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Sunrise… Sunset…

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Sunrise, sunset,
Sunrise, sunset,
Swiftly flow the days,
Seedlings turn overnight to sunflowers,
Blossoming even as we gaze.
(Fiddler on the Roof, Lyrics S. Harnick)

Introduction

Sunrise and sunset are obviously a part of our everyday life. Our connection to the movement of celestial bodies, including the Sun, has been somewhat diminished by modern life, in contrast to the practical interest, fascination or even awe of earlier civilizations. Mathematics and astronomy have a long history of cooperative work, but due to their complexity and technical difficulty, usually those who participated in this endeavor were at the advanced studies level, or occupied high religious positions.

However, today's technological tools can overcome many of these obstacles, and they have the potential to make mathematical–astronomical investigations possible at relatively early stages of education. In our opinion, currently our challenge is to raise student interest and to design appropriate investigative activities in this field. Beyond its social and historical importance, the investigation of the Sun's "movement" (actually, position) in our terrestrial world has many mathematical benefits.

In the following sections we will describe the *Sunrise – Sunset* activity, present some observed classroom reactions to this task, and finally discuss the learning potential of such an activity.

Sunrise – Sunset

The data for the *Sunrise-Sunset* activity were prepared in advance by the teacher, and presented to the students in the format of two spreadsheet tables. The tables contain the local time (in our case, Tel Aviv) for the sunrise and sunset at the first day of each of 24 consecutive months, starting January 1st. The dates were numbered from 1 to 24, and the time was presented both in standard form (figure 1a) and in decimal notation (figure 1b).

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	A	B	C	D	E	F	G	H
1		Month	Sunrise	Sunset		Month	Sunrise	Sunset
2		1	6:41	16:48		1	6.68	16.80
3		2	6:35	17:15		2	6.58	17.25
4		3	6:10	17:38		3	6.17	17.63
5		4	5:30	18:00		4	5.50	18.00
6		5	4:56	18:20		5	4.93	18.33
7		6	4:35	18:41		6	4.58	18.68
8		7	4:37	18:51		7	4.62	18.85
9		8	4:55	18:39		8	4.92	18.65
10		9	5:15	18:07		9	5.25	18.12
11		10	5:33	17:27		10	5.55	17.45
12		11	5:56	16:52		11	5.93	16.87
13		12	6:22	16:38		12	6.37	16.63
14		13	6:41	16:48		13	6.68	16.80
15		14	6:35	17:15		14	6.58	17.25
16		15	6:10	17:38		15	6.17	17.63
17		16	5:30	18:00		16	5.50	18.00
18		17	4:56	18:20		17	4.93	18.33
19		18	4:35	18:41		18	4.58	18.68
20		19	4:37	18:51		19	4.62	18.85
21		20	4:55	18:39		20	4.92	18.65
22		21	5:15	18:07		21	5.25	18.12
23		22	5:33	17:27		22	5.55	17.45
24		23	5:56	16:52		23	5.93	16.87
25		24	6:22	16:38		24	6.37	16.63

Figure 1. Spreadsheet timetables of sunrise and sunset (in Tel Aviv) in standard form (a) and in decimal notation (b).

The work of the students, a class of ninth graders, was observed during two class periods. Some student reactions are presented below (in italics).

1. Getting acquainted and making predictions.

First, the students understood the context and the data of the problem under investigation, made predictions about the shape of the graphs that show the change in time for the sun's rising and setting during two consecutive years, and became involved in the activity. This part consists of two steps:

- Class discussion and comparison (advantages, disadvantages) of the standard and decimal notation in the two tables (figure 1).
- Sketching (i.e. drawing in an unscaled coordinate system) two graphs, in order to show the general change in sunrise and sunset time during two consecutive years.

The students worked in pairs, and then looked and compared their graphs with those of their colleagues. Most of them sketched cyclic graphs (figure 2), and one pair of students sketched two identical graphs – a separate graph for each year.

Students S1 and S2 sketched a graph of straight lines (see figure 2A), and compared their work with the curved graph of their neighbors, S3 and S4 (figure 2B).

S1: I think they are right.
S2: Our [graph] is the same. This is only a sketch.
S1: Theirs is rounded – and this is better.
S2: It's the same – it's only a sketch.
S1: It's rounded…It's better.
S2: What do you mean, "rounded"? What is the difference between these two [graphs]?
S3 (Interrupts her work with her peer in Pair B and points to the neighborhood of the graph's maximum): Ours is slower here and yours drops sharply.
S4: It's the period between going up and going down… We have more points, and in yours it looks as if there is only one point.
S2: ("Giving up"): You want us to round it?
S1: Yes. It's more correct.
S2: Then let's round it off.
S1: They made them this way [showing that the graphs of pair B are parallel].
S2: Ours is OK [the sunrise graph starts by decreasing – in contrast with the phase difference of the sunset graph, which starts by increasing].

Some students drew the sunrise and the sunset graphs as intersecting curves (figure 2C). When asked by the teacher about the meaning of the intersection points, these students changed the position of their graphs.

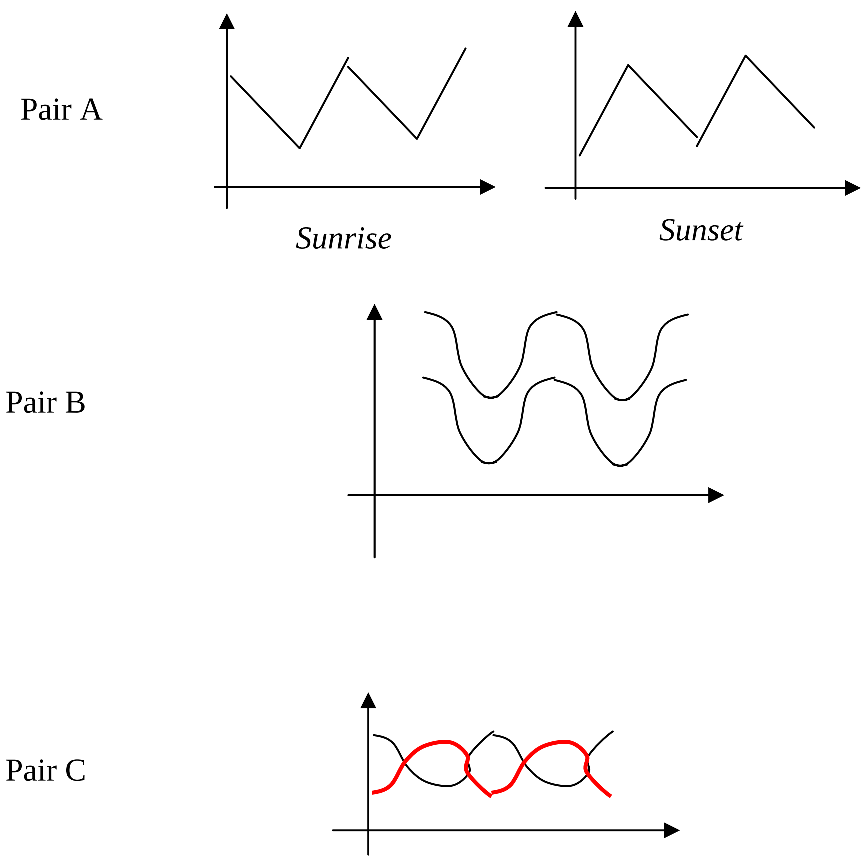


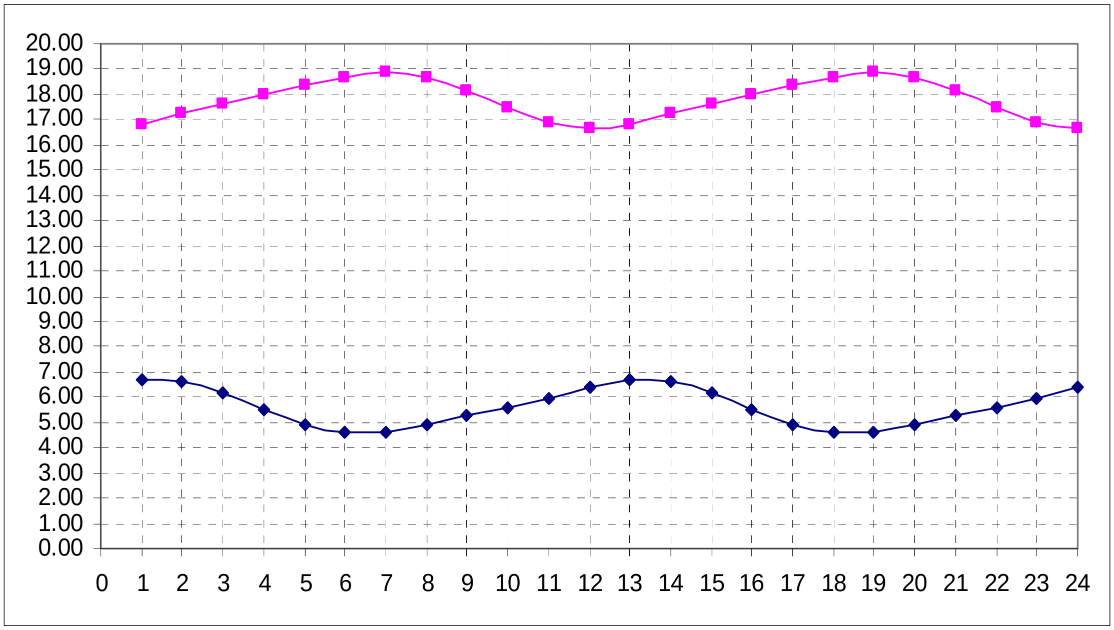
Figure 2. Students' predictions of shape and position of *Sunrise-Sunset* graphs.

- Reflecting on predictions. The issue of shape and position of the two graphs was discussed first in groups, and then in a forum involving the whole class.

2. Analyzing data and drawing conclusions.

In this part, the students produced graphical representations of the data, and analyzed their mathematical, astronomical, and daily-life meaning. This part consisted of several steps.

- Using spreadsheets to construct graphs of the sunrise and sunset times, and comparing the hand-sketched predictions and the spreadsheet graphs. Figure 3 shows the spreadsheet graphs of the sunrise and sunset times located in one coordinate system.



<u>Figure 3.</u> *Sunrise-Sunset* spreadsheet graphs.

An "Aha!" effect could be felt in the class, whenever the spreadsheet graphs of several students turned out to be different from their preliminary sketches.

- Investigating the data (patterns, extremes, cycles, axes of reflection, and rate of change) and making everyday-life interpretations of the given numbers and obtained graphs.

  Some students "talked" in a mathematical language, whereas others used context-bound expressions. Some examples:

  The functions are "monotonous" [intuitively meaning cyclic].

  The functions are opposite one another [intuitively meaning there is a phase difference, and the maximum of one graph corresponds to the minimum of the other].

  The sketch repeats itself.

  The sketch goes up and down.

  The sunset [graph] goes up, as the sunrise [graph] goes down.

  Every twelve months, the sunrise and the sunset occur at the same hours.

- Investigating patterns of daylight hours: How can we find the length of the daylight from the *Sunrise-Sunset* timetable? From the *Sunrise-Sunset* graphs? Construct a daylight column in the spreadsheet table, and produce a corresponding graph. Analyze the change in daylight hours (look again for patterns, extremes, cycles, axes of reflection, and rate of change).

  Students S1 and S2 (pair A) used the formula of the difference between sunrise and sunset (decimal) times to find the length of daylight in their spreadsheet table, and were surprised to receive negative numbers. They corrected their formula, and then wondered whether the "straight" difference provides the desired answer, or whether their formula should be corrected by plus or minus one. Subtracting whole hours and *checking by finger counting helped them decide that the "straight" difference gives the desired answer.*

Most students drew vertical segments between the two graphs as a graphical method of finding the length of daylight (figure 4). The students investigated patterns of daylight by using either the numerical data (the spreadsheet column) or a graphical representation (the vertical distances between the graphs).

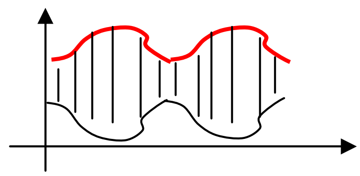


Figure 4. Using the *Sunrise-Sunset* graphs to look for patterns of variation of daylight.

3. Reflecting on the activity and on its mathematical implications.

During the summary in class, the students were asked to reflect on their investigation, and to consider the following issues:

- To what extent are the obtained graphs accurate – i.e. how well do they describe the real situation?

  A student raised the issue of the spreadsheet graphs' lack of symmetry (see figure 3). Most of the class had the intuitive feeling that the graphs should be symmetric. The teacher asked about the meaning of a symmetrical graph (sunrise and sunset times going up and down for equal periods), but the prevailing opinion was still that "the computer is probably right". The teacher indicated that the graphs could be different, if they were based on a more detailed (for example, a daily) timetable, or on monthly dates that are closer to the graphs' real extremes (the 21st or the 22nd of each month).

- If we separate the graphs from their context of sunrise-sunset time, and view them as mathematical "creatures", how can we characterize them?
- How would the *Sunrise-Sunset* graphs look in other places on the globe?
- How would the graphs look if the hours were not adjusted for daylight savings time?
- On what occasions can the *Sunrise-Sunset* data and conclusions be useful?

Conclusion

Finally, we would like to reflect on whether activities like *Sunrise-Sunset* provide satisfactory answers to the following questions.

- ***How can pattern recognition be used to develop students' problem-solving abilities?***

  The *Sunrise-Sunset* activity and similar investigations in other domains frequently require the following steps:

- define a research problem
- collect and organize the data
- predict the results
- analyze data / find patterns / look for solutions
- draw conclusions / compare the findings and predictions / reflect on the process.

This paradigm of work is common to both scientific inquiry and mathematical problem solving.

- ***How can patterns generated through spreadsheets enhance students' understanding of mathematics?***

  Spreadsheets play an important role throughout the *Sunrise-Sunset* activity:
  - They provide a quick, effective, and accurate passage from an extended numerical table to the corresponding graphs.
  - They provide opportunities to make predictions based on raw data, subsequently comparing them to the results of a more extensive analysis.
  - They emphasize global aspects and patterns of complex phenomena, based on a large quantity of local data.
  - They provide a variety of representations (numerical, graphical, and algebraic), and allow the use of these representations, according to the task at hand.
  - They help to perform calculations almost instantaneously, and as a result, they enable students to develop and employ higher-level skills – such as defining new variables, creating and using algebraic formulas, generalizing patterns, monitoring results, and drawing conclusions.

- ***How can the investigation of patterns lead to a better understanding of such concepts of algebra as variable, rate of change of a function and the slope of a graph?***

  When students investigated the *Sunrise-Sunset* problem, we observed spontaneous references to the rate of change (especially with regard to the timetable) and the graphs' increase or decrease of slope. The observed class of ninth graders had not yet encountered a formal definition of a slope or of a linear function. The context allowed and even encouraged students to discuss and use these concepts with regard to natural phenomena, such as change in sunrise, sunset, and daylight time, or to mathematical phenomena, such as the steepness, curvature, and symmetry of a graph.

- ***How can explorations of patterns in a student's earlier experiences be used to develop more sophisticated topics?***

  We tried to show here that our exploratory activity has the potential to provide an informal link in the continuous process of learning mathematical concepts related to the properties and patterns of linear and trigonometric functions.

- ***Where can pattern recognition in other disciplines be connected with familiar mathematical content?***

  Similar activities of pattern recognition can relate to a variety of domains, such as architecture, plants, animals, physics, and geography. In these activities, the data can be represented and analyzed in various ways, and interesting patterns can be observed and explained. Finally, mathematical and context-based conclusions are drawn, and additional issues are perhaps raised. In our case, we observed students intuitively linking the familiar sunrise and sunset phenomenon to important mathematical concepts, later to be formalized in function analysis and trigonometry.