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Consecutive numbers

Steve Humble

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Steve Humble[1]
The National Centre for Excellence, UK

The hidden secrets of our number system can often reveal the magical quality of mathematics. Through the process of discovery and discussion with fellow classmates, the hidden depths of maths takes on new appeal. Consecutive numbers is one such area that gives this excitement.

Starting with the open question:

How many ways can you share out 6 counters?

The children will find cases like these:

1+5, 2+4, 3+3, 1+1+2+2, 1+2+3

[1] drmaths@hotmail.co.uk

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What about 7 counters and other numbers?
Are there any special cases from all these?

Then notice that 6=1+2+3 and 7=3+4 can be written as the sum of consecutive numbers.

Why can 7 be made from two consecutive numbers and 6 from three? Are there patterns like this in other numbers? Can all numbers be written as the sum of consecutive numbers?

After further investigation a proof can be found for all odd numbers.

$$n+(n+1)=2n+1$$

It can be seen that all multiples of 3 can be written as three consecutive numbers from the following proof:

$$(n-1)+n+(n+1)=3n$$

After this the students see the real benefit of using algebra and move to four consecutive numbers.

$$(n-1)+n+(n+1)+(n+2)=4n+2=2(2n+1)$$

So if you double any odd number these can be written as four consecutive numbers.

Therefore since we have just proved that we can write all the odd numbers as consecutive numbers, if we double any odd number we can also write this as a consecutive number. Is that therefore a proof that all numbers can be written as the sum of consecutive numbers?

Odds	1	3	5	7	9	11	13	15	17…
Double2	6	10	14	18	22	26	30	34…	

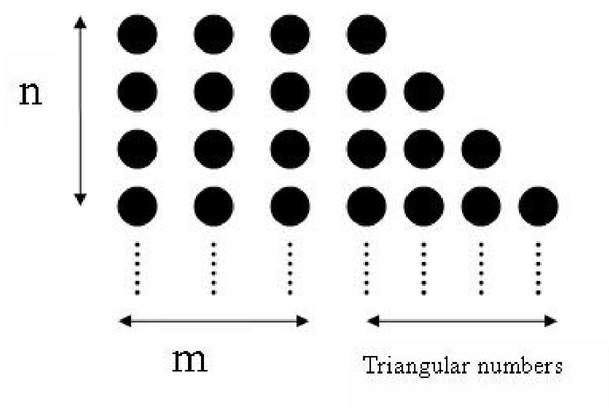
You can see from the previous table that gaps still exist in our logic, for example 4, 8, 12, 16, 20, 24….

We can find more proofs to help solve some of these problems, but 4, 8, 16, 32… still remain. By this time the student may begin to wonder if continuing to prove particular cases will ever prove to be enough!

We need to move to the next level and the crowning glory of this problem. By looking at why 4,8,16,32… cannot be partitioned into the sum of consecutive numbers, we begin to understand the link to triangular numbers and other deeper issues.

Theorem: All consecutive numbers have at least one odd factor.

Proof:



Sum = $nm + \frac{1}{2}n(n+1)$

Sum = $\frac{n}{2}(2m+n+1)$

Either i) n is even, $\therefore$ $2m+n+1$ is odd

ii) $(2m+n+1)$ is even, $\therefore$ n is odd

This problem is an excellent way to motivate thinking about proof and why proof is necessary. It gives students confidence in the use of algebra and the ability to find particular results which can be shown to always be true. Yet its real magic is in this final proof, which shows the need to stand back and look. If we are not careful, sometimes we can lose ourselves in the detail and not see the whole picture.

Steve Humble (aka DR Maths) works for The National Centre for Excellence in the Teaching of Mathematics in the North East of England (http://www.ncetm.org.uk). He believes that the fundamentals of mathematics can be taught via practical experiments. He is the author of the book *The Experimenter's A to Z of Mathematics,* which develops an experimenter's investigative approach to mathematical ideas. Always having had great fun playing with maths, he enjoys teaching this to others.
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