1991

A Montana-Wyoming coal severance tax duopoly model

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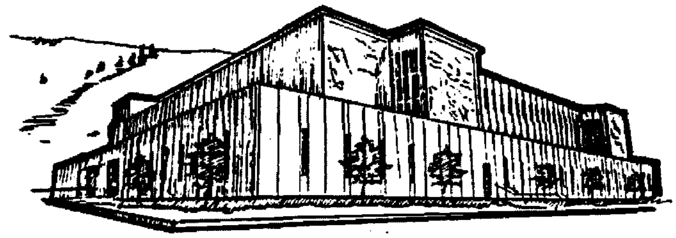

A MONTANA-WYOMING COAL SEVERANCE TAX DUOPOLY MODEL

By

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B. S., University of Montana, 1983

Presented in partial fulfillment of the requirements

for the degree of

Master of Arts

University of Montana

1991

Approved by

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Chairman, Board of Examiners

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April 11, 1991
Date

Tubbs, John E., Master's, March 1991 Economics

A Montana-Wyoming Coal Severance Tax Duopoly Model (129 pp.)

Director: John W. Duffield

The large coal reserves of the Northern Great Plains represent one of the nation's largest energy sources. The states of Montana and Wyoming assert market control over these reserves through state severance taxes. This report identifies optimal severance tax strategies and rates for Montana and Wyoming. In this analysis, optimal rates are defined as those that maximize state coal severance tax revenues.

A variant of the classic Bertrand price duopoly model is used to describe the Northern Great Plains coal market and the role of Montana and Wyoming severance taxes. Bertrand described a case where two producers maximize profits by controlling a commodity's price. In this analysis, severance tax rates are adjusted to maximize state tax revenues. Montana and Wyoming reaction functions are derived for naive and tax-leadership behavioral assumptions. These reaction functions describe how each state would react to a change in the other state's severance tax rate.

Using demand forecasts as an empirical base, the theoretical model is applied and optimal tax rates are calculated. The demand for Montana and Wyoming coal is forecasted using a spatial market model. Numerous coal demand forecasts are made for severance tax rates ranging from 0 to 120 percent in each state. These forecasts are then used to identify demand functions using simple regression techniques. The results suggest that severance tax rates of 75 percent for Montana and 119 percent for Wyoming would maximize each state's coal severance tax revenues.

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ABBREVIATIONS

BN	--	Burlington Northern Railroad
Btu	--	British thermal unit
DNRC	--	Montana Department of Natural Resources and Conservation
DOE	--	U.S. Department of Energy
EPRI	--	Electric Power Research Institute
GWh	--	Gigawatt-hour
MW	--	Megawatt
NCM	--	National Coal Model
NGP	--	Northern Great Plains
PPCC	--	Power Plant Capacity Cost
SMSA	--	Standard Metropolitan Statistical Area

CHAPTER 1

INTRODUCTION

Since the early seventies, Northern Great Plains (NGP) coal production has increased dramatically. Montana coal production went from 3.45 million tons in 1970 (Commission 1980, 93) to 37.77 million tons in 1989 (DOE 1990a, 17). Wyoming coal production went from 7.22 million tons in 1970 (Commission 1980, 93) to 171.45 million tons in 1989 (DOE 1990b, 5). The striking difference in the demand for Montana coal as compared to Wyoming coal is due to the proximity of Wyoming coal reserves to the large population centers in the in the mid-west (Duffield and others 1985, II-7). Along with increasing production, both states initiated substantial severance taxes.

Montana and Wyoming coal severance taxes have been the topic of several research efforts. The analysis presented in this thesis is an extension of a series of reports analyzing the NGP coal demand. In the first report, Projections of Northern Great Plains Coal Mining and Energy Conservation Development, 1975-2000 A.D. (Power and others 1976) a spatial market boundary between NGP coal and Midwest coal reserves was estimated. However, no other market boundaries were estimated. Estimates of a fully bounded NGP spatial market were reported in Projections of Coal Demand from the Norther Great Plains through the Year 2010 (Duffield and others 1982). In a third report, Montana Coal Market to the Year 2000: Impact of Severance Tax, Air Pollution Control, and Reclamation Costs (Duffield and others 1985), coal supply centers for Montana and Wyoming were introduced into the model. This allowed the competition between the coal producing states of Montana and Wyoming to be modeled.

This thesis takes the analysis of the impact of severance taxes presented in the 1985 report the a step further and develops optimal tax strategies. Optimal coal tax strategies are defined as those that maximize state severance tax revenues. The analysis presented in this thesis is not the first analysis of Montana and Wyoming cartel control over NGP coal. In 1983, Charles D. Kolstad and Frank A. Wolak jr. published an article entitled "Competition in Interregional Taxation: The Case of Western Coal" presenting a similar analysis. Michael P. Ward analyzed Montana and Wyoming market control in Coal Severance Taxes: The Effects of Western State's Tax Policy on the U.S. Coal Market (Ward 1982). A third study, The Western Coal Tax Cartel (Zimmerman and Alt 1981) also analyzed the market power of Montana and Wyoming. These reports will be discussed in more detail in Chapter 2.

OVERVIEW

This thesis is broken into five chapters. Chapter 1 provides this overview and a brief survey of Montana and Wyoming coal resources. Chapter 2 is a literature review of coal cartel models and coal demand models. Chapters 3 present the analytic components needed to calculate optimal severance tax rates. A variant of the classic Bertrand duopoly model is used to describe Montana and Wyoming market control. It is important to note that, in this case, the coal severance tax rate is the decision variable, not price. Montana and Wyoming reaction functions are then used to identify optimal severance tax strategies.

Chapter 3 also provides a description of the spatial market model used to forecast coal demand. These demand forecasts provide the empirical data upon which optimal rates are estimated. There are numerous factors affecting the demand for Montana and Wyoming coal. However, the pivotal factor for this analysis is the influence of Montana and Wyoming severance tax rates on the delivered price of coal and, in turn, coal demand. Based on severance taxes ranging from 0 to 120 percent, 169 combinations of Montana and Wyoming coal severance tax rates are used to forecast coal demand.

Chapter 4 presents the empirical analysis. First, the demand forecasts are presented. Using simple regression techniques, Montana and Wyoming coal demand equations are estimated. The OLS parameters are then substituted into the optimal tax strategies derived in Chapter 3. The result is an identification of optimal tax rates which would maximize Montana and Wyoming revenues.

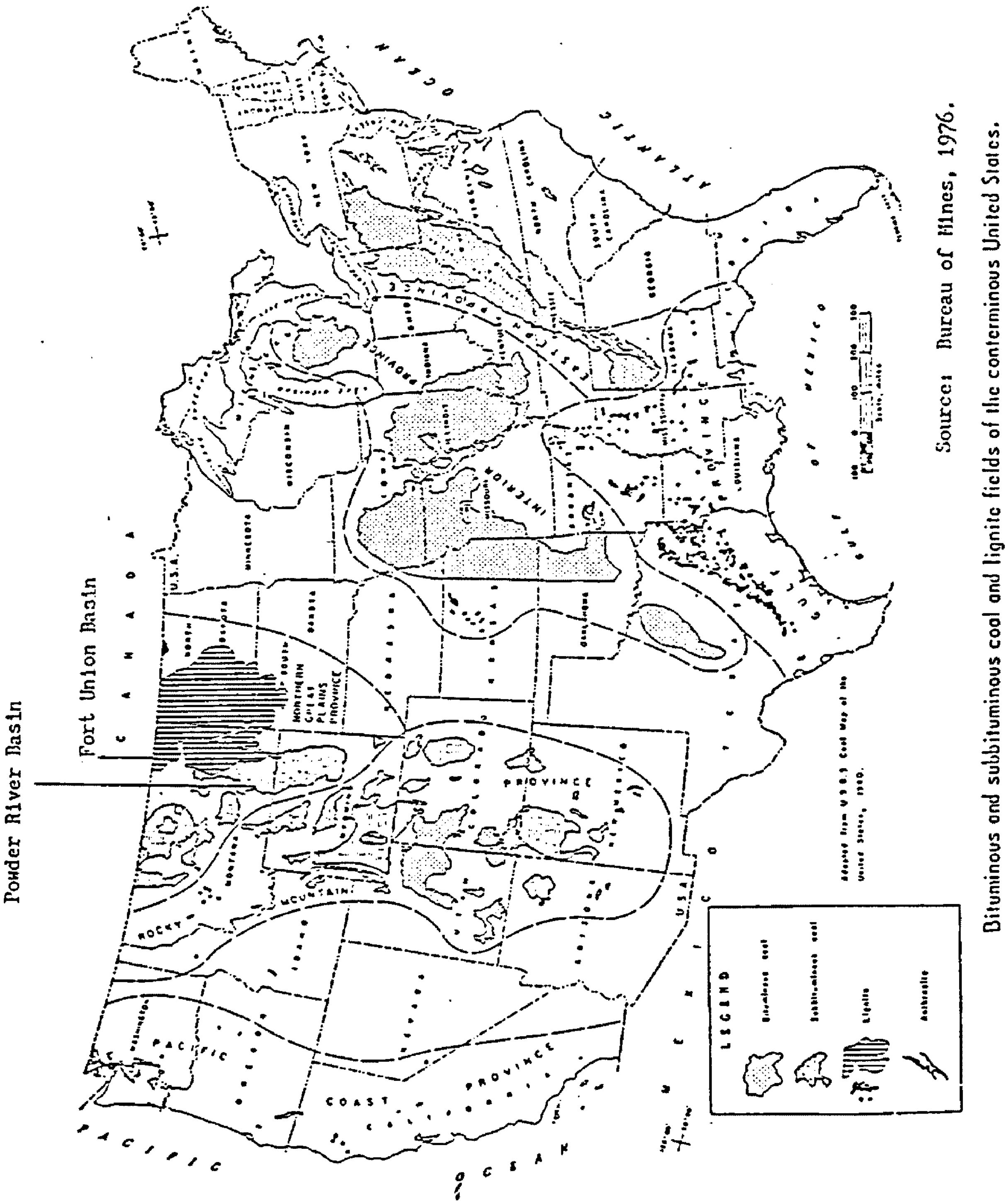
Chapter 5 provides an a review of the major assumptions and a summary of the empirical results as they compare to three previous studies. Chapter 5 also provides a summary of the major conclusions that are reached.

MONTANA AND WYOMING COAL RESOURCES

The coal resources of the world represent approximately half of the energy recoverable from the earth's crust (Silverman 1983, 5). The large, low-sulfur coal reserves of the Northern Great Plains (NGP) are one of the nation's largest energy resources. Figure 1 shows the location of major coal reserves in the United States.

Over ninety-five percent of the coal produced in the NGP is used for steam electric generation (DOE 1988a, 20). The 1982 coal study conducted by Duffield and others identified seven competing coal supply centers. Each of these coal producing regions compete against Montana and Wyoming coal for market shares. The analysis presented in this thesis focuses on the Powder River coals of Wyoming and Montana and treats the Green River coals of Wyoming as a competing supply region.

Figure 1. Coal Reserves of the United States.



Montana Coal Resources

The coal fields in Montana underlie approximately 35 percent of the total land area of the state (Keystone 1989, 526). The Fort Union Formation contains coal deposits ranging from low Btu lignite deposits, in northeast Montana, to subbituminous coal along the northern part of the Powder River Basin (Brown 1983, 205). Approximately, one quarter of the demonstrated reserves in the U.S. are in Montana. This represents 57 percent of the demonstrated subbituminous coal reserves and 35 percent of demonstrated lignite. The 1989 Keystone Coal Industry Manual (Keystone 1989, 527) estimates that remaining coal reserves in Montana total 471,630 million tons, of which about 50 million tons are strippable.

Coal mining in Montana was reported as early as 1807 when a Spanish fur trader heated his trading post with lignite coal (Keystone 1989, 531). Today, nine surface mines produce most, if not all, of the coal produced in Montana (Keystone 1989, 531). Table 1 provides historic coal production for Montana from 1970 through 1989.

Table 1. Montana Coal Production[a]

Year	Production (millions of tons)	Year	Production (millions of tons)
1970	3.447	1980	29.981
1971	7.064	1981	33.332
1972	8.221	1982	27.838
1973	10.725	1983	28.660
1974	14.106	1984	33.054
1975	22.054	1985	33.141
1976	26.231	1986	33.743
1977	29.320	1987	34.377
1978	26.679	1988	38.920
1979	34.454	1989	37.772

[a] Sources: 1970 - 1977 (Commission 1980, 93), 1978 - 1988 (DNRC 1989, 49), 1989 (DOE 1990a, 17).

The first Montana coal tax, a 5 cent per ton license fee, was imposed in 1921. Over 50 years later, Montana's coal tax legislation was "overhauled" and a new 30 percent severance tax was established (Verdon 1988, 53). In 1987, bowing to pressure from the governor and the coal industry, the legislature approved a phased reduction in the severance tax rate. A bench mark level of production was set at 32.2 million tons. In 1988, coal production exceeded this bench mark and the severance tax rate was lowered to 20 percent. Without new legislation, the coal severance tax will be reduced to 15 percent in 1991 (Verdon 1988, 54).

Wyoming Coal Resources

Two major coal bearing provinces cross Wyoming. Coal in the Eastern part of the state falls into the Northern Great Plains province (Keystone 1989, 611). The coals in this province are dominated by subbituminous rank coals. These are the coals that are the focus of the analysis presented in this paper. The deposits in other parts of the state are part of the Rocky Mountain Province (Keystone 1989, 611).

According to the 1989 Keystone Coal Manual, coal has been produced continuously in Wyoming since World War I (Keystone 1989, 617). In 1958, production fell to an all time low of 1.6 million tons. However, beginning in the late 1960s coal production began to increase and now Wyoming is the largest coal producing state in the nation (DOE 1990b, 5). Annual production totals are presented in Table 2 from 1970 through 1989.

Table 2. Wyoming Coal Production[a]

Year	Production (millions of tons)	Year	Production (millions of tons)
1970	7.222	1980	94.033
1971	8.052	1981	101.661
1972	10.928	1982	107.084
1973	14.886	1983	112.213
1974	20.703	1984	130.914
1975	23.804	1985	140.714
1976	30.836	1986	136.820
1977	44.500	1987	146.850
1978	58.328	1988	164.014
1979	70.795	1989	171.454

[a] Sources: 1970 - 1977 (Commission 1980, 93), 1978 (DOE 1980, 11), 1980 (DOE 1981a, 5), 1980 (DOE 1982a, 7), 1981 (DOE 1982b, 2), 1982 (DOE 1983, 2), 1983 (DOE 1984a, 2), 1984 (DOE 1985, 3), 1985 (DOE 1986, 4), 1986 (DOE 1988b, 13), 1987 (DOE 1988c, 8), 1988 (DOE 1989, 15), and 1989 (DOE 1990b, 5).

Wyoming's current severance tax rate is 8 and 1/2 percent. From 1979 through 1986 the tax rate was 10 and 1/2 percent. This two percent reduction took effect when the cumulative coal severance tax revenues exceeded $160 million (39-6-303 Wyoming Statutes Annotated) which occurred in 1986. Prior to 1979 a series of adjustments were made to the severance tax which took it from one percent in 1969 to 10.1 percent in 1978 (Wyoming Department of Revenue 1990).

CHAPTER 2

LITERATURE REVIEW

INTRODUCTION

Two components are needed to study Montana and Wyoming coal severance tax policies. First, a theoretical model of how each state will react to the others coal tax policies is developed. Next, an empirical analysis is preformed to identify optimal tax rates. This is done by first forecasting 1990 coal demand for a range of severance tax rates and then, through regression analysis, use these forecasts to empirically estimate the parameters identified in the theoretical model. The first section of this chapter provides a review and comparison of other studies analyzing Montana and Wyoming cartel control over NGP coal production. The second section provides a review of three coal demand forecasting models.

MONTANA-WYOMING COAL PRODUCTION COMPETITION

Kolstad and Wolak

In many ways the analysis presented in this thesis is fashioned after the duopoly analysis presented in Competition in Interregional Taxation: The Case of Western Coal (Kolstad and Wolak 1983). Kolstad and Wolak develop a theoretical model of Montana and Wyoming competition where the severance tax rate is the decision variable. While the general approach developed in this thesis is similar, the specific construct of the theoretical models and the coal forecasting models differ.

Kolstad and Wolak (1983, 450) establish a five equation model of the competition between these two coal producing states (see Table 3). The five equations are solved simultaneously to identify optimal coal tax policies.

Table 3. Equilibrium conditions - Kolstad and Wolak (1983, 450).

$$p_m = (1 + t_m)\cdot(\alpha_m + \beta_m\cdot q_m) + e_m$$

$$p_w = (1 + t_w)\cdot(\alpha_w + \beta_w\cdot q_w) + e_w$$

$$p_d = a_d + b_d\cdot(q_w + q_m) + e_d$$

$$r_i = c_i + d_i\cdot p_i + n_i \text{ for } i = m,w$$

$$p_d = p_i + r_i \text{ for } i = m,w$$

where:
α, β, a, b, c, and d are coefficients;
e_m, e_w, e_d and n_i are error terms;
P_m = FOB price of Montana coal;
P_w = FOB price of Wyoming coal;
P_d = delivered price of coal;
r_i = transportation cost;
t_m = Montana severance tax rate;
t_w = Wyoming severance tax rate;
q_m = Quantity of Montana coal demanded;
q_w = Quantity of Wyoming coal demanded.

Kolstad and Wolak (1983, 451) found that, for the simplest cartel strategy of adopting a single rate, the optimal coal severance tax rate is 87 percent. If "noncooperative" conditions are assumed, revenue maximizing, equilibrium rates would be 27 percent for Montana and 33 percent for Wyoming. If price-leadership, non-cooperative conditions are assumed, Montana and Wyoming would raise tax rates to approximately 35 percent. Kolstad and Wolak (1983, 453) also report, if standard Cournot conditions are assumed, Montana would set rates at 57 percent and Wyoming at 67 percent. At the end of Chapter 5, these rates will be compared to the optimal rates developed in this thesis.

Ward

Michael Ward's research had a slightly different twist. Ward (1982) analyzed the potential for Montana and Wyoming to impact national coal production and energy prices by extorting economic rents through the coal severance tax. Ward did not impose any duopoly model. Instead, he used a number of coal demand forecasts, based on different severance tax rate scenarios, to identify possible national impacts. In this regard, Ward's analysis is similar to the approach used in the 1985 report Montana Coal Market to the Year 2000: Impact of Severance Tax, Air Pollution Control, and Reclamation Costs (Duffield and others 1985).

Ward's analysis focused on the degree to which higher taxes might reduce coal production in Montana and Wyoming which would, in turn, reduce national production and drive coal and energy prices up (Ward 1982). Secondly, he analyzed what decision rules state governments use in setting their severance taxes. To answer these questions, Ward used five scenarios: "(1) reducing all severance taxes to zero; (2) limiting all severance tax rates to 12.5 percent; (3) lowering Montana's severance tax to that of Wyoming; (4) raising Wyoming's tax to that of Montana; and (5) raising both Montana's and Wyoming's severance tax to 70 percent" (Ward 1982, 37). He also analyzed the effect of transportation costs on the state production taxes.

Based on this analysis, Ward concluded Western tax policies would only have minor impacts on the national coal market. Midwestern utilities would be impacted most; however, substitution of Eastern coal mitigated a large portion of the costs (Ward 1982, 53). Ward also found transportation rates are much more important than state severance taxes to the marketability of western coal. Even in the "extreme" case where both states raise severance tax rates to 70 percent, national production was not impacted substantially. However, in this scenario Montana and Wyoming production fell by approximately two-thirds (Ward 1982, 47).

Table 4 shows the Montana and Wyoming coal severance tax revenues presented in Ward's study. Note that Montana severance tax revenues drop when the rate is lowered. This same result was reported in the 1985 study (Duffield and others 1985) and is also true for the estimates presented in Chapter 5. Revenues for both states are maximized at a rate of 70 percent; the highest rate Ward used.

Table 4. Ward's Forecast of 1990 Gross Revenues from Severance Taxes[a,b].

State	Reference	12.5% Tax Limit	Montana Tax Lowered to Wyoming	Wyoming Tax Raised to Montana	70%,Both Montana and Wyoming
Montana	267.15	144.93	152.58	275.60	489.66
Wyoming	255.27	214.23	234.23	405.25	1,096.52

[a] Source: Ward (1982, 47).
[b] All data is in millions of dollars.

Zimmerman and Alt

Zimmerman and Alt (1981, 26) analyze rates as high as 200 percent. Like Ward (1982), they do not impose a duopoly model. Instead, optimal severance tax rates are identified by maximizing the present discounted value of forecasted tax revenues. Assuming cartel cooperation, the optimal rate is 62.5 percent. They explain that while this rate seems high, "when considered in terms of what railroads have been doing, a 62.5 percent tax is not high" (Zimmerman and Alt 1981, 20).

Like Ward (1982), Zimmerman and Alt (1981) found the impact of raising Montana and Wyoming severance tax rates fell on energy consumers in the Midwest. "The Eastern regions (of the U.S.) bear almost no burden of this optimal tax" (Zimmerman and Alt 1981, 19). A comparison of Zimmerman and Alt's results and the optimal tax rates calculated in this thesis is provided in Chapter 5.

NGP COAL DEMAND MODEL

A spatial market model is used to forecast 1990 coal demand. The coal demand forecasts are then used as an empirical basis upon which optimal tax rates are calculated. The origins of the NGP spatial market model are described below along with a review of the models used by Ward (1982), Kolstad and Wolak (1983), and Zimmerman and Alt (1981).

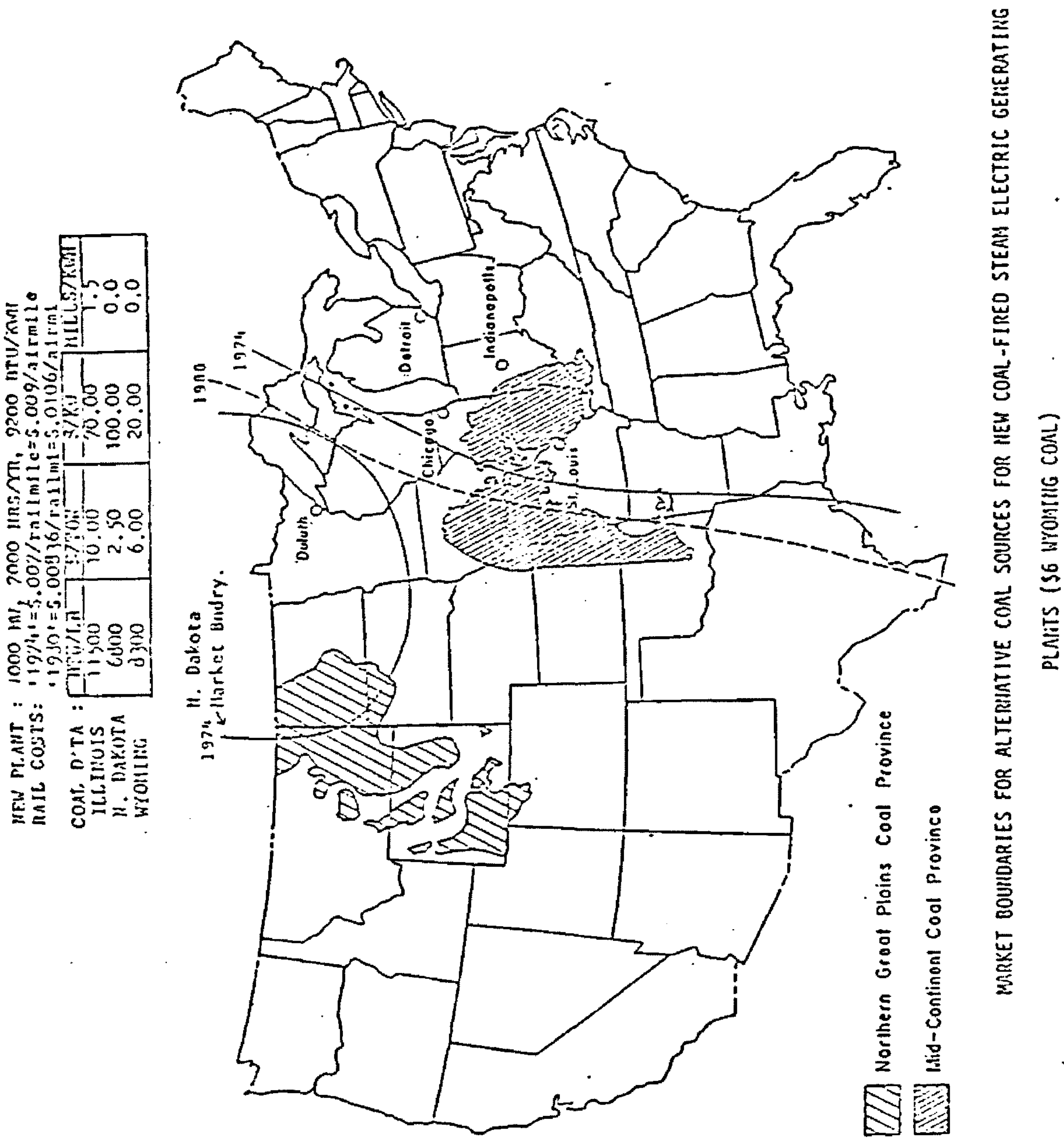
Spatial Market Model

In 1976, the Montana University Coal Demand Study Team forecasted demand for NGP coal from 1975 through the year 2000 (Power and others 1976). Two approaches were used to estimate coal demand in the 1976 study. The first assumed that NGP coal would supply "all the new demand for coal in the market area." A simplistic spatial model of the NGP coal market was also developed to forecast demand.

The spatial market model developed in the 1976 report identified only two competing supply centers (see Figure 2) and did not account for coal production in the south and southwest. Further, no inter-fuel substitution algorithm was developed to account for competing energy sources. Instead, a simplifying assumption that coal would maintain its share of in the energy market was made (Power and others 1976).

Several electric demand growth scenarios were modeled in the 1976 analysis. Three constant, annual electric growth scenarios were modeled: 1, 3, and 5 percent. An econometric forecast of electric demand was also developed which included four scenarios. The key variables in the econometric model were the real prices of electricity and natural gas. The variables impacting electric demand which were analyzed in the 1976 study include: air pollution standards, cost effectiveness of sulfur removal, price and availability of natural gas and petroleum, cost, reliability and acceptability of nuclear power, institutional constraints to the development of coal, transportation and transmission costs, and electric consumption demand elasticity.

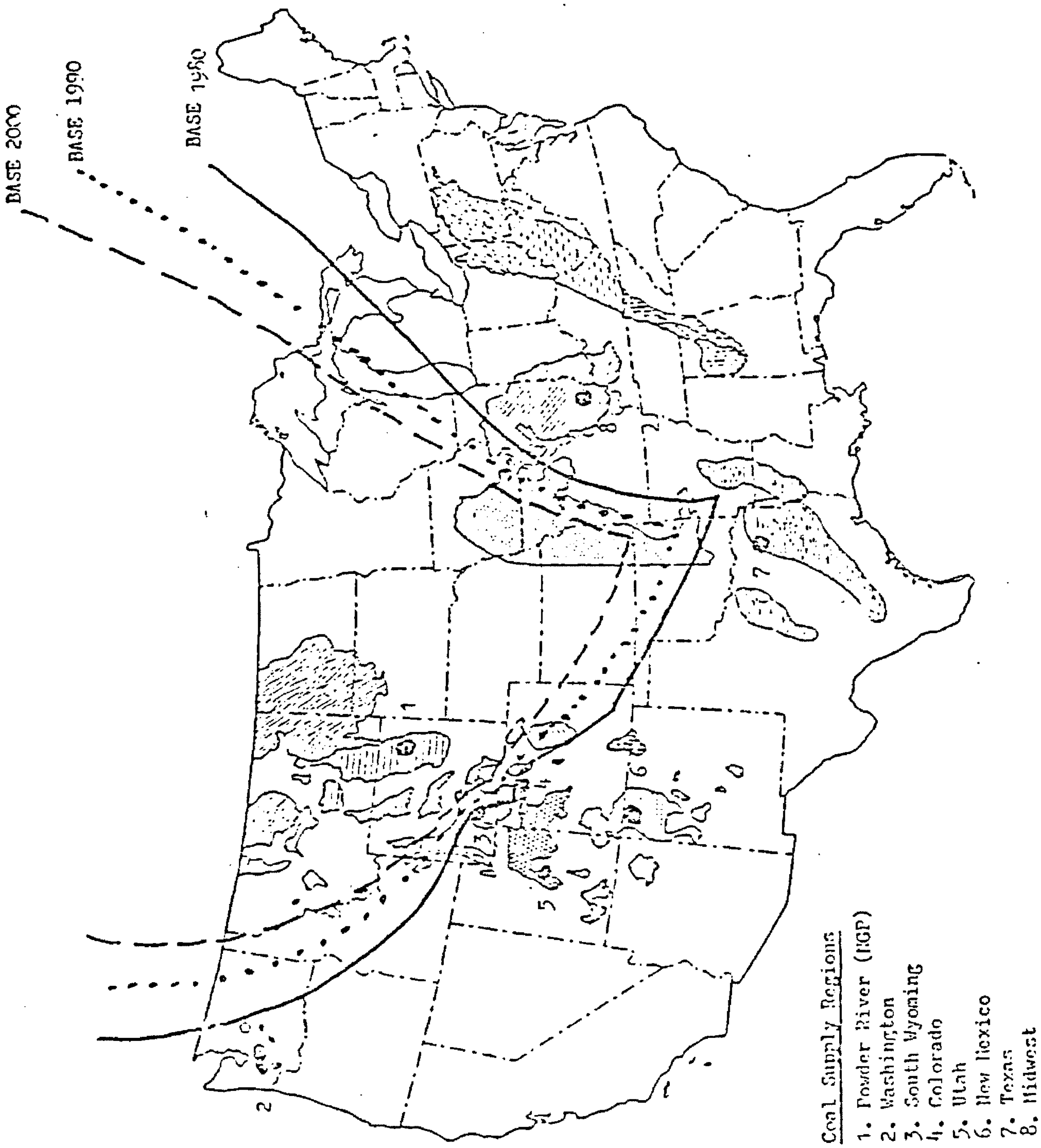
Figure 2. 1976 Spatial Market.



The 1976 spatial analysis was refined in 1982 with the publication of Projections of Coal Demand from the Northern Great Plains through the Year 2010 (Duffield and others 1982). The 1982 NGP spatial market model identified eight competing coal supply centers in Illinois, New Mexico, Southwest Wyoming, Colorado, Texas, Utah, Washington, and the NGP. This was a considerable improvement over the 1976 study and allowed the identification of a fully bounded spatial market (see Figure 3). The spatial market boundaries accounted for all costs associated with burning coal over the life of a model coal fired generation plant. Key variables included coal rank and quality, plant efficiencies, FOB coal prices, and transportation costs (Duffield and others 1982).

An inter-fuel substitution algorithm was also developed for the 1982 study. This algorithm takes the forecasted electric demand and divides it among competing energy sources including coal, nuclear, hydropower, oil, and gas. This was another significant improvement over the 1976 study which assumed constant market share.

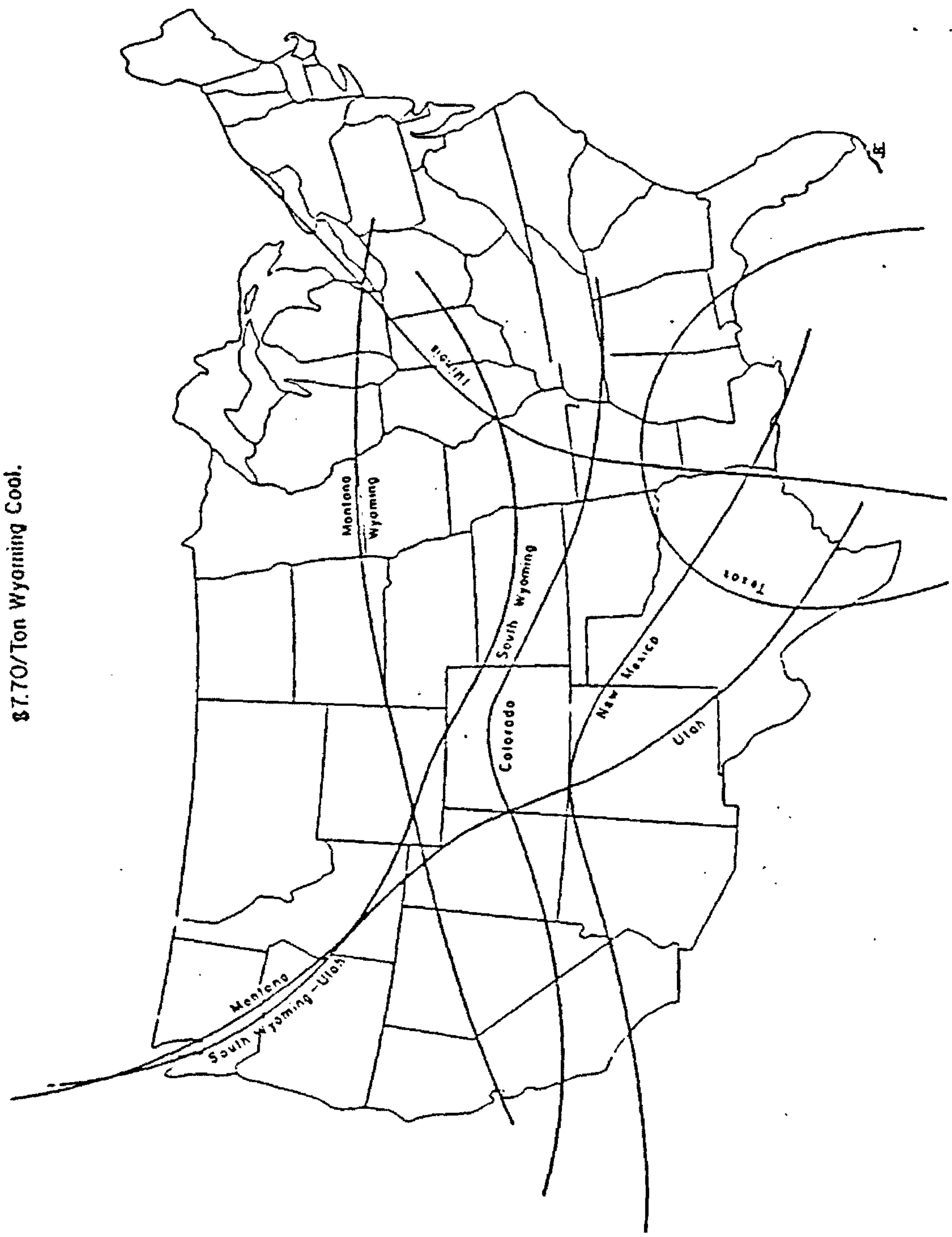
Figure 3. 1982 Spatial Market.



However, The 1982 NGP spatial market model did not identify a market boundary between Montana and Wyoming. In 1985, under a grant from the Montana Economic Development Council, a report analyzing the impacts of severance taxes, air pollution control, and reclamation costs was developed (Duffield and others 1985). This analysis took the NGP spatial market model developed in 1982 and added a Montana-Wyoming market boundary (see Figure 4).

With this addition, forecasts could be made for both Montana and Wyoming Powder River coal demand. The analysis of severance taxes estimated the impact on Montana coal demand and tax revenues that 1, 2, 3, and 4 dollar decreases in Montana FOB prices would have. However, the 1985 analysis presumed that Wyoming would not react to tax changes in Montana. Three electric demand scenarios were used to develop the forecasts: 1, 2, and 3 percent annual growth.

Figure 4. 1985 Spatial Market.



Montana Coal Market Area for $9.50/Ton FOB Against $7.70/Ton Wyoming Coal.

The 1985 study concluded there would be steady annual growth in Montana coal demand and a lowering of the coal severance tax would modestly increase the demand for Montana coal while annual severance tax revenues would drop substantially.

The analysis presented in this thesis uses the forecasting model developed for the 1985 report, which incorporates the improvements that were made to the NGP spatial market model. The key variables in the analysis is the FOB prices of Montana and Wyoming coal. The analysis presented in this thesis also relaxes the assumption that Wyoming will not react to changes in Montana's coal severance tax rate and develops a model to analyze the competition between these two states.

National Coal Model

The Department of Energy's (DOE) national coal model (NCM) is a linear programing model of U.S. coal supply and demand coupled with an electric utility resource decision model (DOE 1982c). This model was used by both Kolstad and Wolak (1983) and Ward (1982). The goal of the linear program is to minimize the total cost of electricity delivered by utilities, and the cost of coal consumed by the non-utility sectors (DOE 1982c, 1). Costs include coal mining, washing and transport, as well as electricity transport and generation costs (Kolstad and Wolak 1983, 457).

The advantage of the NCM is that it uses a true transportation model of coal shipments to utilities. The NCM provides greater accuracy concerning coal shipments in comparison to the NGP spatial model, which relies on a air-to-rail ratio to identify transportation costs to a given geographic area. Further, the NGP spatial market model is based on state wide electric demand forecasts. If a state is bisected by a market boundary, the percent of standard metropolitan statistical areas (SMSA) populations captured within the market are used to weight state electric demand. The national coal model is much more specific about how much power is demanded at each node.

However, even with this greater precision, the NCM is still only as accurate as the base assumptions concerning electric demand growth and the use of competing fuels. The modified version of the national coal model used by Kolstad and Wolak (1983) greatly overestimated the demand for Montana and Wyoming coal in 1990. Kolstad and Wolak

(1983, 451) do not provide a table listing their coal demand forecasts; but their "Figure 1. Activity analysis model" shows forecasted (combined Montana and Wyoming) demand for 1990 no lower than 250 million tons and exceeding 500 million tons at the high end. In 1989 combined Montana and Wyoming coal production was 209 million tons.

Michael Ward (1982, 42) also uses DOE's national coal model. Interestingly, the "Reference" case present in Ward's 1982 analysis is much closer to actual levels of coal demand than the forecasts made by Kolstad and Wolak. Ward reports a forecasted demand for "West Northern Great Plains" of 189.63 million tons in 1990. This suggests that the assumptions Kolstad and Wolak make concerning energy demand and competing energy sources may be driving the high levels of forecasted demand, rather than there being a fundamental problem with the NCM.

Zimmerman and Alt Coal Demand Model

Zimmerman and Alt (1981) also use a linear programing model to forecast coal supply and demand. One of the unique features of their model is that coal production cost is modeled as a function of output. As output accumulates over time, production cost increase. The link is a model of seam thickness which is based on limited data from western Kentucky. Zimmerman and Alt (1981, 6) identify coal shipment costs based on escalated transportation rates. The objective function is to minimize the sum of costs of mining and transportation.

A "Regional Electricity Model" is used to forecast the demand for energy. This model is much more sophisticated than the constant annual electric growth model used in the NGP spatial market model. The proportion of energy demand satisfied by a particular fuel is based on relative prices. The electricity model calculates an optimum configuration of capacity given demand and cost constraints (Zimmerman and Alt 1981, 8). Coal demand and supply are linked by estimating some 360 supply curves and with the demand model through linear programing techniques (Zimmerman and Alt 1981, 10). The linear program minimizes cost of meeting demand on a yearly basis.

Both DOE's and Zimmerman and Alt's coal forecasting models require substantial computing power and are costly to run. The NGP spatial model has the advantage of being rather simple in comparison to both. Kolstad and Wolak (1983, 456) report that the linear program they used contained approximately 4,000 constraints, 25,000 variables and was solved using a CRAY 1 computer. The computing requirements are much smaller for the NGP spatial market model. Further, given the sensitivity of all of the forecasting models to assumptions concerning electric demand growth and the use of competing fuels, the NGP spatial market model can provide very accurate results.

CHAPTER 3

THEORETICAL METHODS

The duopoly model developed to analyze Montana and Wyoming severance tax policies is a variant of the classic Bertrand price duopoly model. The difference between classic duopoly models and the Montana-Wyoming coal tax duopoly model is the use of the coal severance tax rate to exercise market power. Cournot and Bertrand duopoly models are formulated either in terms of direct control over output or price, respectively.

Further, the goal of these classic models is to maximize profit. For the Montana-Wyoming coal tax duopoly model, the goal is to maximize coal severance tax revenues for each state. The severance tax rate, rather than price or quantity, is used as the decision variable. Kolstad and Wolak (1983, 445) also used the coal severance tax as the decision variable stating "it is our view that if tax rates are the actual decision variables, then strategies for setting taxes will be based on how competitors set taxes, not on the indirectly determined output levels of competitors."

This chapter describes the classic Cournot and Bertrand duopoly models. Then using the same general techniques, a Montana-Wyoming coal tax duopoly model is developed. Model specification where each state naively assumes the other will not react to a change in severance tax rates and where one state is a "tax leader" are analyzed. At the end of the chapter, the spatial market model is also described. The spatial market model is used to forecast coal demand. A complete description of this model is provided in the appendices of the thesis.

COURNOT DUOPOLY

Cournot (1838) was one of the first to describe a duopoly model. In this classic duopoly model, there are only two producers of a homogenous product. These producers attempt to maximize profits by controlling the level of output produced. The Cournot model also makes the naive assumption that each producer believes the other will not react to a change in his own output.

A mathematical description of the Cournot model is presented below. First, the demand and revenue functions are described. These are followed by the identification of first order and equilibrium conditions. A linear demand function and, constant marginal costs are assumed.

Cournot Duopoly Model

$P_D(q) = a - b \cdot q$

$q = q_1 + q_2$

$PROFIT_1 = (a - b \cdot q) \cdot q_1 - c \cdot q_1$

where $a, b, c >= 0$;
$P_D(q)$ = Market Demand;
q_1 = Output of Producer 1;
q_2 = Output of Producer 2.

<u>1st Order Conditions -- Producer 1</u>

$$\frac{dProfit}{dq_1} = a - b \cdot (2 \cdot q_1 - q_2) - c = 0$$

$$q_1 = \frac{(a - c)}{2 \cdot b} - \frac{q_2}{2}$$

<u>1st Order Conditions -- Producer 2</u>

$$\frac{dProfit}{dq_2} = a - b \cdot (2 \cdot q_2 - q_1) - c = 0$$

$$q_2 = \frac{(a - c)}{2 \cdot b} - \frac{q_1}{2}$$

The first order conditions describe how each producer will react to one another (reaction functions). In order to maximize profit, each producer not only takes into account market demand and his production, but also the amount produced by his competitor. An equilibrium solution can be calculated by simultaneous substitution of the reaction functions. Profits will be maximized at q^*_1 and q^*_2 output levels.

Equilibrium -- Producer 1

$$q^*_1 = \frac{(a - c)}{2b} - \frac{\frac{(a - c)}{2 \cdot b} - \frac{q^*_1}{2}}{2}$$

$$q^*_1 = \frac{(a - c)}{3 \cdot b}$$

Equilibrium -- Producer 2

$$q^*_2 = \frac{(a - c)}{2b} - \frac{\frac{(a - c)}{2 \cdot b} - \frac{q^*_2}{2}}{2}$$

$$q^*_2 = \frac{(a - c)}{3 \cdot b}$$

BERTRAND DUOPOLY

The Cournot duopoly model describes a situation where the duopolists controlled the market by changing output. An obvious alternative is to use price as the decision variable. This price duopoly model was first described by Bertrand (1883). In Bertrand's model, each producer assumes the other will hold price constant, regardless of his own actions. This is equivalent to the naive assumption in the Cournot analysis where each producer assumed the other will hold output constant. In this example, a slight differentiation between the products of each player is necessary for equilibrium to be achieved. It is further assumed that marginal costs are constant.

Bertrand Duopoly Model

$D_1(p) = a - b \cdot P_1 + c \cdot P_2$

$D_2(p) = \alpha - \beta \cdot P_1 + \mu \cdot P_2$

$PROFIT_1 = (a - b \cdot P_1 + c \cdot P_2) \cdot P_1$

$PROFIT_2 = (\alpha - \beta \cdot P_2 + \mu \cdot P_1) \cdot P_2$

where $a, b, c, \alpha, \beta, \mu >= 0$;
$D_1(p)$ = Demand for producer 1's output;

$D_2(p)$ = Demand for producer 2's output;
P_1 = Price set by producer 1;
P_2 = Price set by producer 2.

As with the Cournot duopoly model, reaction functions are calculated based on the first derivative of the profit function and setting it equal to zero. Each producer will change the price of their product based on both the demand for the product and the price of the competing duopolist's product.

1st Order Conditions -- Producer 1

$$\frac{dPROFIT_1}{dP_1} = a - 2 \cdot b \cdot P_1 + c \cdot P_2 = 0$$

$$P_1 = \frac{a + c \cdot P_2}{2 \cdot b}$$

1st Order Conditions -- Producer 2

$$\frac{dPROFIT_2}{dP_2} = \alpha - 2 \cdot \beta \cdot P_2 + \mu \cdot P_1 = 0$$

$$P_2 = \frac{\alpha + \mu \cdot P_1}{2 \cdot \beta}$$

Similar to the development of the Cournot model, optimal price levels are identified by simultaneous substitution of the two producers reaction functions. By setting prices at P'_1 and P'_2, profits will be maximized for each producer and an equilibrium solution can be identified.

Equilibrium -- Producer 1

$$P'_1 = \frac{a + c \cdot \dfrac{\alpha + \mu \cdot P'_1}{2 \cdot \beta}}{2 \cdot b}$$

$$P'_1 \cdot \left(2 \cdot b - \frac{c \cdot \mu}{2 \cdot \beta}\right) = a + c \cdot \frac{\alpha}{2 \cdot \beta}$$

$$P'_1 = \frac{2 \cdot \beta \cdot a + c \cdot \alpha}{4 \cdot b \cdot \beta - c \cdot \mu}$$

Equilibrium -- Producer 2

$$P'_2 = \frac{\alpha + \mu \cdot \dfrac{a + c \cdot P'_2}{2 \cdot b}}{2 \cdot \beta}$$

$$P'_2 \cdot \frac{4 \cdot \beta - c \cdot \mu}{2 \cdot b} = \frac{\alpha + \mu \cdot a}{2 \cdot b}$$

$$P'_2 = \frac{2 \cdot b \cdot \alpha + a \cdot \mu}{4 \cdot \beta \cdot b - c \cdot \mu}$$

BERTRAND PRICE LEADERSHIP

One weakness of both Cournot's and Bertrand's analyses is the assumption that the competing producer will not react to changes in output or price. This assumption is only true when equilibrium is achieved. A more likely situation would be where one of the two competitors had an advantage over the other and could afford to lead changes. This competitive advantage can be modeled using Bertrand's price duopoly model and assuming that one producer anticipates his competitors action while the other continues to assume the his competitor will not react. In other words, the price leader has full knowledge of his competitors reaction function and the price follower continues to believe his competitor will not change price. Mathematically, this is accomplished by substituting Producer 2's reaction function into the Producer 1's profit equation, assuming Producer 1 is the price leader.

Bertrand Price Leadership Model

$$PROFIT_1 = (a - b \cdot P_1 + c \cdot P_2) \cdot P_1$$

$$PROFIT_1 = a - b \cdot P_1 + c \cdot \frac{\alpha + \mu \cdot P_1}{2 \cdot \beta} \cdot P_1$$

$$PROFIT_2 = (\alpha - \beta \cdot P_2 + \mu \cdot P_1) \cdot P_2$$

<u>1st Order and Equilibrium Condition -- Producer 1</u>

$$\frac{dPROFIT_1}{dP_1} = a - 2 \cdot b \cdot P_1 + \frac{c \cdot \alpha}{2 \cdot \beta} + \frac{2 \cdot c \cdot \mu \cdot P_1}{2 \cdot \beta} = 0$$

$$P_1 \cdot (2 \cdot b - \frac{2 \cdot c \cdot \mu}{2 \cdot \beta}) = a + \frac{c \cdot \alpha}{2 \cdot \beta}$$

$$P_1 = \frac{2 \cdot \beta \cdot a + c \cdot \alpha}{4 \cdot b \cdot \beta - 2 \cdot c \cdot \mu}$$

<u>1st Order Condition -- Producer 2</u>

$$P_2 = \frac{\alpha + \mu \cdot P_1}{2 \cdot \beta}$$

<u>Equilibrium -- Producer 2</u>

$$P'_2 = \frac{\alpha - \mu \cdot \dfrac{2 \cdot a \cdot \beta + c \cdot \alpha}{2 \cdot (2 \cdot b \cdot \beta - c \cdot \mu)}}{2\beta}$$

$$P'_2 = \frac{2\cdot b\cdot\alpha + a\cdot\mu}{4\cdot b\cdot\beta - 2\cdot c\cdot\mu} - \frac{c\cdot\mu\cdot\alpha}{4\cdot\beta\cdot(2\cdot b\cdot\beta - c\cdot\mu)}$$

Assuming that a, b, c, α, β, and μ are all greater than zero, a comparison between the equilibrium solution under Bertrand's classic assumptions versus the price leadership model can be made. As expected, the price leader (Producer 1) increases his profits and less profits are available to the price follower (Producer 2). This is expected because there would be no point to leadership otherwise.

Producer 1

$$\underbrace{\frac{2\cdot\beta\cdot a + c\cdot\alpha}{4\cdot b\cdot\beta - c\cdot\mu}}_{\text{Bertrand Equilibrium}} < \underbrace{\frac{2\cdot\beta\cdot a + c\cdot\alpha}{4\cdot b\cdot\beta - 2\cdot c\cdot\mu}}_{\text{Price Leadership Equilibrium}}$$

Producer 2

$$\underbrace{\frac{2\cdot b\cdot\alpha + a\cdot\mu}{4\cdot\beta\cdot b - c\cdot\mu}}_{\text{Bertrand Equilibrium}} > \underbrace{\frac{2\cdot b\cdot\alpha + a\cdot\mu}{4\cdot b\cdot\beta - 2\cdot c\cdot\mu} - \frac{c\cdot\mu\cdot\alpha}{4\cdot\beta\cdot(2\cdot b\cdot\beta - c\cdot\mu)}}_{\text{Price Leadership Equilibrium}}$$

MONTANA-WYOMING COAL TAX DUOPOLY

The descriptions of the Cournot, Bertrand, and Bertrand price leadership models all follow the same mathematical construct. First, a set of assumptions on how the two duopolists will react to each other and the demand and revenue functions they face are postulated. Based on these assumptions, reaction functions are derived. Then equilibrium conditions are developed by a simultaneous substitution of the two reaction functions.

These basic steps are now used to develop a Montana-Wyoming coal tax duopoly model. The Bertrand model and the price leadership model most closely resemble the coal tax duopoly model for the obvious reason that the coal severance tax rate directly affects the delivered price of coal and only indirectly affects the quantity produced. Initially, it is assumed that the states of Montana and Wyoming will set tax rates based on the naive assumption that the other state will not counter by changing its tax rate. Once equilibrium conditions are identified under this assumption, a price leadership model is developed. It is also assumed that both Montana and Wyoming face linear demand curves and that marginal cost is constant. The first step is then mathematically describing the demand and revenue functions.

Montana-Wyoming Coal Tax Duopoly Model

$$D_{mt} = a - b\cdot(1+t_{mt})\cdot P_{mt} + c\cdot(1+t_{wy})\cdot P_{WY}$$

$$R_{mt} = (a - b\cdot(1+t_{mt})\cdot P_{mt} + c\cdot(1+t_{wy})\cdot P_{WY})\cdot P_{mt}$$

$$TR_{mt} = (1+t_{mt})\cdot R_{mt} - R_{mt} = t_{mt}\cdot R_{mt}$$

$$= t_{mt}\cdot(a - b\cdot(1+t_{mt})\cdot P_{mt} + c\cdot(1+t_{wy})\cdot P_{WY})\cdot P_{mt}$$

$$D_{WY} = \alpha - \beta\cdot(1+t_{wy})\cdot P_{WY} + \mu\cdot(1+t_{mt})\cdot P_{mt}$$

$$R_{WY} = \alpha - \beta\cdot(1+t_{wy})\cdot P_{WY} + \mu\cdot(1+t_{mt})\cdot P_{mt}$$

$$TR_{WY} = (1 + t_{wy})\cdot R_{WY} - R_{WY} = t_{wy}\cdot R_{WY}$$

$$= t_{wy}\cdot(\alpha - \beta\cdot(1+t_{wy})\cdot P_{WY} + \mu\cdot(1+t_{mt})\cdot P_{mt})\cdot P_{WY}$$

where:

<u>Montana</u>:

D_{mt} = Montana coal demand;
P_{mt} = Montana FOB price for coal;
R_{mt} = Montana producer revenue;
TR_{mt} = Montana tax revenue;
t_{mt} = Montana tax rate;
a = Constant for Montana demand (D_{mt});
b = Slope for Montana production (D_{mt});
c = Slope for Wyoming production (D_{mt}).

<u>Wyoming</u>:

D_{WY} = Wyoming Coal demand;
P_{WY} = Wyoming FOB price for coal;
R_{WY} = Wyoming producer revenue;
TR_{WY} = Wyoming tax revenue;
t_{wy} = Wyoming tax rate;
α = Constant for Wyoming demand (D_{WY});
β = Slope for Wyoming production (D_{WY});
μ = Slope for Montana production (D_{WY}).

Now that the demand and revenue functions are described, first order conditions are calculated. These first order conditions describe how each state will react to changes in the other state's severance tax rate. The reaction function for Montana is developed first, followed by the Wyoming reaction function.

First Order Conditions

Montana Reaction Function

$$\frac{dTR_{mt}}{dt_{mt}} = a \cdot P_{mt} - b \cdot P_{mt}^2 - 2 \cdot t_{mt} \cdot b \cdot P_{mt}^2 + c \cdot P_{mt} \cdot P_{WY} + c \cdot t_{wy} \cdot P_{mt} \cdot P_{WY} = 0$$

$$2 \cdot t_{mt} \cdot b \cdot P_{mt}^2 = (a - b \cdot P_{mt} + c \cdot P_{WY} + c \cdot t_{wy} \cdot P_{WY}) \cdot P_{mt}$$

$$t_{mt} = \frac{a - b \cdot P_{mt} + c \cdot (1 + t_{wy}) \cdot P_{WY}}{2 \cdot b \cdot P_{mt}}$$

Wyoming Reaction Function

$$\frac{dTR_{wy}}{dt_{wy}} = \alpha \cdot P_{WY} - \beta \cdot P_{WY}^2 - 2 \cdot t_{wy} \cdot \beta \cdot P_{WY}^2 + \mu \cdot P_{mt} \cdot P_{WY} + \mu \cdot t_{mt} \cdot P_{mt} \cdot P_{WY} = 0$$

$$2 \cdot t_{wy} \cdot \beta \cdot P_{WY}^2 = (\alpha - \beta \cdot P_{WY} + \mu \cdot P_{mt} + \mu \cdot t_{mt} \cdot P_{mt}) \cdot P_{WY}$$

$$t_{wy} = \frac{\alpha - \beta \cdot P_{WY} + \mu \cdot (1 + t_{mt}) \cdot P_{mt}}{2 \cdot \beta \cdot P_{WY}}$$

It is important to note that each state would consider the other states severance tax rate when establishing its own tax rate. Therefore, to describe the equilibrium conditions a simultaneous substitution is needed. Again, it is assumed that the neither duopolist has prior knowledge on how the other state will react and will naively assume that no action will be taken by the other state.

Equilibrium

Montana

$$t_{mt}^{*} = \frac{a - b \cdot P_{mt} + c \cdot \left(1 + \dfrac{\alpha - \beta \cdot P_{WY} + \mu \cdot (1 + t_{mt}^{*}) \cdot P_{mt}}{2 \cdot \beta \cdot P_{WY}}\right) \cdot P_{WY}}{(2 \cdot b \cdot P_{mt})}$$

$$t_{mt}^{*}\cdot 2\cdot b\cdot P_{mt} = \frac{2\cdot\beta\cdot(a - b\cdot P_{mt} + c\cdot P_{WY}) + c\cdot(\alpha - \beta\cdot P_{WY} + \mu\cdot P_{mt}) + \mu\cdot t_{mt}^{*}\cdot P_{mt}}{2\cdot\beta}$$

$$t_{mt}^{*}\cdot P_{mt}\left(2\cdot b - \frac{c\cdot\mu}{2\cdot\beta}\right) = \frac{2\cdot\beta\cdot(a - b\cdot P_{mt} + c\cdot P_{WY}) + c\cdot(\alpha - \beta\cdot P_{WY} + \mu\cdot P_{mt})}{2\cdot\beta}$$

$$t_{mt}^{*}\cdot P_{mt}\cdot\frac{4\cdot b\cdot\beta - c\cdot\mu}{2\cdot\beta} = \frac{2\cdot\beta\cdot(a - b\cdot P_{mt} + c\cdot P_{WY}) + c\cdot(\alpha - \beta\cdot P_{WY} + \mu\cdot P_{mt})}{2\cdot\beta}$$

$$t_{mt}^{*} = \frac{2\cdot\beta\cdot(a - b\cdot P_{mt} + c\cdot P_{WY}) + c\cdot(\alpha - \beta\cdot P_{WY} + \mu\cdot P_{mt})}{P_{mt}\cdot(4\cdot b\cdot\beta - c\cdot\mu)}$$

Wyoming

$$t_{wy}^{*} = \frac{\alpha - \beta\cdot P_{WY} + \mu\cdot\left(1 + \dfrac{a - b\cdot P_{mt} + c\cdot(1+t_{wy}^{*})\cdot P_{WY}}{2\cdot b\cdot P_{mt}}\right)\cdot P_{mt}}{2\cdot\beta\cdot P_{WY}}$$

$$t_{wy}^{*}\cdot 2\cdot\beta\cdot P_{WY} = \frac{2\cdot b\cdot(\alpha - \beta\cdot P_{WY} + \mu\cdot P_{mt}) + \mu\cdot(a - b\cdot P_{mt} + c\cdot P_{WY}) + \mu\cdot c\cdot t^{*}_{wy}}{2\cdot b}$$

$$t_{wy}^{*}\cdot P_{WY}\cdot\left(2\cdot\beta - \frac{\mu\cdot c}{2\cdot b}\right) = \frac{2\cdot b(\alpha - \beta\cdot P_{WY} + \mu\cdot P_{mt}) + \mu\cdot(a - b\cdot P_{mt} + c\cdot P_{WY})}{2\cdot b}$$

$$4\cdot b\cdot\beta - \mu\cdot c \qquad 2\cdot b\cdot(\alpha - \beta\cdot P_{WY} + \mu\cdot P_{mt}) + \mu\cdot(a - b\cdot P_{mt} + c\cdot P_{WY})$$

$$t_{wy}^{*} \cdot P_{WY} \cdot \frac{\qquad}{2 \cdot b} = \frac{\qquad}{2 \cdot b}$$

$$t_{wy}^{*} = \frac{2 \cdot b \cdot (\alpha - \beta \cdot P_{WY} + \mu \cdot P_{mt}) + \mu \cdot (a - b \cdot P_{mt} + c \cdot P_{WY})}{P_{WY} \cdot (4 \cdot b \cdot \beta - \mu \cdot c)}$$

Now that equilibrium conditions have been identified, the naive assumption that the states do not anticipate their competitors actions is relaxed. The two models presented below show the tax leadership equilibriums that would be achieved for Montana and Wyoming. The first set of equations describes the Montana tax leadership model. This is followed by a scenario where Wyoming is the tax leader. The approach is the same as described for the Bertrand price leadership model. The tax leader has full knowledge of the followers reaction function and the follower continues to assume the leader will not react.

Montana Tax Leadership

Optimal Montana Tax Rate

$$TR_{mt} = t_{mt} \cdot (a - b \cdot (1+t_{mt}) \cdot P_{mt} + c \cdot (1+t_{wy}) \cdot P_{WY}) \cdot P_{mt}$$

$$TR_{mt} = t_{mt} \cdot (a - b \cdot (1+t_{mt}) \cdot P_{mt} + c \cdot (1 + \frac{\alpha - \beta \cdot P_{WY} + \mu \cdot (1+t_{mt}) \cdot P_{mt}}{2 \cdot \beta \cdot P_{WY}}) \cdot P_{WY}) \cdot P_{mt}$$

$$TR_{mt} = t_{mt} \cdot a \cdot P_{mt} - t_{mt} \cdot b \cdot P_{mt}^2 - t_{mt}^2 \cdot b \cdot P_{mt}^2 + t_{mt} \cdot c \cdot P_{mt} \cdot P_{WY} + \frac{t_{mt} \cdot c \cdot P_{mt} \cdot (\alpha - \beta \cdot P_{WY} + \mu \cdot P_{mt} + \mu \cdot t_{mt} \cdot P_{mt})}{2 \cdot \beta}$$

$$\frac{dTR_{mt}}{dt_{mt}} = a \cdot P_{mt} - b \cdot P_{mt}^2 - 2 \cdot t_{mt} \cdot b \cdot P_{mt}^2 + c \cdot P_{WY} \cdot P_{mt} + \frac{c \cdot \alpha \cdot P_{mt} - c \cdot \beta \cdot P_{mt} \cdot P_{WY} + c \cdot \mu \cdot P_{mt}^2 + 2 \cdot t_{mt} \cdot c \cdot \mu \cdot P_{mt}^2}{2 \cdot \beta} = 0$$

$$t_{mt} \cdot 2 \cdot b \cdot P_{mt}^2 - \frac{t_{mt} \cdot c \cdot \mu \cdot P_{mt}^2}{\beta} = \frac{(2 \cdot \beta(a - b \cdot P_{mt} + c \cdot P_{WY}) + c \cdot (\alpha - \beta \cdot P_{WY} + \mu \cdot P_{mt})) \cdot P_{mt}}{2 \cdot \beta}$$

$$\frac{t_{mt} \cdot P_{mt}^2 \cdot (2 \cdot b \cdot \beta \cdot - c \cdot \mu \cdot)}{\beta} = \frac{(2 \cdot \beta(a - b \cdot P_{mt} + c \cdot P_{WY}) + c \cdot (\alpha - \beta \cdot P_{WY} + \mu \cdot P_{mt})) \cdot P_{mt}}{2 \cdot \beta}$$

$$t_{mt}^* = \frac{2 \cdot \beta \cdot (a - b \cdot P_{mt} + c \cdot P_{WY}) + c \cdot (\alpha - \beta \cdot P_{WY} + \mu \cdot P_{mt})}{2 \cdot P_{mt} \cdot (2 \cdot b \cdot \beta - c \cdot \mu)}$$

The equilibrium tax rate for the tax follower, Wyoming, is derived by substituting the optimal tax rate for Montana into the Wyoming reaction function.

Optimal Wyoming Tax Rate

$$t_{WY} = \frac{\alpha - \beta \cdot P_{WY} + \mu \cdot (1 + t_{mt}) \cdot P_{mt}}{2 \cdot \beta \cdot P_{WY}}$$

$$t_{wy} = \frac{\alpha - \beta \cdot P_{WY} + \mu \cdot \left(1 + \frac{2 \cdot \beta \cdot (a - b \cdot P_{mt} + c \cdot P_{WY}) + c \cdot (\alpha - \beta \cdot P_{WY} + \mu \cdot P_{mt})}{2 \cdot P_{mt} \cdot (2 \cdot b \cdot \beta - c \cdot \mu)}\right) \cdot P_{mt}}{2 \cdot \beta \cdot P_{WY}}$$

$$t_{wy}' = \frac{2 \cdot (2 \cdot b \cdot \beta - c \cdot \mu) \cdot (\alpha - \beta \cdot P_{WY} + \mu \cdot P_{mt}) + \mu \cdot (2 \cdot \beta \cdot (a - b \cdot P_{mt} + c \cdot P_{WY}) + c \cdot (\alpha - \beta \cdot P_{WY} + \mu \cdot P_{mt}))}{4 \cdot \beta \cdot P_{WY} \cdot (2 \cdot b \cdot \beta - c \cdot \mu)}$$

The equations below depict a scenario where Wyoming is the tax leader and Montana is the follower. Again, the same approach is used to identify optimal tax strategies.

Tax Leadership Wyoming

Optimal Wyoming Tax Rate

$$TR_{WY} = t_{wy} \cdot (\alpha - \beta \cdot (1 + t_{wy}) \cdot P_{WY} + \mu \cdot (1 + t_{mt}) \cdot P_{mt}) \cdot P_{WY}$$

$$TR_{WY} = t_{wy} \cdot \left(\alpha - \beta \cdot (1 + t_{wy}) \cdot P_{WY} + \mu \cdot \left(1 + \frac{a - b \cdot P_{mt} + c \cdot (1 + t_{wy}) \cdot P_{WY}}{2 \cdot b \cdot P_{mt}}\right) \cdot P_{mt}\right) \cdot P_{WY}$$

$$TR_{WY} = t_{wy} \cdot \alpha \cdot P_{WY} - t_{wy} \cdot \beta \cdot P_{WY}^2 - t_{wy}^2 \cdot \beta \cdot P_{WY}^2 + t_{wy} \cdot \mu \cdot P_{mt} \cdot P_{WY} + \frac{t_{wy} \cdot \mu \cdot P_{WY} \cdot (a - b \cdot P_{mt} + c \cdot P_{WY} + c \cdot t_{wy} \cdot P_{WY})}{2 \cdot b}$$

$$\frac{dTR_{WY}}{dt_{wy}} = \alpha P_{WY} - \beta \cdot P_{WY}^2 - 2 \cdot t_{wy} \cdot \beta \cdot P_{WY}^2 + \mu \cdot P_{mt} \cdot P_{WY} + \frac{\mu \cdot a \cdot P_{WY} - \mu \cdot b \cdot P_{mt} \cdot P_{WY} + \mu \cdot c \cdot P_{WY}^2 + 2 \cdot t_{wy} \cdot c \cdot \mu \cdot P^2_{WY}}{2 \cdot b} = 0$$

$$2 \cdot t_{wy} \cdot \beta \cdot P_{WY}^2 - \frac{t_{wy} \cdot c \cdot \mu \cdot P^2_{WY}}{b} = \frac{(2 \cdot b \cdot (\alpha - \beta \cdot P_{WY} + \mu \cdot P_{mt}) + \mu \cdot (a - b \cdot P_{mt} + c \cdot P_{WY})) \cdot P_{WY}}{2 \cdot b}$$

$$\frac{t_{wy} \cdot P^2_{WY} \cdot (2 \cdot \beta \cdot b - c \cdot \mu)}{b} = \frac{(2 \cdot b \cdot (\alpha - \beta \cdot P_{WY} + \mu \cdot P_{mt}) + \mu \cdot (a - b \cdot P_{mt} + c \cdot P_{WY})) \cdot P_{WY}}{2 \cdot b}$$

$$t_{wy}^* = \frac{2 \cdot b \cdot (\alpha - \beta \cdot P_{WY} + \mu \cdot P_{mt}) + \mu \cdot (a - b \cdot P_{mt} + c \cdot P_{WY})}{2 \cdot P_{WY} \cdot (2 \cdot \beta \cdot b - c \cdot \mu)}$$

The equilibrium tax rate for the tax follower, Montana, is derived by substituting the optimal Wyoming tax strategy into the Montana reaction function.

Optimal Montana Tax Rate

$$t_{mt} = \frac{a - b \cdot P_{mt} + c \cdot (1 + t_{wy}) \cdot P_{WY}}{2 \cdot b \cdot P_{mt}}$$

$$2 \cdot b \cdot (\alpha - \beta \cdot P_{WY} + \mu \cdot P_{mt}) + \mu \cdot (a - b \cdot P_{mt} + c \cdot P_{WY})$$

$$t_{mt} = \frac{a - b\cdot P_{mt} + c\cdot\left(1+\frac{}{2\cdot P_{WY}\cdot(2\cdot\beta\cdot b - c\cdot\mu)}\right)\cdot P_{WY}}{2\cdot b\cdot P_{mt}}$$

$$t_{mt}^{*} = \frac{2\cdot(2\cdot\beta\cdot b - c\cdot\mu)\cdot(a - b\cdot P_{mt} + c\cdot P_{WY}) + c\cdot(2\cdot b\cdot(\alpha - \beta\cdot P_{WY} + \mu\cdot P_{mt}) + \mu\cdot(a - b\cdot P_{mt} + c\cdot P_{WY}))}{4\cdot b\cdot P_{mt}\cdot(2\cdot\beta\cdot b - c\cdot\mu)}$$

This concludes the development of the Montana-Wyoming tax duopoly model. The optimal tax strategies identified in this chapter, both equilibrium and tax leadership scenarios, are used to identify optimal tax rates in Chapter 4.

SPATIAL MARKET MODEL

In order to estimate optimal tax rates, a spatial market model is used to forecasts 1990 coal demand. These demand forecasts are then use in a regression analysis to specify Montana and Wyoming coal demand functions. The parameters are then substituted into the optimal tax strategies developed above. The remaining portion of this chapter describes the central core of the spatial market model. A full description of the spatial market model is provided in the appendices.

The spatial market model is used to distribute the forecasted demand for new coal production among the competing coal supply centers. These competing supply centers are located in Colorado, Illinois, New Mexico, Texas, Washington, Utah, South West Wyoming, Montana, and Eastern Wyoming. While this study is focused on the competition between Montana and Wyoming coal production, the other six supply centers must be accounted for so that completely bounded spacial markets can be generated. This is an important point, because there is a basic assumption made in the modeling that the producers in these other states will not react to price changes of Montana and Wyoming coal supplies.

The spacial market model calculates market boundaries between each of the competing coal supply centers. These boundaries are defined by points where the cost of using coal from one supply center equals the costs of using coal from the competing supply center. A simplistic model of the coal boundaries, described in Campbell and Hwang (1978), is shown by the following equilibrium relationship:

(1) $M_A + T_A D_A = M_B + T_B D_B$

(2) $D_B = (M_B - M_A)/T_B + T_A D_A/T_B$

(3) $D_B = k + h D_A$

Where:

M_A = the cost of using coal from supply center A;
M_B = the cost of using coal from supply center B;
D_A = the distance from supply center A to the market boundary;
D_B = the distance from supply center B to the market boundary;
k = $(M_B - M_A)/T_B$;
h = T_A/T_B;
T = variable cost of transportation.

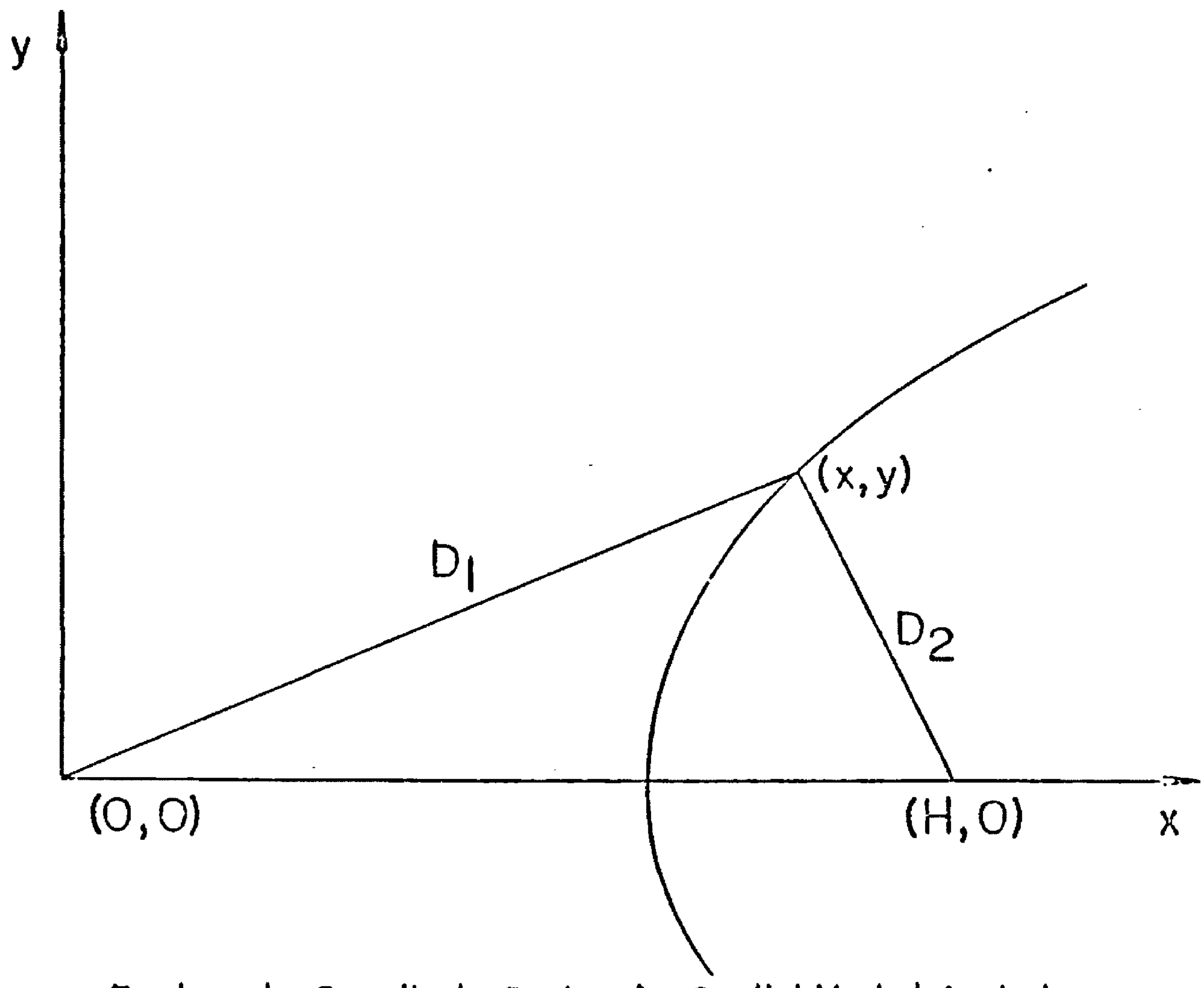
By identifying the distance between the two competing coal supply centers, a spacial constraint to the market is applied. A market boundary can then be generated using the Euclidian distance function. Equation 3 then becomes:

(4) $(h-x)^2 + y^2 = k^2 + 2hk(x^2+y^2)^{1/2} + h^2(x^2 + y^2)$

Where h = distance between competing centers; and y and x are rectangular coordinates.

Figure 5 shows this relationship. However, the model described by Campbell and Hwang (1978) did not take into account other costs associated with using coal from the various supply centers. While the distance relationship remains the center of the spacial market model, all costs must be accounted for before the market boundaries can be defined. The variables used to describe this relationship are presented in Table 5. The values for these variables, used for the forecasts are presented in Appendix A.

Figure 5. Rectangular Coordinate System for Spatial Market Analysis.



Rectangular Coordinate System for Spatial Market Analysis.

Once the market boundaries are identified, the spatial market model must determine the level of coal demand within the spatial market area. This is a fairly straight foreword process of summing the coal demand for each state in the market area. The inter-fuel substitution algorithm provides these estimates of state coal demand. However, when a state is bisected by a market boundary, the state-wide total must also be divided. To do this, SMSA locations and populations are identified (U.S. Department of Commerce 1979). The population of the SMSAs captured within the market boundaries are summed. The sum is divided by the total state SMSA population. This ratio is then used to weight the total state coal demand and determine the proportion of the statewide coal demand captured with in the spatial market.

Table 5. Variable Description[a].

Line #	Coal Supply Center[b]	Description
1	A & B	Power Plant Size (net MW)
2	A & B	Hours Operated at Full Load (hours)
3	A	Power Plant Heat Rate (Btu/KWH)
4	A	Coal Heat Content (Btu/lb)
5	B	Power Plant Heat Rate (Btu/KWH)
6	B	Coal Heat Content (Btu/lb)
7	A	Power Plant Capital Cost ($/KW)
8	B	Power Plant Capital Cost ($/KW)
9	A & B	Fixed Charge Rate (decimal)
10	A	Operating and Maintenance Costs ($/KWH)
11	B	Operating and Maintenance Costs ($/KWH)
12	A	FOB Mine Price ($/ton)
13	B	FOB Mine Price ($/ton)
14	A	Fixed Transportation Cost ($/ton)
15	B	Fixed Transportation Cost ($/ton)
16	A	Variable Transportation Costs ($/ton-air mile)
17	B	Variable Transportation Costs ($/ton-air mile)
18	A & B	Straight Line Distance Between A & B (miles)

[a] Source: Duffield and others (1985, III-8).
[b] A = Either the Montana or Wyoming Supply Center; B = The market competitor of A; one of seven possible coal supply centers.

CHAPTER 4

EMPIRICAL ANALYSIS

In this chapter, optimal severance tax rates are estimated using the duopoly model developed in the previous chapter and coal demand forecasts generated by the spatial market model. The concept is relatively straight forward. The spatial market model is used to forecast coal production for severance tax rates ranging from 0 to 120 percent in each state. As in Kolstad and Wolak's 1983 analysis, the production forecasts provide the base data upon which Montana and Wyoming demand equations are estimated. This is done by regressing the production forecasts on the corresponding prices of Montana and Wyoming coal. The regression estimates are then substituted into the tax strategies developed in the previous chapter and optimal severance tax rates are calculated.

Table 6 and 7 present Montana and Wyoming coal production forecasts. In each table, Wyoming coal severance tax rates increase as you move to the right and are listed across the top. Montana coal severance tax rates increase as you move down the table and are listed along the left side. It is important to note that this is "new" coal production and does not include coal production that is currently under contract. There is insufficient data, specifically concerning Wyoming coal contracts, to conduct an in depth analysis which includes current production contracts.

These tables reflect the effect of the severance tax rate on the demand for each states coal. As expected, the demand for Montana and Wyoming coal decreases as the severance tax for that state is increased. The converse, where demand increases when the tax for the other states coal increases, is strongly reflected in the Montana forecasts but is almost absent in the Wyoming forecasts.

Table 6. New Montana Coal Production in 1990.

WT Tax	0	10	20	30	40	50	60	70	80	90	100	110	120
						(Millions of Tons)							
MT 0	12	12	13	13	13	13	14	16	18	18	18	18	18
Tax 10	12	12	12	13	13	13	13	13	14	18	18	18	18
20	11	12	12	12	13	13	13	13	13	14	18	18	18
30	11	11	11	11	12	13	12	12	12	12	14	14	14
40	10	11	11	11	11	12	11	12	12	12	12	12	12
50	8	9	9	9	9	11	9	10	10	10	10	10	10
60	3	9	9	9	9	9	9	9	10	10	10	10	10
70	3	9	9	9	9	9	9	9	9	10	10	10	10
80	2	9	9	9	9	9	9	9	9	9	9	9	9
90	2	6	6	6	6	9	6	6	6	6	6	6	6
100	2	2	2	2	2	6	2	2	2	2	2	2	2
110	2	2	2	2	2	2	2	2	2	2	2	2	2
120	2	2	2	2	2	2	2	2	2	2	2	2	2

Table 7. New Wyoming Coal Production in 1990.

WY Tax	0	10	20	30	40	50	60	70	80	90	100	110	120
						(Millions of Tons)							
MT 0	18	18	18	18	18	18	18	16	16	8	5	10	9
Tax 10	18	18	18	18	18	18	18	17	17	12	9	5	4
20	18	18	18	18	18	18	18	17	17	12	10	9	5
30	18	18	18	18	18	18	18	17	17	12	10	10	8
40	18	18	18	18	18	18	18	17	17	12	10	10	9
50	18	18	18	18	18	18	18	17	17	12	10	10	9
60	18	18	18	18	18	18	18	17	17	12	10	10	9
70	18	18	18	18	18	18	18	17	17	12	10	10	9
80	18	18	18	18	18	18	18	17	17	12	10	10	9
90	18	18	18	18	18	18	18	17	17	12	10	10	9
100	18	18	18	18	18	18	18	17	17	12	10	10	9
110	18	18	18	18	18	18	18	17	17	12	10	10	9
120	18	18	18	18	18	18	18	17	17	12	10	10	9

Contained in tables 6 and 7 are 169 (13·13) coal production forecasts for each state. These forecasts are used in the regression analysis to estimate the linear demand functions listed below. Coal demand is the dependant variable and the Montana and Wyoming tax rates are the independent variables. The contract sales price for each state is used for price variables P_{mt} and P_{WY} in the equations shown below.

REGRESSION ESTIMATES

Demand Equations

$$D_{mt} = a - b\cdot(1+t_{mt})\cdot P_{mt} + c\cdot(1+t_{wy})\cdot P_{WY}$$

$$D_{WY} = \alpha - \beta\cdot(1+t_{wy})\cdot P_{WY} + \mu\cdot(1+t_{mt})\cdot P_{mt}$$

<u>OLS Estimates</u>

$$D_{mt} = \underset{\substack{(1.58)\\ SE}}{23.38} - \underset{\substack{(0.04)\\ SE}}{1.55}\cdot(1+t_{mt})\cdot 7.47 + \underset{\substack{(0.06)\\ SE}}{0.46}\cdot(1+t_{wy})\cdot 5.37 \qquad R^2=0.89$$

$$D_{WY} = \underset{\substack{(2.12)\\ SE}}{28.13} - \underset{\substack{(0.08)\\ SE}}{1.61}\cdot(1+t_{wy})\cdot 5.37 + \underset{\substack{(0.06)\\ SE}}{0.08}\cdot(1+t_{mt})\cdot 7.47 \qquad R^2=0.70$$

OPTIMAL TAX RATES

The next step in the analysis is to use the coal demand forecasts and the tax strategies to compute optimal tax rates. Optimal tax rates are calculated using the regression estimates listed in the demand equations above. This is done by substituting the regression estimates into the optimal tax strategies derived in Chapter 4. Optimal tax rates are calculated for equilibrium and tax leadership scenarios.

Equilibrium Conditions

Optimal Tax Strategies

$$t_{mt}^{*} = \frac{2\cdot\beta\cdot(a - b\cdot P_{mt} + c\cdot P_{WY}) + c\cdot(\alpha - \beta\cdot P_{WY} + \mu\cdot P_{mt})}{P_{mt}\cdot(4\cdot b\cdot\beta - c\cdot\mu)}$$

$$t_{wy}^{*} = \frac{2\cdot b\cdot(\alpha - \beta\cdot P_{WY} + \mu\cdot P_{mt}) + \mu\cdot(a - b\cdot P_{mt} + c\cdot P_{WY})}{P_{WY}\cdot(4\cdot b\cdot\beta - \mu\cdot c)}$$

Optimal Tax Rates

$$2\cdot 1.61\cdot(23.38 - 1.55\cdot 7.47 + 0.46\cdot 5.37) + 0.46\cdot(28.13 - 1.61\cdot 5.37 + 0.08\cdot 7.47)$$

$$t_{mt}^* = \frac{}{7.47\cdot(4\cdot1.55\cdot1.61 - 0.46\cdot0.08)}$$

$$t_{mt}^* = .74$$

$$t_{wy}^* = \frac{2\cdot1.55\cdot(28.13 - 1.61\cdot5.37 + 0.08\cdot7.47) + 0.08\cdot(23.38 - 1.55\cdot7.47 + 0.46\cdot5.37)}{5.37\cdot(4\cdot1.55\cdot1.61 - 0.08\cdot0.46)}$$

$$t_{wy}^* = 1.19$$

The optimum tax rates identified above are based on the naive assumption that each producer will not react to a change in the other states severance tax rate. This assumption is now relaxed and optimal tax rates for two price leadership scenarios are presented. First, optimal tax rates are identified where Montana is the price leader. This is followed by an analysis of optimal tax rates where Wyoming is the price leader.

Montana Price Leadership

Optimal Tax Strategies

$$t_{mt}^* = \frac{2\cdot\beta\cdot(a - b\cdot P_{mt} + c\cdot P_{wy}) + c\cdot(\alpha - \beta\cdot P_{wy} + \mu\cdot P_{mt})}{2\cdot P_{mt}\cdot(2\cdot b\cdot\beta - c\cdot\mu)}$$

$$t_{wy}^{*} = \frac{2\cdot(2\cdot b\cdot\beta - c\cdot\mu)\cdot(\alpha - \beta\cdot P_{WY} + \mu\cdot P_{mt}) + \mu\cdot(2\cdot\beta\cdot(a - b\cdot P_{mt} + c\cdot P_{WY}) + c\cdot(\alpha - \beta\cdot P_{WY} + \mu\cdot P_{mt}))}{4\cdot\beta\cdot P_{WY}\cdot(2\cdot b\cdot\beta - c\cdot\mu)}$$

Optimal Tax Rates

$$t_{mt}^{*} = \frac{2\cdot 1.61\cdot(23.38 - 1.55\cdot 7.47 + 0.46\cdot 5.37) + 0.46\cdot(28.13 - 1.61\cdot 5.37 + 0.08\cdot 7.47)}{2\cdot 7.47\cdot(2\cdot 1.55\cdot 1.61 - 0.46\cdot 0.08)}$$

$t_{mt}^{*} = 0.75$

$$t_{wy}^{*} = \frac{2\cdot(2\cdot 1.55\cdot 1.61-0.46\cdot 0.08)\cdot(28.13-1.61\cdot 5.37+0.08\cdot 7.47)+0.08\cdot(2\cdot 1.61\cdot(23.38-1.55\cdot 7.47+0.46\cdot 5.37)+0.46\cdot(28.13-1.61\cdot 5.37+0.08\cdot 7.47))}{4\cdot 1.61\cdot 5.37\cdot(2\cdot 1.55\cdot 1.61 - 0.46\cdot 0.08)}$$

$t_{wy}^{*} = 1.19$

Wyoming Tax Leadership

Optimal Tax Strategies

$$t_{wy}^{*} = \frac{2\cdot b\cdot(\alpha - \beta\cdot P_{WY} + \mu\cdot P_{mt}) + \mu\cdot(a - b\cdot P_{mt} + c\cdot P_{WY})}{2\cdot P_{WY}\cdot(2\cdot\beta\cdot b - c\cdot\mu)}$$

$$t_{mt}^{*} = \frac{2\cdot(2\cdot\beta\cdot b - c\cdot\mu)\cdot(a - b\cdot P_{mt} + c\cdot P_{WY}) + c\cdot(2\cdot b\cdot(\alpha - \beta\cdot P_{WY} + \mu\cdot P_{mt}) + \mu\cdot(a - b\cdot P_{mt} + c\cdot P_{WY}))}{4\cdot b\cdot P_{mt}\cdot(2\cdot\beta\cdot b - c\cdot\mu)}$$

Optimal Tax Rates

$$t_{wy}^* = \frac{2\cdot 1.55\cdot(28.13 - 1.61\cdot 5.37 + 0.08\cdot 7.47) + 0.08\cdot(23.38 - 1.55\cdot 7.47 + 0.46\cdot 5.37)}{2\cdot 5.37\cdot(2\cdot 1.61\cdot 1.55 - 0.46\cdot 0.08)}$$

$$t_{wy}^* = 1.19$$

$$t_{mt}^* = \frac{2\cdot(2\cdot 1.61\cdot 1.55+0.46\cdot 0.08)\cdot(23.38-1.55\cdot 7.47+0.46\cdot 5.37)+0.46\cdot(2\cdot 1.55\cdot(28.13-1.61\cdot 5.37+0.08\cdot 7.47)+0.08\cdot(23.38-1.55\cdot 7.47+0.46\cdot 5.37))}{4\cdot 1.55\cdot 7.47\cdot(2\cdot 1.61\cdot 1.55-0.46\cdot 0.08)}$$

$$t_{mt}^* = 0.74$$

Table 8 summarizes the optimal tax rates calculated above.

Table 8. Optimal Tax Rates.

	Montana	Wyoming
Equilibrium	74 %	119 %
Montana Price Leadership	75 %	119 %
Wyoming Price Leadership	74 %	119 %

Matrix Analysis

A simple matrix analysis is presented as a cross-check to the regression analysis. The coal production forecasts presented in tables 6 and 7 can easily be converted to state tax revenue forecasts by multiplying the production level by the corresponding price and severance tax rate. Table 9 presents the corresponding prices and tax rates for both states. Tables 10 and 11 present the resulting tax revenue forecasts. Optimal severance tax rates can be identified by inspection, as discussed below.

Table 9. Price and Tax Rates.

Tax Rate	Montana FOB	Wyoming FOB
0	7.47	5.37
10	8.22	5.91
20	8.96	6.44
30	9.71	6.98
40	10.46	7.52
50	11.21	8.06
60	11.95	8.59
70	12.70	9.13
80	13.45	9.67
90	14.19	10.20
100	14.94	10.74
110	15.96	11.27
120	16.43	11.81

Table 10. Montana Tax Revenue Forecasts. (New Coal Production)

	WY Tax (%)	0	10	20	30	40	50	60	70	80	90	100	110	120
							(million $)							
MT	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Tax	10	9.00	9.00	9.00	9.75	9.75	9.75	9.75	9.75	10.50	13.50	13.50	13.50	13.50
(%)	20	16.39	17.88	17.88	17.88	19.37	19.37	19.37	19.37	19.37	20.86	26.82	26.82	26.82
	30	24.64	24.64	24.64	24.64	26.88	29.12	26.88	26.88	26.88	26.88	31.36	31.36	31.36
	40	29.90	32.89	32.89	32.89	32.89	35.88	32.89	35.88	35.88	35.88	35.88	35.88	35.88
	50	29.92	33.66	33.66	33.66	33.66	41.14	33.66	37.40	37.40	37.40	37.40	37.40	37.40
	60	13.44	40.32	40.32	40.32	40.32	40.32	40.32	40.32	44.80	44.80	44.80	44.80	44.80
	70	15.69	47.07	47.07	47.07	47.07	47.07	47.07	47.07	47.07	52.30	52.30	52.30	52.30
	80	11.96	53.82	53.82	53.82	53.82	53.82	53.82	53.82	53.82	53.82	53.82	53.82	53.82
	90	13.44	40.32	40.32	40.32	40.32	60.48	40.32	40.32	40.32	40.32	40.32	40.32	40.32
	100	14.94	14.94	14.94	14.94	14.94	44.82	14.94	14.94	14.94	14.94	14.94	14.94	14.94
	110	16.98	16.98	16.98	16.98	16.98	16.98	16.98	16.98	16.98	16.98	16.98	16.98	16.98
	120	17.92	17.92	17.92	17.92	17.92	17.92	17.92	17.92	17.92	17.92	17.92	17.92	17.92

Table 11. Wyoming Tax Revenue Forecasts. (New Coal Production)

	WY Tax (%)	0	10	20	30	40	50	60	70	80	90	100	110	120
							(million $)							
MT	0	0.00	9.72	19.26	28.98	38.70	48.42	57.96	60.16	68.80	38.64	26.85	59.00	57.96
Tax	10	0.00	9.72	19.26	28.98	38.70	48.42	57.96	63.92	73.10	57.96	48.33	29.50	25.76
(%)	20	0.00	9.72	19.26	28.98	38.70	48.42	57.96	63.92	73.10	57.96	53.70	53.10	32.20
	30	0.00	9.72	19.26	28.98	38.70	48.42	57.96	63.92	73.10	57.96	53.70	59.00	51.52
	40	0.00	9.72	19.26	28.98	38.70	48.42	57.96	63.92	73.10	57.96	53.70	59.00	57.96
	50	0.00	9.72	19.26	28.98	38.70	48.42	57.96	63.92	73.10	57.96	53.70	59.00	57.96
	60	0.00	9.72	19.26	28.98	38.70	48.42	57.96	63.92	73.10	57.96	53.70	59.00	57.96
	70	0.00	9.72	19.26	28.98	38.70	48.42	57.96	63.92	73.10	57.96	53.70	59.00	57.96
	80	0.00	9.72	19.26	28.98	38.70	48.42	57.96	63.92	73.10	57.96	53.70	59.00	57.96
	90	0.00	9.72	19.26	28.98	38.70	48.42	57.96	63.92	73.10	57.96	53.70	59.00	57.96
	100	0.00	9.72	19.26	28.98	38.70	48.42	57.96	63.92	73.10	57.96	53.70	59.00	57.96
	110	0.00	9.72	19.26	28.98	38.70	48.42	57.96	63.92	73.10	57.96	53.70	59.00	57.96
	120	0.00	9.72	19.26	28.98	38.70	48.42	57.96	63.92	73.10	57.96	53.70	59.00	57.96

In Table 10, Montana tax revenues are maximized at a rate of 80 percent for all columns except where the Wyoming tax rate is zero. In Table 11, Wyoming tax revenues are maximized at a rate of 80 percent in every column. This would suggest that Wyoming, if it were trying to maximize coal severance tax revenues, would set the tax rate at 80 percent. Montana in response would also set the tax rate at 80 percent.

The optimal Montana severance tax rate shown in Table 10 compares favorably with the duopoly model results discussed above. However, the 80 percent rate for Wyoming differs by 40 percentage points when compared to the duopoly model results. This suggests that the regression analysis may be over estimating the optimal severance tax rate.

The matrix analysis provides a rather simple solution in this case. However, this is not always the case. If the combinations were arranged in a more complex fashion, the solution could not be identified by inspection but would require the development of optimal solutions using matrix analysis techniques. Further, the imposition of the duopoly model provides a method to analyze both the equilibrium and leadership scenarios.

CHAPTER 5

SUMMARY AND CONCLUSIONS

Thus far, this thesis has provided a literature review, developed a theoretical model to analyze Montana and Wyoming coal tax policies, and provided estimates of optimal severance tax rates. The analysis now becomes one of interpretation. Built into the Montana-Wyoming coal severance tax duopoly model and the spatial market model are a number of assumptions. These assumptions simplify the issues surrounding Montana and Wyoming severance tax policies and focuses the discussion on how changes in the severance tax rate impact coal tax revenues. In reality, the issues are not simple.

The impact of the major assumptions on the results will be discussed in this chapter. Further, the results of Kolstad and Wolak (1983), Ward (1982), and Zimmerman and Alt (1981) are compared to the optimal rates derived in this thesis. At the end of this chapter, a discussion of the major conclusions that can be reached from this analysis is presented.

REVIEW OF ASSUMPTIONS

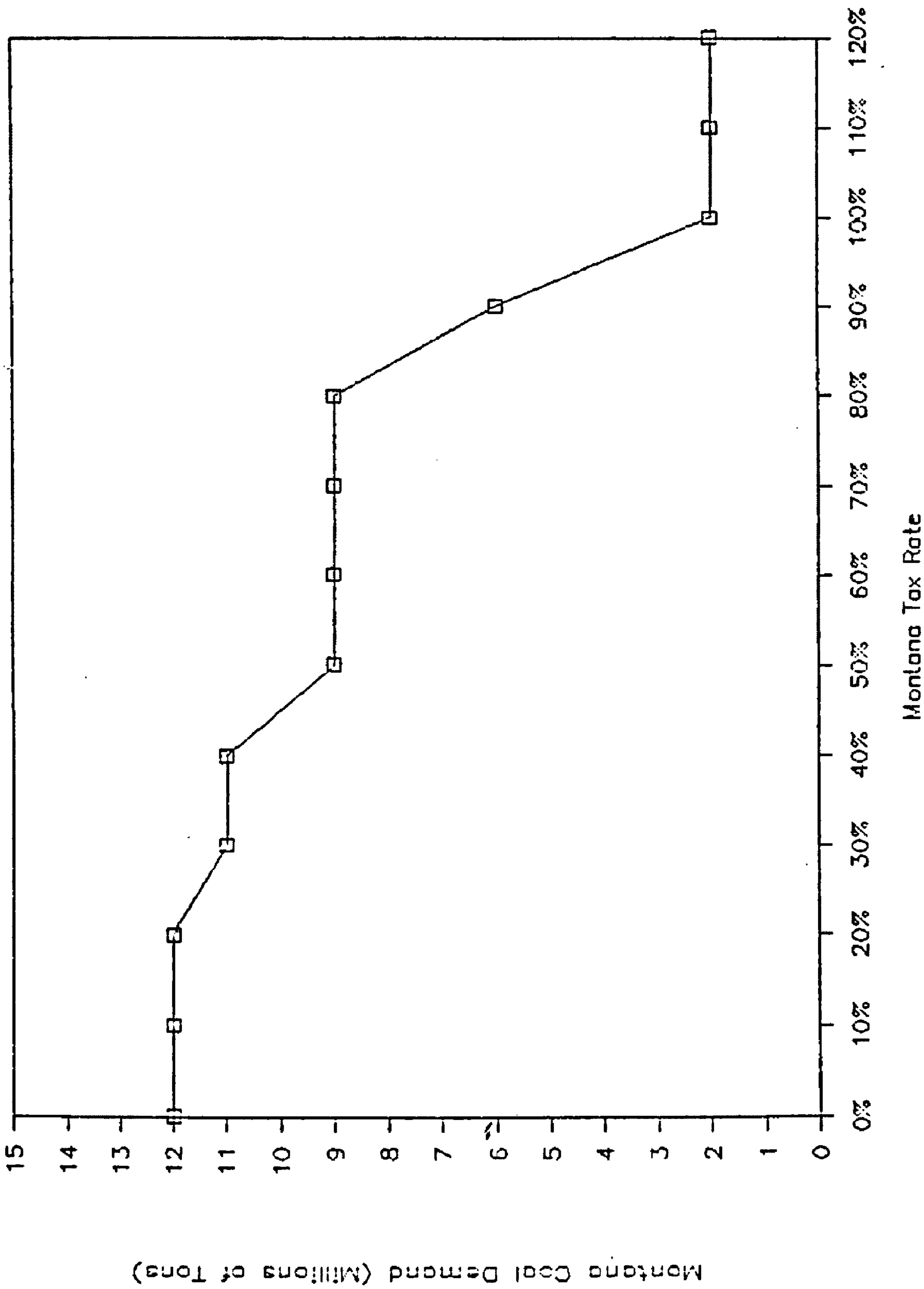
The assumption that each state wants to maximize severance tax revenues underlies the entire analysis.

Clearly, this is a considerable simplification of reality. The fact that both Montana and Wyoming have recently reduced severance tax rates suggests that other considerations are more important to policy makers. On the other hand, the analysis presented in this thesis is still important. Often the discussion of setting severance tax rates is clouded by contradicting reports from proponents and opponents of higher severance tax rates. An objective economic study should provide a basis upon which policy makers can gauge the impact of other considerations such as overall tax revenues, employment, the health of the coal industry, and energy policy, among others.

Another important point that must be recognized is that the analysis is based on the change in demand for "new" production not total production. As discussed in the previous chapter, existing coal contracts are not analyzed because of insufficient data on current contracts. To the extent that coal production under existing contracts changes because of changes in the severance tax rate, optimal rates may change. However, the consistency of the optimal rates calculated in this thesis, as compared to other analyses of optimal coal tax rates, suggests that using the growth in Montana and Wyoming coal demand does provide accurate results (see "COMPARISON WITH OTHER STUDIES", below).

The use of linear demand curves also influences the results of the model. Because of the concentration of coal fired generation plants in given geographical areas, such as the Minneapolis area, the demand function may actually have breaks where, if the tax rate exceeds a certain rate, demand falls dramatically. Figure 6 shows a graph of one column of data presented in Table 6. This column of data reflects the change in demand if Wyoming were to hold their severance tax rate at 10 percent. Notice the breaks between 40 and 50 percent, 80 and 90 percent, and 90 and 100 percent. Once the rate exceeds 90 percent, the only remaining increase in demand for Montana coal is by the coal generation facilities located in Montana.

Figure 6. Montana Coal Demand When Wyoming Severance Tax is 10 percent.



Severance tax revenues are not the only tax revenues the state receives from the coal industry. Both Montana and Wyoming have income taxes and coal company employees pay a portion of these taxes. Raising the severance tax rate will increase severance tax revenues up to the optimum. However, if coal workers are laid off because of declining coal production, income tax revenues will fall. If the goal is to maximize overall state tax revenues, the optimum severance tax rate will probably be smaller due to declining income tax collections at high severance tax rates. However, the importance of this factor should be relatively small given the few number of coal workers that are employed in the industry. Zimmerman and Alt (1981, 17) did analyze wage income and the associated taxes and found that the optimal rate of 62.5 percent for each state was not sensitive to wage income.

Another simplifying assumption is that all other coal producing states and the railroad companies would not react to changes in Montana and Wyoming coal severance tax policies. To the extent that other coal producing states would react by changing their own tax rates, optimal Montana and Wyoming severance tax rates would be reduced.

The railroads may have the strongest market power of all of the potential market participants. In Montana there is only one carrier, BN. In Wyoming, there are two. These railroad companies are regulated and, to the extent that this is effective in controlling cartel actions, regulations would limit the exercise of market power. However, Zimmerman and Alt (1981, 20) did analyze the economic rents that these railroads extract and state that "when considered in terms of what the railroads have been doing, a 62.5% (severance) tax in not high."

It is also implicitly assumed that the total increase in severance taxes is passed on to the electric companies. This would not be the case. With constricting markets, some portion of the tax increase would be taken up by the producers and the rail companies in order to maintain market share. These actions would tend to push the optimal rate higher because the consumers of Montana coal would not feel the full impact of tax increase.

The marginal cost of producing coal is assumed to be constant. Because of economies of scale, reduced production would tend to drive up variable costs. This would mean that production may fall off faster than otherwise. On the other hand, strip mine operations are very capital intensive requiring large and expensive equipment. These costs are largely fixed costs and would tend to outweigh the effect of rising variable costs.

The above discussion is presented in order to provide some perspective concerning the accuracy of the optimal tax estimates. A comparison of the results Montana coal duopoly model and the three other reports that are the subject of Chapter 2 is presented below. At the end of this chapter, the general conclusions that can be reached are stated.

COMPARISON WITH OTHER STUDIES

Table 12 shows the optimal tax rates developed in this chapter and the optimal rates calculated by Kolstad and Wolak (1983), Ward (1982), and Zimmerman and Alt (1981). Because the analysis presented in this thesis most closely followed the Kolstad and Wolak methodology there are comparable rates for all four categories listed. The fourth category, Simple Cartel Rate, needs some explanation. The Chapter 4 result listed for the Simple Cartel Rate category is taken from the matrix analysis presented at the end of that chapter. This number was chosen because it is more comparable to the methods used to identify optimal rates in Ward (1981) and Zimmerman and Alt (1981). As stated in Chapter 2, Ward (1981) and Zimmerman and Alt (1981) did not develop a duopoly model to identify optimal rates. These reports approached the problem by estimating potential tax revenues under a number of different severance tax scenarios and identifying the scenario returning the highest revenue. The simple matrix analysis presented at the end of Chapter 4 takes this same approach.

Kolstad and Wolak (1983) do not present an optimal tax rate calculated in this manner. Instead, they present a "collusion" scenario where the optimal tax rate maximizes the combined severance tax revenues of Montana and Wyoming. This is the number shown in the Simple Cartel Rate category for Kolstad and Wolak (1983).

Table 12. Comparison of Optimal Severance Tax Rates.

Duopoly Assumption	Chapter 4 Results	Kolstad and Wolak (1983)	Ward (1982)	Zimmerman and Alt (1981)
Equilibrium				
Montana	74 %	27	--	--
Wyoming	119	33	--	--
Montana Leadership				
Montana	75	35	--	--
Wyoming	119	35	--	--
Wyoming Leadership				
Montana	74	35	--	--
Wyoming	119	35	--	--
Simple Cartel Rate				
Both States	80	87	70	62.5

CONCLUSIONS

The optimal coal rates identified in this thesis and the other studies suggest that either state would maximize revenues at much higher rates than are currently charged. The rates that are calculated for Montana and Wyoming are stable and do not vary significantly with any of the duopoly model assumptions. The simple matrix approach does differ from the results using the more sophisticated duopoly model approach. This may suggest that for the Wyoming optimal tax rate the duopoly model is overestimating the rate. This may reflect the use of a linear demand curve.

Even if rates as high as 74 percent in Montana and 119 percent in Wyoming are never adopted, it is clear that increasing severance tax rates from current levels will result in increasing severance tax revenues and cutting the severance tax rates in Montana and Wyoming will lead to tax revenue reductions. This can be seen more clearly by inspecting figures 7 and 8.

Figures 7 and 8 show Montana and Wyoming tax revenues for a range of severance tax rates. These figures are simply a graphical representation of the tax revenue functions and empirical data presented in Chapter 4. Moving left to right along the x-axis of Figure 7, the Montana severance tax rate increases from 0 to 150 percent. The family of curves represent Montana severance tax revenues for six different Wyoming severance tax rates (0, 25, 50, 75, 100, and 125 percent). Moving left to right along the x-axis of Figure 8, the Wyoming severance tax rate increases from 0 to 200 percent. The family of curves represent Wyoming severance tax revenues for six different Montana severance tax rates (0, 25, 50, 75, 100, and 125 percent).

Figure 7. Montana Tax Revenues.

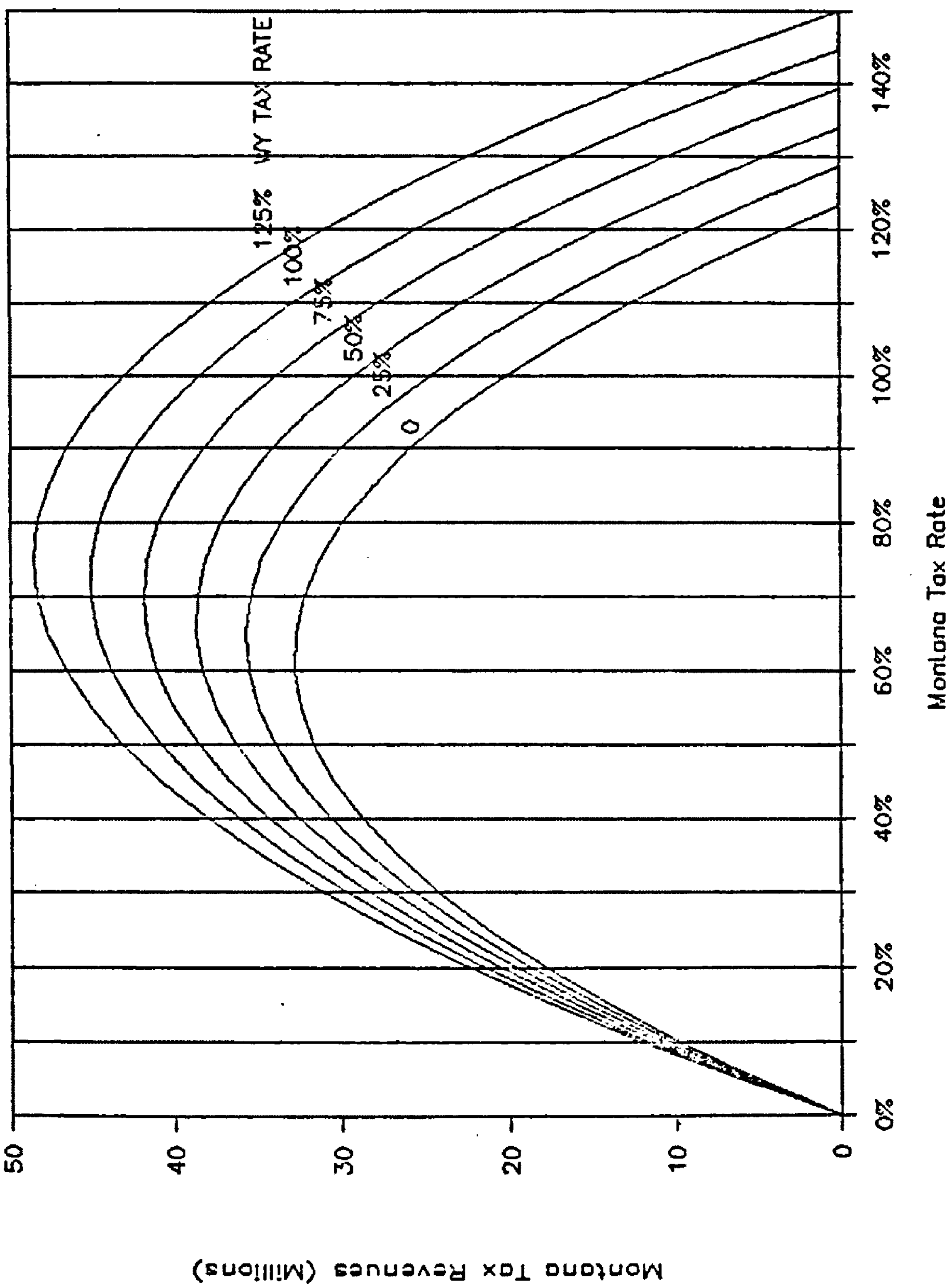
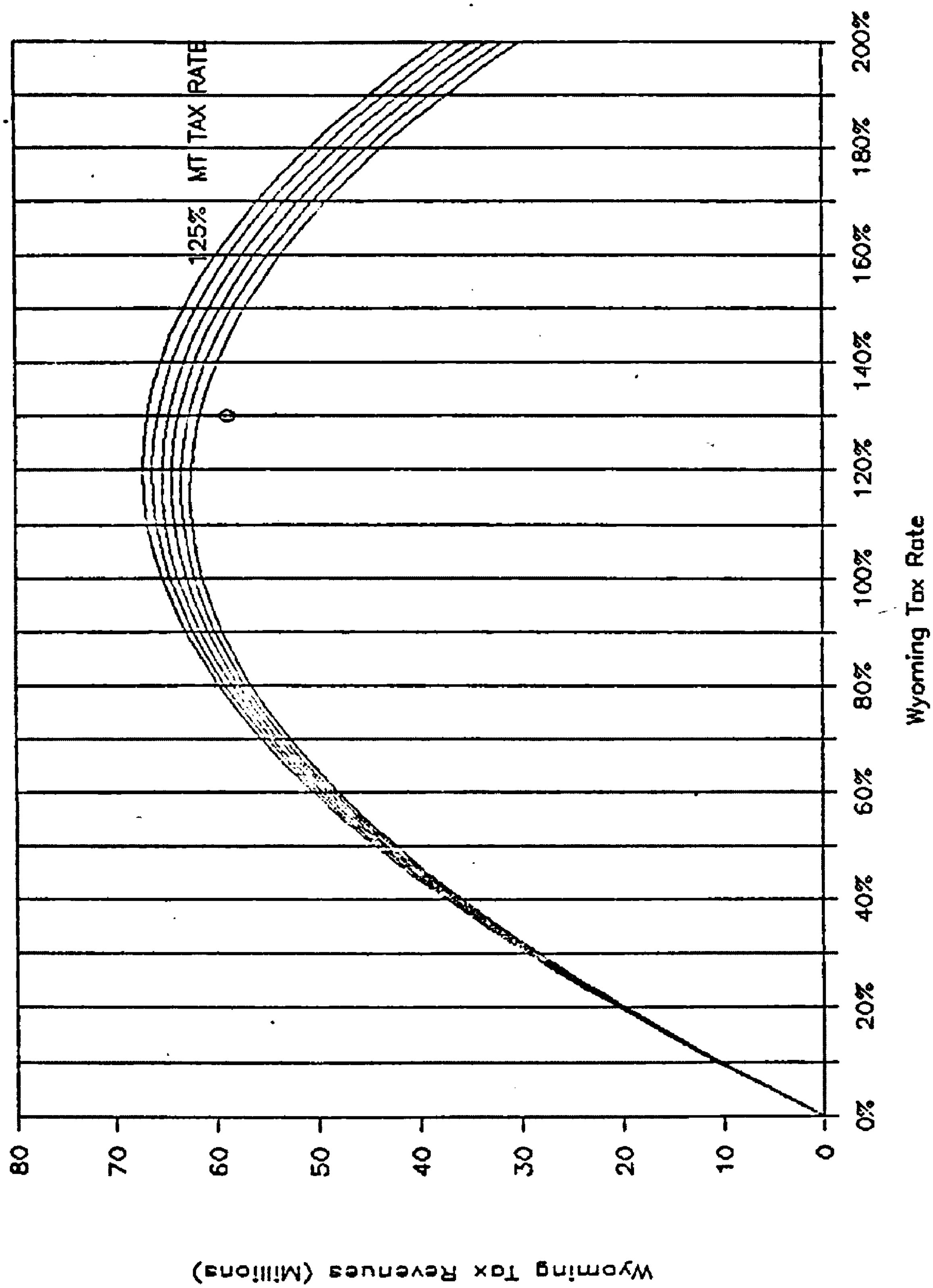


Figure 8. Wyoming Tax Revenues.



If coal production and the health of the coal industry is considered, then the relatively high optimal severance tax rates would be of concern. At the optimal rates, Montana would lose 3 to 6 tons of forecasted new production and Wyoming would lose 9 tons, compared to forecasted production levels at current severance tax rates. These are significant declines in the amount of new coal demand that producers in Montana and Wyoming would lose.

Another consideration that must be taken into account is that as the severance tax rises and demand for NGP coal is reduced, a larger and larger share of the severance tax is paid by electric consumers in Montana and Wyoming. This is another factor that would that would put downward pressure on optimal severance tax rates. However, Zimmerman and Alt (1981) found that even at high rates, out of state energy consumers still pay most of the tax. This is because a large portion of the electricity produced by coal fired generation in Montana and Wyoming is exported to other western states.

APPENDIX A

SPATIAL MARKET MODEL

The demand for Montana and Wyoming coal is a derived demand generated by the demand for electricity. Approximately ninety-five percent of the coal sold from Montana and Wyoming is used to generate electricity (DOE 1988a, 20). In the energy market, Montana and Wyoming Powder River coal producers compete against the coal producing regions of Colorado, Illinois, New Mexico, Washington, Texas, Utah, and South West Wyoming (Duffield and others 1982). Coal also competes with alternative fuels and power sources including nuclear, hydroelectric, oil, and natural gas.

Coal demand forecasts are made by forecasting future electric demand, calculating the portion of this demand supplied by coal, and estimating the share of this market supplied by Montana and Wyoming. The electric demand forecasting model is used to predict 1990 energy demand for each state in the potential NGP market area. A constant, annual electric growth rate of two percent is assumed.

An inter-fuel substitution algorithm is used to identify the proportion of this forecasted electric demand which will be supplied by coal. The approach is relatively simple. Forecasts of nuclear, hydroelectric, gas, and oil generated electricity are subtracted from the forecasted demand for electricity. Then the amount of electricity currently generated by coal-fired plants is subtracted. The residual represents the portion of electric demand forecasted to be supplied by new coal-fired generation. This forecast of the demand for new coal-fired generation (GWh) for each state is then converted to a demand for coal, in tons.

Nine competing coal supply regions are identified in the spatial market model : Illinois, New Mexico, southwest Wyoming, Colorado, Texas, Utah, Washington, Montana, and Eastern Wyoming. Geographical market boundaries between competing coal supply regions are calculated. When all nine supply centers are used, fully bounded spatial markets can be identified for both Montana and Wyoming. Within these market areas, coal from the producing state is the least cost source fuel for coal-fired generation plants. The end result is a forecast of new coal demand for Montana and Wyoming Powder River coal. (Appendix B contains a description of all of the computer programs needed to make the coal demand forecasts. Appendix C provides a line listing of the Fortran program "NEWHYP.FOR". This program calculates the spacial market boundaries.)

ELECTRIC DEMAND FORECASTS

A forecast of 1990 electric demand is made for each state in the potential NGP coal market, based on a two percent constant annual electric growth rate (see Table 15). The two percent growth rate corresponds to the "median" forecast used in Montana Coal Market to the Year 2000: Impact of Severance Tax, Air Pollution Control and Reclamation Costs (Duffield and others 1985). As it turns out, a two percent electric growth rate is also supported by current trends. Energy demand increased by an average annual rate of 2.02 percent between 1983 and 1987. Tables 13 and 14 present actual electric generation for these two years.

Table 13. Net Electric Energy Generation in 1983, by Source.[a]

State	All Fuel	Coal	Nuclear	Hydro	Petroleum (Fired Steam)	Gas (Fired Steam)
	(Gigawatt-hours)					
Iowa	22,253	18,734	2,309	918	47	161
Kansas	23,773	19,592	0	6	198	3,410
Minnesota	29,971	17,054	11,753	929	24	180
Montana	15,097	3,452	0	11,561	10	32
Nebraska	17,052	9,471	6,082	1,346	33	77
North Dakota	19,601	17,182	0	2,377	42	0
Oklahoma	45,711	19,575	0	2,500	13	22,616
South Dakota	7,779	2,274	0	5,494	5	1
Wisconsin	39,348	27393	9299	2298	93	133
Arkansas	30,073	16,042	7,646	3,315	46	3,016
Colorado	25,225	22,243	748	1,870	54	291
Idaho	12,772	0	0	12,771	0	0
Illinois	99,160	66,908	28,021	117	3,095	736
Michigan	70,911	52,016	16,383	1,113	639	618
Missouri	52,705	50,596	0	1,716	118	138
Wyoming	26,258	25,054	0	1,150	41	12
Louisiana	37,042	8,378	0	0	352	28,295
Oregon	49,180	443	3,685	45,049	8	0
Texas	206,214	87,488	0	1,107	1,964	111,905
Washington	95,099	6,111	3,494	85,435	9	0
Indiana	70,863	70,004	0	418	177	224
	996,087	540,010	89,420	181,490	6,968	171,845

[a] Source: DOE 1984b

Table 14. Net Electric Energy Generation in 1987, by Source. [a]

State	All Fuel	Coal	Nuclear	Hydro	Petroleum (Fired Steam)	Gas (Fired Steam)
	(Gigawatt-hours)					
Iowa	25,549	21,704	2,523	970	39	194
Kansas	30,822	23,075	6,471		48	948
Minnesota	34,406	21,627	11,554	9	26	410
Montana	20,884	11,836	0	720	16	57
Nebraska	20,489	10,153	8,489	8,925	43	117
North Dakota	22,620	20,618	0	1,982	19	0
Oklahoma	42,737	21,089	0	2,948	25	16,199
South Dakota	6,265	905	0	5,354	2	2
Wisconsin	44,030	31032	11311	1319	58	110
Arkansas	36,287	19,373	11,369	2,407	5	3,131
Colorado	28,810	26,192	174	1,818	16	599
Idaho	8,105	0	0	8,105	0	0
Illinois	109,887	57,933	50,194	90	1,445	144
Michigan	87,035	71,135	14,389	364	694	360
Missouri	55,087	47,170	6,284	1,447	71	51
Wyoming	37,370	36,535	0	768	59	9
Louisiana	51,309	15,102	12,324	0	60	23,820
Oregon	39,739	0	4,348	35,431	0	0
Texas	212,620	106,405	0	2,158	540	100,665
Washington	83,756	8,166	5,528	69,698	5	0
Indiana	78,575	77,620	0	507		92
	1,076,382	627,670	144,958	145,020	3,171	146,908

[a] Source: DOE 1988a.

Table 15. Net Electric Demand in 1983, 1987, and 1990.

State	Net Electric Generation (Gigawatt-hours)		
	1983[a]	1987[b]	1990[c]
Iowa	22,253	25,549	25,562
Kansas	23,773	30,822	27,308
Minnesota	29,971	34,406	34,427
Montana	15,097	20,884	17,342
Nebraska	17,052	20,489	19,587
North Dakota	19,601	22,620	22,515
Oklahoma	45,711	42,737	52,508
South Dakota	7,779	6,265	8,936
Wisconsin	39,348	44,030	45,198
Arkansas	30,073	36,287	34,544
Colorado	25,225	28,810	28,976
Idaho	12,772	8,105	14,671
Illinois	99,160	109,887	113,904
Michigan	70,911	87,035	81,454
Missouri	52,705	55,087	60,541
Wyoming	26,258	37,370	30,162
Louisiana	37,042	51,309	42,550
Oregon	49,180	39,739	56,492
Texas	206,214	212,620	236,875
Washington	95,099	83,756	109,239
Indiana	70,863	78,575	81,399
	996,087	1,076,382	1,144,191

[a] DOE 1984b, 29.
[b] DOE 1988a, 20.
[c] Forecasted.

Sensitivity to Electric Growth Rates

There is no sensitivity analysis of electric rates conducted for this study. However, Duffield and Others (1982 and 1985) did conduct sensitivity analyses. The results of the 1985 analysis are presented below.

In the 1985 study, Duffield and others forecasted coal demand using 3 different electric growth rates: 1, 2, and 3 percent. Table 16 shows forecasted Montana coal demand (in millions of tons) for five Montana FOB prices and two Wyoming FOB prices, and for forecast years 1990, 1995, and 2000. Table 16 also shows the percent change in coal demand associated with each electric growth rate and price scenario.

For example, the percentage change in demand when the electric growth rate increases form 1 to 2 percent is 5.49 percent, for the 1990 forecast and a Montana FOB of $10.50, and a Wyoming FOB of $6.00. The largest change, 76.77 percent, occurred between the 1 percent and 2 percent electric growth rates, for a Montana FOB of $7.50 and a Wyoming FOB of $7.70 in the year 2000. This reflects both greater sensitivity when Montana and Wyoming

FOB prices are similar and when the forecast period is relatively long.

Table 16. Montana Coal Demand for Three Electric Rates[a,b].

Year	1990			1995			2000		
Electric Growth Rate:	1%	2%	3%	1%	2%	3%	1%	2%	3%
A. Wyoming FOB Price $6.00/ton									
Montana FOB									
$10.50/ton	36.4	38.4	41.3	38.0	44.2	53.0	44.1	51.0	59.9
%change		5.49%	7.55%		16.32%	19.91%		15.65%	17.45%
$9.50/ton	38.1	41.5	42.9	41.9	46.2	64.6	48.3	62.9	85.2
%change		8.92%	3.37%		10.26%	39.83%		30.23%	35.45%
$8.50/ton	38.9	42.5	44.2	42.8	47.2	72.2	48.3	68.0	96.8
%change		9.25%	4.00%		10.28%	52.97%		40.79%	42.35%
$7.50/ton	42.7	44.7	44.7	45.4	47.2	72.2	48.3	68.0	100.1
%change		4.68%	0.00%		3.96%	52.97%		40.79%	47.21%
B. Wyoming FOB Price $7.70									
Montana FOB									
$10.50/ton	37.9	40.8	42.0	41.3	45.3	64.6	48.3	62.9	81.2
%change		7.65%	2.94%		9.69%	42.60%		30.23%	29.09%
$9.50/ton	38.6	41.8	43.3	42.5	46.3	64.6	48.3	62.9	85.4
%change		8.29%	3.59%		8.94%	39.52%		30.23%	35.77%
$8.50/ton	38.9	42.5	44.2	42.8	47.2	73.0	48.4	68.7	98.4
%change		9.25%	4.00%		10.28%	54.66%		41.94%	43.23%
$7.50/ton	42.7	45.8	47.4	46.6	54.4	94.7	50.8	89.8	140.8
%change		7.26%	3.49%		16.74%	74.08%		76.77%	56.79%

[a] All demand forecasts are in millions of tons.
[b] Source: Duffield and others (1985, III-32).

INTER-FUEL SUBSTITUTION

The inter-fuel substitution algorithm takes the forecasted electric demand and allocates it to competing energy sources. Nuclear, hydropower, oil, gas and existing coal fired electric generation, for 1983, are shown in Table 13 above. Forecast of nuclear, hydropower, oil and gas in the year 1990 are based on information provided in the 1982 NGP coal demand study (Duffield and others 1982), as updated using information from DOE (1981b) and U.S. Nuclear Regulatory Commission (1983).

Using the 1990 electric demand forecasts for each state in the NGP market area, forecasted generation for nuclear, hydropower, oil, gas, and existing coal generation is subtracted. The residual represents the demand for new coal-fired generation in the year 1990. The forecasted new coal-fired generation demand, in kilowatt-hours, is converted to demand for coal in tons using a simple conversion (see Appendix B).

SPATIAL MARKET MODEL

The core of the spatial market model is described in Chapter 3. The description presented in this appendix goes further into the detail of the model description. The variables listed in Table 17 below provide a complete description of the costs of operating a coal fired generator. The variables and the values used in the analysis are the subject of the remaining portion of this appendix.

Table 17. Variable Descriptions[a].

Line #	Coal Supply Center[b]	Description
1	A & B	Power Plant Size (net MW)
2	A & B	Hours Operated at Full Load (hours)
3	A	Power Plant Heat Rate (Btu/KWH)
4	A	Coal Heat Content (Btu/lb)
5	B	Power Plant Heat Rate (Btu/KWH)
6	B	Coal Heat Content (Btu/lb)
7	A	Power Plant Capital Cost ($/KW)
8	B	Power Plant Capital Cost ($/KW)
9	A & B	Fixed Charge Rate (decimal)
10	A	Operating and Maintenance Costs ($/KWH)
11	B	Operating and Maintenance Costs ($/KWH)
12	A	FOB Mine Price ($/ton)
13	B	FOB Mine Price ($/ton)
14	A	Fixed Transportation Cost ($/ton)
15	B	Fixed Transportation Cost ($/ton)
16	A	Variable Transportation Costs ($/ton-air mile)
17	B	Variable Transportation Costs ($/ton-air mile)
18	A & B	Straight Line Distance Between A & B (miles)

[a] Source: Duffield and others (1985, III-18).
[b] A = Either the Montana or Wyoming Supply Center; B = The market competitor of A; one of seven possible coal supply centers.

Table 18. Spatial Market Model: Coal Supply Center Data.

	Colorado	Illinois	New Mexico	Texas	Utah	Washington	SW Wyoming	BJ Wyoming
1	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0
2	5694.0	5694.0	5694.0	5694.0	5694.0	5694.0	5694.0	5694.0
3	10486.0	10058.0	10564.0	10251.0	10486.0	10486.0	10486.0	10486.0
4	8700.0	8700.0	8700.0	8700.0	8700.0	8700.0	8700.0	8700.0
5	10341.0	10204.0	10564.0	11045.0	10197.0	10486.0	10341.0	10486.0
6	10700.0	10500.0	10000.0	6300.0	11500.0	8100.0	10500.0	8450.0
7	1329.7	1167.3	1257.5	1049.3	1329.7	1329.7	1329.7	1329.7
8	1307.1	1206.8	1257.5	1357.0	1227.6	1333.5	1307.1	1329.7
9	0.07410	0.07410	0.07410	0.07410	0.07410	0.07410	0.07410	0.07410
10	0.00671	0.00633	0.00665	0.00663	0.00671	0.00671	0.00671	0.00671
11	0.00665	0.00822	0.00665	0.00699	0.00660	0.00775	0.00665	0.00671
12	9.5	9.5	9.5	9.5	9.5	9.5	9.5	9.5
13	21.25	25.52	25.0	11.92	26.3	27.22	16.5	6.0
14	2.02	2.02	2.02	2.02	2.02	2.18	2.02	2.02
15	2.18	5.3	2.02	2.02	2.18	2.02	2.02	2.02
16	0.0246	0.0256	0.0265	0.0265	0.0246	0.0265	0.0246	0.0246
17	0.0265	0.0256	0.0265	0.0265	0.0265	0.0246	0.0246	0.0246
18	MT 395.7	1061.5	663.6	1106.6	521.6	779.1	321.6	252.8
	WY 128.6	852.5	384.0	792.0	341.0	854.5	181.5	252.8

Power Plant Size (net MW)

The model plant used for this study is 500 MW in size. This was based on the information presented in Projections of Coal Demand from the Northern Great Plains Through the year 2010 (Duffield and others 1982). Of 443 coal-fired utility boilers ordered between 1958 and 1980, the average net capacity of was 482 MW per boiler. Furthermore, of the 144 boiler orders expected in 1982, the average capacity was 511 MW per boiler (Duffield and others 1982).

Capacity Factor

Large coal-fired power plants typically provide base load generation. Base load capacity ranges form 50 to 70 percent (EPRI 1979). A 65 percent load is used in this study. This was also the rate used in the 1982 study.

Heat Rate

The heat rate represents the amount of heat, Btu, required to generate one kilowatt-hour. The heat rates for using coal depend on the quality of the coal used. For example, bituminous coal in the west has an average heat rate of 9,772. This compares to an average heat rate of 10,049 for subbituminous coal in the same region. The heat rates used in this study are based on those presented in Projections of Coal Demand from the Northern Great Plains through the Year 2010 (Duffield and others 1982) and a personal interview with staff members of ICF Inc. (1984).

Power Plant Capital Costs (PPCC)

The PPCC is based on costs developed in Duffield and others (1982). The PPCC without sulfur control costs are escalated to 1984 dollars at a real rate of 2.3 percent. The sulfur control costs are escalated to 1984 dollars at a real rate of 0.5 percent. After multiplying the PPCC with out sulfur costs by a capacity penalty, the sulfur costs are added in order to get total PPCC. These costs are levelized within the model when multiplied by the fixed capital recovery factor. In addition to accounting for real changes, all inputs are converted to mid-1984 dollars using the implicit price deflator for GNP.

Fixed Capital Recovery Factor

The fixed capital recovery factor accounts for the costs incurred when a coal fired generator is built. These costs include: interest, depreciation, taxes (income and property), and insurance. A rate of 7.41 percent is used. This rate is based on the analysis presented in the 1982 study. Table 19 provides a listing of the various rates included in the recovery factor.

Table 19. Real Levelized Annual Fixed Charge Rate[a].

Component	Percent
Weighted cost of capital	3.77
Depreciation (sinking fund)	1.85
Insurance and property taxes	2.00
Levelized income tax	1.71
Levelized investment tax credit	(1.03)
Levelized accelerated depreciation	(0.89)
Total	7.41

[a] Source: Duffield and others (1982).

Operation and Maintenance Costs

Operation and maintenance (O&M) costs were again based on the 1982 coal study. Only variable costs are included in this category. The O&M costs were escalated to 1984 dollars based on a real rate of 1.25 percent. This factor does reflect increases in cost associated with using coal containing higher concentrations of SO_2. No distinction was made between Powder River coal produced in Montana compared to Wyoming.

FOB Mine Price

The FOB mine price is the pivotal variable of the analysis. Increasing the severance tax rate increases the price of coal sold by either Montana or Wyoming. All other things being equal, the effect of the severance tax rate on the demand for Montana and Wyoming coal can then be measured. Contract sale prices are established for both Montana and Wyoming. The rates are $7.47 per ton for Montana coal and $5.37 per ton for Wyoming and represent the FOB price of coal if severance tax rates are set to zero. The FOB prices reported in Duffield and others (1982) were used for all of the other supply centers. These prices were escalated to 1984 dollars based on the real escalation rate of 1.25 percent.

By 1984, FOB mine prices had become confidential. Therefore, a number of alternative sources were investigated to develop an estimate of Montana and Wyoming FOB prices. Montana gross proceeds tax forms for 1983 were used by weighting these proceeds by coal production. Proceeds coming from the Spring Creek and Decker mines were separated because of the relatively high Btu content of this coal. Proceeds collected from sales of coal owned by Westmoreland, Western Energy, and Peabody coal were averaged together. The subbituminous coals produced from these mines is similar to the quality of average coal produced from Wyoming Powder River coals. The Btu content per pound range from 8,400 to 8,700 for coal produced at these three mines. The FOB price for these coals varied from $10.77 to $11.13 and averaged $11.01, per ton.

From this information, a base rate of $9.50 per ton was established for Montana coal. However, this rate includes the 1984 effective tax rate of 21.3 percent. Therefore, a rate of $7.47 per ton was used to reflect the 0 percent tax rate scenario.

Price information was more difficult to collect for Wyoming coal. Only "value per ton" is reported by the Wyoming Department of Revenue. However, individuals in the Wyoming Ad Valorem Tax Division were able to supply an estimate of average FOB mine price for mines in the Powder River basin of $9.77 for 1983 (Wyoming Department of Revenue 1984).

While FOB prices based on state tax proceeds are excellent for estimating average prices, this average includes prices based on contracts that were signed in the early seventies. Information published by Coal Network Associates (1984) provided the contract information used to calcualte current FOB prices. Average FOB price for Montana coal (subbituminous coal) averaged about $9.75 per ton. In Wyoming the 1983 FOB price's reported by Coal Network Associates (1984) averaged $6.25 per ton.

Individuals at the Wyoming Geological Survey were able to provide an estimate of Wyoming mine price of $7.70 per ton. Reports published in the Wyoming Quarterly Update (1984) indicated that Omaha Public Power had renegotiated a coal supply price with Exxon resulting in a drop in FOB mine price at the Caballo and Rawhide mines from $8.25 per ton to $5.75.

From this information, a base rate of $6.00 per ton was established for Wyoming coal. Again, the effective Wyoming tax rate of 10.5 percent is included in this FOB price. The without tax rate would be $5.37.

Once the base rates are established, spacial market boundaries are estimated for Montana and Wyoming tax rates ranging from 0 percent to 120 percent. Table 20 presents Montana and Wyoming tax rates and the associated FOB prices.

Table 20. Montana and Wyoming FOB at Tax Rates Ranging from 0 to 120 percent.

Tax Rate	Montana FOB	Wyoming FOB
0	7.47	5.37
10	8.22	5.91
20	8.96	6.44
30	9.71	6.98
40	10.46	7.52
50	11.21	8.06
60	11.95	8.59
70	12.70	9.13
80	13.45	9.67
90	14.19	10.20
100	14.94	10.74
110	15.96	11.27
120	16.43	11.81

Fixed and Variable Transportation Costs

For coal shipments out of the Powder River, the dominant cost is the transportation cost. This why a spacial market model provides an excellent forecasting tool. All coal transportation for Powder River coal is assumed to be by unit train. The fixed and variable transportation costs used in the 1982 coal study (Duffield and others 1982) used a fixed transportation cost of $1.04/ton and a variable transportation cost of $0.0133/ton-mile. In order to estimate current rates, a complete set of Burlington Northern (BN) time-volume/unit train tariffs as of July 1984 was obtained for Wyoming and Montana coal shipments (Burlington Norther Railroad 1984). Based on regression analysis of 120 observations the following linear equation was specified:

$$TARIFF = 1.77 + 0.0166 \cdot MILE$$

(t-statistic) (2.67) (27.80) $R^2 = .88$

The overall R^2 of 0.88 indicates an excellent fit to the data. When "minimum volume" was included as a second independent variable, the estimated coefficient was not significantly different from zero. This indicates a yearly nominal change in real rail tariffs of 9.2 percent or, given the change in the implicit price deflator (mid-1980 to mid-1984), a 3.4 percent real annual increase. This is very close to the historical 3.5 percent change found in the 1982 study as well as being very close to the escalation rate used for levelizing rail transport costs in the earlier study (Duffield and others 1982). However, the current analysis did indicate a possible slowing of the rate of increase to approximately 1 percent per year. This 1 percent rate was used to derive the levelized rail rates for 1984.

The first year variable rail rates were assumed to be $0.0166 per ton-mile. Levelized over 30 years at 1 percent per year and at real weighted cost of capital of 3.77 percent yields a levelized variable cost of $0.0189 per ton-mile. Because the model is run on actual (air mile) rectangular coordinates, this is inflated by the rail to air-mile ratio for each boundary.

In order to account for the substantial additional distance Wyoming coal must travel to the major Montana "low-Btu"' market in Minnesota, a fixed transportation costs equivalent to an extra 200 miles was included in the Wyoming transportation cost function. Similarly, in modeling states just south of the Minnesota and Wisconsin borders, the difference in air to actual miles from the market centers required an 80 air mile addition to Wyoming fixed costs.

APPENDIX B

SPATIAL MARKET MODEL PROGRAMS

This appendix presents "short-hand" documentation for the computer programs used to develop the 1990 demand forecasts. There are ten Fortran programs involved. Each provides specific calculations needed to forecast coal demand. NEWHYPE.FOR is the core model that calculates the hyperbolic spatial market boundaries. This model is listed line by line in Appendix C. ELEC.FOR is the program used to generate electric demand forecasts. POP.FOR computes the percent of each state's SMSA population captured within either the Montana or Wyoming market boundary. These percentages are used to weight the new coal demand forecasts. ONETON.FOR makes use of regional projections of hydroelectric, nuclear, oil, and gas electric generation forecasts, to calculate the growth in coal demand in each state. TON.FOR takes the residual forecasts made by ONETON.FOR and the population weights calculated by POP.FOR to forecast Montana and Wyoming coal demand. DATAM.FOR, MASH.FOR, MATRAN.FOR, TAX.FOR, and FOB.FOR are data manipulation programs.

NEWHYP.FOR (Intra-fuel Substitution)

Calculates the data coordinates for seven market boundaries. The result is a completely bounded spacial market for either Montana or Wyoming coal production. In addition to the supply centers in Montana and Eastern Wyoming, the following supply centers are used: Colorado, Illinois, New Mexico, Texas, Utah, Washington, Southwest Wyoming.

Input files :

FOR22.DAT: Model plant data including O&M FGD, FOB, and transportation costs.

```
     READ(22,100,END=110)((A(I,J),J=1,8),I=1,18)
100  FORMAT(8F)
```

Output files :

FOR21.DAT: Output of market boundary coordinates. These coordinates are used in POP.FOR.

```
     WRITE(21,550,END=560)(AMAP(I,J),J=1,32)
550  FORMAT('  '(32('  ',F9.2)))
```

FOR37.DAT: Catch all file, where the run identifiers and various responses which are output from the model are sent. These files are not integral to the program, so most of the statements are commented out from the program.

FOR22.DAT Variable Description

Line #	Coal Supply Center[b]	Description
1	A & B	Power Plant Size (net MW)
2	A & B	Hours Operated at Full Load (hours)
3	A	Power Plant Heat Rate (Btu/KWH)
4	A	Coal Heat Content (Btu/lb)
5	B	Power Plant Heat Rate (Btu/KWH)
6	B	Coal Heat Content (Btu/lb)
7	A	Power Plant Capital Cost ($/KW)
8	B	Power Plant Capital Cost ($/KW)
9	A & B	Fixed Charge Rate (decimal)
10	A	Operating and Maintenance Costs ($/KWH)
11	B	Operating and Maintenance Costs ($/KWH)
12	A	FOB Mine Price ($/ton)
13	B	FOB Mine Price ($/ton)
14	A	Fixed Transportation Cost ($/ton)
15	B	Fixed Transportation Cost ($/ton)
16	A	Variable Transportation Costs ($/ton-air mile)
17	B	Variable Transportation Costs ($/ton-air mile)
18	A & B	Straight Line Distance Between A & B (miles)

[a] Source: Duffield and others 1985.
[b] A = Either the Montana or Wyoming Supply Center; B = The market competitor of A; one of seven possible coal supply centers.

FOR22.DAT Data

	Colorado	Illinois	New Mexico	Texas	Utah	Washington	SW Wyoming	BJ Wyoming
1	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0
2	5694.0	5694.0	5694.0	5694.0	5694.0	5694.0	5694.0	5694.0
3	10486.0	10058.0	10564.0	10251.0	10486.0	10486.0	10486.0	10486.0
4	8700.0	8700.0	8700.0	8700.0	8700.0	8700.0	8700.0	8700.0
5	10341.0	10204.0	10564.0	11045.0	10197.0	10486.0	10341.0	10486.0
6	10700.0	10500.0	10000.0	6300.0	11500.0	8100.0	10500.0	8450.0
7	1329.7	1167.3	1257.5	1049.3	1329.7	1329.7	1329.7	1329.7
8	1307.1	1206.8	1257.5	1357.0	1227.6	1333.5	1307.1	1329.7
9	0.07410	0.07410	0.07410	0.07410	0.07410	0.07410	0.07410	0.07410
10	0.00671	0.00633	0.00665	0.00663	0.00671	0.00671	0.00671	0.00671
11	0.00665	0.00822	0.00665	0.00699	0.00660	0.00775	0.00665	0.00671
12	9.5	9.5	9.5	9.5	9.5	9.5	9.5	9.5
13	21.25	25.52	25.0	11.92	26.3	27.22	16.5	6.0
14	2.02	2.02	2.02	2.02	2.02	2.18	2.02	2.02
15	2.18	5.3	2.02	2.02	2.18	2.02	2.02	2.02
16	0.0246	0.0256	0.0265	0.0265	0.0246	0.0265	0.0246	0.0246
17	0.0265	0.0256	0.0265	0.0265	0.0265	0.0246	0.0246	0.0246
18 MT	395.7	1061.5	663.6	1106.6	521.6	779.1	321.6	252.8
WY	128.6	852.5	384.0	792.0	341.0	854.5	181.5	252.8

<u>Other important variables</u>:

THETA: The angle from the supply center to one of seven competing supply center.

Value of THETA for Montana and Wyoming Supply Centers

Montana			Wyoming		
Distance	THETA	Competing Supply Center	Distance	THETA	Competing Supply Center
395.7	263.32	Colorado	129.0	238.02	Colorado
1,061.5	328.29	Illinois	853.0	20.37	Illinois
663.6	256.76	New Mexico	671.0	197.87	New Mexico
1,106.6	299.09	Texas	792.0	327.01	Texas
521.6	237.92	Utah	696.0	206.58	Utah
779.1	165.43	Washington	854.5	160.63	Washington
321.6	243.99	SW Wyoming	181.5	171.18	SW Wyoming
252.8	286.79	BJ Wyoming	252.8	106.79	BJ Wyoming

POP.FOR

Computes the percent of a states SMSA population that falls in either the Montana or Wyoming market region.

<u>Input files</u>:

FOR21.DAT: Boundary coordinates output from NEWHYP.FOR.

```
     READ(21,130,END=140)((AMAP(I,J),J=1,8)
     ,I=1,148)
130  FORMAT(32F)
```

FOR69.DAT:SMSA coordinates and population forecasts.

```
     READ(20,110,END120((SMSA(I,J),J=1,8)
     ,I=1,148)
110  FORMAT(8F)
```

FOR69.DAT Data

Coordinates		Population Forecasts					
X	Y	1980	1985	1990	2000	1995	1984
576	604.8	177,850	166,830	166,830	184,537	178,792	168,923
560	-650.4	131,822	132,044	138,044	148,750	143,282	131,999
677.6	-694.4	393,494	395,397	412,815	443,840	428,046	395,014

(148 SMSAs)

Output files:

FOR15.DAT: 2xn X,Y boundary coordinates, where n = the number of boundary locations.

```
     WRITE(15,190,END=200)((ZHYPER(I,J),J=1,2)
     ,I=1,1616
190  FORMAT(2F)
```

FOR17.DAT: The X,Y coordinates of the SMSA's captured within the spacial market.

```
     DO 590 I==1,139
     WRITE(17,580)(SMINXY(I,J),J=1,2)
580  FORMAT(1X,F6.1,1X,F7.1)
590  CONTINUE
```

FOR23.DAT: Population weights for the 21 state market.

```
     WRITE(23,570)(OUT(I),1,21)
570  FORMAT(1X,F)
```

FOR66.DAT Data

Iowa	OUT(1)
Kansas	OUT(2)
Minnesota	OUT(3)
Montana	OUT(4)
Nebraska	OUT(5)
North Dakota	OUT(6)
Oklahoma	OUT(7)
South Dakota	OUT(8)
Wisconsin	OUT(9)
Arkansas	OUT(10)
Colorado	OUT(11)
Idaho	OUT(12)
Illinois	OUT(13)
Michigan	OUT(14)
Missouri	OUT(15)
Wyoming	OUT(16)
Louisiana	OUT(17)
Oregon	OUT(18)
Texas	OUT(19)
Washington	OUT(20)
Indiana	OUT(21)

ELEC.FOR

Generates electric demand forecast for each state in the market area. This is based on the 1983 kwh generated in each state and a constant growth rate.

Input files:

FOR42.DAT: State kwh generation for 1983.

Output files:

FOR61.DAT: Electric demand for each state for the following years: 1983, 1984, 1985, 1990, 1995, 2000.

ONETON.FOR (Inter-fuel Substitution)

Given state electric demands, forecasted production of hydroelectric, nuclear, oil, and gas generation, and current coal generation, calculates excess electric demand. This residual demand represents the potential for growth for coal fired generation. The weights calculated by POP.FOR are used to identify the generation which will be supplied by either Montana or Wyoming coal producers. The result is the generation, kwh which will be generated by one of these two supply centers coal. Finally, the generation is converted into tons of coal.

Important Variables:

```
      DO 1000 I=1,21
      DO 900 J=1,5
      RESID(I,J)=STAKW(I,M)-(AHYD(I,J)+AOIL(I,J)+AGAS(I
      ,J)+ANUK(I,J)+COALT(I,J))
900   CONTINUE
1000  CONTINUE
      NBILL = 1,000,000,000
      DO 1100 I=1,21
      TON(I)=((RESID(I,K)*NBILL*COALT(I,5)/COALT(I,6)
      /2000)1000
1100  CONTINUE
```

Input files:

FOR31.DAT: Historic coal shipments--1983 coal generation (kwh) for each state.

```
      READ(31,100,END=400)((COALT(I,J),J=1,6),I=1,21)
100   FORMAT(6F)
```

FOR43.DAT:Hydropower generation (kwh) for each state.

```
      READ(43,300,END=400)((AHYD(I,J),J=1,5),I=1,21)
```

FOR44.DAT: Oil generation (kwh) for each state.

```
      READ(44,300,END=400)((AOIL(I,J),J=1,5),I=1,21)
```

FOR45.DAT: Gas generation (kwh) for each state.

```
      READ(45,300,END=400)((AGAS(I,J),J=1,5),I=1,21)
```

FOR46.DAT: Nuclear generation (kwh) for each state.

```
      READ(43,300,END=400)((ANUK(I,J),J=1,5),I=1,21)
```

FOR61.DAT: State electric demand forecast--Electric growth forecasts for 1984, 1985, 1990, 1995, and 2000.

```
     READ(61,450)((STAKW(I,J),J=1,6),I=1,21)
450  FORMAT(6F)
```

<u>Output files</u>:

FOR35.DAT: Supply center's coal demand forecast.

```
     WRITE(58,1200)(TON(I),I=1,21)
1200 FORMAT(F15.3)
```

DATAM.FOR

Manipulates an (21*13*13)x1 vector and converts it into a (21*13)x13 matrix. This data manipulation is used to order the population weights so they can be used in TON.FOR.

<u>Input files</u>:

FOR51.DAT: Population weights (21*13*13)x1.

```
     READ(51,40)(POP(I),I=1,3731)
40   FORMAT(6X,F10.7)
```

<u>Output files</u>:

FOR52.DAT: Population weights (21*13)x13.

```
     WRITE(52,60)((XPOP(I,J),J=1,13),I=1,287)
60   FORMAT(13F15.7)
```

MASH.FOR

Takes regional population forecasts and combines them into one final output file. The regions are the Northwest (NW), Northcentral (NC), and the Southcentral (SC).

<u>Important Variables</u>:

```
        DO 400 J=1,13
        DO 500 I=1,287,22
        POP(I,J)=XNWPOP(I,J)
        POP(I+1)=SCPOP(I+1,J)
        POP(I+2)=SCPOP(I+2,J)
        POP(I+3)=XNCPOP(I+3,J)
        POP(I+4)=XNWPOP(I+4,J)
        POP(I+5)=SCPOP(I+5,J)
        POP(I+6)=XNCPOP(I+6,J)
        POP(I+7)=SCPOP(I+7,J)
        POP(I+8)=XNCPOP(I+8,J)
        POP(I+9)=XNCPOP(I+9,J)
        POP(I+10)=SCPOP(I+10,J)
        POP(I+11)=SCPOP(I+11,J)
        POP(I+12)=XNWPOP(I+12,J)
        POP(I+13)=SCPOP(I+13,J)
        POP(I+14)=XNCPOP(I+14,J)
        POP(I+15)=SCPOP(I+15,J)
        POP(I+16)=SCPOP(I+16,J)
        POP(I+17)=SCPOP(I+17,J)
        POP(I+18)=XNWPOP(I+18,J)
        POP(I+19)=SCPOP(I+19,J)
        POP(I+20)=XNWPOP(I+20,J)
        POP(I+21)=SCPOP(I+21,J)
  500   CONTINUE
        400 CONTINUE
```

<u>Input files</u>:

FOR51.DAT: NW population weights.

```
        READ(51,300)((XNWPOP(I,J),J=1,13),I=1,287)
  300   FORMAT(4X,13F15.7)
```

FOR52.DAT: NC population weights.

```
        READ(52,300)((XNCPOP(I,J),J=1,13),I=1,287)
```

FOR53.DAT: SC population weights.

```
        READ(53,300)(SCPOP(I,J),J=1,13)(I=1,287)
```

<u>Output files</u>:

FOR54.DAT: Combined population weights.

```
        WRITE(54,300)((POP(I,J),J=1,13),I=1,287)
```

TON.FOR

This program uses residual coal demand forecasts from ONETON.FOR and multiplies associated population weights to forecast derived demand for Montana or Wyoming supply centers.

Input files:

FOR54.DAT: Combined population weights output from MASH.FOR.

```
     READ(54,100)((POP(I,J),J=1,13),I=1,287)
1100 FORMAT(13F10.7)
```

FOR58.DAT: Residual coal demand.

```
     READ(58,1100)(RESID(K),K=1,21)
```

Output files:

FOR56.DAT:Coal demand forecasts for 13x13 MT,WY price scenarios.

```
     WRITE(56,1000)((TON(L,J),J=1,13),L=1,13)
1000 FORMAT(13F15.2)
```

FOR57.DAT: Coal demand forecasts by state, for 13x13 MT,WY price scenarios.

```
     WRITE(57,1000)((STATON(I,J),J=1,13),I=1,187)
```

MATRAN.FOR

Transposes a 13x13 data matrix.

Input files:

FOR56.DAT :Production forecasts.

Output files:

FOR62.DAT: Transposed Production forecast.

TAX.FOR

Updates the tax rate used to change the MT or WY FOB price in FOR22.DAT. This program is used in conjunction with FOB.FOR.

Input files:

FOR5Ø.DAT: The previous tax rate used to update the FOB price.

Output files:

FOR5Ø.DAT: The current tax rate used to update the FOB price.

FOB.FOR

Modifies the FOB mine price in FOR22.DAT. Once this price is updated, NEWHYP.FOR is run to generate new market boundaries.

Input files:

FOR5Ø.DAT: Current tax rate used to update the previous FOB price.

Output files:

FOR48.DAT: Corrected data file used in NEWHYP.FOR. The file name is changed to FOR22.DAT by a batch file.

APPENDIX C

NEWHYP.FOR

```
C   PROGRAMMER: MICHAEL H. LEE/ECONOMICS DEPARTMENT/U. OF H.
C                  ALTERED BY JOHN TUBBS/1984
C
C       PROGRAM:  THIS PROGRAM CREATES DATA COORDINATES FOR SEVEN
C                 MARKET BOUNDARIES, RESULTING IN A COMPLETELY
C                 BOUNDED NGP COAL MARKET REGION.
C
C                 DATE: OCTOBER  1984
C
      DIMENSION A(18,8)
      DIMENSION AMAP(101,32)
      DIMENSION STORE(101,4)
C
      DATA AA,AHALF,AIRAN,ATBT,ADUA,ADUB,APROD/7*0.0/
      DATA BB,BPROD,BIRAN,CC,DTHETA,DXOLD,DYOLD,DXNEW1/8*0.0/
      DATA DXNEW2,DYNEW1,DYNEW2,DD,DSCHKW,DIFFAB,DISTA,DS1,DS2/9*0.0/
      DATA EE,FIXTRA,FIXTRB,S1,S2,SUM,THETA,TONA,TONB,TKKW,TOTA/11*0.0/
      DATA TOTB,VARB,XCORD,XNEW1,XNEW2,YNEW1,YNEW2,YPOS/8*0.0/
      DATA TOTRA,TOTRB,FLAG,RATIO1,RATIO2,RATIO3,RATIO4/7*0.0/
      DATA SUDIST,TWICE,ANUM,SGNSG,AMKT,DECIDE,DISTAN/7*0.0/
      DATA DISTA1,DISTB1,DISTB/3*0.0/
      DATA RATIO5,CKNUM,CKDEN/3*0.0/
C
      DATA I,J,K,INTGR1,INTGR2,JARAY,NCOUNT,IROW,JCOL/9*0/
      DATA IDISA1,IDISB1/2*0/
C
C INITIALIZE STORE(I,J)
C TEMPORARY STORAGE OF COORDINATES PASSED TO AMAP.
      DO 20 J=1,4
          DO 10 I=1,101
              STORE(I,J)=0.0
 10       CONTINUE
 20   CONTINUE
C INITIALIZE A(I,J)
C ECONOMIC PARAMETERS USED TO GENERATE MARKET BOUNDARIES ARE IN A(I,J).
      DO 40 I = 1,18
          DO 30 J = 1,8
              A(I,J) = 0.0
 30       CONTINUE
 40   CONTINUE
C INITIALIZE AMAP)I,J)
C AMAP(I,J) HAS THE TRANSFORMED X & Y COORDINATES FOR THE 7 BOUNDARIES.
      DO 60 I = 1,101
          DO 50 J = 1,32
              AMAP(I,J) = 0.0
 50       CONTINUE
 60   CONTINUE
C
      WRITE(5,70)
 70   FORMAT(//,1X,'TYPE IN A FIVE LETTER CODE IDENTIFYING THIS RUN:')
      ACCEPT 80,ALPHA
 80   FORMAT(A5)
      WRITE(5,90)ALPHA
 90   FORMAT(//,1X,'THIS RUN IS: ',1X,A5,////)
C UNIT FOR22.DAT HAS INPUT DATA READ INTO A(I,J)
      READ(22,100,END=110)((A(I,J), J=1,8),I=1,18)
 100  FORMAT(8F)
 110  CONTINUE
C
      WRITE(5,120)


 120  FORMAT(/,6X,'COLORADO',6X,'ILLINOIS',8X,'NEW MEXICO',7X,
     1'TEXAS',9X,'UTAH',10X,'WASHINGTON',6X,'WYOMING',6X,'MONTANA',/)
      WRITE(5,130,END=140)((A(I,J),J=1,8),I=1,18)
 130  FORMAT(' ',8F)
 140  CONTINUE
C
      DO 540 JARAY = 1,8 ! LOOP THROUGH ALGORITHM FOR EACH OF 8 BDYS
          GOTO(150,160,170,180,190,200,210,215)JARAY
C
 150      CONTINUE
          THETA = 248.38          ! COLORADO
          GOTO 220
 160      CONTINUE
          THETA = 333.46          ! ILLINOIS
          GOTO 220
 170      CONTINUE
          THETA = 247.85          ! NEW MEXICO
          GOTO 220
 180      CONTINUE
          THETA = 300.06          ! TEXAS
          GOTO 220
 190      CONTINUE
          THETA = 222.69          ! UTAH
          GOTO 220
 200      CONTINUE
          THETA = 157.77          ! WASHINGTON
          GOTO 220
 210      CONTINUE
          THETA = 217.97          ! WYOMING
          GOTO 220
 215      CONTINUE
          THETA = 116.01          !MONTANA
 220      CONTINUE
C
          S1 = COSD(THETA)
          S2 = SIND(THETA)
C
```

```
C   THE NUMBER OF TONS OF COAL FOR ANNUAL OPERATION OF A GIVEN
C   COAL-FIRED POWER PLANT IS COMPUTED.
          AA = A(1,JARAY)*A(2,JARAY)
          BB = A(4,JARAY)*2000.00
          CC = A(3,JARAY)*1000.0
          DD = A(5,JARAY)*2000.0
          EE = A(6,JARAY)*1000.0
          TONA = (AA/DB)*CC
          TONB = (AA/DD)*EE
C
C FIXED TRANSPORTATION COSTS FOR A GIVEN QUANTITY OF COAL PRODUCTION
C ARE COMPUTED.
          FIXTRA = TONA*A(14,JARAY)
          FIXTRB = TONB*A(15,JARAY)
C
C   ADDITIONAL COSTS ARE COMPUTED.
          DSCMKW = A(1,JARAY)*1000.0*A(9,JARAY)
          TMKW = A(2,JARAY)*A(1,JARAY)*1000.0
          ADDA = ( A(7,JARAY)*DSCMKW) + (A(10,JARAY)*TMKW)
          ADDB = ( A(8,JARAY )*DSCMKW) + (A(11,JARAY)*TMKW)
C
C   PRODUCTION AND TRANSPORTATION COSTS ARE COMPUTED.
          APROD = A(12,JARAY)*TONA
```

```
          BPROD = A(13,JARAY)*TONB
          ATRAN = A(16,JARAY)*TONA
          BTRAN = A(17,JARAY)*TONB
          TOTA = APROD + ADDA + FIXTRA
          TOTB = BPROD + ADDB + FIXTRB
C
C   THE FOLLOWING DETERMINES WHETHER THE MARKET BOUNDARY
C   INTERSECTS THE X-AXIS ON THE NEAR OR FAR SIDE OF MARKET B.
          DISTAB = A(18,JARAY)
          AMKT = TOTA + (ATRAN*DISTAB)
          DECIDE = AMKT - TOTB
  230     CONTINUE
          IF(DECIDE.LT.0.0) GOTO 370
C
C   DISTA IS COMPUTED FOR THE CASE THE MARKET BOUNDARY INTERSECTS
C   THE X-AXIS BETWEEN MARKETS A AND B.
          DIFFAB = TOTB-TOTA
          VARB = BTRAN * DISTAB
          SUM = DIFFAB + VARB
          ATBT = ATRAN + BTRAN
          DISTA = SUM / ATBT
C
C      TOTAL TRANSPORTATION COSTS ARE COMPUTED.
          TOTTA = FIXTRA + A(16,JARAY)*DISTA
          TOTTB = FIXTRB + A(17,JARAY)*(DISTAB-DISTA)
C
C   COMPUTE DATA USED IN GENERATING BOUNDARIES.
          AHALF = DISTAB / 2
          DISTB = DISTAB - DISTA
          IF(DISTA.EQ.DISTB)  GOTO 460
          RATIO1 = ATRAN/BTRAN
          RATIO2 = (TOTA-TOTB)/BTRAN
          RATIO3 = 1/RATIO1
          RATIO4 = (TOTB-TOTA)/ATRAN
          SQDIST = DISTAB**2
          TWICE  = 2 * DISTAB
          IF(DISTA.LT.AHALF)  GOTO 300
  240     CONTINUE
C
C   BELOW LOOP IS FOR THE CASE WHEN A BOUNDARY OPENS TO MARKET B.
          INTGR1 = INT(DISTA + 5)
          INTGR2 = INTGR1 + 1000
          I = 0
          FLAG = 0.0
          DO 290 IDISAI = INTGR1,INTGR2,10
             I = I + 1
             IF(I.GT.101) GOTO 290
             J = 1
             IF(FLAG.GT.1.0) GOTO 250
             XCORD = DISTA
             YPOS = 0.0
             GOTO 270
  250        CONTINUE
             DISTBI = (IDISAI * RATIO1) + RATIO2
  260        CONTINUE
             ANUM = IDISAI**2 - (DISTBI**2) + SQDIST
             XCORD = ANUM/TWICE
             SQNEG = IDISAI**2 - (XCORD**2)
             IF(SQNEG.LT.0.0) GOTO 280 !FOR ELLIPSES AND CIRCLES
             YPOS = SQRT(SQNEG)
```

```
270          CONTINUE
             CALL EUCLID(XCORD,YPOS,XNEW1,YNEW1,XNEW2,YNEW2,S1,S2)
280          CONTINUE
             STORE(I,J)=XNEW1
             J=J+1
             STORE(I,J)=YNEW1
             J=J+1
             STORE(I,J)=XNEW2
             J=J+1
             STORE(I,J)=YNEW2
             FLAG = 100.0
290       CONTINUE
          GOTO 400
C
C  BELOW LOOP IS FOR THE CASE WHEN A BOUNDARY OPENS TO MARKET A.
300       CONTINUE
          INTGR1 = INT(DISTB + 5)
          INTGR2 = INTGR1 + 1000
          I = 0
          FLAG = 0.0
          DO 360 IDISBI = INTGR1,INTGR2,10
             I = I + 1
             IF(I.GT.101) GOTO 360
             J = 1
             IF(FLAG.GT.1.0) GOTO 310
             XCORD = DISTA
             YPOS = 0.0
             GOTO 340
310          CONTINUE
             DISTAI = ( IDISBI*RATIO3) + RATIO4
320          CONTINUE
             ANUM = DISTAI**2 - (IDISBI**2) + SQDIST
             XCORD = ANUM / TWICE
             SQNEG = DISTAI**2-(XCORD**2)
             IF(SQNEG.LT.0.0) GOTO 350
             YPOS = SQRT(SQNEG)
330          CONTINUE
340          CONTINUE
             CALL EUCLID(XCORD,YPOS,XNEW1,YNEW1,XNEW2,YNEW2,S1,S2)
350          CONTINUE
             STORE(I,J)=XNEW1
             J=J+1
             STORE(I,J)=YNEW1
             J=J+1
             STORE(I,J)=XNEW2
             J=J+1
             STORE(I,J) = YNEW2
             FLAG = 100.0
360       CONTINUE
          GOTO 400
C
C  FIND THE INTERSECTION POINT OF THE MARKET BOUNDARY WITH
C  THE X-AXIS.
370       CONTINUE
          CKNUM = BTRAN*A(6,JARAY) ! HEAT CONTENT B
          CKDEN = ATRAN*A(4,JARAY) ! HEAT CONTENT A
          RATIO5 = CKNUM/CKDEN
          IF(RATIO5.GT.1.0) GOTO 440
          DIFFAB = TOTB-TOTA
          VARB = BTRAN * DISTAB


          SUM = DIFFAB - VARB
          ATOT = ATRAN-BTRAN
          DISTA = SUM / ATOT
C
          IF(DISTA.GT.0.0) GOTO 390
          WRITE(6,380) JARAY
380       FORMAT(' ',////,'DISTA IS NEGATIVE FOR JARAY<',1X,I)
          GOTO 520
390       CONTINUE
C
C  COMPUTE TOTAL TRANSPORTATION COSTS
          TOTFA = FIXTRA + A(16,JARAY)*DISTA
          TOTFB = FIXTRB + A(17,JARAY)*(DISTA-DISTAB)
C  COMPUTE DATA FOR GENERATING THE MARKET BOUNDARY
          RATIO1 = ATRAN/BTRAN
          RATIO2 = (TOTA-TOTB)/BTRAN
          SQDIST = DISTAB**2
          TWICE  = 2*DISTAB
C
```

```
C   [illegible] ON THE OTHER SIDE
C   OF MARKET B.
            INTGR1 = INT(DISTA + 5)
            INTGR2 = INTGR1 + 1000
            I = 0
            FLAG = 0.0
            DO 430 IDISAI = INTGR1,INTGR2,10
                I = I + 1
                IF(I.GT.101) GOTO 430
                J = 1
                IF(FLAG.GT.1.0) GOTO 400
                XCORD = DISTA
                YPOS = 0.0
                GOTO 410
  400           CONTINUE
                DISTBI = ( IDISAI*RATIU1 ) + RATIO2
                ANUM = IDISAI**2 - (DISTBI**2) + SQDIST
                XCORD = ANUM / TWICE
                SQNEG = IDISAI**2 - (XCORD**2)
                IF(SQNEG.LT.0.0) GOTO 420
                YPOS = SQRT(SQNEG)
  410           CONTINUE
                CALL EUCLID(XCORD,YPOS,XNEW1,YNEW1,XNEW2,YNEW2,S1,S2)
  420           CONTINUE
                STORE(I,J) = XNEW1
                J=J+1
                STORE(I,J) = YNEW1
                J=J+1
                STORE(I,J) = XNEW2
                J=J+1
                STORE(I,J) = YNEW2
                FLAG = 100.0
  430       CONTINUE
            GOTO 490
C
  440       CONTINUE
            WRITE(5,450)JARAY
            WRITE(5,450)JARAY
  450       FORMAT(' ',///,1X,'THE MARKET BOUNDARY FOR JAFAY "QUAL TO',1X
     1,I,1X,'DOES NOT EXIST')
            GOTO 520




C
  460       CONTINUE
            WRITE(5,470) JARAY
  470       FORMAT(' ',///,'THE MARKET BUY FOR JARAY EQUAL TO:',1X,I,1X,'I
     1S A STRAIGHT LINE.')
            GOTO 520       ! ENTER LOOP AND PROCESS NEXT STATE
C
  490       CONTINUE
            WRITE(5,190) JARAY,TONA,TONB,ADDA,ADDB,APROD,BPROD,DIFFAB
     1,TOTRA,TOTRB,DISTA
  490       FORMAT(///,' ','STATE NUMBER',3X,I,//,' ','THE QUANTITY OF COA
     1L REQUIRED FOR THE ANNUAL OPERATION',/,' ','OF A FUSEL COAL
     2 FIRED GENERATING PLANT FOR COAL FROM MARKETS A & B EQUALS',
     3 /,' ',2(1X,F11.2),/,' ','THE ADDITIONAL COSTS FOR COAL FROM
     4 MARKETS A & B EQUALS:',/,' ',2(1X,F11.2),/,' ','PRODUCTION
     5 COSTS FOR COAL FROM A & B EQUALS:',/,' ',2(1X,F11.2),/,' ',
     6 'THE DIFFERENCE IN TOTAL COST EQUALS:',1X,F11.2,/,' ','TOTAL
     7 TRANSPORTATION COSTS FOR A & B EQUALS',2(1X,F11.2),/,' ',
     8 'DISTA EQUALS:',1X,F8.2,///)
C
C   FOR EACH STATE, THE X & Y VALUES OF THE BOUNDARY ARE READ FROM
C   STORE(I,J) INTO ARRAY 'AMAP'.
            DO 510 IROW=1,101
                DO 500 J=1,4
                    JCOL=K + J
                    AMAP(IROW,JCOL)=STORE(IROW,J)
  500           CONTINUE
  510       CONTINUE
  520       CONTINUE
            K=K+4
  530       CONTINUE
  540   CONTINUE
        DO 560 I=1,101
            WRITE(21,550,END=560)(AMAP(I,J),J=1,32)
  550       FORMAT(' '(32(' ',F9.2)))
  560   CONTINUE
  570   STOP
        END
C*************************************************************
C
C   SUBROUTINE EUCLID CREATES NEW COORDINATES FOR MARKET BOUNDARIES
C   BY ROTATING THE STANDARD BASIS VECTERS BY 'THETA' DEGREES.
        SUBROUTINE EUCLID(DXOLD,DYOLD,DXNEW1,DYNEW1,
     1DXNEW2,DYNEW2,DS1,DS2)
  999   DXNEW1 = ( DS1*(DXOLD)-DS2*(DYOLD) )
        DYNEW1 = ( DS2*(DXOLD)+DS1*(DYOLD) )
        DXNEW2 = ( DS1*(DXOLD)-DS2*(-DYOLD))
        DYNEW2 = ( DS2*(DXOLD)+DS1*(-DYOLD) )
        RETURN
        END
```

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