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Antoine-Augustine Cournot: His duopoly problem with a re-appraisal of his duopoly price and his influence upon Leon Walras

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ANTOINE-AUGUSTINE COURNOT: HIS DUOPOLY PROBLEM WITH A RE-APPRAISAL OF HIS DUOPOLY PRICE AND HIS INFLUENCE UPON LEON WALRAS

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TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

This thesis begins with the assumption that all "neo-classical" economics represented an early stage of the long, slow development--still going on today--of "mathematical economics" or what may be called a gradual "mathematization" of economic theory.[1] In a real sense the origins of this development are very old; only the "fruits" have been slow to appear and "ripen." Many of the central problems of economic theory always have been, in one aspect, mathematical in nature, i.e., problems needing to be studied with the aid of mathematics. There, even in early times there were gropings for something concrete to build upon.

An example of the common lack of command of mathematics on the part of former economists can be demonstrated by the Ricardian-classical "law of diminishing returns." The use of differential calculus would have provided the needed insight to explain the diminishing additional returns of produce from more intensive cultivation of agricultural land. As it was, Ricardo used words and arithmetical illustrations. It later turned out that the "law of

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[1]For a recent book dealing with the early development of mathematical economics, see Theocharsis, Early Developments in Mathematical Economics.

diminishing returns" was only one special case of a much more general principle of marginal analysis which led to the marginal revolution, mathematical economics, and all "neo-classical" economic theory.

Another example that illustrates the relative shortness of the distance separating "classical" economic theory from "mathematical" theory is the fact that Malthus foresaw the latter and expressed his belief that "fluxions" (as the branch of mathematics that is now called "the calculus" was then called) would some day come into use in economics, as it already had in physics and other sciences.[2]

Although the development of the marginal concept and the introduction into economics of formal mathematical techniques do not coincide perfectly, there is enough coincidence to make their separate treatment extremely awkward. Among the pioneers of the marginal concept--for example, Herman Gossen, William Jevons, and Leon Walras--were also pioneers in the use of mathematics in economics. Among the earliest economists to use mathematics, only A. A. Cournot and A. J. E. Dupuit seem to have overlooked the marginal concept.

Augustin Cournot's pioneer work in mathematical

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[2]Overton H. Taylor, History of Economic Thought, (New York: McGraw Hill, 1960), p. 328.

economics is called *Recherches sur les Principes Mathematiques de la Theorie des Richesses* (1838). Although he never had a really systematic theory, he covered most of the problems later taken up by Walras and Pareto, and for the first time utilized infinitesimal calculus in treating economic phenomena. (The founders of the general equilibrium mathematical method were Walras, a Frenchman, and Pareto, an Italian. Since they both taught at the Swiss University of Lausanne, the label "Lausanne School" is often applied to them and their students.)

As was mentioned, Cournot pioneered in the use of calculus, especially differential equations, to represent the functional relationship between prices and quantities sold in a competitive market. Cournot's theory of the firm corresponds with the modern analysis of firm equilibrium. Cournot assumed that total cost and total revenue vary continuously with output and that the equilibrium position is that of the maximization of net revenue--total revenue minus total cost. In the contemporary version of firm profit maximization, marginal cost is equated to marginal revenue. Both treatments are essentially the same.[3]

Cournot's book was bold for its day, and some claim

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[3]Philip C. Newman (ed.), *Source Readings in Economic Thought*, (New York: W. W. Norton & Co., 1954), p. 451.

it has no equal in the history of economic theory. He was the first writer to define and graph a demand function. Although not interested in utility theory, Cournot assumed as a matter of course that the demand curve was negatively sloped.[4] As the total quantity increased the added utility from an additional unit decreased. In his analysis, Cournot begins with pure monopoly instead of competition and works toward unlimited competition.

The purpose of this paper is to demonstrate the extent to which Cournot's Recherches[5] has influenced the development of economic thought and the writings of Leon Walras in particular. This will be attempted by first giving the reader a little background of Cournot's life and works and then by explaining his famous duopoly problem aroused interest among later economists. Cournot's solution is based upon certain explicit assumptions, which Cournot failed to spell out clearly. Chapter III covers some of these assumptions that later economists read into Cournot's theory and the confusion that resulted when the market failed to achieve equilibrium.

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[4]Mark Blaug, Economic Theory in Retrospect, (Homewood, Illinois: Irwin, Inc., 1968), p. 318.

[5]A. A. Cournot, Recherches sur les principes mathematiques de la theorie des richesses (Paris: Hachette, 1838). English translation by N. T. Bacon in Economic Classics (New York: Macmillan, 1897).

to be greater today than was earlier believed. His duopoly problem that could be expanded to oligopoly is very timely today in a business world swaying between pure monopoly and free competition. The influence of Cournot's concepts and methods is further demonstrated by tracing the later development of the famous economist Leon Walras and his work on general equilibrium analysis.

CHAPTER II

BIOGRAPHICAL SKETCH OF AUGUSTIN COURNOT

Every neophyte in economics is well acquainted with the fundamental notion that quantity demanded is a function of price. He is also familiar with the idea that a demand curve can be drawn relating the two variables--price and quantity, and will be convex to the origin. Few if any, however, are aware of the origin of these concepts in economics. For their introduction we owe credit to an obscure French mathematician, Antoine-Augustin Cournot. Further, as we shall see, we must credit Mr. Cournot with a great many other contributions as well.

Augustin Cournot was born at Gray, in Haute-Saone, France, in 1801. There he received his earlier training, and later took special instruction in mathematics at the Lycee de Besancon. He continued his mathematical studies at the Ecole Normale in Paris, in 1834 he became Professor of Mathematics at Lyon, and the following year Rector of the Academy at Grenoble. His most important work, The Mathematical Principles of the Theory of Wealth, was published in 1838. He was summoned to Paris as Inspector General of Studies until 1854, when he became Rector of the Academy at Dijon. In 1862 he retired from active teaching, and remained an active researcher and writer until his

death in 1877.[1]

Although Cournot was a mathematician by training, he is equally well known as a philosopher and educator. His mathematical writing included his Traité Élémentaire de la Théorie des Fonctions et du Calcul Infinitésimal, Exposition de la Théorie des Chances et des Probabilités, and De l'Origine et des limites de la Correspondence entre l'Algebre et la Géométrie. Among his philosophical works we find Traité de l'Enchainement des Idées Fondamentales dans les Sciences et dans l'Histoire and Considérations sur la Marche des Idées et des Événements dans les Temps Modernes. His influence on Philosophy was substantial:

> When Cournot is mentioned in histories of Philosophy he is most often referred to... as a rather obscure figure, standing outside the philosophic tradition of his time and place, yet exercising a considerable influence on the development of the later philosophy in France. He seems to be best known for his conception of chance as inherent in reality in an objective way. But chance so conceived imposes definite limitations upon science, and Cournot is credited also for his effort to analyze and criticize rigorously the methods, the significance, and the bearing of economic truths.[2]

However, it is not as a philosopher, educator, or mathematician that Cournot is best remembered today, but as an economist. He is fulfilling Jevon's prediction that he

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[1]Cournot, Researches, op. cit., p. v.

[2]Chester Townsend Ruddick, "Cournot's Doctrine of Philosophical Doctrine," The Philosophical Review, XLIX (July, 1940), p. 415.

"must occupy a remarkable position in the history of the subject."[3] Irving Fisher, who wrote the introduction to the English translation of Cournot's most important work, Researches sur les Principes Mathématiques de la Théorie des Richesses, which in English is The Mathematical Principles of the Theory of Wealth, paid him high tribute:

> Although some score of writers had preceded him in attempting to apply mathematical processes to political economy, he was the first to win substantial results. He alone of the early writers exerts today a powerful influence on economic thought. It is with him, therefore that any survey of modern mathematical economics should begin.[4]

It seems only fitting in view of Fisher's comments that 35 years later, in 1933, the first article to appear in the first issue of volume one of Econometrica, the journal of the Econometric Society, whose purpose is the advancement of economic theory in its relation to statistics and mathematics, was a tribute to Augustin Cournot. In this same issue Joseph Schumpeter said:

> ...still higher tribute is due to Antoine-Augustin Cournot who, without encouragement or lead, in what was then a most uncongenial environment, in 1838 fully anticipated the econometric program by his Researches, one of the most striking achievements of true genius, to which we pay respect to this day by nearly always

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[3] W. S. Jevons, Theory of Political Economy (4th ed.; London: Macmillan and Co., 1924) p. xxix.

[4] Irving Fisher, "Cournot and Mathematical Economics," The Quarterly Journal of Economics, XII (January, 1898), p. 135.

starting out from them.[5]

While it seems that Cournot's influence as an economist is unquestionable, his Mathematical Principles of the Theory of Wealth was completely ignored by economists for nearly 40 years. Thanks to Walras, Jevons, and Boccardo, Cournot's work was discovered and his theory and methods spread. Apparently discouraged by the lack of success of his Mathematical Principles, Cournot later paraphrased it using a non-symbolic approach and published it as Principes de la Théorie des Richesses in 1863. In the year before his death he published still another revised version as Revue Sommaire des Doctrines Économiques. But neither of these works were as interesting and important as the original, and have since fallen into disuse.

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[5]Joseph Schumpeter, "The Common Sense of Econometrics," Econometrica, I January, 1933), p. 9.

CHAPTER III

COURNOT'S DUOPOLY PROBLEM

Augustin Cournot is credited with discovering the distinctive feature of the oligopoly problem. He also showed that with certain assumptions a determinate equilibrium solution is obtained for the duopoly problem, the market condition of only two sellers, and that this solution can be extended to oligopoly.[1] The main significance arising from Cournot's work lies not in the determinateness of the solution, though, but in connection with legitimate criticisms of Cournot's solution. To understand why, we have to start by considering how the determinate solution is obtained. A realistic approach to oligopoly problems cannot be based on Cournot's theory, yet, to understand an oligopoly theory an examination of Cournot's basic construction is necessary.

Cournot's solution is based on the assumption that each duopolist believes that his rival will on on producing a definite quantity irrespective of the quantity he himself produces. It is apparent that each duopolist thinks he can calculate the quantity he should produce in order to maximize his profits when these conditions exist. Each producer can obtain his own individual demand function under these

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[1]See chapters vii and ix of Cournot's Researches.

circumstances by deducting the fixed quantity of the rival's output from the total quantity indicated by the market demand function for each price. From here he proceeds to equate his individual marginal revenue to his marginal cost. This is not, however, the reason the solution is determinate. Neither duopolist is likely to make the correct guess as to the quantity his rival will produce, because the rival produces the quantity which maximizes his profits on the same assumption with respect to his rival's behavior, and therefore does not keep his output constant. The pair of outputs obtained at first cannot last, since each firm will change its output.

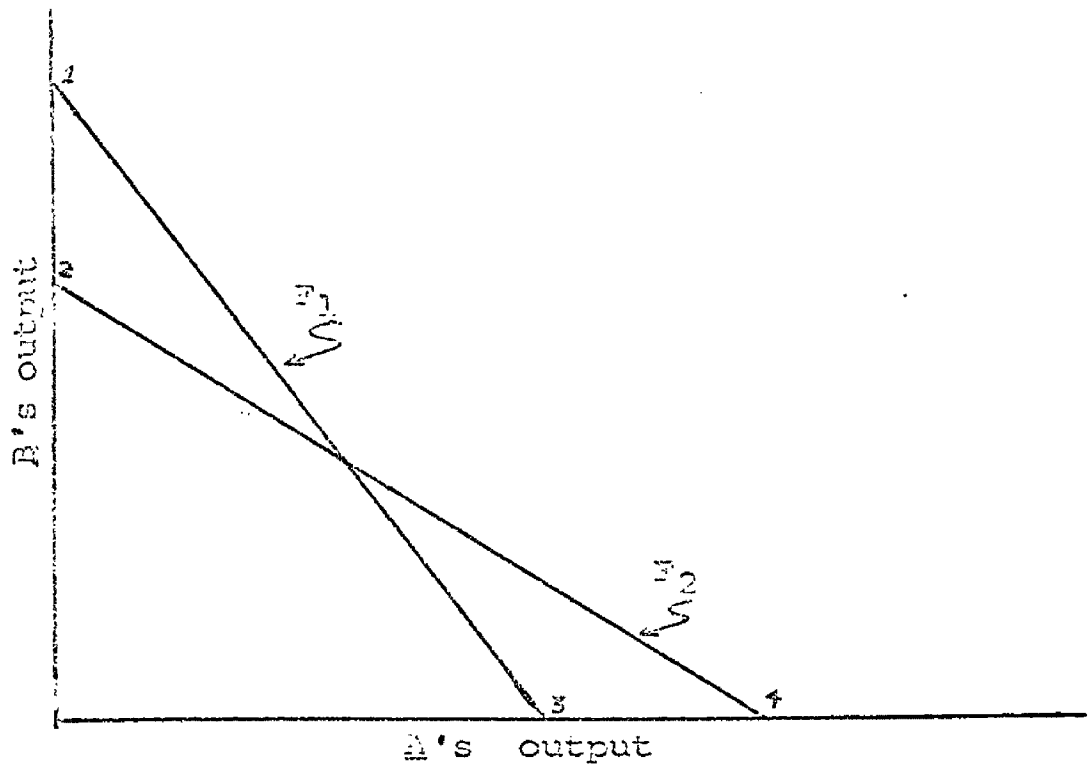
The main feature of Cournot's model is that each duopolist continues to assume that the other will not change his rate of output. As the two approach equilibrium they will be wrong, but when they reach equilibrium they are right. A produces a quantity which maximizes his profits on the assumption that B will go on producing his present output. Now B adjusts his output so as to maximize his profits on the assumption that A will go on producing his present output, which induces A to adjust his output, and so on. What Cournot proved is that these adjustments finally result in an output for A which he can go on producing on the assumption that B will continue to produce his "present" output because B will continue to produce an output

Which justifies A's output on the assumption he makes..This is a circular argument: B must be in equilibrium to be producing an output which is truly justified on the assumption he makes about A's behavior.

In this model the two producers are "right" for the wrong reasons. Each assumes his rival is following a policy of fixed output while in reality each producer is continually adjusting his own output to profit maximization. If they both have adjusted their output to the simultaneous output of the other it is because each producer has <u>not</u> followed a policy of producing a fixed output disregarding his rival's behavior.

For a graphical proof of this refer to figure 1.

Figure 1



The output of A is measured along the abscissa and the output of B is measured along the ordinate. The curve indicating A's output as a function of B's output is F_1, and the curve F_2 expresses B's output as a function of A's output. The shape of these reaction curves is given by the postulate that each duopolist maximizes his individual profits on the assumption that the other will go on producing his "present" output. Let us further assume that the demand function is linear (which Cournot did not assume) and that producers face constant costs at identical levels (which Cournot did assume).

Both functions decline monotonically, or, the more the one produces, the less will the other produce because the more to the left will his assumed individual demand function lie. The intersection of the two curves marks stable equilibrium. To the left of the intersection A's output falls and B's rises, while to the right of the intersection A's output tends to rise and B's to fall, so that equilibrium is restored in the event of disturbances. This must be so, for F_1 must intersect with the ordinate above F_2, while it must intersect with the abscissa to the left of F_2. Point 1 shows that output of B which would induce A to produce nothing, while point 2 shows the output which B produces if A produces nothing. The first of these two quantities is the competitive output for the industry and the

second quantity is B's monopoly output. The competitive output is greater than the monopoly output. The same analysis applies to A's output.

Since we assumed a linear demand function and constant costs, the reaction functions of F_1 and F_2 are linear. Therefore, the stable equilibrium point is the only intersection point. In addition, given identical cost functions, the outputs of the duopolists will be equal. From these assumptions the aggregate duopoly output is two-thirds of the competitive output, while the monopoly output is one-half of the output under pure competition. On analogous assumptions for undifferentiated oligopoly, the aggregate oligopoly output is $\frac{r}{r+1}$ times the competitive output, where r is the number of producers.[2]

Reaction functions may be linear even if the demand and cost functions are not as simple as assumed above. We begin by dropping the simplifying assumptions of the market demand function and the cost functions, except that we require the monopoly output of each firm should be smaller than will induce the other producer to produce zero. Without this assumption the Cournot problem is meaningless because there would be monopoly. With this assumption points 1, 2, 3, and 4 are still as in Figure 1. These points

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[2]William Fellner, Competition Among the Few, Oligopoly and Similar Market Structures, (New York: Knopf, 1949), p. 60.

stand for the outputs which the assumption predetermined. The reaction curves will not be linear, but they will still slope downward. This follows from Cournot's premise of rival behavior where a higher rival output means the lowering of one's assumed individual demand function.

Therefore, the reaction functions will intersect, and at least one of these intersections will be stable. (There may be only one intersection.) Cournot even implied that points 1, 2, 3, and 4 were arranged in this manner. He also implied one unique and stable intersection.[3]

It is most important to note, that as long as firms make the Cournot assumptions of their rival's output, the analysis cannot be adjusted to make firms right for the right reasons, but instead the analysis will turn out right for the wrong reasons. This leads to the problem of "leadership."

If producer A knows that producer B is producing along reaction curve F_2, he would not react along F_1. Producer A would instead select the point on F_2 which would be optimal from A's point of view. This point would ordinarily be where A's output is larger than B's and would result in a higher total output. Likewise, if B knows A is reacting along F_1, then, instead of reacting along F_2, he will select the point on F_1 which is optimal from his point of view. This would

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[3]Ibid., p. 63.

probably be to the left of the intersection point. Such procedures do not result in equilibrium, because when we assume they know each other's reactions, we assumed away the earlier behavior assumptions.

It is possible, though, that a producer would act on the assumption that the other reacts along his reaction curve which in fact happens. This results in leadership equilibrium. If A knows that B reacts along F_2 but B, instead of anticipating reactions along F_1, believes that A will go on producing whatever output he actually produces at the moment, then A will be able to select the point along F_2 which is optimal from his point of view, and this point will mark equilibrium. Likewise, if B anticipates reactions along F_1, and A reacts along F_1, then B's optimum point along F_1 will mark equilibrium. In the first case A is the leader, in the second case B is. These leadership equilibria are different from Cournot's intersection-point equilibrium.

In its essential points, the Cournot model can be regarded as the parent model for oligopoly analysis. This is not a result of Cournot's equilibrium propositions, though. The intersection point in Figure 1 will not express equilibrium unless firms react (which is doubtful) along F_1 and F_2. It is possible that a firm would assume that the other is following a policy of fixed output, but on the way to the equilibrium intersection-point it is bound to realize that its assumption was wrong. This would destroy the validity

of Cournot's reactions functions and the analysis based on them. In addition, no single equilibrium position would last for long in a world of shifting demand and cost functions.

Cournot's stability proposition is related to conditions which move the system away from the intersection-point without affecting the reaction functions. Equilibrium will be restored from such disturbances. The doubts which producers have of the realism of their assumptions of a rival's behavior will cause a disturbance which makes the system unstable. If these firms, for any reason, change their assumptions about their rival's behavior patterns, there is no tendency to restore the Cournot behavior pattern. For example, if one producer anticipates a small output adjustment of his rival instead of the fixed output policy, there will be no reason to return to the former Cournot behavior pattern. The Cournot assumption is merely one of the many assumptions a firm can make about his rival.

It would appear that similar criticism could be held against equilibrium theories in general abstract certain properties of the real world and their fruitfulness depends on how convenient it is to isolate these properties and to treat them separately as problems of disequilibrium. A person may be opposed to such arbitrary methodological distinctions, but he would have to admit that they are not meaningless distinctions because these problems of disequilibrium

are articulate and significant. On the other hand, to propose that the problems excluded from Cournot's duopoly analysis should be treated separately from those included in the model would be meaningless. This implies that it is fruitful to develop a separate kind of disequilibrium analysis of the disturbances arising from firms assuming their rivals do not follow a fixed-output policy. This further implies that the Cournot theory of the equilibrium point is not important, as "pure theory," but as a theory of imperfections.

Other analytical problems arise when we refer to the leadership equilibria developed from the Cournot model. Here no firm acts on incorrect and arbitrary assumptions. The follower believes correctly that the leader has adopted a policy of fixed output. The leader has adopted a policy of fixed output. The leader selects this level in full knowledge of the follower's reaction function. In short, the leader has the desire and the power to indicate the quantity he will produce and to keep on producing this quantity. The follower maximizes his profits in full knowledge of this. The leader selects his output quantity realizing that the follower will maximize his profits at that output. The leader chooses his output in view of the follower's behavior, and he also maximizes his profit at that output. Both firms in this case act on rival behavior based on correct assumptions.

On Cournot's assumptions the equilibrium is

determinate not only for duopoly but for oligopoly with any number of sellers. It is determinate not only for producers selling substitutes but also for differentiated products. If there is product differentiation, the boundaries of the oligopolistic industry may be defined as the extent to which conjectural interdependence applies. Cournot did not discuss differentiated oligopoly but the reaction functions will have to be downward sloping for substitute products too.

All intersection points of Cournot's reaction functions share the weaknesses of the intersection for undifferentiated duopoly. If conditions are changed by the introduction of more producers and by product differentiation, the equilibrium point still represents a position from which each producer believes his rivals will not change their output. Unfortunately no producer acts in this fashion and equilibrium will not be maintained.

A further problem that enters when more producers and product differentiation exist is the leadership equilibrium. If there are more than two producers the superiority of the leadership equilibria over the Cournot equilibrium with its intersection of reaction functions no longer exists. The Cournot reaction function of each follower is based on the assumption that all his rivals produce a definite quantity, no matter what he does. This is true for the leader but not for the followers. If each follower sets up Cournot

reaction functions and lets the leader choose his output in view of these, then each follower acts on incorrect assumptions about the other followers.

The Cournot intersection remains subject to the criticism discussed above, but leadership equilibria have the same properties as under undifferentiated duopoly. The leadership equilibrium rests on correct assumptions if it is postulated that the leader has the desire and the power to keep on producing a definite quantity of his own choosing. Yet, it is quite arbitrary to postulate that the leader has the desire and the power to act this way. The leader might test the reaction of his rival to other methods of asserting his will.[4]

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[4]Ibid., pp. 58-71. Professor Fellner's review of Cournot's problem is quite clear and complete. He rightly criticizes the classical duopoly theory for postulating individual profit maximization on specific but completely arbitrary assumptions. Later in his book he acknowledges that the rivals recognize their interdependence but this does not mean that there is even a tendency towards joint maximization.

CHAPTER IV

RE-APPRAISAL OF COURNOT'S DUOPOLY PRICE

More than a hundred years ago, Antoine Augustin Cournot gave an unappreciative world his Recherches. Since that time many of his ideas have been absorbed into everyday economic thought. Alfred Marchall wrote in the preface to the First Edition of his Principles of Economics:

> ....two kinds of influences have affected, more than any other, the substance of the views expressed in the present book; but their form has been most affected by mathematical conceptions of continuity, as represented in Cournot's Principes mathematiques de la theorie des richesses.
>
> Under the guidance of Cournot, ...I was led to attach great importance to the fact...that... the demand for a thing is a continuous function.[1]

In spite of the special attention that Cournot has received in recent years, it seems to several economists that the strength and weakness of one important feature of Cournot's work is not yet fully understood.[2] The feature that they refer to is Cournot's treatment of duopoly price.[3]

Professor A. J. Nichol of Duke University claims that the long-prevailing disagreement[4] on this subject is in

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[1]Alfred Marchall, Principles of Economics. (1st ed.; New York: Macmillan Co., 1890) pp. ix-x.

[2]A. J. Nichol, "A Re-Appraisal of Cournot's Theory of Duopoly Price," Journal of Political Economy, Vol. 42 (February, 1934), p. 80.

[3]Cournot, Researches, chapter vii, pp. 79-89.

[4]Beginning with J. Bertrand's review of Walras and Cournot Journal des Savants (September, 1883) pp. 499-508.

reality an unrecognized disagreement regarding a certain "institution" of the market. (1) In some large commodity exchange markets, prices are both bid and asked; the prices are determined by bargaining between buyers and sellers. (2) In the second class of markets, e.g., any retail market, prices are almost always named by sellers. Buyers scarcely ever get the opportunity to bid. In this market the buyer decides how much is to be purchased at any price quoted by the sellers. In this manner the buyer has an indirect influence on the price, but does not have a direct voice in price determination. (3) The third class of markets is where buyers name prices. Sellers are left to determine the quantity to be sold at any particular price named by buyers. An example of this last class is the southern tobacco farmer, bringing his product to market. A buyer from one of the large tobacco companies offers him a price. If the farmer does not like the price he can try to get a higher price from another buyer or he can take his tobacco back to the farm.[5]

In a free competitive market any equilibrium price, whether temporary or permanent, is unaffected by the particular way in which prices are named. When buyers and sellers are numerous, and each individual buyer or seller has little effect on price, an equilibrium price is determined where

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[5]Nichol, _op. cit._, p. 81.

the demand and supply curves intersect. If the market price rises above this intersection, the total quantity offered exceeds the quantity demanded. Then reductions in the price offers of sellers, or lower bidding by buyers tend to bring the market price back to the intersection. If the market price falls below the intersection, the quantity demanded is greater than the supply. Higher bidding or advances in price tend to bring the price back to the intersection. Whatever the prices bid or asked, the market price tends toward the same equilibrium level.

In a purely monopolistic market, the way in which prices are named is of little significance from the viewpoint of general analysis. A monopoly may either directly name the price or indirectly control the bidding by limiting the quantity offered to the buyers. The final profit is theoretically the same, regardless of the manner in which prices are announced.

It also appears on first glance of little significance in the competitive dealings of two sellers with many buyers, whether sellers or buyers name prices. This turns out tobe a key to the interpretation of Cournot's duopoly theory. If only buyers name prices, his theory is easy to understand. Assume that in a given market, the highest price (p), bid among buyers for any particular quantity (D) of a given commodity conforms with the following equation:

$$p = -\frac{1}{5000} D + 90.$$

The demand for the commodity may be represented as in Table I.

TABLE I

Demand Schedule

Quantity offered	Highest price bid (cents)
150,000	60
175,000	55
200,000	50
225,000	45
250,000	40
275,000	35
300,000	30
325,000	25
350,000	20
375,000	15
400,000	10

We assume that it is the custom in this market that only buyers bid and the sellers only determine how much to offer. To illustrate Cournot's theory, we assume that there are only two producers, M and N, who could produce unlimited quantities under constant costs. M's costs are 20 cents per unit; N's only 10 cents. If the two producers combine into a monopoly only M's plant would be operated. To make the greatest profit then, M and N could offer 200,000 units and would accept the highest price of 50 cents. If N's cheaper facilities are divided among a large number of smaller producers, 400,000 will be offered, and the price bid would be 10 cents, equal to the cost of production per unit. If M

and 2 independently try to make as much money as possible in competition with each other, the price, according to Cournot, will finally settle at an intermediate level between 50 cents, the monopoly price, and 10 cents, the price of unlimited competition.

In this market structure neither seller has a direct opportunity to name a price. Each seller has thepower to influence the bidding by adjusting the quantity he offers for sale. Each seller may independently try to exert his influence to his advantage. If he is unable to predict his rival's future output, Cournot claimed each seller bases his calculations at any given time on the quantity which his rival is offering at that time. A series of adjustments may be necessary, and the ultimate result of these adjustments is the same, whether they are made concurrently or alternately. When the equilibrium price is reached, each seller makes a maximum profit in view of the quantity which his rival is selling..[6]

The same solution is obtained by applying cournot's formulas[7] to this problem.

(1) $f(D) + D_1 f'(D) - \phi'_1(D_1) = 0.$

(2) $f(D) + D_2 f'(D) - \phi'_2(D_2) = 0.$

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[6]Cournot, op. cit., p. 81.

[7]Ibid., p. 85

(3) $D = D_1 + D_2$.

($f(D) + D_1 f'(D)$ represents marginal revenue. $\phi'_1(D_1)$ is the marginal cost.)

The equivalent of f(D) in this problem is given by $-\frac{1}{5000}D + 90$.

With constant unit costs, $\phi_1(D_1) = 20D_1$; $\phi_2(D2) = 10D_2$.

Therefore,

(4) $\phi'_1(D_1) = 20$; $\phi'_2(D_2) = 10$

Solving:

(5) $D_1 = 100,000$; $D_2 = 150,000$.

(6) $p = 40$

Professor Chamberlin presented a new theory of duopoly in which he assumed that sellers always anticipate each other's actions.[8] Chamberlin uses a model where there are no costs of production. His conclusions may apply to cases in which both sellers' cost schedules are exactly the same, but in most cases producers do not operate under the same cost conditions. For this reason the new theory may not be completely accurate. Cournot's theory is applicable, no matter how great the difference in costs of production. It stands as the best general analysis of limited competition when only buyers name prices.[9]

Did Cournot intend to limit his theory of duopoly to

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[8]E. H. Chamberlin, "Duopoly: Value Where Sellers Are Few," The Quarterly Journal of Economics, vol. 44, 1930, p.83.

[9]Nichol, op. cit., p. 87.

where only buyers name prices? In Cournot's treatment of pure monopoly, the monopolist seller named prices while buyers determined quantities.[10] When Cournot wrote the chapter on limited competition he included nothing to suggest that he might have changed any procedures. Many theorists speak of market price as if it were obedient to abstract principles; that it needed no human voice or hand to proclaim it.

Professor Nichol interprets Cournot as making an unconscious change in his assumptions.[11] Nichol claims there is an indication of the change in the form of Cournot's equation of demand. When treating monopoly,[12] Cournot assumed $D = F(p)$; in treating duopoly,[13] the revers notation, $p = f(D)$. These two equations may represent the same relationship, but when values of p in $D = F(p)$ are substituted to determine corresponding values of D, it is an indication that the calculator is thinking of amounts which buyers will take at given prices, i.e., established by sellers. Cournot's analysis of monopoly confirms this. Opposed to this, if one substitutes values of D in the right-hand side of $p = f(D)$, to solve for p, this indicates one is thinking of prices

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[10] Cournot, Researches, p. 56.

[11] Nichol, op. cit., p. 87.

[12] Cournot, op. cit., p. 56.

[13] Ibid., p. 90.

which buyers will pay for given quantities. No conclusion should be based on such thin evidence, but it is clear that Cournot had in mind a situation where sellers determined quantities but did not compete with each other by quoting different prices. "In this case," said Cournot, "the price is necessarily the same for each producer."[14]

This price is not one named by either seller, for if sellers name prices, the fact that one of them changes his price does not compel the other to do likewise. If sellers name prices either one may maintain his price at any level. Edgeworth showed in his problem,[15] when sellers name prices, and sellers are few, a perfectly uniform price is not a necessity. The perfect uniformity of price in Cournot's model is not explained, unless prices are directly determined by buyers. Any agreement between sellers is carefully abstracted.[16] Cournot's discussion is logical only if it is assumed that both sellers accept the highest price bid for their combined offerings. Then, one duopolist can influence buyers by a change in the quantity he offers. This is the same as forcing the price he desires upon his rival.

Cournot's chief defender, Luigi Amoroso, also assumed

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[14]Ibid., p. 79.

[15]F. Y. Edgeworth, Mathematical Psychics, (London, 1881), reprinted by London School of Economics and Political Science (1932).

[16]Cournot, op. cit., pp. 79-80, 83.

price changes were brought about by changes in quantity. On this, Amoroso confirmed Cournot's theory, adding new details of his own.[17] As Edgewater pointed out, Amoroso, too, was unaware of the implied assumption that only buyers name prices.[18]

Pareto's criticism of Cournot's mathematics is familiar tothose acquainted with the literature of the subject. His view of the conditional validity of Cournot's theory has not been discussed by any other writer.[19] Pareto was not aware of the underlying assumption that only buyers name prices. He therefore introduced another assumption which, he felt, was a departure from pure competition, but was necessary to correct Cournot's problem. According to Pareto, each competitor is to let his rival sell any particular quantity the rival desires. For this, though, it is necessary to inquire how he may prevent the rival from doing so, especially in a market in which buyers name prices. The methods of prevention and their shortcomings are as follows:

(1) Competitive advertising is not necessary, since in the Cournot problem the products offered by the two competitors are identical. (2) No seller can keep his rival

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[17]See Luigi Amoroso, Lezioni di economic matematica, (Bologna, 1921), pp. 254. See also Economic Journal, "The Mathematical Economics of Professor Amoroso," EJ Vol. 32, p.402.

[18]Edgeworth, Economic Journal, XXXII, p. 402.

[19]Chamberlin, op. cit., p. 95-96.

from selling what he wants, even if he accepts lower bids. If either seller strays from Cournot's assumption and accepts less than the highest price bid, the other can do likewise and still dispose of his product. One seller may injure the other but this will not prevent th sale of any given quantity. (3) If one seller does dump a large quantity of his product on the market thus forcing down the bidding, this does not force his rival to sell a smaller quantity. In the long run the rival may choose to restrict output, but in Cournot's market, only immediate effects are considered in adjustments. In conclusion, it is false to say that Cournot's market "allows" the other producer to sell any quantity. Pareto was incorrect, for neither seller can prevent his rival from selling. For lack of power, each seller is forced to acquiesce in a division of sales. To monopolize the market by accepting a loss, or a profit less than the maximum, is discounted by Cournot's assumption that each seller maximizes profit on his immediate transactions.

Cournot's assumptions would have been much clearer had he had the advantage of competent criticism during his own lifetime. Much controversy could have been avoided. As it was, the French economist Bertrand, ignored Cournot's Conclusions and thought of a situation in which sellers named prices.[20]

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[20]Irving Fisher, "Cournot and Mathematical Economics," The Quarterly Journal of Economics, XII, (January, 1898), p. 124.

Edgeworth considered Bertrand's discussion limited by "uniformity of price."[21] In Cournot's market, a uniform price was practically a necessity. Pareto must not have understood the simplicity of Cournot's analysis and therefore added an unnecessary assumption.

Opposed to this, some readers have given Cournot more credit than his model deserves because they have seen examples of his analysis in the business world. Considerable ingenuity has been used to fit Cournot's theory to markets in which sellers name prices. Such efforts fail to give a complete picture of the situation for a detail here or there is adjusted. To show how ridiculous such a market becomes can best be shown by an example of one of Cournot's interpreters. Professor Zeuthen wrote:

> The characteristic feature of Cournot's solution is really that if it is to be correct, it must be based on the assumption that a price reduction on the part of one monopolist will attract the unsatisfied wants more easily and quickly than it would attract the customers of his competitors."[22]

Professor Zeuthen is a poor interpreter of Cournot. As pointed out, Zeuthen missed one of the most fundamental features of Cournot's model:

> Cournot does not consider price changes on the part of the monopolists as motivating influences. Initial causes in his problem are always variations in quantity, which automatically bring

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[21] Edgeworth, op. cit., p. 47.

[22] Frederik Zeuthen, Problems of Monopoly and Economic Warfare, (Kelley: London, 1930), p. 29.

about changes in price.[23]

Professor Zeuthen also missed another important point. Cournot was very explicit when stating that both producers always sold at the same price. Edgeworth's market has the tendency of buyers drifting "en masse" from one seller to another if there is a difference in price.[24] Zeuthen assumed differences in price,[25] and then attempted to explain what Cournot has explained by perfectly uniform prices. Changes in Cournot's market are not small changes, either. Professor Zeuthen considered only very small price changes,[26] which if for no other reason, make his treatment inadequate. When price changes are very slight and one duopolist lage behind his competitor in reducing his price, it may be, as Zeuthen suggests, that the price leader gets all the new business while the other duopolist only keeps the business he has been doing.

If price changes are more pronounced, it becomes increasingly improbable that the seller who is slow to reduce his price will retain all his previous sales. In Cournot's model it may be assumed that the duopolists reduce prices simultaneously to the same level, or that one immediately

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[23] Erich Schneider, Reine Theorie monopolistischer Wirtschaftsformen, p. 173.

[24] F. Y. Edgeworth, Papers Relating to Political Economy, (Franklin B., 1925), Vol. I, p. 119.

[25] Zeuthen, op. cit., pp. 29-31.

[26] Ibid., pp. 30-31.

follows a reduction of the other. With these conditions, neither one has any appreciable advantage over his rival.

Another fault with Zeuthen's revision of Cournot is the process of oscillation toward an equilibrium price. This oscillation involves not only reductions in price, but also advances. Zeuthen only considered half of his own problem. He introduced an original assumption regarding the behavior of buyers when the price is lowered, but omitted their behavior when the price is raised. The only way to test the applicability of Cournot's solution in a market where sellers name prices is to consider all the necessary conditions. Such conditions become infinite, but since many critics see more than Cournot's analysis entails, we will examine the possibilities available.

To illustrate the possible interrelationships let us assume the same numerical example as earlier. M's costs are 20 cents per unit, and N's are 10 cents per unit. The equation for demand may be expressed as D = -5000p + 450,000. The demand schedule is thus:

Demand Schedule

Price Named by Sellers (cents)	Quantity Taken
60	150,000
55	175,000
50	200,000
45	225,000
40	250,000
35	275,000
30	300,000
25	325,000
20	350,000

In the first place, both sellers must always sell at the same price, otherwise a complication is introduced which is foreign to Cournot's problem. This assumption is generally made when two competitors compete.

In the second place, a very artificial method of changing prices must be assumed. In Cournot's original theory, the price bid by buyers changes automatically with changes in quantity. At the same time, the price bid for the output of both ducpolists is always the same. If cournot's theory is to be restated, in order to apply to a market where sellers name prices, the two competing sellers must move their prices in the same direction at the same time and by the same amount. Without competitive bidding among buyers, there is no third force to bring M's price and N's price to any particular uniform level. The uniformity of price that Cournot's equilibrium necessitates, can be maintained only by some sort of agreement or understanding between sellers. The agreements can be as follows:

1. In each particular stage of adjustment, one seller begins a price change, and the other immediately follows. In succeeding stages of adjustment M and N alternate in accepting each other's prices until an equilibrium is reached.

2. In each stage of adjustment it may be imagined that both sellers compromise in the matter of price. The equality between the total amount offered and the total amount demanded is not a necessity in any market except at an

equilibrium price. Rivals do not name prices with the purpose of insuring the sale of each other's stock. Such agreements at disequilibrium prices do form a series which finally results in an equilibrium price.

3. A combination of steps 1 and 2 may also lead to Cournot's equilibrium.[27]

Changes in price from any of these conditions, instead of through competitive bidding among buyers, are governed by assumptions completely foreign to Cournot's theory. In fact, Cournot carefully eliminated any agreement or understanding between sellers.[28] These conditions listed are highly improbably for the amount of co-operation required would be more naturally invested in a perfectly monopolistic combination.

Professor Nichol claims that this difficulty alone is sufficient to invalidate Cournot's solution in any market in which sellers name prices.[29] Some admirers of Cournot try to pass over this difficulty, so attention is now directed to a special system of limitation of sales necessary to the correctness of Cournot's solution when sellers name prices. Vilfredo Pareto put forth this system. Pareto thought that sellers named prices in Cournot's market.

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[27]Nichol, op. cit., p. 95.

[28]Cournot, Researches, pp. 79-80, 83.

[29]Nichol, op. cit., p. 96.

Pareto first stated his technical mathematical objections to Cournot's theory,[30] and then in simpler terms tried to show that Cournot had really only solved an insignificant artificial problem. Pareto's expose falls down in a market where buyers name prices. He does deserve credit for outlining some of the conditions which have to be met when sellers name prices in Cournot's problem.

The crux of Pareto's notion is mutual limitation of sales in deference to certain arbitrarily recognized "rights" of one's rival. According to Pareto, if Cournot's theory is correct, changes in sales of either competitor are made alternately. In step toward the final equilibrium each seller is assumed to limit his own sales so as not to interfere with the other's sales. When sellers name prices it is not enough to assume, as Chamberlin suggests, that the less active seller "will hold his supply fixed."[31]

If Cournot's solution is correct, the other seller also must not take his rival's momentarily recognized share of the business away from him. To reach equilibrium the following assumptions must be carried out:

1. Each duopolist, in his turn, leads in changing the price.

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[30]See Encyclopedie des sciences mathematiques, Tome I, Vol. IV, p. 608.

[31]Chamberlin, op. cit., p. 72.

2. The rival duopolist in each step immediately follows the price change.

3. With each adjustment, the price leader only takes up the "slack" in the new demand price. He does not interfere with his rival's old quantity sold.

4. In each new adjustment the less active duopolist sell no more than he sold in the period immediately preceding the adjustment.

5. Subject to these, each duopolist calculates a new price to maximize profits.

Edgeworth assumed each seller would cut into the sales of his rival by quoting a slightly lower price. Ignoring this assumption, the improbability of any such series of events leading to equilibrium can be shown by contrast with a similar series of steps without artificial limitation of sales. Assume price is quoted and that neither competitor makes any effort to attract business away from his rival by quoting a lower price. Further assume that each seller sells at the price of the moment the quantity which buyers ask of him. Since buyers have no preference for either seller when the product is identical, they may be assumed to divide their purchases between the two sellers equally.

The fact that one seller may be slower than the other in changing his price does not alter this result for few buyers become immediately aware of differences in price. In addition, if we assume that buyers act rationally, the fact

that one duopolist offers buyers more than half the total quantity demanded at the price of the moment will not by itself cause buyers as a group to depart from an equal division of their total purchases. Therefore, neither duopolist can sell more than half of the total quantity demanded at any price at which his rival also is selling, unless his rival's sales are limited to less than half of the total quantity demanded. Either duopolist can prevent his rival from selling more than half if he offers half himself, assuming the both will sell at the same price.[32]

In Cournot's original problem there was no need to use different symbols for the quantity offered and the quantity sold. Price was determined by competitive bidding so all goods offered may be assumed to be sold, no matter what price. The use of D_1 and D_2 to represent quantities offered and quantities sold must be restated when applied to a market in which sellers name prices. When sellers name prices the total quantity offered is not necessarily equal to the total quantity sold, except at equilibrium.

Cournot's theory assumed that each seller determines the quantity he will produce with the purpose of influencing the bidding of buyers. When applied to a market where sellers name prices the determination of quantities must depend on the

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[32]Nichol, *op. cit.*, p. 99.

arbitrary concessions on the part of one or both of the competitors. In this case, each duopolist has a direct and predictable influence on the sales of his rival, even when they sell at the same price. Either one may sell more than half of the total quantity demanded, if his rival limits his offerings to less than half the total quantity demanded.

Professor Nichol therefore asserts that it isincorrect to apply Cournot's solution to any market in which sellers name prices. Under these conditions, the solution is dependent upon (1) an improbable system of changing prices, and (2) a more improbable system of limitation of sales. Sellers may mention or suggest prices, but this is not important as long as prices are directly determined by competitive bidding among buyers.[33]

For many years economists accepted Edgeworth's unfavorable judgment of Cournot's theory, not realizing Edgeworth failed to recognize the conditional validity of Cournot's conclusions.[34] More recently, however, Edgeworth's conclusions have been criticized by many economists.[35] Edgeworth should be given some credit for he was consistent in assuming that

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[33]Nichol, op. cit., p. 103.

[34]Uniformity of price in Cournot's problem is dependent on the more fundamental assumption that only buyers name prices.

[35]See Joseph Schumpeter, E.J., Vol. 38, p. 369-70; E. H. Chamberlin, AER, Vol. 44, p. 72-79 (op. cit.)

sellers name prices, and buyers determine quantities. The difference between Edgeworth's[36] and Cournot's duopoly discussion boils down to the supposition that one man's sales remain constant, or that his price remains constant. [37] The question is one of whether the model is realistic inasmuch as it conforms with the institutions of buying and selling.

It is a common observation that when a few deal with many, the few almost always name prices. One among many does not expect any price he may offer to be taken seriously. The lack of organization keeps the multitude from having a direct voice in price determination. Its influence arises from the quantities it buys or sells. Bertrand and Edgeworth recognized this fact, as Cournot must have. Perhaps this is the reason Cournot failed to explicitly state it as one of his assumptions. This middle-ground that Cournot's duopoly covers lies between pure monopoly and pure competition. For this reason any statement pertaining to either one must be modified when applied to duopoly.

Cournot deserves credit for recognizing, long before anyone else, that duopoly was a problem distinct from monopoly or unlimited competition. It appears, though, that Cournot

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[36] Edgeworth, op. cit., p. 119.

[37] Chamberlin, The Quarterly Journal of Economics, Vol. 44, op. cit., p. 72.

actually got no further than a description of a sort of double or multiple auction sale.[38]

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[38]Edgeworth, Economic Journal, XXXII, op. cit., p. 402.

CHAPTER V

COURNOT'S INFLUENCE UPON ECONOMIC THOUGHT AND LEON WALRAS

Paul A. Samuelson once remarked on the role of Cournot in the history of economic thought:

> Who would want to deny that Cournot, writing in 1838, had an analytical power and freshness that is breathtaking? But who in his right mind could argue that Cournot had been a great force on the history of ideas...? Except through possible indirect influence of his teachings, Cournot's impact on ideology must surely have been negligible.[1]

A preliminary comparison of Antoine Augustin Cournot's Recherches (1838) to Leon Walras' Elements (1878) suggested that Cournot's influence on the history of ideas is much greater than Samuelson believed.

Although all of Cournot's publications and some of his mathematical treatises give evidence of economic thought, it is scattered, fragmentary, often merely illustrative or suggestive. His works were usually written to express some other noneconomic idea, and generally were of little significance for the development of economic ideas. Three of Cournot's economic works would therefore provide the logical focus of attention for any such investigation.[2] Further

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[1]Paul Samuelson, "Economists and the History of Ideas," American Economic Review, March 1962, p. 5.

[2]Jon V. Plantz, "Cournot's Recherches: Some Insights on its Influence upon the Development of Economic Thought," The Southern Economic Journal, Vol. XI, no. 3, Summer 1964, p. 195.

study shows that Cournot's fundamental economic doctrines were contained in his Recherches,[3] with merely different forms in the Principes[4] and Revue Sommaire.[5] In pointing out the influence of Cournot's doctrine and mathematical method on Walras, attention will be concentrated on Cournot's Recherches. In looking at Walras, we will examine his Elements,[6] since this contains the foundation of his reputation in the development of economic ideas.

Cournot recognized the complexity of the economic system and the interdependency of its variables. In order to solve them he realized that it was indispensable to take the entire system into consideration. This is precisely what Walras later did. Like the post-Marshallians, Cournot further believed that this surpassed the powers of mathematical analysis and practical methods of calculation, since the number of variables was infinite.

In order to fully appreciate Cournot's performance in his famous chapters on the Law of Demand and on Monopoly, it is necessary to remember that mathematically-oriented works were not received as such, but were judged on literary

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[3]A. A. Cournot, Recherches sur les principes mathematiques de la theorie des richesses (paris: Hachette, 1838). English translation by N. T. Bacon in Economics Classics (New York: Macmillan Company, 1897).

[4]Cournot, Principes de la theorie des richesses (Paris: Hachette, 1863).

[5]Cournot, Revue sommaire des doctrines economiques (Paris: Hachette, 1877).

[6]Leon Walras, Elements of Pure Economics, (Great Britain: Royal Economic Society, 1954) trans. William Jaffe.

style. This has special significance for mathematical works of Cournot's time, since lack of sufficient literary translation may have tended to suppress dissemination of concepts originating from the mathematical economists, and retarded development of economics in general.

Cournot was aware of this problem when he published his Recherches but still intended to treat the subject matter of political economy in a theoretical manner by means of mathematical analysis.

> But the title of this work sets forth not only theoretical researches; it shows also that I intend to apply to them the forms and symbols of mathematical analysis. This is a plan likely, I confess, to draw on me at the outset the condemnation of theorists of repute.[7]

Indeed it did. It is therefore surprising that although his Recherches received such a chilly reception in economic circles, its influence had a direct and demonstrable impact upon Walras and his Elements. The remainder of this chapter will attempt to analyze the extent of Cournot's influence upon Walras.

Before Walras ever thought of becoming an economist he studied Cournot's Recherches. At 19, in an attempt to correct a mathematical deficiency that kept him from entering engineering school, Walras read, learned, and digested Cournot's

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[7]Cournot, op. cit., p. 2.

Recherches. Considering his later economic achievements it is fortunate that he failed the exam. Perhaps the study of the Recherches only aided in the study of theoretical economics.[3]

Ten years later Walras published one of his earliest economic works. It was intended to be a review of Cournot's Principes, but mainly covered the Recherches. In the years that followed, Walras investigated the Recherches several more times, but these investigations were never published.[9] In these studies he was concerned with Cournot's criticism of supply and demand. The methods used in these unpublished manuscripts is nothing on the level of the analysis contained in Walras' Elements.

Walras was always anxious to share his ideas with Cournot on the applicability of mathematical principles in economics. As Jaffe discovered,[10] Walras wrote to Cournot in 1873 and invited him to comment on his newly written mathematics of exchange. In the same letter Walras asked for assistance in preparing his Elements for publication. The next year his Elements was published and Walras

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[3]Plantz, op. cit., p. 196.

[9]William Jaffe, "Unpublished Letters of Leon Walras," Journal of Political Economy, Vol. 43 (1935), pp. 192-93.

[10]William Jaffe, "Unpublished Letters of Leon Walras," Econometrica Applicata, January 1952, pp. 5-33.

dedicated the folio to Cournot.[11]

In the Elements he discussed its relation to the Recherches, demonstrating that this new work used mathematics on a simpler scale. Walras also pointed out that his objective was to treat the interdependency of economic variables under static conditions. In addition, competition was used as the general case and monopoly considered special. In conclusion, Walras claims to have borrowed nothing from Cournot but his method, which prompted him to mention Cournot, since Cournot's work had received so little recognition.

> I am indebted to...Augustin Cournot for the idea of using the calculus of functions in the elaboration of this doctrine. I have publicly acknowledged this fact in my first essays and on every suitable occasion ever since.[12]

Walras' work, like Cournot, was not well received in France. This might have tended to make Walras feel closer than ever to Cournot since they both attempted to find useful application of mathematical principles in economics.[13]

Cournot had developed a static partial equilibrium analysis to show that the prices that result were an approximation that took little account of secondary changes in economic variables. The ceteris paribus assumptions limited price effects to what he called changes of the first

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[11]Planta, op. cit., p. 197.

[12]Leon Walras, Elements of Pure Economics, (Great Britain: Royal Economic Society, 1954) trans. Wm. Jaffe, p.37.

[13]Planta, op. cit., p. 198.

order. The influence of a change in demand for a good on the prices and quantities of other goods transacted were noted but omitted for the sake of simplicity. Cournot was aware that his system was merely an approximation, but argued that important price influences were derived by the interaction of demand and supply on the good itself. Other influences introduced error into the price determination, but since their importance is continually decreasing, the error is slight. Such a partial equilibrium approach was able to render a useful approximation of prices.[14]

Cournot started with his doctrine of mutual interdependency of economic variables and consistently repeated it throughout his Researches. The development of the mathematical principles of the theory of wealth was based on Cournot's concept of relative value which emphasizes mutual interrelationship.

> Just as we can assign situation to a point by reference to other points, so we can assign value to a commodity by reference to other commodities. In this sense there are only relative values. But when these relative values change, we perceive plainly that the reason of the variation may lie in the change of one term or of the other or of both at once.[15]

It is interesting to note that Cournot's entire chapter on relative and absolute value was reprinted in

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[14] Cournot, *op. cit.*, pp. 150-51.

[15] *Ibid.*, pp. 18-19.

Walras' Elements in the first (pp. 163-8) and second (pp. 451-7) editions.

At the end of his price theory analysis, Cournot reviewed his achievements, showing their partial equilibrium nature.

> So far we have studied how, for each commodity by itself, the law of demand in connection with the conditions of production of that commodity determines the price of it and regulates the incomes of its producers. We considered as given and invariable the prices of other producers; but in reality the economic system is a whole of which all the parts are connected and react upon each other...It seems, therefore, as if, for a complete and rigorous solution of the problems to some parts of the economic system, it were indispensable to take the entire system into account.[16]

Realizing that adjustments in one market might disturb equilibrium in another, Cournot employed a partial equilibrium analysis under ceteris paribus assumptions. Since Walras had studied the Recherches he expected Cournot to expand the theory to encompass the entire economy. When this did not happen with Cournot's publication of Principes in 1863, Walras criticized him for not completing a definite theory of exchange value.[17]

The reason Cournot did not complete such a theory was because he believed it mathematically impossible.

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[16]Ibid., p. 146.

[17]Leon Walras, "Compte-rendu des 'Principes de la theorie des richesses' par M. Cournot," Independent de la Semaine (13 juillet 1863). Appendix to the 1938 reprint of Cournot's Researches, op. cit., p.231.

Compared to Cournot, though, Walras was a poor mathematician. Nevertheless he was able to algebraically set up an equilibrium system by simply using different assumptions. Cournot's general equilibrium system was based on explicit empirically grounded equations for all the goods and services in real markets.[18]

> But this would surpass the powers of mathematical analysis and of our practical methods of calculation, even if the values of all the constants could be assigned to them numerically.[19]

The reason given for Cournot's inability to expand his model of determining exchange rates to formulate a general equilibrium model was the poor reception of his Recherches.[20] Cournot's analysis consisted of proceeding from the simple to the complex. When his partial equilibrium analysis was rejected he saw no reason to proceed to a more complex theory. Cournot had hoped that the essence of the Recherches might appeal to contemporary mathematicians, but this also was not the case. He therefore turned to writing textbooks and mathematical studies until he developed a cronic eye disorder.[21]

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[18]Plantz, op. cit., p. 199.

[19]Cournot, op. cit., p. 127.

[20]Plantz, op. cit., p. 199.

[21]Ibid., p. 200.

Cournot never completed his general equilibrium analysis so it was left to Walras. Eleven years after reviewing Cournot's Researches in 1863 Walras published his Elements, which took Cournot's analysis and developed it to show the interaction of economic variables within a system.

In attempting to expand on Cournot's demand theory Walras ran into problems. Cournot's theory established, on empirical bases, a functional relationship between the price of a good and the quantity taken. This made it a ceteris paribus construct and therefore of little use in developing a system of mutual interrelationships among variables including quantities demanded and prices paid. In the second place, Walras thought Cournot's demand function did not explain the origin of exchange value. Walras realized that Cournot's demand function pertained to two goods. When the demand curve, representing the amounts demanded as a function of price, was used for two or more goods it became merely an approximation. Cournot was aware of this and indicated that demand was a function of the price of the good and of all other goods in each market.

Cournot's demand was a function of utility and the price of the good. Since demand was derived from the market, whatever factors caused it, there was no necessity to explain demand behavior in the market. He thought it impossible to discover the causes of demand because utility was unmeasurable.

Opposed to this, Walras began with total utility as a function of the quantity consumed. This rareté (synonymous with marginal utility and diminishing as consumption increased required each consumer to maximize his satisfaction (total utility) in consumption by altering the quantities consumed at different prices. This made demand a dependent variable. The demand function of a consumer for two goods is thereby derived from his utility schedule.[22]

Walras tackled the problem of measuring utility after Cournot failed. He devotes but one page to the problem and solution. This is in Lesson 8 with his utility or "want curves."

> The above analysis is incomplete; and it seems impossible at first glance, to pursue it further, because intensive utility, considered absolutely, is so elusive, since it has no direct or measurable relationship to space or time, as do extensive utility and the quantity of a commodity possessed. Still this difficulty is not insurmountable. We need only assume that such a direct and measurable relationship does exist, and we shall find ourselves in a position to give an exact mathematical account of the respective influences on prices of extensive utility and the initial stock possessed.
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> I shall, therefore, assume the existence of a standard measure of intensity of wants or intensive utility...[23]

This is as far as Walras ever went with the utility measurement problem. Not only did he accept Cournot's

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[22]Ibid., p. 201.

[23]Walras, op. cit., p. 117.

conclusions, but he compared his derived demand function to Cournot's.

> This is precisely the equation which Cournot posits a priori in his Recherches sur les principes mathematiques de la theorie des richesses (1838), and which he calls an equation of demand or sales (debit.) It has a wide range of useful application.[24]

While working on his demand function, Walras encountered another problem that Cournot had covered. This was the problem of continuity. Cournot had held that, as the market expanded and became more efficient, people with different tastes and incomes assure continuity in the demand function. But Walras' demand function was based on nonaggregative utility functions rather than observations. He therefore fell back on the excuse that Cournot used in his mathematical models--that the aggregation of individual demand functions only approximated a continuous function.[25]

In Walras' two-goods model of equilibrium he used Cournot's demand function. However, there was a major difference in the ways in which each used it. Both Cournot and Walras dealt with the quantity demanded as a function of price. Walras did this because there were only two goods, one of which was the numeraire (value set for the good in terms of price). Walras' demand function worked for any

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[24] Ibid., p. 109.

[25] Ibid., p. 95.

number of goods since the quantity of the good consumed was a function of the prices of all other goods facing the consumer. Therefore, Walras developed a utility approach with price a dependent variable. For three or more goods he assumed given given prices. However, Walras felt that consumption alone did not entirely determine individual utility functions and thereby demand curves. He therefore returned to his observations of consumer behavior as the source of the demand function.

As shown, Cournot provided Walras with a statement of the general equilibrium problem and the demand theory for its development. From this Walras was able to create his theory of subjective value. The influence of Cournot upon Walras can further be seen if we look into the concepts and method of the Recherches that were later used in Walras' Elements.

Cournot used the mathematical approach to make political economy more scientific. In market models preceding Cournot, demand, supply, and price had been clearly stated. Cournot, however, wanted to go beyond the specific market model approach and build some general propositions around demand, supply and price. To do this, he stated the economic relationships of demand, supply and price by the use of mathematical symbols. By expressing the variables this way, he was able to show the functional relationship of one or more of the other variables without identifying the particular

form of the function. Therefore, Cournot indicated that price was a function of the demand for a commodity with the equation $P = F(D)$. Cournot did not have to specify all the exact forms of the function of demand upon price, although he could state the function in a mathematical equation and then discuss some of the characteristics of the function. Such functional notations have the added advantage of clearly indicating mutual interdependencies of economic variables. Simultaneous equations also clearly show that demand and cost are in reality schedules. The origin of this functional notation and thinking is attributed to Cournot since it did not appear before the nineteenth century. The study of such mathematical notation in the Researches is claimed to have given Walras the foundation upon which he built his analysis of general equilibrium.[26] Schumpeter noted that:

> Cournot founded a different kind of mathematical economics which is based on the fact that the forms of thought of higher analysis can be applied very well to a number of economic propositions. These forms of higher analysis allow the investigation to be continued at such a point where ordinary logic fails because of its clumsiness. The concept of function exists patently or latently in most purely economic arguments and as far as the task consists in comprehending the general relationships between variable quantities and in deducing from the latter as much as is possible with regard to their variation, mathematical analysis is absolutely the suitable instrument. Moreover, the description of economic relationships in systems of simultaneous equations

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[26] Blaug, *op. cit.*, p. 203-4.

> in itself provides a survey of the former that cannot be obtained in such a precise form in any other way.[27]

Walras also employed techniques of graphing that Cournot had developed. This is clearly seen throughout his Elements.

It is clear, then that Walras obtained much from Cournot in the way of concepts and methods. The extent to which Cournot was helpful in this respect in the writing of Walras' Elements can be summed up as follows: 1) the discussion of relative and absolute value, 2) the extensive use of Cournot's demand function, 3) the solution of the problem of continuity, 4) the explanatory value assigned the first derivative of a function, 5) the immeasurability of utility, 6) the use of a numeraire, 7) the combining of rational behavior with the maximization principle, 8) the general equilibrium theorem (arbitrage--to insure uniform prices within the market), 9) the definition of competition, 10) the reproduction of Cournot's monopoly theory, and 11) the paraphrase of Cournot's analysis of foreign exchange.

J. R. Hicks once wrote that both Marshall and Walras profited decisively from Cournot's Recherches.

> Yet in fact there is a clear historical reason for it (the similarity of Elements and the Grundsätze,) one decisive influence we

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[27] Joseph A. Schumpeter, Economic Doctrine and Method, (London: George Allen and Urwin, 1954), p. 174.

> know to have been felt by both. Each of them had read Cournot.
>
> Now although each makes a specific acknowledgement to Cournot it is in each case couched in very general terms. They each tell us that Cournot showed them how to use the differentical calculus in economics, and this may mean much or little. But it is at least striking that certain very significant elements of Cournot's mathematical economics, going far beyond the mere idea of using mathematical method, appear in Walras and appear in Marshall.[28]

The method used in Walras' general equilibrium model came from Cournot's chapter on foreign exchange. Walras admits this himself in his Elements when he refers his readers to Cournot's general formula of exchange.

> Cournot devotes a special chapter to exchange in his Researches [and his Mathematical Principles of the Theory of Wealth]. I beg to refer the reader to this chapter for a fuller development of the theory and to confine myself to a restatement of Cournot's general formula of exchange.[29]

Schumpeter, too, commented that one could build a model of mutual interdependency if Cournot's methods were followed.[30]

The Researches presented the problem, concepts, techniques, but only foreshadowed the method to be used. It took a lesser mathematician like Walras, using a

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[28]Hicks, *op. cit.*, p. 339.

[29]Walras, *op. cit.*, p. 370.

[30]Schumpeter, History of Economic analysis, *op. cit.*, p. 959.

simpler form, to complete the analysis. But just how was this done?

As is pointed out in Lessons 5-7 of the Elements, Walras' Theory of Exchange assumes a competitive market with only two goods. Buyers and sellers compete for good A and good B. In Walras' market there are prices but no monetary unit was used. Pa stands for the price of good A, and Pb for the price of good B. If m of A is exchanged for n of B the following ratios result:

Pa = n/m and Pb = m/n, therefore Pa = 1/Pb or Pa · Pb = 1.

The principle arrived at from this exercise is that the price of A is the inverse of the price of B. If:

the quantity of A demanded is Da
the quantity of B demanded is Db
the quantity of A offered is Oa
the quantity of B offered is Ob, then another

principle emerged. This says the demand for A is equal to the offer of B times the price of B, (or Da = Ob · Pb). This is true from the assumption that all income is spent, so with only two goods available B must decrease if A is to increase. The same relationship of supply and demand holds for B as well as A with Db = Oa · Pa, or Oa = Db/Pa. But since 1/Pa = Pb it follows that Oa = Db · Pb. Thus:

Oa = Db · Pb
Ob = Da · Pa

and; Db/Oa = Ob/Db = 1.

At this point, Walras wondered whether if the unknowns might be solved simultaneously. The unknowns numbered six--Da, Db, Pa, Pb, Oa, and Ob. The equations also totaled six:

1. Da = Fa(Pa)
2. Db = Fb(Pb)
3. Da = ObPb
4. Db = OaPa
5. PaPb = 1
6. Oa = Da

From this Walras showed that these six equations were independent and the six unknowns could be ascertained. (But there was no way to determine if these six equations are consistent. Six independent equations are no indication of a meaningful solution.)

From this simple beginning Walras expanded his work. Proceeding as Cournot had taught him, he developed the demand function from individual utility schedules where satisfaction is being maximized. To expand his model he added more goods in the market and again solved for the unknowns using simultaneous equations.[31]

Cournot's exchange theory was based on the assumption that money was not shipped between countries in exchange for goods traded, and on the premise that the value of exports equaled the value of imports. Surplus goods were

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[31]Blaug, op. cit., p. 206.

always exchanged for other products of the trading country in order to maximize total satisfaction. Walras followed Cournot's assumptions exactly. Goods were traded for other goods to maximize satisfaction. The demand of any individual for a good was equal to the value of the good times the price of the goods he sold. Walras and Cournot used the same method to determine value. Walras used the price of one good in relation to another, where Cournot used the exchange rate between countries.

Cournot only solved for one unknown--the exchange rate of a country. He took m -- the indebtedness of a country -- as given. In this way, he solved only for the exchange rate or price, not for the quantities traded. His equations listed the indebtedness of one country to all other countries:

1. $m_{12} + m_{13} + \cdots + m_{1r} = m_{21}c_{21} + m_{31}c_{31} + \cdots + m_{r1}c_{r1}$,

2. $m_{21} + m_{23} + \cdots + m_{2r} = m_{12}c_{12} + m_{32}c_{32} + \cdots + m_{r2}c_{r2}$,

and so on for the r centers. Equation 1 shows that the total indebtedness of country m is made up of what was brought from country 2 + country 3, + ...+ country r, and is equal to the value of its sales to country 2 times the exchange rate between it and country 2. This is repeated for all the countries.[32]

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[32]Cournot, op. cit., p.117-126.

Walras' exchange equations are thus:

1. $Dab + Dac + \ldots + Dan = DbaPba + DcaPca + \ldots + DnaPna$

$Dba + Dbc + \ldots + Dba = DabPab + DcbPcb + \ldots + DnbPnb$

and so on, for n number of goods. Equation 1. shows the total demand of the owner of good "a" as being made up of his purchased of b + c +...+ N, and is equal to sales he made of good "a" to the holder of good "b" (Dba), times the price of "a" in terms of "b",(Pba), etc., for "n" number of goods.[33]

Walras' equations are quite similar to Cournot's. in fact there is but one important difference between the two. Cournot believed the exchange rate among the various countries to be dependent, subject to simultaneous determination. On the other hand, Walras incorporated both quantities and prices as dependent variables. Cournot invented his concept of arbitrage to demonstrate the mutual dependency of all exchange rates. Walras used the same concept but called it the general equilibrium theorem It established an interrelationship within the system of prices such that arbitrage was not profitable.

After this comparison of Cournot's and Walras' works it can definitely be claimed that Cournot was a major influence upon Walras and his Elements. Towards the end of Walras'

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[33]Walras, op. cit., p. 162.

life he spent a great deal of time reviewing his accomplishments. He sadly discovered that the claims to fame he once thought were his alone were now subject to question because of newly discovered predecessors. He clung to the one theory that he claimed to have invented-- his general equilibrium analysis. He even chose to have this fact chiseled on his tombstone.

Leon Walras is very deserving of the honors bestowed upon him, for his general equilibrium analysis is a much broader, more detailed, highly refined, and complicated model. But Cournot must be given credit for originating the concept, defining the problem, and providing the concepts and methods later adopted in the [illegible].[34]

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[34]Blaug, op. cit., p. 301.

CHAPTER II

CONCLUSION

The primary purpose of this thesis has been to show that Augustin Cournot was very modern for his day. Before 1933 the economic profession considered it theoretically useful to deal only with the two market extremes of pure monopoly and pure competition. Classical and neoclassical economists usually assumed firms were purely competitive. Classical economists realized monopoly existed, but they considered the competitive model to be the most useful tool for economic analysis. Pure competition may have been easier to work with than duopoly and oligopoly, but such a market classification as pure competition did not apply to the real world.

Cournot's duopoly problem did provide a market classification that corresponded more closely with the real world than earlier classifications. With the expansion of his duopoly theory to oligopoly, Cournot was as modern as today. He realized in 1838 that neither pure competition nor pure monopoly were likely to last. Some form of market classification must provide insights for the solution of real business problems. Cournot's duopoly theory was such an attempt. It is still used today as the starting point of all oligopoly theory.

In analyzing Cournot's duopoly theory and Cournot's influence upon later economists I have tried to show that many of our present basic economic principles have originated

from this once unknown Frenchman. Cournot's functional notation of the demand curve relating the two variables--price and quantity-- is fully accepted today in all modern economic theory. Cournot's behavioral assumption that the economic man is a maximizer is also generally accepted. From Cournot's partial equilibrium analysis Walras developed a general equilibrium theory that could be applied to the entire economic system. Cournot was also interested in probability theory which has been expanded today into Game Theory.

Cournot can be credited with much when he analyzed a dominant real-world market form. His assumptions tended to limit the conclusions possible, but it was a beginning of truly useful analysis. Since Cournot's work we have discovered that the number of assumptions which might be made about the behavior of oligopolists is large. Cournot's model was determinant, but as the model becomes more realistic the theory becomes less determinant.

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