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UNDERGRADUATE STUDENT DIFFICULTIES WITH INDEPENDENT AND MUTUALLY EXCLUSIVE¹ EVENTS CONCEPTS

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Abstract: The concepts of disjunctive events and independent events are didactical ideas that are used widely in the classroom. Previous observations of attitudes in assessments given to students at university level who attended the introductory Statistics course helped to detect the confusion between disjunctive and independent events, and indicate the spontaneous ideas that students tend to elaborate about both concepts in different situations in which these appear. However the didactical relation between these ideas and their formal definitions is not known in detail. In this work, we analyze students' misconceptions, their persistence, and the process by which the student confronts his misconceptions by applying theoretical concepts. The aim is to improve the teaching of these topics.

Keywords: independent events; mutually exclusive events; probability; statistics education; undergraduate mathematics education

1. Introduction

Statistical education is not very well researched in Argentina. However Statistics is a science which gets more emphasis as we move from basic education to mathematics education in post-graduate levels onto career paths.

Therefore, in education we must ask ourselves: What happens with knowledge in Statistics?

The problem starts from reality and for the student it is a real problem to associate reality with formal concepts. Ernesto Sanchez says: in education it is good to ask in which conditions and how an individual person changes a conception, a belief, an intuition or a spontaneous idea about a pre-determined situation, by virtue of using a scientific instrument.

This work is based on the analysis of answers to common problems encountered by university students going into mathematics and social science careers.

This study, carried out with students from Statistics I, a course for majors in Accountancy, is an analysis to the responses to a problem where there is the concept of mutually exclusive and independent events and the confusion they have about these concepts.

According to Sanchez E. (who wrote Ph.D. thesis about independent events³) the problem starts with:

¹ The term disjunctive is interchangeably used with the words mutually exclusive

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- the belief that independent and disjunctive events are the same;
- confusion between independent events and independent experiences.

Besides, it is understood that independence is only quantitatively proven by the product rule. These concepts are simple when they are defined. However it has been proven through interviews that the confusion persists in many university students attending Statistics I. The phenomenon appears in students with different mathematical backgrounds.

Are there techniques of teaching and learning good enough to take into account the spontaneous concepts of the probability notions while developing their formal knowledge?

Some studies about attitudes and responses in exams indicate that students use intuitive ideas to analyze independent events and mutually exclusive events in different situations where these notions play a role. But the relation between these intuitive concepts and the formal definitions is not known, at least in Argentina at the tertiary level.

2. Research Problem

A first course in Statistics typically covers the minimal contents required for a basic understanding of statistics in day to day use. Most text books include probability topics for introducing the following concepts: events, probability definition, conditional probability and independence, random variables and probability distributions. There are very few didactical studies on such topics or proposals on the teaching and learning of such topics. This study seeks to understand students' confusions between mutually exclusive events and independent ones, with the goal of providing some recommendations to deal with this confusion and implementation in education.

3. Research topic

Historically according to the renowned probabilist Kolmogorov the concept of independence represents a crucial concept which provides probability theory numerous pedagogical illustrations. I will focus on the concepts of: mutually exclusive events and independent ones, because of general reasons such as:

Firstly, it is common the confusion which associates disjunctive to independent, and it is well known that only if one of them is empty both are confirmed, in the context of finite sample spaces.

Secondly independence is confused with individual experiences without an explanation about the difference between both ideas.

As Sanchez (1996) found in his thesis, this confusion is due to causality.

³ Sánchez, E. (1996) *Conceptos teóricos e ideas espontáneas sobre la noción de independencia estocástica en profesores de bachillerato : Un estudio de casos*. Tesis de Doctorado, Departamento de Matemática Educativa, Cinvestav-IPN, México.

3.1 General questions

The problem starts in probability which has always been considered, according to teachers experience, a topic of difficulty for most students. Even though the independent and mutually exclusive events concepts are apparently simple, people spontaneous ideas give rise to wrong answers. These misconceptions have become interesting for researchers not only in psychology but also in didactics. Therefore among the various questions posed by researchers, I have chosen the following:

What are the relations between subjective or intuitive conceptions and those which are transmitted in the classroom, and which compose the formal knowledge of probability?

Are there optimal teaching and learning techniques which consider the spontaneous conceptions that individuals hold about probability ideas while they develop their formal knowledge in courses?

In fact previous observations of attitudes have shown some spontaneous ideas which tend to elaborate about independent and mutually exclusive events in different situations where this idea is involved, but it is unknown what is the relation between these intuitive conceptions and the formal definitions they encounter? Sánchez also poses the following questions .

- What happens with an individual's misconceptions about independence in determined situations, when discussing independent and mutually exclusive event definitions in a probability course?
- What is the process via which an individual can confront their wrong conceptions?

4. Some Prior Findings

In a probability survey by Sánchez administered to 44 mathematics professors who had some probability and statistics knowledge , they were asked to answer the following question⁴:

A card from an American deck is taken by chance: It is "A" the event "it was taken a clover" and B "it was taken a queen". Are A and B independent events? Justify

It was expected that they would calculate the probabilities of A and B and $A \cap B$ and then check the product rule by performing $P(A \cap B) = P(A) \cdot P(B)$.

However, this simple solution was found by only 4 professors. Standard answers were:

Answer 1: Independence or mutually exclusive events are the same. They are not independent because there is a clover queen.

Answer II: Solution depends on an application of a succession, namely

If a card to verify the event A is taken and it is placed in the card deck to verify event B, so A and B are independent.

If it is taken to verify A and it is not placed again so they are dependent

⁴ Editorial note: The language of translated exercises, and responses have been left in the native (English) formulation of the author. The mathematical context allows for no misunderstandings.

5. Hypothesis to explain the mistakes

The mistakes in the answers noted above could come from various factors:

- They forgot independence concepts
- Terminology confusion
- We must consider that the problems which involve independent events notion are immersed in a wider range of problems, which leads us to the area that gives the most general idea of stochastic.
- Epistemological problems about the meaning

6. History and epistemology

The concept of independence emerges in the analysis of hazard games “without replacement” given by De Moivre (1718-1756) and by Bayes (1763). Before them Bernoulli used this concept to formulate his theory without realizing it.

There were no changes in the intuitive concept of independence with the improvement made by Laplace and Moivre. The concept of independence was understood only in the context of independent experiences as is shown with the definitions of classical authors such as De Moivre (1756):

“Two events are independent when there is no connection between them and what happens in one of them does not occur in the other one.”

“Two events are dependent when they are connected in such a way that the probability that one occurs is altered by the occurrence of the other”

Laplace does not define in any explicit way independent events and their properties. In this period drawing with and without replacement in successive trials was identified with independent and dependent events respectively.

At present numerous difficulties arise from these classical authors' concepts.

Some theorists like von Mises reject the formal definition of independence. He considers that in the axiomatic theory of Kolmogorov there are events that are independent but are not seen as independent one of each other, in the intuitive sense that “they do not influence each other”.

“When two characters are considered to influence each other or not, it is given a notion of independence. Nevertheless, a definition based on the multiplication rule is no more than the weak generalization of a concept full of meaning”.

This problem of the inversion of content and the mathematics definition plays an important role in the teaching process. In some books the deduction of the independence formula appears as a consequence of conditional probabilities.

These difficulties appear in the historical development of the concept. During the eighteenth century, the task was to legitimize and delimit the object of mathematical studies; most diverse methods were accepted to analyze this object. In the nineteenth century, the relation was inverted. The object became arbitrary, and the task was to confirm the methods and define strict procedures to allow the abstraction of the objects and so, an extension of the applications.

Feller (1983) comments:

Generally the correct intuition that certain events are stochastically independent is felt, because if it is not like that, the probability model would be absurd. Nevertheless [...] there exist situations in which the stochastic independence is discovered just from calculus. (p. 137)

Steinbring (1986) analyzes the historical development of stochastic independence from an epistemological perspective, to find elements for a didactic perspective. In the historical development there is an inversion of content of the concept and its mathematics definition.

- Firstly, there is an association of concrete representations of dependency with real facts.
- Secondly, the concepts have been defined formally in mathematics by the multiplication rule.

These statements are usually not connected properly. Consequently, it may produce a confusion about the concepts of independency (or dependency).

7. Theoretical framework

Some studies on student's misconceptions have shown that:

1-It seems that what characterizes students' misconceptions is its steadiness in time, its relative internal coherence and its acceptance among the larger community of students.

2-Ideas are not by chance but in relation with what they know and their thinking characteristics and abilities; that is, the idea that a child has, implies a determined knowledge about how things are, and happen, and a determined intellectual functioning, a way of explaining not only a particular concept but also others in relation with it.

3- The number of different conceptions that classroom students depict about a fact or a situation is not limited, whereas several common patterns among them are found.

Beliefs and conceptions are intuitively acquired by students from their experience of interaction with reality and with ideas, as a consequence of the learning processes which they are formally exposed. Cornu (1991) designates spontaneous conceptions of a mathematical idea to the group of intuitions, images and prior knowledge which are made in the individual person from their daily experience and from the semantic contexts in which these ideas arise.

According to Cornu these spontaneous conceptions are made before the formal learning processes, so it does not mean that they disappear after those processes, but are usually mixed in with the new ones. They are modified and they end up as hybrids. Impediments appear in the phylogenesis and ontogenesis of concepts (Sriraman & Törner, 2008) so it can be noticed in the historical developments of those concepts, and also in the individual construction processes of those.

8. Methodology

8.1 Economic sciences career

An exam was developed on the topics of probability and random variables. In the first exercise there were concepts of probability organized in items. One of them included the concepts of mutually exclusive and independent events. The students were asked if the proposition was false or true and to justify their answer. The statement was false.

The analyzed exercise is the following:

Let (S, P) be a probability space, A and B events in S so that $P(A) > 0$ and $P(B) > 0$. Decide whether the following statement is true or false. If it is true, justify your conclusion; if it is false, state the right expression.

“If A and B are mutually disjunctive, the probability of at least one of them occurring is: $P(A).P(B)$ ”.

8.1.1 Responses

Among the responses, the student is given the total mark for the exercise if he or she answers “false: and writes the correct expression: that is, the following:

“The probability that at least one of them occurs, when the events are disjunctive, is:

$$P(A \cup B) = P(A) + P(B).”$$

Table 1 shows the results of the 97 students.

This apparent easy response was answered correctly by 14 students, of whom 10 passed. As it is seen, 12 students do not answer, of whom only 4 passed. Incorrect, with response true, 14. Responses F but with wrong justification or without justification and had no total mark were 57, only 19 students passed.

Our target is to analyze the false answers which were wrongly justified.

Table 1

student	correct	no answer	F no Justified	F Justified wrong	F Justified regular	True Justified	True no Justified	Total
passed	10	4	1	14	5	3	0	37
failed	4	8	6	26	5	7	4	60
Total	14	12	7	40	10	10	4	97

8.1.2 Analysis of responses of the students

An analysis was made of the responses of each student, particularly of the 40 that considered F and justified incorrectly. Nearly half of them (17) justified it in this way:

$A \cap B = \emptyset \Rightarrow$ the probability of occurrences is given by $P(A).P(B)$; that is

$$P(A \cap B) = P(A).P(B).$$

This was the most repeated mistake in the justifications.

The confusion of the concepts followed the patterns proposed by Sanchez. The question was not direct, which can produce misunderstanding. This shows that the students make mistakes systematically because they do not have a clear concept of both notions.

A second type of response that was presented was:

Si A y B son me $A \cap B = \emptyset$ luego $P(A) = 1 - P(B)$ ó $P(B) = 1 - P(A)$

entonces $P(A) \cdot P(B) = P(A \cap B) = \emptyset$

If A and B are disjunctive $A \cap B = \emptyset$ then $P(A) = 1 - P(B)$ or $P(B) = 1 - P(A)$; then

$P(A) \cdot P(B) = P(A \cap B) = \emptyset$

The third answer that also was present with several students was:

$P(S) = P(A) + P(B)$.

We can say that the second and third responses are associated because the students consider that the sample space is formed by two sets. They do a Venn diagram including these sets in S. According to Duval's commonly known finding that there is a problem in the translation from graphs to symbols. Students represent one thing and write another.

This problem of symbol representations is repeated among the answers that we can associate because of the wrong symbolic representation. Half of the students wrote $P(A \cap B) = \emptyset$.

Other students wrote $P(A) \cup P(B)$, $A \cap B = 0$

They tend to misunderstand symbols. On the one hand they considerer union of probabilities. On the other hand they associate the empty set with the number 0 (zero). They equate the probability of intersection of disjunctive events to the empty set, and they equate the intersection set to zero. It is recurrent in students from all the careers.

8.2 Political Science case

The following conceptions were taken from probability surveys which were done by students in the humanities who are attending statistics.

Example: A behaviour test in a large number of drug addicts indicated that after their treatment re-incidence (relapse) occurred within the two years after treatment. This relapse could depend on the socio-economical level they belong to, as shown in the following contingency chart:

		Condition within the two years after the treatment	
		Reincidence(R)	No reincidence(NR)
Socio-Economical level	Superior (S)	10	50
	Inferior (I)	30	10

a) Which is the probability that he gets back and belongs to a superior level?

b) Whis is the probability that he belongs to I socio-economical level or not get back?

- c) Are the events R and S independent? Justify.
- d) If the chosen interviewed belongs to the superior socio-economical level, which is the probability he gets back?
- e) Are the events R and S mutually exclusive? Justify with the definition.

8.2.1 Analysis of responses of the students

These were the answers given by some students to questions c and e :

- 1) c) *No because two events are independent ,because when S happens it does not modify that R happens*
e) *They are not mutually exclusive because if S event happens it can not happen event R*
 $S \cap R = \emptyset$
- 2) c) *No because one events depends on the other one.*
e) *No because they have common elements*
- 3) c) *Justify with $P(S \cap R) \neq P(S) \cdot P(R)$*
e) *They are not mutually exclusive because they are different $S \neq R$*
- 4) c) *They are not independent because they are not the same*
e) *They are because they do not happen at the same time*
- 5) c) *No because the intersection is not empty*
e) *R y S are mutually exclusive because they can happen simultaneously*
 $S \cap R = \emptyset$
- 6) c) *They are not independent because the rule is not followed $P(S \cap R) = P(S) \cdot P(R)$*
e) *They are mutually exclusive because there is intersection between R and S*

Among the 54 students only 8 answered the items c) and e) properly well and used the form.

8.2.2 Observations:

- It was noticed that most of the students confused the product rule with the addition one trying to show the independence .
- Others generalized the product rule considering that the events are independent
 $P(A \cap B) = P(A) \cdot P(B)$
- It is curious that in any case they used the conditional $P(A/B) = P(A)$ to prove the independence.
- In the case of justifying mutually exclusive events it was detected the mistake
 $P(A \cap B) = \emptyset$

9. Discussion

The difficulty of the subject is depicted in students' answers. It was present not only in students coming from careers with a better background in mathematics (like Economics, Business, etc.) but also in those like Politics, Sociology, etc. It could be thought that students with some mathematical knowledge are less confused, however because of the concept complication it is not always the case. There is again a symbol and meaning problem.

10. Conclusions

The concepts of disjunctive events and independency persist in the students in a mistaken way. In a way, these come from games of chance but they have a more complex relationship in probability calculus. From the historic studies with De Moivre (1756) a wrong concept of independent events may be inferred, if it is not analyzed exhaustively: “*Two events are independent when they have no connection to each other and what happens to one does not affect the occurrence of the other.*”

When we say they have *no connection*, we are talking improperly; this persists at present. The difficulty in the case of independence is to place the concepts in opposition. On the one hand, there is a theoretical mathematical definition. On the other hand, there are numerous intuitive representations. The symbolic representation associated with the graph presents difficulties too. Although the students know the formal symbolic definition of each concept separately, in exercises such as the ones we have analyzed, they cannot distinguish one from the other. Teachers must be conscious that the idea of independence has a meaning only in a probability context while the one of disjunctive events may be considered with no knowledge of this.

Endnote:

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