Strengthening Elementary Students' Understanding of Factors

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STRENGTHENING ELEMENTARY STUDENTS’ UNDERSTANDING OF FACTORS

By

JORDAN ROSEMARY FROTZ

Undergraduate Thesis
presented in partial fulfillment of the requirements
for the University Scholar distinction

Davidson Honors College
University of Montana
Missoula, MT

May 2016

Approved by:

Dr. Matt Roscoe, Faculty Mentor
Department of Mathematical Sciences
ABSTRACT

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Elementary Education

STRENGTHEN ELEMENTARY STUDENTS’ UNDERSTANDING OF FACTORS

Faculty Mentor: Dr. Matt Roscoe

Research on pre-service elementary school teachers’ understanding of the multiplicative structure of the natural numbers demonstrates an under-utilization of unique prime factorization in the identification of a number’s factors. For example, Zazkis and Campbell (1996b) found that a majority of teacher candidates employed trial division to analyze factor-candidates of a number, even when both were presented in prime-factored-form. Recent studies (i.e. Roscoe & Feldman, 2015) have shown that teachers’ understanding of factor can be strengthened by engaging in a sequence of instructional tasks that explore the relationship between a number’s prime factorization and its factors. This study seeks to extend the scope of investigation in this area to the population of elementary school students. The research questions addressed by the study are:

1. To what extent do elementary school students under-utilize unique prime factorization in the identification of a number’s factors?
2. How is elementary students’ usage of unique prime factorization in the identification of a number’s factors similar to, or different from, that of pre-service elementary school teachers as identified in the research literature?
3. Which instructional tasks strengthen elementary school students’ understanding of the use of prime factorization in the identification of a number’s factors?

Researchers conducted teaching experiments in two elementary school mathematics classrooms. A mix-methods analysis of quantitative pre- and post-test data and qualitative student work was employed.
STRENGTHENING ELEMENTARY STUDENTS’ UNDERSTANDING OF FACTORS

Introduction

Consider the equation N=2x3x5x19 and the set of numbers: {5, 11, 38, 20}. Which of these numbers are factors of N? One way to think of this problem is to compute N, and trial divide the four numbers. Without any computation errors, you would be able to successfully identify the factors through the understanding of the definition of a factor: a number would be a factor of N if it evenly divides N. Using this method, you could successfully identify that 5 and 38 are both factors of N, while 11 and 20 are not. This trial division method is the one most often taught in schools, and, arguably, the one most likely employed by the average person faced with solving this problem.

The second method of approaching this problem is through the use of prime factorization. This method employs a richer understanding of divisibility and factor concepts, and is less prone to computation errors. An important result in number theory, known as the Fundamental Theorem of Arithmetic (FTA), tells us that each natural number excluding 1, any number of the set {2, 3, 4, …}, is uniquely determined by its prime factorization. No two natural numbers share the same prime factorization; it can be thought of as a number’s “DNA.” In this instance, N has been presented in its prime factor form: all of the numbers multiplied together are primes. By the FTA, N’s factors can only be made up by some combination of the prime numbers 2, 3, 5, and 19. Thus, any number that is not made up of those primes cannot be a factor of N. A factor must only contain primes in the original number, with respect to the total number of each prime. Thinking this way, it is easy to see that 5 is a factor of N; it is in its prime factorization. Similarly, it is straightforward to see that 11 is not a factor of N; nowhere does the prime number 11 exist in N. Moving to think about composite factors requires more information. We must know the prime factorization of the composite numbers to know if they share primes with N. The number 38 is 2x19; since both 2 and 19 are in the prime factorization of N, 38 is a factor of N. The number 20, however, is 2x2x5; N contains the prime factors of both 2 and 5, but the prime factorization of 20 contains two 2’s, where N only has one. This means 20 cannot be a factor of N.

This example shows the nuances associated with numbers. In prime factored form, identifying factors and non-factors is easier and less likely to be faulted by conceptual errors.
compared to trial division. The DNA of the number is available for viewing and can be used easily; it is transparent. However, using this method correctly requires complete understanding of the FTA, specifically the uniqueness of prime factorization, the communicative property of multiplication, and prime and composite numbers. It has been shown that pre-service elementary school teachers (PSTs) struggle to make use of prime factorization and the uniqueness to identify factors (Zazkis & Gadowsky, 2001), given this, and the lack of research done on elementary school students, there is a gap in learning that needs to be filled. This paper summarizes an intervention conducted with 4th and 5th grade students to develop their abilities in this area.

**Literature Review**

In 1996, Zazkis conducted clinical interviews with 21 PSTs and found that 15 of them exhibited limited and procedural understandings of divisibility (Zazkis & Campbell, 1996a). These PSTs admitted to needing to compute the whole number and then trial dividing; this shows a misunderstanding of prime factorization and divisibility. Zazkis (1998) supported this finding, showing that PSTs relied on long division or application of divisibility rules with little ability to use prime factorization as a tool for reasoning about factors. Zazkis and Gadowsky (2001) demonstrated that PSTs’ fail to make use of the transparent features of prime factorization. They did not take into account the transparent nature of numbers represented in prime factored forms. For example, the prime factored representation of N=2x3x5x19, makes the fact that 5 is a factor easily seen, or transparent, PSTs fail to make this connection. Other studies have identified PSTs’ misconceptions about factors and prime numbers; such as the notion that bigger numbers have more factors or that prime numbers are small (Zazkis & Campbell, 1996b; Zazkis & Gadowsky, 2001).

Some studies have characterized the extent of PSTs’ knowledge of number theory topics. Zazkis (2005) found that PSTs use negative descriptions to define prime numbers (e.g., “prime numbers ‘cannot be divided’, ‘cannot be factored’ or ‘wouldn’t have/are not having any other factor’” (p. 208)), which may be an obstacle to achieving a robust conceptual understanding of prime number. Researchers have also noted that PSTs tend to have an easier time identifying factors than non-factors, and are better able to recognize prime factors than composite factors (Zazkis & Campbell, 1996a, 1996b). Zazkis and Campbell (1996b) noted that PSTs’ difficulty with identifying non-factors may be due to a lack of appreciation for the uniqueness feature of the FTA: “Whereas the existence of prime decomposition may be taken for granted, the
uniqueness of prime decomposition appears to be counterintuitive and often a possibility of different prime decompositions is assumed” (p. 217). Liljedahl and colleagues (2006) found that the use of a computer program known as Number Worlds, which allowed PSTs to experiment with different arrays of the natural numbers, “thickened” student’s understandings of factors, multiples, and primes. The authors say, “this use of the adjective ‘thick’ to describe a learner’s layered, rich, contextual, and often affective understanding of a mathematical concept” (Liljedahl, Sinclair, & Zazkis, 2006, p. 254). This study shows that the use of arrays help PSTs to develop a “thicker” understanding of number theory. Roscoe and Feldman (2015) provide an intervention with PSTs in a mathematics content university course. They conducted a three-week intervention, with three in-class lessons and two homework assignments. This intervention produced statistically significant results in developing PST’s understandings of factors and prime factorization, specifically in identifying prime factors, prime non-factors, composite factors, and composite non-factors. Roscoe and Feldman, Liljedahl, Zazkis, and colleagues’ research focuses on PSTs, and not much research has focused on elementary school children. However, the research completed on PSTs is significant because if PSTs do not understand the implications of the FTA, they cannot teach this content to their future students.

Currently, the literature related to elementary school students on this topic is naught. Burkhart (2009) has provided an article in Mathematics Teaching in the Middle School that provides an intervention possibility without the backup of quantitative data. He explores using actual building blocks to create a visual representation of prime factorizations. This allows students to physically explore the concepts, and transparency, of prime numbers. Burkhart used blocks to help his 6th graders explore how numbers are made up of their unique set of primes. Students then analyze the patterns and structure of the counting numbers to fifty from the appearance of their prime factorizations. Furthermore, Burkhart (2009) allowed his students to explore multiplication, division, exponents, factors, multiples, greatest common factors, and least common multiple. While he provides a description of teaching tasks to help students understand these difficult concepts, there is no quantitative evidence of students’ gains presented.

**Methodology**

This study is viewed as a teaching experiment aimed at providing students with a robust understanding of the implications of prime factorization as an aid to identifying the factors of a number. This study is an extension of research conducted by Matt Roscoe and Ziv Feldman
(2015) with PSTs. Our intervention follows a similar course of study; however, ours has been adapted to meet the instructional and developmental needs of 4th and 5th grade students. The study has three research questions:

1. To what extent do elementary school students under-utilize unique prime factorization in the identification of a number’s factors?
2. How is elementary students’ usage of unique prime factorization in the identification of a number’s factors similar to, or different from, that of pre-service elementary school teachers as identified in the research literature?
3. Which instructional tasks strengthen elementary school students’ understanding of the use of prime factorization in the identification of a number’s factors?

This study’s intervention was conducted in two different classrooms, one fourth grade and one fifth grade. The fourth grade classroom completed the intervention in one and a half hour long mathematics lesson. They completed the same worksheets and homework as the fifth graders did. The fifth graders intervention was conducted over two forty-five minute periods. The fourth graders were not familiar with exponents, so prime factorizations were written as such: 8=2x2x2; fifth graders wrote 8=2^3. As researchers, we also noticed the fifth grade students were more open in their discussion with each other; the discussions from this intervention will obviously vary from class to class. The most important part of this intervention is that there was no overt teaching regarding the ability to find factors using only primes and not trial division.

All of the understanding of this concept was derived from student discussion. Students were challenged to look at patterns within the numbers and realize the prime factorization of a number’s factors are simply subsets, or groups of, the prime factorization of the number in question. Students were also challenged to look at how to produce the factors of a number when given the number in prime factor form.

The first day of the intervention focused on the necessary definitions and skills needed to

The Sieve of Eratosthenes is a simple algorithm to identify all the prime numbers up to any value.

Ignoring 1, the first prime is 2, so start by marking off all the other numbers which are divisible by 2.

The next prime is 3, so mark off all the other numbers which are divisible by 3.

The next prime is 5, and then 7. Proceed in the same way.

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The next prime is 3, so mark off all the other numbers which are divisible by 3.

The next prime is 5, and then 7. Proceed in the same way.
fully understand and appreciate prime factorizations of numbers. We began by leading a short
discussion about the definitions of prime, composite, and 1. Posters were made which defined
prime as “a number whose only factors are 1 and itself”; composite as “a number with more than
two factors”; and 1 as “a special number that is neither prime nor composite.” Next, students
were lead through the Sieve of Eratosthenes; Eratosthenes was a Greek philosopher who first
proposed the method over 2000 years ago. This Sieve is a simple process of elimination that
produces all of the prime numbers. See Figure 1 for an explanation of the Sieve. For our
intervention, the fourth grade class completed the Sieve with their classroom teacher, and was
observed by one of the researchers to maintain consistency in the intervention.
Students were then taught how to make factor
trees. Figure 2 shows a sample of the board
work done with students to teach this skill.
Students were expected to take notes on
factor trees and a discussion was lead about if the prime factorization found at the end
would be the same no matter how the tree began. Students practiced this skill with a
partner on the numbers 36 and 28. One student did one number and the other did the next, and
then they taught their partner how to make the factor tree for their number. To finish out the first
day, students were split into six groups and completed the factor trees of the numbers 1-50.
Students wrote these numbers on index cards, which were taped to a 10 by 5 grid. Table 1 shows
the numbers that were assigned to each group. The groupings were based on difficulty in the
factor tree and were assigned to split up the primes as evenly as possible. On every index card,
students wrote out the factor tree, identified the number as prime or composite, and wrote out the
prime factorization for the number. Once the researcher showed an example, the students worked
on completing the chart. After completing the grid and discussing the different difficulties of
prime factor trees, students were asked to begin the first day’s homework assignment.
Table 1

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
<th>Group 5</th>
<th>Group 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>14</td>
<td>10</td>
<td>8</td>
<td>13</td>
<td>4</td>
</tr>
<tr>
<td>19</td>
<td>16</td>
<td>12</td>
<td>9</td>
<td>22</td>
<td>17</td>
</tr>
<tr>
<td>20</td>
<td>18</td>
<td>26</td>
<td>25</td>
<td>27</td>
<td>21</td>
</tr>
<tr>
<td>34</td>
<td>23</td>
<td>29</td>
<td>30</td>
<td>36</td>
<td>28</td>
</tr>
<tr>
<td>40</td>
<td>33</td>
<td>42</td>
<td>31</td>
<td>37</td>
<td>35</td>
</tr>
<tr>
<td>45</td>
<td>44</td>
<td>43</td>
<td>32</td>
<td>38</td>
<td>41</td>
</tr>
<tr>
<td>47</td>
<td>49</td>
<td>46</td>
<td>39</td>
<td>50</td>
<td>48</td>
</tr>
</tbody>
</table>

Note: 11 and 24 reserved as teacher examples.

The second day began with a discussion of the homework from the night before. Questions were answered and difficulties discussed (note: the fourth graders had the two days combined). The beginning of the lesson for the second day involved explaining factor pairs to the students.

As a class, all the factor pairs for the number 24 were found. Students then paired-up and found the factor pairs for 36 and 28. The most important part of the intervention was when the students found the prime factorization for their number’s factors and compared that to the prime factorization for their actual number. They were challenged to look for patterns in the numbers and a discussion followed with the goal to elicit from students that all factors are subsets of a number’s prime factorization. Figure 3 shows how the factors of 24, in prime factor form, are subsets of the prime factorization of 24. After this vital discussion was complete on the second day, students were given a chance to begin their homework. The students turned in worksheets from both days as well as completed homework.
Pre- and post-tests were given. The assessments were based on tests given by Zazkis in her research (1996a). The pre- and post-test assessments contained the exact same wording with the numbers changed; see table 2 for assessment wording.

Table 2

Assessment:

**Directions:** Answer each question below. Read the directions carefully. Do not use a calculator. Recall that a factor is any number that evenly divides another number. For example, 6 is a factor of 24 but 10 is not a factor of 24.

1. \( N = 2 \times 2 \times 5 \times 7 \times 13 \)
   a) Is 7 a factor of \( N \)? How do you know?
   b) Is 3 a factor of \( N \)? How do you know?
   c) Is 14 a factor of \( N \)? How do you know?
   d) Is 45 a factor of \( N \)? How do you know?

2. If \( M = 3 \times 5 \times 7 \) can you find all the factors of \( M \)? Show how you found them.

A rubric was developed to assess the level of change from the pre-assessment to the post-assessment. Both the researchers developed the rubric and conducted a test of inter-rater reliability. All the scores given to students were agreed upon by both the researchers and the rubric adjusted as conversations and discussions surfaced. See the table 3 for the full rubric used.

The assessments were scored out of a total of 18.

Table 3

<table>
<thead>
<tr>
<th>Question</th>
<th>0pts.</th>
<th>1pt.</th>
<th>2pts.</th>
<th>Total pts.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>Not Correct</td>
<td>Correct-Uses Other Method (i.e. long division)</td>
<td>Correct-Uses Primes (prime factorization)</td>
<td>/2</td>
</tr>
<tr>
<td>1b</td>
<td>Not Correct</td>
<td>Correct-Uses Other Method (i.e. long division)</td>
<td>Correct-Uses Primes (prime factorization)</td>
<td>/2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1c</td>
<td>Not Correct</td>
<td>Correct-Uses Other Method (i.e. long division)</td>
<td>Correct-Uses Primes (prime factorization)</td>
<td>/2</td>
</tr>
<tr>
<td>1d</td>
<td>Not Correct</td>
<td>Correct-Uses Other Method (i.e. long division)</td>
<td>Correct-Uses Primes (prime factorization)</td>
<td>/2</td>
</tr>
</tbody>
</table>

Section 1-Total pts. /8

<table>
<thead>
<tr>
<th></th>
<th>0pts.</th>
<th>1-8pts.</th>
<th>Total pts.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2a</td>
<td>No Factors</td>
<td>Total Factors Found: (Factors include factors used within work, i.e.: 3x5=15 (3, 5, and 15) are all factors)</td>
<td>/8</td>
</tr>
</tbody>
</table>

Section 2A- Total pts. /8

<table>
<thead>
<tr>
<th></th>
<th>0pts.</th>
<th>1pt.</th>
<th>2pts.</th>
<th>Total pts.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2b</td>
<td>No written or visual representation of reasoning.</td>
<td>Shows written or visual representation of reasoning using other methods of reasoning. (long division)</td>
<td>Shows written or visual representation of reasoning using prime factorization. (prime factorization) Identifying the use of prime factorization includes: (arrows pointing toward numbers in the equation, parenthesis around numbers in the equation-showing use of these numbers, multiplication with use of factors from the equation-i.e.: 3x5=15 (3, 5, and 15)</td>
<td>/2</td>
</tr>
</tbody>
</table>

Section 2B-Total pts. /2

**Results**

The results were examined using a paired t-test with an alpha level set at .05. This study uses N=33 for analysis. Students were removed from the sample if they did not attend both days of the intervention, did not complete both the pre- and post-assessment, or completed less than 50% of one or both of the assessments.
Table 4

<table>
<thead>
<tr>
<th></th>
<th>Pre-Assessment</th>
<th>Post-Assessment</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>8.30303</td>
<td>12.06061</td>
<td>3.757576</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>4.111606</td>
<td>3.999526</td>
<td>4.437273</td>
</tr>
</tbody>
</table>

$T(33) = 4.864615744; p=2.94419E-05$

Table 4 shows the average and standard deviation for the pre-assessment, post-assessment and the difference between the two for the total scores on the assessments. The p-value is substantially less than the alpha level of .05. This means that the students had statistically significant gains in their ability to answer all of the questions on the test with a higher success rate after the intervention. Overall, this data shows the intervention is associated with an increase in ability to use prime factorization to find factors of numbers.

Table 5

<table>
<thead>
<tr>
<th></th>
<th>Pre-Assessment</th>
<th>Post-Assessment</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>3</td>
<td>5.363636</td>
<td>2.363636</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>1.436141</td>
<td>2.19115</td>
<td>2.329407</td>
</tr>
</tbody>
</table>

$T(33) = 5.828976; p=1.78E-06$

Table 5 shows the average and standard deviation for the pre-assessment, post-assessment, and the difference between the two for the score on question 1 of the assessments. Question 1 of the assessment tested the student’s ability to distinguish between factors and non-factors of a number. Students were also expected to show the use of prime numbers in their explanation for why a number was a factor or not. The p-value for table 5 is much less than the alpha level of .05. This indicates that the intervention is associated with student gains in abilities to identify factors and non-factors of a number using the prime factorization of a number to help them with the identification process.

Table 6

<table>
<thead>
<tr>
<th></th>
<th>Pre-Assessment</th>
<th>Post-Assessment</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>3.787879</td>
<td>5.272727</td>
<td>1.484848</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>2.654684</td>
<td>2.225881</td>
<td>2.751377</td>
</tr>
</tbody>
</table>

$T(33) = 3.100195; p=0.004015$
Table 6 shows the average and standard deviation for the pre-assessment, post-assessment, and the difference between the two for the score on question 2a of the assessments. Questions 2a tested the student’s ability to identify factors of a number presented in prime factorization form. The p-value for table 6 still meets the alpha level p less than .05; this means that the intervention is associated with students being able to find more factors of a number after the intervention.

Table 7

<table>
<thead>
<tr>
<th></th>
<th>Pre-Assessment</th>
<th>Post-Assessment</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>1.515152</td>
<td>1.424242</td>
<td>-0.09091</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.870388</td>
<td>0.867118</td>
<td>1.259058</td>
</tr>
</tbody>
</table>

T(33)= -0.41478; p=0.68107

Table 7 shows the average and standard deviation of the pre-assessment, post-assessment, and the difference between the two on question 2b. Questions 2b tests the student’s ability to use the prime factors of a number to identify the factors. Students were assessed on their explanation abilities. As the data shows, students were actually less successful at explaining how they found the factors for the number. However, the p-value indicates that the intervention is not associated with any change in ability in this area.

Conclusion

The results of this study are very promising. The statistics show that students had great gains in all areas of the assessment, with the exception of explaining how they found the factors for the last question (i.e. question 2b). The results from the pre-assessment showed that students did not use the uniqueness of prime factorizations to help them in identifying factors and non-factors of a number presented in prime factor form. The results found show that students share a similar misunderstanding of identifying prime factors that PSTs have shown in previous research. Furthermore, the research conducted by Roscoe and Feldman (2015) showed improvement in PSTs understandings of this topic, which was replicated in this study with students in the 4th and 5th grades. This seems to indicate that the instructional tasks used during the intervention helped to improve students’ abilities in this area of mathematics.

The results point toward the need for more studies to develop the instructional tools to help students with their explanation abilities. The hope is for students to be able to successfully show and produce a written explanation for how prime factors of a number help to produce all of
the factors for a number. It may be helpful to have student verbalize their understanding and then ask them to write it down after a successful discussion. However, the ultimate goal would be for students to be able to write out their understandings without needing to verbalize it first. Importantly, the Common Core State Standards in Mathematics (2010) have called for a focus on developing student’s abilities to successfully reason through and explain difficult mathematical concepts. The mathematical practices for 4th graders that this intervention most closely worked towards are: reason abstractly and quantitatively; construct viable arguments and critique the reasoning of others; look for and make sure of structure (Common Core State Standards Initiative, 2015). Students were challenged to construct their own knowledge and use the knowledge in new and unique situations. This challenge will hopefully lead to a richer and deeper understanding of factors and how numbers are uniquely made up of primes.

Secondly, it may be helpful and important to include some of Burkhart’s (2009) ideas into this intervention. The use of physical blocks to build understanding of prime factorization and the creation of factors could help students develop a model for what is happening with the mathematics. Students would be able to physically move around different pieces, primes, of the number to produce the factors. While the intervention, as is, was successful, it is always important for educators to think of more ideas to help students understand these important mathematical concepts and explain them.

This study produced unique and significant information about mathematical tasks for 4th and 5th grade students to develop their understandings of factors. While the study needs improvement in developing student’s ability to explain their work, the results show that students were successful at developing a richer understanding of factors. The research points towards instructional methods that teachers can readily adopt to help them provide a deeper understanding of factors for their students.

**Acknowledgements**

I would like to express gratitude to: Dr. Matt Roscoe of the University of Montana, Department of Mathematical Sciences, for his expertise and help with this intervention; Heather Vallejo for her hard work and extraordinary ideas, this intervention would not have been possible without her; The University of Montana’s Davidson Honors College for funding for this project; The National Undergraduate Research Conference 2015 for allowing me to present my exciting research; and Tonya Froitz and Johnathan Bush for editing assistance.
References


# Appendix

## Teacher Guide

### Teaching Factors – Lesson 1

<table>
<thead>
<tr>
<th>Segment</th>
<th>Description</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Introduction to Prime and Composite</strong></td>
<td>The teacher writes the following numbers on the board: 1, 11, 15, 23, 24. The teacher says, “Today we are going to start learning about prime numbers. Does anyone know what a prime number is? Can anyone tell me which of the numbers that I have written on the boards are prime numbers? Why are these numbers considered prime numbers?” Once the primes and composites and the number 1 have been discussed, then, the teacher turns over a poster that has the definition of prime and composite numbers as well as a statement about 1 being neither prime nor composite.</td>
<td>5 min</td>
</tr>
<tr>
<td><strong>Sieve</strong></td>
<td>The teacher says, “One way that we can find all of the prime numbers is using a process of elimination. We have in front of us a chart with the first 100 numbers. Let’s cross off all the non-prime numbers.” The teacher then asks any of the following questions: 1. Should I cross off one? 2. Should I cross off two? 3. How about three? 4. What about four? (b/c two divides it) 5. What other numbers are going to be divisible by two? 6. Can we do the same thing for three? 7. Let’s look at five.... Once all of the primes have been identified, the teacher explains that this process of elimination has a special name: The Sieve of Eratosthenes. The sieve is named after a famous Greek philosopher who first proposed the method over 2000 years ago.</td>
<td>10 min</td>
</tr>
<tr>
<td><strong>Introduction to Prime Factor Trees</strong></td>
<td>Teacher hands out in-class worksheet. Teacher says, “One interesting thing about prime numbers is that they can be used to write other numbers using multiplication. Let’s take a look at the number 24....” Teacher does an example of a factor tree of 24 starting with 2 times 12. Teacher asks, “What would happen if I had started the prime factor tree with 4 times 6 instead? Would we get the same result?” “Can you start the factor tree a different way? Do you get the same result?” After discussion and exploration of these two prime factor trees, the teacher asks students to try one: 36 and 28. In pairs students teach each other about their prime factor tree. Teacher writes a prime factor tree for both 36 and 28 on the board.</td>
<td>10 min</td>
</tr>
<tr>
<td><strong>Finding Factor Trees 1-50</strong></td>
<td>Teacher says, “Now we are going to find the rest of the prime factor trees. Each group is going to be given some number written on note cards. Your job is to decide if the number is prime or composite (....teacher points at Sieve results). If the number is prime, write “prime” on the first line and the number in the bottom space and put it on the big chart in its slot. If the number is composite, make a prime factor tree on the note card and then write “composite” on the first line and the number as a multiplication of primes in the space below.” Teacher shows two example cards: 11 and 24 and puts them on the board.</td>
<td>20 min</td>
</tr>
<tr>
<td><strong>Start Homework</strong></td>
<td>Once groups’ numbers have been placed on the board, student hand in in-class worksheet and then homework practice is handed out to the group and students can get started on this activity.</td>
<td>Variable</td>
</tr>
</tbody>
</table>
### Sample Board Work

<table>
<thead>
<tr>
<th>Number</th>
<th>Divisors</th>
<th>Prime?</th>
<th>Composite?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>11</td>
<td>1,11</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>16</td>
<td>1,4,8,16</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>23</td>
<td>1,23</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>24</td>
<td>1,2,3,4,6,8,12,24</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

\[
24 = 2^3 \times 3
\]

\[
24 = 2^2 \times 2 \times 3
\]

\[
36 = 2^2 \times 3^2
\]

\[
36 = 2^2 \times 3 \times 3
\]

\[
28 = 2^2 \times 7
\]

\[
28 = 2 \times 2 \times 7
\]
### Group Number Assignments

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
<th>Group 5</th>
<th>Group 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>14</td>
<td>10</td>
<td>8</td>
<td>13</td>
<td>4</td>
</tr>
<tr>
<td>19</td>
<td>16</td>
<td>12</td>
<td>9</td>
<td>22</td>
<td>17</td>
</tr>
<tr>
<td>20</td>
<td>18</td>
<td>26</td>
<td>25</td>
<td>27</td>
<td>21</td>
</tr>
<tr>
<td>34</td>
<td>23</td>
<td>29</td>
<td>30</td>
<td>36</td>
<td>28</td>
</tr>
<tr>
<td>40</td>
<td>33</td>
<td>42</td>
<td>31</td>
<td>37</td>
<td>35</td>
</tr>
<tr>
<td>45</td>
<td>44</td>
<td>43</td>
<td>32</td>
<td>38</td>
<td>41</td>
</tr>
<tr>
<td>47</td>
<td>49</td>
<td>46</td>
<td>39</td>
<td>50</td>
<td>48</td>
</tr>
</tbody>
</table>

Note: 11 and 24 reserved as teacher examples.
<table>
<thead>
<tr>
<th>Segment</th>
<th>Description</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Check Homework</td>
<td>Teacher asks students to spend the first few minutes of class checking homework with group-mates. Teacher answers questions raised by homework. Students are then asked to hand in homework 1.</td>
<td>5 min</td>
</tr>
<tr>
<td>Factor Pairs Introduction</td>
<td>Teacher hands out in-class worksheet. Teacher says, “Today we are going to explore how a number’s prime multiplication form helps us to understand more about a number’s factors. Any number that evenly divides another number is called a factor. Factors are often given in pairs. Let’s take a look at the number 24 again. One of the 24’s factor pairs is 1 and 24 because 1X24=24. Another of 24’s factor pairs is 2 and 12 because 2X12=24. Can anyone give me another pair of factors for 24?” Teacher leads activity until all factor pairs are found. Reorganizes (if necessary) the factor pair table so that it is well-ordered. Asks groups to find all the factor pairs of 36 and 28. In pairs students teach each other about their number’s factor pairs. Student volunteer or teacher writes factor pairs for both 36 and 28 on the board.</td>
<td>15 min</td>
</tr>
<tr>
<td>Factor Pairs and Prime Factors Activity</td>
<td>Teacher says, “Now we are going to take a look at how a number’s factors are related to that number’s prime multiplication form. Let’s look at 24 again. Its prime multiplication form is 2X2X2X3. Let’s write all of its factors in prime multiplication form as well....” Teacher writes 24=2X2X2X3 at the top of the board and then creates a table with headings: factors, prime multiplication form of factors. Using the table created in lesson one, the teacher fills in the table. Teacher then asks students to create a similar chart on their handout using their number: 28 or 36. Students are encouraged to look for patterns and share results in small groups. Groups who finish first are asked to write their results on the board. Teacher leads a large-group discussion of patterns. Goal: elicit from students that all factors are subsets (i.e. parts, smaller parts, make-up-of pieces, etc.) of a number’s prime factorization. Guiding Questions: 1. Talk to your partner, what did they notice? 2. Do you notice anything the number and its factors have in common? 3. What is true about every factor’s prime multiplication form in relation to the number’s prime multiplication form? 4. Why is not 8=2x2x2 a factor of each number?</td>
<td>20 min</td>
</tr>
<tr>
<td>Start Homework</td>
<td>Teacher hands out homework.</td>
<td>Variable</td>
</tr>
</tbody>
</table>
Sample Board Work

**Factor Pairs of 24**

- 3, 8
- 4, 6
- 2, 12
- 1, 24

Unordered:

- 3, 8
- 4, 6

Ordered:

- 1, 24
- 2, 12

---

24 = 2 × 2 × 2 × 3

<table>
<thead>
<tr>
<th>Factors</th>
<th>Prime Mult. Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>2 × 2</td>
</tr>
<tr>
<td>6</td>
<td>2 × 3</td>
</tr>
<tr>
<td>8</td>
<td>2 × 2 × 2</td>
</tr>
<tr>
<td>12</td>
<td>2 × 2 × 3</td>
</tr>
<tr>
<td>24</td>
<td>2 × 2 × 2 × 3</td>
</tr>
</tbody>
</table>
\[
\begin{array}{c}
36 \\
1, 36 \\
2, 18 \\
3, 12 \\
4, 9 \\
6, 6 \\
\end{array}
\quad \begin{array}{c}
28 \\
1, 28 \\
2, 14 \\
4, 7 \\
\end{array}
\]
Worksheet Day 1

Name: _________________________________

Teacher Example

Please write-out the teacher's example of a prime factor tree for your reference.
Now try your own number. The partner whose birthday is earlier in the year will get the number 36; the other partner will use the number 28.

My Number is: _________________  My Partner's Number is: ____________

My Number’s prime factor tree:  My Partner’s prime factor tree:

My number’s prime factorization:  My Partner’s number’s prime factorization:
Worksheets Day 2

Name: _________________________

Write out all of the factors for your number. The partner with the earliest birthday in the year gets 28; the other partner gets 36.

My number is: __________________________
Write out your number’s prime factorization, list the factors of your number, and list the prime factorization of those numbers in the second column. Write what you notice about this chart at the bottom of the page.

The prime factorization of my number is: _____________________

<table>
<thead>
<tr>
<th>My number’s factors</th>
<th>Prime multiplication form of factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>**</td>
</tr>
</tbody>
</table>

What I noticed:__________________________________________________________________________
_______________________________________________________________________________________
_______________________________________________________________________________________
_______________________________________________________________________________________
_______________________________________________________________________________________

**Remember that 1 is a special number that is neither prime nor composite, so it does not have its own prime factorization.
Fill out this page as before with your partner’s number.

The prime factorization of my partner’s number is: ________________

<table>
<thead>
<tr>
<th>My partner’s number’s factors</th>
<th>Prime multiplication form of factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>**</td>
</tr>
</tbody>
</table>

What I noticed: ____________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

__________________________________________________________________________

**Remember that 1 is a special number that is neither prime nor composite, so it does not have its own prime factorization.
Let’s practice prime factor trees.

Complete a prime factor tree for the following two numbers.

134  131

Select your own three-digit number and complete a prime factor tree for it.

Look at the following factor trees. Decide if the factor tree is a prime factor tree, and if it is not, complete it.

```
166
  /\  \
2   83
```

```
255
  /\  \
5   51
```
Look at the following numbers and decide if they are prime or composite.

52: __________

83:___________

69:___________

97:___________

Challenge!

Group the following numbers according to their prime factorizations. Explain your groupings.

6, 12, 15, 18, 20, 38
Day 2 Homework

1. Find all the factors for the following numbers. Show how you found them all.

   a) 42

   b) 64
2. N = 3x5x7x19

a) Is 7 a factor of N? How do you know?

b) Is 21 a factor of N? How do you know?

c) Is 23 a factor of N? How do you know?

d) Is 11 a factor of N? How do you know?
3. Given this number: 2x3x13, can you list all its factors?

4. Given this number: 2x2x2x2x5, can you figure out how many factors the number has?