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Abstract: *Mathematical modeling of real world conditions should be part ofl mathematics classroom activities. In this paper I argue that when real world problems are taught at schools learners are not able to cope on their own, without the assistance of their educator. There is very little or no emphasis placed on this aspect of mathemtics at schools, although it is just beginning to make an appearance in our new Outcomes Based Curriculum. I also discuss an experiment conducted with Grade 10 learners (15 year old) and their responses to real world problems and the conditions that need to be considered. There is ample evidence that a lot of work on mathematical modeling of real world problems has been done elsewhere in the world, but not much has been done in South Africa. This experiment was fully conducted using Sketchpad as a mediating tool. This in itself was a difficult task because our learners have not been really exposed to dynamic geometry environments.*

1. The Teaching Experiment

This paper reports on a modeling experiment that was conducted, to teach concepts such as perpendicular bisectors and concurrency, with grade 10 learners, using a pseudo real world problem. The learners selected for this research were from a school in Chatsworth in KwaZulu-Natal, South Africa. These learners came from lower to middle income families and were all average to below average performers in their school mathematics examinations and tests. In this particular school, very little or no modeling was done with them, although certain word problems may have, to a small extent, required some modeling activity. The learners themselves were selected randomly from a computer studies class by their computer studies educator. The problem given to them was contextualized within the South African rural background. The question was:

> *In a developing country like South Africa, there are many remote villages where people do not have access to safe, clean water and are dependent on nearby streams or rivers for their water supply. With the recent outbreak of cholera in these areas, untreated water from these streams and rivers has become dangerous for human consumption. Suppose you were asked to determine the site for a water reservoir and purification plant so that it would be the same distance away from four remote villages. Where would you recommend the building of this plant?*

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2. The learners' ability to create and use mathematical models to solve the specific real world problem

Rather than immediately starting with *Sketchpad* (a dynamic geometry software), the researcher first asked the learners to attempt to find a solution on their own, using any previous knowledge. All learners "guessed" a solution somewhere in the "middle" of the quadrilateral, but none could find a precise solution. More importantly, few of them even thought of or tried to test their solution. This extract from the interview with Pravanie (see below) was typical of several interviews. The learners seemed to feel that this type of question was not within their ability to solve it and in some cases the learners explicitly said that this was because this type of question had never been taught or asked of them. This is a very significant point. Learners seemed to feel that they could not solve problems not seen before. Moreover it seemed that a real-world context such as this was rather novel to them.

RESEARCHER	Where do you think that we should build the reservoir?
PRAVANIE	I don't know … all we are only given is this diagram …
RESEARCHER	Do you think that you will be able to find the most suitable point? You can use any method you know, to do so.
PRAVANIE	I don't know sir … this is too difficult … please don't interview me (pleading).
RESEARCHER	Are you saying that you cannot find any way of solving this problem?
PRAVANIE	I can't ... I'm not so good in maths … maybe at the centre here (pointing to the middle of the quadrilateral).
RESEARCHER	Will you be able to justify your answer? Will you be able to tell me why?
PRAVANIE	(silence)…… not really …
RESEARCHER	Don't you even want to try?
PRAVANIE	I don't know what to do …

From the above it is clear that Pravanie was very uncomfortable being faced with an unknown problem not seen before, and that she seemed to have been so intimidated that she did not even want to continue with the interview. She seems a typical product of the traditional approach where learners acquire a "learned helplessness" that is, an unwillingness to attempt problems on their own.

Christina's approach though was different (see below). Although she could not determine a precise way of finding a satisfactory solution, she was initially prepared to use pencil and ruler. After she discovered that using the pencil and ruler only gave an approximate solution she simply gave up.

RESEARCHER	Where do you think that we should build the reservoir?
CHRISTINA	Can I use my ruler and pencil?
RESEARCHER	You can use any method you know, to do so.
CHRISTINA	(after a while) I thought it will be here … (pointing to the middle) …(measuring the distances) … but when I measure the distances, it's not the same.
RESEARCHER	Are you saying that you cannot find any way of solving this problem?

CHRISTINA I think it is in the middle … maybe at the centre here (pointing to the middle of the quadrilateral but not continuing further).

When asked to explain why she thought the solution was somewhere in the middle, she emphatically refused to provide an explanation or to continue looking for a better solution.

RESEARCHER Will you be able to justify your answer? Will you be able to tell me why?
CHRISTINA No! (emphatic)
RESEARCHER Don't you want to try?
CHRISTINA No!

Faeeza displayed good self-confidence initially (see below). Her comments such as : " *It's easy to understand*" and "*Ya … can I measure with my ruler*?" showed that she felt that she was capable of solving the problem. Although she was sure that she could prove that her guess (somewhere in the middle of the quadrilateral) was correct by measuring, she suddenly lost some of her confidence when she discovered that her guess was not correct. Yet again, just like the others, she could not think of another method. It is interesting to note that the learners became irritated when prompted by the researcher to try and find another method. Faeeza's teacher dependence acquired from traditional teaching is also clearly highlighted by her question "… *what method should I use*?"

RESEARCHER Faeeza, you've read the question, do you understand it?
FAEEZA It's easy to understand.
RESEARCHER Where do you think that we should build the reservoir?
FAEEZA About here (pointing to the centre of the quadrilateral).
RESEARCHER Can you prove to me that this is the correct point?
FAEEZA Ya … can I measure with my ruler?
RESEARCHER Yes you may.
FAEEZA (After a while) I don't think I'm measuring correctly …am I? …
RESEARCHER Do you want to perhaps try another method?
FAEEZA I don't know … what method should I use?
RESEARCHER Do you know of any method?
FAEEZA No!
RESEARCHER Are you sure?
FAEEZA I really don't know (irritably).

Similarly, Nigel, Roxanne and Schofield had no idea about what they should be doing.

RESEARCHER Where do you think that we should build the reservoir?
NIGEL There (pointing to the middle of the quadrilateral).
RESEARCHER Can you prove that your answer is correct?
NIGEL No, I can't.
RESEARCHER Are you saying that you cannot find **any** way of proving this?
NIGEL I can't.
RESEARCHER Don't you want to just try?
NIGEL I wouldn't know were to start … I can't.

In all cases the learners could not think of any method (not even successive trial and error) that they could use to verify their guesses. The fossilized teacher-dependence of the learners is aptly summarized by Schofield's comment that: "*We didn't do this in class before … I can't do it!*" Clearly all these learners seemed to have been only accustomed to teaching-learning situations in which the teacher would always first present a new method or technique and all that is required of them is to practice it. It did not appear at all as if these learners had ever been exposed to a problem-centred (or modeling) approach before, namely, where they are expected to regularly tackle problems not seen before.

Roxanne's interview.

RESEARCHER	Are you saying that there is no way to do it or is it that you don't know how to do it?
ROXANNE	I can't. There might be a way to do it but I can't.
RESEARCHER	Wouldn't you want to try?
ROXANNE	I don't know what to do!

Schofield's interview.

SCHOFIELD	I … I don't know …
RESEARCHER	Do you think that there is a way of showing why your answer is correct?
SCHOFIELD	I don't think so.
RESEARCHER	Don't you want to try?
SCHOFIELD	We didn't do this in class before … I can't do it!

3. Learners' conjectures and their justifications

It is perhaps not surprising to note that all learners conjectured that the most suitable point was somewhere in the 'centre', presumably relying on their visual intuition to locate an approximate point equidistant to the vertices.

FAEEZA	Ya. It should be situated towards the middle.
CHRISTINA	In the centre
NIGEL	Towards the middle.
ROXANNE	In the middle.

The inaccuracy of their visual perception created surprise when the learners later discovered that their conjectures were not correct. This surprise in turn created some level of curiosity.

Despite their difficulty or reluctance to use successive guessing and testing with ruler or compass, they nonetheless were able to realize in the *Sketchpad* environment that the distances should be measured from each vertex to the constructed point, and that by dragging the point around one could change the distances until they were the same.

RESEARCHER	In the centre would mean about there in the middle? Is that correct? (Christina nods her head). Now how will we be able to determine whether that is the correct point?
CHRISTINA	We measure from that point to that (pointing to the vertices of the quadrilateral).
RESEARCHER	(after Christina measures the distances) What do you observe?
CHRISTINA	The distances are different…
RESEARCHER	How then can we find a suitable point?
CHRISTINA	Move the point around.
RESEARCHER	Move the point around. (After awhile) Is it easy to find this point?
CHRISTINA	No.
RESEARCHER	So what happens if we move the point around?
CHRISTINA	The distances will change.

Faeeza, who showed much confidence during the interview, was not intimidated by the fact that she was in a new environment (doing mathematics using the computer was new to them). Her reaction to questions and the tone of her replies conveyed the impression that she was quite comfortable with using *Sketchpad*. Other learners displayed similar behaviour, although they hesitated at some stages with their responses.

RESEARCHER	Now, Faeeza, draw a point somewhere in this quadrilateral … this is where you suggested the building should be. How will we know what the distances are from that point to the various villages?
FAEEZA	Obviously you need to measure it.
RESEARCHER	Will you measure it then? (after a while) Ok…there are all the distances from the point that you chose to the different villages. So what do we do now?
FAEEZA	Drag the point around until we get those distances equal.

Schofield also showed that he was comfortable with using *Sketchpad*.

SCHOFIELD	It is obvious that it must be in the middle. We can show this by measuring the distances from the village to the point that is chosen.
RESEARCHER	Do just that then….(after a while) … what do you observe?
SCHOFIELD	(silence)… the distances are different…
RESEARCHER	What should we do then to get the distances to be equal?
SCHOFIELD	Move this point around?
RESEARCHER	Go ahead and do that.
SCHOFIELD	(after a long while)… this must be the point.

Roxanne was the only one who initially stated that the ideal point should be at the centre and thereafter asked for the point to be shifted to the side. Although she had not found the accurate point, her moving of the point towards the right was getting closer to the correct position.

RESEARCHER	So do you think that it should be in the middle here? (pointing)
ROXANNE	Ehhh…maybe more towards the side here…(indicating a shift to the right)
RESEARCHER	All right, then how do you think we should go about checking whether it is correct?

ROXANNE By measuring the distances.
RESEARCHER Quickly do that … (after a while) okay. These are all the measurements. But the distances are not equal. How can we get then to be equal?
ROXANNE You can take the point around and try to find the spot for which the distances will be equal.

4. The recognition of real world conditions when modeling

At this point the child was told that real world situations are extremely complex and they usually must be simplified before mathematics can be applied to it. The learners were then asked to give some of the assumptions that they thought may have been made in order to simplify the problem that may not be true in real life. The responses received here showed that given an opportunity learners would be able to reflect on real life conditions as compared to traditional classroom situations where these assumptions are usually not discussed.

The learners being interviewed were able to recognize factors that could have affected the position they chose. At the expense of belaboring this point it may be essential to list the responses of all the learners just so that a clear impression can be obtained with regard to the way the learners construct their reality.

PRAVANIE Apart from the fact that there might be a valley over there, there could be a mountain, there could be a building already constructed. There could be endangered species.
RESEARCHER … endangered species of what?
PRAVANIE Any plants or animals and stuff like that … a nature reserve.

Rivers and mountains were common responses but the endangered species of plants and animals was an interesting response. It must be remembered that these learners had not seen this question before so their responses were spontaneous.

Christina's responses were similar to earlier responses but Faeeza's and Roxanne's responses about the chief's house or the chief's kraal being situated there was highly realistic.

CHRISTINA There might be… a hard rock.
RESEARCHER Yes….
CHRISTINA Other buildings
RESEARCHER What kind of buildings do you think?
CHRISTINA Police station or something….
RESEARCHER Any other reasons?
CHRISTINA If there's like a stream or river you won't be able to build

FAEEZA Maybe there is building …….
RESEARCHER What kind of building do you think?
FAEEZA Could be a school, I don't know…. Could be a shopping complex.
RESEARCHER Remember that these are remote villages ……
FAEEZA Might be the chief's house……
RESEARCHER Yes…

FAEEZA There might be a mountain there … anything, like a big rock. The cost factor must be too great to get rid of the rocks. It won't be cheap to build there.

ROXANNE Maybe there is a mine or a building.
RESEARCHER Remember that these villages are remote villages. What kind of building might be there?
ROXANNE There might be kraals there or the chief's house…
RESEARCHER Do you think that there might be other reasons?
ROXANNE The place might be a mountain, with hard rocks…

Nigel's responses below about a mine being at the exact spot or hazardous nuclear waste being stored there, were responses that were not expected and it showed that given the opportunity, learners could be very creative. This suggests the viability of a more concerted effort be made to encourage learners to think creatively about the real-world and its relationships with mathematics in the mathematics class.

NIGEL There might be a mine ……
RESEARCHER Yes, any other reasons?
NIGEL If the area is very rocky or has mountains…….maybe they are storing hazardous nuclear waste nearby…….

Perhaps a response more suited to a country like South Africa was that of Schofield. He spoke of financial constraints and the social problems by referring to the fact that the people themselves might be unhappy with the location. This showed that learners themselves could be politically mature enough to consider a wide range of issues.

SCHOFIELD There might be buildings like a school
RESEARCHER Remember that these are remote villages.
SCHOFIELD … there may be a river close to the point… if there are no roads then it is going to cost more money to first build roads…the people in the villages may say that they don't want the reservoir at that point…what about very hard rocks…

A matter of concern regarding the learners' responses regarding the unsuitability of the chosen position was the fact that none of the learners focused on the issues more specifically related to mathematics. With the exception of Pravanie none of the other learners stated that it was assumed that the land was flat and though a chosen position may be mathematically ideal, it may not be correct if one considered the possibility of hills and valleys. None of the learners realized that the relative sizes of villages might have some influence on the chosen position. For example, if one village was substantially larger than the others it may make practical sense to put the reservoir closer to it. It is also implicitly assumed that the positions of the reservoir and the villages can be represented by points, that is, their sizes are insignificant compared to the distances in question. If not, this raises several questions. Were the distances being calculated related to the centres of the villages or to the outer boundaries of the villages? Would this therefore not affect the position chosen?

These are critical questions that learners need to focus on when working with real world problems. It seems that their lack of experience in working with real world problems played a role.

The learners' responses to the real-life problems that could be experienced indicated a reasonable level of understanding. In fact, three of the learners directly indicated that mathematics alone cannot always be used for solving real life problems, that is, problems can be experienced.

NIGEL That sometimes in real-life we may not be able to use exact mathematics to solve problems.

ROXANNE We are saying that we can work out a place on the computer or by calculating it but it does not mean that it will be the right place.

SCHOFIELD Maths might be one thing but reality is another … sometimes we can't use maths on its own.

The other learners could see that a mathematically determined point may be unsuitable in the real world. This does not in any way indicate that mathematics is not effective but it does show that they had some understanding that with real life problems other factors must be considered.

PRAVANIE Very often we think a certain place will work but when we go there we notice that there is a problem.

CHRISTINA Sometimes finding the point might not work because … because the place might not be suitable.

FAEEZA Because we are trying to show that we can choose a point but that point is not always good… maybe we should only choose the point after we see the place.

5. Conclusions

Given the results of this experiment it was obvious that learners had not previously been exposed to real world problems which they had to solve using modeling strategies. In fact some learners clearly felt threatened and inadequately prepared to solve such a problem. One learner went as far as asking the researcher not to interview her because the work seemed too difficult. Another learner indicated that this type of problem was too difficult whilst another felt that he did not do this type of problem in class. Evidently learners seem to be good imitators of their educators in class. If the educator does a particular type of example then the learners are able to copy those strategies. Unfortunately it currently seems that few educators are engaging learners in mathematical modeling of problems. As a result learners may successfully answer ordinary mathematics questions, but may encounter difficulties when facing questions of the real world.

None of the learners interviewed in this research knew or could devise a precise method of finding a solution to the problem. All some of them could do was to attempt a trial and error

approach and measure the distances using a ruler. Although this leads to an approximate solution, it is rather time consuming (and often not accurate). It however generally shows that learners can understand and cope with real world, at least at the trial and error level. This finding therefore shows that teaching via modeling is possible as any real world problem can at least be approached in a trial and error fashion. Indeed, not all real world problems can be solved in a precise way. In many cases, the best solution can only be obtained by trial and error methods.

There is little doubt that much work still has to be done in encouraging the development of mathematical modeling skills in learners. Furthermore, this development must start at an early age. It is often difficult to start encouraging learners to use different modeling strategies at the end of their schooling careers if they were not exposed to such methods already. The Grade 10 learners interviewed in this experiment showed considerable unease initially because of the lack of knowledge of strategies to work with this problem. It is this unease that transmitted itself to the researcher as mild irritation. Their emphatic "*I don't know what to do*" or "*I really don't know*" was said in a tone that could not be captured on the transcript. They were visibly frustrated that the researcher was attempting to coax a solution out of them. This is possibly the result of inadequate preparation for not only tackling real world problems, but also on having to rely on their own ingenuity to invent an appropriate strategy.

The researcher is convinced that exposure to various modeling strategies and dynamic computer software would create a conducive environment for the solution of real world problems and would instill in learners greater confidence when working with different problems. It certainly would prevent learners from becoming overwhelmed by the seemingly insurmountable nature of the problem ("*I don't know sir…. This is too difficult… please don't interview me* (pleading)"). The fact that the learners were able to eventually, through a guided interview, arrive at correct solutions to problems 1, 2 and 3, indicates that with the correct guidance, available strategies and confidence, they could become more successful at solving such problems.

In conclusion of this section it must also be stated that learners showed immense awareness of real world conditions. They were able to recognize why mathematics alone cannot be used in real life, but has to be interpreted and adapted taking local conditions into consideration when considering the suitability of the solutions. It is the opinion of the researcher that this aspect of learning and teaching is often neglected and it might be a useful way of involving learners in a problem whilst at the same time it offers them a look at real world conditions – aspects of which they will encounter as engineers, land surveyors, business people and so on.

Learners obtain a vast amount of experience as they interact with the world and environment around them. When providing reasons for why a particular position was not suitable learners gave reasons that were interesting and realistic such as the position may be the actual homestead of the Chief of the tribe living there. For political and tribal reasons one cannot simply move a Chief out to build a reservoir. Educators ought to start utilizing these experiences to direct the learners' thinking when solving real world problems to finding real world solutions. For example, a problem involving bags of cement may result in a solution of 3,5 bags (50 kilograms each) of cement. Learners are already aware that cement cannot be purchased in smaller quantities – they are only sold in 50-kilogram units. Thus the solution must be 4 bags of cement.

So we know now that children do realize that there are real world conditions. The question that arises is, if and when they do recognize real world conditions, how do they cope with them mathematically? This is an interesting research topic which the researcher has just touched upon with these few problems.

References

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