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Early Insurance Mechanisms and Their Mathematical Foundations

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Abstract:
This article gives a historical survey of early insurance mathematics and its development in relation to the maritime industry and the origins of the life insurance industry. The history of the probability and statistics and its intricate connection to the actuarial sciences and modern insurance can be traced to the time period described in this paper

Keywords: Actuarial sciences; Anders Hald (1913-2007); Insurance mathematics; Life insurance; Logarithms; risk management; history of probability; history of mathematics

Background

Popular history tells the story of how insurance began at a coffeehouse in London called Lloyds, where merchant shipmen sat around the table drinking coffee and discussing their future voyages. One merchant explained that he was afraid that he would lose everything if his next ship was lost at sea, and another merchant offered to share in his risk – for a fee. Around the table they went, dividing up the risk of loss and the profits of the voyage, and giving birth to insurance. Or, so the story goes. In truth, insurance existed in many early forms long before that conversation could have taken place at Lloyds coffeehouse. This article considers the factual basis for the history of early insurance mechanisms, provides a survey of their general forms, and discusses their foundations in mathematics. Particular emphasis will be placed on the insurance mechanisms developed and popularized during the 13th through the 17th centuries.

In today's world insurance is a part of everyday life. Modern insurance relies heavily on sophisticated mathematics. Greene, in his 1961 article highlights some applications of mathematics to the insurance field. His examples include utility analysis, game theory, statistical decision making, and actuarial science. In actuarial science in particular, the ties between insurance and advanced mathematics are very tight.

This paper was born out of a curiosity to know whether the strong ties of mathematics and insurance trace back to the industry's origins. The study was separated into two parts to consider the earliest examples of Life Insurance and Property Casualty Insurance. Both areas are deserving of analysis, although the depth of previous study on each varies greatly.

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Life Insurance

The earliest ancestors of life insurance as it exists today are annuities, which have existed since ancient times. Annuities are life incomes given or sold by one party to another party called the annuitant. An annuitant would be paid a certain sum yearly until his death, but as some men lived longer than others the value of an annuity could vary greatly. The earliest documentation of attempts to valuate annuities can be traced to Ancient Rome. Rules were developed in 40 B.C. for the handling of annuities in the event of the death of the testator, or grantor of the annuity. In order to settle the estate and disburse the inheritance among heirs, the expected value of outstanding annuities needed to be ascertained. Hald (1990) provides translation of tables developed by Roman jurist D. Ulpian for the calculation of the expected or maximum value of the annuity based on the annuitant's age. These tables present our earliest known records of the consideration and calculation of life expectancies. While it is unlikely that they are mathematically sophisticated or represent relevant estimations, they are important as the foundation of an idea that would later become the basis of life insurance mathematics.

Annuities and "bequests of maintenances" (Hald 1990) continued as a general practice throughout history. However, no documented references are available to indicate further attempts to quantify the value of an annuity until the plague years. It was during that time that mathematicians laid their hands on mortality records as a data source through which to explore the emerging field of probability theory.

The Black Death is the largest known outbreak of plague, but plague was fairly common in Europe throughout the 1300s-1700s. It was fear of plague, and action of the Church of England that can be credited with the collection and compilation of mortality data in a manner that allowed mathematicians to analyze the data. In the city of London in the 1530s the church instituted measures to record the number of weddings, deaths and christening that took place within the various parishes, including the categorization of deaths by cause. This data was used to tell where plague was occurring, and give warning as to where it might reach epidemic threat. Beginning in 1604 (Hald, 1990) the Company of Parish Clerks began publishing and selling these statistics weekly as bills of mortality. The demand for these bills is truly a testament to the impact plague had on society.

John Graunt (1620-1674), a London city politician and businessman was the first person to realize the potential of the data provided in the bills of mortality. In 1662 he published his first edition of *Natural and Political Observations Made Upon the Bills of Mortality*. This publication is credited as being the first statistical analysis of demographic data. Graunt's genius must be understood in the context of his environment; as the son of a tradesman, he received an ordinary education, certainly nothing that would foreshadow his mathematical achievement. Following his father in trade he developed a keen mind for the simple mathematics of running a business. From such simple mathematics he devised the advanced idea of statistical analysis. The significance of his contribution and the high regard with which it was viewed is evidenced by his eventually membership in The Royal Society.

Graunt also fundamentally contributed to the development of statistics and probability by his thorough critical analysis of the validity of his data. His *Observations* clearly highlighted flaws he saw in the bills of mortality, and showed the assumptions and methods he applied to mitigate those flaws. Among the most significant weaknesses Graunt saw in his data was the lack of total population numbers. Graunt held that it was likely that the population was unstable due to movement of people into and out of the city as a result of plague scares.

Hald, in both his 1990 book and 1987 article, gives an excellent detailed account of how Gaunt devised his various observations, including his work to estimate the life expectancy of the London population. It is with this particular part of Graunt's Observations that we concern our discussion of the development of life insurance mathematics. For centuries to follow, mathematicians and politicians would refer to Graunt's tables and comment on their implications. Among the great mathematicians to take up Graunt's work were the Bernoulli family, the brothers Huygens, and fellow Royal Society member Edmond Halley.

Graunt's invention was immediately put to use by his friend Sir William Petty, a fellow politician. Petty led a varied and educated life, and explored several intellectual subjects. He is best known for his work on statistics and political economics, and is credited with being the first to coin the term "political arithmetik." An innovator, Petty was also a founding member of the Royal Society in 1662, and his work and Graunt's are credited with having influenced the establishment of statistical offices.

Independent of Graunt, mathematician Christiaan Huygens had taken up the study of probability theory in the Netherlands. A talented scholar, Huygens spent much of his life engaged in research in mathematics and physics as well as other areas of scientific enquiry. In 1656 he published his first work on probability, the title of which translates to *On Games of Chance*. In the following years he continued his exploration of probability theory and the work of fellow mathematicians Pascal and Fermat. In 1657 he published his work *De Ratiociniis in Ludo Aleae*, a text on the theory of probability.

Living as he did so far from London, Huygen's first knowledge of Graunt's invention came from his friend Sir Robert Moray in 1662. He does not appear to have concerned himself overly with the development until 1667, when his brother Ludwig wrote to him to initiate discussion on the usefulness of Graunt's tables to the application of annuities. The correspondence of the brothers is well documented among the twenty-two volumes of Huygens collected writings, the *Oeuvres Completes* (Hald 1990). Despite Ludwig's interest in life annuities, the brothers' correspondence centered on the broader probabilistic interpretations of Graunt's tables. Huygen's major contribution to the field of insurance mathematics came in the interpretation of Graunt's tables as a continuous distribution, and is credited as being the first to produce a graph of statistical data as a continuous distribution function. This step would lead to developments in curve fitting of statistical data to generate fully predictive functions.

Jan DeWitt, a contemporary of Christian Huygens who studied under the same mathematics professor, made additional contributions to the development of early life insurance mathematics. DeWitt was well educated in mathematics and law, and became the prime minister of the Dutch Republic in 1653 at the tender age of twenty-eight. It was a combination of both his education and political knowledge that made his contributions the life insurance mathematics possible. As a political leader in times of war DeWitt understood that governments need to raise capital to finance their armed forces. The sale of annuities was a popular method employed by governments for that purpose. In 1671, based on the work of Graunt and Huygens, he wrote the *Value of Life Annuities in Proportion to Redeemable Annuities*. Rather than a paper of solely mathematical interest, this was an application of mathematics to a particular problem facing the Dutch Republic. The government sorely needed to raise funds, and DeWitt sought to assure that annuities would be priced in a mathematically accurate manner.

DeWitt's probabilistic analysis of the distribution of the number of deaths divided the data into four segments, beginning at age three which would be the youngest age at which an annuity could be purchased. He further considered that annuities were paid twice annually, and for the purposes of his analysis regarded that a man was no more likely to die in the first half year than the second. Through his mathematical computations, which were reviewed and approved by fellow Dutch statistician Jan Hudde, DeWitt calculated that previous annuities had been sold at an undue discount of two years' purchase.

Under DeWitt's leadership, the Dutch Republic became the first country to offer annuities based on the annuitants age. However, despite his valuations, the annuities were offered at significantly less than his indications. As a political economist, DeWitt was acutely aware that an annuity offered at a price unpalatable to the market would be of no value to the government. Despite its remarkable contribution to the field of insurance mathematics, DeWitt's work is not believed to have been widely distributed outside the political environment of the Dutch Republic. This lack of distribution may be a result of his fall from power and execution by a mob a year later (1672) after the French invaded the Dutch Republic.

The next great mathematician to take up the topic of valuation of life annuities was Edmond Halley, better known for his work on astronomy and in defense of Newton. In something of a departure from his traditional fields of inquiry, Halley undertook an analysis of the valuation of annuities at the behest of the Royal Society which had just received new mortality data. This new mortality data, the first since the London Bills used by Graunt, had been compiled in the city of Breslau. Halley took up the challenge and in1693 presented his paper *An Estimate of the Degrees of the Mortality of Mankind, drawn from curious Tables of the Births and Funerals at the City of Breslaw; with an Attempt to ascertain the Price of Annuities upon Lives*.

Halley utilized Graunt's work extensively in establishing the distribution of mortality of persons aged six and younger. Further, he created a table of the number of deaths at reported ages and, like DeWitt before him, divided the data into four age intervals. For each age interval he established a minimum and a maximum number of deaths to occur per year. Like Graunt, Halley was highly analytical and critical of the validity of his data. Halley addressed the questionable areas of the data through comparison to other sources of comparable data. This practice is heavily utilized by actuaries today as a means of shoring up the statistical significance of an individual insurer's proprietary data. Hald (1990) provides clear reproductions of Halley's tables. Through manipulation of the data, Halley developed the distribution of deaths as a piecewise function calculated on age intervals.

Members of the famed Bernoulli family of mathematicians were also engaged in the study of probability. James (or Jacob) Bernoulli had already demonstrated his mathematical prowess through his work on infinite series and the new calculus of Liebnitz. Turning to probability, he began work on his masterpiece *Artis Conjecturi,* which would not be published during his life. He made intervening publications on the topic, including one in 1686 in which he refers to Graunt's tables. Nicholas Bernoulli, James' nephew and student, worked closely with his uncle on the study of probability theory and produced results similar to those generated by Huygens which he published in his thesis on probability and law, *De Usu Artis Conjecturi in Jure* (The Art of Conjecture in Law) in 1711.

Three decades later the topic would be further developed by a pair of controversial mathematicians. Thomas Simpson was a self-taught mathematician, sometime teacher, and author of texts on mathematical concepts. Unfortunately, it is this latter occupation that got Simpson in trouble. Simpson's first genius was not in the development of new mathematical ideas, but in his ability to clarify and simplify the work of those before him. He published eleven text books, but was criticized for having grossly plagiarized most of them. One example is his book *The Nature and Laws of Chance. The Whole After a new, general and conspicuous Manner, And illustrated with A great Variety of Examples*, published in 1740 of which Hald (1990) states "There is, however, nothing 'new, general and conspicuous' in Simpson's book; it is simply plagiarism of the mathematical parts of the Doctrine," (p 414) in reference to Abraham de Moivre's groundbreaking Doctrine of Chances, originally published in 1718.

De Moivre was, understandably, outraged at Simpson's appropriation of his work In 1742 when Simpson published a text book on life insurance mathematics, de Moivre was convinced that Simpson had once again copied his own work, from his *Annuities upon Lives: or, The Valuations of Annuities upon any Number of Lives; as also of Reversions. To which is added, An Appendix concerning the Expectation of Life, and Probabilities of Survivorship*, published in 1725. While it is generally held that Simpson's work on life insurance mathematics was indeed founded upon the work of de Moivre, it is in this field that Simpson truly found his own unique genius and made original contributions.

Both de Moivre and Simpson delved into great detail of the intricate valuation methodologies of variations on annuities including reversions at marriage and succession, and the valuation of annuities upon multiple lives. These topics had been left unresolved by Halley, and it was de Moivre who is credited with having identified that Halley's distribution could be linearly approximated as a piecewise function. This presented to de Moivre a means of approximating the more complicated annuities which he had been otherwise unable to estimate. De Moivre advocated the use of linear approximation to calculate multi-life annuities.

Simpson had an advantage on de Moivre when he published his *The Doctrine of Annuities and Reversions, Deduced from General and Evident Principles: With Useful Tables, Shewing the Values of Single and Joint Lives, etc. at different Rates of Interest* in 1742. De Moivre had developed his calculation on Halley's data on the mortality rates of Breslau, and had not had access to any new mortality data. Simpson, on the other hand, had data that became available after DeMoivre's treatise, the newly published London Bills of Mortality. He fundamentally disagreed with de Moivre's linear approximation method and instead advocated the use of tables for deriving the value of annuities on multiple lives. His tables were far more advanced than any previously produced, and it is with Simpson's tables that the insurance industry sided. For hundreds of years to follow, mortality tables would be produced in similar form.

These advances in the understanding of the financial mathematics of life insurance helped fuel the generation of private life insurers. While previously annuities had only been offered for sale by governments, now the burgeoning capitalism of Europe created an environment ripe for privatization. This shift was not without troubles. Many insurance companies formed in the early 1700s failed, to the financial detriment of their annuitants. Insurance companies came and went for the next several centuries, and were held in great suspicion by the populace. So much so, in fact that in 1843 author Charles Dickens created a fictitious and villainous life insurance company, the Anglo-Bengalee Disinterested Loan and Life Assurance Company in his serialized novel *The Life and Adventures of Martin Chuzzlewit* (Hickman, 2004). Dickens is not the only literati to turn his pen to topics of insurance. A century later, great English playwright George Bernard Shaw would write an essay entitled *The Vice of Gambling and the Virtue of Insurance*. Though not, by his own estimation, a talented mathematician, Shaw proved to have an excellent grasp of the vagaries of insurance, and its mathematical precepts.

The privatization of insurance and general suspicion of insurance companies, lead to the creation of the actuarial profession, one of many new professions that were born during the Victorian Era. In his *History of Actuarial Profession*, James Hickman gives an excellent account of the specifics of the requirements of the new profession. By Hickman's account, the actuarial profession was created to serve a public purpose - the reasonable protection of purchasers of life insurance products. Actuaries were changed with assuring adequacy of assets for the fulfillment of life insurer obligations. While the actuarial profession's origins trace to life insurance, the role of actuaries has expanded to included property and casualty insurance. Actuaries now work publicly and privately, both at the side of the insurance companies, and with the governments that regulate insurance.

Marine Insurance

Like life insurance, the precursors to modern property insurance can be traced back to ancient times. The concept of spreading risk and providing for the assurance of monetary aid in the event of loss are common to many cultures. The first written records of what could be considered early insurance law are found in the Code of Hammurabi (circa 1760 B.C.) as promulgated by Hammurabi, the sixth King of Babylonia. Guilds also played a significant role in the evolution of modern property insurance. Membership in a guild brought protection; it also meant mandatory participation in the funding of this protection. Early caravan traders are also purported to have developed schemes of protection of property that was at risk to over-land travel.

These early ancestors of insurance as we know it today demonstrate the concept of risk sharing or risk distribution, one of the fundamental concepts underlying modern insurance. In many ways, the agreements among guilds and traders for the protection of assets under threat of risk during transport bear simplistic similarity to such modern insurance forms as Risk Retention Groups. In both cases, the risk is transferred and distributed among parties of interest who all participate in the same business (affinity groups) and each participant in the agreement not only agrees to pay losses incurred by another, but also contributes their own risk to the pool.

The risks faced by early overland traders were minimal compared to the risks faced by merchants of seafaring trade in the 13th and 14th centuries. While land traders were exposed to risk of loss from theft and spoilage, seafaring traders faced all the same risks, plus the increased risk of piracy and general sea risk. These risks required more sophisticated insurance mechanisms. It is in these early years of ocean trade that we find the first examples of property insurance as we consider it in modern times. In early marine insurance mechanisms we see some signs of the types of risk-sharing exhibited by earlier insurance ideas. The changing nature of the trade soon resulted in the emergence of true risk transfer, or the insurer as unaffiliated third party.

The exact origin of marine insurance is a subject of some disagreement among scholars. Most properly, the evolution of the industry should be credited to medieval Italy, and its ports of trade. The early Italian insurance mechanisms are thought to have found their roots in the contracting of loans, specifically the sea loan.

Another form of loan that is sometimes considered to be an ancestor of marine insurance is the loan of bottomry. A loan of bottomry was an agreement between the owner of a ship and the ship's master, in the event that the two parties were different. A loan of monies was made by the ships owner to pay for costs of repair to the ship and its components in its destination port. The reason loans of bottomry are considered to be a form of early insurance is that the agreement would generally stipulate that the amounts were only repayable upon the safe transit of the ship and its cargo. However, if the party loaning the money (the ship owner) is also the party that would stand the risk of financial detriment in the event of loss of the ship there is no true risk transfer, without which it does not meet the standard of a true scheme of insurance.

The sea loan, on the other hand, exhibits risk transfer and is therefore a monre proper ancestor of marine insurance. These loans faced significant challenge from the canonical prohibition on usury. To try to skirt the prohibition, loans were falsely recorded as having been made *gratis et amore* (without interest). In fact, the amount repaid would be in excess of what was actual received by the borrower, the additional payment constituting both interest on the loan and payment for the risk taken by the lender. Such practices are believed to have been common for many financial transactions during the prohibition on usury. Regardless of the misleading records, the sea loan was condemned as usurious in 1236 by Pope Gregory IX (De Roover, 1945). De Roover (1945) indentifies the *cambium nauticum* and the *foenus nauticum* as two early forms of insurance loans practiced in medieval Italy.

The *foenus nauticum* was a loan made by a lender to the seafaring merchant which was repayable only if the ship completed its journey. Merchants often used the borrowed money to purchase cargo, and their financial success was therefore dependant on their ability to sell the cargo at a price greater than the repayment. Under such loans, the lender may be regarded as the actually owner of the cargo (by virtue of its use as collateral), which brings into question whether or not it truly constitutes risk transfer. Perhaps due to fear of condemnation of usury, such loans were not terribly popular, and were replaced in practice by *cambium nauticum*.

The *cambium nauticum* was likely a more successful instrument for avoiding accusations of usury. *Cambium*, in general, were contracts of exchange made through the sale of bills of exchange. Money was advanced in one currency which would be repaid by the borrower in the currency of a different port. In the case of cambium nauticum, this sum was only payable if the ship reached its foreign port safely. Critically speaking, such agreements simultaneously reduced a merchants risk while exposing him to the additional exchange risk. While protected in the event of loss of cargo or ship, the merchant risked loss from a change in the value of the terminal port's currency from that assumed upon the issuance of the bill of exchange. Scholars believe that lenders selected high estimations of the port currency's value to ensure that exchange risk was one-sided and born exclusively by the merchant.

The 13th century saw the advent of partnerships among two or more cooperating merchants. These partnerships can be identified as the roots of the modern multi-national corporation, and like their modern descendents required the division of duties due to geographic factors. One party, generally a person possessing greater capital and familial ties, acted as the "headquarters" of the partnership; he would not travel aboard the ship, but would remain on land and manage the relationship of the partnership with suppliers and creditors. The other partner would act as the managing partner abroad, accompanying their ship and cargo, and negotiating contracts of sale and resupply at the destination port.

Demand for insurance increases when the party to be insured possesses additional assets that would be exposed to the risk of a financial loss. Within the new partnerships, the loss of a ship at sea (and potentially one of the partners) could expose the sedentary merchant's personal assets to being seized by creditors. The sedentary partner most likely also possessed greater financial decision-making authority within the partnership. The development of partnerships created a condition of increased demand for marine insurance.

Despite their obvious drawbacks, *cambium nauticum* persisted as the most common form of marine insurance until they were replaced by standard *cambium* contracts in the 1300s. Perhaps the lenders became unwilling to accept the risk of loss at sea. A standard *cambium* was repayable regardless of the possible loss of the ship or cargo. Without the previously available form of insurance, a new market was required to help merchants mitigate their sea risk. This vacuum of coverage, coupled with the increasing demand, provided the right conditions for the development of the first true contracts of insurance.

A great body of research has been conducted by legal scholars on the earliest known true contracts of insurance. This research has provided no common opinion as to the exact date of the first contract. Because the modern contract of insurance evolved from various risk-sharing instruments such as sea loans, it is difficult to pinpoint when exactly insurance as a separate financial contract truly developed. Scholars generally agree that in order to be considered insurance in the modern context, the contract must contain risk transfer, risk distribution, and the provision of premiums. While the early loan-based mechanisms provided for transfer of risk to the lender, they failed to meet the threshold of risk distribution or payment of premium and are therefore not considered true contracts of insurance from a legal perspective. De Roover (1945) attributes the earliest contract of premium insurance to certain texts, records of account, and statutes originating in the period of 1318-1320. Holdsworth (1917) credits the first contract as one existing in the archives of Genoa dating from 1347

Within the history of the contract of marine insurance we also find reference to the origins of the concept of insurable interest. Holdsworth (1917) discusses the common abuse of insurance as a vehicle for gambling during the 15th century. Parties otherwise un-related to the business of a ship could use an insurance policy to wager on whether or not a ship would be lost at sea. This clearly violates the intent of the contract of insurance to provide for indemnity upon a loss. We find in the actions of the legislature at Genoa in the late 15th century, and the statutes of Barcelona in 1484 the first attempt to create in law the provision that contracts of insurance are only binding when there exists an insurable interest. These laws became the foundation of insurance law, and provisions for insurable interest are still found today in both insurance regulation and insurance contract language.

While the histories of marine insurance provide ample discussion on the origins of early insurance mechanisms, they do not address their foundation in mathematics. Very little research is available to indicate the particulars of how premiums were calculated prior to the creation of Lloyd's of London. Straus (1938), describes Lloyds as "the most famous single name in all the world." Lloyds' history is well documented, from its beginnings as a London coffee house where merchants met to discuss business, to its emergence as the world's largest private insurer. The history of Lloyds also tells of the role of the insurance underwriter in the establishment of insurance premiums.

The term underwriter was first coined to describe the individual who agreed to insure a risk. He agreed to the contract by writing his name *under* the terms of the contract, thus the name underwriter. It was the underwriter who determined the value of the potential loss, and the corresponding premium to charge for insuring it. In the early days of marine insurance the risks to be insured were very large and possessed unique characteristics such as trade route, crew experience, and cargo. This made the job of pricing marine insurance particularly varied and subjective to the individual determinations of the underwriter.

Unfortunately, though exhaustive, the histories of Lloyds fail to document the mathematics used by early underwriters. The likelihood that underwriters employed formal, consistent mathematical models to evaluate risk seems unlikely. In the absence of any evidence to the contrary, it is easy to speculate that premium determination in early marine insurance was far more like gambling than any type of actuarial science as known today. Such speculation is born up by Lloyds's early organizational ties to wealthy British gentlemen for whom gambling was reported to be a common pastime.

Early marine underwriting of a small number of individually large risks contrasts with the majority of modern insurance rating which involves risks that tend to be larger in number, smaller in relative value, and generally homogenous within each line or type of insurance. Further, in modern insurance, the process of determining insurance premiums is far more complicated than was encountered by early underwriters. In the U.S. and other countries, the process is heavily regulated. Insurance today would not be possible without the contribution of actuaries, insurance mathematicians who quantify the value of risk. Actuaries utilize a vast array of advanced mathematical tools including calculus, probability theory and statistics to model data and analyze probable value of losses an insurer might expect to pay on a risk. Underwriting of modern insurance involves considering the individual variations of a particular risk and selection of an actual premium based on the analysis of actuaries.

Observations

This brief survey of insurance mathematics highlights an interesting difference between the level of sophistication of early life insurance mathematics and that of early marine insurance. There are several probable explanations for this disparity. First, the respective eras that saw their development were dramatically different. The mathematics of the 13th and 14th centuries were far less advanced than the mathematics of the 17th and 18th centuries. The 17th and 18th centuries were a time of immense expansion of mathematical knowledge. From Napier's development of logarithms to Newton and Leibnitz and their work on the development of the Calculus, the 1600s were an exciting time for mathematics. It certainly makes sense that life insurance, which came into being during and after this scientific renaissance, would have more advanced mathematics attributed to it than earlier insurance mechanisms.

Further, the types of insurance considered in this paper are in and of themselves very different. As previously mentioned, early marine insurance involved individual risks that were generally large, but restricted in number. Life insurance, even early annuities, on the other hand, involved a larger number of risks that were comparatively smaller in individual value. The development of life insurance mathematics also hinged significantly on the availability and value of mortality data. While records of ships lost at sea were also kept, their smaller number may have made the data less adaptable to probabilistic interpretation.

In this context we consider one more fundamental contribution to insurance mathematics, James Bernoulli's posthumously published *The Art of Conjecture*. Within this seminal work of probability theory, Bernoulli identifies and proves what has come to be known as the Law of Large Numbers, or Bernoulli's theorem. Bernoulli's theorem is of paramount importance to the practice of modern insurance mathematics. Through the collection, compilation, and analysis of large amounts of data, actuaries are able to develop far more accurate probabilistic interpretations of insurer's losses. This creates a more scientific basis for the industry of insurance, and further distances it from its gambling-like origins.

The impact of the quantity and relative size of the risk on the analysis of data warrants further research. Additional analysis of the evolution of property insurance, and an identification of when it became more scientific in application would be particularly valuable. Research into the origins of fire and personal property insurance might lend a greater understanding of this stage in property insurance's evolution, and create a more thorough understanding of similarities and differences between the mathematical origins of modern life and health versus property and casualty insurance. Further research into the evolution of the actuarial profession, and its transition from life insurance to all lines of insurance might also provide valuable insight into this topic. Such research however is outside the scope of this paper.

In the study of the development of Life Insurance Mathematics, we see an interesting phenomenon in the interplay of contributions made by practical versus theoretical mathematicians. There is little question that the development of the mathematics of annuities played a significant part in the greater picture of the study and development of proability theory and statistics. It is intriguing that while important contributions were made by famous figures from the mathematical world (the Bernoullis, de Moivre, Halley, et al.), contributions of at least equal import were made by practical men with little formal mathematical training. Examples of these practical mathematicians are John Graunt and Thomas Simpson. We recall that neither Graunt nor Simpson received formal mathematics training. Graunt appears to have developed his genius through practical inspirations and his knowledge of basic bookkeeping, while Simpson's knowledge was almost exclusively self-taught. Both men came from humble, working-class origins. Neither supported himself primarily through his mathematical endeavors (although Simpson would be able to do so later in life).

Conversely, we consider the giants of mathematical theory from the same era: Halley, Huygens, and the Bernoulli's. These men were true geniuses of academia, raised by families of generous means, and educated in the finest of classical and mathematical pedagogy. In Huygens case, we know that he lived for many years supported by his father while he dedicated his time to intellectual pursuit. Later, he would move to Paris and become a member of the Academie de Royales des Sciences where he was supported by Louis XIV. Halley was the son of a very wealthy London family. His father provided him not only with an extensive education, but also the resources and instruments he needed to isolate himself in his scientific observations.

The Bernoulli's are well known for their success in capitalizing on their mathematical developments. Acting as private tutors and professors, they were able to carve out a comfortable place for themselves entirely upon their intellectual pursuits. Nicholas Bernoulli, the nephew of James (Jacob) and John (Johann) would also turn to the study of law, but even there his pursuits were of a far more theoretical than practical bent. The 16^{th} through the 18^{th} centuries were not an era of intellectual popularism – advanced education of the type enjoyed by our giants of mathematical theory was reserved for young men of a certain degree of privilege. The humble nature of Simpson and Graunt's origins and education make their contributions to probability and statistics even more remarkable. These practical mathematicians were instrumental in the development of what is the practical mathematics of Probability and Statistics.

Probability and Statistics are only limited in their application to insurance by their supposition that past behavior is indicative and predictive of future behavior. In our ever changing soci-political and increasingly globalized society, this fundamental precept of probability is sometimes open to challenge. In the following centuries insurance mathematics will further evolve as application of game theory and other advanced mathematics are developed to continue the traditions detailed in this paper. The influence of mathematics on insurance is firmly established. Only time will tell if insurance will again provide the type of influence on the development of new mathematics that it did in the early days of life insurance mathematics.

Acknowledgements

This paper was inspired by my experience in the insurance industry and my academic forays into advanced mathematics, during which I became drawn to the history of how the advanced mathematical concepts were devised. My curiosity as to the origins of insurance mathematics began with references to the applicability of Napier's logarithms to finance mathematics and eventually led to my discovery of the work of Anders Hald, formerly Professor of Statistics at the University of Copenhagen in Denmark. My humble survey of life insurance mathematics pales in comparison to the depth of detail exhibited in Professor Hald's work. His book *A History of Probability and Statistics and Their Applications Before 1750* is a must read for anyone interested in developing a greater understanding of the technical and computational aspects of the mathematics referenced in this paper. Hald, in turn, makes acknowledgement to other authors whose work has made our understanding of this topic possible.

It is with great sadness that I note that my discovery of Professor Hald's work coincides to the month with his passing at the age of ninety-four. I dedicate this paper to his memory and the hope that future generations of mathematicians and insurance academics will know and honor his contributions to their fields.

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