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Abstract: This article explores problem solving in elementary classrooms while focusing on how children use (perform tasks) manipulatives and/or tools in problem solving while working on mathematical tasks. Ways for teachers to assess children's learning through performance-based tool (manipulative) use will also be examined and suggested. Current research reveals that teachers need to teach and assess children's mathematical knowledge in ways that will allow them to show (perform) what they really understand. And, teachers must be able to see beyond obvious correct or incorrect answers into children's thinking processes by testing with "tests that allow students the opportunity to show what they know" (Van de Walle, 2003, p. 73).

Key words: Automacity; Classroom pedagogy; Manipulatives; Problem solving; Teacher practices..

1. Purpose and Introduction

In recent years and with the refinement of the *Principles and Standards for School Mathematics* (NCTM, 2000), it has become clear that standards-based mathematics teaching and learning is not only multi-faceted for the teacher, but also for the student. Acquisition of mathematical knowledge through problem solving and with manipulatives has long been considered to be time-consuming and labor intensive for many classroom teachers who are seemingly overwhelmed with high-stakes testing, published test scores, and unacceptable achievement scores on international measures (National Science Foundation, 2004). This has since encouraged educators to study ways in which teaching and learning occurs in the elementary classroom, particularly when a primary goal is to teach students fluency and flexibility with numbers and strategies for using those numbers (NCTM, 2000).

This article explores problem solving in elementary classrooms while focusing on how children use (perform tasks) manipulatives and/or tools in problem solving while working on mathematical tasks. Ways for teachers to assess children's learning through performance-based tool (manipulative) use will also be examined and suggested. The term, *manipulative*, will be defined as any tangible object, tool, model, or mechanism that may be used to clearly demonstrate a depth of understanding, while problem solving, about a specified mathematical topic or topics. *Performance-Based Assessment* implies that the measure encourages students to perform, create, and produce solutions while using contextualized problem solving and higher level thinking (Hatfield, Edwards, Bitter, & Morrow, 2005).

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Developing higher level thinking skills and fluency and flexibility with numbers in young students supports the idea for implementing manipulative-based problem solving in the classroom. While more traditional drill-and-practice, non-manipulative-based methods for teaching and learning mathematics might imply that understanding beyond automaticity does occur, it would seem that multiple and varied methods for teaching children mathematics should be explored.

Van de Walle (2004) describes automaticity as performing a task mindlessly and quickly (p.86), which implies that drill and repetition of isolated skills may not be as essential as it once was. There are certain mathematical skills that many believe must be committed to memory automatically (e.g. rote counting, basic facts, ordinal positioning, etc.); however, what a problem centered approach to mathematics teaching brings to the forefront is a connection to real life problem solving and connects with more students more of the time. "Developing flexible thinking strategies requires adequate opportunity with varied numbers and contexts…the development of different methods of thinking requires problem-based tasks" (Van de Walle, 2003, p. 87). If we want children to learn to think deeply and ponder real mathematics as well as to be able to use in depth thinking in real life scenarios, we must teach them and assess their knowledge in ways that will allow them to show us what they really understand about the tasks being tested. Additionally, teachers also need to be able to see beyond the correct or incorrect answers and garner a larger, more diversified view of each child's mathematical understanding. These teacher-skills seem to develop quicker and more fully in classrooms where a problem-based approach is in place (Van de Walle, 2004).

However, growth in problem-based teaching techniques seems to evolve slowly. In analyzing how teachers teach mathematics, results from the TIMSS (Third International Math and Science Study) Video Study of Teaching revealed that of the two-thirds of teachers who felt that they were implementing real-world applications and group work with problem solving, only 19 percent actually implemented activities involving problem solving (Hiebert & Stigler, 2000). Additionally, the Program for International Student Assessment (PISA), which assessed student competency following mandatory education and looked at mathematical literacy and problem solving skills applied to real-life situations, reported mathematical skills scores for American 15 year olds that were below average, ranking them 24th out of 29 countries (NCTM, 2005). This has led mathematical experts to conclude that teachers are not teaching problem solving within and among real-world applications (NCTM, 2005) and that more work needs to done in strategizing ways in which to facilitate and guide teacher's mathematics instruction.

2. Problem Solving in Elementary Mathematics

Lambdin (2003) describes problem solving as somewhat cyclical and interdependent with understanding. "Understanding enhances problem solving… learning through problem solving develops understanding" (as cited in Lester & Charles, Eds., NCTM, 2003, p. 7). As Lambdin (2003), Wilson, Fernandez, & Hadaway (1993) also imply that problem solving is cyclic in nature. When a student focuses on a problem, thinks s/he understands it, and devises a solution plan, a series of steps (processes) are initiated and revisited as the student's thinking continues to evolve. Yet when implementing the plan of action, s/he discovers a misconnection in her/his understanding of the problem which requires revisiting the problem. Thus, it seems that problem solving is an iterative process and, if this is the case, all elementary, middle, and high school teachers might be well advised to include problem solving in the majority of their mathematics teaching. Van de Walle (2004) calls problem solving "a principle instructional strategy" (p.36) used to fully engage students in important mathematical learning; thus, it seems that problem solving might permeate not only almost every mathematical task, but also life in general.

The real question is: How do children use problem solving and what might their selection of tools, manipulatives, or materials for creating and showing their solutions look like? Reusser (2000) argues that, "children are active individuals who genuinely construct and modify their mathematical knowledge and skills through interacting with the physical environment, materials, teachers, and other children" (p.18). In reality, how many times are decisions made through problem solving right in the child's home at the breakfast table when five people choose to eat cereal for breakfast and only one cup of milk remains in the carton or when you get in your car to drive to work and suddenly find a flat tire? Aren't these everyday situations in nearly everyone's lives that imply some knowledge of problem solving skill? And, when needed, wouldn't most people solve either of these problems relatively efficiently? It seems that we do use problem solving on a regular basis, and, usually our problem solving tasks appear to be connected to some form of tool, tangible, or manipulative (tires, air pressure gauges, cereal types, sizes, and characteristics, and so on…). If this kind of problem solving were connected both directly and indirectly to problem solving with numbers and mathematics, might this become a natural conduit for a deepening sense of knowledge in mathematics? The goal here would be to guide elementary children to take the performance-based knowledge gained from choosing and using manipulative-based materials, as described previously, into the creation of models which are, subsequently, connected to numbers and algorithms.

Young children are immersed in problem solving as they experiment with and intuitively contemplate how things work, what makes it warm or cold outside, and what series of hand manipulations are required to tie the laces on a pair of shoes. If teachers and parents were to facilitate this type of experimentation with models, diagrams, and, finally numbers, it would seem that the fine line between everyday problem solving *with* tools and manipulatives and mathematical problem solving *through* tools and manipulatives would fade away. In other words, the mathematics would optimally gain fluency and flexibility and, at the same time, automaticity, with less traditional drill.

Realistically, sometimes adults assume far too much about the connections piece of the puzzle and hurriedly skip over a short lesson on the sequence of steps needed to tie a shoe (ordinal positioning) or what temperature, in numbers (degrees), really represents. For the latter, temperature is often reported to children as hot or cold, cloudy or sunny, windy or humid, rather than numerically with clear and accurate descriptions. Children are very able to decipher why 18º Fahrenheit is "cold" and 88º Fahrenheit is "hot", numerically and mathematically, when allowed to use problem solving in such situations with the guidance of the teacher. When the teacher uses accurate mathematical and meteorological language to describe the *difference* (70º) between the two temperatures as a part of the process, it most likely will become a functional part of the child's problem solving process and offer them more opportunities for developing real understanding.

3. The Role of the Teacher in Problem Solving

Innately, most teachers seem to teach problem solving as a series of steps and/or in linear fashion, while most students need not just the linear set of steps, but also a full array of ongoing, supported opportunities to indirectly develop and hone problem solving techniques. This in depth development of problem solving does not denote full understanding of the mathematical task at hand, nor does it imply that it is done in isolation, rather it is usually accomplished through engaging problems in which children connect new and previous information (Lambdin, 2003).

One of the foremost criteria for enhancing problem solving skill in children appears to involve being taught by teachers who are able to facilitate rather than direct the learning. Desoete, Roeyers, and Buysse (2001) studied the relationship between metacognition and problem solving in 80 third graders and found that a connection between these two variables occurred more significantly among above-average than novice students. This might imply that children who experience more metacognitive or introspective thinking opportunities may subsequently become stronger problem solvers, which might suggest that more interactive approaches to teaching could develop stronger thinkers and problem solvers. It is also important to ponder varied strategies for guiding teachers who are reluctant to embrace manipulatives and problem-based methods in their classrooms.

Part of what happens when teachers are reluctant to use perceived innovative, manipulative- and problem-based approaches relates back to perceptions of what mathematical problem solving and learning looked like in their own learning – most likely an abstract word problem or an algorithm written on the chalkboard. In reality, the latter technique is a relatively far-fetched approach to the problem solving that will occur everyday. This becomes the impetus for striving to apply realistic, everyday situations to the mathematical skill development in the classroom using a *situational context* or real-world context to teach and learn within, implying that the elementary mathematics teacher should observe, facilitate, and foster problem solving within and among academic disciplines to guide a natural and non-threatening approach to everyday problems. Such an approach also reduces the abstractness of the problem solving, moving it into a more concrete, reality-based setting. Concrete and less abstract situations may arise during reading groups where children will need to determine the number of pages read each day in order to complete a chapter book in a two-week period or how many children would need to be in each group if there were thirty-one children in all and each group needed five members. When children are encouraged to think like problem solvers on a regular, everyday basis, they will most likely become effective and confident mathematical problem solvers during mathematics instruction and use.

4. Strategies for Teaching and Assessing Problem Solving with Manipulatives

Good elementary teachers are masters at modeling appropriate strategies for children. They bring their own automaticity, or fluidity, to master instructional skill in mathematics when they not only use the numbers, algorithms, and processes needed to solve a specific problem, but also show, through interactive modeling, what the problem actually looks like. For many children, this is the moment of enlightenment in their mathematics learning because they are actually able to see and touch "the problem" while associating the "model" with the numbers. This is the essence of tool- and problem-based teaching and learning which, in turn, affords children multiple opportunities to construct mathematical knowledge while making reasonable connections to everyday tasks. This is the real bridge between the concrete and abstract.

Although formal testing of knowledge will continue to be evident in determining how much a person knows about a particular topic, it “need not be a collection of low-level skills exercises” (Van de Walle, 2003, p. 72). Assessment of mathematics learning should be cohesively connected to mathematics instruction, which is often done using models and/or manipulatives. Van de Walle (2004) argues that within a well-constructed test “much more information can be found than simply the number of correct or incorrect answers” (p.72). This is important since good instruction (prior to testing) should have included performance-based use of models, drawings, and other representational depictions through which students developed further relational understanding between and among mathematical concepts. Then, during testing, the same models and/or manipulatives should be included as a relevant piece of the assessment and not simply test fragments of learning in isolation (performance-based assessment) or “tests that allow students the opportunity to show what they know” (Van de Walle, 2003, p. 73).

Investigation of manipulative use in problem solving within a classroom setting often reveals teachers who are invested in meeting the diverse needs of all students (Equity Principle, *Principles and Standards for School Mathematics*, NCTM, 2004), but who are averse to using manipulatives for varied reasons.

First of all, teachers need to know when, why, and how to use manipulatives effectively in the classroom as well as opportunities to observe, first-hand, the impact of allowing learning through exploration with concrete objects. Constructivism has evolved from theorists such as Jean Piaget (1965) and Lev Vygotsky (1962). Piaget (1965) approached the construction of knowledge through questioning and building on children’s answers while they constructed knowledge while Vygotsky (1962) felt that children could be guided to stronger mathematical understandings as they progressively analyzed complex skills on their own with the teacher nearby to scaffold or facilitate as needed.

Prior to discussing specific strategies, it is important to delineate some of the necessary benchmarks for effective manipulative use. First, it is essential for teachers to realize the impact of referring to manipulatives as *tools* to help students learn math more efficiently and effectively rather than as *toys or play things*. If manipulatives are referred to as "toys", students will see them as something to play with rather than as tools to work with to better understand mathematics. Second, manipulatives must be introduced in a detailed format with a set of behavior expectations held firmly in place for students to begin to develop a respectful knowledge-base about using manipulatives for math learning. Third, manipulatives need to be modeled often and directly by teachers in order to help students see their relevance and usefulness in problem solving and communicating mathematically. And, finally, manipulatives should be continuously included as a part of an exploratory workstation or work time once open explorations have been completed.

Teachers who consistently and effectively model manipulatives in front of all students will automatically offer all students a belief that using concrete objects to understand abstract concepts is acceptable and expected. Just as encouraging students to develop mental models, Venn diagrams, flowcharts, or matrices helps to broaden and enhance complex mathematics learning, so do manipulatives. If students feel that manipulatives are only used by those who are can't or don't understand or are less able, they will develop a negative attitude about manipulatives and an unwarranted stigma toward manipulatives will be launched.

5. Effectively Introducing and Implementing Manipulatives in Performance-Based Tasks

Preparing students to use concrete objects in mathematical exploration and problem solving is often overlooked, but is, truly one of the essential elements of successful implementation of a manipulative-based math program. Following these ten essential steps will help to establish a manipulative standard for the classroom (Author, 2002):

1. *Clearly Set and Maintain Behavior Standards for Manipulatives*

Students need to have a clearly established criteria for effectively handling and using manipulatives in the classroom. Without a clear set of expectations, students may misuse materials and teachers will become frustrated and disillusioned about manipulative use and, most likely, discontinue their use in the classroom. Rules for specific activities that incorporate manipulatives must be clearly articulated by the teacher, posted in the classroom, and re-affirmed consistently as needed during the manipulative lesson. In short, students need to be meaningfully guided to use and understand the purpose of the manipulative for the specific math task at hand, and then, it will gain relevance for them as mathematicians.

2. *Clearly State and Set the Purpose of the Manipulative Within the Mathematics Lesson*

If students know why the teacher has a certain expectation for a lesson, he/she will be much more likely to attend to the purpose of the task and handle the lesson manipulative correctly. It is important to remember that most math manipulatives are colorful, enticing, and closely resemble what most students have previously referred to as "toys". Since this is a natural association, it is of primary importance that teachers consciously facilitate understanding of the difference between math manipulative or tool and toy. If this is done carefully and effectively at the beginning of the academic year, students will be much less apt to misuse or mishandle mathematics manipulatives.

3. *Facilitate Cooperative and Partner Work to Enhance Mathematics Language Development*

The nature of manipulative use encourages interaction with not only objects but also with people since it usually involves an *action* on an object. Being able to learn and use mathematical language effectively helps to lay a strong foundation for conceptualizing and using abstract math skills in everyday life. It also helps students to develop and feel mathematical power as they become more able to articulate, both verbally and in written form, their math thinking processes. Using partner work with manipulatives to construct mathematical meaning also allows the more reticent math student a supported opportunity to explore strategies from both the observer and participant viewpoints.

4. *Allow Students an Introductory Timeframe for Free Exploration*

Once the purpose and behavior expectations have been established, students need to be given an opportunity to become familiar with the manipulative, discover its properties and limitations, and experiment with it in a variety of contexts. This, too, encourages cooperative work, language development, and risk-taking. Free exploration gives less active students individual opportunities to construct their own meaning and develop confidence in using the manipulative to solidify and enhance their math understanding.

5. *Model Manipulatives Clearly and Often*

Modeling on the overhead or in large or small group sessions will help students see how a particular manipulative can facilitate understanding. For example, when students are beginning to learn about measurement and non-standard or arbitrary units of measure, it is essential that they have a large variety of manipulatives (Cuisenaire rods, Unifix cubes, links, paper clips, pencils, etc.) with which to measure commonly used items (desks, chalk board ledges, window sills, etc.). In doing so, students will be developing real number sense about measurement. As a means for developing number sense, Marilyn Burns (1997) suggests that teachers include as much measurement as possible in their mathematics teaching as it is the foundation upon which strong mathematical understanding is built.

6. *Incorporate a Variety of Ways to Use Each Manipulative*

Offering students different ways to view the same problem will ensure that more of the students will gain a deeper and richer understanding of mathematics. In turn, they will further develop their own levels of fluency and flexibility with numbers as suggested by the new *Principles and Standards for School Mathematics* (2000). And, showing students how to use the same manipulative in a variety of ways will only tend to strengthen their everyday use and understanding of mathematics. For example, students may use pattern blocks initially to learn colors, shapes, or patterns and eventually create and decipher equal and unequal fractional parts.

7. *Support and Respect Manipulative Use by All Students*

The Equity Principle (NCTM, 2000) clearly states that high expectations and strong support for all students must be evident for excellence in mathematics education. Be sure to clearly and positively "set the stage" in your classroom for inclusion of all students in manipulative use. If you model and use manipulatives, mental models, and other tangible materials to problem-solve, your students will be much more apt to do the same. In turn, when teachers openly express less positive feelings about using manipulatives to solve problems, those who require tangible objects to reach success will be less likely to use them and, subsequently, less likely to gain a firm grasp of the math skill or concept in front of them.

8. *Make Manipulatives Available and Accessible*

In order to facilitate manipulative use at any grade level, the chosen and/or required manipulative must be stored in such a way that it will be physically reachable by all students, plentiful enough (in number) to allow each student to have access to a complete set, and labeled correctly with clear instructions as needed based on the intended purpose, which is not to say that creative manipulative use should not be sanctioned.

9. *Support Risk-Taking and Inventiveness in Both Students and Colleagues*

Teachers who model risk taking and are open to mistakes and re-thinking will enhance student's abilities to move into uncharted territory. Supporting risk-taking and inventiveness in students leads them to explore unknowns and strive to reach unanswered questions as it facilitates open-mindedness and creativity. Students should be supported in seeking and using their own processes in problem-solving. Manipulatives are natural conduits for successful, interactive construction of knowledge through problem-solving.

10. *Establish a Performance-Based Assessment Process*

Since manipulative use is based on constructing or performing an action with a tangible object or set of objects, finding out what students know must also be based on active teacher observation and a set criteria of expected outcomes or, usually, a rubric-style assessment tool. Assessing hands-on inquiry can be a challenging task that implies a commitment of time and energy beyond paper-pencil measures of achievement. Authentically taking into account what a student knows and can do (perform) following a manipulative- and inquiry-based task requires keen observation skills and patience from the teacher. What the teacher actually *sees* the student do with a manipulative-based task is nearly as important as the mathematical thinking that the student can verbally organize and communicate coherently and clearly to teachers, peers, and parents. In assessing hands-on inquiry with manipulatives, the ten standards for school mathematics (NCTM, 2000) are actualized through verbal and non-verbal means. Developing rubric-style assessments for manipulative-based activities with students and colleagues helps to assure that the assessment actually measures what was taught and practiced, to bring strong student investment in the teaching-learning process, and develop real mathematical learning.

6. Conclusion

Early examples of the benefits of a manipulative-based mathematics program can be seen in kindergarten and primary classrooms where young children are using manipulatives, such as Algebocks, to learn algebraic concepts such as patterns and functions. In turn, bubbleology and materials, like Zometools, plastic Polydrons, and connected drinking straws, are helping very young children learn about the properties of angle, shape, and congruence in geometry. Realizing this, one might ask – "Would it be as easy to teach young children these more abstract concepts without manipulatives?" Probably not, and, in turn, isn't it plausible to surmise that middle and high school students who might struggle with math and discontinue math classes altogether after requirements are met, might also benefit from teacher encouragement through manipulative use? If manipulative use becomes an integral part of the academic structure for all students in mathematics classrooms, it may keep more students in higher-level math classes through college and beyond.

Students quickly pick up on what teachers verbal and body language tells them very quickly, particularly when it effects overall self-esteem and confidence. If teachers send messages to students that "only less able students need manipulatives" or "you don't need those anymore, do you?", students will be begin to devalue and stop using manipulatives and reduce their own chances for success in mathematics overall. Usually students will naturally choose tools that will best help them learn a particular concept, and, if manipulatives help to make mathematics "come to life", they should be encouraged.

Finally, with research on performance-based manipulative use seeming to be varied and limited, it would be beneficial for longitudinal study on the year-to-year transfer of children's problem solving abilities with manipulatives to be examined. Not only would it benefit teachers and parents, but also future employers and businesses.

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