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# Mathematics for Middle School Teachers: Choices, Successes, and Challenges

## *How do we get there from here?*

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Abstract:
The opportunities for mathematics department faculty from institutions of higher education (IHEs) to work with middle school mathematics teachers are on the rise; the U.S. Department of Education has allocated almost $800 million for mathematics and science partnerships since 2004 that require collaboration between faculty from IHEs departments of arts and sciences and school districts. Changes in credentialing requirements due to the No Child Left Behind Act of 2001 mean that middle school mathematics teachers must take more mathematics content courses, yet current offerings for teachers in many mathematics departments typically focus on elementary or high school mathematics and often neglect the needs of middle school teachers. This article talks about the efforts of a group of mathematics faculty and school district personnel to develop mathematics courses that would help middle school teachers develop a Profound Understanding of Middle School Mathematics. An explanation of the curriculum structure – highlighted by the teachers' responses – is given here.

Keywords: middle school mathematics; in-service teacher training; mathematics content; mathematics pedagogy; No Child Left Behind;

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**Introduction**

Amelie,[3] a middle school teacher, was reflecting back on a week of intensive algebra. We asked her to comment on the themes of the course, and she wrote, "I feel these themes have helped me to make many connections that I have never made before. It is fascinating to see how interrelated math really is, like how completing the square is really doing the steps of the quadratic formula, or how eliminating and substituting are really two different ways of arriving at the same solution." A day earlier, the class had struggled to complete the square for the equation $ax^2 + bx + c = 0$; several participants were able to do this for themselves, and most of the rest were able to follow the class demonstration by one of the instructors. At the end, the class erupted in applause—they got it, and in the words of another participant, "It was so helpful to see the relationship between completing the square and the quadratic formula. That was so cool."

In November 2006, our third cohort of middle-grade teachers finished the third of three math courses specifically designed for them. The courses were taught by different combinations of instructors, each adapting and refining the materials in an effort to improve them. But for each iteration of each course, our goal was always been the same: to help lay the foundations for what we call here a Profound Understanding of Middle School Mathematics (a specialization of Liping Ma's notion of a Profound Understanding of Fundamental Mathematics, or PUFM). Ma states:

> A teacher with PUFM is aware of the "simple but powerful" basic ideas of mathematics and tends to revisit and reinforce them. He or she has a fundamental understanding of the whole elementary mathematics curriculum, thus is ready to exploit an opportunity to review concepts that students have previously studied or to lay the groundwork for a concept to be studied later. (1999, p. 124)

It is interesting that Ma's notion of PUFM resonates with both research mathematicians and mathematics educators. Her book provides an access point for mathematicians into the issues of teacher knowledge for teaching precisely because they are able to recognize in her work a view of mathematics that reflects their own. And mathematics educators see that Ma's description of PUFM includes other critical components of mathematics learning, for instance, how mathematical topics should be organized in order to be best learned. In PUFM, mathematical and pedagogical knowledge are entwined. In his review of Ma's book, Roger Howe said,

> It seems that successful completion of college course work is not evidence of thorough understanding of elementary mathematics. Most university mathematicians see much of advanced mathematics as a deepening and broadening, a refinement and clarification, an extension and fulfillment of elementary mathematics. However, it seems that it is possible to take and pass advanced courses without understanding how they illuminate more elementary material, particularly if one's understanding of that material is superficial…. [I]t seems also that the kind of knowledge that is

[3] Names have been changed where we know them—in this case, these comments were made on an anonymous end-of-course evaluation, so we don't know which of our participants wrote it (but probabilistically speaking, it was a woman).

> needed is different from what most U.S. teacher preparation schemes provide, and we have currently hardly any institutional structures for fostering the appropriate kind of understanding. (1999)

While many teacher education programs do have mathematics courses for elementary teachers, courses specifically for middle school teachers are relatively uncommon. Richard Askey elaborates:

> Courses for prospective elementary school teachers, for example, frequently slight material dealing with fractions since whole number arithmetic is the main focus in our elementary schools. Middle school teachers frequently fall between the cracks. The material they will be teaching is not taught in detail to either prospective elementary school teachers or to prospective high school teachers; there are no courses specifically for middle school teachers. (1999)

Since he wrote this, new courses for middle school teachers have sprung up around the US, but the national discussion of what these courses should look like is still quite new. This was the context in which we initially set out to design our mathematics courses for middle school teachers.

When we were designing and refining our courses, we were aware of several resources, some of which directly address the mathematical preparation of teachers such as Conference Board (2001), Howe (2006), Madden (n.d.), Milgram (2005), Wu (1999), and some of which addressed the middle school curriculum more directly, such as the NCTM Standards (2000) and the state standards that our teachers were required to address in their own teaching. But these resources range from philosophical considerations to encyclopedic coverage of vast areas of mathematics[4], and there are many decisions that need to be made when creating an extended curriculum. For those of us who have had the opportunity to design mathematics courses for middle school teachers, we have had to make concrete and immediate decisions about what mathematical topics to cover and how to address them so that teachers have a chance of developing PUMM. Our challenge was to find a pathway that leads toward a deeper understanding of middle school mathematics, and to identify the curricular and instructional components necessary to achieve this.

Like many other mathematicians and mathematics educators, we struggled with these issues. Before settling on course materials, we considered the backgrounds of the teachers, how to balance pedagogy and mathematical content, and how to present the content in a way that we hoped would lead to substantial development of PUMM. Our results were encouraging. What follows is the story of these decisions and the lessons we learned in the process.

## Guiding principles

In the summer of 2003, one of us visited the Vermont Mathematics Initiative (VMI) summer institute for elementary teachers at the invitation of its founder, Dr. Ken Gross. Dr. Gross said that the two guiding principles for the VMI were to

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[4] The two exceptions that we know of are the set of curriculum materials developed specifically for middle school mathematics teachers by Ira Papick and his colleagues on the Connecting Middle School and College Mathematics Project, see Papick (n.d.), and the set of curriculum materials developed by Jim Lewis and his colleagues at the Mathematics in the Middle project, see Lewis (n.d.), but these were being developed or just being published around the time we were creating our own materials and were not readily accessible to us at that time.

1. Be true to the mathematics, and
2. Treat teachers as professionals with their own intellectual needs.

This philosophical stance struck a deep chord with us. More importantly, the clear success of the VMI approach was apparent in the mathematical work of the participating teachers. These elementary teachers were working on challenging mathematical problems and using mathematical language more sophisticated than many university calculus students use. Inspired, we held our first summer institute the very next year.

Because we were working primarily with middle school teachers, we still had a lot of work to do developing the curriculum, but the pedagogical approach (from team-teaching down to paper covering the tables so that participants and instructors could write mathematics on every nearby surface) was very much inspired by what we saw at the VMI.

It was important to us to develop courses specifically designed for middle school teachers. We wanted to address the topics that form the core of the middle school curriculum, such as fractions, pre-algebra, basic geometry, and topics in data analysis. Culling this list of topics to a two-week institute (and even the extended 6 course version of our program) required making many choices. We chose topics that were prominent in the middle school curriculum and that were highly connected to each other and to topics that anchor the high school curriculum.

Teaching teachers requires different strategies than teaching the same material to children. Adult learners have a broader range of mathematical and life experiences that shape what they currently know. One can be quite certain that they have all seen integer arithmetic and linear equations, but many may have only a partial understanding or hold misconceptions about the mathematics that they have studied. One must consider how to build upon adult learner's previous experience and structure the work so that they see that the mathematics is deep and important.

Most of our participants had a solid ability to do basic arithmetic and to solve problems using a guess-and-check strategy; there were very few other mathematical topics with which all participants were equally comfortable. Developing materials for such an audience requires careful planning, but also offers certain advantages. On the one hand, it requires beginning with material that is easily accessible but can be developed at deeper levels. On the other hand, capitalizing on and connecting these different levels can help bootstrap teachers who may struggle with more advanced material.

**Building on prior knowledge**

"Build on prior knowledge" is almost a mantra in mathematics education. In this section we elaborate on how we attempted to do this in the context mathematics courses for middle school teachers.

First, we introduced many topics by using a contextual problem. These are not meant to be "real-world problems," but are situated in an idealized context that our participants can readily understand yet are naturally represented by the mathematical objects that we are studying. This allowed our participants to see that the abstractions of mathematics are related to things they already know and understand. There is another reason that we chose to use contexts extensively throughout the curriculum. People often have good intuition and common sense in everyday quantitative situations, but then fail to apply those reasoning skills in more formal mathematical situations. The contexts we provide help them to face this conflict, and ultimately lead them to apply what they already understand to the mathematical abstractions. In other words, the context is not just meant to help them appreciate the usefulness of the mathematics, but is designed to lead them to a deeper understanding of the mathematics itself.

Certain problems and contexts were chosen because they can be used to illustrate several different important topics. For example, the first problem that participants tackled on the first day of class was used to illustrate the idea that different approaches can (and should) lead to the same solution, the value of different mathematical representations, the importance of taking one's time to understand a problem thoroughly, and later to introduce arithmetic sequences, the notion of a function, and systems of equations. Some of our problems span multiple courses. For example, one problem that participants solved in the algebra course was revisited in both the geometry and calculus courses as well. In this way, we are assured that all of our participants have a common experience on which to draw when learning new mathematics. As one participant said at the end of one of the summer institute, "I believe that taking the same problems and using them throughout the two weeks, but in different ways, really helped me see some of the concepts differently."

Let us illustrate this principle with an example. Our first summer institute began with the following problem, borrowed from the VMI. It is important to note that this problem is given to the participants with no mathematical introduction, so from their perspective, it is open-ended.

1. Photo Developing.[5]

> There are two photography stores in town, Perfect Picture (abbreviated PP) and Dynamic Developers (abbreviated DD) that do custom film developing. At PP the cost to develop one roll of specialty film is \$12 but any additional rolls of film cost only \$10. At DD the cost of developing one roll of film is \$24 but each additional roll is developed at a cost of only \$8.
>
> For what number of rolls of film is the cost of developing the same at PP and DD?

In our experience, almost everyone gets an answer, either by doing some systematic guessing or by setting up a system of equations. For participants who are more comfortable with systematic guessing, their understanding of the problem and its solution will help them to understand the equations that represent the problem equally well. For participants who are comfortable with the algebraic representation of the problem, it is instructive for them to see how others struggle with writing equations.

The next question, while related in its mathematical structure (a system of linear equations or a linear inequality will represent it quite nicely), requires a more substantial mathematical understanding in order to interpret the result.

2. More Photo Developing.

> Suppose you feel strongly that PP does a much better job of developing your film; in fact, so much better that you are willing to develop the film at PP unless the cost is more that double the cost at DD. How many rolls of film do you have to develop in order for the

[5] ©1999-2007 Vermont Mathematics Initiative, Kenneth I. Gross. All rights reserved.

> cost at PP to be more than double the cost at DD? What do you think of this "double-cost" rule?

Here the solution is, "If you are developing the same number of rolls, the cost at PP will never be more than twice the cost at DD." So in this case, the problem cannot be solved by systematic guessing (although this can help with understanding the solution). Furthermore, even participants who solve it algebraically have a difficult time interpreting the answer they get (since the value where one is double the other is negative), and typically they see it best if they also see an alternate representation for the problem (such as a table of positive values that "never work out" or a graph of a related system of equations). Seeing that the equations and graphs which represent the problem are much more powerful tools than systematic guessing helps to motivate the purposeful study of these mathematical structures later in the course.

In summary, there are two strategic ways we attempt to build on our participants' prior knowledge when introducing new mathematical ideas. The first is to initially set problems in contexts that are likely to be familiar and to let them solve it in any way they see fit. The second is to strategically include certain problems early in the curriculum that they can then build on later. An example of the first situation is the initial experience with the photo developing problem. After participants have solved this problem, however, the solution methods we subsequently discuss, such as using equations and graphs, become part of the participants' knowledge base on which we were able to build later in the program. This is a key to the success of the curriculum.

**Increasing the intellectual load**

For many people, the photo developing problems look like they belong in a long set of exercises at the end of a section on systems of equations. But it is the context in which they are given that creates the appropriate *intellectual load*. The intellectual load can be very different for the same problem depending on when it is given and who is "doing the work." Most participants (and even some university faculty) have considered these problems to be substantial open-ended problems because of the context in which they were considering it. Initially, some participants were frustrated with this approach, and sometimes complained that we shouldn't start things out with a problem that "doesn't work." But the solidity of understanding that comes with struggling through it themselves soon becomes more important to the participants. For example, in the first follow-up meeting after the first summer institute, we gave them a typical problem involving common factors of two whole numbers. Given with no context, however, it becomes an open-ended problem:

> Iggy is filling a bulletin board with pictures. The bulletin board measures 144 cm by 96 cm. If Iggy wants to hang square pictures of the same size (and that measure a whole number of cm on each side), what size picture can Iggy use? (Assume he doesn't want the pictures to overlap or have gaps between them.)

An interesting natural experiment occurred the first time we gave this problem. It was intended to be given with no introduction so that it might serve as a common context for a discussion of common factors, but because of miscommunication between the instructors, one gave a mini-lecture on common factors right before the participants were given the problem. During the class period, one of the participants commented, "I knew I was supposed to use the greatest common factor, but I didn't know why." (In fact, the intention was to discuss all common factors, not just the greatest common factor.) On the daily evaluation, another said, "I'm not sure telling what we are going to see was effective. I think the discovery approach will bring it out better." Still another said, "I really like to have more time to work on the problems and a little less explanation. It was interesting to realize that I often want to explain too much before allowing my students to struggle." This was one of the days we felt our greatest sense of accomplishment. Our participants had gone from wanting us to give them answers more directly to wanting more time to think about it for themselves.

**How can we improve?**

During the coursework, we did not discuss pedagogy directly; we assumed that teachers would learn best how to teach mathematics by learning it themselves in a deep, coherent way. We hoped that they would then be able to adapt the strategies for use in their own classrooms. In fact, when asked what has benefited them most about their experiences in the program, teachers were almost as likely to cite knowledge of effective teaching as content knowledge gains[6]. For example, at the end of the 2004 summer institute, one of the participants wrote:

> I never knew quite where to start teaching. I see how the text books are organized, but when my students have such a hard time with abstract thought, I've had a hard time prioritizing. I mean, place value is just a lot of talk unless it can be used. Now that I see how concepts can be developed slowly and shown by illustration and with manipulatives, I see that I can make something like place value important. I need patience and a slow pace for something to sink in, but it will be worth it…. I've learned a lot more about teaching and my own math skill has improved greatly."

While we feel that the program had a very positive impact on teachers' attitudes about teaching, it is an open question to what extent it impacts their actual practice. Informal observations of their classrooms have shown us that the extent to which their experiences in our program have actually impacted their practice in easily observable ways varies greatly from teacher to teacher. This has caused us to consider how we can improve our program so that teachers make more frequent and direct connections with their own teaching practice. Recent conversations with Mark Thames have broadened our thinking about this subject. He states, "Studies attempting to link students' achievement to teachers' content knowledge consistently suggest that mathematical knowledge more closely related to practice–for instance, related to specific curricula or to the work teachers do–is more likely to have a positive effect on teaching and learning." (2008) Deborah Ball and her colleagues have elaborated this view of "mathematical knowledge for teaching," and their work provides us with deep insights into the nature of such knowledge (see Ball, 2002, Ball, Hill, Bass, 2005). We feel that our next step is for us to find or create materials that help teachers develop this kind of knowledge as well.

**The intellectual needs of teachers**

What is deep or challenging depends on what you already know and how it is presented. For many teachers, an opportunity to look more deeply into the content that they teach and the mathematics of nearby grades can be very intellectually stimulating. As one

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[6] They also often cited content knowledge gains, and their scores on content pre- and post-tests confirmed this, see Umland (2006).

participant said, "I am overjoyed that it was understandable for me, and yet it was also very stretching and challenging." It is also important to provide materials that are clearly about real mathematics. Another participant said, "I have greatly improved my conceptual knowledge…We didn't get any 'cutsie' activities… It's that 'give them a fish… teach them to fish' thing."

The participants in our program had a wide range of backgrounds and mathematical expertise. Our goal was to create a course that would contribute to the growth of all of these teachers. Below are excerpts from the essays of two participants, one who was very anxious about mathematics and another who was very confident on entering the program (their pretest scores showed that they were indeed at the bottom and top in terms of entering knowledge, respectively).

> [H]ow did I end up in [the program]?... [It] seemed suited to helping me grow in my understanding of the why [of math]. Yet I was very apprehensive in signing up because I sensed I would be very much out of my comfort zone. And was I ever!... [Yet] I experienced a great deal of satisfaction getting through the two weeks. The morning after the last day, I woke up with a new insight about slope and how it is used!
>
> When I read the description of [the program], I was really skeptical about the program. I thought I knew everything I needed to know about math…. I decided to try it mostly because I was very interested in being a part of a lesson study. I now know that I made one of the best decisions of my teaching career. Not only have I learned so much more about the basics and foundations of algebra, I have also learned the importance and validity of multiple approaches to the same problem…. After spending a good part of my life focusing on getting the right answers quickly, I now know that to be a better teacher I need to help my students go beyond learning how to just get the right answer.

Carefully chosen problems and content can benefit teachers at all places along the mathematical spectrum, as long as one balances challenge and support. Maintaining high standards while giving the conceptual and emotional support that teachers (and probably most math students) need is the key to starting where teachers are and helping them reach for PUMM. As one participant said, "What I kept thinking was why didn't they teach it this way when I was in school. I have a much better understanding of how everything fits together."

We often hear people talk about teaching mathematics in a "connected, coherent way." But this is a very abstract idea. How does this translate into choices for actual topics and problems? Our answer was to take the content of middle school mathematics and a couple of layers up and to show, in very specific ways, how these ideas fit together. So when Amelie talked about her insights into the relationship between completing the square and the quadratic formula or between different algebraic methods of solving systems of linear equations, she was seeing the important structures of middle school mathematics. Our next challenge is to help teachers like Amelie provide the same kinds of experiences for their own students.

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