps increases. It could be reduced by using double precision for all variables used in many steps. Since this requires more core storage, however, it is preferable to reduce the step size in the integration scheme, since the error associated with a fourth-order Runge-Kutta method varies as the fifth power of the change in step size.

Although the increased number of steps would increase round-off error, the result is still a gain in accuracy. To see the effect that changing the step size has on the results, model 67 was run twice for 200 years using step sizes of 0.01 year and 0.10 year. It was found that in this relatively stable model the two trials gave differences in position vectors for each body of less than 3×10^{-7} A.U. in each year and differences in velocity vectors of less than 1×10^{-7} A.U./YR in each year. Since the accuracy gained by reducing the step size by a factor of ten is very small in a stable case, the larger step size may be used to expedite the trial fitting procedure. When a near fit is found, the step size may be reduced to obtain the final fit.

An additional refinement to the code NTERAC would be to introduce a subroutine which would continually adjust the integration step size to a maximum value which would still maintain an acceptable error tolerance. This could be done by making a parallel integration at each step in time using a lower-order scheme such as Simpson's method. A comparison of the results of the two methods at the end of each step would determine if the next step should be integrated with the same or an even larger step size, or whether the previous step should be repeated using a reduced step size. The computation time lost in making the additional calculations is outweighed by the gain resulting from the use of the largest allowable step size (Strack, 1963). This method would be most important in cases where close encounters require very small step sizes to maintain accuracy.

Minimizing companion interactions

The analysis which has been presented has considered only those cases in which the companion motions are coplanar with the same direction of revolution as the central pair. It is possible that two companions could revolve in opposite directions about a binary. During a close encounter in such a case, each body would spend less time within the other's sphere of influence, and therefore the interaction would be less severe. If the constraint of coplanarity is removed, the probability of a close encounter is decreased. These two additional degrees of freedom could be used to fit O-C residual curves in systems where otherwise the trial procedure suggests that no stable configuration fits the O-C residuals.

Predictions

A test of how well a hypothetical model describes an actual system is its ability to make predictions. Figure 11 and 12 show that the trend of both models 67 and 127 is that eclipses of RZ Cassiopeiae which occur shortly after the ephemeris Julian date will be observed to occur slightly earlier than predicted by the linear elements of eq. (1.1). Since the ephemeris Julian date (i.e. model year = 0) corresponds to late 1974, indication of whether either model correctly predicts the observed trend can be expected by 1980 or 1985.

As more observations are made for an eclipsing binary, the O-C residual curve will further reveal its behavior. Continual revision of the fitting parameters will then better define the system. Although the controversy of explaining the apparent variation in the eclipse period of RZ Cassiopeiae continues, the results given in the preceding chapter give support to the hypothesis that the binary is experiencing a perturbation caused by two red dwarf companion stars.

CHAPTER V

SUMMARY

The observed time residuals for some eclipsing binary stars may be caused by a light travel-time effect produced by unseen companion bodies. An object which has sufficient mass to cause a measurable perturbation of its central binary pair may be unseen because it is "of low luminosity, a faint dwarf, close to and lost in the image of the primary star or too faint to be recorded photographically, even at a large angular separation from the primary" (Van de Kamp, 1975). While the companion hypothesis for explaining eclipse time residuals has often been postulated, the determination of the masses and dynamics of such companions has not been rigorously pursued.

A method of deducing the masses and dynamics of unseen companions associated with eclipsing binary stars is presented in Chatper II. The general technique applied uses a physical model to generate light travel-time residuals which fit the observations by adjusting the assumed values of companion masses and their initial positions and velocities. Models employing circular or elliptical orbits of two companions about the central binary pair are considered and discarded since a substantial interaction between the two companions is not included by superimposing the simultaneous two-body analyses. Allowance for the mutual interaction among all bodies in the system results in a three-body problem

which can only be solved numerically. The Runga-Kutta method of integration is encoded into the Fortran program NTERAC (appendix 2), which solves the eight simultaneous equations of motion (eqs. 2.24, 2.25) to describe the dynamics of the system at each step in time.

Application of the mutual-interaction model to the eclipsing binary star RZ Cassiopeiae is described in Chapter III. These results indicate that the companions, whose presence is postulated in order to explain the observed eclipse time residuals of that system, have masses between 0.26 and 0.33 solar masses. If the companions are considered to be main-sequence stars, they would be of spectral type M3-M5.

A hypothesis reveals its validity through its ability to make correct predictions. The fit of a model to the observed eclipse time residuals of RZ Cassiopeiae has been extrapolated into the future (figure 11). A discernible change in the eclipse time residuals and in the radial velocity of the binary pair is predicted by 1985.

APPENDIX 1

O-C RESIDUAL FILES

Observations of eclipsing variable star minima are generally reported as heliocentric Julian dates. Quite often an observer will fail to include in his publication the accuracy of his results. By noting the method of observation, a typical value for its accuracy may be assigned through reference to table 5.

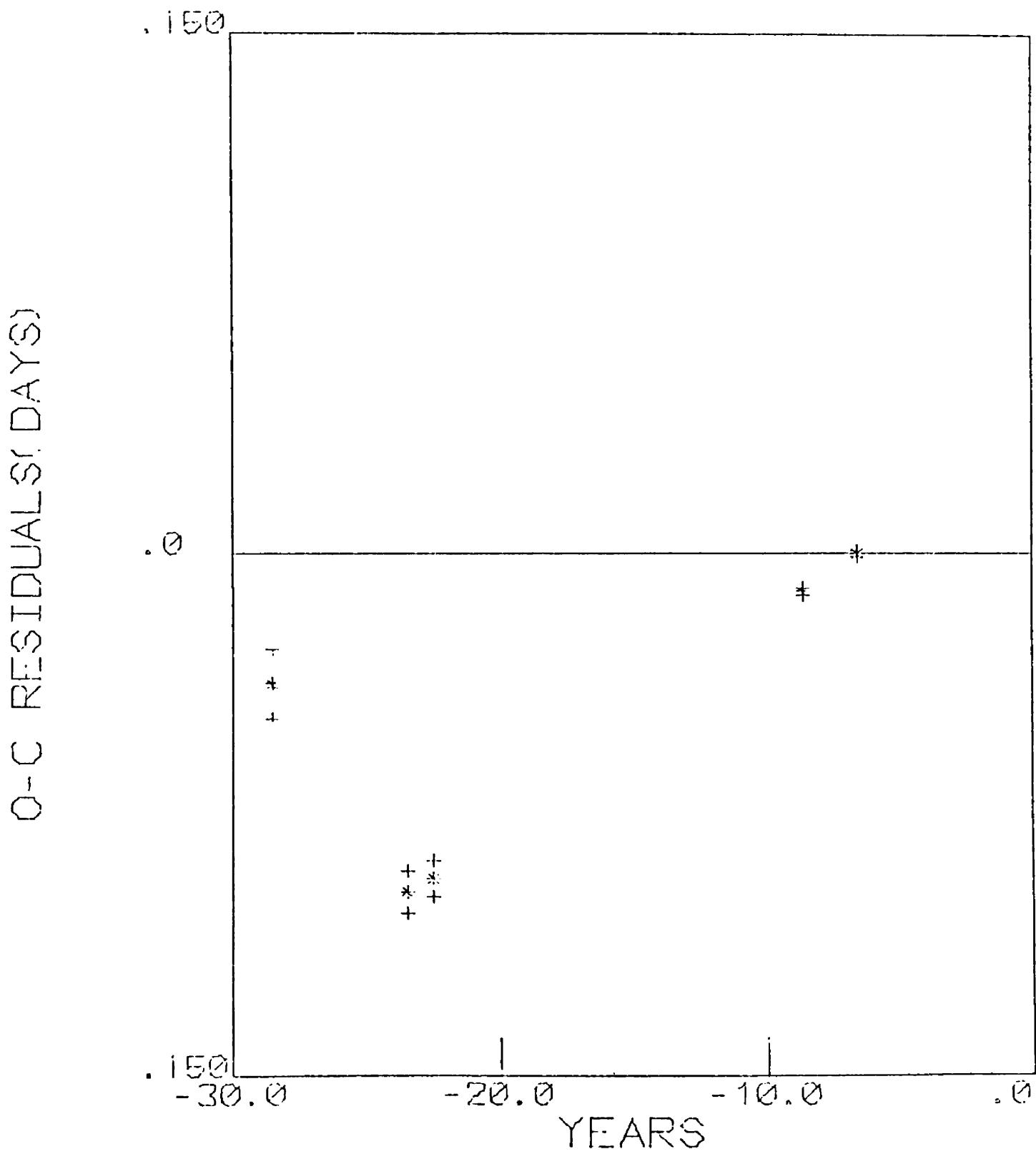
TABLE 5
ACCURACY OF OBSERVATIONS

METHOD	ACCURACY (days)
Visual	±0.01
Photographic	±0.004
Photovisual	±0.002
Photoelectric	±0.001
Normalizing	±0.0005

In the following plots of the O-C residuals of eclipsing binaries which exhibit variable periods, year zero corresponds to January 1, 1980. Residual values are indicated by an asterisk (*) with crosses (+) located above and below to indicate the observational accuracy. The eclipse periods are given in units of days.

In the tables of eclipsing binary minima, the observational accuracies and O-C residuals are given in units of days. Julian dates are truncated to show only five digits to the left of the decimal (e.g. 2442000.1234 will be entered as 42000.1234).

KOAOL PERIOD = 2.86395400

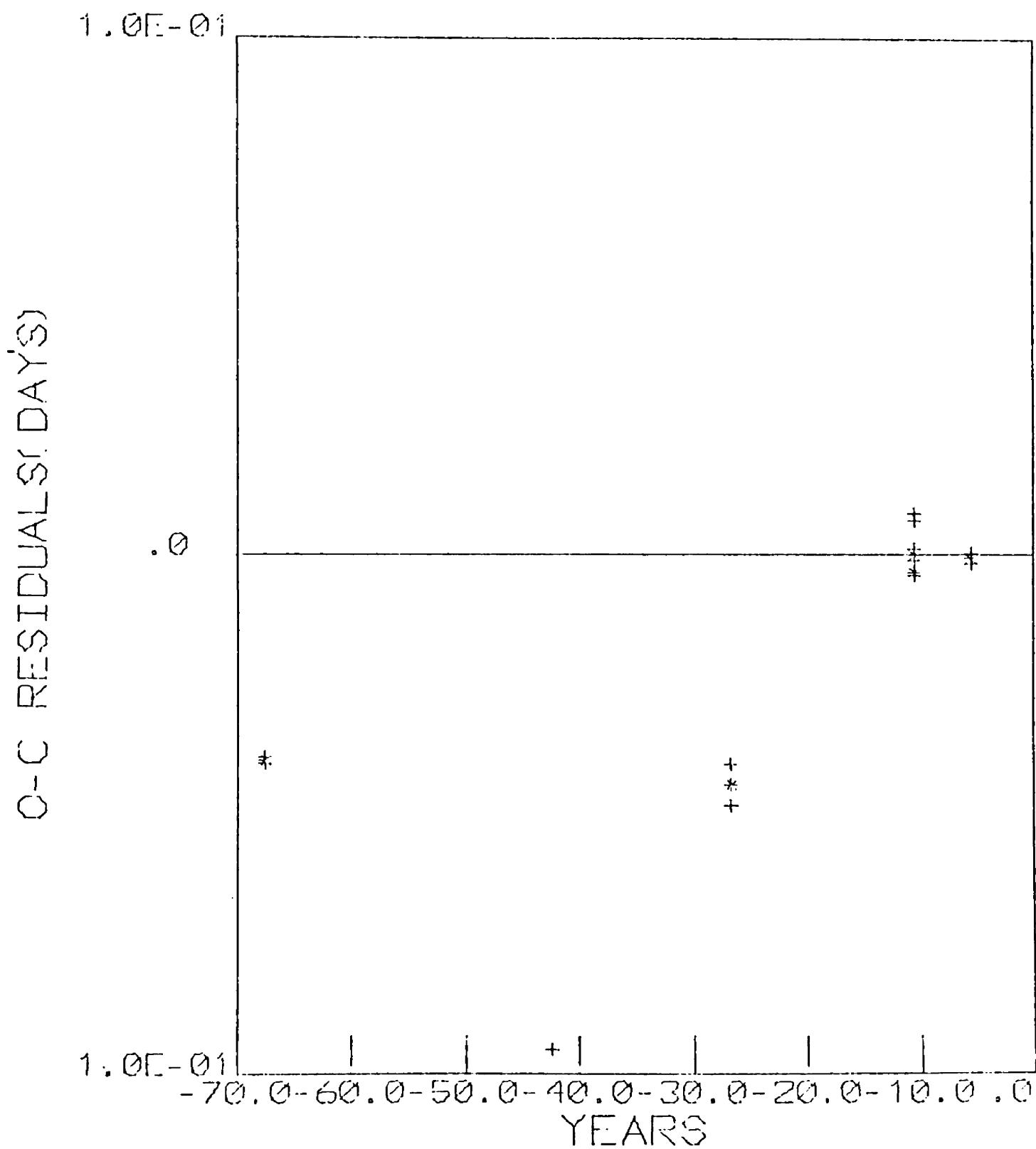


CHRONOLOGY OF 9 OBSERVATIONS OF KOASL

TCALC = JD 41837.4710 + E * 2.86395400

HELIOPCENTRIC JULIAN DATE	+/-	O-C RESIDUAL	OBSERVER	REFERENCE
33833.4100	0.010	-0.0375	DOMKE, JAHN	AN 281(3), P. 113
35713.4170	0.006	-0.0971	SZAFRANIEC	A.A. 7, P. 188
36073.5510	0.005	-0.0934	SZAFRANIEC	A.A. 8, P. 189
37107.4650	0.001	-0.0668	GERHART	AN 288, P. 69
33937.5280	0.010	-0.0704	MULLER, KRAUSSER	AN 289, #4, P. 192
40435.4076	0.001	-0.0337	CALISKAN, IBANOGL	IBVS #456
41148.5600	0.001	-0.0129	HOLZL	IEVS #647
41837.4710	0.001	0.0000	IBANOGLU	IBVS #937
42245.4798	0.001	0.0145	GUDUR, ERTUKEL	IBVS #1053

RYAQR. PERIOD.. 96660700



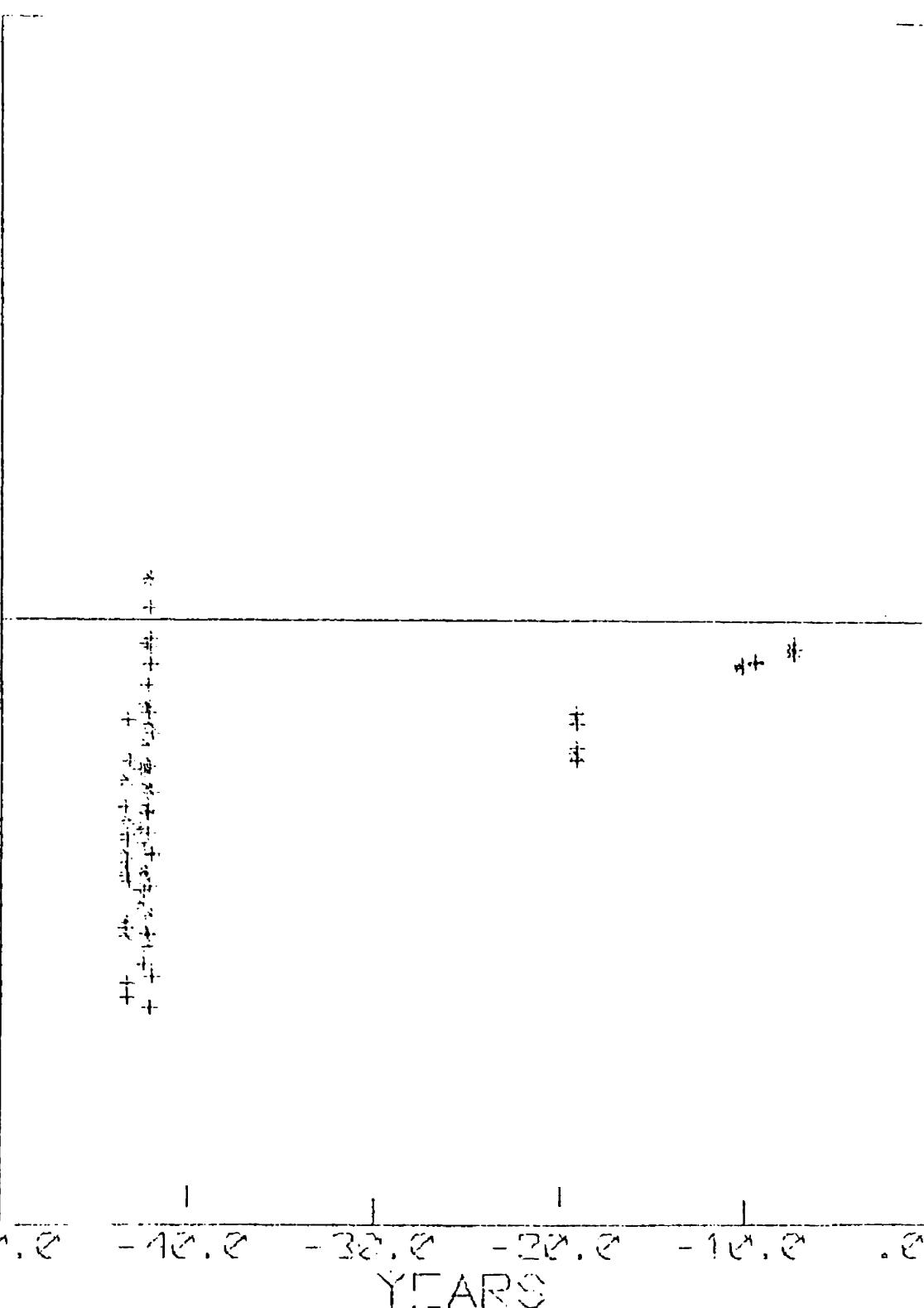
TCALC = JD 48476.3240 + E * 1.26668700

HELIOCENTRIC JULIAN DATE	+/-	O-C	RESIDUAL	OBSERVER	REFERENCE
19613.4585	0.001	-0.0397	ZINNER		AJ 4679, P. 460
28304.4260	0.010	-0.0355	SZAFRANIEC		AJ 67, P. 7
34543.9260	0.004	-0.0445	KOCH, KOCH		AJ 67, P. 7
40446.3110	0.010	-0.0139	MONSKE		IBVS #795
48454.6330	0.010	-0.0033	HAZEL		IBVS #795
40456.6740	0.003	0.0161	MONSKE		IBVS #795
40476.3240	0.001	0.0000	IBANOGLU, IZMIR		IBVS #456
42303.3012	0.001	-0.0007	GULGEN, ALKAN, SE		IBVC #1053
42203.3015	0.001	-0.0004	GULGEN, ALKAN, SE		IBVC #1053

EF AUR PERIOD - 60001700

RESIDUALS (DAYS)

1.EE-01



CHRONOLOGY OF 33 OBSERVATIONS OF EFAUR

65

TCALC = JD 41752.4560 + E * 1.53321700

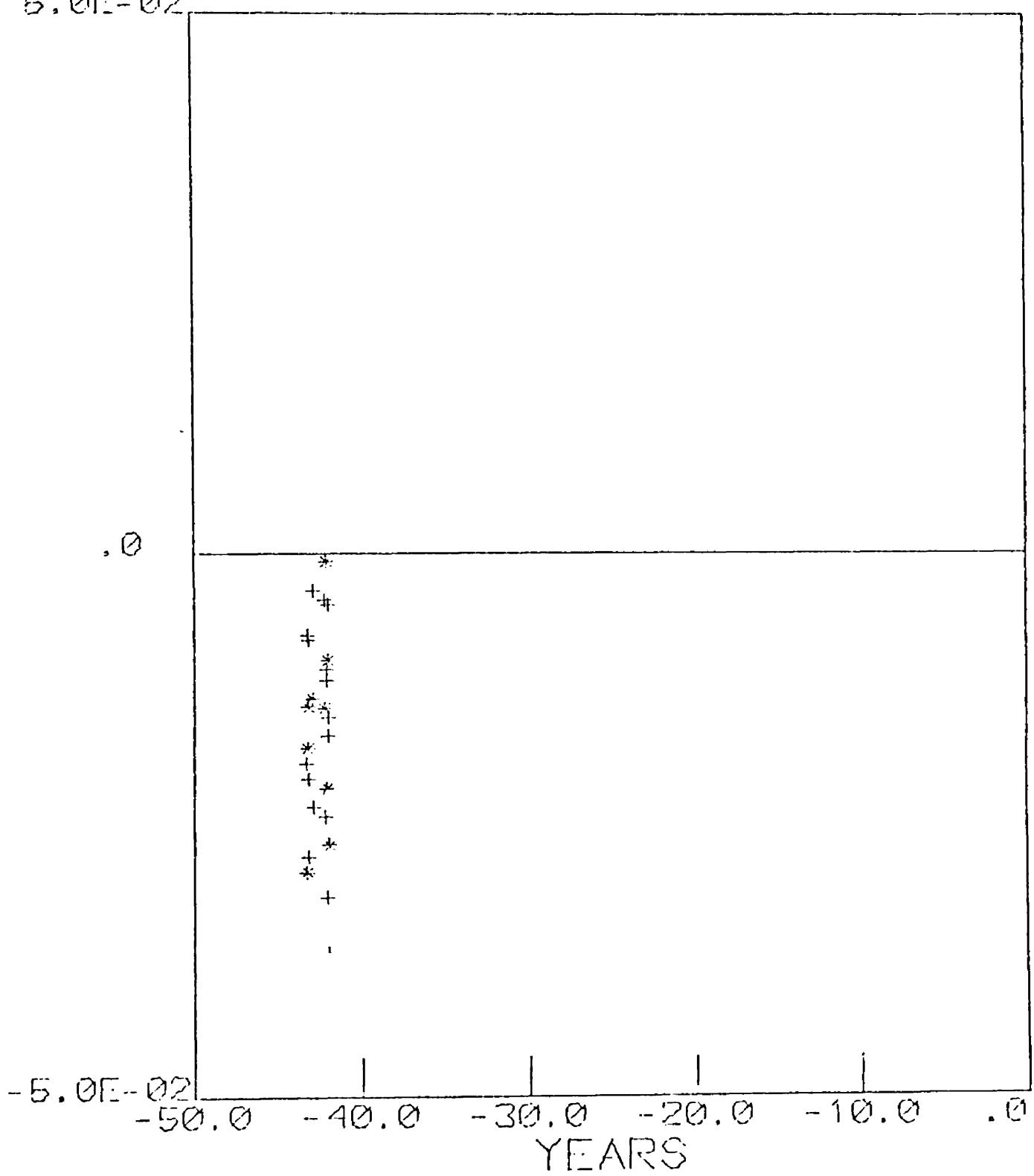
HELIOCENTRIC JULIAN DATE	+/-	O-C RESIDUAL	OBSERVER	REFERENCE
28497.7220	0.010	-0.0413	LAUSE	AN 6404, P. 324
28513.5430	0.010	-0.0524	LAUSE	AN 6404, P. 324
28521.4727	0.003	-0.0388	LAUSE	AN 6404
28521.4350	0.010	-0.0265	LAUSE	AN 6404, P. 324
28534.1270	0.010	-0.0503	LAUSE	AN 6404, P. 324
28556.3090	0.010	-0.0333	LAUSE	AN 6404, P. 324
28779.5410	0.010	-0.0349	LAUSE	AN 6404, P. 324
28833.3580	0.010	-0.0473	LAUSE	AN 6404, P. 324
28860.2850	0.010	-0.0350	LAUSE	AN 6404, P. 324
28868.1940	0.010	-0.0420	LAUSE	AN 6404, P. 324
28882.4710	0.010	-0.0140	LAUSE	AN 6404, P. 324
28917.2950	0.010	-0.0203	LAUSE	AN 6404, P. 324
28920.4400	0.010	-0.0422	LAUSE	AN 6404, P. 324
28920.4542	0.004	-0.0230	LAUSE	AN 6404
28936.2390	0.010	-0.0254	LAUSE	AN 6404, P. 324
28950.5090	0.010	-0.0543	LAUSE	AN 6404, P. 324
28955.3250	0.010	0.0120	LAUSE	AN 6404, P. 324
28963.2360	0.010	0.0069	LAUSE	AN 6404, P. 324
28966.3730	0.010	-0.0175	LAUSE	AN 6404, P. 324
28974.2970	0.010	-0.0146	LAUSE	AN 6404, P. 324
28977.4220	0.010	-0.0490	LAUSE	AN 6404
29004.3640	0.010	-0.0287	LAUSE	AN 6404
37368.5060	0.001	-0.0221	RUDOLPH	AN 283, P. 163
37376.4230	0.001	-0.0162	RUDOLPH	AN 283, P. 163
40567.4101	0.001	0.7336	BATTISTINI, BONI	IBVS #317
40623.3645	0.001	-0.0074	BATTISTINI, BONI	IBVS #317
40916.5107	0.001	-0.0067	BATTISTINI, BONI	IBVS #317
40931.5513	0.001	0.7349	BATTISTINI, BONI	IBVS #317
40939.4671	0.001	0.7846	BATTISTINI, BONI	IBVS #817
41668.5426	0.001	-0.0049	BATTISTINI, BONI	IBVS #817
41684.3730	0.001	-0.0047	EBERSBERGER	IBVS #937
41691.4976	0.001	0.7371	BATTISTINI, BONI	IBVS #817
41752.4560	0.001	0.0000	EBERSBERGER	IBVS #937

BFAU2: PERIOD=

58321700

5.0E-02

O-C RESIDUALS(DAYS)



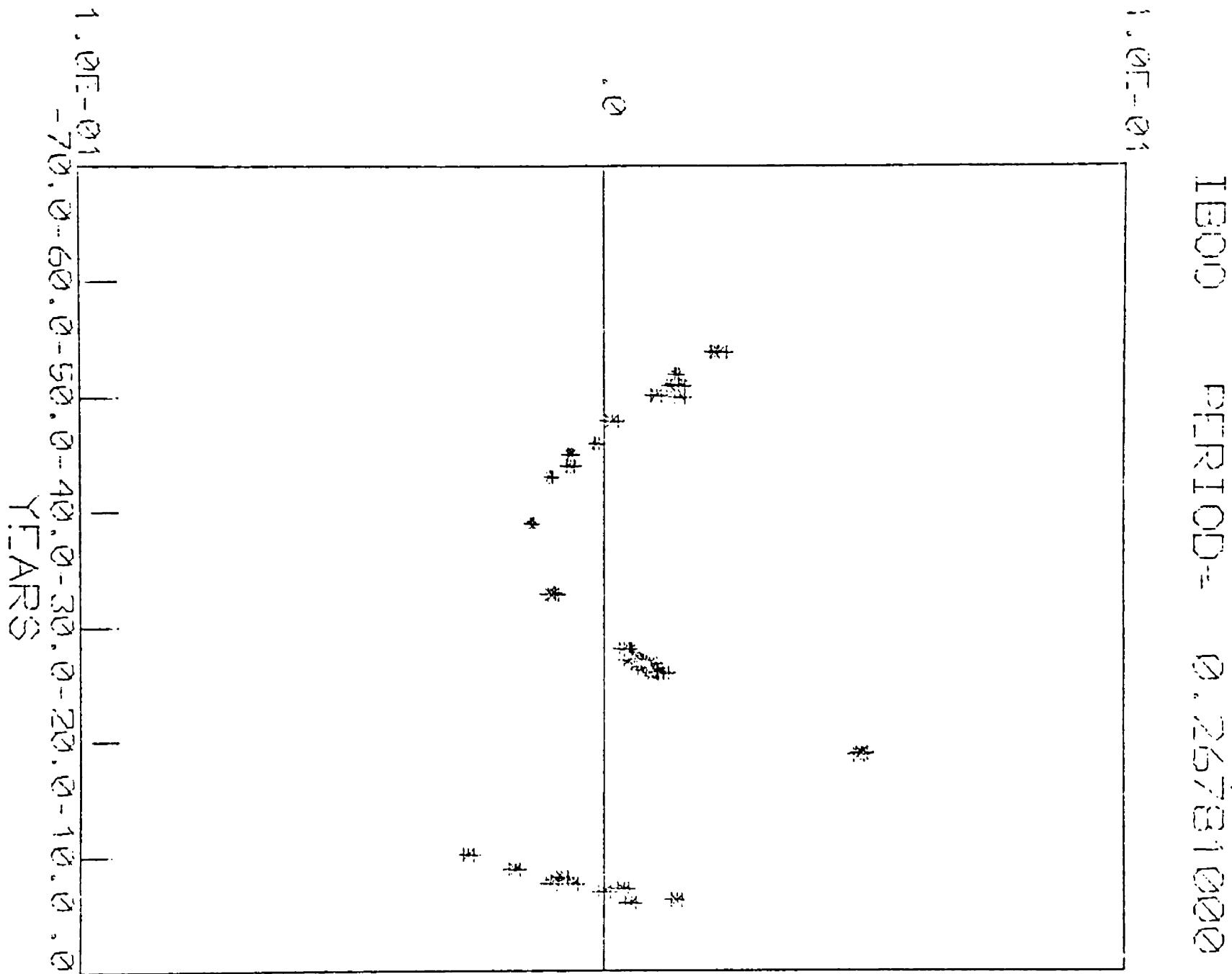
CHRONOLOGY OF 11 OBSERVATIONS OF EAU2

67

$$TCALC = JD \ 23931.4140 + E * 1.53321728$$

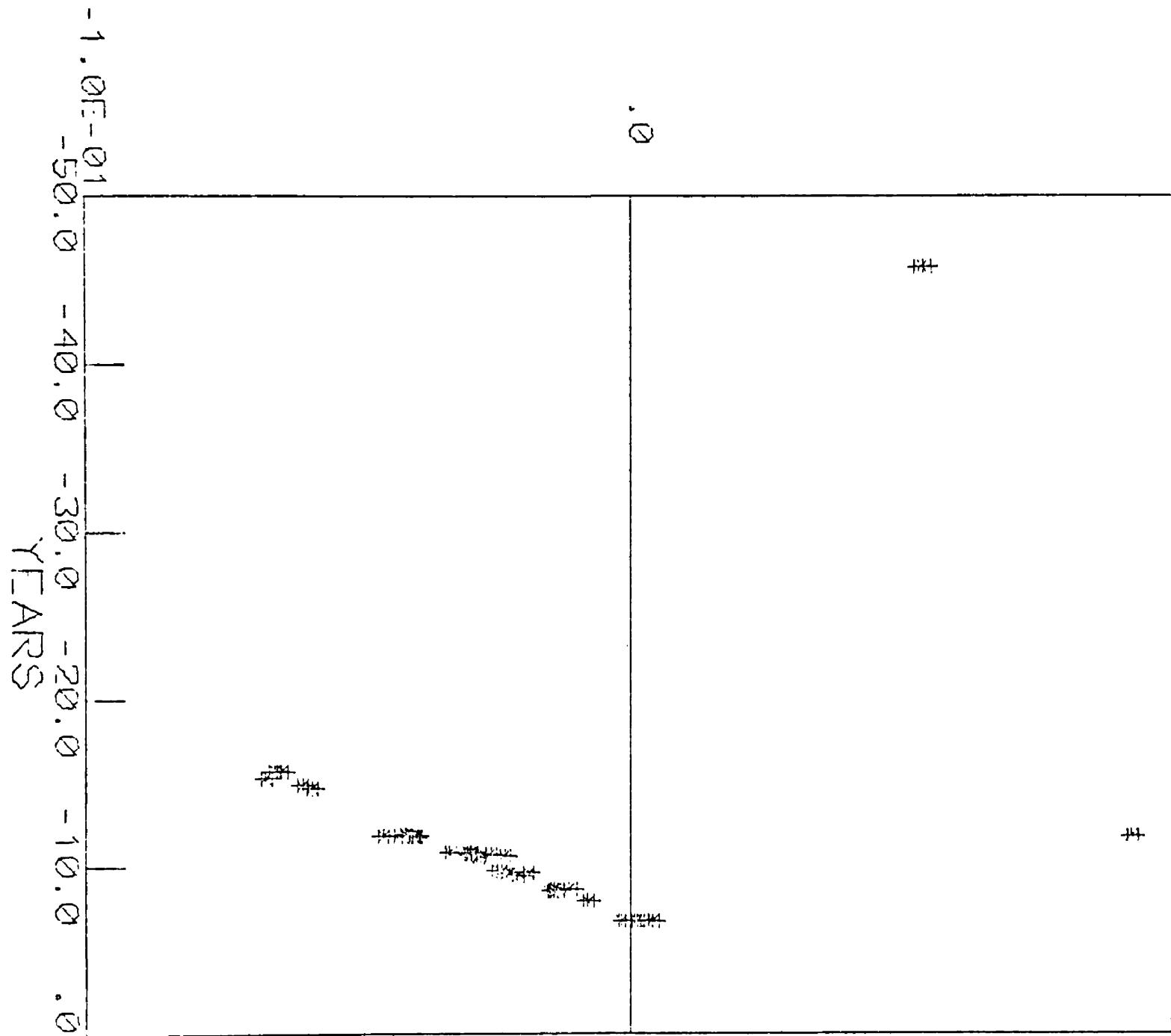
HELIOPCENTRIC JULIAN DATE	+/-	O-C RESIDUAL	OBSERVER	REFERENCE
28495.3370	0.010	-0.0294	LAUSE	AN 6404
28514.3470	0.010	-0.0130	LAUSE	AN 6404
28533.3494	0.007	-0.0142	LAUSE	AN 6404
28612.5110	0.010	-0.0134	LAUSE	AN 6404
28831.6570	0.010	-0.0143	LAUSE	AII 6404
28913.3350	0.010	-0.0007	LAUSE	AN 6404
28935.4790	0.010	-0.0217	LAUSE	AN 6404
28962.4055	0.005	-0.0099	LAUSE	AN 6404
28973.4710	0.010	-0.0269	LAUSE	AN 6404
28973.2790	0.010	0.0314	LAUSE	AN 6404
28931.4140	0.010	0.0000	LAUSE	AN 6404

O-C RESIDUALS (DAYS)



HELIOCENTRIC JULIAN DATE	+/-	O-C RESIDUAL	OBSERVER	REFERENCE
34353.7347	0.001	0.0033	BINNENDIJK	AJ 62, P. 357
34393.6339	0.001	0.0093	BINNENDIJK	AJ 60, P. 357
34393.6335	0.001	0.0099	BINNENDIJK	AJ 62, P. 356
34931.6743	0.001	0.0104	BINNENDIJK	AJ 60, P. 357
34913.7265	0.001	0.0112	BINNENDIJK	AJ 60, P. 357
35930.3457	0.001	0.0236	SZAFRANIEC	A.A. 10, P. 111
37443.7668	0.001	0.2496	WEHLAU	JRASC 56, P. 163
37449.6577	0.001	0.8495	WEHLAU	JRASC 56, P. 123
37472.6339	0.001	0.8490	WEHLAU	JRASC 56, P. 123
40661.4273	0.001	-0.0263	ENDPES	IBVS #530
41138.4068	0.001	-0.0172	GROBEL	IBVS #647
41334.5323	0.001	-0.0033	GULMEN	IBVS #937
41322.4320	0.001	0.1250	GORE, HOLZL	IBVS #937
41435.4166	0.001	-0.0079	GUDUR	IBVS #937
41462.4642	0.001	-0.0093	AKINCI	IBVS #937
41535.3220	0.001	-0.0061	HUCK	IBVS #937
41599.3149	0.001	-0.0102	HUCK	IBVS #937
41758.4970	0.001	0.0037	VOGEL	IBVS #937
41793.4448	0.001	-0.1303	SCHAROLD, VOGEL	IBVS #937
41319.4648	0.001	0.0000	WEBER	IBVS #937
42095.4540	0.001	-0.1221	: SCHELLEMANN	IBVS #1053
42122.3710	0.001	0.0139	: GROBEL, SCHAROL	IBVS #1053
42137.4402	0.001	0.0051	: EBERSBERGER, GR	IBVS #1053
42219.4470	0.001	-0.1251	GULMEN	IBVS #1053
42268.3230	0.001	0.0094	GULMEN	IBVS #1053

O-C RESIDUALS(DAYS)

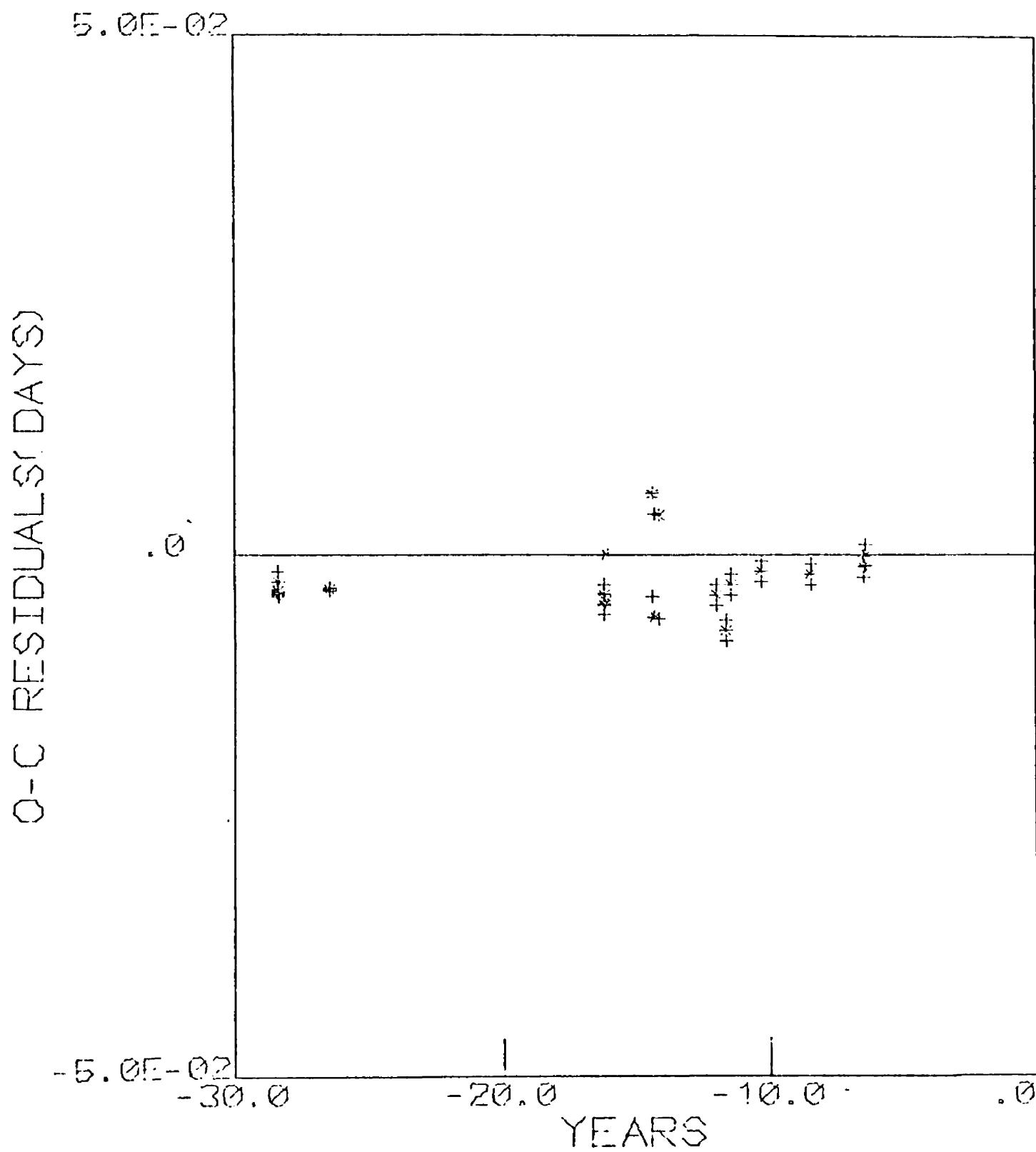


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27563.4092	0.002	0.0536	MUNCH-PLAUT	EAN 321
33512.3470	0.001	-0.0642	POHL	AN 289, #4, P. 191
33513.4170	0.001	-0.0655	POHL	AN 289, #4, P. 191
33663.3392	0.001	-0.0671	HELD, FLEISCHER	AN 289 #4, P. 191
38333.4550	0.001	-0.0604	GORZ, ENDRES	AN 289, #4, P. 191
38882.4660	0.001	-0.0537	GORZ, ENDRES	AN 289, #4, P. 191
39922.9250	0.001	-0.0415	BROWN	AJ 74, P. 542
39945.4222	0.001	-0.0403	MINTI	IBVS #322
39948.3630	0.001	-0.0455	BICKEL, NURNBERG	IBVS #456
39952.9210	0.001	-0.0402	BROWN	AJ 74, P. 542
39956.4000	0.001	-0.0423	GORZ, MEIER	IBVS #456
39959.3490	0.001	-0.0397	GORZ, ENDRES	IBVS #456
39959.4310	0.001	0.0923	GORZ, ENDRES	IBVS #456
39963.4530	0.001	-0.0412	BICKEL, MEIER	IBVS #456
39977.2920	0.001	-0.0399	KURUTAC, POHL	IBVS #456
40316.3460	0.001	-0.0334	POHL, GUDUR	IBVS #456
40319.2960	0.001	-0.0293	POHL, KURUTAC	IBVS #456
40325.4554	0.001	-0.0293	DUMITRESCU	IBVS #419
40331.3460	0.001	-0.0307	GUDUR	IBVS #456
40339.3816	0.001	-0.0295	DUMITRESCU	IBVS #419
40339.6493	0.001	-0.0296	RUDNICK	IBVS #739
40346.6161	0.001	-0.0258	RUDNICK	IBVS #739
40363.4342	0.001	-0.0300	BAUMBACH, MEIER	IBVS #456
40332.7676	0.002	-0.0287	SCARF	AJ 76, P. 50
40320.3026	0.001	-0.0236	SCARF	AJ 76, P. 50
40392.6320	0.001	-0.0232	RUDNICK	IBVS #789
40420.7971	0.001	-0.0282	SCARF	AJ 76, P. 50
40700.3943	0.001	-0.0246	DUMITRESCU	IBVS #503
40714.5391	0.001	-0.0238	RUDNICK	IBVS #739
40753.4231	0.001	-0.0222	GULMEN	IEVS #530
40762.7628	0.001	-0.0189	RUDNICK	IBVS #789
40730.4710	0.001	-0.0231	HOLZL	IBVS #530
40331.3266	0.001	-0.0223	NIEHAUS	IBVS #844
40312.3737	0.001	-0.0204	NIEHAUS	IBVS #844
41102.6556	0.001	-0.0140	RUDNICK	IBVS 27, #789
41110.9603	0.001	-0.0114	SCARF	IBVS #844
41131.3469	0.001	-0.0139	NIEHAUS	IBVS #844
41139.3450	0.001	-0.0145	GULMEN	IBVS #647
41141.4386	0.001	-0.0134	MEIER	IBVS #647
41147.9143	0.001	-0.0151	SCARF	IBVS #844
41392.4320	0.001	-0.0080	GORZ	IBVS #937
41793.4440	0.001	0.0041	VOGEL	IBVS #937
41814.7746	0.001	-0.0017	BARLOW	IBVS #844
41816.9193	0.001	0.0005	BARLOW	IBVS #844
41318.7946	0.001	0.0011	SCARF	IEVS #844
41327.9021	0.001	0.0031	BARLOW	IBVS #844
41829.7737	0.001	0.0000	SCARF	IBVS #844

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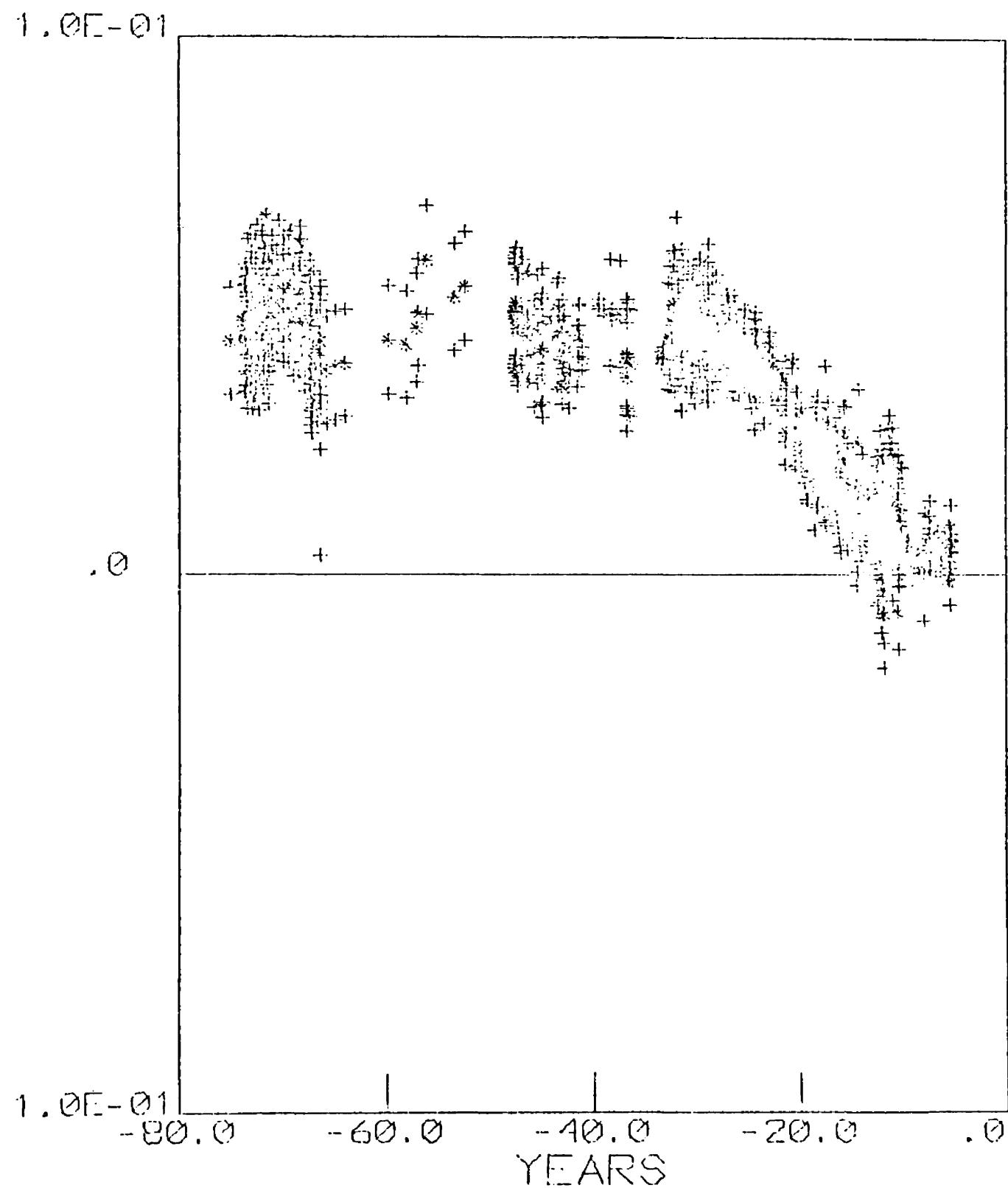
HELIOPCENTRIC JULIAN DATE	+/-	O-C RESIDUAL	OBSERVER	REFERENCE
33926.4573	0.001	-0.0034	K'EE	BAJ 485, P. 134
33931.2355	0.001	-0.0026	SCHNELLER	AN 231(1), P. 25
33935.3575	0.001	-0.0039	K'EE	BAJ 485, P. 134
34636.4560	0.001	-0.0033	K'EE	BAJ 485, P. 134
33379.5242	0.001	-0.2039	WINKLER	AJ 71 #1, P. 44
33383.6313	0.001	-0.0043	WINKLER	AJ 71 #1, P. 44
33399.3790	0.010	-0.0044	ORLOVIUS	AN 239 #4, P. 192
33414.4460	0.010	-0.0000	ORLOVIUS	AN 239 #4, P. 192
39031.3360	0.010	0.0059	HAZER, BOZKURT	AN 239 #4, P. 192
39070.3500	0.010	-0.0060	KIZILIRMAK	AN 239 #4, P. 192
39122.3945	0.010	0.0039	BOZKURT, AKYOL	AN 239 #4 P. 192
39917.2340	0.001	-0.0033	BICKEL, MEIER	IBVS 27, #456
40051.4750	0.001	-0.0073	IEANOGLU, KIZILI	IBVS 27, #456
40114.4637	0.001	-0.0029	KIZILIRMAK	IBVS 27, #456
40518.4230	0.001	-0.0015	GUDUR	IBVS 27, #456
41200.3499	0.001	-0.0019	SENGONCA	IBVS 27, #647
41335.3666	0.001	-0.0011	SEZER	IBVS #937
41960.2310	0.001	0.0000	VOGEL	IBVS #937
42405.3623	0.001	-0.0011	EBERSBERGER, WEB	IBVS #1053

RZCAS: PERIOD=

19525189

1.0E-01

O-C RESIDUALS(DAYS)



CHRONOLOGY OF 613 OBSERVATIONS OF EZCAS

76

TCALC = JD 42339.7331 + E * 1.19525139

HELIOPCENTRIC JULIAN DATE	+/-	O-C RESIDUAL	OBSERVER	REFERENCE
16886.8310	0.010	0.0369	EPHEMERIS	ODESSA IZVEST.4, P17
17355.4200	0.010	0.0372	MULLER, KEMPF	VAR. STAR 9.2, P. 125
17355.4233	0.001	0.0405	GRAFF	VJS 1914
17410.4030	0.010	0.0386	NIJLAND	AN 4211
17417.5730	0.010	0.0371	NIJLAND	AN 4211
17429.5260	0.010	0.0375	NIJLAND	AN 4211
17435.5020	0.010	0.0373	NIJLAND	AN 4211
17437.8960	0.004	0.0408	PARKHURST, JORDA	P. ALLEGHENY 3.16, P14
17441.4320	0.010	0.0410	NIJLAND	AN 4211
17447.4560	0.010	0.0388	NIJLAND	VAR. STAR 9.2, P. 125
17454.6300	0.004	0.0412	PARKHURST, JORDA	P. ALLEGHENY 3.16, P14
17479.7350	0.004	0.0460	PARKHURST, JORDA	P. ALLEGHENY 3.16, P14
17491.6320	0.010	0.0424	PARKHURST, JORDA	VAR. STAR 9.2, P. 126
17495.2730	0.010	0.0457	NIJLAND	AN 4211
17502.4330	0.010	0.0342	NIJLAND	AN 4211
17564.5870	0.004	0.0351	PARKHURST, JORDA	P. ALLEGHENY 3.16, P14
17582.5190	0.010	0.0383	NIJLAND	AN 4211
17587.3030	0.010	0.0413	NIJLAND	AN 4211
17599.2610	0.010	0.0463	NIJLAND	AN 4211
17630.3300	0.010	0.0392	NIJLAND	P. ALLEGHENY 3.16, P14
17630.3330	0.010	0.0422	NIJLAND	AN 4211
17649.4490	0.010	0.0342	NIJLAND	AN 4211
17667.3370	0.010	0.0434	GRAFF	VAR. STAR 9.2, P. 125
17685.3080	0.010	0.0356	NIJLAND	AN 4211
17771.3770	0.010	0.0465	NIJLAND	AN 4211
17772.5620	0.010	0.0362	NIJLAND	AN 4211
17777.3470	0.010	0.0402	NIJLAND	AN 4211
17778.5400	0.010	0.0330	NIJLAND	AN 4211
17783.3250	0.010	0.0420	NIJLAND	VAR. STAR 9.2, P. 125
17791.6910	0.010	0.0412	PARKHURST, JORDA	VAR. STAR 9.2, P. 126
17791.5960	0.004	0.0462	PARKHURST, JORDA	P. ALLEGHENY 3.16, P14
17815.5030	0.010	0.0432	NIJLAND	AN 4211
17839.5020	0.010	0.0421	GRAFF	VAR. STAR 9.2, P. 125
17840.6970	0.004	0.0419	PARKHURST, JORDA	P. ALLEGHENY 3.16, P14
17863.3290	0.010	0.0341	NIJLAND	BAN 58
17864.6010	0.010	0.0408	YENDELL	VAR. STAR 9.2, P. 126
17942.2910	0.010	0.0395	NIJLAND	BAN 58
18010.4280	0.010	0.0471	NIJLAND	BAN 58
18022.3820	0.010	0.0456	NIJLAND	BAN 58
18035.5250	0.010	0.0438	YENDELL	VAR. STAR 9.2, P. 126
18047.4740	0.010	0.0403	NIJLAND	BAN 58
18047.4750	0.010	0.0413	NIJLAND	VAR. STAR 9.2, P. 125
18043.6710	0.010	0.0421	NIJLAND	BAN 58
18053.4470	0.010	0.0370	NIJLAND	BAN 58
18090.5060	0.010	0.0432	NIJLAND	BAN 58
18145.4330	0.010	0.0437	NIJLAND	BAN 58
18157.4570	0.010	0.0601	NIJLAND	BAN 58
18182.5360	0.010	0.0389	NIJLAND	BAN 58

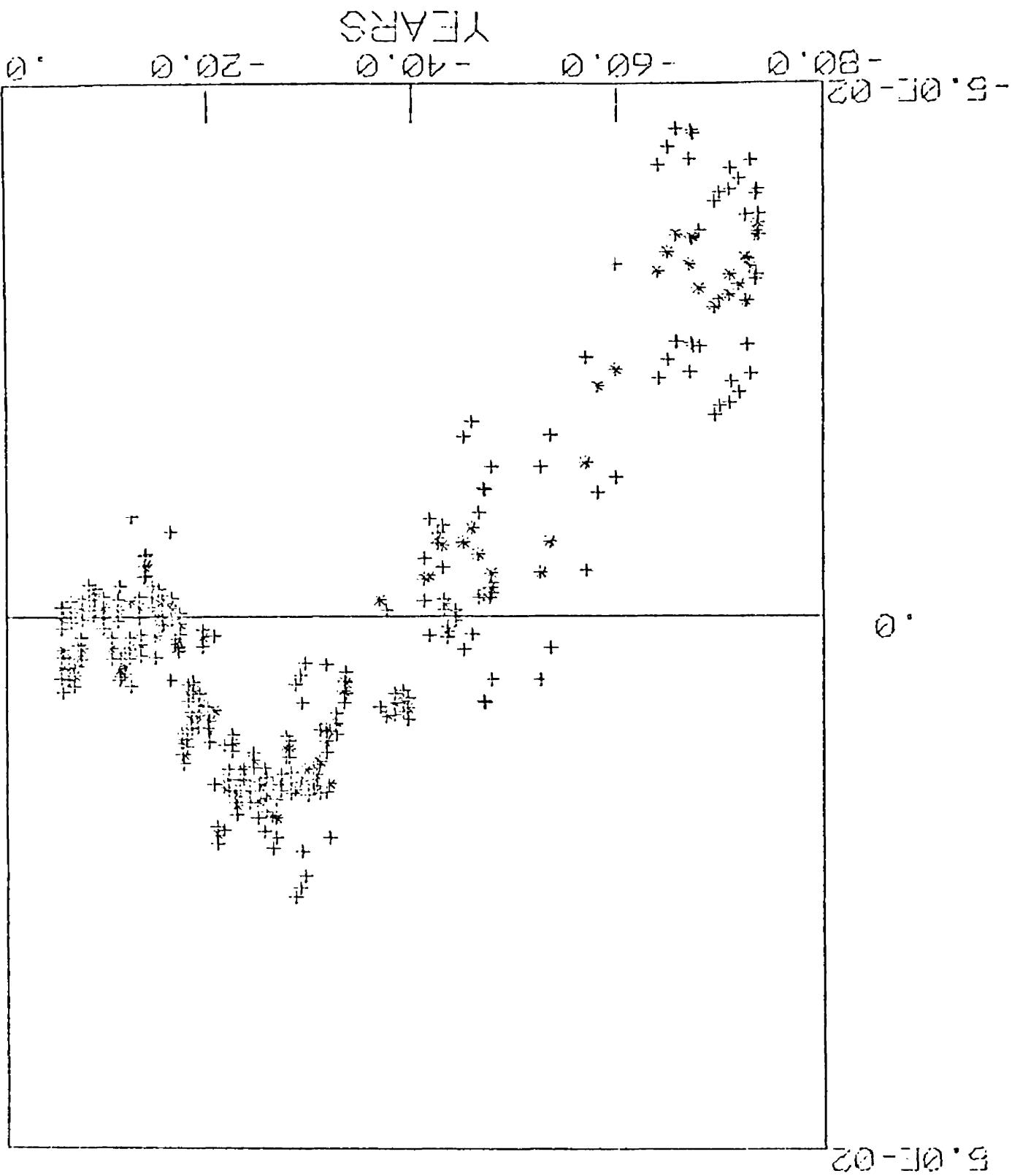
	+/-	O-C RESIDUAL	OBSERVER	REFERENCE
39046.3166	0.001	0.0025	HECKATHORN	P. GOODSELL OBS. #15
39050.4180	0.010	0.0181	KIZILIRMAK	AN 289#4, P. 191
39052.7928	0.001	0.0024	STOKES	VAR. STAR 16, P. 39
39057.5737	0.001	0.0023	HECKATHORN	P. GOODSELL OBS. #15
39062.3562	0.001	0.0038	MINTI, DRAGUSIN	IBVS #148
39068.3316	0.001	0.0029	MINTI, DRAGUSIN	IBVS #148
39141.2450	0.010	0.0060	LEITMEIRE	AN 289#4, P. 191
39246.4200	0.001	-0.0012	SCHUBERT, GROBAN	AN 292, P. 185
39327.6970	0.001	-0.0013	STOKES	IBVS #180
39362.3630	0.001	0.0074	SCHUBERT, GROBAN	AN 292, P. 185
39436.4720	0.001	0.0057	SCHUBERT, GROBAN	AN 292, P. 185
39511.7690	0.001	0.0019	STOKES	IBVS #221
39694.6460	0.010	0.0053	BALDWIN	IBVS #795
39701.3100	0.010	-0.0022	BALDWIN	IBVS #795
39737.6737	0.001	0.0040	STOKES	IBVS #247
39737.6750	0.010	0.0053	BALDWIN	IBVS #795
39749.6255	0.001	0.0033	STOKES	IBVS #247
39763.7520	0.003	0.0057	SANNER	IBVS #795
39774.7230	0.003	0.0005	SANNER	IBVS #795
39784.2876	0.002	0.0030	MINTI	IBVS 27, #322
39785.4834	0.001	0.0036	ENDRES, GORTZ	AN 291, P. 111
39793.3490	0.003	0.0024	SANNER	IBVS #795
39803.4170	0.001	0.0084	SCHUBERT, GROBAN	AN 292, P. 185
39810.5870	0.003	0.0069	BORTLE	IBVS #795
39817.7500	0.010	-0.0016	COOK	IBVS #795
39822.5450	0.008	0.0124	SANNER	IBVS #795
39830.5920	0.010	-0.0074	CRAGG	IBVS #795
39835.6730	0.010	-0.0074	CRAGG	IBVS #795
39835.6860	0.010	0.0056	COOK	IBVS #795
39845.2470	0.001	0.0046	SCHUBERT, GROBAN	AN 292, P. 185
39847.6400	0.007	0.0071	SVANBERG	IBVS #795
39856.0050	0.010	0.0053	SANNER	IBVS #795
39859.5910	0.010	0.0056	BALDWIN	IBVS #795
39860.7760	0.010	-0.0047	SANNER	IBVS #795
39860.7340	0.010	0.0033	COOK	IBVS #795
39872.7400	0.010	0.0068	COOK	IBVS #795
39877.5140	0.001	-0.0002	?????	IBVS #285
39873.7120	0.010	0.0025	COOK	IBVS #795
39882.2930	0.001	0.0038	SCHUBERT, GROBAN	AN 292, P. 185
39884.6850	0.010	-0.0007	COOK	IBVS #795
39884.6390	0.010	0.0033	CRAGG	IBVS #795
39884.6920	0.003	0.0063	BALDWIN	IBVS #795
39890.6650	0.003	0.0030	LUCAS	IBVS #795
39890.6690	0.007	0.0070	BALDWIN	IBVS #795
39896.6470	0.003	0.0033	BALDWIN	IBVS #795
39902.6050	0.010	-0.0095	DOBROWSKI	IBVS #795
39903.5350	0.003	-0.0058	THOMPSON	IBVS #795
39903.5930	0.003	0.0072	BORTLE	IBVS #795
39915.7660	0.008	0.0037	BALDWIN	IBVS #795
39919.3520	0.001	0.0110	SCHUBERT, GROBAN	AN 292, P. 185
39945.6420	0.003	-0.0016	SANNER	IBVS #795
39945.6460	0.007	0.0024	THOMPSON	IBVS #795

HELIOCENTRIC JULIAN DATE	+/-	O-C RESIDUAL	OBSERVER	REFERENCE
39957.5320	0.010	-0.0141	DOBKOVSKI	IBVS #795
39963.5770	0.003	0.0247	BALONEK	IBVS #795
40037.6320	0.003	0.0040	BALONEK	IBVS #795
40054.4163	0.001	0.0053	KIZILIRMAK,KURU	IBVS 27, #456
40056.3070	0.003	0.0250	BALDWIN	IBVS #795
40080.7070	0.003	-0.0000	BALONEK	IBVS #795
40093.3022	0.002	0.0094	SURKOVA	IBVS #394
40095.4965	0.002	0.0035	SKATOVA	IBVS #394
40095.4973	0.002	0.0093	SURKOVA	IBVS #394
40097.4453	0.002	0.0052	SURKOVA	IBVS #394
40098.6390	0.003	0.0032	BORTLE	IBVS #795
40098.6410	0.003	0.0052	BALONEK	IBVS #795
40098.6490	0.010	0.0132	HAZEL	IBVS #795
40116.5750	0.010	0.0104	HEASLEY	IBVS #795
40117.7640	0.010	0.0042	NOLTHENIUS	IBVS #795
40121.3564	0.002	0.0103	SURKOVA	IBVS #394
40127.3272	0.021	0.0053	IBANOGLU	IBVS 27, #456
40129.7150	0.003	0.0026	SWANBERG	IBVS #795
40129.7200	0.003	0.0076	BALDWIN	IBVS #795
40139.2373	0.002	0.0134	SURKOVA	IBVS #394
40147.6470	0.010	0.0059	HAZEL	IBVS #795
40147.6490	0.010	0.0079	BALDWIN	IBVS #795
40151.2413	0.002	0.0149	SURKOVA	IBVS #394
40173.7200	0.003	0.0023	NOLTHENIUS	IBVS #795
40173.7220	0.003	0.0043	BALDWIN	IBVS #795
40182.3089	0.001	0.0055	ROSSATI	ASTRON.AP. 30, P.261
40184.6990	0.003	0.0051	SWANBERG	IBVS #795
40189.4301	0.001	0.0051	ROSSATI	ASTRON.AP. 30, P.261
40196.6523	0.001	0.0053	LANDIS	IBVS #795
40196.6540	0.003	0.0075	BORTLE	IBVS #795
40202.6221	0.001	0.0064	LANDIS	IBVS #795
40203.6055	0.001	0.0065	LANDIS	IBVS #795
40203.6090	0.003	0.0102	BORTLE	IBVS #795
40220.5500	0.010	-0.0015	SIMMONS	IBVS #795
40220.5570	0.003	0.0055	BORTLE	IBVS #795
40227.7320	0.008	0.0290	CRAGG	IBVS #795
40233.7120	0.003	0.0127	SWEETSR	IBVS #795
40245.6530	0.003	0.0062	BALDWIN	IBVS #795
40270.7600	0.007	0.0079	BALDWIN	IBVS #795
40274.3375	0.001	-0.0003	SURKOVA	IBVS 27, #501
40282.7100	0.010	0.0054	CRAGG	IBVS #795
40294.6640	0.003	0.0069	BALDWIN	IBVS #795
40318.5630	0.010	0.0059	BORTLE	IBVS #795
40429.7070	0.007	-0.0136	SWANBERG	IBVS #795
40429.7220	0.003	0.0014	BALONEK	IBVS #795
40436.3970	0.008	0.0049	BALDWIN	IBVS #795
40442.8740	0.010	0.0057	BALDWIN	IBVS #795
40447.6500	0.003	0.0006	BALONEK	IBVS #795
40453.6240	0.007	-0.0016	BALONEK	IBVS #795
40453.6220	0.010	0.0034	HAZEL	IBVS #795
40453.6330	0.007	0.0074	ANDERSON	IBVS #795
40454.8200	0.008	-0.0009	BALONEK	IBVS #795

HELIOCENTRIC
JULIAN DATE
O-C
+/- **RESIDUAL** **OBSERVER** **REFERENCE**

40454.3220	0.012	0.0011	S'EETSI R	IBVS #795
40466.7760	0.003	0.0026	BALDWIN	IBVS #795
40502.6350	0.003	0.0040	ORTWEIN	IBVS #795
40519.3669	0.001	0.0024	DEMIRCAN	IBVS 27, #530
40533.7130	0.003	0.0055	S'EETSI R	IBVS #795
40533.7130	0.003	0.0055	GREEN	IEVS #795
40539.6370	0.010	0.0033	BALDWIN	IBVS #795
40557.6139	0.001	0.0014	LANDIS	IEVS #795
40557.6140	0.010	0.0015	SWANBERG	IBVS #795
40557.6150	0.010	0.0025	BALDWIN	IBVS #795
40746.4607	0.001	-0.0016	KIZILIMAK	IEVS 27, #530
40753.4125	0.001	-0.0023	DEMIRCAN	IBVS 27, #530
40789.4902	0.001	-0.0014	SURKOVA	IBVS 27, #501
41319.3702	0.001	-0.0025	DEMIRCAN	IEVS 27, #530
41433.5193	0.001	-0.0041	HERCZEG, FRIEBE	ASTRON.AP. 32, P.261
41144.4760	0.001	-0.0052	HERCZOG, FRIEBE	ASTRON.AP. 30, P.261
41162.4060	0.001	-0.0040	IBANOGLU	IEVS 27, #647
41181.5297	0.001	-0.0043	BOZKURT, IBANOGL	ASTRON.AP. 30, P.261
41192.4533	0.001	-0.0045	SENGONCA	IBVS 27, #647
41333.3257	0.001	-0.0053	GORG	IEVS #937
41353.6457	0.010	-0.0053	KLIMEK	IEVS #637
41511.4206	0.001	-0.0029	AKINCI	IBVS #937
41542.4930	0.004	-0.0021	CABAN	IBVS #742
41560.4240	0.001	-0.0049	EBERSBERGER	IBVS #937
41560.4333	0.003	0.0041	ZALUSKI	IBVS #746
41566.4022	0.001	-0.0029	EBERSBERGER	IEVS #937
41566.4041	0.001	-0.0010	DURBECK	ASTRON.AP. 30, P.261
41573.3533	0.004	0.0003	SEDEIELOWSKI	IEVS #742
41534.3326	0.001	-0.0013	AKINCI	IEVS #937
41597.4301	0.001	-0.0016	DURBECK	ASTRON.AP. 32, P.261
41669.4329	0.001	-0.0013	DURBECK	ASTRON.AP. 32, P.261
41763.6392	0.001	-0.0009	CHAMBLISS	IEVS #333
41726.5630	0.001	-0.0009	CHAMBLISS	IEVS #333
41732.5442	0.001	-0.0009	CHAMBLISS	IEVS #333
41360.4375	0.001	0.0004	HUCK	IEVS #937
41921.3940	0.001	-0.0009	HOLZL	IBVS #937
41933.3450	0.001	-0.0025	BAYGUN	IBVS #937
41934.5418	0.001	-0.0009	EBERSBERGER	IBVS #937
41954.3561	0.001	-0.0059	MARGRAVE, LUKES	IBVS #1019
41990.7138	0.001	-0.0007	CHAMBLISS	IBVS #333
42004.7056	0.001	-0.0009	CHAMBLISS	IBVS #333
42235.7330	0.004	-0.0032	KROBUSEK	IEVS #954
42235.7430	0.004	-0.0032	HALLAMA	IBVS #954
42265.6260	0.004	-0.0015	KROBUSEK	IEVS #954
42289.5308	0.001	-0.0017	SCHE	IEVS #1053
42300.2384	0.001	-0.0014	IB/HN/ER	IEVS #1053
42303.3754	0.001	-0.0001	KARLE, VAUCHER	PASP 87, P. 909
42314.6350	0.004	0.0022	KROBUSEK	IBVS #954
42325.3889	0.001	-0.0012	ER/BY	IEVS #1053
42339.7265	0.001	-0.0066	DOOLITTLE, EVANS	IBVS #1019

O-C RESIDUALS (DAYS)



RZABER; PERIOD= 19524783

HISTORIOLOGY OF 123 OBSERVATIONS OF PLUTO

 $T_{CALC} = JD\ 42339.7265 + E * 1.10534733$

HELIOPCENTRIC JULIAN DATE	0-C RESIDUAL	OBSERVER	REFERENCE
17355.4233	0.001	-0.0363	GRAFT
17437.3260	0.004	-0.0362	PARKHURST, JORDA
17454.6309	0.004	-0.0357	PARKHURST, JORDA
17657.3372	0.010	-0.0323	GRAFF
17791.5360	0.004	-0.0296	PARKHURST, JORDA
17840.6370	0.004	-0.0337	PARKHURST, JORDA
18035.5250	0.010	-0.0311	YENDELL
18324.3368	0.010	-0.0321	VIJLAUD
18421.5210	0.010	-0.0302	YENDELL
18733.5512	0.010	-0.0299	GRAFF
18922.4100	0.010	-0.0291	BETTENDORF
19442.3322	0.005	-0.0302	LEINERT
19631.2777	0.010	-0.0355	GRAFF
19725.6010	0.010	-0.0356	YENDELL
19822.4133	0.010	-0.0329	LEINERT
20275.6100	0.010	-0.0359	DUGAN
20557.4950	0.010	-0.0342	DUGAN
20949.5330	0.010	-0.0325	DUGAN
22412.5303	0.010	-0.0231	DUGAN
23100.2253	0.010	-0.0217	DUGAN
23512.3320	0.010	-0.0144	
24305.4330	0.010	-0.0271	
25104.2430	0.010	-0.0241	
26393.5340	0.010	-0.0242	RIGGIER
26907.3734	0.001	-0.0023	RIGGIER
26933.9553	0.001	-0.0023	RIGGIER
27112.4330	0.010	-0.0026	ELLSWORTH
27156.4085	0.010	-0.0022	SKOBERLA
27345.3360	0.004	-0.0059	KORDYLEWSKI
27603.5070	0.010	-0.0084	ELLSWORTH
27372.4322	0.010	-0.0070	ELLSWORTH
28165.2317	0.001	-0.0022	SZAFRAJNEC
28434.4145	0.001	0.0014	TECZA
28551.3457	0.001	-0.0013	TECZA
28643.3744	0.002	-0.0067	TECZA
28833.1920	0.001	-0.0075	TECZA
29273.4475	0.006	-0.0033	TECZA
29261.3206	0.002	-0.0036	TECZA
29375.6202	0.001	0.0036	HUFER, KOPAL
30223.6312	0.001	0.0073	HUFER, KOPAL
30322.7125	0.001	0.0032	HUFER, KOPAL
30633.2520	0.010	0.0092	PAGACZEWSKI
30360.5643	0.010	-0.0015	BANACHIEWICZ
32035.7011	0.001	0.0061	HUFER, KOPAL
32152.6353	0.001	0.0070	HUFER, KOPAL
32452.5460	0.001	0.0099	HUFER, KOPAL
32641.5010	0.025	0.0153	SZAFRAJNEC
32770.5324	0.066	0.0164	SZAFRAJNEC

HELIOCENTRIC JULIAN DATE	+/-	O-C RESIDUAL	OBSERVER	REFERENCE
32246.4200	0.001	-0.0050	SCHUBERT, GROBAN	AN 292, P. 135
39327.6270	0.001	-0.0049	STOKES	IBVS #130
32436.4720	0.001	0.0026	SCHUBERT, GROBAN	AN 292, P. 135
32511.7590	0.001	-0.0010	STOKES	IBVS #221
32774.7230	0.003	-0.0016	SANNER	IBVS #795
32845.2470	0.001	0.0028	SCHUBERT, GROBAN	AN 292, P. 135
40054.4168	0.001	0.0042	KIZILIRMAK, KURU'	IBVS 27, #456
40127.3272	0.001	0.0045	IBANOGLU	IBVS 27, #456
40132.3039	0.001	0.0043	ROSSATI	ASTRON.AP. 30, P.261
40220.5570	0.003	0.0050	DORTLE	IBVS #795
40274.3375	0.001	-0.0007	SURKOVA	IBVS 27, #501
40519.3669	0.001	0.0029	DEMIRECAN	IBVS 27, #530
40739.4230	0.001	0.0000	SURKOVA	IBVS 27, #501
40819.3702	0.001	-2.0010	DEMIRECAN	IBVS 27, #530
41033.5193	0.001	-0.0017	HERCZEG, FRIEBOE	ASTRON.AP. 30, P.261
41162.4060	0.001	-0.0013	IBANOGLU	IBVS 27, #647
41199.4533	0.001	-0.0017	SENGONCA	IBVS 27, #647
41333.3257	0.001	-0.0021	GORE	IBVS #937
41560.4240	0.001	-0.0009	EBERSBERGER	IBVS #937
41629.4329	0.001	0.0023	DURBECK	ASTRON.AP. 30, P.261
41703.6392	0.001	0.0036	CHAMBLISS	IBVS #883
41860.4375	0.001	0.0054	HUCK	IBVS #937
41954.3561	0.001	-0.0006	MARGRAVE, LUKES	IBVS #1019
41990.7138	0.001	0.0047	CHAMBLISS	IBVS #883
42094.7056	0.001	0.0049	CHAMBLISS	IBVS #883
42235.7430	0.004	0.0031	MALLAMA	IBVS #954
42239.5303	0.001	0.0047	SCHE	IBVS #1053
42339.7265	0.001	0.0000	DOOLITTLE, EVANS	IBVS #1019

TWCAS: PERIOD= .42832800

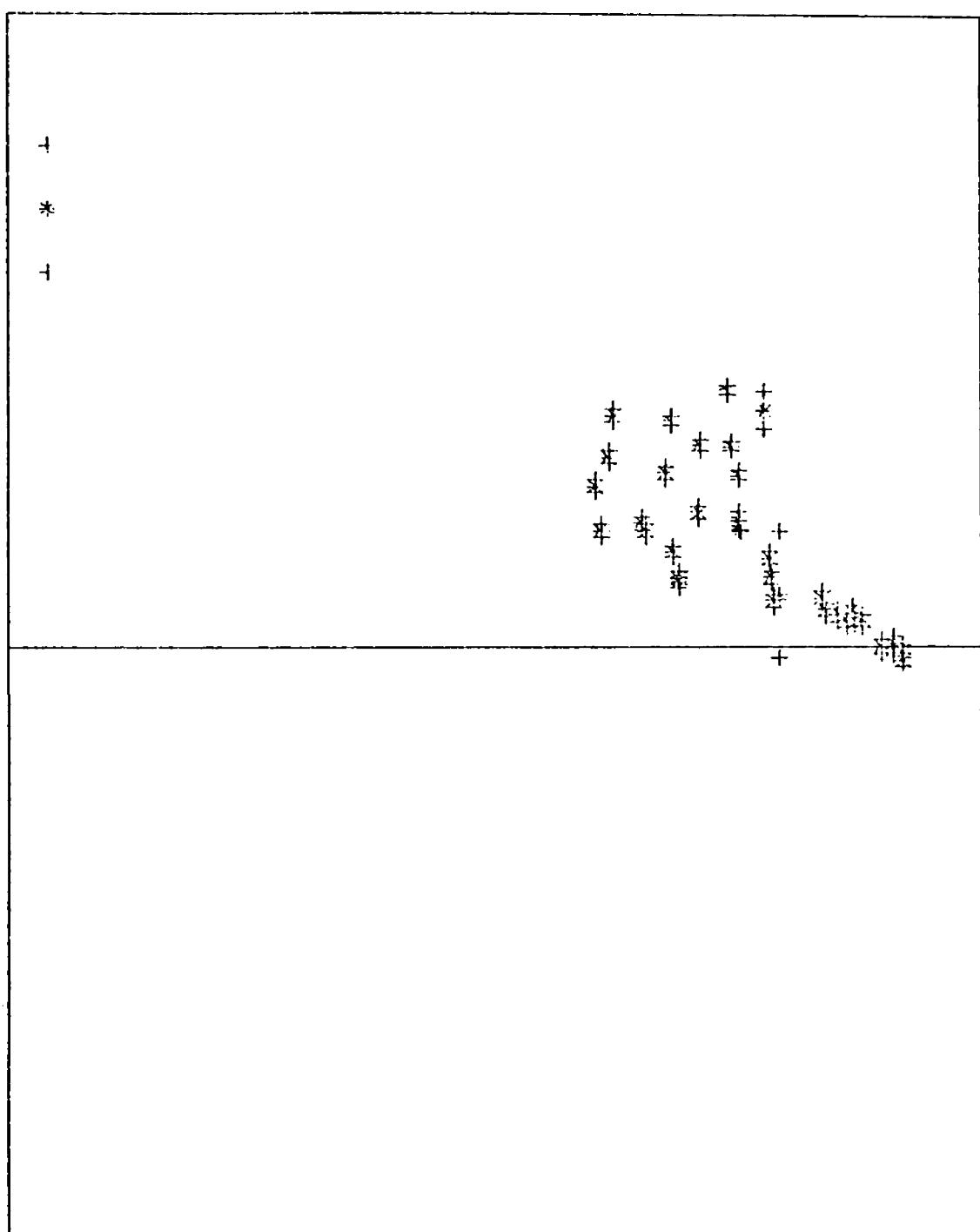
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O-C RESIDUALS(DAYS)

1.0E-01

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YEARS

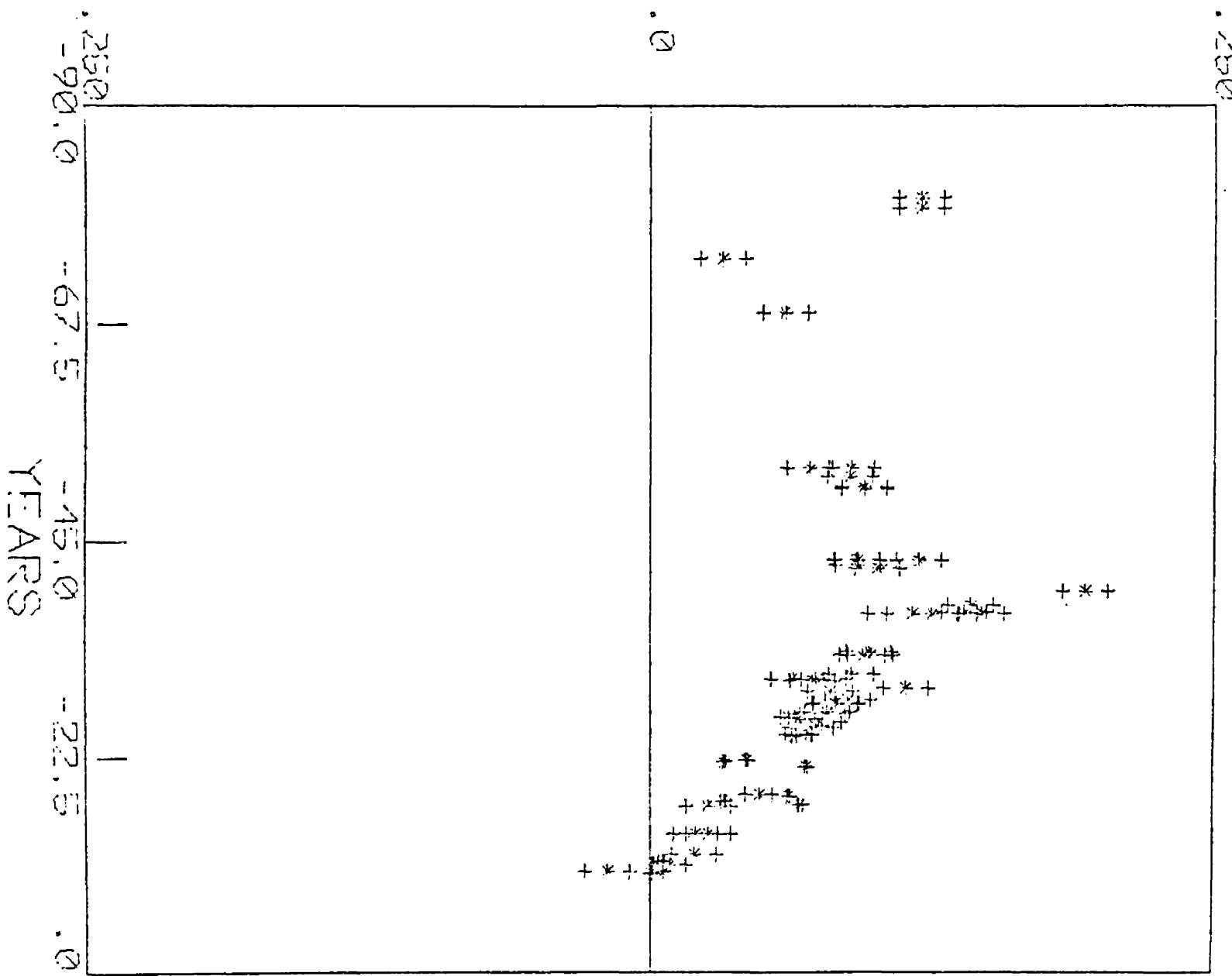


TCALC = JD 41671.3072 + E * 1.423323E0

HELIOPCENTRIC JULIAN DATE	+/-	O-C RESIDUAL	OBSERVER	REFERENCE
19819.3792	0.010	0.0693	ZINNER	AN 4679, P. 454
34135.4670	0.001	0.0255	POHL	AN 232 P. 235
34253.2260	0.001	0.0183	POHL	AN 232 P. 235
34455.4170	0.001	0.0301	POHL	AN 232 P. 235
34605.3930	0.001	0.0366	POHL	AN 232 P. 235
35332.4000	0.001	0.0197	RUDOLPH, JAHN	AN 285 P. 163
35442.3800	0.001	0.0134	POHL	AN 235 P. 163
35959.4440	0.001	0.0277	RUDOLPH	AN 285 P. 163
36106.5700	0.001	0.0359	RUDOLPH	AN 235 P. 163
36166.5390	0.001	0.0151	RUDOLPH, BRAUNE,	AN 285 P. 163
36226.5120	0.001	0.0103	RUDOLPH	AN 236 P. 210
36306.5110	0.001	0.0110	:BRAUNE	AN 286, P. 210
36342.2350	0.001	0.0213	BRAUNE	AN 286, P. 210
36359.2950	0.001	0.0320	BRAUNE	AN 236 P. 210
37583.4660	0.001	0.0407	FERNANDEZ	AN 288 P. 169
37696.2250	0.001	0.0313	FERNANDES	AN 288 P. 169
37903.3900	0.001	0.0193	KIZILIRMAK	AN 288 P. 69
37903.3980	0.001	0.0273	FERNANDES	AN 288 P. 169
37923.3830	0.001	0.0207	BRAUNE	AN 288 P. 169
37933.3840	0.001	0.0134	MASUCH	AN 288 P. 169
33590.4340	0.003	0.0375	FLIN	A.A. 17, P. 61
33740.3850	0.001	0.0141	KRAUSSER	AN 289#4, P. 191
33753.2370	0.001	0.0111	POHL	AN 289#4 P. 191
33820.3630	0.001	0.0074	KRAUSSER	AN 289#4 P. 191
33977.4317	0.210	0.0033	KIZILIRMAK	AN 289#4 P. 191
40104.4322	0.001	0.0073	IBANOGLU, GUDUR	IBVS #456
40204.4130	0.001	0.0059	BICKEL	IBVS #456
40534.3560	0.001	0.0051	DEMIRCAN	IBVS #530
40751.4610	0.001	0.0042	ENDRES	IBVS #530
40904.2934	0.001	0.0055	KAFACAN	IBVS #647
40964.2320	0.001	0.0044	MEIER	IBVS #647
41201.3342	0.001	0.0041	HOLZL	IBVS #647
41671.3000	0.001	0.0000	DEMIRCAN	IBVS #937
41983.3395	0.001	0.0007	PATKOS	IBVS #1065
41991.2455	0.001	0.0000	PATKOS	IBVS #1065
42003.3350	0.001	-0.0024	PATKOS	IBVS #1065
42265.4322	0.001	-0.0022	IBANOGLU, GULMEN	IBVS #1053
42265.4336	0.001	-0.0003	IBANOGLU, GULMEN	IBVS #1053

EXCERP. PERIOD = 2.3373520

O-C RESIDUALS(DAYS)

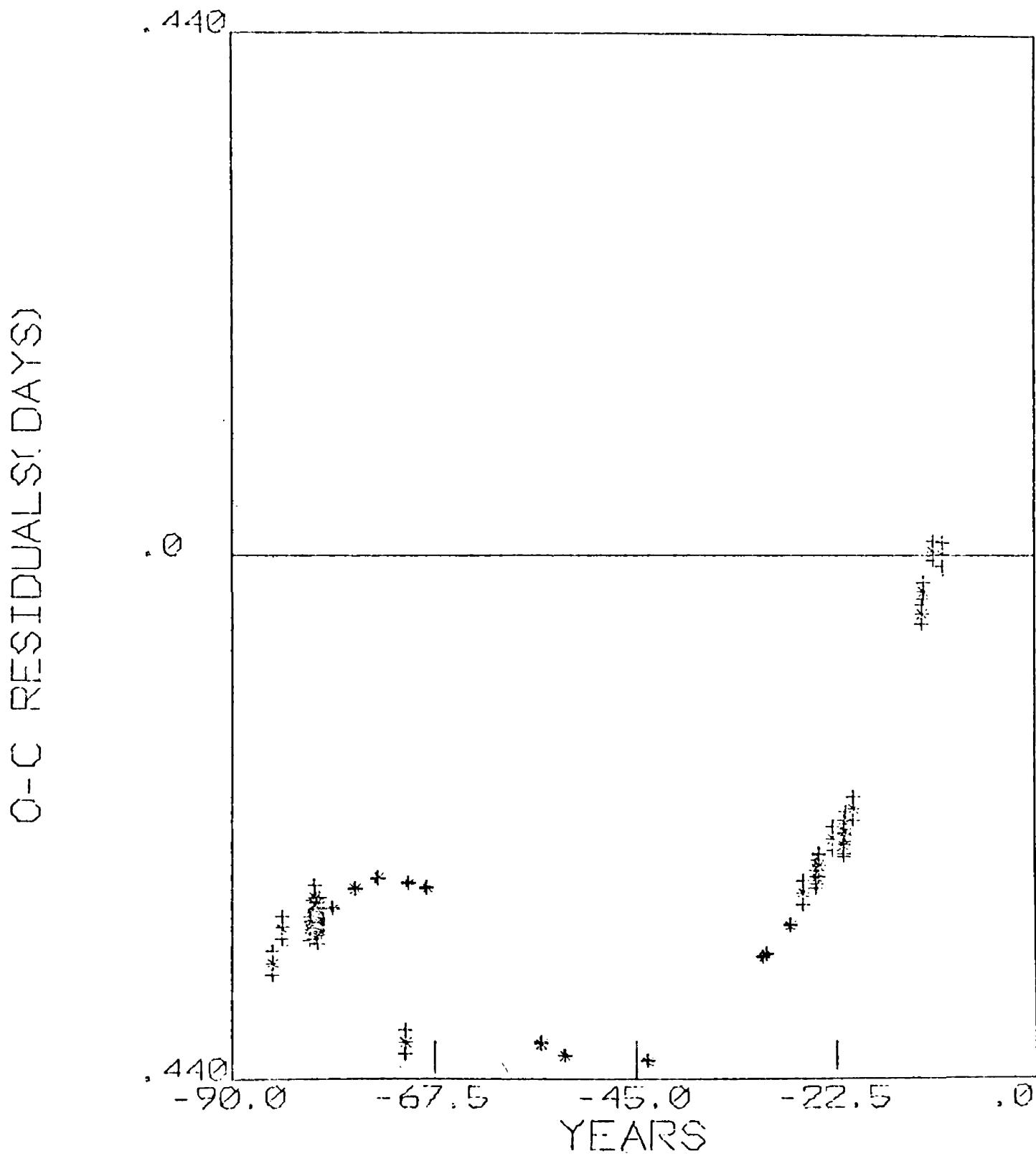


TCALC = JD 42520.4260 + E * 2.33733520

HELIOCENTRIC JULIAN DATE	+/-	O-C RESIDUAL	OBSERVER	REFERENCE
14931.4000	0.010	0.1193	LAUROV	VAR. STAR 12, P21
15291.3500	0.010	0.1201	LAUROV	VAR. STAR 12, P21
17196.1900	0.010	0.0320	LAUROV	VAR. STAR 12, P21
19255.4100	0.010	0.0597	LAUROV	VAR. STAR 12, P21
25124.4700	0.010	0.0710	LAUROV	VAR. STAR 12, P21
25131.5600	0.010	0.0390	LAUROV	VAR. STAR 12, P21
25442.3650	0.010	0.0384	LAUROV	VAR. STAR 12, P21
25351.4050	0.010	0.0347	FRESA	AJ 67 P. 7
25353.4170	0.010	0.0347	FRESA	AJ 67, P. 7
25574.3930	0.010	0.0922	LAUROV	VAR. STAR 12, P21
25595.4330	0.010	0.0912	LAUROV	VAR. STAR 12, P21
23607.1430	0.010	0.1195	LAUROV	VAR. STAR 12, P21
23810.4630	0.010	0.0923	LAUROV	VAR. STAR 12, P21
23926.3320	0.010	0.1006	LAUROV	VAR. STAR 12, P21
23775.3320	0.010	0.1929	LAUROV	VAR. STAR 12, P21
30318.1600	0.010	0.1421	LAUROV	VAR. STAR 12, P21
30539.2230	0.010	0.1393	LAUROV	VAR. STAR 12, P21
30603.2290	0.020	0.1162	:LAUROV	VAR. STAR 12, P. 21
30610.2500	0.020	0.1253	:LAUROV	VAR. STAR 12, P. 21
32617.2340	0.010	0.1472	LAUROV	VAR. STAR 12, P21
32059.3720	0.010	0.0974	FRESA	AJ 67, P. 7
32204.2650	0.010	0.0976	FRESA	AJ 67, P. 7
32232.3320	0.010	0.0946	FRESA	AJ 67, P. 7
32612.0920	0.010	-1.1310	FRESA	AJ 67, P. 7
32954.5610	0.010	0.0890	FRESA	AJ 67, P. 7
33099.4640	0.010	0.0773	FRESA	AJ 67, P. 7
33134.5100	0.010	0.0632	FRESA	AJ 67, P. 7
33155.5550	0.010	0.0722	FRESA	AJ 67, P. 7
33445.4262	0.010	0.1136	LAUROV	VAR. STAR 12, P21
33537.9720	0.010	0.0822	LAUROV	VAR. STAR 12, P21
33332.4940	0.010	0.0330	LAUROV	VAR. STAR 12, P21
34023.2780	0.010	0.0325	LAUROV	VAR. STAR 12, P21
34041.4150	0.010	0.0322	LAUROV	VAR. STAR 12, P21
34060.1140	0.010	0.0325	LAUROV	VAR. STAR 12, P21
34061.3020	0.001	-1.0663	LAUROV	AJ 67, P. 7
34062.4510	0.010	0.0822	LAUROV	VAR. STAR 12, P21
34083.1620	0.010	0.0325	LAUROV	VAR. STAR 12, P21
34337.3370	0.010	0.0736	LAUROV	VAR. STAR 12, P21
34394.3490	0.010	0.0736	LAUROV	VAR. STAR 12, P21
34457.4550	0.010	0.0765	LAUROV	VAR. STAR 12, P21
34543.9210	0.003	0.0611	KOCH, KOCH	AJ 67, P. 7
34623.3950	0.001	0.0657	FRESA	AJ 67, P. 7
34630.4150	0.001	0.0737	FRESA	AJ 67, P. 7
34758.3190	0.010	0.0749	LAUROV	VAR. STAR 12, P21
34903.3330	0.003	0.0735	KOCH, KOCH	AJ 67, P. 7
34949.4680	0.001	-1.0832	FRESA	AJ 67, P. 7
34983.3500	0.010	0.0711	LAUROV	VAR. STAR 12, P21
35240.4520	0.006	0.0662	KORDYLEWSKI	SAC 23, P. 123

HELIOCENTRIC JULIAN DATE	+/-	O-C RESIDUAL	OBSERVER	REFERENCE
35247.4680	0.001	0.3702	FRESA	AJ 67, P. 7
35275.5100	0.001	0.0642	FRESA	AJ 67, P. 7
36231.4590	0.001	0.2431	RUDOLPH	AN 236, P. 210
36308.5310	0.001	0.0330	RUDOLPH	AN 286, P. 210
36460.5440	0.001	0.0692	DORR	AN 286, P. 210
37547.3340	0.006	0.0484	KUBICA	A.A. 17, P. 61
37561.4210	0.001	0.0614	:POHL	AJ 238, P. 70
37721.3490	0.001	1.0506	BRAUNE	AN 238, P. 170
37790.4510	0.001	2.0325	DUEBALL	AN 288, P. 170
37935.4000	0.001	0.0667	POHL	AN 283, P. 70
37936.7300	0.010	0.0254	:ANGIONE	PASP 75, P. 407
39057.2740	0.010	0.0193	:KIZILIRMAK	AN 289#4, P. 192
39257.2300	0.010	0.0258	:KIZILIRMAK	AN 289#4, P. 192
39321.5820	0.010	0.0192	BALDWIN	IBVS #795
40097.3730	0.001	0.0247	IBANOGLU	IBVS 27, #456
40139.4420	0.001	0.0016	IBANOGLU	IBVS 27, #456
40237.6170	0.007	0.0036	BORTLE	IBVS #795
40473.6600	0.010	-0.0193	HAZEL	IBVS #795
40513.4190	0.001	0.0050	BAUMBACH, GORZ	IBVS 27, #456
40520.4260	0.001	0.0000	GUDUR	IBVS 27, #530

SWCYG: PERIOD= 4.57284000



TCALC = JD 40454.6350 + E * 4.57234E03

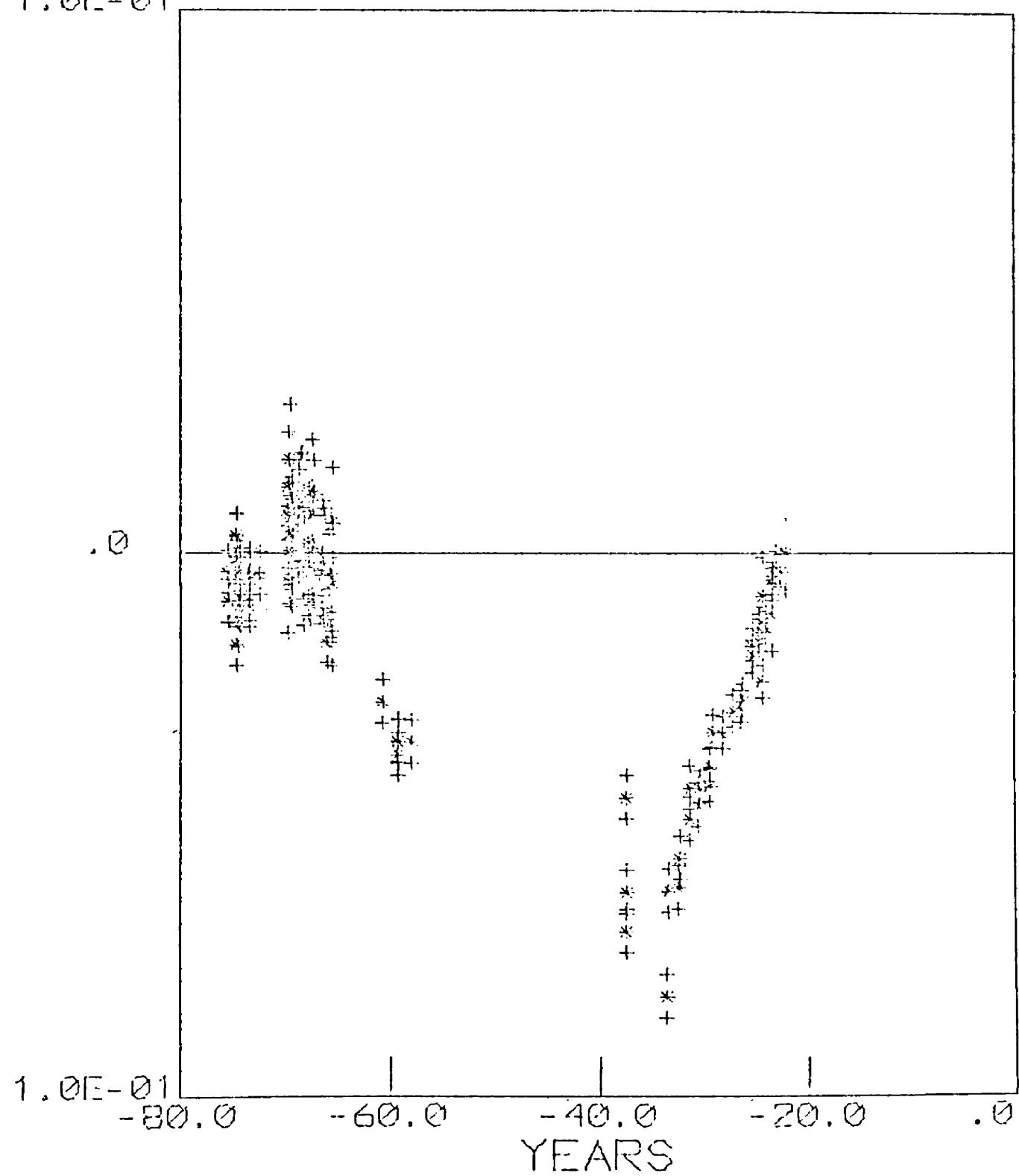
HELIOPCENTRIC JULIAN DATE	+/-	O-C	RESIDUAL	OBSERVER	REFERENCE
13031.3220	0.010	-0.3423	SCHNELLER, H.	AN 237	
13456.3240	0.010	-0.3136	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14553.3050	0.010	-0.3142	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14653.3360	0.001	-0.3036	SLOVOCKOTOVA	VAR. STAR 10 #1(85)25	
14736.7320	0.010	-0.3008	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14732.4430	0.010	-0.3132	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14796.1920	0.010	-0.2878	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14305.3180	0.010	-0.3074	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14329.3910	0.008	-0.3073	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14814.4610	0.008	-0.3101	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14323.1920	0.010	-0.3076	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14332.7520	0.007	-0.3105	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14337.3270	0.010	-0.3083	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14346.4700	0.010	-0.3110	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14846.4740	0.007	-0.2070	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14355.6240	0.010	-0.3027	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14360.1260	0.007	-0.3035	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14364.7640	0.010	-0.3034	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14362.3460	0.010	-0.2992	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14373.4350	0.003	-0.3059	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14901.3540	0.010	-0.3011	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14919.6270	0.003	-0.3194	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14923.7370	0.001	-0.3051	SLOVOCKOTOVA	VAR. STAR 10 #1(85)25	
14933.3560	0.007	-0.3090	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14933.3600	0.710	-0.3050	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14942.5640	0.010	-0.3066	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
14947.0350	0.010	-0.2935	SLOVOCKOTOVA	VAR. STAR 10.1, P.22	
15491.2540	0.001	-0.2974	SLOVOCKOTOVA	VAR. STAR 10 #1(85)25	
16433.2760	0.001	-0.2325	SLOVOCKOTOVA	VAR. STAR 10 #1(85)25	
17366.1440	0.001	-0.2718	SLOVOCKOTOVA	VAR. STAR 10 #1(85)25	
18440.6230	0.010	-0.4102	SLOVOCKOTOVA	AN 287	
18619.6980	0.001	-0.2760	SLOVOCKOTOVA	VAR. STAR 10 #1(85)25	
19341.6230	0.001	-0.2797	SLOVOCKOTOVA	VAR. STAR 10 #1(85)25	
21472.4370	0.001	-0.3392	SLOVOCKOTOVA	VAR. STAR 10 #1(85)25	
23996.6230	0.001	-0.4108	SLOVOCKOTOVA	VAR. STAR 10 #1(85)25	
24261.4320	0.001	-0.4211	SLOVOCKOTOVA	VAR. STAR 10 #1(85)25	
23322.5150	0.001	-0.4255	SLOVOCKOTOVA	VAR. STAR 10 #1(85)25	
33937.5010	0.001	-0.3383	SLOVOCKOTOVA	VAR. STAR 10 #1(85)25	
33160.6690	0.001	-0.3362	?????	AJ 64(1272) P. 259	
34157.5720	0.001	-0.3123	SLOVOCKOTOVA	VAR. STAR 10 #1(85)25	
34683.4760	0.010	-0.2849	SCHNELLER	AN 287	
35250.5210	0.006	-0.2721	SZCZEPA NOWSKA	SAC 28, P. 103	
35250.5210	0.010	-0.2721	SCHNELLER, H.	AN 287	
35296.2540	0.010	-0.2675	SCHNELLER, H.	AN 287	
35346.5600	0.009	-0.2627	SZCZEPA NOWSKA	SAC 28, P. 103	
35346.5600	0.010	-0.2627	SCHNELLER, H.	AN 287	
35963.9160	0.010	-0.2401	SCHNELLER	AN 287	
36402.9030	0.010	-0.2458	SCHNELLER	AN 287	

HELIOCENTRIC JULIAN DATE	+/-	O-C RESIDUAL	OBSERVER	REFERENCE
36430.3430	0.010	-0.2423	SCHNELLER	AN 237
36439.4960	0.010	-0.2355	SCHNELLER	AN 237
36457.7950	0.010	-0.2278	SCHNELLER	AN 287
36787.0520	0.010	-0.2153	SCHNELLER	AN 287
39640.6690	0.003	-0.0505	HAZEL	IBVS #795
39704.7030	0.007	-0.0312	HAZEL	IBVS #795
40079.7150	0.003	0.0029	HAZEL	IBVS #795
40454.6350	0.010	0.2000	HAZEL	IBVS #795

WWCYG: PERIOD= 3.31771000

1.0E-01

O-C RESIDUALS(DAYS)



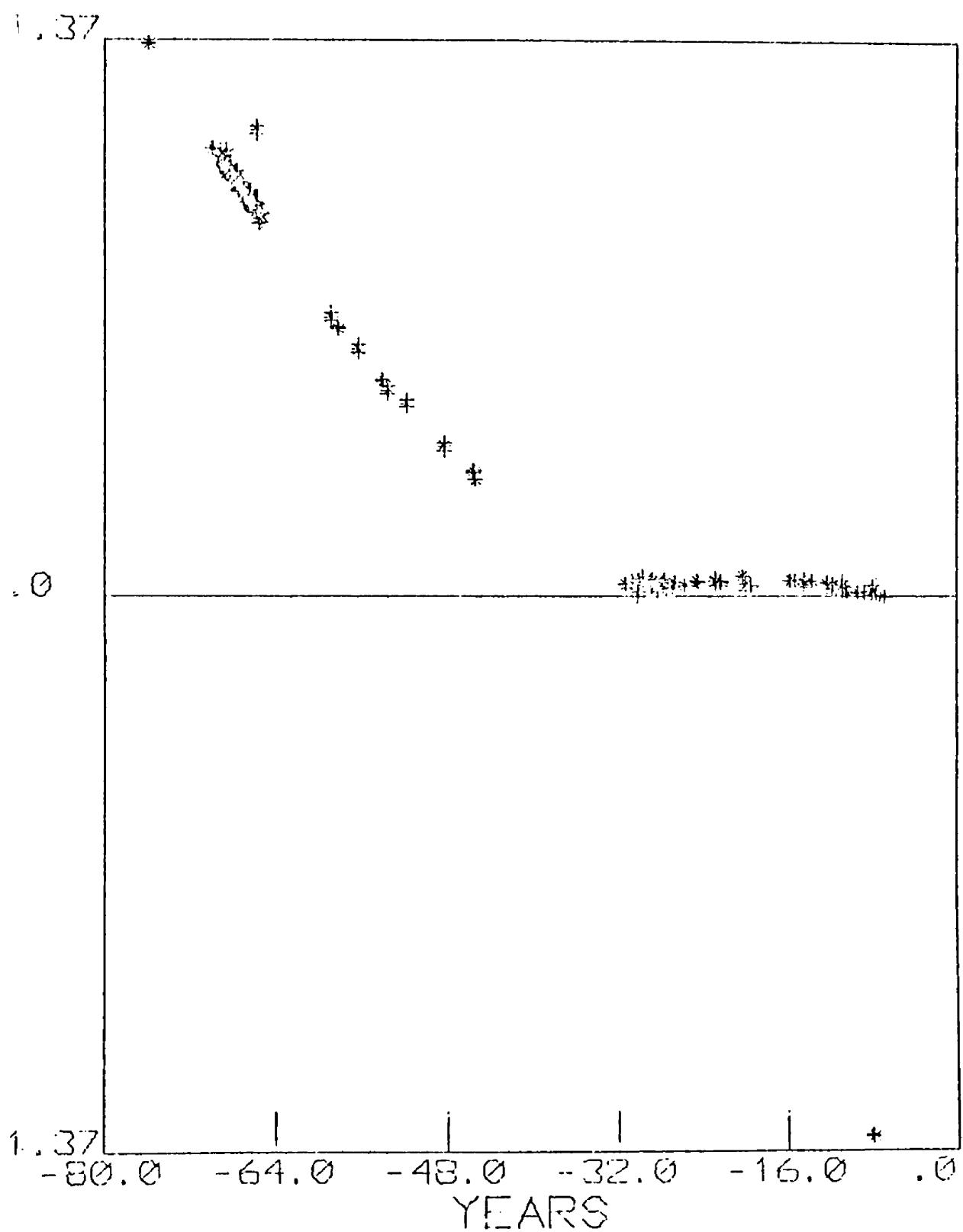
TCALC = JD 36164.3150 + E * 3.31771000

HELIOCENTRIC JULIAN DATE	+/-	O-C RESIDUAL	OBSERVER	REFERENCE
16722.5310	0.004	-0.0034	BLAZKO	AN 5270
16729.1610	0.004	-0.0088	BLAZKO	AN 5270
16752.3850	0.004	-0.0088	BLAZKO	AN 5270
16981.2990	0.004	-0.0168	DUGAN, WRIGHT	CONT. PRINCE. 1
16981.3140	0.004	-0.0013	GRAFF	AN 5270
16991.2720	0.004	0.0031	GRAFF	AN 5270
16994.5900	0.004	0.0034	GRAFF	AN 5270
17064.2450	0.004	-0.0135	GRAFF	AN 5270
17107.3350	0.004	-0.0038	GRAFF	AN 5270
17117.3370	0.004	-0.0049	GRAFF	AN 5270
17130.6030	0.004	-0.0097	GRAFF	AN 5270
17137.2450	0.004	-0.0032	GRAFF	AN 5270
17160.4640	0.004	-0.0031	GRAFF	AN 5270
17452.4220	0.004	-0.0086	GRAFF	AN 5272
17462.3300	0.004	-0.0037	BLAZKO	AN 5270
17472.3320	0.004	-0.0049	GRAFF	AN 5270
17495.5510	0.004	-0.0098	BLAZKO	AN 5270
17502.1930	0.004	-0.0033	GRAFF	AN 5270
17827.3230	0.004	-0.0033	GRAFF	AN 5270
13789.4800	0.010	0.0123	NIJLAND	BAN 53
18799.4230	0.010	0.0071	NIJLAND	BAN 58
13332.5930	0.010	-0.0050	NIJLAND	BAN 53
13342.5600	0.004	0.0089	GRAFF	AN 5270
13372.4170	0.004	0.0065	GRAFF	AN 5270
13332.3310	0.010	0.0174	NIJLAND	BAN 53
13395.6350	0.010	0.0005	NIJLAND	BAN 53
13235.4470	0.004	0.0000	GRAFF	AN 5270
13935.4510	0.010	0.0040	NIJLAND	BAN 53
18945.4000	0.010	-0.0001	NIJLAND	BAN 53
18945.4860	0.004	0.0059	GRAFF	AN 5270
18953.6740	0.010	0.0031	NIJLAND	BAN 53
13995.1700	0.004	0.0042	GRAFF	AN 5270
12144.4630	0.010	0.0053	NIJLAND	BAN 53
19164.3762	0.010	0.0070	NIJLAND	BAN 53
19237.3670	0.010	0.0084	NIJLAND	BAN 58
19260.5210	0.010	0.0034	NIJLAND	BAN 53
19320.2930	0.010	-0.0033	NIJLAND	BAN 58
19363.4330	0.010	0.0014	NIJLAND	BAN 53
19519.3660	0.010	0.0021	NIJLAND	BAN 58
19522.6300	0.010	-0.0016	NIJLAND	BAN 53
19572.4450	0.010	-0.0023	NIJLAND	BAN 53
19635.4850	0.010	0.0022	NIJLAND	BAN 53
19653.7030	0.010	0.0003	NIJLAND	BAN 58
19665.3540	0.010	0.0108	NIJLAND	BAN 53
19668.6680	0.010	0.0071	NIJLAND	BAN 58
19673.6150	0.010	0.0010	NIJLAND	BAN 58
19635.2510	0.010	0.0016	NIJLAND	BAN 58
19927.4390	0.010	-0.0033	NIJLAND	BAN 53

19960.6190	0.010	-0.0004	NIJLAND	BAN 53
20010.3810	0.004	-0.0040	GRAFF	AJ 5270
20023.6540	0.010	-0.0019	NIJLAND	BAN 58
20053.5120	0.010	-0.0032	NIJLAND	BAN 53
20166.3010	0.004	-0.0164	GRAFF	AN 5270
20242.6130	0.010	-0.0067	NIJLAND	BAN 58
20252.5740	0.010	-0.0038	NIJLAND	BAN 58
20315.6200	0.010	0.0057	NIJLAND	BAN 58
20325.5630	0.010	-0.0045	NIJLAND	BAN 58
20335.5150	0.010	-0.0056	NIJLAND	BAN 58
20355.4160	0.010	-0.0103	NIJLAND	BAN 53
22100.5150	0.004	-0.0273	GRAFF	AN 5270
22611.4350	0.004	-0.0346	GRAFF	AJ 5270
22621.3380	0.004	-0.0348	GRAFF	AN 5270
22641.2920	0.004	-0.0370	GRAFF	AN 5270
23079.2320	0.004	-0.0343	GRAFF	AN 5270
30550.6300	0.004	-0.0697	WHITNEY	AJ 55(1139), 230
30573.9110	0.004	-0.0626	WHITNEY	AJ 55(1139), 230
30580.5640	0.004	-0.0451	WHITNEY	AJ 55(1139), 230
31930.6010	0.004	-0.0317	WHITNEY	AJ 55(1189), 230
32053.6100	0.004	-0.0623	WHITNEY	AJ 55(1139), 230
32353.3400	0.004	-0.0616	WHITNEY	AJ 55(1189), 230
32431.3350	0.004	-0.0562	WHITNEY	AJ 55(1189), P. 230
32736.3370	0.004	-0.0492	WHITNEY	AJ 55(1189), 230
32806.7400	0.004	-0.0435	WHITNEY	AJ 55(1189), 230
33058.8920	0.004	-0.0464	WHITNEY	AJ 55(1139), 230
33151.7910	0.003	-0.0433	WHITE	AN 62(1254), 372
33463.6560	0.003	-0.0431	WHITE	AN 62(1254), 372
33506.7900	0.003	-0.0393	WHITE	AN 62(1254), 372
33539.7390	0.003	-0.0330	WHITE	AN 62(1254), 372
33964.6400	0.003	-0.0333	WHITNEY	AN 62(1254), 372
34329.5920	0.003	-0.0294	WHITNEY	AN 62(1254), 372
34591.6220	0.003	-0.0285	WHITNEY	AN 62(1254), 373
34634.5890	0.003	-0.0273	WHITNEY	AN 62(1254), 373
35019.6330	0.003	-0.0170	WHITNEY	AN 62(1254), 373
35029.6390	0.003	-0.0192	WHITNEY	AN 62(1254), 373
35253.5650	0.003	-0.0152	SZAFRANIEC	SAC 28, P. 108
35263.5190	0.003	-0.0143	SZAFRANIEC	SAC 28, P. 103
35331.5460	0.003	-0.0238	WHITNEY	AN 62(1254), 373
35341.5150	0.007	-0.0079	SZAFRANIEC	SAC 28, P. 103
35364.7340	0.003	-0.0129	WHITNEY	AN 62(1254), 373
35467.5820	0.003	-0.0139	WHITNEY	AN 62(1254), 373
35686.5600	0.003	-0.0048	WHITNEY	AN 62(1254), 373
35709.7330	0.003	-0.0057	WHITNEY	AJ 62(1254), 374
35726.3660	0.007	-0.0113	SZAFRANIEC	SAC 28, P. 103
35802.6820	0.003	-0.0026	WHITNEY	AJ 62(1254), 374
36011.6970	0.003	-0.0033	WHITNEY	AJ 62(1254), 374
36164.3150	0.007	0.0000	SZAFRANIEC	SAC 30, P. 106

TWDR: PERIOD= 2.80687000

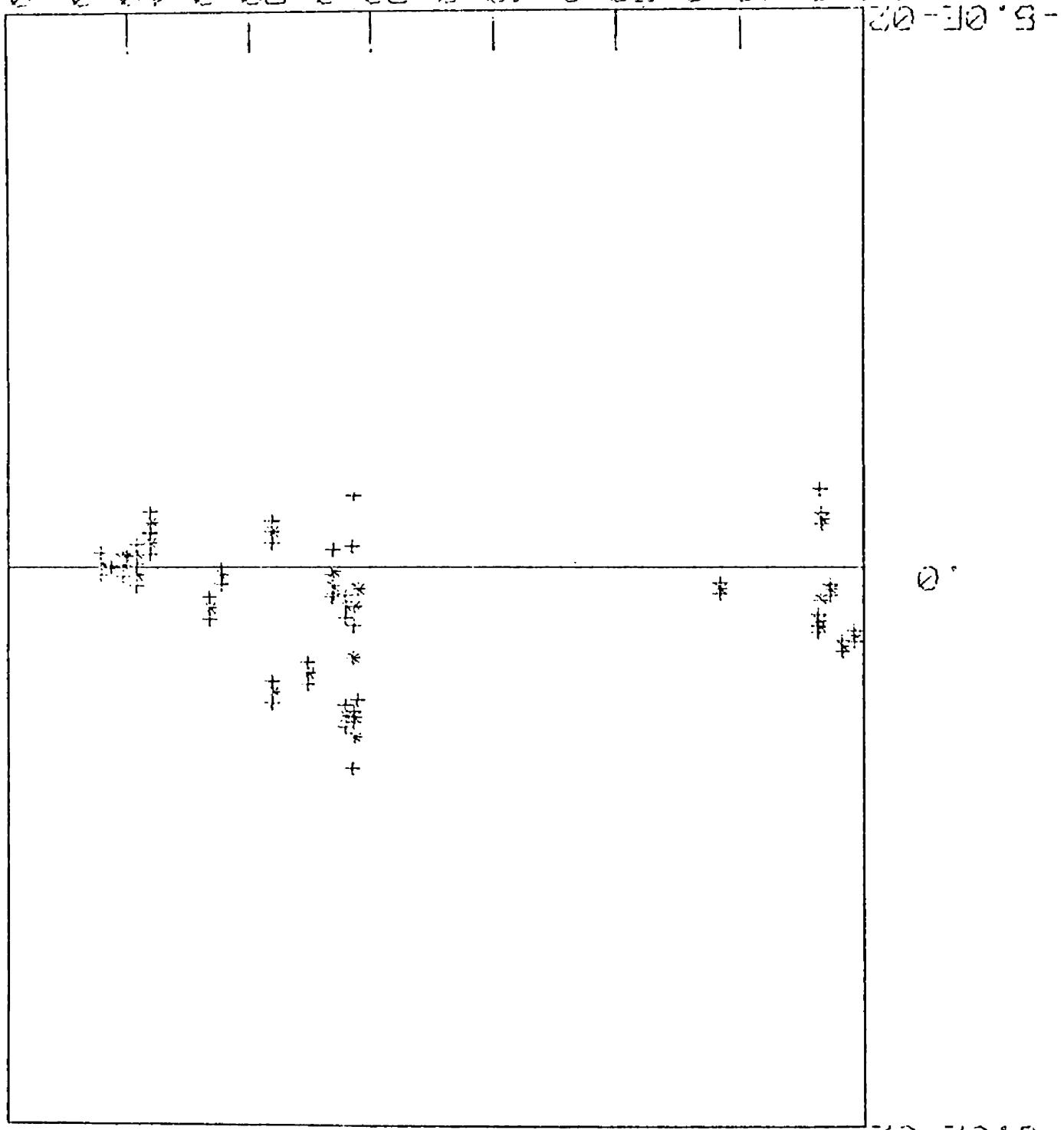
O-C RESIDUALS(DAYS)



HELIOCENTRIC JULIAN DATE	+/-	O-C RESIDUAL	OBSERVER	REFERENCE
20040.3610	0.010	0.9704	HOFFMEISTER	AN 4723
20054.3350	0.010	0.9700	NIJLAND	DAN 58
20054.3420	0.010	0.9770	HOFFMEISTER	AN 4723
20068.3810	0.010	0.9817	HOFFMEISTER	AN 4723
20121.6390	0.010	0.9592	NIJLAND	DAN 58
20138.5260	0.010	0.9549	NIJLAND	DAN 58
20169.4010	0.010	0.9544	NIJLAND	DAN 58
20169.4070	0.007	0.9604	POHL	IBVS #443
20239.5760	0.008	0.9576	NIJLAND	DAN 58
20242.3790	0.010	0.9537	NIJLAND	DAN 58
20259.4130	0.010	1.1465	NIJLAND	DAN 58
20270.4470	0.010	0.9531	NIJLAND	DAN 58
20357.4330	0.010	0.9261	NIJLAND	DAN 58
20335.5020	0.010	0.9264	NIJLAND	DAN 58
20453.4850	0.010	0.9303	NIJLAND	DAN 58
22310.3980	0.010	0.6367	POHL	IBVS #443
23015.2720	0.001	0.6592	SLOVOCKOTOVA	VAR. STAR 10.1(85)P32
23711.3240	0.010	0.6074	POHL	IBVS #443
24567.3420	0.001	0.5301	SLOVOCKOTOVA	VAR. STAR 10.1(85)P32
24746.9580	0.010	0.5064	POHL	IBVS #443
25364.4330	0.003	0.4750	MERGENTALER	A.A. 1, P. 36
26627.4220	0.010	0.3675	POHL	IBVS #443
27629.4140	0.004	0.3069	POHL	IBVS #443
27724.3250	0.001	0.2343	SLOVOCKOTOVA	VAR. STAR 10.1(85)P32
32824.6510	0.001	0.0275	SLOVOCKOTOVA	VAR. STAR 10.1(85)P32
33060.4270	0.010	0.0265	POHL	AN 279, P. 178
33265.3080	0.010	0.0060	POHL	AN 279, P. 178
33310.2330	0.010	0.0260	POHL	IBVS #443
33310.2420	0.010	0.0300	POHL	AN 279, P. 178
33324.2640	0.010	0.0177	POHL	AN 279, P. 178
33436.5510	0.010	0.0299	POHL	AN 279, P. 178
33436.5590	0.010	0.0379	DOMKE	AN 279, P. 178
33509.5250	0.010	0.0253	DOMKE, POHL	AN 281(3), P. 114
33700.3860	0.001	0.0191	DOMKE, POHL, JAHN	AN 281(3), P. 114
33745.3130	0.001	0.0362	DOMKE, POHL, JAHN	AN 281(3), P. 114
33756.5330	0.010	0.0237	DOMKE	AN 281(3), P. 114
33759.3400	0.010	0.0288	POHL	IBVS #443
33798.6370	0.001	0.0296	SLOVOCKOTOVA	VAR. STAR 10.1(85)P32
33888.4570	0.001	0.0298	DOMKE, JAHN	AN 281(3), 114
33947.3890	0.010	0.0175	DOMKE	AN 281(3), P. 114
34079.3220	0.021	0.0277	POHL	AN 282, P. 236
34135.4610	0.001	0.0292	POHL	AN 282, P. 236
34163.5330	0.001	0.0326	DOMKE	AN 282, P. 236
34203.4380	0.010	0.0276	POHL	IBVS #443
34253.3350	0.001	0.0147	POHL	AN 282, P. 236
34455.4350	0.001	0.0201	POHL	AN 282, P. 236
34542.4600	0.001	0.0321	DOMKE	AN 282, P. 236
34876.4683	0.001	0.0229	POHL	AN 286, P. 210
35283.4770	0.001	0.0354	POHL	AN 285, P. 163
35370.4350	0.001	0.0304	RUDOLPH	AN 285, P. 163
35951.5100	0.010	0.0334	POHL	IBVS #443
35951.5120	0.001	0.0354	QUESTER	AN 285, P. 163

HELIOCENTRIC JULIAN DATE	+/-	O-C RESIDUAL	OBSERVER	REFERENCE
36111.5010	0.001	0.0328	RUDOLPH	AN 235, P. 163
36123.3350	0.004	0.0255	SZAFRANIEC	SAC 30, P. 107
36855.3350	0.001	0.0462	:BRAUNE	AN 286, P. 210
37015.3030	0.014	0.0276	SZAFRANIEC	A.A. 13.1
37172.4370	0.003	0.0219	FLIN	A.A. 17, P. 61
38539.4451	0.001	0.0343	:JOBGENS,KRAUSS	AN 289 #4, P. 192
38539.4451	0.001	0.0343	NURNBERG	IBVS #443
38671.3693	0.001	0.0361	NURNBERG	IBVS #443
38991.3455	0.010	0.0286	KIZILIRMAK	AN 289 #4, P. 192
39033.4520	0.008	0.0321	SZAFRANIEC	A.A. 16, P. 158
39266.4245	0.001	0.0344	HUBSCHER,BRAUNE	AN 292, P. 185
39805.3390	0.001	0.0299	SCHUBERT	AN 292, P. 185
39962.5210	0.001	0.0271	SCHUBERT	AN 292, P. 185
39979.3515	0.001	0.0164	POHL	IBVS #456
40080.3995	0.001	0.0171	GUDUR,IBANOGLU	IBVS #456
40324.6040	0.010	0.0239	BORTLE	IBVS #795
40333.6300	0.008	0.0156	BORTLE	IBVS #795
40439.6720	0.003	0.0102	MONSKE	IBVS #795
40473.3520	0.001	0.0078	POHL,MEIER	IBVS #456
40877.5390	0.001	0.0055	ENDRES	IBVS 27, #530
41068.4066	0.000	0.0060	BATTISTINI,BONI	IBVS #817
41357.5200	0.010	0.0117	KLIMEK	IBVS #637
41395.4670	0.002	-1.3374	BATTISTINI,BONI	IBVS #817
41503.4655	0.001	0.0000	GROBEL	IBVS #937
41764.5000	0.000	-1.0857		
42253.5032	0.001	-0.0044	GROBEL	IBVS #937
		-0.0103	:EBERSBERGER	IBVS #1053

O-C RESIDUALS(DAYS)

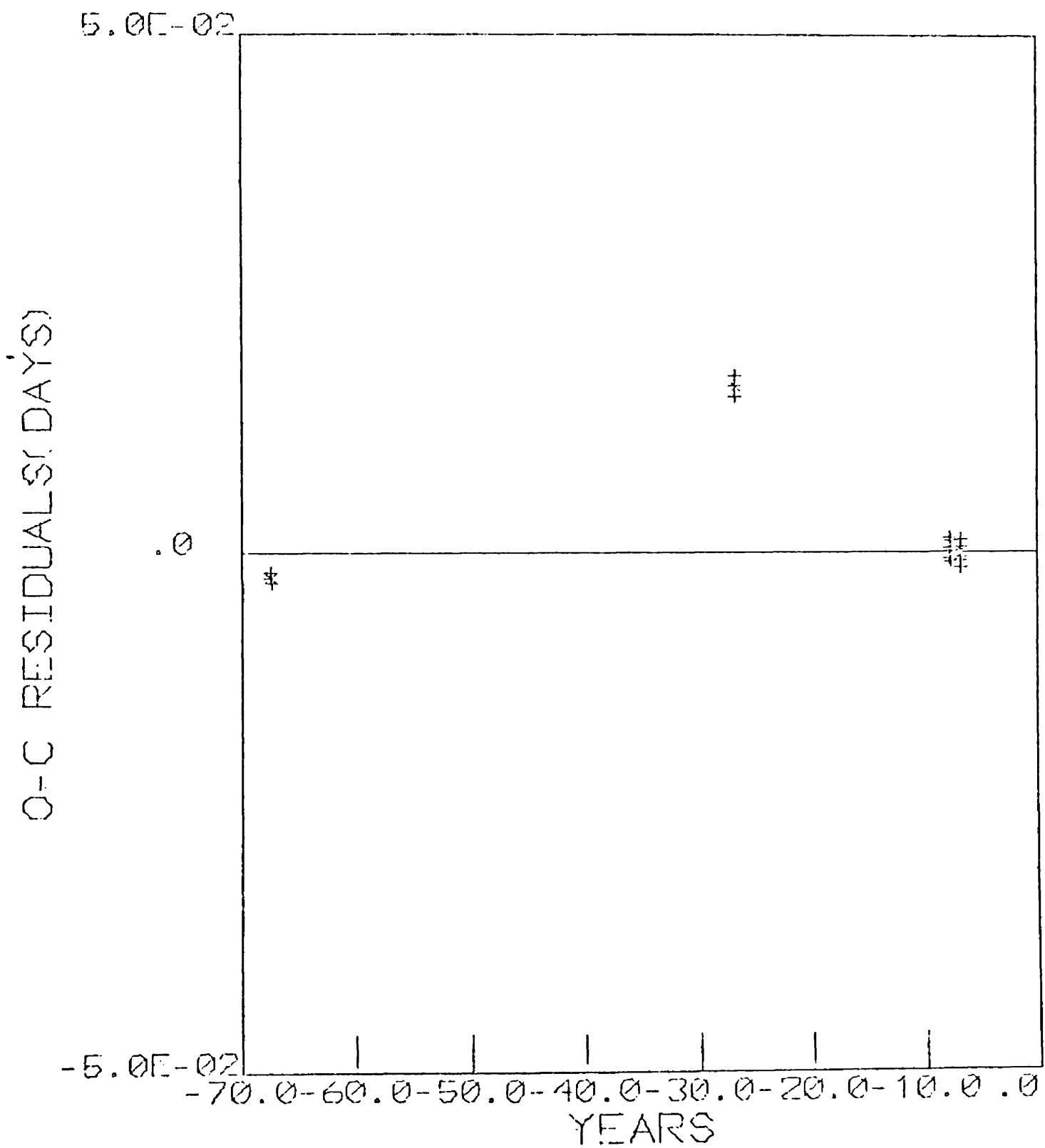


TXHDF. EPOCH = 2.05931000

TCALC = JD 41132.4511 + E * 2.0593103E

HELIOPCENTRIC JULIAN DATE	+/-	O-C RESIDUAL	OBSERVER	REFERENCE
13957.4064	0.001	0.0061	LAZZARINO	AN 4711
19340.2330	0.001	0.0071	BALANOVSKY	AN 5159
19636.2760	0.001	0.0021	BALANOVSKY	AN 5159
19954.2440	0.001	-0.0043	LAZZARINO	AN 4711
19999.3680	0.010	0.0029	BALANOVSKY	AN 4769, P. 282
20026.4470	0.001	0.0054	BALANOVSKY	AN 5159
20063.2240	0.001	0.0043	BALANOVSKY	AN 5159
22959.3140	0.001	0.0020	HELLERICH	AN 5159
33779.4260	0.010	0.0020	DOMKE	AN 281(3), P. 114
33840.5310	0.010	0.0035	DOMKE	AN 281(3), P. 114
33830.4480	0.010	0.0154	JAHN	AN 281(3), P. 114
33832.4930	0.021	0.0135	JAHN, DOMKE	AN 281(3), P. 114
33911.2300	0.010	0.0032	DOMKE	AN 281(3), P. 114
34110.3760	0.001	0.0134	DOMKE	AN 232, P. 236
34121.4260	0.001	0.0036	DOMKE	AN 232, P. 236
34477.7700	0.002	0.0204	FITCH	AJ 69, P. 316
34535.4460	0.001	0.0013	DOMKE	AN 232, P. 236
35227.5500	0.001	0.0096	LICHTENKNECKTER	AN 285, P. 163
36226.5930	0.001	0.0112	RUDOLPH	AN 236, P. 210
36325.4160	0.001	-0.0031	RUDOLPH	AN 236, P. 210
37374.3070	0.001	0.0003	FERNANDES	AN 238, P. 172
33220.4430	0.001	0.0037	POHL	AN 238, P. 71
39979.5180	0.001	-0.0041	GUDUR	IBVS #456
40023.3570	0.001	-0.0021	GUDUR	IBVS #456
40330.4230	0.001	-0.0011	GUDUR	IBVS #456
40426.5015	0.001	0.0007	HOLEZL	IBVS #456
40735.4714	0.001	-0.0029	BALTISTINI	IBVS #951
40334.3431	0.001	-0.0001	YILDIZ	IBVS #530
41132.4511	0.001	0.0000	BALTISTINI	IBVS #951
41491.4223	0.001	-0.0003	GANEA	IBVS #931
42279.3930	0.001	-1.0263	: IBANOGLU, EBERS	IBVS #1053
42230.3332	0.001	0.0034	GUDUR	IBVS #1053

TXHE2: PERIOD= 2.05981000

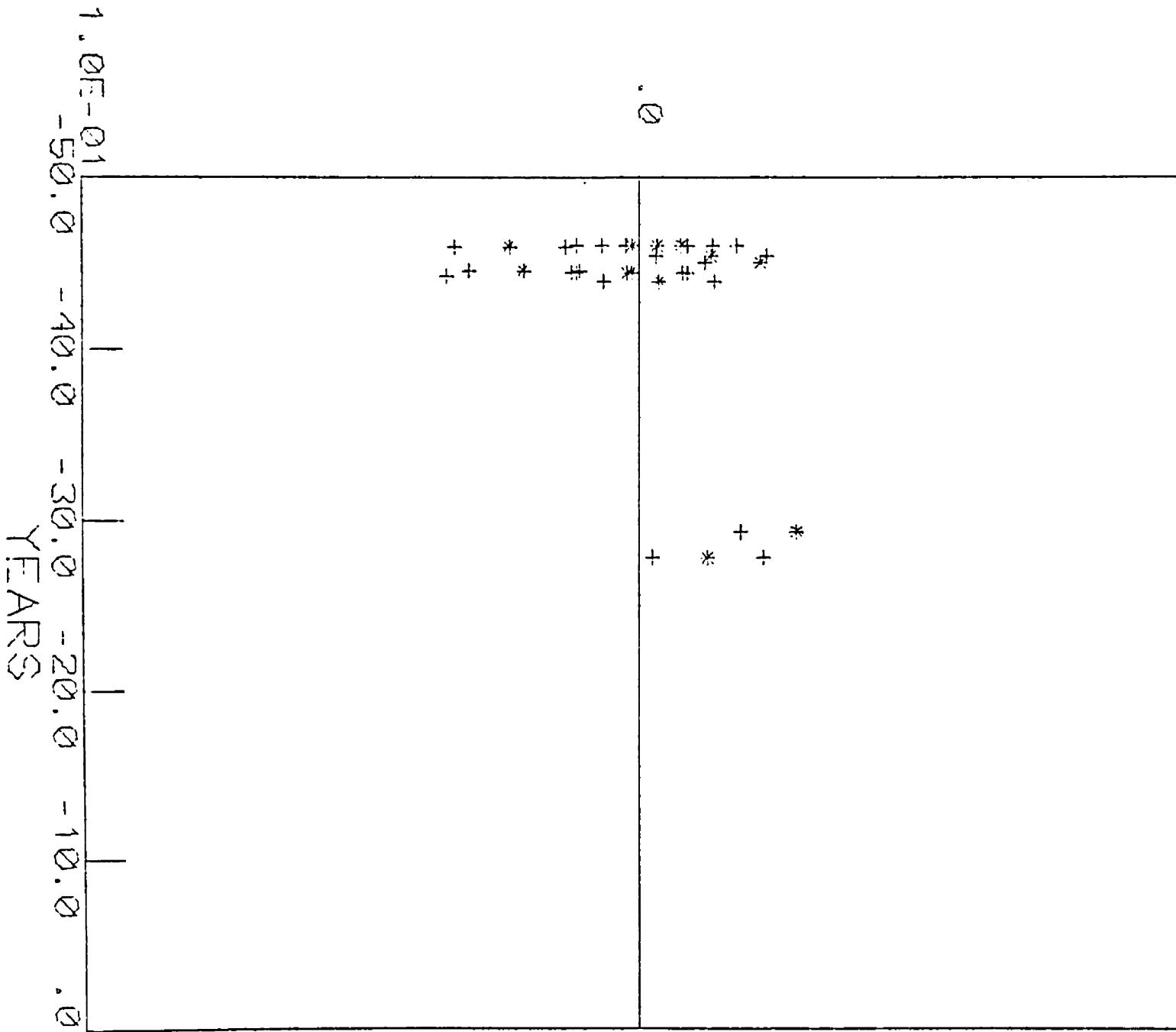


TCALC = JD 41768.4674 + E * 2.05931000

HELIOPCENTRIC JULIAN DATE	+/-	O-C RESIDUAL	OBSERVER	REFERENCE
19639.9262	0.001	-0.0024	LAZZARINO	AN 4711
34499.4140	0.001	0.0161	JAHN	AN 282, P. 236
41455.3767	0.001	0.0004	PEKUNLU	IBVS #937
41492.4530	0.001	0.0001	PEKUNLU	IBVS #937
41768.4674	0.001	0.0000	IDANOGLU	IBVS #937

ZORI PERIOD = 5.20329000

1.0E-01



CHRONOLOGY OF 15 OBSERVATIONS OF ZORI

TCALC = JD 37267.5240 + E * 5.20329000

HELIOPCENTRIC JULIAN DATE	+/-	O-C RESIDUAL	OBSERVER	REFERENCE
27480.1430	0.010	0.0075	LAUSE	AN 260, P. 292
27485.3373	0.010	-0.0015	LAUSE	AN 260, P. 292
27485.3420	0.010	0.0032	LAUSE	AN 260, P. 292
27516.5350	0.010	-0.0235	LAUSE	AN 260, P. 292
27724.7030	0.010	0.0129	LAUSE	AN 260, P. 292
27370.4040	0.010	0.0217	LAUSE	AN 260, P. 292
28026.4600	0.010	-0.0210	LAUSE	AN 260, P. 292
28073.3033	0.010	-0.0023	LAUSE	AN 260, P. 292
28073.3090	0.010	-0.0016	LAUSE	AN 260, P. 292
28151.3150	0.010	-0.0449	LAUSE	AN 260, P. 292
28250.2260	0.010	0.0036	LAUSE	AN 260, P. 292
29004.7570	0.010	0.0575	BOCHKOREVA	VAR. STAR 15#4, P.437
33533.6230	0.010	0.0233	BOCHKOREVA	VAR. STAR 15#4, P.437
34140.3590	0.010	0.0123	BOCHKOREVA	VAR. STAR 15#4, P.437
37267.5240	0.010	0.0000	BOCHKOREVA	VAR. STAR 15#4, P.437

ATPEG: PERIOD:

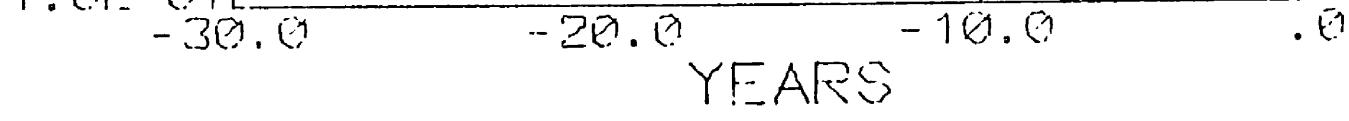
14609600

1.0E-01

O-C RESIDUALS(DAYS)

.0

1.0E-01



TCALC = JD 42645.7662 + E * 1.14603600

HELIOCENTRIC JULIAN DATE	+/-	O-C	RESIDUAL	OBSERVER	REFERENCE
33504.5250	0.001	0.0205	DOMKE, POCHER	AJ 281(3), P. 115	
33558.3370	0.010	0.0159	DOMKE	AJ 281(3), P. 115	
33338.4630	0.010	0.0163	DOMKE	AJ 281(3), P. 115	
34272.4060	0.001	0.0171	POHL	AJ 282, P. 236	
34303.3500	0.021	0.0165	DOMKE	AJ 282, P. 236	
35019.6660	0.004	0.0225	WHITNEY	AJ 62(1254), P. 373	
35034.5720	0.004	0.0293	WHITNEY	AJ 62(1254), P. 373	
35332.5450	0.001	0.0173	JAHN, RUDOLPH	AJ 285, P. 164	
35388.7100	0.004	0.0236	WHITNEY	AJ 62(1254), P. 373	
35726.3100	0.003	0.0253	WHITNEY	AJ 62(1254), P. 373	
36035.5270	0.001	0.0143	RUDOLPH	AJ 285, P. 164	
36100.4340	0.001	0.0220	LICHTENKNECKE	AJ 285, P. 164	
36103.4540	0.001	0.0193	RUDOLPH	AJ 285, P. 164	
37175.4730	0.003	0.0230	FLIN	A.A. 17, P. 62	
37544.5070	0.006	0.0141	KUBICA	A.A. 17, P. 62	
37544.5150	0.003	0.0221	KUZMINSKI	A.A. 17, P. 62	
37872.3190	0.001	0.0426	POHL	AJ 288, P. 71	
37904.4020	0.001	0.0349	KIZILIRMAK	AJ 288, P. 72	
37911.2870	0.001	0.0433	ASLAM	AJ 288, P. 72	
38233.3470	0.001	0.0378	BECKER	AJ 288, P. 72	
39387.4060	0.001	-0.0093	BRAUNE	AJ 292, P. 137	
40407.4380	0.001	-0.0023	IBANOGLU	IBVS #456	
40433.3330	0.001	-0.0023	IBANOGLU	IBVS #456	
40877.3370	0.001	-0.0031	ENDRES, SENGONCA	IBVS #530	
41661.2729	0.001	0.0031	ALKAN	IBVS #937	
42645.7662	0.001	0.0000	DOOLITTLE	BLUE MT. OBSERVATORY	

APPENDIX 2

FORTRAN CODE LISTINGS

The Fortran codes listed in this section are stored on magnetic tape at the UM Department of Physics and Astronomy. Also available are detailed descriptions of the codes, which may be consulted during execution or modification attempts.

Program FILCHG

This Fortan code determines O-C residuals for the observed times of minima of eclipsing binary stars, using the linear elements of eq. (1.1). A disk file containing the information exhibited in appendix 1 is created by the initial execution. Later executions allow for the addition or deletion of observations and for the residuals to be recalculated using new values of the ephemeris date and period.

```

C          FTLCHG.F7
C
C THIS PROGRAM CREATES, UPDATES, AND MODIFIES ECLIPSING BINARY
C OBSERVATION FILES. PRESENT MAXIMUM NUMBER OF ENTRIES IS 700.
C DOUBLE PRECISION TT,PERIOD,TNEW,TCH.CK,TSAVE,PDM,T1,TDB,FN
C 1,TCALC,DFLT,SAVT
C DIMENSION TT(700),DOBS(700),SAVT(700),HOLD(700),NEPOCH(700)
C 1,OP(7,700),OBNEW(7),SAVD(700),SAVOS(7,700),ID(20),
C 20BINPT(7),OBLAST(7)
C BLANK=' '
C ICOUNT=0
C ITAB=0
C DO 10 I=1,700
C TT(I)=0.
C DOBS(I)=0.
C SAVT(I)=0.
C HOLD(I)=0.
C NEPOCH(I)=0
C SAVD(I)=0.
C DO 10 M=1,7
C OB(M,I)=BLANK
10   SAVOS(M,I)=BLANK
C DO 20 I=1,7
C OBNEW(I)=BLANK
C OBINPT(I)=BLANK
20   OBLAST(I)=BLANK
C TYPE 30
30   FORMAT(' SPECIFY FILE NAME:',$)
C ACCEPT 40,FILE
40   FORMAT(A5)
C TYPE 50
50   FORMAT(' CREATE NEW FILE(0)?//, UPDATE FILE(1)?//,
C 1' CHANGE LINEAR ELEMENTS(-1)?')
C ACCEPT 60,ILE
60   FORMAT(I)
C IF(ILE)230,70,230
C ++++++ENTER NEW FILE ++++++++
70   TYPE 80
80   FORMAT(' SPECIFY NO. OF OBSERVATIONS')
C ACCEPT 60,IMAX
C TYPE 90
90   FORMAT(' ENTER EPHEMERAL JD:',$)
C ACCEPT 130,T1
C TYPE 100
100  FORMAT(' ENTER TIMES OF MINIMA IN CHRONOLOGICAL ORDER.')
C TYPE 110
110  FORMAT(' OBSERVERS NAME MAY BE 15 LETTERS MAXIMUM.',/
C 1' REFERENCE NOTE MAY BE 20 LETTERS MAXIMUM.',/
C 2' <CR> MAY BE USED WHEN INPUT IS SAME AS FOR PRECEDING ENTRY.')
C DO 210 I=1,IMAX
C TYPE 120,I
120  FORMAT(/I5,' ENTER JD:',$,)
C ACCEPT 130,TT(I)
130  FORMAT(D)
C TYPE 140
140  FORMAT(' ENTER ACCURACY(DAYS):',$)
C ACCEPT 150,DINPT
150  FORMAT(F)
C DOBS(I)=DINPT
C IF(DINPT.LT.0.00001)DOBS(I)=DOBS(I-1)
C TYPE 160

```

```

160  FORMAT(' ENTER OBSERVER NAME.',/15X,' <')
170  ACCEPT 170,(OBINPT(M),M=1,3)
170  FORMAT(4A5)
180  DO 180 M=1,3
180  OB(M,I)=OBINPT(M)
180  IF(OBINPT(1).EQ.BLANK)OB(M,I)=OB(M,I-1)
180  CONTINUE
180  TYPE 190
190  FORMAT(' ENTER REFERENCE.',/20X,' <')
190  ACCEPT 170,(OBINPT(M),M=4,7)
190  DO 200 M=4,7
190  OB(M,I)=OBINPT(M)
190  IF(OBINPT(4).EQ.BLANK)OB(M,I)=OB(M,I-1)
200  CONTINUE
210  CONTINUE
210  TYPE 220,(TT(I),DOBS(I),(OB(J,I),J=1,7),I=1,TMAX)
220  FORMAT(///700(I20.10,F10.5,2X,3A5,2X,4A5/))
220  GO TO 540
C++++++RECALL EXISTING FILE ++++++-----+
230  CALL IFILE(1,FILE)
230  READ(1,630),IMAX,PERIOD,T1,
230  1(NFPOCH(I),TT(I),DOBS(I),HOLD(I),(OB(J,I),J=1,7),I=1,IMAX)
230  CALL OFILE(1,'FILBAK')
230  WRITE(1,630),IMAX,PERIOD,T1,
230  1(NEPOCH(I),TT(I),DOBS(I),HOLD(I),(OB(J,I),J=1,7),I=1,IMAX)
230  END FILE 1
230  TYPE 240,FILE,IMAX,T1,PERIOD
240  FORMAT(' FILE ',A5,' CONTAINS',I5,' OBSERVATIONS',
240  1/' TCALC=',F12.5,' + E ',F12.9)
240  TYPE 250
250  FORMAT(' NEW EPHEMERAL J)(1) OR NOT<CR>?',S)
250  ACCEPT 60,NEPH
250  IF(NEPH.EQ.0)GO TO 260
250  TYPE 90
250  ACCEPT 180,T1
260  CONTINUE
260  IF(ILE.EQ.-1)GO TO 540
C.....DELETE OBSERVATION.....<CR> TO PROCEDE.....
270  TYPE 270
270  FORMAT(/' CAUTION: DELETE ONLY IN ASCENDING ORDER! ')
270  TYPE 280,(I,TT(I),I=1,IMAX)
280  FORMAT(///700(I10,I20.10/))
290  TYPE 300
290  FORMAT(/// ENTER INDEX OF LINE TO BE DELETED:,I)
290  ACCEPT 60,INDEX
290  IF(INDEX.EQ.0)GO TO 330
290  INDFX=INDEX-ICOUNT
290  IMAX=IMAX-1
290  JT0P=IMAX
290  IF(INDEX.GT.IMAX)JT0P=INDEX
290  DO 320 J=INDEX,JTOP
290  TT(J)=TT(J+1)
290  DO 310 K=1,7
290  OB(K,J)=OB(K,J+1)
290  DO 310 K=1,7
290  OB(K,J)=OB(K,J+1)
290  CONTINUE
290  ICOUNT=ICOUNT+1
290  GO TO 290
C.....ENTER NEW OBSERVATION .....<CR> TO PROCEDE.....
330  TYPE 340

```

340 FORMAT(' ENTER NEW OBSERVATIONS IN ANY ORDER.')

119

TYPE 110

MNUM=0

350 MNUM=MNUM+1

TYPE 120,MNUM

ACCEPT 130,TNEW

IF (TNEW.LT.1.) GO TO 530

C INCLUDE FOLLOWING STATEMENT IF NEW OBSERVATIONS GIVEN AS N.E.A.

C TNEW=TN_W+378860.5-400000.0

TYPE 140

ACCEPT 150,DINPT

DOBLST=DOBNEW

DOBNEW=DINPT

IF (DINPT.LT.0.00001) DOBNEW=DOBLST

TYPE 160

ACCEPT 170,(OBINPT(M),M=1,3)

DO 360 M=1,3

OBLAST(M)=OBNEW(M)

OBNEW(M)=OBINPT(M)

IF (OBINPT(1).EQ.BLANK) OBNEW(M)=OBLAST(M)

360 CONTINUE

TYPE 190

ACCEPT 170,(OBINPT(M),M=4,7)

DO 370 M=4,7

OBLAST(M)=OBNEW(M)

OBNEW(M)=OBINPT(M)

IF (OBINPT(4).EQ.BLANK) OBNEW(M)=OBLAST(M)

370 CONTINUE

DO 480 K=1,IMAX

IF (TT(K).LT.TNEW) GO TO 460

MCOUNT=0

DO 380 M=1,IMAX

TCHECK=OABS(TNEW-TT(M))

IF (TCHECK.GT.PERIOD) GO TO 380

MCOUNT=MCOUNT+1

ID(MCOUNT)=M

380 CONTINUE

IF (MCOUNT.EQ.0) GO TO 420

TYPE 390

390 FORMAT(' SIMILAR MINIMA:')

400 TYPE 400,(TT(ID(M)),DOBSS(ID(M)),(OB(J, ID(M)),J=1,7),M=1,MCOUNT)

410 FORMAT(20(F12.5,F10.5,2X,3A5,2X,4A5/))

TYPE 410

410 FORMAT(' TYPE (1) TO DISREGARD NEW ENTRY, <CR> TO PROCEDE:',\$)

ACCEPT 60,MGO

IF (MGO.EQ.1) GO TO 350

420 CONTINUE

SAVT(K)=TNEW

SAVD(K)=DOBNEW

DO 430 M=1,7

430 SAVOB(M,K)=OBNEW(M)

LBEG=K+1

IMAX=IMAX+1

DO 450 L=LBEG,IMAX

SAVT(L)=TT(L-1)

SAVD(L)=DOBSS(L-1)

DO 440 M=1,7

440 SAVOB(M,L)=OB(M,L-1)

450 CONTINUE

GJ TO 500

```

460      SAVT(K)=TT(K)
        SAVD(K)=DOBS(K)
        DO 470 M=1,7
470      SAVOB(M,K)=OB(M,K)
        CONTINUF
        IMAX=IMAX+1
        SAVT(IMAX)=TNEW
        SAVD(IMAX)=DOBNEW
        DO 490 M=1,7
490      SAVOB(M,IMAX)=OBNEW(M)
        DO 520 I=1,IMAX
          TT(I)=SAVT(I)
          DOBS(I)=SAVD(I)
          DO 510 M=1,7
510      OB(M,I)=SAVOB(M,I)
        CONTINUF
        GO TO 350
530      TYPE 220,(TT(I),DOBS(I),(OB(J,I),J=1,7),I=1,IMAX)
540      TYPE 550
550      FORMAT(' ENTER PERIOD.')
        ACCEPT 130,PDUM
        IF (PDUM.EQ.0.0)GO TO 560
        PERIOD=PDUM
560      CONTINUE
C++++++LINEAR DETERMINATION OF O-C ++++++
570      SJM=0.
        TEST=PERIOD/2.
        DO 600 I=1,IMAX
          T0B=TT(I)
          NBEG=DABS(T0B-T1)/PERIOD-?
C++++++CALCULATE TIME OF MINIMA+++++
        DO 580 N=NBEF,1000000
          FN=N
          IF (T0B.LT.T1)FN=-FN
          NEPOCH(I)=FN
          TCALC=T1+FN*PERIOD
          DELT=DABS(T0B-TCALC)
          IF (DELT.GT.TEST)GO TO 580
          OMING=T0B-TCALC
          GO TO 590
580      CONTINUE
590      CONTINUE
        HOLD(I)=OMING
        SUM=SUM+OMING*OMINC
600      CONTINUF
        FMAX=IMAX-1
        DEV=SQRT(SUM/FMAX)
C++++++OUTPUT STAGE ++++++
610      TYPE 620,PERIOD,DEV
620      FORMAT(4X,' PERIOD=',1P015.9,' GIVES MINIMUM RMS
        1DEVIATION=',1PE12.6)
        CALL OFILE(1,FILE)
        PRINT 240,FILE,IMAX,T1,PERIOD
        PRINT 630,IMAX,PERIOD,T1,
        1(NEPOCH(I),TT(I),DOBS(I),HOLD(I),(OB(J,I),J=1,7),I=1,IMAX)
        WRITE(1,630),IMAX,PERIOD,T1,
        1(NEPOCH(I),TT(I),DOBS(I),HOLD(I),(OB(J,I),J=1,7),I=1,IMAX)
        FORMAT(I10,2D20.10/,700(I7.017.10,F8.5,F9.5,2X,3A5,2X,4A5/))
630      END FILE 1
640      ENO

```

Program AVEPER

This code employs a method of iteration to find a value of the eclipse period for which the sum of all positive and negative O-C residuals considered approaches zero. The average period thus obtained is determined to nine decimal places and can be used to modify the original data file through the execution of PERCHG.

```

C
C COMPUTES ECLIPSE PERIOD WHICH GIVES AVERAGE OF ALL
C 0-C VALUES NEARLY EQUAL TO ZERO.
C IMPLICIT DOUBLE PRECISION (A-H,P-Z)
C DIMENSION TT(800),OB(7)
ICOUNT=0
ITAB=0
TYPE 10
10 FORMAT(' SPECIFY FILE NAME:',B)
ACCEPT 20,FILE
20 FORMAT(A5)
CALL IFILE(1,FILE)
READ(1,140),IMAX,PERIOD,T1,
1(DUM,TT(I),DUM,DUM,(OB(J),J=1,7),I=1,IMAX)
TYPE 30,FILE,PERIOD,IMAX
30 FORMAT(' INPUT FILE ',A5,' WRITTEN WITH PERIOD=',1PD15.9,/
1' AND CONTAINS ',I3,' TIMES OF MINIMA.')
PERIOD=PERIOD*1.0004
IPER=PERIOD
PERIOD=IPER
PERIOD=PERIOD*1.0D-04
LINEAR DETERMINATION OF 0-C ++++++ ++++++ ++++++ ++++++
ISTOP=0
PERSTP=1.0E-04
40 SJM=0.
TEST=PERIOD/2.
DO 70 I=1,IMAX
TOB=TT(I)
NBEG=DABS(TOB-T1)/PERIOD-2.
CALCULATE TIME OF MINIMA
DO 50 N=NBEG,100000
FN=N
IF(TOB.LT.T1)FN=-FN
TCALC=T1+FN*PERIOD
DELT=DABS(TOB-TCALC)
IF(DELT.GT.TEST)GO TO 50
DIF=TOB-TCALC
GO TO 60
50 CONTINUE
CONTINUE
SUM=SUM+DIF
CONTINUE
70 ISENSE=1
IF(SUM.LT.0.)ISENSE=-1
IF(ISTOP.EQ.0)LSENSE=ISENSE
ISTOP=ISTOP+1
IF(ISTOP.LT.100)GO TO 90
TYPE 80
FORMAT(' STOPPED AT 100 TRIALS')
STOP
90 CONTINUE
P9=P0
P0=PERIOD
S9=S0
S0=SUM
TYPE 100,ISENSE,LSENSE,PERIOD,SUM
FORMAT(2I5,2(1PD20.12))
100 IF(ISENSE.EQ.LSENSE)GO TO 110
PERIOD=PERIOD-1.1*PERSTP
PERSTP=PERSTP/10.

```

110 PNEW=PERIOD+PERSTP
SEND=PERSTP
IF (SEND.LT.1.E-10)GO TO 120
PERIOD=PNEW
GO TO 40
C
120 OUTPUT STAGE +++++++
TYPE 130,P9,S9,ISTOP
130 FORMAT(//', PERIOD=',1PD15.9/, ' ABSOLUTE DEVIATION=',
11PD15.6/,I5,' TRIALS.')
FORMAT(I10,2D20.10/,800(I7,17.10,F8.5,F9.5,2X,3A5,2X,+A5/))
140 STOP
END

Program DETECT

This Fortran code is used to determine the detectability of a low luminosity companion which is in the proximity of a brighter star or close binary. The input parameters which must be specified by the user are the distance in parsecs from the earth to the stellar system and the apparent visual magnitude of the visible central star(s). The output is given in a data file which may easily be plotted on the Calcomp plotter as shown in figure 3.

PROGRAM TO DETERMINE VISUAL DETECTABILITY OF COMPANIONS.
 STEPS THROUGH MASSES FROM 0.01 TO 1.50 (M/MSUN) AND
 FINDS SEPARATIONS (AU) AT WHICH DETECTABILITY INDEX C=1.0.
 DIMENSION BOLM(30),BOLC(30),FMASS(150),SFPAU(150)
 1,FLM(30),ALPH(30)
 DATA BOLM/22.0,13.1,12.0,11.5,11.0,10.5,9.7,8.9,8.7,8.4,7.9,7.5
 1,7.2,6.8,6.6,6.3,6.1,6.0,5.8,5.5,5.1,4.9,4.6,4.4,3.5,3.3,3.0
 2,2.4,2.2,2.1/
 DATA BOLC/-5.8,-5.8,-4.6,-4.0,-3.4,-2.9,-2.6,-2.3,-2.0,-1.71
 1,-1.45,-1.17,-0.89,-0.62,-0.50,-0.40,-0.30,-0.24,-0.19,-0.13
 2,-0.09,-0.07,-0.06,-0.05,-0.04,-0.03,-0.01,0.01,-0.02,-0.03/
 DATA FLM/-3.0,-1.0,-0.8,-0.6,-0.4,-0.2,-0.0,23*0.0/
 DATA ALPH/2.9,2.9,3.13,3.33,3.75,4.0,4.0,23*4.0/
 TYPE 10
 10 FORMAT(' ENTER DISTANCE(PC):'3)
 ACCEPT 20,DIST
 FORMAT(e)
 TYPE 30
 30 FORMAT(' ENTER APPARENT VISUAL MAGNITUDE AT'
 1 MID-POINT OF PRIMARY ECLIPSE:')
 ACCEPT 20,APTMAG
 DO 40 I=1,150
 FI=I
 FMASS(I)=FI*0.01
 FLOGM=ALOG10(FMASS(I))
 DETERMINE DIFFERENCE IN MAGNITUDE
 J=7
 CALL INTRP(FLM,ALPH,FLOGM,ALPHA,J)
 XLUMI=FMASS(I)**ALPHA
 BOLMAG=4.77-2.5*ALOG10(XLUMI)
 J=30
 CALL INTRP(BOLM,BOLC,BOLMAG,BC,J)
 ABMAGI=BOLMAG+BC
 APMAGI=ABMAGI+5.*ALOG10(DIST/10.)
 DELMAG=ABS(APMAGI-APTMAG)
 SEPLOG=0.22*DELMAG-1.0
 SEPSEC=10.**SEPLOG
 SEPAU(I)=SEP SEC*DIST
 40 CONTINUE
 50 FORMAT(150(2F10.2/))
 C WRITE FILE FOR CALCOMP PLOTTER.....
 CALL OFILE(1,'GRAPH')
 WRITE(1,60)
 60 FORMAT('GRAPH01',/
 1'TITLE13 DETECTABILITY',/
 2'XAXIS15 SFPARATION (AU)',/
 3'YAXIS23 COMPANION MASS (M/MSUN)',/
 4'SPECS0.0 150.0 0.0 1.50 1 1 15 15 .02 3 3',/
 5'DATA 150')
 WRITE(1,50),(FMASS(I),SEPAU(I),I=1,150)
 WRITE(1,70)
 70 FORMAT('PLOT 020107',/
 1'STOP')
 END FILE 1
 STOP
 END
 SUBROUTINE INTRP(X,Y,X1,F1,J)
 LINEAR INTERPOLATION SUBROUTINE.
 J=NUMBER OF ENTRIES IN ARRAY.

C GIVEN ARRAYS X(N) AND Y(N), SUBROUTINE WILL FIND
C FOR EACH VALUE OF X1 THE CORRESPONDING VALUE OF Y SUCH
C THAT F1=Y(X1).

DIMENSION X(30),Y(30)

IORDER=0

IF (X(1).GT.X(J)) IORDER=1

I=1

10 IF (IORDER.GT.0) GO TO 20

IF (X(I+1)-X1)30,40,40

20 IF (X(I+1)-X1)40,40,30

30 I=I+1

IF (I+1-J)10,40,40

40 F1=(X(I+1)-X1)*Y(I)+(X1-X(I))*Y(I+1)

F1=F1/(X(I+1)-X(I))

RETURN

END

Program NTERAC

This code makes the dynamical calculations of the three-body mutual interaction model and displays the resulting light travel-time effect on a plot of the O-C residuals observed for an eclipsing binary star. As shown in figure 15, the main routine CTRL directs the logical flow of information through several subroutines.

The initial position, velocity, and mass must be known for each body before the dynamical calculations begin. This may be accomplished through either subroutines PARAM or RECTIN. Subroutine PARAM accepts the elements which describe the orbit of a binary about the two-body center-of-mass due to a companion, since these may be inferred from the observed O-C residual curve's cyclic pattern. The companion mass and orbital semi-major axis are found by subroutine MASS where the solution of the quartic equation (eq. 2.16) is found by the subroutine ZEROS, which is provided through the courtesy of its author, R. J. Hayden. Subroutine PARAM then proceeds to calculate the companion's initial position through a converging iteration of Kepler's Equation as described previously (eqs. 2.6, 2.11). The rectangular components of the position are found in the usual manner. The expressions which yield the velocity components are obtained by differentiating the position equations as outlined in a NASA Technical Note (Strack, 1963). The analysis is repeated to account for the other companion. Finally the fractional masses are determined for each companion and are used to determine the initial rectangular position coordinates of the binary based on the three-body center-of-mass (eq. 2.27). Since the dynamical motion of the

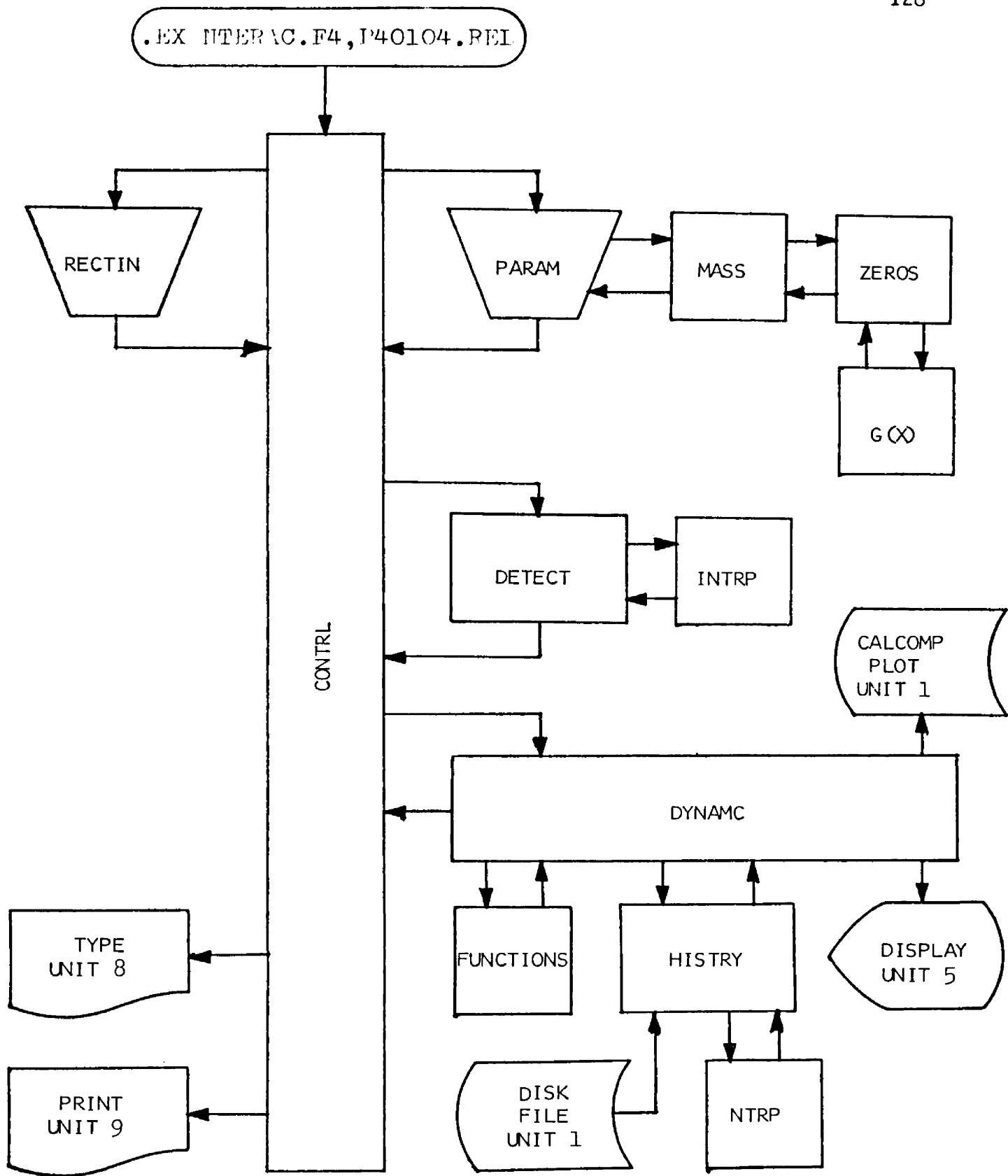


Fig. 15. Interaction model flow chart

three-body system follows open paths rather than the constraint of orbits, the superpositioning of simultaneous two-body systems is used in subroutine PARAM only to define the initial configuration.

A more direct way of defining the initial conditions of the system is to simply enter the position, velocity, and mass of each body through subroutine RECTIN. In practice, it is difficult to choose a set of rectangular input parameters at random which will describe the initial conditions of a system which will remain stable. The subroutine RECTIN is, however, quite useful for continuing the dynamical calculations of systems whose masses, positions, and velocities are known.

The subroutine DETECT determines detectability thresholds which are the maximum separations between each companion and the binary, as measured perpendicular to the line of sight, at which the companions could be located without being detected visually. During the dynamical calculations the separation of each companion from the central binary pair is repeatedly checked against its particular threshold so that detectable cases may be recognized.

The dynamical calculations are made by subroutine DYNAMC which employs a fourth-order Runge-Kutta integration scheme. The eight functions which are called are the equations of motion (eqs. 2.24, 2.25). A plot of the motion in time of all of the bodies is made on a graphical CRT display. The displacement as a function of time of the eclipsing binary pair along the line of sight is used to generate light travel-time residual data. This is plotted on a CRT display along with the observationally-derived O-C residual file which is recalled from the disk by subroutine HISTRY. An rms deviation is calculated in order to

determine the accuracy of the fit by numerically comparing the model residual curve with the observations through the linear interpolation subroutine NTRP. An evaluation of how closely a simulated curve fits the system's past behavior guides in the selection of the input parameters for the next trial so as to achieve a better fit. The parameters which give a best fit are then considered to describe, based on the initial assumptions, a possible multiple-body system which is capable of satisfying the existing eclipsing binary star residual observations.

The code creates a permanent record of the input data for each trial as is shown in table 6. Also listed in this log are the final rectangular parameters, which are useful if further calculations are to be made. The comments are a subjective appraisal of the model.

To run the code, the source program NTERAC.F4 must be accompanied in execution by the graphical display subroutine package P40104.REL.

TABLE 6
TYPICAL MODEL LOG

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TRIAL NO. 127

BINARY MASS = 2.40000

DISTANCE (AU) = 75.00

APPARENT MAGNITUDE = 7.98

INCLINATION (DEG) = 82.14

BINARY ORBIT 1

ORBITAL PERIOD = 17.00000

SEMI-MAJOR AXIS (AU) = 1.10000

ECCENTRICITY = 0.00000

PERIAPSE PASSAGE (YR) = 0.00000

LONGITUDE OF PERIAPSE (DEG) = 125.00000

COMPANION ORBIT 1

COMPANION MASS = 0.23721

ORBITAL PERIOD (YR) = 17.00000

SEMI-MAJOR AXIS (AU) = 9.19179

ECCENTRICITY = 0.00000

PERIAPSE PASSAGE (YR) = 0.00000

LONGITUDE OF PERIAPSE (DEG) = 335.00000

BINARY ORBIT 2

ORBITAL PERIOD = 88.00000

SEMI-MAJOR AXIS (AU) = 3.80000

ECCENTRICITY = 0.00000

PERIAPSE PASSAGE (YR) = 0.00000

LONGITUDE OF PERIAPSE (DEG) = 238.00000

COMPANION ORBIT 2

COMPANION MASS = 0.32983

ORBITAL PERIOD (YR) = 88.01000

SEMI-MAJOR AXIS (AU) = 27.65034

ECCENTRICITY = 0.00000

PERIAPSE PASSAGE (YR) = 0.01000

LONGITUDE OF PERIAPSE (DEG) = 58.00000

NO. OF YEAPS = 300

STEP SIZE (YR) = .150

NTH POINT PLOTTED = 20

STARTING RECTANGULAR PARAMETERS:

1 GAMA = 0.11967

X = 5.27219

Y = -7.52947

VX = 2.78305

VY = 1.94871

2 GAMA = 0.13743

X = 14.65245

Y = 23.44882

VX = -1.57434

VY = 1.04624

BINARY TO COMPANION A MAXIMUM SEPARATION (AU) = 17.124

BINARY TO COMPANION B MAXIMUM SEPARATION (AU) = 43.941

MAXIMUM RADIAL VELOCITY (KM/SEC) = 4.776

XTTRANS (AU) = -2.64467

YTTRANS (AU) = -2.32152

FINAL RECTANGULAR PARAMETERS:

1 X = -3.39828

Y = 11.77710

VX = -1.33644

VY = -0.87506

2 X = -34.72274

Y = 9.67988

VX = -2.76683

VY = -1.34830

COMMENTS:

FROM 0 TO +300 YR.....PLOT ON CALCOME.....

```

MASTER CONTROL FOR N-BODY PROBLEM SUBROUTINES.
COMMON/CONTRL/ITRIAL,G,PI,X(3),Y(3),VX(3),VY(3),FMASS(3)
1,XTRANS,YTRANS
COMMON/STAR/DIST,APTMAG,FINC
DIMENSION WORD(35)
PI=3.14159265
G=6.6732E-08*3.1558E07*3.1558E07/1.49597893E13/
11.49597893E13*1.989E33/1.49597893E13
ISTRTR=0
WRITE(5,10)
10 FORMAT(' INPUT: ORBITAL ELEMENTS(0)? OR RECTANGULAR COORDINATE
1S(1)?',\$)
READ(5,20),ISWTCH
20 FORMAT(I)
WRITE(5,30)
30 FORMAT(' ENTER BINARY MASS:',\$)
READ(5,40),FMASS(1)
40 FORMAT(E)
WRITE(5,50)
50 FORMAT(' ENTER DISTANCE(PC):',\$)
READ(5,40),DTST
WRITE(5,60)
60 FORMAT(' ENTER APPARENT VISUAL MAGNITUDE AT
1 MTD-POINT OF PRIMARY ECLIPSE:',\$)
READ(5,40),APTMAG
WRITE(5,70)
70 FORMAT(' ENTER INCLINATION OF BINARY ORBIT(DEG):',\$)
READ(5,40),FINC
WRITE(5,80)
80 FORMAT(' LAST TRIAL NO.=',\$)
READ(5,20),ITRIAL
90 PAUSE
ITRIAL=ITRIAL+1
WRITE(8,100),ITRIAL,FMASS(1),DIST,APTMAG,FINC
WRITE(9,100),ITRIAL,FMASS(1),DIST,APTMAG,FINC
100 FORMAT('1TRIAL NO.',I4,' BINARY MASS=',F10.5/,
1' DISTANCE(PC)=',F8.2/, ' APPARENT MAGNITUDE=',F8.2/,
2' INCLINATION(DEG)=',F8.2)
IF(ISWTCH.EQ.0)GO TO 110
CALL RECTIN
GO TO 120
110 CALL PARAM
120 CALL DETECT(FMASS(2),VIS1)
CALL DETECT(FMASS(3),VIS2)
CALL DYNAMIC(VIS1,VIS2,ISTRTR,TIME)
130 WRITE(9,140),XTRANS,YTRANS
140 FORMAT(' XTRANS(AU)=',F10.5,' YTRANS(AU)=',F10.5)
WRITE(9,150),TIME
150 FORMAT(' FINAL RECTANGULAR PARAMETERS:      Y=AR=',F9.2)
J2=1
J3=2
WRITE(9,160),(J2,X(2),Y(2),VX(2),VY(2),
1J3,X(3),Y(3),VX(3),VY(3))
160 FORMAT(2(/I2,5X,' X=',F10.5,/7X,' Y=',F10.5,
1/6X,' VX=',F10.5,/6X,' VY=',F10.5/))
WRITE(5,170)
WRITE(8,170)
WRITE(9,170)
170 FORMAT(' COMMENTS:')
```

```
180 READ(5,190), (WORD(J), J=1, 36)
      FORMAT(36A4)
      WRITE(8,190), (WORD(J), J=1, 36)
      WRITE(9,190), (WORD(J), J=1, 36)
190 FORMAT(2(1X,18A4)/)
      ISTRT=1
      GO TO 90
      END
```

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```

C ACCPTS INITIAL OSCULATING ORBITAL PARAMETERS AND
C CONVERTS THEM TO RECTANGULAR COORDINATE PARAMETERS.
COMMON/CONTRL/ITRIAL,G,PT,X(3),Y(3),VX(3),VY(3),FMASS(3)
1,XTRANS,YTRANS
COMMON/STAR/DIST,APT,MAG,FINC
DIMENSION PERIOD(2),A(2),ECC(2),T0(2),W(2),FN(2),GAMA(2)
N=3
N=N-1.
FM12=FMASS(1)
DO 80 J=1,N
INPUT PARAMETERS ARE OF BINARY BARYCENTER ORBIT.
WRITE(5,10)
10 FORMAT(//' PERIOD(YR)? SEMI-MAJOR AXIS(AU)? ECCENTRICITY?',
1/' PERIAPSE PASSAGE(YR)? LONGITUDE OF PERIAPSE(DEG)?')
READ(5,20),TFMP1,TEMP2,TEMP3,TEMP4,TEMP5
20 FORMAT(5F)
WRITE(8,30),TEMP1,TEMP2,TEMP3,TEMP4,TEMP5
30 FORMAT(5F8.2)
<CR> IF INPUT DATA IS THE SAME AS LAST TRIAL.....
IF(TEMP1.LT.0.01) GO TO 80
PERIOD(J)=TEMP1
ECC(J)=SQRT(1.-(1.-TEMP3*TEMP3)*SIND(FINC)*SIND(FINC))
T0(J)=TEMP4
W(J)=TEMP5+180.
IF(W(J).GT.360.)W(J)=W(J)-360.
WRITE(9,40),J,TEMP1,TEMP2,TEMP3,TEMP4,TEMP5
FORMAT(/' BINARY ORBIT:',I2,' ORBITAL PERIOD=',15X,F10.5,
1/' SEMI-MAJOR AXIS(AU)=',10X,F10.5,
2/' ECCENTRICITY=',17X,F10.5,
3/' PERIAPSE PASSAGE(YR)=',9X,F10.5,
4/' LONGITUDE OF PERIAPSE(DEG)=',3X,F10.5)
TEMP2=TFMP2/SIND(FINC)
CALL MASS(FM12,TEMP2,PERIOD(J),FM3,A(J))
FMASS(J+1)=FM3
50 WRITE(9,50),J,FMASS(J+1),PERIOD(J),A(J),ECC(J),T0(J),W(J)
FORMAT(/' COMPANION ORBIT:',I2,' COMPANION MASS=',15X,F10.5,
1/' ORBITAL PERIOD(YR)=',11X,F10.5,
2/' SEMI-MAJOR AXIS(AU)=',10X,F10.5,
3/' ECCENTRICITY=',17X,F10.5,
4/' PERIAPSE PASSAGE(YR)=',9X,F10.5,
5/' LONGITUDE OF PERIAPSE(DEG)=',3X,F10.5)
W(J)=W(J)*PI/180.
C ELLIPTICAL ORBITAL ELEMENTS ARE CONVERTED TO RECTANGULAR
C COORDINATES OF POSITION AND VELOCITY.
FN(J)=2.*PI/PERIOD(J)
Q=-T0(J)
ANMEAN=FN(J)*Q
E1=ANMEAN+ECC(J)*SIN(ANMEAN)+0.5*ECC(J)*ECC(J)*SIN(2.*ANMEAN)
EOLD=0.
DO 60 L=1,1000
DE=(ANMEAN-(E1-ECC(J)*SIN(E1)))/(1.-ECC(J)*COS(E1))
E1=E1+DE
DELT=ABS(EOLD-E1)
IF(DELT.LE.1.E-6)GO TO 70
EOLD=E1
CONTINUE
CONTINUE
60 R=A(J)*(1.-ECC(J)*COS(E1))
70 SJB=COS(E1/2.)

```

```

ASUB=ABS(SUB)
IF (ASUB.LT.1.0E-30)ASUB=1.0E-30
IF (SUB.LT.0.)SUB=-ASUB
IF (SUB.GE.0.)SUB=ASUB
TEST=SQRT((1.+ECC(J))/(1.-ECC(J)))*SIN(F1/2.)/SUB
F=ATAN(TEST)
F=2.*F
WF=F+W(J)
THE FOLLOWING DETERMINES X,Y,VX,VY.
X(J+1)=R*COS(WF)
Y(J+1)=R*SIN(WF)
FMU=G*(FMASS(1)+FMASS(J+1))
P=R*(1.+ECC(J)*COS(F))
QN=ECC(J)*COS(W(J))+COS(WF)
QD=ECC(J)*SIN(W(J))+SIN(WF)
VX(J+1)=-SQRT(FMU/P)*QN
VY(J+1)=SQRT(FMU/P)*QD
CONTINUE
GAMA(1)=FMASS(2)/FMASS(1)
GAMA(2)=FMASS(3)/FMASS(1)
DETERMINE COORDINATE TRANSFORMATIONS: ORIGIN AT INITIAL (X1,Y1)
X(1)=-GAMA(1)*X(2)-GAMA(2)*X(3)
Y(1)=-GAMA(1)*Y(2)-GAMA(2)*Y(3)
XTRANS=X(1)
YTRANS=Y(1)
RETURN
END

```

```
SUBROUTINE MASS(BIMASS,AINPUT,PINPUT,FM3,A3)
SUBROUTINE TO DETERMINE 'PLANET' MASS AND SEMI-MAJOR AXIS.
DIMENSION XFM3(100)
COMMON/ZERO/XFM3,JMAX,LMAX,M,BOT,TOP
COMMON/C/PM12,P,A12
JMAX=100
LMAX=6
BOT=0.0001
TOP=10.0
FINC=90.
P=PINPUT
PM12=BIMASS
FORMAT(F)
10 A12=AINPUT/SIN(FINC)
CALL ZEROS
IF(M.EQ.0)GO TO 30
IF(M.GT.1)WRITE(5,20),M
20 FORMAT(' MULTIPLE SOLUTIONS TO MASS. M=',I4)
FM3=XFM3(1)
A3=A12*PM12/FM3
RETURN
30 WRITE(9,40)
40 WRITE(5,40)
FORMAT(' NO MASS FOUND')
STOP
END
```

C
C SUBROUTINE ZEROS
C G(X)=FUNCTION WHOSE ZEROS ARE DESIRED. MUST BE WRITTEN IN
C FUNCTION SUBPRO.
C BOT=LOWER LIMIT OF X INVESTIGATION RANGE
C TOP=UPPER LIMIT OF X INVESTIGATION RANGE
C JMAX=NUMBER OF INVESTIGATION INTERVALS
C LMAX=NUMBER OF SQUEEZES BY FACTOR OF 10
C M IS THE NUMBER OF ROOTS LOCATED IN THE INTERVAL.
C RT(1),RT(2),---RT(M) ARE THE ROOTS FOUND.
C DIMENSION RT(100)
C COMMON/ZERO/RT,JMAX,LMAX,M,BOT,TOP
A=BOT
B=TOP
FJMAX=JMAX
M=0
DO 150 J=1,JMAX
FJ=J
X=A+(FJ-1.)*(B-A)/FJMAX
Y=A+FJ*(B-A)/FJMAX
IF (G(X)) 10,120,10
10 IF (G(X)*G(Y)) 20,150,150
20 DO 60 L=1,LMAX
DO 40 K=1,10
FK=K
U=X+(FK-1.)*(Y-X)/10.
V=X+FK*(Y-X)/10.
IF (G(U)) 30,130,30
30 IF (G(U)*G(V)) 50,40,40
40 CONTINUE
50 X=U
Y=V
50 CONTINUE
S=(X+Y)/2.
TEST TO THROW OUT INFINITIES
IF (G(S)*G(X)) 70,100,80
70 IF (G(S)*G(Y)) 150,110,90
80 IF ((G(S)-G(X))/G(X)) 100,100,150
90 IF ((G(Y)-G(S))/G(Y)) 150,100,100
100 R=S
GO TO 140
110 R=Y
GO TO 140
120 R=X
GO TO 140
130 R=U
GO TO 140
140 M=M+1
RT(M)=R
150 CONTINUE
IF (G(B)) 170,160,170
160 M=M+1
RT(M)=B
170 RETURN
END
FUNCTION G(X)
COMMON/C/FM12,P,A12
G=(X*X*X*X+FM12*X*X*X)*P*P-((FM12+X)*A12)**3
RETURN
END

SUBROUTINE RECFTI

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C UNITS:

G	AU**3/YR**2/MSIIN
X & Y	AU
VX & VY	AU/YR
FMASS	SOLAR MASSES

COMMON/CONTRL/ ITrial,G,PI,X(3),Y(3),VX(3),VY(3),FMASS(3)
 1,XTRANS,YTRANS
 DIMENSION GAMMA(2)

C++++++INPUT MASSES & POSITIONS ++++++ ++++++ ++++++ ++++++ ++++++

N=3

DO 30 I=2,N
IC=I-1

WRITE(5,10),IC

10 FORMAT(I2,' : ENTER GAMMA, X, Y:',B)
READ(5,20),GAMMA(I-1),X(I),Y(I)

20 FORMAT(3E)

30 CONTINUE

C++++++DETERMINE CRITICAL VELOCITIES. ++++++ ++++++ ++++++ ++++++ ++++++

DO 60 I=2,N

IC=I-1

FMASS(I)=GAMMA(I-1)*FMASS(1)

TOTMAS=FMASS(1)+FMASS(I)

X(1)=-GAMMA(I-1)*X(I)

Y(1)=-GAMMA(I-1)*Y(I)

R=SQRT((X(I)-X(1))**2+(Y(I)-Y(1))**2)

VCIR=SQRT(G*TOTMAS/R)

VCIRX=-VCTR*(Y(I)-Y(1))/R

VCIRY= VCTR*(X(I)-X(1))/R

VESCX=1.4142*VCIRX

VESCY=1.4142*VCIRY

WRITE(5,40),IC,VCIRX,VCIRY,VFSCX,VFSCY

40 FORMAT(I2,' : CIRCULAR VELOCITY: VX=',F10.5,5X,' VY=',F10.5,
1// ' ESCAPE VELOCITY : VX=',F10.5,5X,' VY=',F10.5)

50 WRITE(5,50)

FORMAT(' ENTER VELOCITY COMPONENTS VX AND VY:',3)

READ(5,20),VX(I),VY(I)

60 CONTINUE

X(1)=-GAMMA(1)*X(2)-GAMMA(2)*X(3)

Y(1)=-GAMMA(1)*Y(2)-GAMMA(2)*Y(3)

WRITE(5,70)

70 FORMAT(' ENTER XTRANS,YTRANS:',5)

READ(5,20),XTRANS,YTRANS

RETURN

END

PROGRAM TO DETERMINE VISUAL DETECTABILITY OF COMPANIONS.
 FINDS SEPARATIONS (AU) AT WHICH DETECTABILITY INDEX C=1.0.
 COMMON/STAR/DIST,APTMAG,FINC

```

DIMENSION BOLM(30),BOLC(30),FLM(30),ALPH(30)
DATA BOLM/22.0,13.1,12.0,11.5,11.0,10.5,9.7,8.3,8.7,8.4,7.9,7.5
1,7.2,6.8,6.6,6.3,6.1,6.0,5.8,5.5,5.1,4.9,4.6,4.4,3.5,3.3,3.0
2,2.4,2.2,2.1/
DATA BOLC/-5.8,-5.8,-4.6,-4.0,-3.4,-2.9,-2.6,-2.3,-2.0,-1.71
1,-1.45,-1.17,-0.89,-0.62,-0.50,-0.40,-0.30,-0.24,-0.19,-0.13
2,-0.09,-0.07,-0.06,-0.05,-0.04,-0.03,-0.01,0.01,-0.02,-0.03/
DATA FLM/-3.0,-1.0,-0.8,-0.5,-0.4,-0.2,-0.0,23*0.0/
DATA ALPH/2.9,2.9,3.13,3.33,3.75,4.0,4.0,23*4.0/
FLOGM=ALOG10(FMASS)
DETERMINE DIFFERENCE IN MAGNITUDE
J=7
CALL INTRP(FLM,ALPH,FLOGM,A_PHA,J)
XLUMI=FMASS**ALPHA
BOLMAG=4.77-2.5*ALOG10(XLUMI)
J=30
CALL INTRP(BOLM,BOLC,BOLMAG,BC,J)
ABMAGI=BOLMAG+BC
APMAGI=ABMAGI+5.*ALOG10(DIST/10.)
DELMAG=ABS(APMAGI-APTMAG)
SEPLOG=0.22*DELMAG-1.0
SEPSEC=10.*SEPLOG
SEPAU=SEPSEC*DIST
RETURN
END
```

SUBROUTINE INTRP(X,Y,X1,F1,J)

LINEAR INTERPOLATION SUBROUTINE.

J=NUMBER OF ENTRIES IN ARRAY.

GIVEN ARRAYS X(N) AND Y(N), SUBROUTINE WILL FIND
 FOR EACH VALUE OF X1 THE CORRESPONDING VALUE OF Y SUCH
 THAT F1=Y(X1).

```

DIMFNSION X(30),Y(30)
IORDER=0
IF(X(1).GT.X(J)) IORDER=1
I=1
IF(IORDER.GT.0) GO TO 20
IF(X(I+1)-X1)30,40,40
IF(X(I+1)-X1)40,40,30
I=I+1
IF(I+1-J)10,40,40
F1=(X(I+1)-X1)*Y(I)+(X1-X(I))*Y(I+1)
F1=F1/(X(I+1)-X(I))
RETURN
END
```

SUBROUTINE DYNAMIC(VIS1,VIS2,ISTRT,TIME)
 PROGRAM TO SOLV. 3-BODY PROBLEM USING RUNGE-KUTTA INTEGRATION.
 UNITS:

T	YR
C	AU/DAY
G	AU**3/YR**2/MSUN
X# & Y#	AU
VX# & VY#	AU/YR
FMASS#	SOLAR MASSES
H	YR
F1 - F4	AU/YR**2
F5 - F8	AU/YR
C#1 - C#4	AU/YR
C#5 - C#8	AU

DIMENSION XSAVE(3,301), YSAVE(3,301), VSAVE(301), TSAVE(301)
 COMMON/HIST/T(2,301), OMINC(2, 301), ODBS(1501), IBAR, TMAX, ISTART
 1, DIF(301)

COMMON/FUNCT/FACTOR, GAMA(2)

COMMON/CONTRL/ITRIAL, G, PI, X(3), Y(3), VX(3), VY(3), FMASS(3)

1, XTRANS, YTRANS

COMMON/STAR/DIST, APTMAG, FINC

ISTART=ISTRT

ITRIP=0

NYR=0

ISEE1=0

ISEE2=0

C=2.9979E08*8.6400E04/1.49598E11

CONTINUE

FACTOR=FMASS(1)*G

GAMA(1)=FMASS(2)/FMASS(1)

GAMA(2)=FMASS(3)/FMASS(1)

*****DYNAMICAL CALCULATIONS *****

WRITE(5,30)

30 FORMAT(' ENTER NUMBER OF YEARS, STEP LENGTH, AND')

1 NTH POINT PLOTTED.')

READ(5,40), NYEAR, H, NTHPT

40 FORMAT(I,F,I)

WRITE(9,50), NYEAR, H, NTHPT

50 FORMAT(' NO. OF YEARS=',I5,' STEP SIZE(YR)=',F5.3,
 1// NTH POINT PLOTTED=',I5)

NPERYR=1./ABS(H)

NTOP=NYEAR*NPERYR

IF (H.LT.0.) NYEAR=-NYEAR

TIME=0.

60 NJM=1

NCOUNT=0

DO 70 J=1,3

XSAVE(J,NJM)=(X(J)-XTRANS)*SIND(FINC)

70 YSAVE(J,NJM)=Y(J)-YTRANS

VX(1)=-GAMA(1)*VX(2)-GAMA(2)*VX(3)

VSAVE(NJM)=VX(1)*SIND(FINC)/0.2104

TSAVE(NJM)=TIME

VMAX=0.

SEP1=0.

SEP2=0.

XMIN=0.

XMAX=0.

YMIN=0.

YMAX=0.

WRITE(9,80), TIME

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```

30 FORMAT(// ' STARTING RECTANGULAR PARAMETERS: YEAR=',F9.2)
J2=1
J3=2
WRITE(9,90), (J2,GAMA(1),X(2),Y(2),VX(2),VY(2),
1 J3,GAMA(2),X(3),Y(3),VX(3),VY(3))
90 FORMAT(2/I2,2X,' GAMA=',F10.5,/7X,' X=',F10.5,/7X,' Y=',F10.5,
1/6X,' VX=',F10.5,/6X,' VY=',F10.5/))
DO 200 N=1,NTOP
TIME=TIME+H
C11=H*F1(X(2),Y(2),X(3),Y(3))
C12=H*F2(X(2),Y(2),X(3),Y(3))
C13=H*F3(X(2),Y(2),X(3),Y(3))
C14=H*F4(X(2),Y(2),X(3),Y(3))
C15=H*F5(VX(2))
C16=H*F6(VY(2))
C17=H*F7(VX(3))
C18=H*F8(VY(3))
C21=H*F1(X(2)+.5*C15,Y(2)+.5*C16,X(3)+.5*C17,Y(3)+.5*C18)
C22=H*F2(X(2)+.5*C15,Y(2)+.5*C16,X(3)+.5*C17,Y(3)+.5*C18)
C23=H*F3(X(2)+.5*C15,Y(2)+.5*C16,X(3)+.5*C17,Y(3)+.5*C18)
C24=H*F4(X(2)+.5*C15,Y(2)+.5*C16,X(3)+.5*C17,Y(3)+.5*C18)
C25=H*F5(VX(2)+.5*C11)
C26=H*F6(VY(2)+.5*C12)
C27=H*F7(VX(3)+.5*C13)
C28=H*F8(VY(3)+.5*C14)
C31=H*F1(X(2)+.5*C25,Y(2)+.5*C26,X(3)+.5*C27,Y(3)+.5*C28)
C32=H*F2(X(2)+.5*C25,Y(2)+.5*C26,X(3)+.5*C27,Y(3)+.5*C28)
C33=H*F3(X(2)+.5*C25,Y(2)+.5*C26,X(3)+.5*C27,Y(3)+.5*C28)
C34=H*F4(X(2)+.5*C25,Y(2)+.5*C26,X(3)+.5*C27,Y(3)+.5*C28)
C35=H*F5(VX(2)+.5*C21)
C36=H*F6(VY(2)+.5*C22)
C37=H*F7(VX(3)+.5*C23)
C38=H*F8(VY(3)+.5*C24)
C41=H*F1(X(2)+C35,Y(2)+C36,X(3)+C37,Y(3)+C38)
C42=H*F2(X(2)+C35,Y(2)+C36,X(3)+C37,Y(3)+C38)
C43=H*F3(X(2)+C35,Y(2)+C36,X(3)+C37,Y(3)+C38)
C44=H*F4(X(2)+C35,Y(2)+C36,X(3)+C37,Y(3)+C38)
C45=H*F5(VX(2)+C31)
C46=H*F6(VY(2)+C32)
C47=H*F7(VX(3)+C33)
C48=H*F8(VY(3)+C34)
X(2)=X(2)+(C15+2.*C25+2.*C35+C45)/6.
Y(2)=Y(2)+(C16+2.*C26+2.*C36+C46)/6.
X(3)=X(3)+(C17+2.*C27+2.*C37+C47)/6.
Y(3)=Y(3)+(C18+2.*C28+2.*C38+C48)/6.
VX(2)=VX(2)+(C11+2.*C21+2.*C31+C41)/6.
VY(2)=VY(2)+(C12+2.*C22+2.*C32+C42)/6.
VX(3)=VX(3)+(C13+2.*C23+2.*C33+C43)/6.
VY(3)=VY(3)+(C14+2.*C24+2.*C34+C44)/6.
X(1)=-GAMA(1)*X(2)-GAMA(2)*X(3)
Y(1)=-GAMA(1)*Y(2)-GAMA(2)*Y(3)
VX(1)=-GAMA(1)*VX(2)-GAMA(2)*VX(3)
VY(1)=-GAMA(1)*VY(2)-GAMA(2)*VY(3)
VTEST=ABS(VX(1)*SIND(FINC)/0.1204)
IF(VTEST.GT.VMAX)VMAX=VTEST
NCOUNT=NCOUNT+1
IF(NCOUNT.NE.NTHPT)GO TO 200
C++++++SAVE SPATIAL COORDS, RADIAL VELOCITY, & CHECK DETECTABILITY+++++
NCOUNT=0
NUM=NUM+1

```

```

TSAVE(NUM)=TIME
DO 100 J=1,3
  XSAVE(J,NUM)=(X(J)-XTRANS)*SIND(FINC)
  YSAVE(J,NUM)=Y(J)-YTRANS
  IF(XSAVL(J,NUM).GT.XMAX)XMAX=XSAVE(J,NUM)
  IF(YSAVF(J,NUM).GT.YMAX)YMAX=YSAVE(J,NUM)
  IF(XSAVE(J,NUM).LT.XMIN)XMIN=XSAVE(J,NUM)
  IF(YSAVE(J,NUM).LT.YMIN)YMIN=YSAVE(J,NUM)
100 CONTINUE
  VSAVE(NUM)=VX(1)*SIND(FINC)/9.2104
  SEP1=ABS(YSAVE(1,NUM)-YSAVE(2,NUM))
  SEP2=ABS(YSAVE(1,NUM)-YSAVE(3,NUM))
  IF(SEP1.GT.SEPMX1)SEPMX1=SEP1
  IF(SEP2.GT.SEPMX2)SEPMX2=SEP2
  IF(SEP1.LT.VIS1.OR.ISEE1.NE.0)GO TO 120
  ISEE1=1
  WRITE(5,110),TIME
  WRITE(8,110),TIME
  WRITE(9,110),TIME
110 FORMAT(' COMPANION A VISIBLE AT TIME=',F8.2)
  GO TO 140
120 IF(SEP1.GT.VIS1.OR.ISEE1.NE.1)GO TO 140
  ISEE1=0
  WRITE(5,130),TIME
  WRITE(8,130),TIME
  WRITE(9,130),TIME
130 FORMAT(' COMPANION A NON-VISIBLE AT TIME=',F8.2)
140 IF(SEP2.LT.VIS2.OR.ISEE2.NE.0)GO TO 160
  ISEE2=1
  WRITE(5,150),TIME
  WRITE(8,150),TIME
  WRITE(9,150),TIME
150 FORMAT(' COMPANION B VISIBLE AT TIME=',F8.2)
  GO TO 180
160 IF(SEP2.GT.VIS2.OR.ISEE2.NE.1)GO TO 180
  ISEE2=0
  WRITE(5,170),TIME
  WRITE(8,170),TIME
  WRITE(9,170),TIME
170 FORMAT(' COMPANION B NON-VISIBLE AT TIME=',F8.2)
180 IF(XMIN.GT.-200..AND.YMIN.GT.-200..AND.XMAX.LT.200..
1 AND.YMAX.LT.200.)GO TO 200
  IF(ITRIP.GT.0)GO TO 200
  ITRIP=1
  WRITE(5,190),TIME
  WRITE(8,190),TIME
  WRITE(9,190),TIME
190 FORMAT(' SPATIAL EXCURSION GREATER THAN 200 AU AT TIME=',F8.2)
200 CONTINUE
  WRITE(9,210),SEPMX1,SEPMX2,VMAX
210 FORMAT(' BINARY TO COMPANION A MAXIMUM SEPARATION(AU)=',F8.3,/
1' BINARY TO COMPANION B MAXIMUM SEPARATION(AU)=',F8.3,/
2' MAXIMUM RADIAL VELOCITY (KM/SEC)=',F8.3)
C++++++SPATIAL CONFIGURATION PLOTTING ++++++ ++++++ ++++++ ++++++
C PROJECTED ON PLANE INCLUDING LINE OF SIGHT.
220 AYMIN=ABS(YMIN)
  IF(YMAX.GT.AYMIN)GO TO 230
  YMAX=AYMIN
230 YMIN=-YMAX
  AXMIN=AES(XMIN)

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        IF (XMAX.GT.XMIN) GO TO 240
        XMAX=XMIN
        XMIN=-XMAX
        IF (YMAX.GT.XMAX) XMAX=YMAX
        IF (YMIN.LT.XMIN) XMIN=YMIN
        YMAX=XMAX
        YMIN=XMIN
        XMIN=XMIN*1.34
        XMAX=XMAX*1.34
        DINCX=1.
        IF (XMAX.GT.10.) DINCX=10.
        IF (XMAX.GT.100.) DINCX=100.
        DINY=DINCX
        MODE1=0
        MODE2=1
        ORIGIN=0.
        PAUSE
        CALL ERASE
        CALL SETUP(XMIN,YMIN,XMAX,YMAX,MODE1,MODE2)
        CALL AXIS(ORIGIN,ORIGIN,DINCX,DINY)
        CALL HOME
        NYR=NYR+NYEAR
        250   WRITE(5,250) ITrial,DINCX,NTHPT,NYR
              FORMAT(//' TRIAL NO.',T4,//' SCALE(AU)=',F5.0,
              1//' NO. OF TIME STEPS=',I6,
              1//' TIME(YR)=',T6)
        MODE=-1
        DO 260 I=1,NUM
        DO 260 J=1,3
        CALL TPLOT(XSAVE(J,I),YSAVE(J,I),MODE)
        260   CONTINUE
        CALL HOME
C++++++DETERMINE RESIDUALS FROM DYNAMICAL MODEL ++++++
        DO 270 I=1,NUM
        OMINC(2,I)=-XSAVE(1,I)/C
        270   CONTINUE
        CALL WAIT(NWORD)
        IF (NWORD.EQ.'G') GO TO 60
        WRITE(5,280)
        280   FORMAT(' SAVE SPATIAL COORDS ON DISK<1> OR NOT<CR>?',B)
        READ(5,40),ISPACE
        IF (ISPACE.EQ.0) GO TO 340
        WRITE(5,290)
        290   FORMAT(' ENTER BEGINING AND ENDING TIMES(YR):',*)
        READ(5,300),TBEG,TEND
        300   FORMAT(2F)
        JTBEGL=TBEG
        JTEND=TEND
        QPLTX=50.
        IF (YMAX.GT.50.) QPLTX=100.
        IF (YMAX.GT.100.) QPLTX=150.
        IF (YMAX.GT.150.) QPLTX=200.
        IF (YMAX.GT.200.) QPLTX=YMAX
        QPLTY=QPLTX*7./5.
        PLTX=-QPLTX
        PLTY=-QPLTY
        LINES=NUM/2+1
        LINFLT=LINES-1
        CALL OFITLE(1,'SPACE')
        WRITE(1,310),ITrial,JTBEG,JTEND,PLTX,QPLTX,PLTY,QPLTY,LINES

```

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310 FORMAT('VERSNO3',/
1'GRAPH01',/
2'TITLE29MODEL #',I3,';',I6,' TO',I6,' YR',/
3'XAXIS32PROJECTED ON PLANE INCLUDING LOS',/
4'YAXIS04(AU)',/
5'SPFC5',4F6.0,' 2 2 2 2 .05 2 2',/
6'DATA',I4)
      WRITE(1,320)((XSAVE(I,J),YSAVE(I,J),J=1,3),I=1,NUM-1)
320  FORMAT(150(12F6.1/))
      WRITE(1,330),(LINPLT,I=1,10)
330  FORMAT('PLOT 010220',I5,/
1'PLOT 030420',I5,/
2'PLOT 030409',I5,/
3'PLOT 050620',I5,/
4'PLOT 050609',I5,/
5'PLOT 070820',I5,/
6'PLOT 091020',I5,/
7'PLOT 091009',I5,/
8'PLOT 111220',I5,/
9'PLOT 111209',I5,'STOP')
      END FILE 1
340  IF(NWORD.EQ.'B')RETURN
      CALL ERASE
      CALL HOME
C++++++PLOT O-G CURVE ++++++ ++++++ ++++++ ++++++ ++++++ ++++++
350  IF(ISPACE.GT.0)GO TO 380
360  WRITE(5,370)
370  FORMAT(' ENTER BEGINNING TIME(YR): ',\$)
      READ(5,300),TBEG
380  IF(TIME)390,390,400
390  TMAX=TBEG
      TMIN=TBEG+TIME
      GO TO 410
400  TMIN=TBEG
      TMAX=TBEG+TIME
410  CONTINUE
      TMAX=TMAX
      IF(TMAX.LT.10.)TMAX=10.
      IF(TMIN.GT.-100.)TMIN=-100.
      DO 420 I=1,NUM
420  T(2,I)=TSAVE(I)+TBEG
      CALL HISTRY(NUM)
      RMIN=-0.1
      RMAX=0.1
      DTINCT=10.
      DINCR=0.01
      CALL SETUP(TMIN,RMIN,TMAX,RMAX,MODE1,MODE2)
      CALL AXTS(ORIGIN,ORIGIN,DTINCT,DINCR)
      CALL HOME
      WRITE(5,430),ITRIAL
      FORMAT('// TRIAL NO.',I4)
      DO 460 I=1,IMAX
      IF(IBAR)440,440,450
440  MODE=-1
      CALL TPLOT(T(1,I),OMINC(1,I),MODE)
      GO TO 460
450  MODE=0
      OCTOP=OMINC(1,I)+D0BS(I)
      OCBOT=OMINC(1,I)-D0BS(I)
      CALL TPLOT(T(1,I),OCTOP,MODE)

```

```

      ,NUM
470  IF (I1.GT.1) MODE=1
      CALL TPLOT(T(2,I),OMINC(2,I),MODE)
      CONTINUE
      CALL HOME
      CALL WAIT(NWORD)
      IF (NWORD.EQ.'G') GO TO 60
      IF (NWORD.EQ.'B') RETURN
      IF (NWORD.EQ.'R') GO TO 360
C++++++RADIAL VELOCITY RESIDUAL CURVE ++++++
      VTOP=5.
      VBOT=-VTOP
      DINCV=1.
      CALL ERASF
      CALL SETUP(TMIN,VBOT,TMAX,VTOP,MODE1,MODE2)
      CALL AXIS(ORIGIN,ORIGIN,DINCT,DINCV)
      CALL HOME
      WRITE(5,430),ITRIAL
      MODE=-1
      J=2
      DO 480 I=1,NUM
      CALL TPLOT(T(J,I),VSAVE(I),MODE)
480  CONTINUE
      CALL HOME
      CALL WAIT(NWORD)
      IF (NWORD.EQ.'G') GO TO 60
      IF (NWORD.EQ.'R') GO TO 360
C+++++WRITE RESIDUAL DATA FILE FOR CALCOMP PLOT+++++
      WRITE(5,490)
490  FORMAT(' SAVE RESIDUALS ON DISK(1) OR NOT<CR>?',B)
      READ(5,40),IRES
      IF (IRES.EQ.0) GO TO 530
      LINES=IMAX+1
      IF (NUM.GT.IMAX) LINES=NUM+1
      LINPLT=LINES-1
      CALL OFILE(1,'RESID')
      WRITE(1,500),ITRIAL,TMIN,TMAX,LINES
500  FORMAT('VERSNO3',//GRAPH01',//TITLE25MODEL #',I3,' O-C
      1RESIDUALS',//XAXIS05YEARS',//YAXIS36EARLY          0-C(DAYS)
      2      LATE',//SPECS',2F7.0,' -0.1 0.1 2 2 2 2 0.08 2 2',
      3//DATA',I4)
      WRITE(1,510)((T(J,I),OMINC(J,I),J=1,2),VSAVE(I),DIF(I),
      1I=1,LINPLT)
510  FORMAT(301(6F10.4/))
      WRITE(1,520),IMAX,NUM,ITRIAL,TMIN,TMAX,NUM,ITRIAL,NUM
520  FORMAT('PLOT 010207',I5,
      2//PLOT 030420',I5,//GRAPH01',//TITLE39MODEL #',I3,' RADIAL
      3 VELOCITY RESIDUALS ',//XAXIS05YEARS',//YAXIS12VRAD(KM/SEC'),
      4//SPECS',2F7.0,' -10. 10. 2 2 2 2 .08 2 2',
      5//PLOT 030520',I5,//GRAPH01',//TITLE36MODEL #',I3,' REDUCED
      6 O-C RESIDUALS ',//XAXIS05YEARS',//YAXIS36EARLY          0-C
      7(DAYS)      LATE',//SPECS-100. 0. -0.01 0.01 2 2 2 2
      80.08 2 2',//PLOT 010620',I5,'STOP')
      END FILE 1
530  RETURN
      END

```

```

FUNCTION F1(X2,Y2,X3,Y3)
COMMON/FUNCT/FACTOR,GAMA(2)
X1=-GAMA(1)*X2-GAMA(2)*X3
Y1=-GAMA(1)*Y2-GAMA(2)*Y3
R23=SQRT((X3-X2)**2+(Y3-Y2)**2)
R12=SQRT((X2-X1)**2+(Y2-Y1)**2)
F1=FACTOR*((X1-X2)/R12**3+GAMA(2)*(X3-X2)/R23**3)
RETURN
END

FUNCTION F2(X2,Y2,X3,Y3)
COMMON/FUNCT/FACTOR,GAMA(2)
X1=-GAMA(1)*X2-GAMA(2)*X3
Y1=-GAMA(1)*Y2-GAMA(2)*Y3
R23=SQRT((X3-X2)**2+(Y3-Y2)**2)
R12=SQRT((X2-X1)**2+(Y2-Y1)**2)
F2=FACTOR*((Y1-Y2)/R12**3+GAMA(2)*(Y3-Y2)/R23**3)
RETURN
END

FUNCTION F3(X2,Y2,X3,Y3)
COMMON/FUNCT/FACTOR,GAMA(2)
X1=-GAMA(1)*X2-GAMA(2)*X3
Y1=-GAMA(1)*Y2-GAMA(2)*Y3
R23=SQRT((X3-X2)**2+(Y3-Y2)**2)
R13=SQRT((X3-X1)**2+(Y3-Y1)**2)
F3=FACTOR*((X1-X3)/R13**3+GAMA(1)*(Y2-X3)/R23**3)
RETURN
END

FUNCTION F4(X2,Y2,X3,Y3)
COMMON/FUNCT/FACTOR,GAMA(2)
X1=-GAMA(1)*X2-GAMA(2)*X3
Y1=-GAMA(1)*Y2-GAMA(2)*Y3
R23=SQRT((X3-X2)**2+(Y3-Y2)**2)
R13=SQRT((X3-X1)**2+(Y3-Y1)**2)
F4=FACTOR*((Y1-Y3)/R13**3+GAMA(1)*(Y2-Y3)/R23**3)
RETURN
END

FUNCTION F5(VX2)
F5=VX2
RETURN
END

FUNCTION F6(VY2)
F6=VY2
RETURN
END

FUNCTION F7(VX3)
F7=VX3
RETURN
END

FUNCTION F8(VY3)
F8=VY3
RETURN
END

```

SUBROUTINE HISTP (NJM)

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PROGRAM TO CORRELATE OBS. DATA TO MODEL DATA.

NOTE: MAXIMUM NUMBER OF NTRIES IN O-C FILE IS 150.

DOUBLE PRECISION QT,QY,DT1

DIMENSION QT(150),QY(150),DUMOB(7)

COMMON/HIST/T(2,301),OMINC(2,301),DOBS(150),IBAR,IMAX,ISTART

1,DIF(301)

IF(ISTART.GT.0)GO TO 100

DO 20 I=1,300

T(1,I)=0.

OMINC(1,I)=0.

CONTINUE

ISTART=1

WRITE(5,30)

FORMAT(' SPECIFY INPUT FILE NAME:',\$)

READ(5,40),FILE

FORMAT(A5)

CALL IFILE(L,FILE)

READ(1,50),IMAX,ECLPER,DT1,

1(NDUM,QT(I),DOBS(I),QY(I),(DUMOB(J),J=1,7),I=1,IMAX)

FORMAT(10,2D20.10/,150(I7,D17.10,F8.5,F9.5,2X,3A5,2X,4A5/))

WRITE(9,60),FILE,ECLPER

FORMAT(' INPUT FILE ',A5,' WRITTEN WITH PERIOD=',1PD15.9)

WRITE(5,70)

FORMAT(' ERROR BARS(1) OR NOT<CR>:',\$)

READ(5,80),IBAR

FORMAT(I)

DO 90 I=1,IMAX

T(1,I)=(QT(I)-QT1)/365.25

OMINC(1,I)=QY(I)

CONTINUE

CALL ERASE

*****FITTING ACCURACY CALCULATION. *****

100 SUMDIF=0.

COUNT=0.

IF(T(2,NUM).LT.T(2,1))GO TO 110

KTP=1

KBT=NUM

GO TO 120

110 KTP=NUM

KBT=1

CONTINUE

DO 130 I=1,IMAX

IF(T(1,I).LT.T(2,KTP).OR.T(1,I).GT.T(2,KBT))GO TO 130

CALL NTRP(T(1,I),F1,NUM)

COUNT=COUNT+1.

DIF(I)=OMINC(1,I)-F1

SUMDIF=SUMDIF+DIF(I)*DIF(I)

CONTINUE

IF(COUNT.EQ.1.)GO TO 150

RMSFIT=SQRT(SUMDIF/(COUNT-1.))

WRITE(8,140),COUNT,RMSFIT

WRITE(9,140),COUNT,RMSFIT

140 FORMAT(' RMS FITTING ACCURACY OF',F5.0,' OBSERVATIONS=',F10.6)

150 RETURN

END

SUBROUTINE NTRP(X1,F1,J)

C LINEAR INTERPOLATION SUBROUTINE.

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C J=NUMBER OF ENTRIES IN ARRAY.

C GIVEN ARRAYS X(N) AND Y(N), SUBROUTINE WILL FIND
FOR EACH VALUE OF X1 THE CORRESPONDING VALUE OF Y SUCH
THAT F1=Y(X).

DIMENSION X(301),Y(301)

COMMON/HIST/T(2,301),OMINC(2,301),N0BS(150),IBAR,IMAX,ISTART
1,DIF(301)

DO 10 I=1,301

X(I)=T(2,I)

Y(I)=OMINC(2,I)

10 CONTINUE

IORDER=0

IF (X(1).GT.X(J)) IORDER=1

I=1

20 IF (IORDER.GT.0) GO TO 30

IF (X(I+1)-X1)40,50,50

30 IF (X(I+1)-X1)50,50,40

40 I=I+1

IF (I+1-J)20,50,50

50 F1=(X(I+1)-X1)*Y(I)+(X1-X(I))*Y(I+1)

F1=F1/(X(I+1)-X(I))

RETURN

END

Program BINOBS

When planning an observational program for eclipsing binary stars, it is advantageous to know well in advance the time when an eclipse is expected to occur. The Fortran code BINOBS produces a day-by-day chronological listing of times of occurrence for as many as 20 systems. It is useful to execute the code prior to the observational season each year.

A sample of the output from this code is given in table 7. The format is such that the schedules of minima for any one night are confined to a single page. This allows the observer to carry with him to the observatory only information which is relevant to that night's viewing.

TABLE 7
 ECLIPSING VARIABLE SCHEDULE OF MINIMA
 FOR NIGHT BEGINNING ON 7/17
 JULIAN DATE: 42977.0000

STAR	JD OF MINIMA	UNIVERSAL TIME	STANDARD TIME	DAYLIGHT TIME
ATPEG	0.0183	12:26	5:26	6:26
DOCAS	0.0565	13:21	6:21	7:21
XXCEP	0.0599	13:26	6:26	7:26
TWDRA	0.0971	14:19	7:19	8:19
I BOO	0.1858	16:27	9:27	10:27
I BOO	0.4537	22:53	15:53	16:53
I BOO	0.7215	5:18	22:18	23:18
DOCAS	0.7412	5:47	22:47	23:47
BFAUR	0.8443	8:15	1:15	2:15
RYAQR	0.9609	11:03	4:03	5:03
I BOO	0.9893	11:44	4:44	5:44
RZCAS	0.9910	11:47	4:47	5:47

PROGRAM TO CALCULATE ECLIPSING VARIABLE OBSERVATION SCHEDULE.
IMPLICIT DOUBLE PRECISION (A-H,O-Y)
DIMENSION ZSTAR(20),TKNOWN(20),PERIOD(20),MAXDAY(12)
DIMENSION THOLD(50),ZSHOLD(50),TSAVE(50),ZSSAVF(50)
NOTE: ITZONE IS STD. TIME DIF. BETWEEN OBSERVATORY AND GMT.
ITZONE=7
DATA (MAXDAY(J),J=1,12)/31,28,31,30,31,30,31,31,30,31,30,31/
10 TYPE 20
20 FORMAT('1HOW MANY BINARIES BEING CONSIDERED?',\$)
ACCEPT 30,NSTAR
30 FORMAT(I)
IF (NSTAR.GT.20.OR.NSTAR.LT.1)GO TO 10
DO 80 N=1,NSTAR
TYPE 40
40 FORMAT(' ENTER STAR NAME:',\$)
C NOTE: MAXIMUM FIELD WIDTH IS 5.
ACCEPT 50,ZSTAR(N)
50 FORMAT(A5)
TYPE 60
60 FORMAT(' ENTER EPHEMERAL JD, AND PERIOD:',\$)
ACCEPT 70,TKNOWN(N),PERIOD(N)
70 FORMAT(2D)
80 CONTINUE
DO 100 N=1,NSTAR
TYPE 90,ZSTAR(N),TKNOWN(N),PERIOD(N)
PRINT 90,ZSTAR(N),TKNOWN(N),PERIOD(N)
90 FORMAT(A10,2D15.8)
100 CONTINUE
TYPE 110
110 FORMAT(' AT WHAT JD DOES THIS FORECAST BEGIN?',\$)
ACCEPT 30,IBEG
TBEG=IBEG
TYPE 120
120 FORMAT(' FOR HOW MANY DAYS?',\$)
ACCEPT 30,LTOP
TYPE 130
130 FORMAT(' BEGINNING JD IS WHAT MONTH, DAY, YEAR?(12,31,75):',\$)
ACCEPT 140,MON,IDATE,IYR
140 FORMAT(3I)
IDATE=IDATE-1
IYR=IYR-72
LEAP=MOD(IYR,4)
IF (LEAP.EQ.0)MAXDAY(2)=29
FOLLOWING UPDATES EPHEMERAL JD TO BEGINNING OF FORECAST ++++++
DO 170 N=1,NSTAR
DO 150 I=1,100000
E=I
START=TKNOWN(N)+E*PERIOD(N)
IF (START.GE.TBEG)GO TO 160
150 CONTINUE
160 TKNOWN(N)=START
170 CONTINUE
DAY LOOP +++++++
DO 310 L=1,LTOP
EL=L-1
ICOUNT=0
IDATE=IDATE+1
IF (IDATE.LE.MAXDAY(MON))GO TO 180
MON=MON+1

```

IF(MON.GT.12)MON=1
IDATE=1
PRINT 190,MON,IDATE
190 FORMAT('1',12(/),33X,' ECLIPSING VARIABLE SCHEDULE OF MINIMA',
1/37X,' FOR NIGHT BEGINNING ON',I3,' /',I3)
DAY=TBEGL+FL
DAY1=DAY+1.0
ZDAY=DAY
PRINT 200,ZDAY
200 FORMAT(40X,' JULIAN DATE:',=15.4//)
PRINT 210
210 FORMAT(23X,' STAR',4X,' JD OF MINIMA',4X,'UNIVERSAL TIME',
13X,'STANDARD TIME',3X,'DAYLIGHT TIME',/)
-
DETERMINE ALL MINIMA FOR THIS DAY ++++++++
DO 240 N=1,NSTAR
DO 220 I=1,10
E=I-1
TEMP=TKNOWN(N)+F*PERIOD(N)
IF(TEMP.GE.DAY1)GO TO 230
ICOUNT=ICOUNT+1
THOLD(ICOUNT)=TEMP
ZSHOLD(ICOUNT)=ZSTAR(N)
220 CONTINUE
230 TKNOWN(N)=TEMP
CONTINUE
IF(ICOUNT.NE.0)GO TO 260
PRINT 250
250 FORMAT(20X,' NO ECLIPSES FOR ANY OF THESE PROGRAM STARS TODAY.')
GO TO 310
C FOLLOWING PUTS MINIMA IN CHRONOLOGICAL ORDER ++++++++
260 DO 280 M=1,ICOUNT
TSAVE(M)=9.9020
DO 270 J=1,ICOUNT
IF(THOLD(J).GT.TSAVE(M))GO TO 270
TSAVE(M)=THOLD(J)
ZSSAVE(M)=ZSHOLD(J)
JMIN=J
270 CONTINUE
THOLD(JMIN)=9.9021
280 CONTINUE
DO 300 M=1,ICOUNT
TSAVE(M)=TSAVE(M)-DAY
HOUR=TSAVE(M)*24.
IHOUR=HOUR
FHOUR=IHOUR
MIN=(HOUR-FHOUR)*60.
IHOUR=IHOUR+12
LOCLST=THCUR-ITZONE
LOCLDT=LOCLST+1
IF(IHOUR.GE.24)IHOUR=IHOUR-24
IF(LOCLST.GE.24)LOCLST=LOCLST-24
IF(LOCLDT.GE.24)LOCLDT=LOCLDT-24
Z=TSAVE(M)
PRINT 290,ZSSAVE(M),Z,IHOUR,MIN,LOCLST,MIN,LOCLDT,MIN
290 FORMAT(19X,A10,F12.4,4X,I10,':',I2,I13,':',I2,I13,':',I2)
CONTINUE
300 CONTINUE
TYPE 320
320 FORMAT(///' PICK UP LINE PRINTER OUTPUT')
END

```

Program OCPLLOT

To obtain a Calcomp plot of an O-C residual file, execute the Fortran code OCPLLOT. The only input required is the name of the file which is to be accessed. Plotting specifications and titles are determined automatically and a disk file named GRAPH.DAT is written. To complete the plotting, simply run the library code SPLLOT which reads this file.

.RUN SPLLOT (2302,11)

Examples of these plots are given in appendix 2. The residual values are represented by asterisks (*). Located above and below each residual is a cross (+) which indicates the observational accuracy of that value. The maximum number of O-C entries which may be plotted by this code is 800. Year zero corresponds to January 1, 1980 and therefore all eclipses observed prior to this date will be plotted as negative dates.

```

OCPLOT.F4
CREATE DATA FILE FOR O-C PLOT ON CALCOMP PLOTTER....
IS JANUARY 1, 1981..... .
CISION PERIOD,T1,TT
NFPPOCH(800),TT(80),DOBS(800),HOLD(800),OB(7,800)

10  TYPE 10
FORMAT(' ENTER FILENAME:',$)
ACCEPT 20,FILE
20  FORMAT(A5)
CALL IFILE(1,FILF)
READ(1,30),TMAX,PERIOD,T1,
1(NFPPOCH(I),TT(I),DOBS(I),HOLD(I),(OB(J,I),J=1,7),I=1,TMAX),
30  FORMAT(I10,2D20.10/,800(I7,D17.10,F8.5,F9.5,2X,3A5,2X,4A5/))
YMX=0.
DOBMAX=0.
XMN=0.
DO 40 I=1,TMAX
IF (DOBS(I).GT.DOBMAX) DOBMAX=DOBS(I)
IF (ABS(HOLD(I)).GT.YMX) YMX=ABS(HOLD(I))
T1(I)=(TT(I)-44298.)/365.25
IF (TT(I).LT.XMN) XMN=TT(I)
CONTINUE
YMX=YMX+DOBMAX
YMAX=0.05
IF (YMX.GT.0.05) YMAX=0.10
IF (YMX.GT.0.10) YMAX=0.15
IF (YMX.GT.0.15) YMAX=0.20
IF (YMX.GT.0.20) YMAX=0.25
IF (YMX.GT.0.25) YMAX=YMX
YMIN=-YMAX
XMIN=-10.
KK=1
IF (XMN.GT.-10.) GO TO 50
XMIN=-20.
KK=2
IF (XMN.GT.-20.) GO TO 50
XMIN=-30.
KK=3
IF (XMN.GT.-30.) GO TO 50
XMIN=-40.
KK=4
IF (XMN.GT.-40.) GO TO 50
XMIN=-50.
KK=5
IF (XMN.GT.-50.) GO TO 50
XMIN=-60.
KK=6
IF (XMN.GT.-60.) GO TO 50
XMIN=-70.
KK=7
IF (XMN.GT.-70.) GO TO 50
XMIN=-80.
KK=4
IF (XMN.GT.-80.) GO TO 50
XMIN=-90.
KK=4
IF (XMN.GT.-90.) GO TO 50
XMIN=-100.
KK=5
50  CALL OFILE(1,'GRAPH')

```

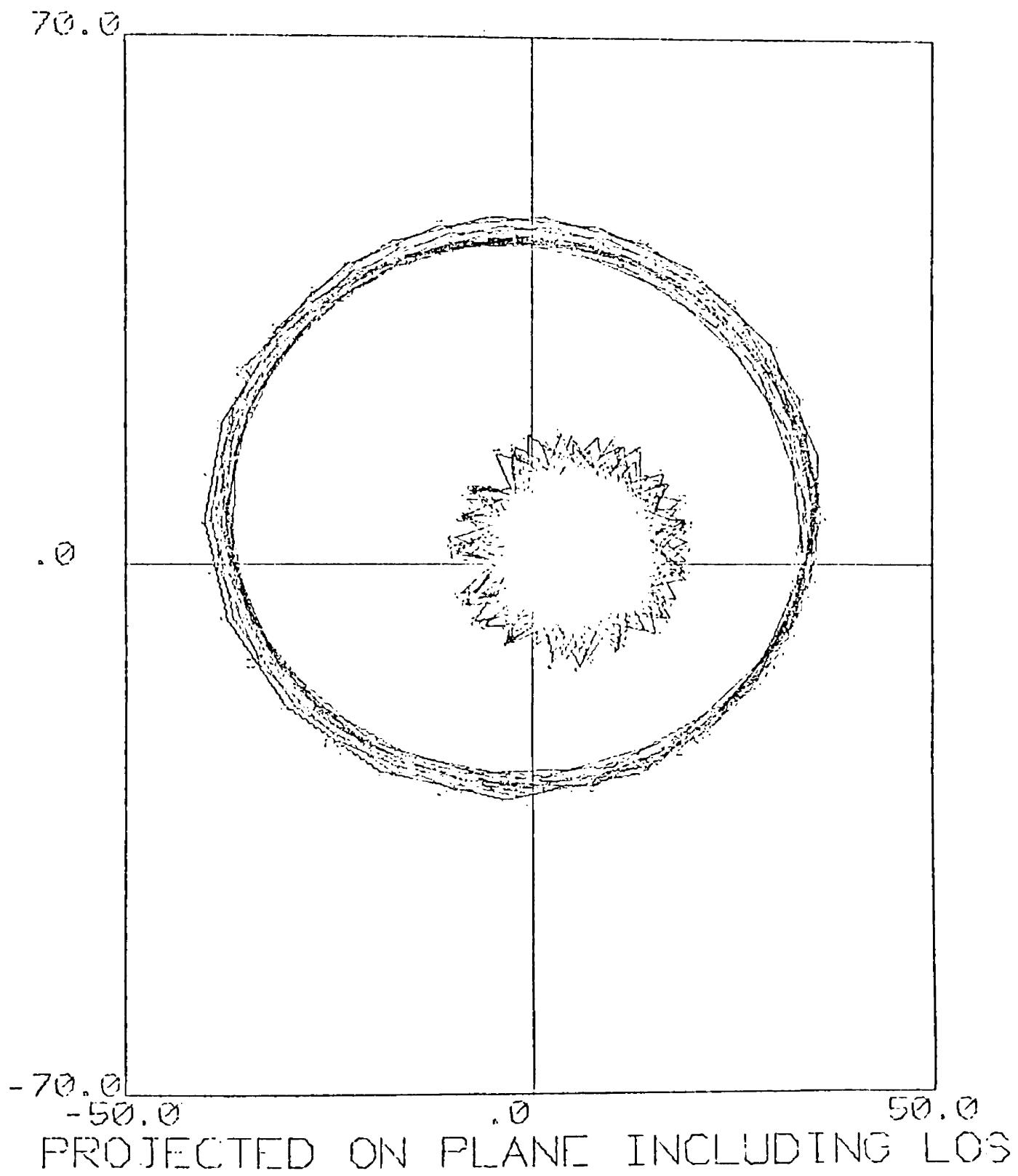
MAXI=IMAX/2+1
60 WRITE(1,60),FILE,PERIOD,XMIN,YMIN,YMAX,KK,KK,MAXI 155
FORMAT('VERSN03',//GRAPH)1',//TITLE26',A5,'; PERIOD=,
1F12.8,//'XAXIS05YFARS',//'YAXIS190-C RESIDUALS(DAYS)',/
2'SPECS',F6.0,' 0. ',2F5.2,' 1 2 ',I1,' 2 .08 ',I1,' 2 ',/
3'DATA ',I3)
WRITE(1,70)(TT(I),HOLD(I),DBBS(I),I=1,IMAX)
70 FORMAT(400(6F10.5/))
IMAX=IMAX/2
WRITE(1,80),IMAX,IMAX,IMAX,IMAX,IMAX,IMAX
80 FORMAT('PLOT 010207',I5,//'PLOT 040507',I5,//'ALTER02034107',//
2'PLOT 010705',I5,//'ALTER05064107',//'PLOT 040705',I5,
3//'ALTER02034207',//'PLOT 010705',I5,
4//'ALTER05064207',//'PLOT 040705',I5,//'STOP')
END FILE 1
END

APPENDIX 3

SPATIAL CONFIGURATION PLOTS

In the following plots, the direction to the earth is that of the positive abscissa.

MODEL # 67: 100 TO 1600 YR

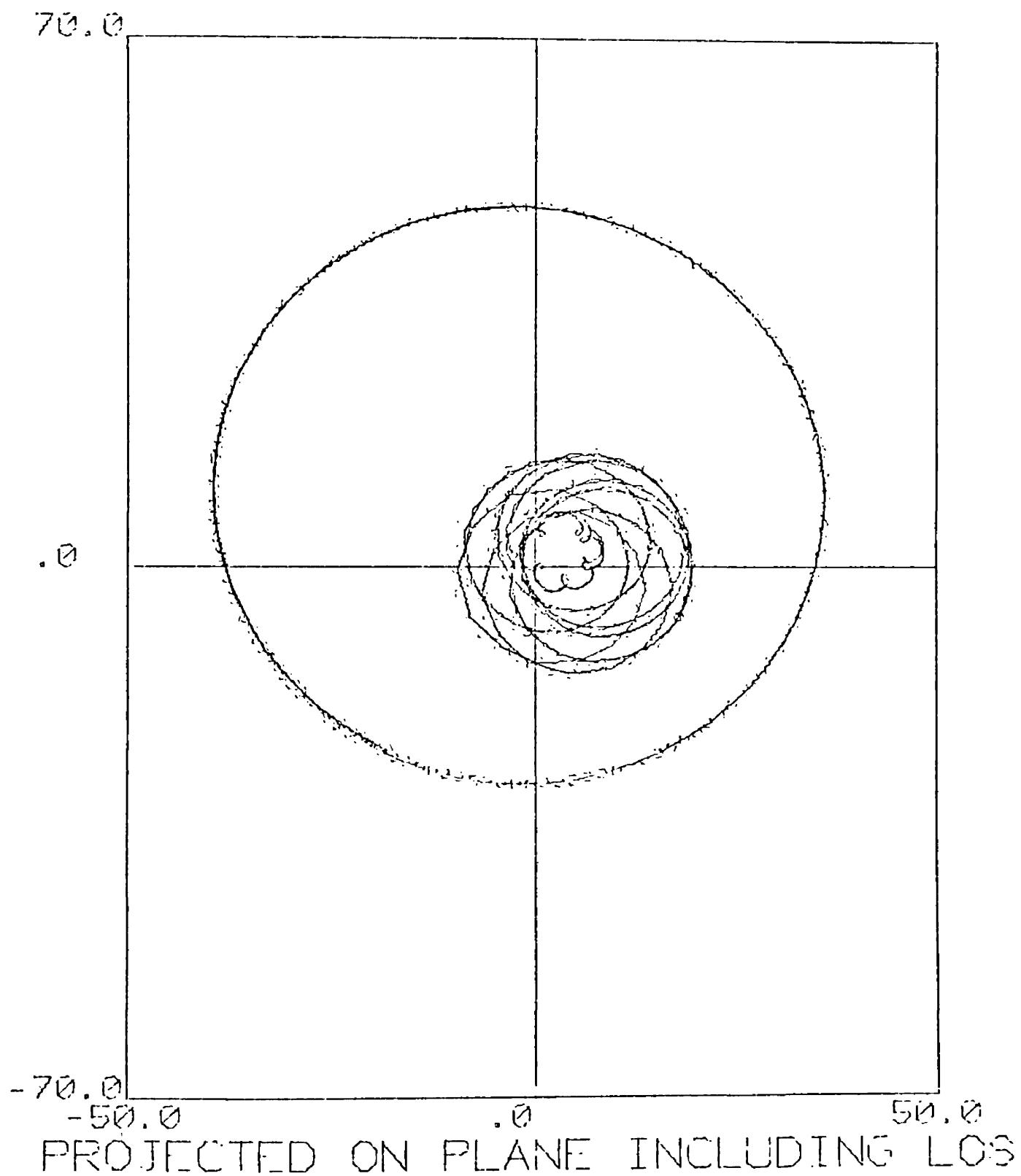


MODEL

67:

-100 TO

100 YR



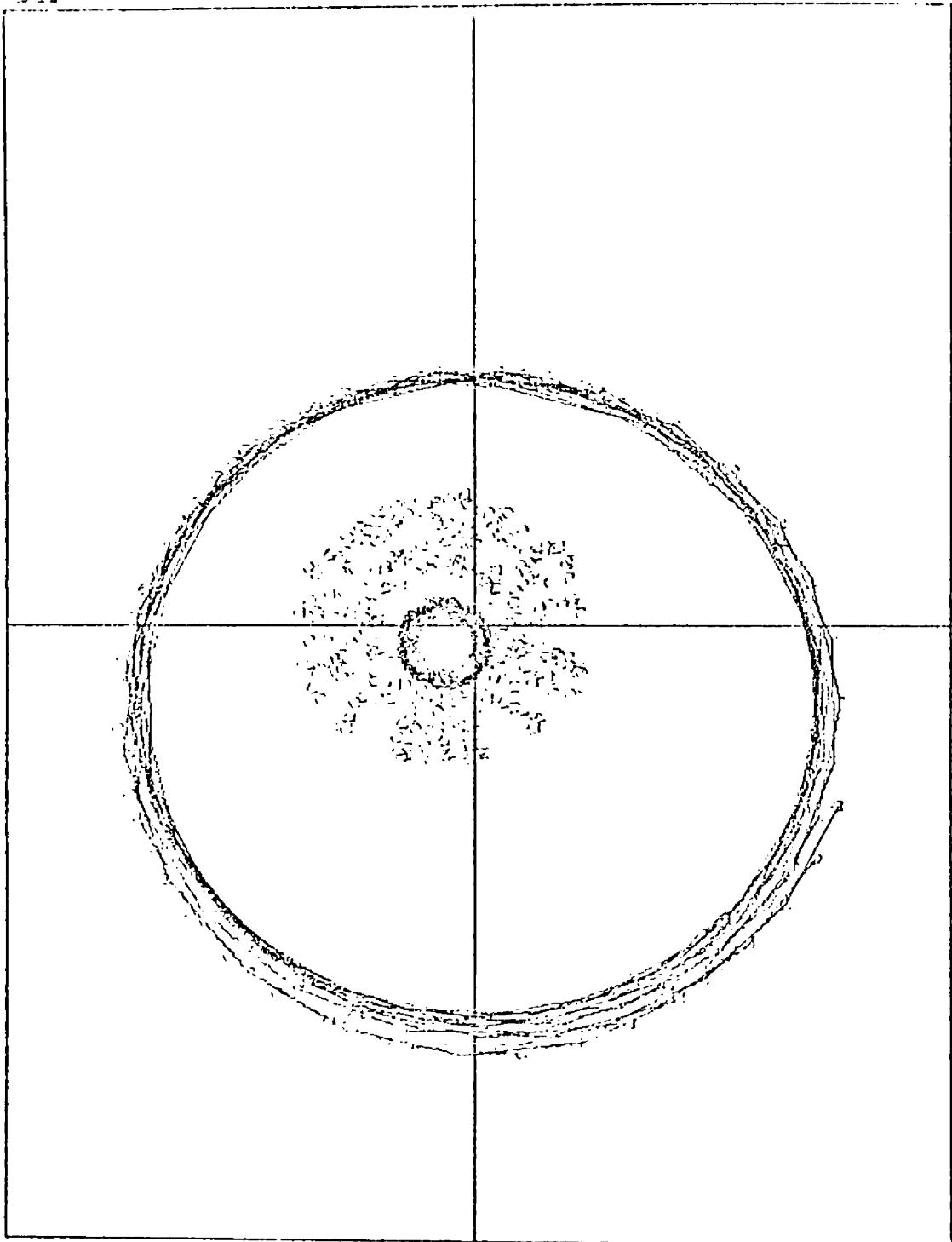
PROJECTED ON PLANE INCLUDING LOS

50.0

0

-50.0

-70.0



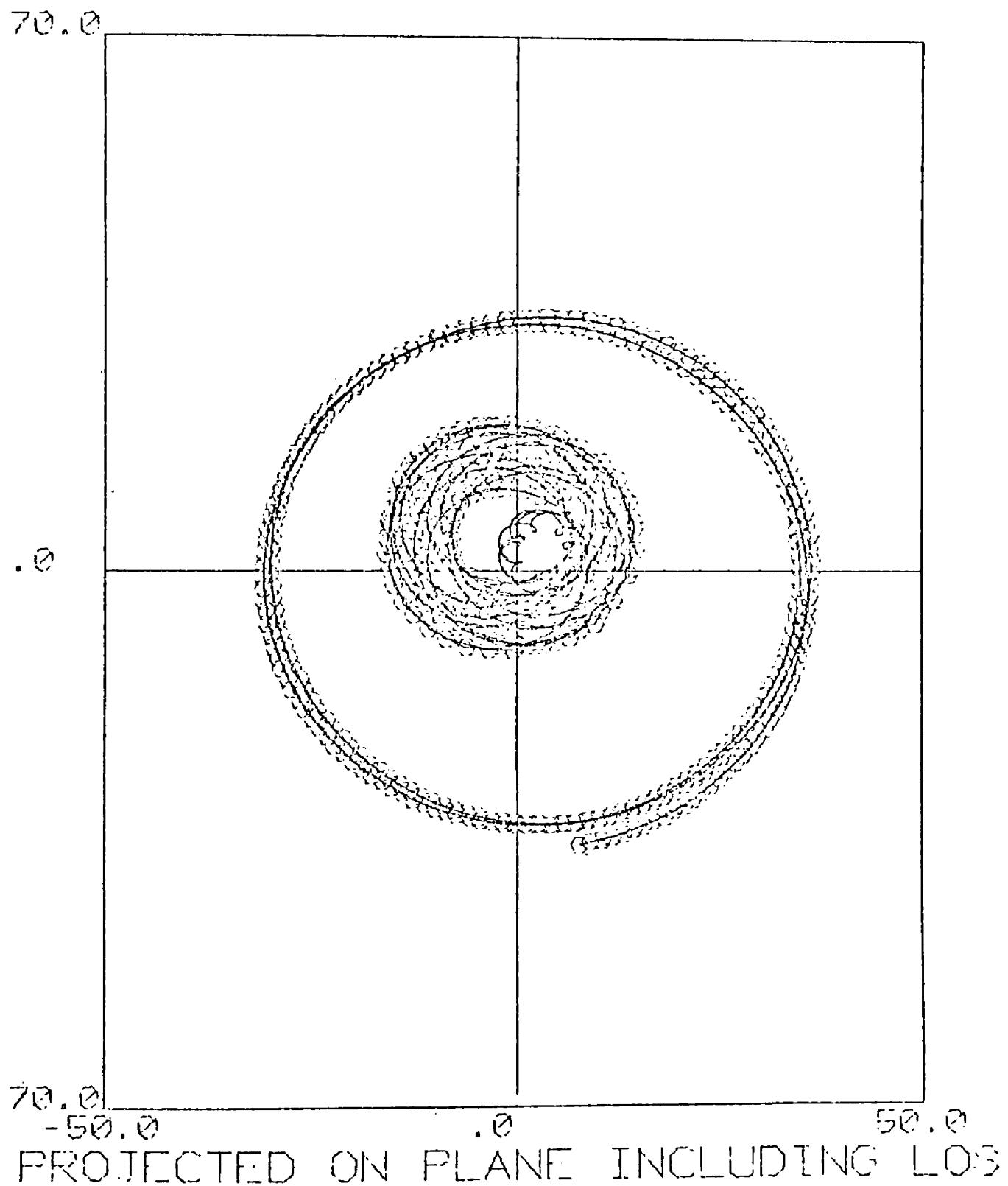
0

(AU)

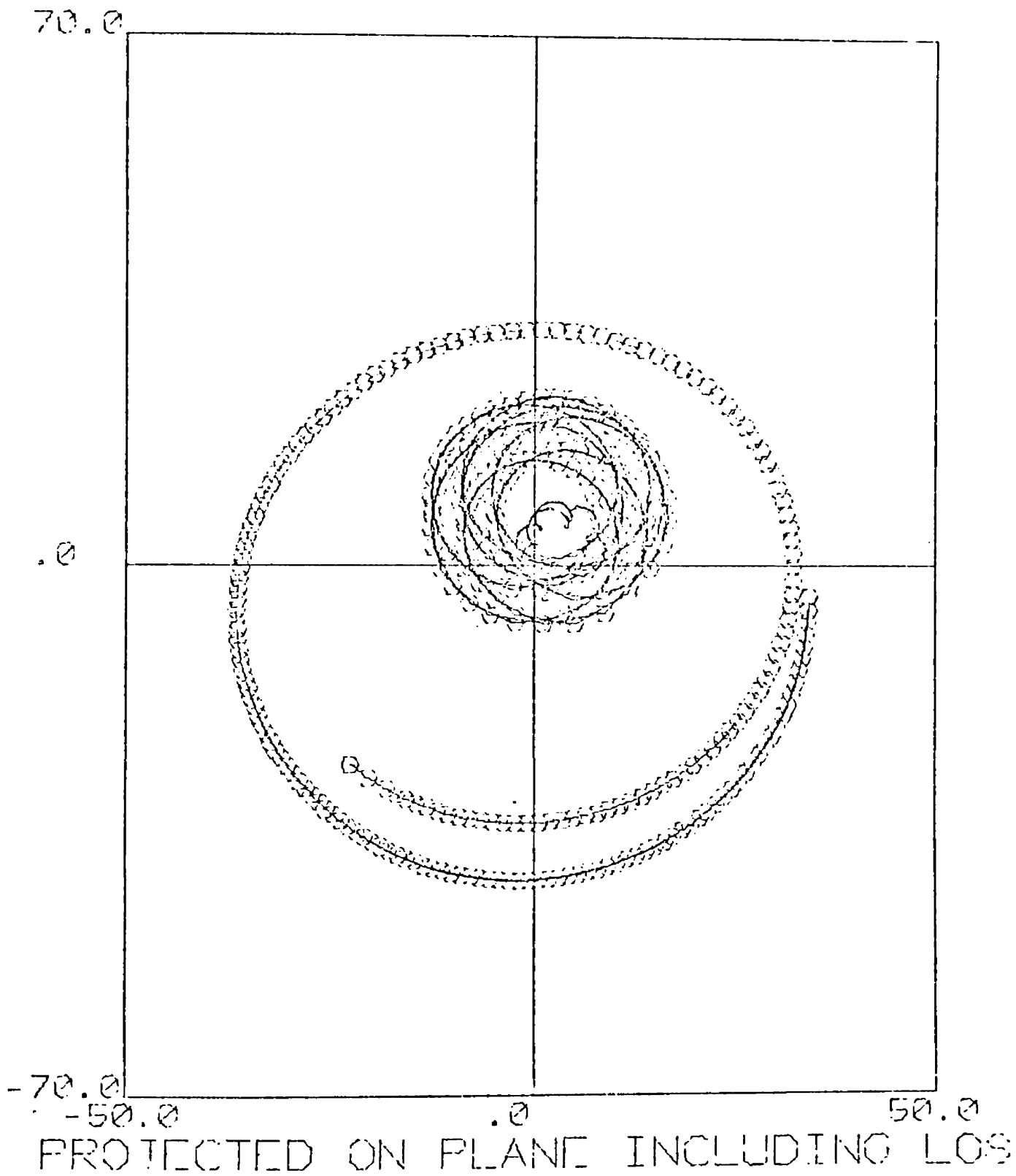
MODEL # 67 100 TO 1600 YE

MODEL 127

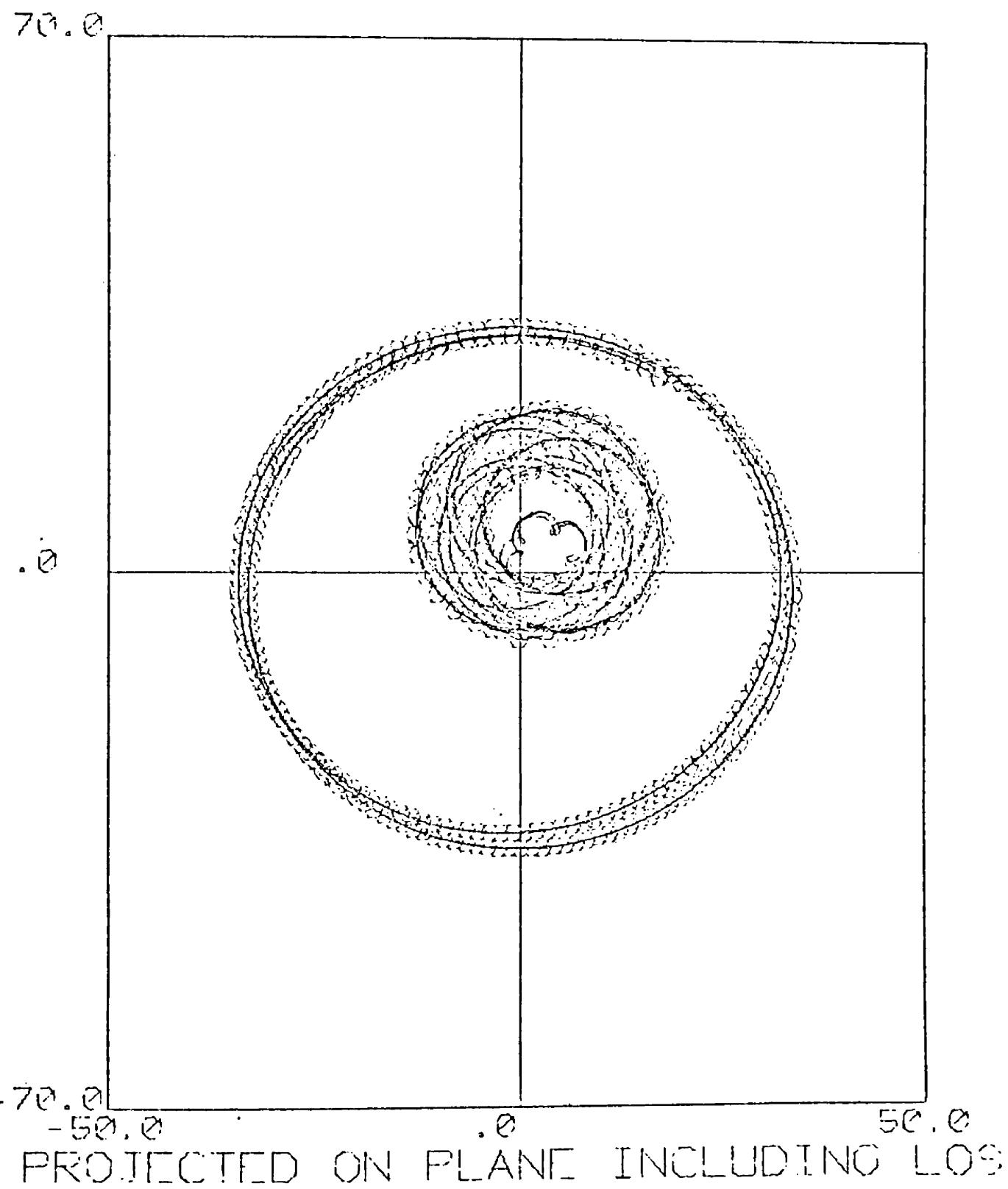
RZ CASSIOPEIAE: +500 TO +300 YR



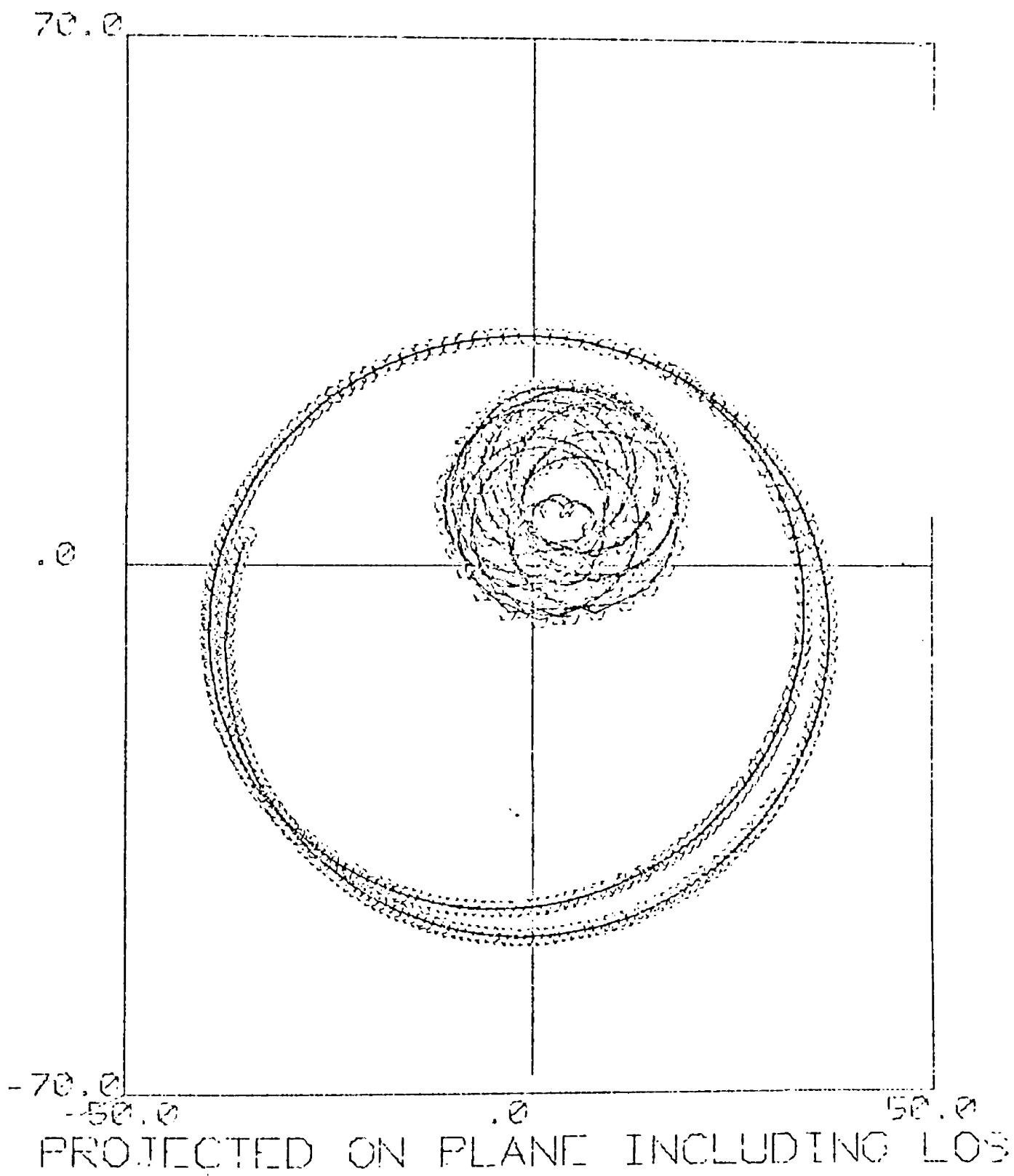
RZ CASSIOPEIAE: +300 TO +500 YE



RZ CASSIOPEIAE: 0 TO +300 YR

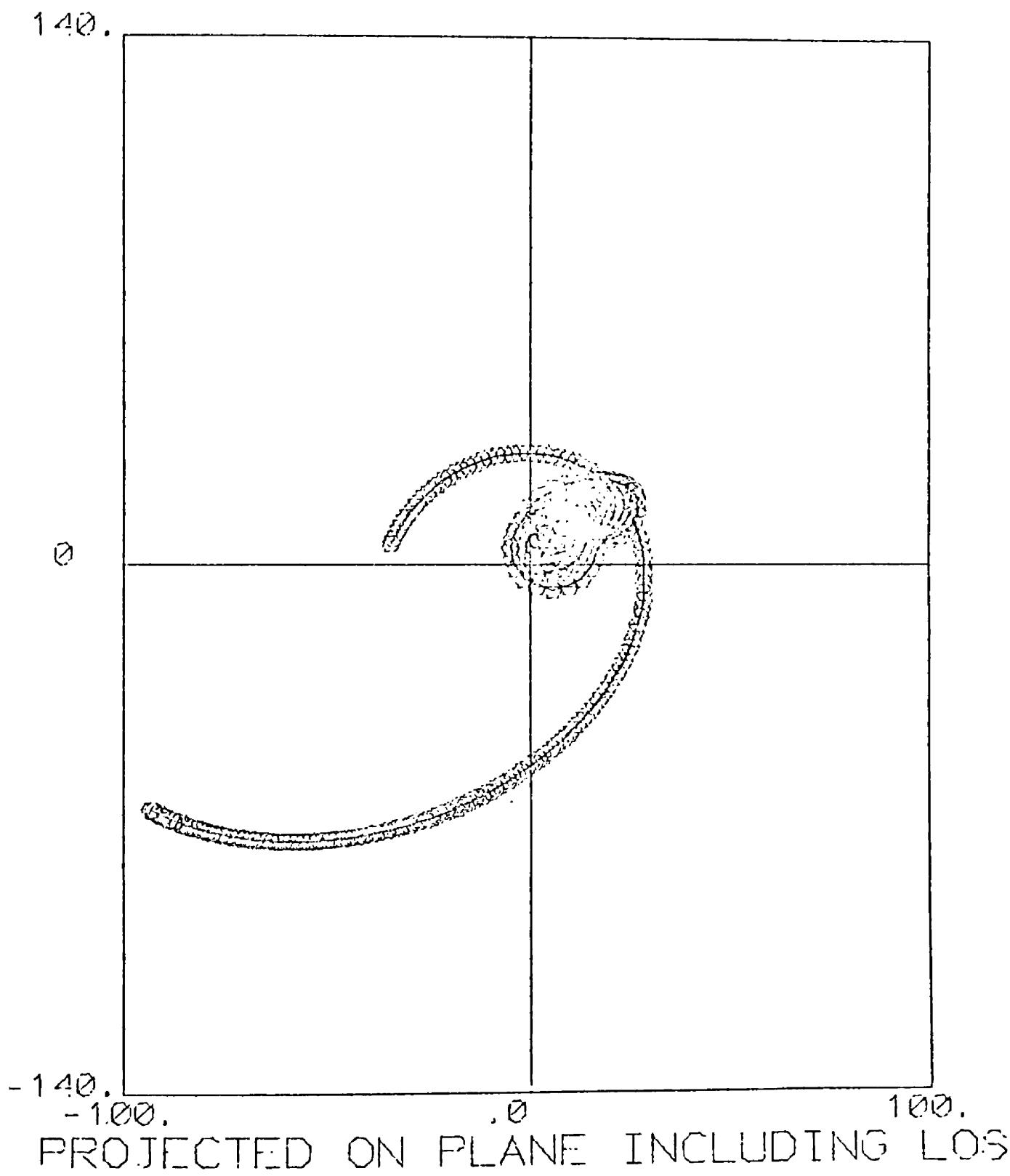


RZ CASSIOPEIAE: 0 TO 300 YR

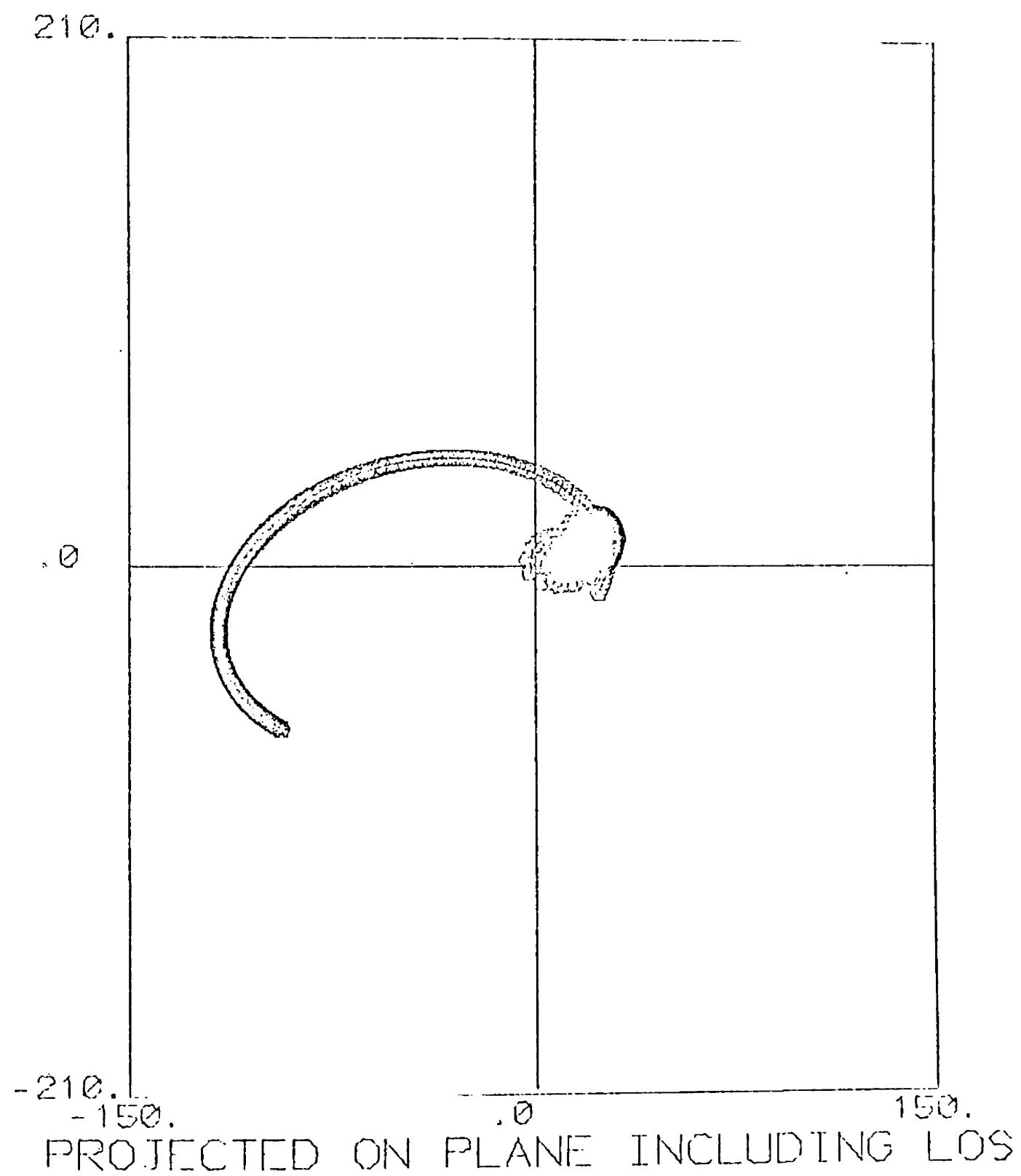


PROTECTED ON PLANE INCLUDING LOS

RZ CASSIOPEIAE: -300 TO -500 YR

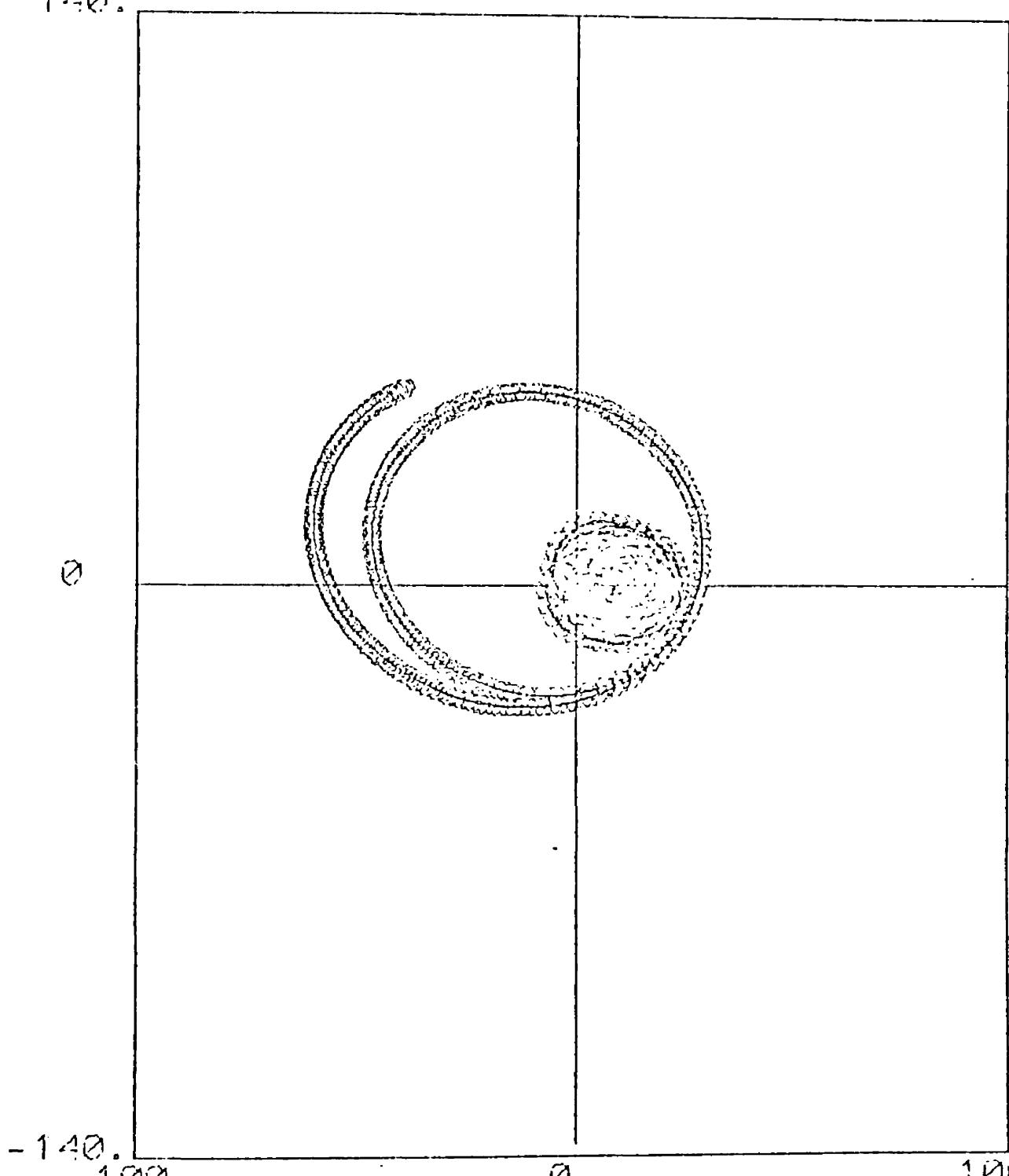


RZ CASSIOPEIAE: -500 TO -300 YR



RZ CASSIOPIAE: -800 TO 1100 YR

140.



-140.

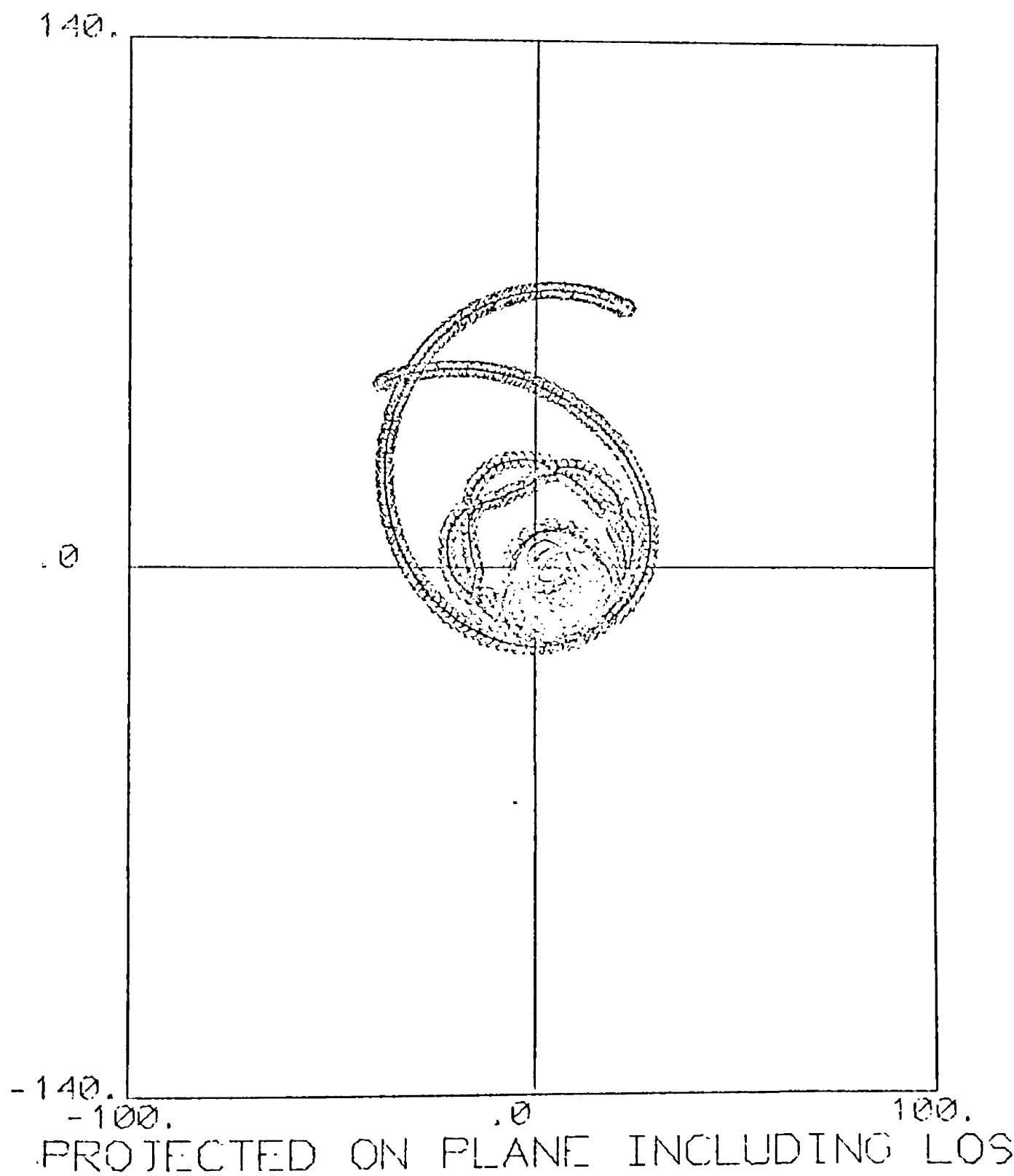
-100.

0

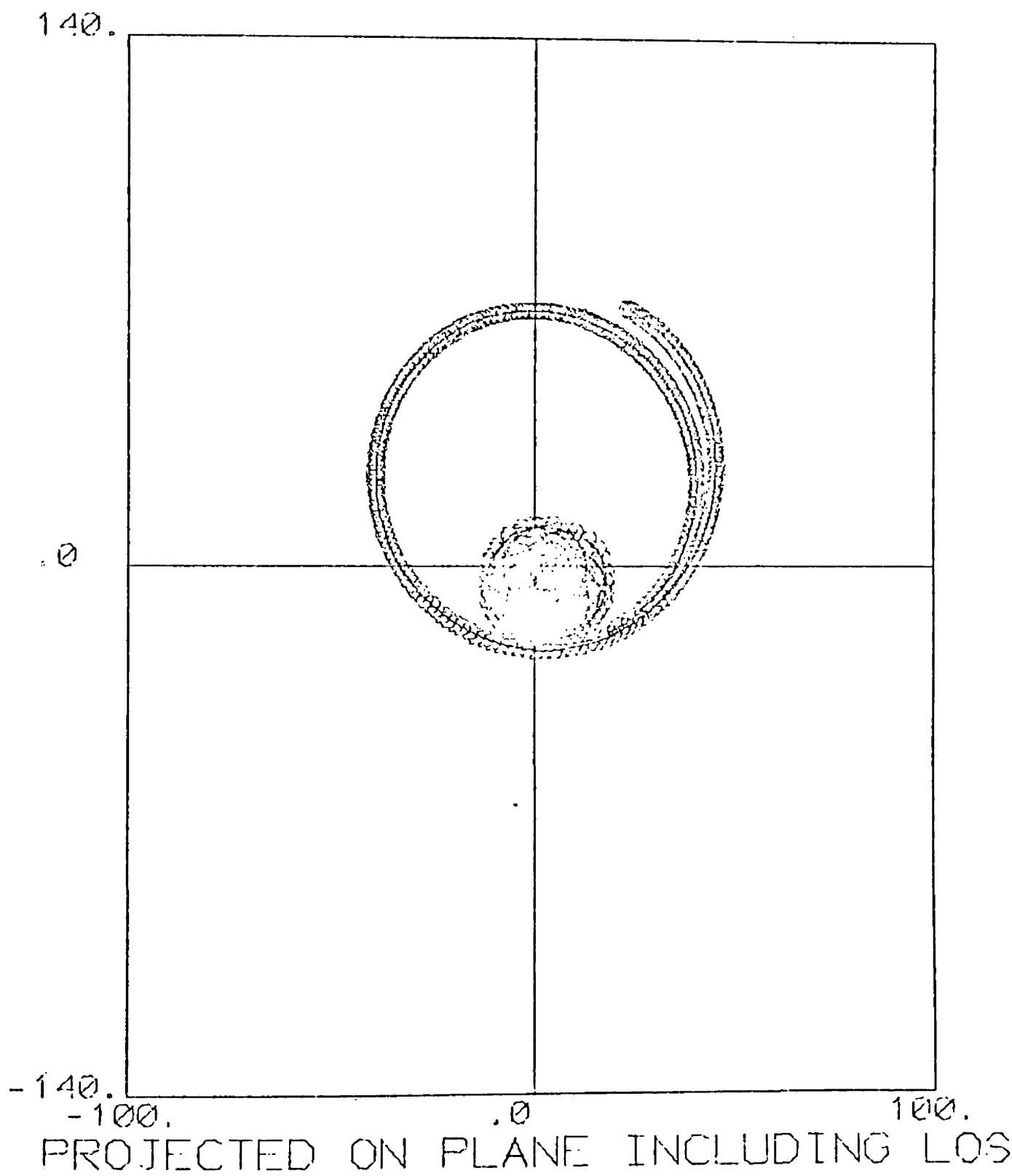
100.

PROJECTED ON PLANE INCLUDING LOS

RZ CASSIOPEIAE: -1100 TO -1500 YR

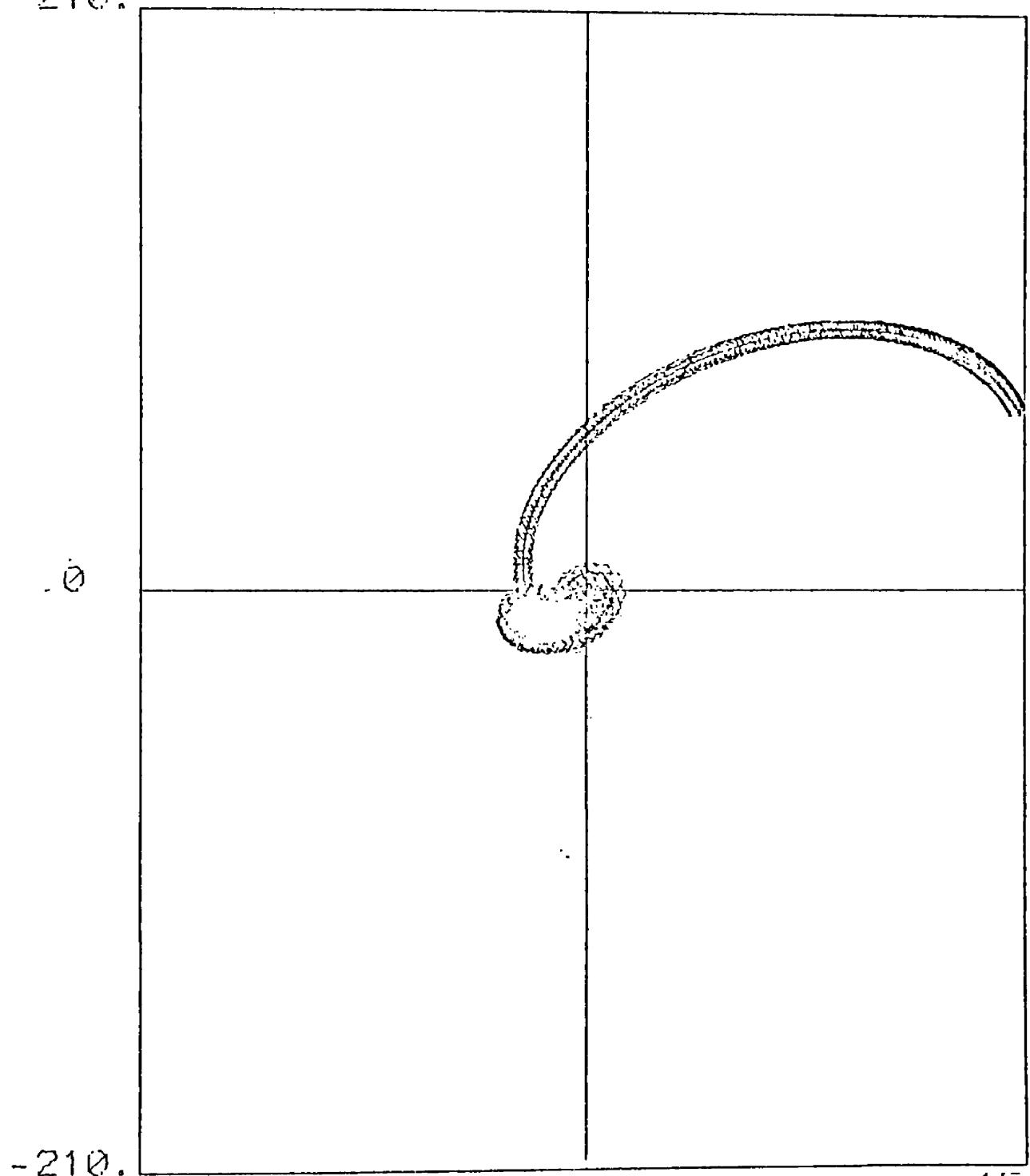


RZ CASSIOPEIAE: -1400 TO -1700 YR



RZ CASSIOPEIAE: 1700 TO -2000 YR

210.



PROJECTED ON PLANE INCLUDING LOS

RZ CASSIOPEIAE: -2000 TO -2300 YR

-280.

0

-280.

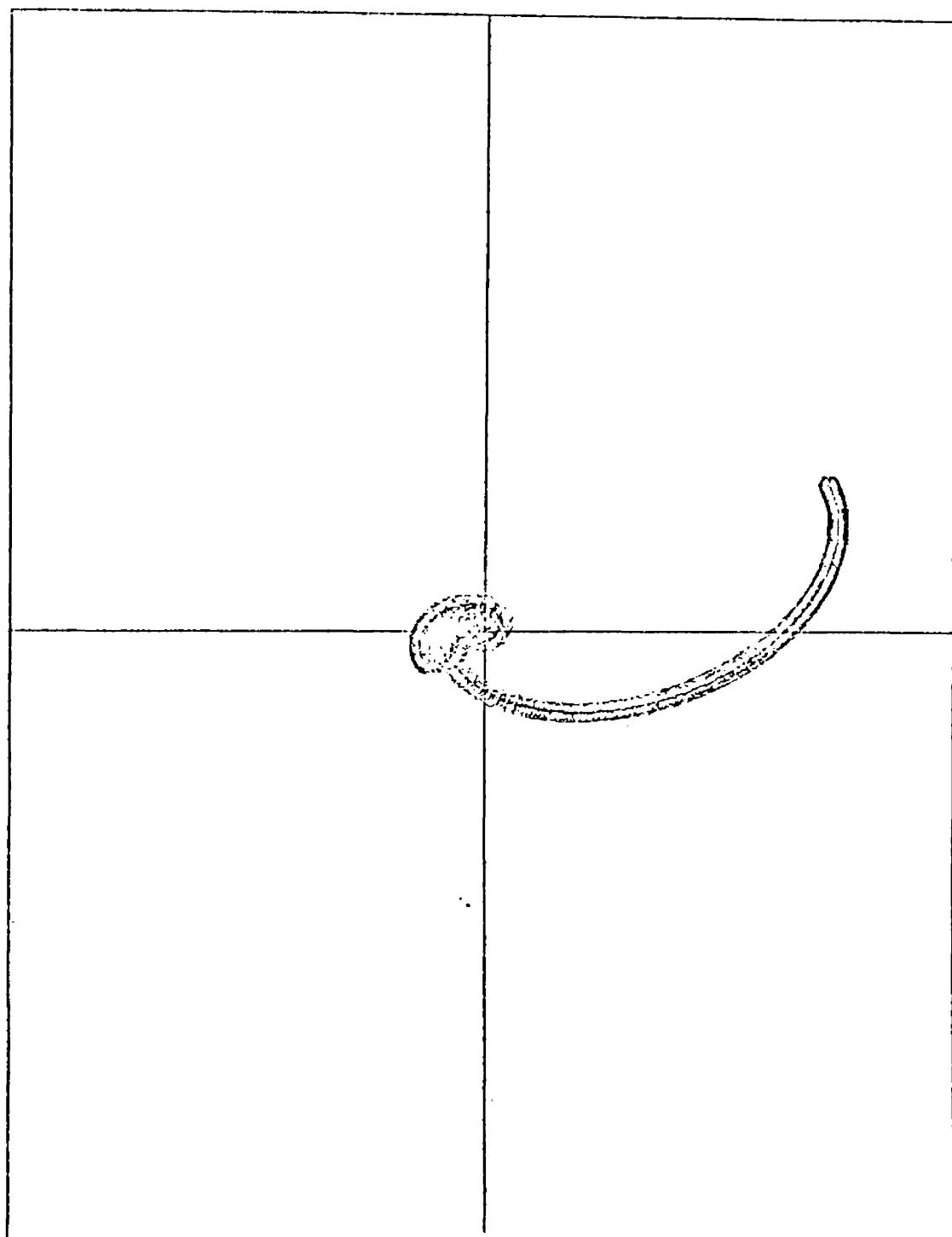
-200.

0

280.

PROTECTED ON PLANE INCLUDING LOS

(AU)



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