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ABSTRACT. The aim of this paper is to investigate the appropriateness, concurrent and construct validity of action map as a tool for assessing situated problem solving performance. Action map is rooted in activity theory whose stipulations are compatible with situated problem solving. Thirty-one last year secondary students were given three tasks with real –world context. Based on the analysis of the written solutions and interviews, evidence is presented on the appropriateness and validity of action map as an instrument to assess situated problem solving performance.

Despite many calls for including applications as a major goal of teaching mathematics citing a variety of social, psychological, pedagogical reasons and justifications, assessment lagged behind in developing appropriate tools to assess situated problem solving (de Lange, 1996). Existing assessment taxonomies, rubrics, and models are lacking in that they are not embedded in a theory that adequately explain the complexity of interaction with reality in situated problem solving. We believe that the action map is an appropriate assessment tool for situated problem solving and at the same time is embedded in activity theory (Leont'ev, 1981) that stipulates that human behavior and thinking are inseparable and occur within meaningful contexts as people conduct purposeful goal-directed activities. The aim of this paper is to describe the action map as an instrument for assessing situated problem solving and to present evidence in support of its construct and concurrent validity. The action map is based on activity theory whose conceptual framework is compatible with situated problem solving. Concurrent validity will studied in relation to an assessment rubric for problem solving.

Activity Theory

Activity theory was developed by Leont'ev (1981). He defined activity as: "…the unit of life that is mediated by mental reflection. The real function of this unit is to orient the subjects in the world of objects. In other words, activity is not a reaction or aggregate of reactions, but a system with its own structure, its own internal transformations, and its own development." (p.46). A central assertion of activity theory is that our knowledge of the world is mediated by our interaction with it, and thus, human behavior and thinking occur within meaningful contexts as people conduct purposeful goal-directed activities. This theory strongly advocates socially organized human activity as the major unit of analysis in psychological studies rather than mind or behavior. Leont'ev (1981) identified several interrelated levels or abstractions in theory of activity. Each level is associated with a special type of unit. The first most general level is associated with the unit of *activity* that deals with specific real activities such as work, play, and learning. The second level of analysis focuses on the unit of a *goal-directed action* that is the process

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subordinated to a conscious goal. The third level of analysis is associated with the unit of *operation* or the conditions under which the action is carried out. Operations help actualize the general goal to make it more concrete.

Human activity can be realized in two forms: "mental" activity or internal activity and practical objective or external activity (Leont'ev, 1981). The fundamental and primary form of human activity is external and practical. This form of activity brings humans into practical contact with objects thus redirecting, changing and enriching this activity. The internal plane of activity is formed as a result of internalizing external processes. "Internalization is the transition in which external processes with external, material objects are transformed into processes that take place at the mental level, the level of consciousness" (Zinchencho & Gordon, 1981, p.74).

Three types of actions in mental activities had been identified: perceptual, mnemonic, and cognitive (Zinchencho & Gordon, 1981). Perceptual actions are those by which the human being maintains contact with the environment. They are initiated by stimuli from the environment and enriched on the basis of prior experience. Mnemonic actions refer to actions, which involve recognition, reconstruction, or recall (Piaget & Inhelder as cited in Zinchencho & Gordon, 1981). Cognitive actions involve thinking in terms of images of real objective processes (Gal'perin cited in Zinchencho & Gordon, 1981).

Activity theory was selected as a conceptual model in this study because it advocates socially organized human activity as the major unit of analysis in psychological studies rather than mind or behavior and because it makes the assumption that thinking and doing are inseparable

Action Map

The action map is a schematic representation of organization and sequence of the actions of the objective content of an activity (see Figure 1) using the method of structural-analysis (Zinchencho & Gordon, 1981). This method was used because it puts forward an operational analytic method derived from activity theory itself. It provides a way for representing the structure of activity as a system of interconnected units with potential relationships among them and among types of connections. In the systematic-structural approach, it is assumed that the structure of actions and operations, the internal transitions from one action to another, and their sequential organization depend on the objective content of activity. Thus the identification of the organization and sequence of the actions of the objective content of an activity provides a characterization of its level, form, and type (Zinchencho & Gordon, 1981).

A number of studies used activity theory to investigate work activities (Millroy, 1992 ; Masingila, 1996 ; Pozzi et al, 1998). Jurdak and Shahin (2001) used activity theory to compare work and learning activities. It is in the last study that structural analysis was used systematically and action map was used as a tool without actually using the name 'action map'.

Methodology

Sample

The sample consisted of 31 grade 12 students selected from four private schools in Beirut, Lebanon. Their teachers nominated the students as being from the highest achievers in mathematics in their classes. All the students were in the last grade of secondary school and were in the general science stream, which prepares students for university studies in mathematics, sciences, and engineering.

Problem Tasks

The problem tasks were constructed to meet the three criteria set for situated problem solving. First, the problem situation has to be *real* to the population of the students concerned. By that we mean that the situation is within the current experiential space of students. Second, the problem has to be formulated in a context in the sense that the problem solver may have to put boundary conditions or introduce assumptions and data and to engage in a process of mathematization to formulate the problem in mathematical terms. Third, the problem task should lend itself to multiple approaches and different levels of treatments.

The process of searching, constructing, and screening resulted in three tasks (Appendix A) that were judged by the researcher to be meaningful and satisfy the three criteria. The *Car Loan* Task presents a situation where two options for payment in installments for a car. The student is to decide which option is better and why. The *Cell Phone Task* presents a situation where two actual offers for a cell phone from two companies are presented with all the specifications as advertised. The student is to decide which offer is better and to rationalize the decision. In the *BMI Task,* the formula for the body mass index together with a table of norms and BMI chart. The student is asked to rationalize how the chart was produced from the table of norms**.**

Assessment Rubric

A rubric adopted from the QUASAR project (Lane, 1993) was used to assess the solutions of the problems. The rubric assesses mathematical knowledge, strategic knowledge, and communication. *Mathematical knowledge* is defined as the degree to which the student shows understanding of the task's mathematical concepts and principles; uses appropriate mathematical terminology and notations; and executes algorithms completely and correctly. *Problem solving* is defined as the degree to which the student may use relevant outside information of a formal or informal nature; identifies all the important elements of the task and shows understanding of the relationships between them; reflects an appropriate and systematic strategy for solving the task; and gives clear evidence of a solution process, and solution process is complete and systematic. *Communication* is defined as the degree to which the student gives a complete response with a clear, unambiguous explanation and/or description which may include an appropriate and complete diagram; communicates effectively to the identified audience; presents supporting arguments which are logically sound and complete; may include examples and counter-examples. The five scale points were defined as follows:
0 (*No Answer*), 1 (*Inadequate*), 2 (*Minimal*), 3 (*Competent*), 4 (*Exemplary*).

Procedure

In each of the four schools, the selected students were asked to come to a designated room in the school. The investigator explained the purpose of the study to them and their queries were addressed. Each student was asked to read the three tasks and choose one of them. While solving the task, each student was asked by the investigator about how their approach of the solution of the task. The problem solving session lasted for 60 to 90 minutes. Students were allowed to use calculators and computers and to ask questions about the task during the session. All interviews were audio-taped. It should be mentioned that the tapes were not used for the purpose of this study and were intended for another aspect of the study which focuses on studying situated problem solving as an activity. All what the students wrote during the problem solving session was collected and properly identified. The written solutions constituted the basic documents that were subjected to documentary analysis in two distinct ways. First, using the assessment rubric, two raters assessed each solution and a comparison of a sample of the two ratings showed a high degree of agreement between the two raters (at least 90% in the three categories).

Second, the written solutions were subjected to structural analysis (Zinchencho & Gordon, 1981). This was an iterative process in which a researcher reviewed the written solution of each student to identify the actions and putting a short description of each action. The sequence of these actions, as they unfolded based on the written solution, were identified. The descriptions of the actions were put in boxes and connected with arrows to indicate the sequence of actions. The actions were then classified into one of three categories (perceptual, mnemonic, and cognitive). The constructed action map was then validated against the written solution and modified accordingly. This iterative process continued until the action map was judged as accounting for almost all the actions, their sequence, and their type. Another researcher did a second validity check by comparing the written solution with the constructed action map. The final product was a figure similar to those in Figure 1, which represents two rather contrasting action maps for two students who the cell- phoned task.

Data Analysis

Five variables were identified from the action map: Relative frequency of mnemonic actions (R/MN), relative frequency of cognitive actions (R/COG), relative frequency of perceptual actions (R/PER), number of actions (ACTIONS), and number of loops (LOOPS) (a loop was defined as a triangle formed by the arrows that indicate the sequence of actions). Four variables were identified from the assessment rubric as follows: Math knowledge, problem solving, communication, and total (the sum of the three variables).

Two statistical analyses were done. A stepwise multiple regression was performed to identify the variables in the action map that predict problem solving performance as measured by the assessment rubric. Second a factor analysis with a Quartimax rotation was done to examine the construct validity of the action map by identifying the structure of the action map and the factors therein. To illustrate these variables we calculated their values for the two examples in Figure 1 (Table 1)

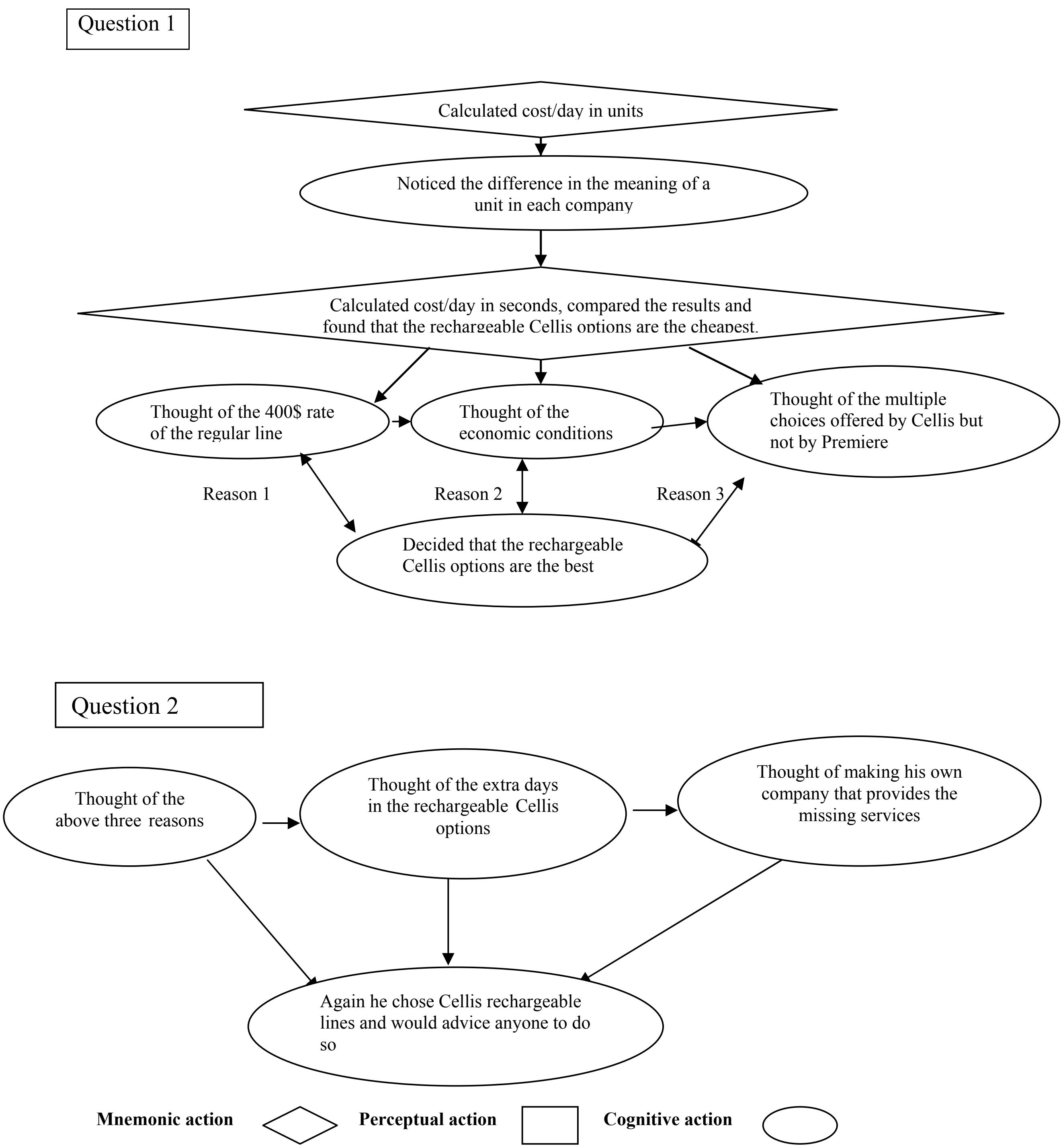
Table 1: Values of the variables for examples 1 &2

	Example 1	Example 2
R/MN[1]	2/11	14/16
R/COG[2]	9/11	2/16
R/PER[3]	0/11	0/16
ACTIONS[4]	11	16
LOOPS[5]	6	2
Math Knowledge	4	2
Problem Solving	4	2
Communication	4	2

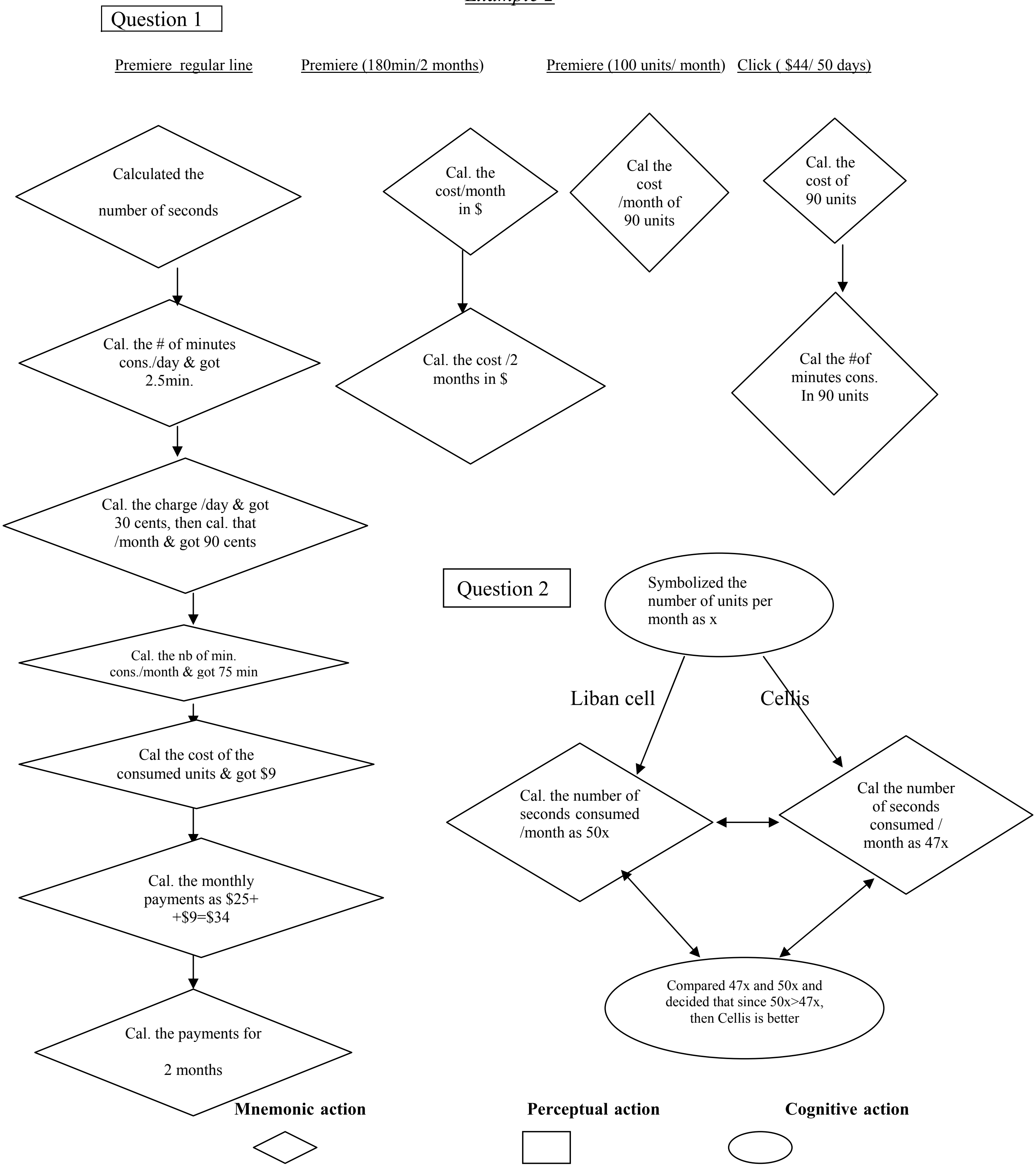
[1]Relative frequency of mnemonic actions
[2]Relative frequency of cognitive actions
[3]Relative frequency of perceptual actions
[4]Number of actions
[5]Number of loops

Figure1. An example of an action map (Cell Phone Task)

Example 1



Example 2


Question 1
Premiere regular line
Premiere (180min/2 months)
Premiere (100 units/ month)
Click ($44/ 50 days)
Calculated the number of seconds
Cal. the # of minutes cons./day & got 2.5min.
Cal. the charge /day & got 30 cents, then cal. that /month & got 90 cents
Cal. the nb of min. cons./month & got 75 min
Cal the cost of the consumed units & got $9
Cal. the monthly payments as $25+ +$9=$34
Cal. the payments for 2 months
Cal. the cost/month in $
Cal. the cost /2 months in $
Cal the cost /month of 90 units
Cal. the cost of 90 units
Cal the #of minutes cons. In 90 units
Question 2
Symbolized the number of units per month as x
Liban cell
Cellis
Cal. the number of seconds consumed /month as 50x
Cal the number of seconds consumed / month as 47x
Compared 47x and 50x and decided that since 50x>47x, then Cellis is better
Mnemonic action
Perceptual action
Cognitive action

The action map seems to be an adequate tool for assessing situated problem solving in at least two ways. First, it provides a representation of not only the product but also the process of problem solving in the sense that it maps the actions and their sequence as they unfold in the problem solving process thus providing a visual representation of the internal structure of the activity. Second, it captures the interaction between the problem solver and reality because it describes the sequence of actions as they occur simultaneously in the internal plane (thinking) as well as the external plane (doing).

Reliability

Cronbach α for the rubric across its five levels and for the action map across its variables (relative frequency of mnemonic actions, relative frequency of cognitive actions, relative frequency of perceptual actions, number of actions, and number of loops) are reported in Table 2. In spite of the small sample in this study, Cronbach α was moderate high, indicating a reasonable internal consistency for the action map.

Table 2.Cronbach α for the Rubric and Action Map

Task	*Rubric*	*Action Map*
Car	.63	.73
Cell phone	.67	.69
BMI	.88	.52

Concurrent Validity

The concurrent validity of the action map relative to the assessment rubric seems to be quite high. The results of the stepwise multiple regression (Table 3) indicate that, as measured by the action map, the predictors of mathematical knowledge, problem solving, communication, and overall performance, as measured by the rubric, fall into two categories. The first category consists of the relative frequency of the type of

Table 3. Results of Regression Analysis

Variable	Math knowledge		Problem solving		communication		Total	
	R	R^2	R	R^2	R	R^2	R	R^2
R/MN[1]	-	-	.54	.30	.45	.21	.59	.35
R/COG[2]	.57	.33	-	-	-	-	-	-
R/PER[3]	-	-	-	-	-	-	-	-
ACTIONS[4]	.70	.49	.77	.59	-	-	.76	.57
LOOPS[5]	-	-	.80	.64	-	-	-	-

[1]Relative frequency of mnemonic actions
[2]Relative frequency of cognitive actions
[3]Relative frequency of perceptual actions
[4]Number of actions
[5]Number of loops

actions (relative frequency of mnemonic actions, relative frequency of cognitive actions, relative frequency of perceptual actions) and the second of the structure of the action map (number of actions and number of loops). For mathematical knowledge, the relative frequency of cognitive actions, and number of actions, account for 49% of the variance. For problem solving, relative frequency of mnemonic actions, number of actions, and number of loops account for 64% of the variance. For communication, relative frequency of mnemonic actions account for 21% of the variance. For the overall performance (total score), relative frequency of mnemonic actions and the number of actions account for 57% of the variance. In general, performance in problem solving increases with the increase in the relative frequency of cognitive actions (or the decrease in frequency of mnemonic actions since this is negatively correlated with the relative frequency of cognitive actions as indicated in Table 4) and the increase in number of actions and number of loops. In other words the quality of problem solving is dependent on the frequency of cognitive actions and the complexity of the structure of the action map.

It is quite remarkable that these two categories of variables in the action map (a tool embedded in activity theory) account for high percentage of problem solving as measured by the assessment rubric, which has different assumptions.

Table 4.Correlation Matrix

	R/MN[1]	R/COG[2]	R/PER[3]	ACTIONS[4]	LOOPS[5]
R/MN[1]	1	-.85	-.53	.10	.01
R/COG[2]	-.85	1	.23	.03	.14
R/PER[3]	-.53	.23	1	-.25	-.26
ACTIONS[4]	.10	.03	-.25	1	.62
LOOPS[5]	.01	.14	-.26	.62	1

[1]Relative frequency of mnemonic actions
[2]Relative frequency of cognitive actions
[31]Relative frequency of perceptual actions
[4]Number of actions
[5]Number of loops

Construct Validity

We examined further the structure of the of the action map by performing a factor analysis with Quartimax rotation on the variables derived from the action map. The analysis provided support to the two- factor structure (Table 5): Factor 1 with high
loadings on the type of action (relative frequency of mnemonic actions, relative frequency of cognitive actions, relative frequency of perceptual actions) and Factor 2 with high loadings on the structure of the action map which reflect the complexity of the activity(number of actions and number of loops)

Table 5: Factor Structure of the action map

Variable	Factor	
	1	2
-Ratio of cognitive actions to total number of actions(R/COG)	.90	.18
-Ratio of perceptual actions to total number of actions (R/PER)	.59	-.45
-Number of actions (ACTIONS)	-.05	.86
-Number of loops (LOOPS)	.05	.89
-Ratio of mnemonic actions to total number of actions (R/MN)	-.97	.04
%of variance	42.17	35.46

The usability of the action map calls for addressing practical questions as to what and how the it may be used as an assessment tool. This study has demonstrated that the action map may be used as a theory-embedded alternative tool to the rubric in assessing performance on mathematical problem solving by trained researchers in a research context. One would conjecture that the action map may be used by teachers to assess problem solving performance of situated problem tasks outside the classroom, assuming that teachers are trained in using action map. It remains an open question whether the action map can be constructed from an audio tape of problem solving through the thinking –aloud technique. The decision to use the action map as an alternative to the rubric is to be mediated by curricular goals of mathematics as well the comparative costs and benefits of the two tools. It should be mentioned that we are not making any claim that action map may be used to assess traditional procedural knowledge or conceptual understanding.

This study has also demonstrated that the action map may be constructed from the written solutions of students only. Our experience shows that the action map requires less time to construct than a rubric, however, the construction of an action map for an individual student will require much more than to administer than an already available rubric.

In conclusion, the action seems to be a promising tool for assessing situated problem solving. It is a tool which is embedded in a theory compatible with the assumptions of situated problem solving and the same time is usable in assessment of problem solving in mathematics classes as a viable alternative to rubrics.

Endnote:

This paper was originally presented at ICME-10 to TSG27 on the topic "Recent Developments in Assessment and Testing in Mathematics Education", Copenhagen, July, 2004

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Appendix A
Context Problem Tasks
A.1 *Car Loan Task*

Rasamny Youniss Company is making a special offer on Nissan-Almera cars, model 1999, and automatic/full option for $13950 cash. Now, you have two options for payment in installments, either through the bank or through the company itself. Through the bank, and with a down payment of $5,000, you can pay with a 12% annual interest on the balance, $305 at the end of each month. However, the second option, and with a down payment of $5,000 you can repay, in equal monthly installments for 36 months at an annual interest rate of 7.5% on the total.

1) Suppose you wanted to pay the whole remaining amount after 6 months. In each option, how much do you have to pay to close your account?
2) Which is the most convenient option for paying for the car?

A.2 *Cell Phone Task*

If you want to get a mobile phone, Libancell and Cellis offer multiple services. Both can give you a regular line for $400 with $25 fixed monthly payment and the call will be charged 12 cents/ min. An alternative plan is providing monthly rechargeable cards with a certain number of units. While Libancell offers a Premiere line, Cellis provides a Click line. To get a Premiere line you have to pay $75 a fixed amount for the line and you can recharge it every two months for 103000 L.L. (180 units with duration of 50 second/unit) or 68000L.L (180 units with duration of 50 seconds/unit). To get a Click line you have to pay $75 a fixed amount for the line and you can recharge it through buying separate cards with prices varying according to the time it serves. A $22 (90 units) rechargeable cards serves for 15 days with 5 extra days for receiving calls only, $33 (135 units) cards serves for 25 days with 10 extra days for receiving calls only, and $44 (180 units) card serves for 40 days with 10 extra days for receiving calls only. With Cellis click line, the unit duration is 47 seconds.

1) Suppose that you consume 3 units per day on the average. Which of the options is the cheapest? Explain.
2) Given the number of the units consumed daily, which of the three options is the cheapest? Explain.

A.3 *BMI Task*

Finding out your body mass index (BMI) is a quick way to figure out if your weight is healthy for your height. Nutritionists have developed refined ways to interpret BMI values, for instance, different BMI values can mean you are underweight, ideal weight, slightly overweight or obese. BMI can be calculated as W/h^2, W=weight (kg), h=height (m).Given the following norms, find a mathematical way that may be used to transform these norms into the chart below:

Symbol	A	B	C	D
BMI	< 20	20 ---25	25---27	27>
Condition	Underweight	Correct	Overweight	Obese