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K- theory for concrete C*-algebras

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In 2018 I was awarded a grant of $4,800 through the University Grant Program to study "K-theory for concrete C^*-algebras." This grant was used to fund my research visit to Australia and New Zealand in June 2019, and to host Prof. Caleb Eckhardt (Miami University) at the University of Montana in both August 2018 and June-July 2019. In broad strokes, the objective of the research undertaken via this grant is to increase our understanding of the mathematical objects known as C^*-algebras, by exploring the structure of certain C^*-algebras arising from relatively concrete data.

The goal of my trip to Australia and New Zealand was to work with Prof. Aidan Sims (University of Wollongong, Australia) and Dr. James Fletcher (Victoria University Wellington, New Zealand) on a new approach to computing the K-theory for the C^*-algebras of higher-rank graphs and related objects. During my June 2019 visit, we completed the research necessary to finalize a preprint, titled " Homotopy of product systems and K-theory of Cuntz–Nica–Pimsner algebras," which will be posted at `www.arXiv.org` and be submitted to a peer-reviewed journal for publication in October 2019. Because the C^*-algebras of higher-rank graphs are examples of Cuntz–Nica–Pimsner algebras, this paper provides a new lens—namely, the notion of a homotopy of product systems—through which to analyze their K-theory. A key tool for studying C^*-algebras, K-theory repackages a large amount of information about the C^*-algebra in a simple-seeming way. This simplicity can be deceptive, however. In the case of higher-rank graph C^*-algebras, there is no explicit formula for computing their K-theory. We therefore expect that our new perspective on K-theory for the C^*-algebras of higher-rank graphs will shed new light on the relationship between the higher-rank graph, its C^*-algebra, and its K-theory.

During Prof. Eckhardt's visits to the University of Montana, we investigated the structure of the twisted C^*-algebras associated to nilpotent groups. We established several years ago [2] that if one removes the adjective "twisted," then the K-theory of these C^*-algebras contains nearly all the information about them. (To be precise, the simple C^*-algebras associated to nilpotent groups are classified by their Elliott invariant, which consists of their K-theory plus a bit more information.) However, the approaches used to analyze the structure of these untwisted C^*-algebras of nilpotent groups (cf. [4, 2, 3]) seem intractable, at least in full generality, in the setting of twisted C^*-algebras of nilpotent groups. We are currently exploring the decomposition rank (a type of dimension) for the twisted C^*-algebra associated to the 2-step Heisenberg group, and we are optimistic that this research will shed more light on the relationship between decomposition rank and other C^*-algebraic constructions such as crossed products, as well as elucidate the challenges confronting our initial approach to the structure of the twisted C^*-algebras of nilpotent groups.

Both my travels, and Prof. Eckhardt's visits, proved unexpectedly beneficial for the research project I pursued with University of Montana students during summer 2019. Through this summer project, funded by my grant "Structural invariants for higher-rank graphs" from the National Science Foundation, two undergraduate mathematics majors (K. Fieldhouse and I. Gonzales) and one master's student (D. Gent) worked together on establishing analogues for higher-rank graphs of the C^*-algebra-preserving "moves" for graph C^*-algebras of [1, 5]. Due to conference travel and obligations under other grants (combined with the fact that the originally planned dates for my visit to Australia/New Zealand proved impossible due to the early start of the Spring 2019 semester), I spent a large part of the summer away from Missoula, so I asked Prof. Eckhardt to spend six weeks

in Missoula as a co-mentor for this research project. The students found his presence and insight incredibly helpful, and the two weeks when Prof. Eckhardt and I were both in Missoula proved beneficial in furthering our joint research, described above.

Moreover, during my visit to the University of Wollongong in Australia, I spoke with Prof. David Pask about this research project and his existing work on "moves" for graph C^*-algebras and related topics. These conversations proved so fruitful that Prof. Pask is now a coauthor on the preprint "Moves on higher-rank graphs preserving Morita equivalence" arising from the summer research project. We anticipate that this preprint will also be posted at `www.arXiv.org` and be submitted to a peer-reviewed journal for publication in October or November 2019.

References

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