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Wolff-Michael Roth

Mathematical Cognition and the *Final Report* of the *National Mathematics Advisory Panel*: A Critical, Cultural-historical Activity Theoretic Analysis

***Wolff-Michael Roth*[1]**
University of Victoria, Canada

Abstract
In its *Final Report*, the *National Mathematics Advisory Panel* has depicted a stark image of mathematical competencies and achievement among U.S. students. The *Advisory Panel* notes the lack of research with "truly scientific" rigor and, mentioning Vygotsky's cultural-historical activity theory in passing, suggests its utility to be untested. In reading the report, I noted the limited understanding of the mathematics education literature it articulates and a complete failure to draw on established, "tried and proven," theory and practice in mathematics education founded upon an encompassing cultural-historical activity theory. This theory is comprehensive and encompassing, because it retains activity in its entirety as the unit of analysis, which leads to an integrated and integral consideration of those "factors" that are taken to be external to mathematical cognition in the *Final Report*. In this article, I articulate the current state-of-the-art understanding of cultural-historical activity theory and then use it to provide a critical perspective on the report, its recommendations, and its conclusions as these pertain to the learning of mathematics.

Keywords: Cultural-historical activity theory; National Mathematics Advisory Panel*;* mathematics education research; socio-cultural perspectives; Vygotsky

One of the first thoughts that arose in my mind while reading the *Final Report* of the *National Mathematics Advisory Panel* was the French saying "Plus ça change, plus ça change [pas]" (the more it changes, the more it doesn't change).[2] The more educational research finds out, the less educational policy changes, as it plays up to the powerful who tend to desire the reproduction of the status quo rather than to bring about changes of life conditions that lead to differences that make a difference.[3] Because the *Final Report* does not articulate a coherent position on what constitutes mathematical knowing, there is little that holds together the statements and recommendations—the report constitutes a disconnected collection of sentences that play into the hands of (simple-minded?) politicians and policymakers rather than providing a comprehensive framework for the mathematical

[1]Lansdowne Professor, Applied Cognitive Science, University of Victoria, Victoria, BC, V8Z 7G6, Canada. mroth@uvic.ca

[2] Perhaps more frequently, the saying may be "Plus ça change, plus c'est la même chose" (the more it changes, the more it remains the same).

[3] Locking thieves up in prisons is easier than to change society so that the poverty from which many thieves issue becomes a thing of the past. Changing society would mean that those who have will have to share with those of have not. Ideologically, this is not where an individualist society wants to head.

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education of current and future generations. A comprehensive framework would have take into account research evidence—such as that issuing from the many ethnographic studies of mathematical knowing in the everyday world—that the panel oh so lightheartedly disqualified as non- or insufficiently scientific. More so, the *Final Report* provides considerable evidence that its findings are not in themselves legitimate but that to the contrary, its legitimacy has to be articulated from the outside by means of statements about its own scientificity and scientific rigor. The report further justifies its own conclusion by taking a deficit perspective on mathematical knowing generally and the mathematical knowing of certain parts of society more specifically, failing to take into account the research on the knowing and learning of mathematics in everyday life and therefore failing according to its own criteria of scientific and scholarly rigor. Notably, the *Final Report* does not interrogate the political dimension of schooling and therefore remains oblivious to the fact that schooling produces deficits, hierarchies, and inequities that differentially lead the children of specific classes into a hierarchy of career and into the for a globalized capitalist economy necessary structural unemployment. The *Final Report* thereby plays into the hand of a particular individualist ideology of the worst kind, continually produced and reproduced by and under the current U.S. administration.

In this essay, I take up a number of problematic points concerning mathematical learning processes as these become apparent throughout the report. Perhaps the greatest problem with the *Final Report* is that it leaves unstated its theoretical framework for mathematical knowing and learning, which, inherently, constitutes the framework within which the value statements and deficit perspectives are framed (e.g., "Far too many middle and high school students lack the ability to accurately compare the magnitudes of such numbers" [p. 27]). The authors of the *Final Report* do not have or articulate a theoretical framework that would allow us to understand why, according to their own account, the students coming from "low parental education levels, low incomes, and single parents" (p. 25) end up bringing "less foundational knowledge for learning school mathematics" (p. 25) and, as a result of lower achievement at the end of schooling, end up in the same societal classes as their parents. I begin by articulating a comprehensive theoretical framework of knowing and learning that does take into account why human beings do what they do and how learning accrues from their actions. Such a comprehensive framework is necessary, because only then can we answer questions about why and how students actually take up and identify with the motives of school mathematics and therefore what the grounds of reason they have for doing what they do in school generally and school mathematics more specifically. The theoretical framework I outline is grounded in a lineage of social psychology that begins with Lev S. Vygotsky, who is mentioned in the report but only briefly but whose work and legacy remains unappreciated and misunderstood in the text of the *Final Report*. The framework I develop generally is referred to as cultural-historical activity theory and I show how it already provides answers to questions that the *Final Report*—I wish it were the final, that is, last of its kinds—can only recommend for further study. I then take up a number of claims and issues made in the *Final Report*, making explicit what it leaves out, especially the research on knowing and learning in mathematics that we are familiar with and that does not seem to have been accounted for in the writing of the report.

Cultural-Historical Activity Theory

Cultural-historical activity theory provides a framework for understanding human activities (Roth & Lee, 2007). In English, the term "activity" is used to render two very different concepts from the Russian and German languages in which much of the theoretical framework was developed: The term aktivnost'/Aktivität means being busy, doing something, whereas the term deiatel'nost'/Tätigkeit means societally motivated and society sustaining human endeavors. Thus,

children doing school tasks engage in the former, whereas farming, manufacturing tools, and schooling constitute activities in the second sense. Tasks do not require knowing *why* one does what one does, whereas participation in societal activity does. In fact, students *do* participate in an activity, but it is, as I show below, concerned with *schooling* rather than with education and knowing.

Cultural-historical activity theory fundamentally assumes the dialectical relationship between agency and structure. On the one hand, agency mobilizes material and social structures, but it is itself enabled by and the product of these structures. It therefore does not make sense (a) to speak of agency independent of the enabling and mobilized structures or (b) to speak of structures independent of the agency that picks out and therefore defines relevant structure. Following one interpretation of cultural-historical activity theory (Engeström, 1987), many researchers draw on one particular, triangular representation to make salient the structural moments and mediational effects of human activity systems (Figure 1).

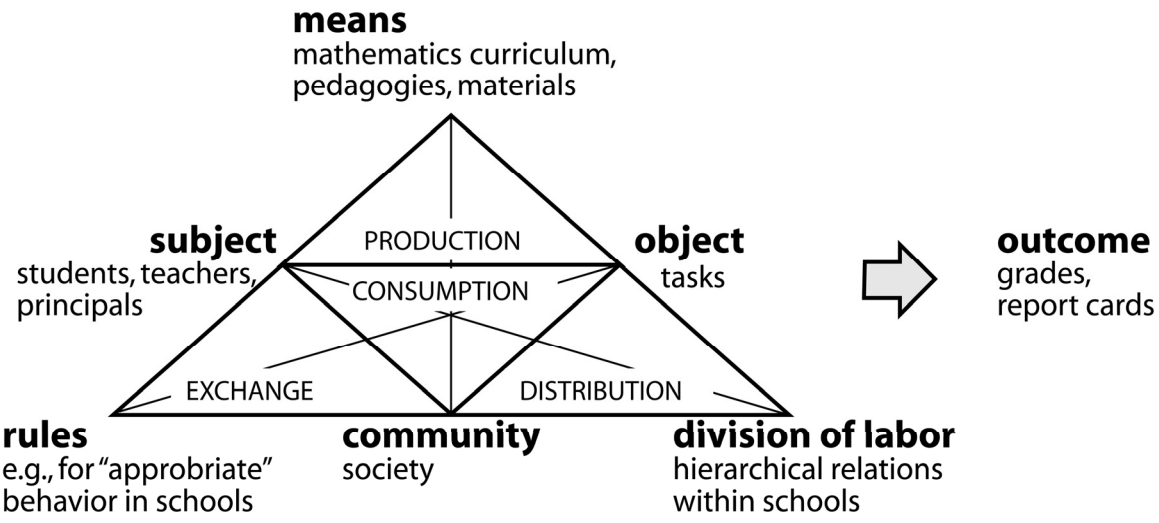

Figure 1. Structural perspective on the cultural-historically evolved activity of schooling, which contributes to the production and reproduction of society, including its hierarchical relations of ruling.

In Figure 1, the structural *moments* of schooling are articulated. (A different example I could have used is the production of the *Final Report*, which would have included its authors as the "subject," existing research, computers, libraries, and meeting facilities as the "means," and the U.S. president, his policymakers, and other politicians as part of the relevant "community.") The activity system as a whole and the different *inner* structures identified—subject, object, means, rules, community, division of labor, and outcome—cannot be reduced to one another. In other words, a structural component such as the "subject" does not constitute an *element* but an irreducible *moment* so that the relevant subject is constituted by the activity as a whole generally and its other moments more specifically. The object of activity cannot be reduced to the intentions (motives, goals) of the acting subject, but rather, subject and object mutually constitute (determine, imply) each other. Each moment denoted in Figure 1 is constitutive of the activity as a whole, but it is only as part of an existing activity that these moments become relevant in the first place. Each moment only constitutes a one-sided image of the activity as a whole, though it may be used, in synecdochical (metonymic) fashion to denote the whole.

Cultural-historical activity theory organizes our gaze concerning human activities by highlighting outcomes of activity. What do students walk away with from their school experience? The answer is: grades and report cards. Some readers may contest this answer and argue that "knowledge" is the true outcome of schooling. We can, however, immediately see that the latter answer is in contradiction with observations according to the *Advisory Panel*'s own account: American students generally and those from low-income, African American, and single parent families specifically do not come away from schooling with the forms of knowledge that the *Advisory Panel* desires them to come away with. More so, research on mathematics in the everyday life shows that mathematical performance of supermarket shoppers (Lave, 1988) or among child street vendors of candy (Saxe, 1991) is unrelated to the number of years someone has studied mathematics or the grades someone has achieved.

Another argument against knowledge being the outcome of schooling can be gleaned from the generally observed fact that students at all levels memorize some facts for exams, which they are frequently proud of having forgotten only hours or days later. It is not knowing something that is valued but "what'ya get?," that is, the grade that the instructor provides in exchange for the exam or assignments handed in on the part of the students. Whatever the student does and learns is less important and in fact irrelevant; what counts are the individual grades, the course grades, and the grade point averages. "Cheating" and "plagiarism" are typical forms of actions that are enabled by a system focusing on grades rather than on knowledge. If learning, that is, the expansion of action possibilities and room to maneuver were the outcome, it is hard to understand why students would "cheat" and "plagiarize" given that they would go against their own interests. "Cheating" and "plagiarizing" work *in their interest* of getting higher grades—at least as long as they do not get caught. Cheating and plagiarizing therefore are action possibilities in and made available by a system rather than deviancies located in the characters and minds of specific individuals.

Further structure for our gaze is provided in the focus that cultural-historical activity theory has on the usage of the outcomes of an activity system within the system or in other systems to which they are "traded" (exchanged) and how the outcomes are distributed within the relevant community. Thus, it is immediately evident that high school graduates use their grades and report cards to garner access to universities; it is common knowledge that higher grade point averages provide graduates with more choices and access to more prestigious institutions. Grades, report cards, and GPAs constitute a form of (symbolic) capital that students can use to open doors; and entry through these doors provides them with many more opportunities to gain cultural, symbolic, and financial capital. Those students with the lowest grade point averages, on the other hand, might find themselves in manual labor and service profession where the results of schooling do not matter. Interestingly, as the *Final Report* shows, children from low-income and single parent families, for example, come with "less knowledge" to school and end up with less, because "the mathematical knowledge that children bring to school influences their math learning for many years thereafter, and probably throughout their education" (p. 25). Why might this be? The simple and unsophisticated answer is that prior knowledge determines future learning. This answer is unsophisticated, because it does not take into account the ways in which culture and language at school and the cultures and languages from which children issue mediate one another: there is research that does show how schools and schooling embody middle class culture leading to an enhancing resonance phenomenon for children and students from the middle class and negatively mediating enculturation, socialization, and learning of working class students (Eckert, 1989). The *Final Report* does not provide evidence about whether panel members are aware of such research or whether, if they are, simply discounted it.

Cultural-historical activity theory is capable of accounting for both phenomena, the inequitable *distribution* of grades and grade point averages available from the students' report cards and the resulting *exchange* relations in which students trade their these, their symbolic capital, against access (Figure 1). These grades allow a hierarchical ordering of students, provide them with differential access to further resources and opportunities, and lead to the production and reproduction of social stratification (Foucault, 1975). Another example may further show the power of cultural-historical activity theory in articulating structures that mediate the attainment of grades. Thus, the *rules* (Figure 1, lower left) regulating "appropriate behavior" in schools are modeled on those that are characteristic of the middle class, including those mediating interactions with teachers, the use of language, and time and temporality. Students who come from poor African American produce time and temporalities very different from those of middle class students and they end up being punished (e.g., excluded from accessing schools) for being late; or they use "foul" language that leads teachers and administrators to suspend students (Roth, 2005). Here, then, the normative rules teachers, administrators, and often police enforce in schools mitigates against the engagement with school tasks, resulting in lower grades, lower achievement, more years in school, and so forth.

Production is a function of the degree to which the subject in an activity system takes up the collective motive of activity. For example, the intended outcome of taking an algebra class supposedly is mathematical knowledge. However, many students do not take up this intention: they are said to be unmotivated. Or they fake motivation but resort to all sorts of learning other than mathematical learning that promises them not to be punished or penalized—such learning is defensive, for it is motivated by avoiding negative repercussions. To make up for this chasm between intentions and their uptake, schools and teachers are supposed to provide incentives that motivate students. Now tested widely, some school districts in the US provide cash incentives for students to work for getting higher grades (Ash, 2008). Why would students who see their room to maneuver and action possibilities increase *not engage* in learning? Why would students act against their own interests not engaging in an activity that expands their control over their life conditions and enhances their opportunities for acting? Not surprisingly, the *Final Report* does not articulate a comprehensive model in which motivation and emotions are *internal* moments of cognition but that are mere factors—which, as all factors, inherently are outside and therefore *indifferent* to the phenomenon (Hegel, 1806/1977).

To understand the for task relevant moments of goal formation and meaning, another dimension of cultural-historical activity theory needs to be articulated. Thus, whereas activity characterizes a societal formation, any activity is concretely realized in and through the goal-directed actions of the subject. But the (individual, collective) subject acts because of the activity. Thus, what teachers in schools are oriented to and do, and what makes their orientations and doings meaningful, *is* the activity of schooling. Lecturing or asking students to complete an algebra task is meaningful *because* teachers find themselves in schools, with their students, and the process of accomplishing the overarching motive—the production of grades and, hopefully though not necessary, the production of some mathematical knowledge. But precisely the same actions realize schooling in a concrete way. The actions are a condition for schooling to emerge and exist. The activity that organizes and motivates goals and the actions that realize it therefore are mutually constitutive. Actions are meaningful to the extent that they are constitutive moments of activity; but activities only exist because of the meaningfulness of actions.

Not every aspect of an action is conscious like the goals. Rather, unconscious operations realize actions and their goals, which nevertheless have occasioned the production of the operations. Thus, we may have the intent to contribute to a conversation, but we do not have to choose the words—the production of words is something unconscious, unless we struggle and actively search for words in some instances. Similarly, writing an algebra test, we do not have to think about writing the numbers, letters, and symbols in the equation $5a + 3a = ?$. Rather we think and the letters, symbols, and numbers are produced in a process that itself, as its objects, remains unconscious. That is, operations are conditioned by the context, including the current state of the action with respect to the ultimate goal.

All three levels are involved when it comes to emotions and affect. The motive of the activity and the goals subservient to them are mediated by short-term costs and long-term payoffs, evaluated in emotional-volitional terms (Bakhtin, 1993). A person may put up with short-term negative effect on emotion—e.g., studying hard, engaging in repetitive tasks—if these actions promise likely-to-be-attained long-term payoffs that increase the person's room to maneuver (being able to do more, better, receiving money that one can purchase things otherwise unattainable, etc.). The bodily states that contribute to emotion also mediate performance, expressing the irreducible relationship between quality of performance and emotional constitution. In the extreme case, getting a rush from engaging in mathematical tasks increases the likelihood of performing much higher than if the engagement is indifferent and certainly than if the engagement is merely for the purpose of avoiding punishment and other negative effects (e.g., low or failing grades).

In this theoretical framework, learning is actually an epiphenomenon arising from the fact that in pursuing and realizing the current activity, thereby reproducing it to a certain extent in form and content, the subject changes becoming more adept in producing relevant entities. Doing something better and having more alternatives at one's disposition than previously here characterize learning. That is, learning is constitutive and the outcome of an expansion: in control and action possibilities. In the ideal case, students learn one or more alternatives of doing certain algebra tasks. But this does not have to be. It is immediately evident that this framework better than others explains why one possible outcome of schooling is a greater number of increasingly better "cheaters" or why students are focused on getting grades rather than on really knowing whatever the curriculum specifies as that to be learned.

Societal Mediation of Poverty and the Reproduction of Inequity

The *Final Report* takes an approach that tinkers with symptoms rather than with the causes of mathematical knowing and achievement. For example, the panel notes that it is unfortunate that

> most children from low-income backgrounds enter school with far less knowledge than peers from middle-income backgrounds and the achievement gap in mathematical knowledge progressively widens throughout their PreK–12 years. (p. xviii)

The first question we may raise—about this and similar statements—is about the "fortune" that leads children from low-income backgrounds to come with "less knowledge" and to face an increasing rather than decreasing gap. The word fortune derives from and relates to the Latin words *fors*, chance, and *ferre*, to bear. Children from these backgrounds do *bear* poverty as a burden. But this burden does not come by chance (i.e., fortune): it is a direct result of exploitation, societal stratification, and the effects of radically individualist capitalist society. (Sweden, a social democratic

country, has low child poverty rates whereas the US, one of the two richest countries in the world, has among the highest child poverty rates.)
The report does not provide an hypothesis or explanation for the increase in the gap between children from well-to-do and poor families. What lies at the origin of the *widening* gap? Why does a school system supposedly providing equal opportunities to *all* children create increasing inequities? What are the social and cognitive forces that lead to those differences? It is quite clear to me that a model of learning that focuses on the mental storage of procedural and declarative knowledge individuals acquire cannot handle the ways in which the participation in cultural practices, such as schooling, not only produces (changes) the practices but also reproduces them including the structures that enable the practices and that are mobilized in these practices.

The discursive ideology of the report is quite clear and not unlike the medical discourses concerning obesity, coronary heart diseases, or cancer: fix the symptoms rather than enact much less expensive (but less lucrative for multinationals) prevention that get rid of the causes. Thus, rather than putting kids into "Head Start and other programs serving preschoolers from low-income backgrounds" (p. 25), recommendations toward fixing unemployment and poverty, the structural prerequisites and outcomes of extreme capitalist markets, would get closer to the *causes* of "less functional knowledge for learning school mathematics" (p. 25). Rather than dealing with societal problems generally, which lead poor children to eat the worst foods on the market (the cheapest foods are those offered by fast-food chains), they offer "school lunches" and think of it as a true accomplishment. Getting rid of exploitation and poverty would mediate having to hand out school lunches in the first place.

The simple-mindedness of the report clearly is notable in statements such as those about how to deal with the lack in "foundational knowledge for learning school mathematics" that characterizes students from families with low-parental education levels, low incomes, and single parents. Thus, the report recommends "training" teachers in how to implement "a variety of promising instructional programs [that] have been developed to improve the mathematical knowledge of preschoolers and kindergarteners, especially those from at-risk backgrounds" (p. 25). There is much evidence that such programs do not eliminate or even marginally mediate school achievement differences and subsequent differential access to further schooling opportunities, jobs, and other desirable dimensions of society. Thus, in France with a generous pre-school daycare system, the production and reproduction of the cultural elite remains one of the outstanding features of the society. Not only does the father's education and level of job correlate with a child's educational prospects, but also there are also differential effects on the gender of the child (Bourdieu & Passeron, 1979). Single-minded cause–effect reasoning as advocated in the *Final Report* will not lead to changes, and there is (non-scientific?) research that already provides evidence of this.

Model of Cognition: Knowing and Learning

In the cultural-historical activity theoretic model outlined above, the three levels of an activity—motives, goals, conditions—are integrally related and cannot be reduced to one another or to some other simpler element. The activity in its entirety is the appropriate unit of analysis.

In contrast, the *Final Report* addresses cognition in terms of disconnected facets, which raises questions about *why* there is thinking and consciousness in the first place. Thus, for example, the Report states that "computational facility with whole number operations rests on the automatic recall of addition and related subtraction facts, and of multiplication and related division facts" (p. 26). The Report goes on note "that many contemporary U.S. children do not reach the point of fast

and efficient solving of single-digit addition, subtraction, multiplication, and division with whole numbers, much less fluent execution of more complex algorithms" (p. 26). And the Report concludes as a surprise that in the US, "many never gain such proficiency" (p. 26).

Here, the report makes statements about cognition generally and about the relationship between computational facility, on the one hand, and automatic recall of addition and related subtraction facts. This may be the case under certain conditions, for example, when research participants subjected to constraints that do not exist in other everyday situation outside of the psychological experiment; this also may be the case in school classrooms, which decontextualize in a similar way motives, goals, and conditions. But it certainly is not the case in out-of-school everyday contexts where the motives of the activity come to be the overarching leitmotiv that allows the general sense of actions to emerge as well as make the situation meaningful *as an everyday pursuit*. Thus, for example, supermarket shoppers perform very differently on ratio-and-proportion (fraction) requiring best-buy tasks if the situations are changed from their normal shopping *in* the supermarket, *in front of* the supermarket with selected products made available on a table, or on paper-and-pencil tests presenting equivalent problems (Lave, 1988). When persons shopping for groceries were asked while choosing certain products about the "best buys" available, many concerns entered the decision that had very little to do with ratio and proportion, and prices also were evaluated in terms of the shelf life of a product once opened—mitigating against a larger and less-pricy product that would spoil because the family could not consume it quick enough—personal taste, and others. When a range of products was taken from the shelf and presented on a special table in front of the supermarket, the conditions already had changed, where the framing already conditioned like a psychological task rather than one in which the person truly made a choice and, for this, had specific goals to be addressed. Not surprisingly, with changing conditions the (tacit, unconscious) operations enacted also change, just as activity theorists have postulated from the beginning (e.g., Leont'ev, 1978). Finally, the same shoppers' performance dropped from near perfect score in the market to about 50% correct on paper-and-pencil tasks asking for the best buy among to products of which nothing more was known than the price and weight. Again, the operations mobilized in the attempts to provide answers to the researchers' questions differed, conditioned by the different context in which goals were framed.

In contrast to the efficiency and competency on best-buy tasks in everyday out-of-school life, the shoppers did poorly on school-related fraction tasks. The *Advisory Panel* notes difficulties with such tasks among American students: "Difficulty with the learning of fractions is pervasive" (p. 28). The *Advisory Panel* also notes that algebra teachers rated "students as having very poor preparation in 'rational numbers and operations involving fractions and decimals'" (p. 28). If the *Advisory Panel* had taken into account the result of ethnographic work in workplaces, it might have begun to hypothesize that the school and the school-nature of the task that is responsible for producing the results it so deplores. Perhaps the difficulties with learning fractions are endemic to the schooling system guided by inappropriate theoretical framework rather than the result of faulty teaching or unwilling and incapable students? Do the statements about students learning difficulties and lack of knowledge not constitute an indictment of decades of research in the psychology or didactics of mathematics?

In a similar way, a significant number of research scientists did not correctly interpret graphs culled from the materials in an introductory course of their own discipline (Roth, 2003). Here again, the scientists did not mobilize basic graphing skills to be recalled and applied in and to the particular task. Many did not focus on the required graph feature, such as slope or height, and therefore made

the same kinds of "confusions" that high school students enact on traditional paper-and-pencil problems (Leinhardt, Zaslavsky, & Stein, 1990). On the other hand, when asked to read a graph directly related to the topic of their own research, then the scientists not only provided highly sophisticated answers that required the recall of simple facts, but they also mobilized their intimate understanding with the motive of activity (knowledge production versus technological application), the contextual particulars under which the research was conducted, strength and fallibility of the equipment and instrumentation employed in the data collection, and so on. That is, under given conditions, we can make any scientists look bad and exposing, like students, "poor knowledge of the core [. . .] concepts" (p. 26).

The authors of the *Final Report* complain that "[F]ew curricula in the United States provide sufficient practice to ensure fast and efficient solving of basic fact combinations and execution of standard algorithms" (p. 26). Again, the theoretical framework underlying the report treats "basic fact combinations" as elements to be deployed in concrete situation, recalled as declarative or procedural knowledge and applied in the situation at hand. The research in everyday contexts shows however that whatever basic skills are mobilized to solve the problematic issues arising for particular people in particular contexts is a function of the activity as a whole, with its affordances and constraints on action possibilities that emerge in a dialectical fashion. Thus, in everyday situations, it is not the application of a particular skill that is important but arriving at a resolution of the problem—which, when it comes to price comparisons, may be a quick look at the unit price that is frequently printed in almost invisible size at the bottom or in a corner of the shelf price tag.

There is a lot of research on mathematics in a variety of workplaces—among street vendors and bookies, scientists, technicians, dairy factory workers, supermarket shoppers—that underscores the "situated" nature of cognition. By this sometimes-overused term situated I mean here that productive activity always is oriented toward a motive, which is realized in the formation of goals and by means of goal-directed actions all of which are mediated by the activity as a whole. Each action and each operation that contributes to realizing one or more actions is made possible, constrained, and made intelligible by the motive that provides the guiding light for anything that realizes the activity. Any goal-directed action takes into account and is mediated by the available tools, the forms of division of labor available, the community for which the outcomes are produced, and the rules that are to be adhered relative to the different pairwise relation within activity (Figure 1).

This research on knowing and learning mathematics in the everyday life outside of schools also raises questions about the possibility of transfer, thereby questioning the very position that the *Advisory Panel* takes on this issue. Thus, whereas the *Advisory Panel* notes that "people's ability to make links between related domains is limited" (p. 30), it continues to assume that the transfer of "basic skills—the fast, accurate, and effortless processing of content information" (p. 30) is possible and should be fostered. I do agree that we need to support students' *education* (which is not the same as schooling), but the essential question about the role of the collective motive of activity and the personally relevant goals that make for the conditions that determine the mobilization of any unconscious operation ("basic skill") is not posed. More so, cultural-historical activity theory already has a model for the transformation of conscious actions into unconscious operations and the reverse transformation during the critical analysis of actions in conditions of breakdown—i.e., when a familiar way of doing things fails.

"The Sociocultural Perspective of Vygotsky"

The *Final Report* notes that the "sociocultural perspective of Vygotsky also has been influential in education" (p. 30). However, the *Final Report* immediately misrepresents what the sociocultural perspective is about in suggesting that the theory "characterizes learning as a social induction process through which learners become increasingly independent through the tutelage of more knowledgeable peers and adults" (p. 30). The Report concludes in stating that the "utility in mathematics classrooms and mathematics curricula remains to be scientifically tested" (p. 30).

First, there has been more than one study from a Vygotskian perspective, and this existing work has "invariably produced descriptions stressing the ways in developmentally primary and expanded forms of cognition are transformed into new, abbreviated, and more complicated forms" (Davydov & Andronov, 1981, p. 4). Here, we find precisely the kinds of answers that the *Final Report* suggests that little is known about—e.g., to answer how students achieve "fast, accurate, and effortless" automaticity of operations to produce the "more complex aspects of problem solving" (NMAP, p. 30). More so, cultural-historical activity theory has a model what happens to "abbreviated operations" that seem to disappear into automaticity: in fact, they "do not simply disappear. They take on a status in which they are treated as if they had been performed and are hence being 'kept in mind'" (Davydov & Andronov, 1981, p. 5). There is, at least in the Russian literature—and contrary to what the *Final Report* claims ("students must eventually transition from concrete [hands-on] or visual representations to internalized abstract representations. The crucial steps in making such transitions *are not clearly understood at present* and need to be a focus of learning and curriculum research" [NMAP, p. 29])—ample evidence "to describe in detail the external features of abbreviated (ideal, cognitive) acts in contrast to their material, object prototypes" (pp. 5–6). Thus, making reference to some examples concerning addition, the authors summarize that the

> ideal act of addition, based on an objective symbol, takes place in the form of a real <u>motor</u> act (the sweeping movement of the hand as the cardinal number is pronounced in a drawl). Then, in a compact reduced form, this movement itself becomes the symbol of the number, manifested in unique gestures and in the accented pronunciation of the cardinal numbers. (p. 25, original underline)

In this way, internalization is but the other coin of externalization, because "the ideal is immediately realized in a symbol and through a symbol, i.e., through the external, sensuously perceived, visual or audible body of a word" (Il'enkov, 1977, p. 266). In the study of the emergence and transformation of hand movements as a basic component of ideal acts (e.g., mental addition), "initially the symbol was the hand's movements together with articulation, and subsequently only the reduced articulation as a basic component of the word in designating the number" (Davydov & Andronov, 1981, p. 25).

It might have been excusable in the 1970s and early 1980s not to know about the developments of Vygotskian theory and educational practices in mathematics. With the wide availability of original and translated research from the former USSR, today it is inexcusable that the *Advisory Panel* is unaware of the considerable work concerning the development of routine skills in the pursuit of meaningful activity as made available, for example, in *Types of Generalization in Instruction* (Davydov, 1990) published by the National Council of Teachers of Mathematics.

With respect to the development of algorithmic knowledge and skills, Vygotskian theory has considerable explanatory power and supportive evidence. There are curricula not only in Russia (now and during the Soviet Union) but also in the US that implement educational strategies for developing algorithmic knowledge that does take into account the conditions in which these skills are mobilized. *Conditional learning* is an outstanding feature of mathematics curricula based on cultural-historical activity theory. Thus, for example, children are encouraged to solve addition and subtraction tasks in a variety of ways and are then asked to select the ways in which the calculations are done easiest (Schmittau, 12004). The children taught according to Vygotskian principles "find it advantageous to add numbers in column form, applying standard algorithm for multi-digit addition (a method which they themselves have derived)" (p. 25), leading them to employ rounding methods when asked which manner of solving the problems constitute the "easiest way." Similarly, the children develop rounding algorithms when asked to mentally solve orally presented addition tasks such as 242 + 37 + 118. That is, when the task involves "finding the easiest way" as motive for identifying among one's different ways of going about a task, then children begin to develop algorithmic knowledge that allows them to flexibly adapt to the demands of the tasks. Modifying and providing for a motive of activity is precisely the kind of work that cultural-historical activity theory has spawned in the USSR and Russia.

From a Vygotskian perspective, rather than teaching symbols that absorb the genesis of the concept they denote, a genetic analysis is required for the design of appropriate curriculum. A requirement for the curriculum is an understanding of how, ontogenetically, semiotic forms absorb into themselves the experience of objective acts, which they represent in abbreviated form, a process that I have denoted elsewhere as the emergence of signifying moments of a situation that denote the situation as a whole in a (metonymic, synecdochical) pars-pro-toto fashion (Roth, 2004). The construction of curriculum requires the historical study of ontogenetic development because "the basic structure of the concrete, objective acts cannot be reconstructed, if what is given is only their semiotic form" (Davydov, 1988, p. 179). Thus, "It is necessary to follow the whole 'history' of different solutions of one and the same problem, in order to see in the abbreviated forms of thought its original course" (p. 179). Once this development is seen and experienced, we come to understand the ways in which automatization emerges from purposeful activity as a whole because "to see in the abbreviated forms of thought its original course . . . one should uncover the laws and rules of this abbreviating and then to 'recapitulate' the full structure of the processes of thought being analyzed" (p. 179).

According to cultural-historical activity theory—which generations of Soviet psychologists have extensively developed and used to study knowing and learning since Vygotsky initially framed it—the real object-oriented activity of collaborating individuals lies at the heart of a psychological understanding of all mental functions. These mental functions are ideal forms of real human relations, and, in a truly dialectical fashion, these ideal forms are realized in the "material and spiritual products that *objectify* the internal psychological conditions of their realizations" (p. 180). Every psychological structure underlying a mathematical concept is the result of an activity, and students develop the relevant and necessary structure by participating in a relationship with material and social reality that corresponds to the concept. It is this reality, social and material, that comes to be synecdochically denoted by the mathematical sign. That is, by participating in activity with internal structure such as shown in Figure 1, the appropriate structures of consciousness that accompanies and is prerequisite of the activity also develop. This perspective has consequences in the way we understand mental activity and its development:

> If the forms of mental activity—concepts in particular—are regarded as the idealization of certain modes of concrete activity, and if in the products of activity one finds the conditions of their social actualization, that determine the future behavior of man, then such attitudes lead inevitably to discarding of the naturalist conception of acquisition and, in the final analysis, to overcoming of passive sensualism, conceptualism, and associationism. Thereby it also becomes clear that the absolutization of formal generalization is unjustifiable. An alternative to it is the generalization of a *contentful* character. (p. 180)

The development of procedural skills and abstraction from concrete material action is not achieved by leaving behind the concrete and enter the world of the ideal. Rather, the ideal and material are relevant to the development and enactment of higher order concepts because they are *never* separated in the first place but exist in and as of a continual exchange between the continually developing processes of thinking and, at a minimum, speaking (Vygotsky, 1986). Our more recent work shows that not only thinking and speaking but more so thinking and communicating generally are in a continually developing dialectical relationship denoted by the term "meaning."

I now briefly address the conclusion that the *Advisory Panel* arrived at, that is, that the utility of a Vygotskian sociocultural perspective remains to be "scientifically" tested. There is nothing in the *Final Report* that would provide evidence for stating that the *Advisory Panel* was aware of the research on Vygotskian curricula in mathematics education and the extensive amount of "scientific" research that has been conducted. There is nothing that would support the claim that the *Advisory Panel* actually understood Vygotsky's cultural-historical activity theory or that it has informed the panel in making sense of mathematical learning and the design of appropriate curriculum. As pointed out here, given the way in which cultural-historical activity theory articulates consciousness and the material world, it makes no sense to make a distinction between the material (also the objectivity of social relations) and the ideal (mental), the inside/interior and the outside/exterior of the subject. All human activity is understood as a production that is mediated by the social and material relations that embed and contextualize the individual subject, who, in its production, also reproduces the activity itself. Performance is never understood independent of the material, social, and contextual particulars—as illustrated in the notion of *conditional learning,* which allows learners to develop efficient and fast algorithms as they reflect upon alternative actions so that component operations come to achieve automaticity and the actions themselves are "demoted" to become conditioned operations.

Sociality, Motivation, and Emotion: More than (External) Influences

In the previous section, I raise doubt about the *Advisory Panel*'s understanding of Vygotsky's (1986) cultural-historical approach to cognition. This is especially highlighted in the manner in which the *Final Report* separates mathematical cognition from sociality, motivation, and emotion (affect). I write separate, because the *Final Report* denotes sociality, motivation, and affect as factors; and factors are never more than *external* aspects of a phenomenon (Hegel, 1806/1977). They cannot enter the dynamic of the system itself or the internal dynamic of the concept at work. Vygotsky operated differently, theorizing systems as a irreducible wholes, where the "factors" are thought of as mutually constitutive and subordinate moments. Not only has Vygotsky advocated unit analysis over analyses that reduce complex systems into elements that are said to constitute the system, which can be reconstructed by a combination of these elements, which do not loose their basic properties in the process. Whereas Vygotsky was mostly concerned with the role of the word in communication, leading him to identify word meaning as the continually developing process that

mediates the exchange between thinking and speaking, those following him articulated the unit as activity. None of the structures identifiable within an activity—subject and object of production, means of production (tools, instruments), recipient of production (community), and the reigning forms of sociality (division of labor)—can be understood on its own but only as mediated by activity as a whole and by the other moments specifically. Consciousness, realized in the developing relation of developing speaking and thinking, always "is practical consciousness-for others and, consequently, consciousness-for-myself" (p. 256). Thus, consciousness, as the word, is nothing individual, subjective, and singular but always and already "becomes a reality for two" (p. 256). Sociality of human beings is not an external influence enhancing or decreasing the efficiency of thinking and learning. Rather, it is the very condition for thought. In speaking, even the uninstructed student is and expresses sociality that makes anything like a lesson possible in the first place. In speaking, gesturing, using symbols, whether with others or for oneself in private, the inherently societal nature of thinking expresses itself. It is not that students express *their* thinking in words and symbols, but the very fact of using words and symbols is an expression of the *sociality* of thought. Mind is as much in society as society is in the mind: Mind and society are irreducibly the same (Bakhtine/Volochinov, 1977; Vygotsky, 1978). Recent neuroscientific research nicely supports what early on was but a hypothesis and presupposition. Thus, any action makes sense because its production requires recognition as an action of others, and the recognition of an action on the part of others requires competency in executing the action (Gallese, 2003). This includes such phenomena as empathy, which is the largely human capacity to feel what another person is feeling.

Recent neuroscientific work shows that affect generally and emotion specifically, too, cannot be separated from cognition: Affect is the very condition for thought (Damasio, 2000). In considering affect as an influence rather than as something constitutive of thinking, the *Advisory Panel* exhibits a limited understanding and conceptualization of thinking generally and enculturation more specifically. More importantly, the *Advisory Panel* excludes for itself the very possibility to understand the nature of thinking, which is based on the close interrelation of thought and affect. Their separation has been, as Vygotsky (1986) noted, "a major weakness of traditional psychology" (p. 10). Vygotsky wrote this in the early 1930s, and there is little that appears to have been learned since; and the hypothesis lies near that the separation of thought and affect in the design of mathematics curriculum is one of the great hindrances to make mathematics generally and algebra more specifically core competencies that students *want to* and do develop. The separation of the study of affect and thought is a weakness, "since it makes the thought process appear as an autonomous flow of 'thoughts thinking themselves,' segregated from the fullness of life, from the personal needs and interests, the inclinations and impulses, of the thinker" (p. 10). That is, affect is the regulating element that underlies the motive of activity, which orients all forms of activity, and which is regulated by the emotional volitional moments of being (Bakhtin, 1993).

With the role of collective motives as orienting moments of activity, and the emotional-volitional moment that serves in the evaluation of actions that realize the activity in concrete form, there is no longer a need for the concept of motivation—which, in any case, is a pseudo-scientific concept because nothing more than an everyday concept that has been operationalized in the service of a bourgeois psychology serving the interests of an oppressive class (Holzkamp-Osterkamp, 1976). Especially the research in a Western offshoot of cultural-historical activity theory, the Berlin school of social psychology denoted by the name *Critical Psychology*, exhibited the weaknesses of the traditional motivation concept, which they showed to be an instrument in the hands of capitalists and teachers to make others do what they do not normally want or intend to do. When conceptualized within cultural-historical activity theory, however, motivation simply is an expression

of the fact that the acting subject has taken up the collective motive and, to expand his or her action possibilities and room to maneuver, engages in expansive learning. Being able to have more control over one's life conditions and being able to do more *inherently* belong to the motive of activity and therefore do not have to be theorized as outside factors that impact and affect cognition either positively (as belief in one's ability) or negatively (leading to performance-decreasing anxiety). That is, in the cultural-historical activity theoretic model, the emotional and motivational "factors" of mathematical achievement are already inherent and therefore have to be taken into account for to understand all levels of performance. The issue, for me, therefore is not the "development of promising interventions for reducing serious mathematics anxiety" (p. 31) but a change in the very activity that leads to the ontogenetic development of mathematical practice.

In thinking about and asking for the development of interventions that "reduce serious mathematics anxiety," the *Advisory Panel* does what critical psychologists have articulated as the goal of traditional psychology, which is but a means of the bourgeoisie in the control of the working class, its productive means and exploited resource. Thus, rather than considering changing school conditions, task forms, and other aspects of schooling, the *Advisory Panel* recommends the development of interventions that constitute outside influences and forces on the individual rather than creating conditions where the anxiety problem does not even surface. Again, the approach is one of fixing the symptoms, here mathematics anxiety, rather than dealing with the causes of a phenomenon. That the *Advisory Panel* has interests of the type indicated, that is, the modification of others to serve their and their employers goals also can be seen in the way the *Advisory Panel* writes about changing the beliefs of others.

The problem in the mistaken approach to questions of affect in the *Final Report* derives, so Vygotsky, from the failure to recognize it as an internal moment of activity generally and thought particularly. Thus, only when thought and affect are theorized as internally connected can we understand the *mutually constitutive* nature and mediation of thought and affect. "The old approach precludes any fruitful study of the reverse process, the influence of thought on affect and volition" and, thereby, the "door is closed on the issue of the causation and origin of our thoughts, since deterministic analysis would require clarification of the motive forces that direct thought into this or that channel" (p. 10).

Coda

In my reading, the *Final Report* constitutes the worst imaginable scenario, one that plays into the hands of simplistic-slogan-using politicians interested more in election and reelection than in the change of existing, inequitable societal conditions that are produced and reproduced by current forms of schooling. The *Final Report* constitutes a piecemeal approach, presenting what we know about knowing and learning in mathematics in a highly selective and disconnected manner, therefore realizing itself as nothing more than selected "knowledge in pieces." In its excessive, president G. W. Bush-rhetoric-supporting and evidence-eliminating discourse on the nature of true scientific research, the *Advisory Panel* fails to provide directions for truly democratic and democracy-enhancing ways of changing mathematics education. Instead, the *Final Report* plays into the hands of those who advocate *training*—e.g., basic skills, automaticity, rote, and routine learning—over *education*, which constitutes an emancipatory and expansive process of learning that people engage in because it enhances their control and room to maneuver.

A way out of the impasse that the *Final Report* leads us into is one that Vygotsky already recommended for better understanding cognition, knowing, and learning: Unit analysis. This form of analysis is advantageous because

> it demonstrates the existence of a dynamic system of meaning in which the affective and the intellectual unite. It shows that every idea contains a transmuted affective attitude toward the bit of reality to which it refers. It further permits us to trace the path from a person's needs and impulses to the specific direction taken by his thoughts, and the reverse path from his thoughts to his behavior and activity. (pp. 10–11)

In sum, I suggest that the *Final Report* provides us with a rather limited and limiting perspective of what it takes to transform mathematics education. I would say that it is its own utility that ought to be scientifically tested—if it were not for the undesirable consequence that we would actually have to implement its program at least partially to subject it to such tests. From my perspective, the recommendations are more like a curse. I do not wish on anyone the experience and consequences of educational practice that the *Final Report* calls for. As I am writing these lines, I wish this were a *final* report, one never to be repeated, one that we could relegate to oblivion, rotting in the musty cellars of the U.S. Department of Education. I do hope for the nation as a whole and for the students who would be subject to the changes brought about on the basis of the *Final Report*'s recommendation that the winds of change bring about a change not only in the political leadership but also in the national position on education.

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