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Abstract: The research discussed in this article comes from an ongoing multifaceted program for the teaching and learning of early statistical reasoning in Cyprus. The initial stage of the program was concerned with the design of a line of instructional materials for the development of statistical reasoning. Central to this design was the functional integration with existing core curricular ideas of the recently developed dynamic statistics software Tinkerplots@, which provides young students with the opportunity to model and investigate real world problems of statistics. Next, professional development seminars for the teaching of statistics with the use of Tinkerplots® were designed and organized. The article discusses insights gained from the professional development seminars regarding the ways in which computer visualization tools can enhance teachers' content and pedagogical knowledge of statistics and how this, in turn, might impact student learning.

Keywords: Cyprus**;** elementary education**;** dynamic statistics software; professional development; statistical reasoning; technology; Tinkerplots; visualization

1. INTRODUCTION

New values and competencies are necessary for survival and prosperity in the rapidly changing world where technological innovations have made redundant many skills of the past (Ghosh, 1997). The Lisbon European Council of March 2000 placed the development of a knowledge-based society at the top of the Union's policy agenda, considering it to be the key to the long-term competitiveness and personal aspirations of its citizens. Statistics education has a

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crucial role to play in this regard. In a world where the ability to analyze, interpret and communicate information from data are skills needed for daily life and effective citizenship, statistical concepts are occupying an increasingly important role in mathematics curricula. Statistics education is becoming the focus of reformers in mathematics education as a vital aspect of the education of citizens in democratic societies (National Council of Teachers of Mathematics [NCTM], 2000).

Despite the larger place for statistics in school and university curricula, the research literature indicates that people continue to have poor statistical reasoning even after having formally studied the subject. Most college-level students and adults have little understanding of data beyond the simple – and often misleading – bar-charts and pie-charts encountered in the media (Rubin, 2002), and exhibit a strong tendency to attribute deterministic explanations to situations involving chance (e.g. Hirsch & O' Donnell, 2001).While university level statistics instruction can indeed be successful in helping students improve their stochastical reasoning (e.g. Meletiou-Mavrotheris & Lee, 2002), poor intuitions and biases acquired early on can be extremely difficult to change (Fischbein, 1975). It is now widely recognized by leaders in mathematics education that the foundations for statistical reasoning should be built in the earliest years of schooling rather than being reserved for high school or university studies (NCTM, 2000).

Statistics has already been established as a vital part of the K-12 mathematics curriculum in many countries. However, instruction of statistical concepts is, similarly to the college level, still highly influenced by the formalist mathematical tradition. Deep-rooted beliefs about the nature of mathematics "as a subject of deterministic and hierarchically-structured knowledge" (Makar & Confrey, 2003) are imported into statistics, affecting instructional approaches and curricula and acting as a barrier to the kind of instruction that would provide students with the skills necessary to recognize and intelligently deal with uncertainty and variability. Intuition and mindset about data and variation are systematically ignored in mathematics classroom (Makar & Confrey, 2003). There is over-emphasis on center criteria and a tendency to underestimate the effect of variation in real world settings (Meletiou-Mavrotheris & Stylianou, 2003a). This tendency is related to the emphasis of the traditional mathematics curriculum on determinism and its orientation towards exact numbers. Since centers are often used to predict what *will* happen in the future, or to compare two different groups, the incorporation of variation into the prediction would confound people's ability to make clean predictions or comparisons (Shaughnessy, 1997). The formalist tradition prepares students to search for the one and only correct answer to a problem – a condition that can easily be satisfied by finding measures of center such as the mean and the median. Variation though rarely involves a "clean" numerical response. Standard deviation, the measure of variation on which statistics instruction over-relies, is computationally messy and difficult for both teachers and curriculum developers to motivate to students as a good choice for measuring spread.

One of the most important factors in any educational change is the change in teaching practices. The direct relationship between improving the quality of teaching and improving students' learning in mathematics is a common thread emerging from educational research (Stigler & Hiebert 1999). For it is what a teacher knows and can do that influences how she or he organizes and conducts lessons, and it is the nature of these lessons that ultimately determines what students learn and how they learn it. Statistics has been introduced into mainstream mathematics curricula without adequate attention paid to teachers' professional development. There is substantial evidence of poor understanding and insufficient preparation to teach statistical concepts among both pre-service and practicing teachers (Carnel, 1997; Begg &

Edward, 1999). As Lajoie and Romberg (1998) point out, statistics may be as new a topic for teachers as for children. Most teachers are likely to have a weak understanding of the statistical concepts they are expected to teach and relatively deterministic epistemological sets, often sharing the same misconceptions regarding the stochastic as their students (Carnel, 1997). As a result, they tend to focus their instruction on the procedural aspects of probability and statistics, and not on conceptual understanding (Nicholson & Darnton, 2003; Watson, 2001).

The arid, context-free landscape on which so many examples used in statistics teaching are built ensures that large numbers of students never see, let alone engage in, statistical reasoning. In order to make statistical thinking accessible by all students, there ought to be fundamental changes to the instructional practices, curricular materials, tools and cognitive technologies employed in the classroom to teach statistical and probabilistic concepts. If the statistics classroom is to be an authentic model of the statistical culture, it should provide ample opportunities for exploration and experimentation with stochastical ideas in varied contexts. It should encourage statistical inquiry and data modeling rather than teaching methods and procedures in isolation (Lehrer & Schauble, 2004). The emphasis should be on the statistical process. The teaching of the different statistical tools should be achieved through putting students in a variety of authentic contexts where they need those tools to make sense of the situation. Rather than having students repeatedly practice how to calculate measures such as the mean and median, instruction should focus on helping them understand how one could use these measures in making comparisons, predictions and generalizations (Rubin, 2005). It is only through exploration and experimentation that students will appreciate the wide applicability and practical usefulness of statistical tools, and will come to view statistics as a powerful means for modeling and describing their physical and social world.

Advances of technology provide us with new tools and opportunities for the teaching of statistical concepts to young learners. These new technological tools are, in fact, designed explicitly to facilitate the visualization of statistical concepts by providing a medium for the design of activities that integrate experiential and formal pieces of knowledge, allowing the user to make direct connections between physical experience and its formal representations (Pratt, 1998; Meletiou-Mavrotheris, 2003; Paparistodemou & Noss, 2004). Having such a set of tools widely available to students has the potential to significantly change the curriculum—to give students access to new mathematical topics and insights by removing computational barriers to inquiry (Rubin, 1999). Students can experiment with statistical ideas, articulate their informal theories, use them to make conjectures, and then use the experimental results to test and modify these conjectures. There is evidence that use of such software in the statistics classroom promotes conceptual change in students and leads to the development of a more coherent mental model of key statistical and probabilistic concepts (Bakker, 2003; Hammerman & Rubin, 2003).

This article reports some of the insights gained from a case study of a group of teachers that participated in professional development seminars for the teaching of statistics with the use of the recently developed dynamic statistics software Tinkerplots®. This research is part of a multifaceted program for the teaching and learning of early statistical reasoning, which has as an overall aim to enhance the quality of statistics education offered in Cypriot elementary schools by facilitating professional development of teachers using contemporary technological tools and exemplary materials and resources.

2. BACKGROUND TO THE STUDY

In this section, we provide some information on Cypriot elementary schools with regards to the technology use and statistics teaching in mathematics classrooms, in order to help him/her appreciate what is described in the next sections. We also describe some of the main features of Tinkerplots®, the dynamic statistics software employed in the study.

2.1 STATISTICS INSTRUCTION AND USE OF TECHNOLOGY IN CYPRIOT SCHOOLS

2.1.1. Technology use in Cypriot mathematics classrooms

Technology is still not central to mathematics teaching in almost all countries, and Cyprus is no exception. Almost in their entirety, Cypriot teachers at both the elementary and secondary school level report that they rarely use computers in their classrooms when teaching mathematics (U.S. Department of Education, National Center for Education Statistics, 2001; Meletiou-Mavrotheris & Stylianou, 2003b; Mavrotheris, Meletiou-Mavrotheris, & Maouri, 2004). Even when computers are used, this use is usually confined to performing routine calculations, practicing skills and procedures, and checking answers. Students rarely or never use technology to solve complex problems, discover mathematics principles and concepts, process and analyze data, produce graphical representations of data, or develop models through simulations (Mavrotheris, Meletiou-Mavrotheris, & Maouri, 2004).

A main factor limiting use of technology in the mathematics classroom is the lack of professional development opportunities for teachers. Professional development is a necessary condition for technology implementation. It includes both learning how to use the computer itself and learning how to effectively integrate technology into mathematics teaching and learning (Rubin, 1999). In Cyprus, the opportunities available to teachers are, for the main part, limited to the former. A large number of teachers have or are currently attending professional development courses, and though these programs are useful in building teachers' computer-literacy skills, they do not prepare teachers to apply technology in instruction. In a recent survey (Mavrotheris, Meletiou-Mavrotheris, & Maouri, 2004), eighty-two percent of elementary school and sixty-four percent of secondary school mathematics teachers noted that they never had any training on computer use in mathematics instruction.

There are several other underlying factors limiting technology use in mathematics instruction. One such factor is the lack of integration of technology into the curriculum. Although teachers' guides encourage teacher use of calculators and computers, there are no specific suggestions on how to integrate them in the teaching and learning process, or recommendations about what software to use. The following factors have also been rated by the majority of both elementary and secondary school Cypriot teachers (see Mavrotheris, Meletiou-Mavrotheris, & Maouri, 2004) as limiting computer use in math instruction to a large extent: lack of support by specialists regarding ways to integrate technology into the curriculum, an oversized curriculum, shortage of computers, shortage of suitable software, and lack of knowledge about suitable software. And, while, these conditions are not unique to Cypriot education (on the contrary, they describe the reality of many educational settings around the world), the fact remains that technology is yet to be integrated functionally in mathematics teaching in Cyprus at any level in K-12 education.

2.1.2 Statistics instruction in Cypriot schools

The elementary mathematics textbooks in Cyprus place strong emphasis on data analysis.

These textbooks present numerous tasks on statistical concepts and also often make connections with concepts from other mathematical domains. However, most of these tasks focus exclusively on low-level data analysis skills. The majority of items included in the textbook are of the type Konold & Khalil (2003) refers to as "encode/decode" items, i.e. items which ask students to convert raw data into a statistic or a tabular or graphical display, or to do the reverse — to determine from a data display or a statistic the corresponding frequencies or data values. There is lack of any real "doing statistics" tasks, that is completely open-ended tasks which develop higher-level skills. Key ideas in data analysis (e.g. choosing between different measures of center based on context, sampling, scaling, predicting and making data-based decisions, etc.) that should ideally be at the focus of statistics instruction are missing. Considering the central role that the textbook plays in the mathematics classroom – the majority of Cypriot teachers report that the textbook is the major source they use in deciding how to present a topic to their classes and in assigning tasks for homework (U.S. Department of Education, National Center for Education Statistics, 2001) – it becomes obvious that the curricula currently used do not adequately nourish the development of statistical literacy in students. Students are not given the opportunity to develop the necessary conceptual understanding for analyzing data using the statistical techniques they are taught (Bakker, 2003).

The problem of inadequate professional development opportunities for mathematics teachers is particularly serious when it comes to statistics instruction. The majority of Cypriot teachers have been trained in very traditional mathematics classrooms with little or no exposure to statistical concepts, and, as a result, have very limited knowledge of statistical content and its pedagogy. Many of the senior teachers have never formally studied statistics. Younger teachers may have taken an introductory statistics course at college, such a course however does not typically adequately prepare future teachers to teach statistics in ways that develop students' intuition about data and uncertainty (Makar and Confrey, 2003). College-level statistics courses are often lecture-based courses that do not allow future teachers to experience the model of data-driven, activity-based, and discovery-oriented statistics they will eventually be expected to adopt in their teaching practices. As a consequence, teachers tend to have a weak understanding of the statistical concepts they are expected to teach and relatively deterministic epistemological sets, often sharing the same misconceptions regarding the stochastic as their students (Carnel, 1997).

2.2. FEATURES OF THE DYNAMIC STATISTICS SOFTWARE TINKERPLOTS®

Use of technology is essential to learning data analysis/statistics. Technology can illuminate key statistical concepts by allowing students to focus on the process of statistical inquiry – on the search and discovery of trends, patterns, and deviations from patterns in the data, and on the communication of findings to others. Choosing the right software however is of paramount importance. Ben-Zvi (2000) argues that statistics instruction ought to employ the use of technological tools which support active knowledge construction, provide opportunities for students to reflect upon observed phenomena, and contribute to the development of students' metacognitive capabilities. Most existing tools for young students do not posses these qualities. They are basically simplified versions of professional tools that have been developed top-down (Biehler, 1995), i.e. from the perspective of the statistician rather than bottom-up from the perspective of statistical novices. They provide a subset of conventional plots and are simpler than professional tools only in that they have fewer options (Konold, 2002).

In response to the need for statistics software specifically designed to meet the learning needs of younger students, Konold and his team recently developed Tinkerplots® (Konold, 2005), a dynamic statistics data-visualization software package intended primarily for elementary and middle grades. Tinkerplots® is a tool designed "from the bottom up", building on the foundation of what young learners already understand (Konold, 2002). The design of the software drew on current constructivist theories of learning as well as five years of academic research about the way young students learn and process statistical concepts and the main difficulties they face.

Tinkerplots® offers an easy-to-learn interface that encourages student activity. Although a complete data analysis tool, Tinkerplots®, unlike any other software, allows students to create their own graphs and plots "from the ground up" (Ben-Zvi, 2000). Using Tinkerplots®, young learners can start exploring data without having knowledge of conventional types of graphs, or of different data types. Rather than choosing from a menu of ready-made plot types, the software allows students to progressively organize their data using a construction set of intuitive operators. Through performing simple actions such as ordering data according to the values of a variable or sorting data into categories, children can develop a wide variety of both standard graphical displays (e.g. pie charts, histograms, boxplots and scatterplots), but also unconventional data representations of their own invention (Ben-Zvi, 2000). They can progressively organize data to answer their questions.

Tinkerplots® aims at genuine data analysis with multivariate data sets from the start, by beginning with students' own ideas and working towards conventional statistical notions and graphs (Bakker, 2002). The software's design allows even young students to use what they already know to search for and detect group differences and trends. By using features such as differences in icon size, color, and sound (e.g. the user can highlight information by the value of an attribute), students can detect subtle relationships in multivariate data in powerful and intuitive ways.

Tinkerplots® belongs to the new family of educational software in the teaching of statistics that came to be known as dynamic software, which offer an environment that permits the construction and flexible usage of multiple data representations. All of the software's objects are continuously connected and, thus, selection of data in one representation means the same data are selected in all representations. Changes in a data point in one representation are reflected in all related representations. Thus, students can interact with the data and see the immediate impact that their actions will have on the different representations of the data on the screen.

The software was built upon the foundation of Fathom® Dynamic Statistics, a highly acclaimed educational software for students in secondary higher education. Both Tinkerplots® and Fathom® are based on the same design principles, encouraging interactivity and empowering students through exploration, simulation, and dynamic visualization of data, to investigate and understand abstract statistical concepts. Both packages share a common code base and allow young learners using Tinkerplots® to make a natural progression to more advanced learning with Fathom®.

Dynamic statistics software such as Tinkerplots® and Fathom® do much more than producing fancy graphs; they facilitate the discovery of patterns through exploratory data analysis. Strong research support exists for the efficacy of dynamic computer graphics as instructional media that support active construction of knowledge by learners rather than forcing them to accept information provided by the computer without deep processing (e.g., Yu & Behrens, 1994; Hoyles & Noss, 1994; Meletiou-Mavrotheris, 2003; Bakker, 2003; Paparistodemou & Noss, 2004). Attributes like the ability to link multiple representations, to provide immediate feedback, and to transform a whole representation into a manipulable object, have affordances towards more constructive pedagogical approaches than traditional packages. The direct manipulation of mathematical objects and synchronous update of all dependent objects facilitates learning by allowing users to ask “what if…?” questions, make conjectures, and then easily test and see these conjectures in action (Ben-Zvi, 2000). Technology goes far beyond the role of mere means for data display and visualization to become a tool for thinking and problem solving.

3. METHODOLOGY

3.1 CONTEXT AND PARTICIPANTS

At an initial stage, we designed a line of research-based instructional materials for the development of overall statistical reasoning that meet curriculum objectives for elementary school. Central to this design was the functional integration of technology with existing core curricular ideas, and specifically the integration of the dynamic statistics software Tinkerplots®. We designed data-centered activities, in contexts familiar to children, which would provide them with opportunities to model and investigate real world problems of statistics using technology. Most of the activities were embedded in the existing elementary school mathematics curriculum and aimed to enrich it using technology.

Next, we designed and organized professional development seminars for the teaching of statistics with the use of technology. The design of the seminars was based on current pedagogical methodologies utilizing statistical investigation, exploration with interactive problem-solving activities, and collaboration. Acknowledging the fact that teachers are at the heart of any educational reform effort, the program aimed to offer high-quality professional development experiences to elementary school teachers that would enable them to effectively integrate technology into their teaching of statistical concepts and ideas.

Twelve in-service elementary school teachers (9 females, 3 males) participated in the professional development seminars which lasted for three weeks (15 hours in total). Teachers varied in their level of comfort with computers. Some had knowledge of only the basic computer applications, while others were very proficient with technology. The teachers also had varied background in statistics. Some had very limited exposure to statistics and had never formally studied the subject, while others had taken a university-based statistics course. The teachers were all experienced educators who had taught mathematics for several years.

During the professional development seminars, we worked with the teachers to help them see how their teaching and, subsequently, their students’ learning of statistical concepts could be enhanced using a technological tool like Tinkerplots®. We adopted Tinkerplots® as the software we would use during the professional development seminars, because we aimed at helping teachers see how the use of a powerful dynamic software could improve their students' learning opportunities and empower them as data analysts. We hoped that by giving teachers exposure to an inquiry-based environment that captures learners’ interest, we would encourage them to adopt teaching practices that would allow their students to develop their data literacy skills and competencies through their own thinking and exploration rather than receiving it predigested from teachers and books (Rubin, 1999). Additionally, we wanted to show teachers how dynamic statistics environments could also be effectively integrated into the teaching of general mathematics topics, as well as subjects outside mathematics (geography, science, etc.). We wished them to experience some of the ways in which dynamic statistics software could bring data analysis into the mathematics classroom in meaningful, relevant and accessible ways that could help convince students of the usefulness of statistics; how it could, through a data-driven perspective, help students internalize key mathematical concepts across the school curriculum while at the same time developing data literacy skills.

The emphasis during the seminars was on enriching the participants' content and pedagogical knowledge of statistics by exposing them to similar kinds of learning situations, technologies, and curricula to those they should employ in their own classroom. Teachers worked in group activities to explore a variety of data sets using Tinkerplots®. Through computer-based practice and experimentation, intensive use of simulations and visualizations, feedback from each other and reflection, we aimed at helping teachers to gain better understanding of some of the bedrock concepts in probability and statistics that should be integrated into the mathematics curriculum. In addition to computer activities, there were also discussions focusing on children's learning and what is required to involve them in learning about statistics. We explored a broad range of topics of interest to the statistics teacher, including curriculum issues (e.g. role of statistics in the national and international mathematics curricula) and statistics education research (development of statistical reasoning in children, common student misconceptions, etc.). Teachers brought in examples based on their own experiences and suggested ways in which their students' learning could be improved through using the tools provided by Tinkerplots®.

3.2 THE STUDY DESIGN

A case study design was employed in the research project. It was judged that this research strategy was well suitable to exploring, discovering, and gaining insight into teachers' perceptions, actions and interactions with the dynamic software Tinkerplots®. The study was exploratory in nature, and thus its purpose was not to prove or disprove hypotheses, but rather to generate descriptions based upon in-depth investigation of teachers' interactions with the technological tool, and of the impact this might have on their content and pedagogical knowledge of statistics. These descriptions, while of limited generalizability, may be used to understand similar situations (Stake, 1995) and can inform future research.

3.3 INSTRUMENTS, DATA COLLECTION AND ANALYSIS PROCEDURES

Consistent with the case-study methodology, the research team collected and analyzed a wealth of data on the development of the teachers' confidence and ability to work with the topic of statistics using technology. One videocamera was used to record all group activities and whole-class discussions taking place during the seminars, with different groups of teachers being filmed at different times. Other data consisted of participant observation, mini-interviews with teachers, and samples of teacher work.

The videotapes of group activities were first globally viewed and brief notes were made to index them. The goal of this preliminary analysis was to identify representative parts of the videotapes indicative of teachers' approaches and strategies when performing specific tasks and of the ways in which use of the dynamic statistics software influenced their thinking. The selected occasions were transcribed and viewed several times. We carefully studied and analyzed teachers' talk and actions. The method of analysis involved inductively deriving the descriptions and explanations of how the teachers interacted with the software and approached selected ideas of statistics. We attempted to verify or refute the conjectures inherent in the design of Tinkerplots® by investigating the degree to which the features and structure of the software influenced teachers' approaches to statistical investigations, and how this, in turn, affected their individual schemes for key statistical concepts and their instruction. These interpretations were corroborated by the insights gained from examining the data collected from other sources. The descriptions derived formed the study findings that are outlined in the next section.

4. RESULTS

The analysis of the data collected during the professional development seminars has provided us with rich insights into how teachers think and learn about statistics and how technology might impact their statistical reasoning and, subsequently, their teaching practices and their students' learning.

Initial observations in this setting confirmed earlier findings of the research literature indicating that teachers have a weak knowledge base in statistics. When they came to the professional development seminars, most of the teachers did not seem to have a global view of the features of a data distribution. They knew how to calculate measures like the mean and the median, but did not have a robust image of what these measures mean or how they are used (Hammerman & Rubin, 2003). When analyzing or interpreting data distributions, they tended to focus on measures of central tendency and avoided to take variation into account. Such an approach is not adequate since meaningful statistical analysis of a data distribution involves simultaneously attending to its center and the variation around that center. Comprehending what an average value or a distribution is about in relation to the variation around that value or distribution necessitates integration of the ideas of center and spread.

During the seminars, teachers' endeavors with Tinkerplots® brought about important changes in their ways of approaching statistical problems. The presence of the dynamic statistics software increased their interest in actively pursuing problems involving data. We also have some evidence for higher cognitive involvement, for improved overall comprehension of statistical concepts.

All the teachers, regardless of statistical background, became fully engaged in data explorations using the dynamic statistics software. They were enthusiastic about Tinkerplots® and the affordances it offers for delving deeply into the data to make sense of the situation at hand. They came to view technology as an indispensable tool in statistical endeavors and expressed eagerness to use it in their own classroom. At the same time, use of technology affected teachers' perceptions of data. Through their continuous participation in a variety of interesting computer activities that elicited conceptions of variability and difference rather than center and sameness, they "discovered the richness and complexity of data" (Hammerman & Rubin, 2003). We observed an improvement in teachers' intuitions about variation and its effects, accompanied by a parallel development of global perception of a data distribution as an entity with typical characteristics such as shape, center, and spread, and sample size (Ben-Zvi, 2003).

We present here an example indicative of the nature of the activities used during the professional development seminars and of the types of interaction teachers had with technology. This short example of statistical reasoning about a relatively simple dataset demonstrates the power of technology in supporting statistical reasoning — not just to calculate measures, but to generate visualizations that can reveal the structure of data. It illustrates how use of the dynamic statistics software could drastically change the culture of the mathematics classroom and support the development of data literacy skills by providing access to tools that allow one to see and manipulate data in ways that are impossible without technology (Hammerman & Rubin, 2003)

The following table was taken from your Geography textbook:

Country	Area (km^2)	Population (1990)	Population density	% Urban popul..	Per Capita (US$)	Av. Life expectancy		Patients per doctor	Students per teacher	TV sets (% households)	Literacy rate
						Males	Females				
Great Britain	244000	57000000	234	89	16900	72	78	611	22	98	98
France	551000	56000000	102	74	21000	72	80	403	21	98	99
Germany	357000	78000000	221	84	18800	71	77	360	14	93	99
Denmark	43000	5000000	119	87	25500	73	78	390	11	96	99
Switzerland	41000	6700000	160	60	34000	74	80	360		88	99
Greece	132000	10000000	78	62	6600	74	78	300	23	97	94
Italy	301200	57700000	191	70	18500	72	79	233	16	94	97
Norway	324000	4200000	13	75	25000	74	80	440	18	99	100
Russia	17000000	140000000	12	66		64	74				
Sweden	450000	8600000	19	84	26400	74	80	320	9	90	99
Finland	388000	5000000	15	60	27500	71	79	440	15	72	100
Cyprus	9251	710000	77	68	10000	73	78	510	22	95.4	95

Use the table to do the following:

a) Find the mean life expectancy of the twelve European countries in the table, separately for men and for women. Compare this with the life expectancy for Cypriot men and women.

b) Find the mean per capita income of the twelve European countries. Compare this with the per capita income for Cyprus.

c) Find the mean for two other measures provided in the table. You can choose whatever interests you. Compare the overall mean with the corresponding value for Cyprus.

Figure 1: "Population in Europe task"

"Population in Europe" task

The "Population in Europe" task appears in the 5th grade mathematics textbook (ages 10-11). In this task, students are given a table representing information about the population in twelve

European countries, and are asked to calculate overall means for different variables and compare them to the corresponding values for Cyprus. This activity is indicative of the nature of the mathematical tasks through which 5th grade students in Cyprus develop their understanding of the notion of the mean of a dataset. Although the table in Figure 1 contains plentiful information and lends itself to rich data analysis, students are only asked to calculate and compare means. While students use algorithms to compute the mean, they do not get the opportunity to explore how this measure should be interpreted in the context of other characteristics of the data.

Teachers were asked to approach the task first using traditional paper-and-pencil means as investigation tools, and subsequently, the dynamic statistics package. Our goal in giving this task was to investigate the role of technology (specifically, the role of this dynamic statistics tool) in shaping teachers' approaches and strategies. We were interested in what effect the visualization affordances of the technology would have on their perceptions of center, spread, and distribution. Thus, our data analysis paid particular attention to the processes teachers used when they were actually solving this problem with and without the help of technology.

Teachers worked collaboratively on the task in groups of two or three. Next, we describe the way teachers approached the task, both as they began to analyze the data on paper and then as they moved to Tinkerplots®. Our analysis of the data revealed important differences in teachers' approaches to the problem – these differences will be the focus of our analysis and report.

***Paper-and-pencil Stage*:** During the paper-and-pencil stage, teachers focused primarily on numeric strategies to complete the task. They perceived the provided table as a way to obtain the numbers needed to calculate means and respond to the task questions. None of the teachers attempted to approach the task using a visual strategy. Hence, none of the teachers took the initiative to investigate the problem situation visually by constructing a graph of the data values to gain a better perspective in solving the problem.

Here we share a description of the exploration of two female teachers, Anna and Sophia (these are pseudonyms), whose approach was typical of how most of the teachers approached the problem, first without and then with use of technology. Anna and Sophia spent only a few minutes on completing the task using paper-and-pencil. In the first question, they simply calculated the overall mean life expectancies of men and women and compared them to the corresponding life expectancies for Cyprus. They concluded that mean life expectancies for both genders (72 years for men, 78.4 years for women) were close to the corresponding values for Cyprus (73 years for men, 78 years for women). Also, they noted that women, both in Cyprus and throughout Europe, had a much higher life expectancy than men. In the second question, again all they did was to calculate the overall mean per capita income ($20927) and compare it to the per capita income for Cyprus ($10000). They concluded that the per capita income in Cyprus was much lower than the average per capita income of the European countries under consideration. In the last question, asking them to find the mean of any two other measures provided in the table and to compare them with the corresponding data values for Cyprus, they looked at two variables of particular interest to educators: "number of students per teacher", and "literacy rate". They did the calculations and found that the mean number of students per teacher in Cyprus was bigger than the corresponding overall mean (22 students vs. 17.1 students), whereas the literacy rate in Cyprus was lower than the mean literacy rate (95% vs. 98.1%).

Dynamic Statistics Stage. Teachers employed very different strategies during the dynamic statistics software assisted stage of instruction. Use of technology facilitated the use of advanced cognitive levels of statistical problem solving. It provided the means for teachers to focus on statistical exploration and not on recipes and formal derivations, which became secondary in importance. At this stage, teachers did not limit their analysis to calculating means like they did during the paper-and-pencil stage. In order to get better understanding of the problem situation, they now explored the entire distribution of data values using a combination of visual and numerical strategies. These explorations went much beyond the task requirements and generated more questions for teachers to investigate. Teachers made conjectures about observed trends in the data, and actively searched for evidence to support their claims by creating, transforming, and interpreting graphical data representations. Using a variety of techniques afforded by Tinkerplots® like categorizing data into a small number of bins, imposing cut points, or clumping similar values together and declaring them the same, they were able to view and manipulate the data, to make comparisons, and to draw conclusions.

The two teachers discussed earlier, Anna and Sophia, next proceeded to using the dynamic statistics software Tinkerplots® as an aid in responding to the same task. This time the two teachers initiated the use of a graph and their approach to the first question was now much richer. Their comparisons were done visually, using a variety of graphical representations. First, they produced the graph in Figure 2-left, which is a dotplot of the life expectancy for men. They highlighted the data point corresponding to Cyprus and looked for the location of men's mean life expectancy. Once again, they concluded that life expectancy for Cypriot men (73 years) was close to the mean life expectancy (72 years). However, this time they looked for additional evidence (besides mean comparison) so they divided the values of life expectancy variable into four groups ("bins"). They got Figure 2-right. Looking at the location of the largest cluster of data points, Sophia noted: *"The fact that 9 of the 12 countries are in the 72-74 bin shows that Cyprus has a life expectancy similar to the rest of the European countries"*. This new view supported their previous argument.

The two teachers followed a similar procedure when computing the women's life expectancy and, based on their displays (Figure 3), concluded that Cypriot women had a life expectancy similar to that of the other European countries. Now, though, while looking at Figure 3-left, Anna observed that the life

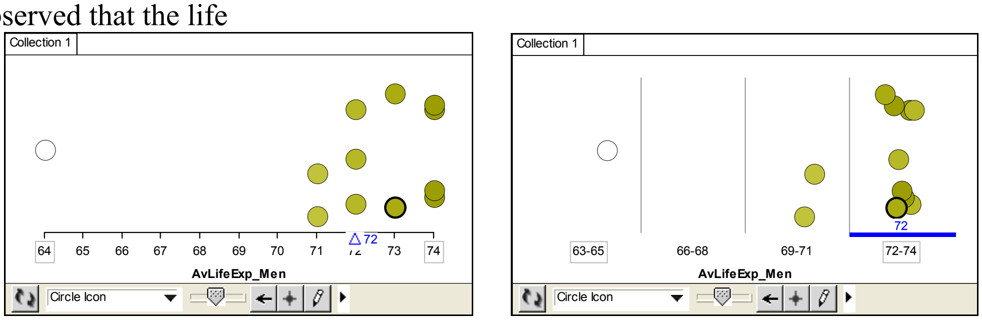

Figure 2: Dotplots of life expectancy for men

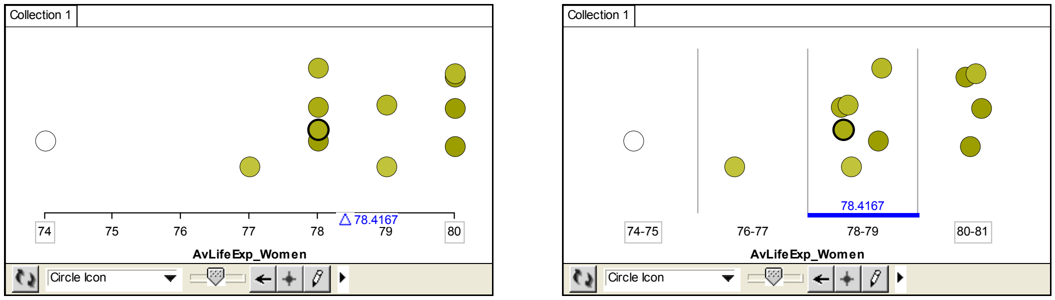

Figure 3: Dotplots of life expectancy for women

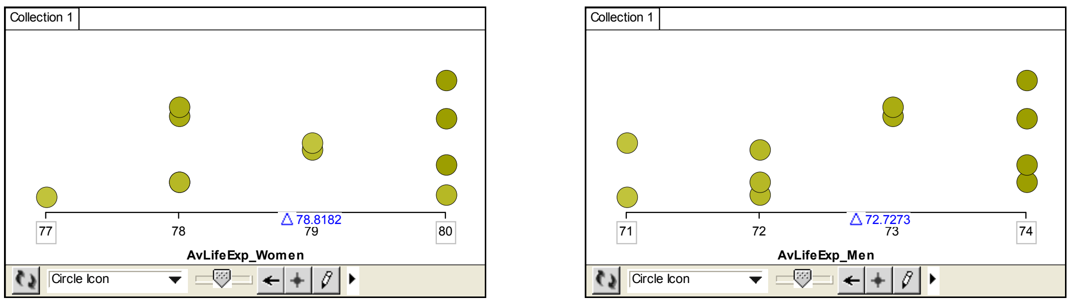

Figure 4: Life expectancy of women and men after deleting Russia

expectancy for one country (Russia, with a life expectancy of 74 years) was particularly low and that this might possibly lower the mean life expectancy for women. Sophia suggested to delete the data point corresponding to Russia and observe what happens. After doing so (Figure 4-left), they noticed that the mean life expectancy for women increased *"by almost half a year"* (from 78.4 to 78.8 years). They repeated their experiment with the men's data (Figure 4-right), and, once again, they observed that *the mean increased more for men (*from 72 to 72.7 years) *because men's life expectancy in Russia is only 64 and this lowered the mean a lot"*.

The two teachers also explored the relationship between life expectancy for men and life expectancy for women. Whereas during the paper-and-pencil stage the only observation they had made was that women tend to live longer than men, now access to technology allowed them to make more sophisticated comparisons. They drew a scatterplot of average age of men vs. average age of women (Figure 5-left), and looking at it concluded: *"Life expectancy for men tends to be higher in countries where life expectancy for women is also high"*. To further test their argument, they split the women's life expectancy variable into four groups (74-75, 76-77, 78-79, 80-81) and compared the women's mean life expectancy in each of these groups to the corresponding men's mean life expectancy (Figure 5-right). Looking at the sequence of means of these distributions in the "sliced" scatterplot made it easier for

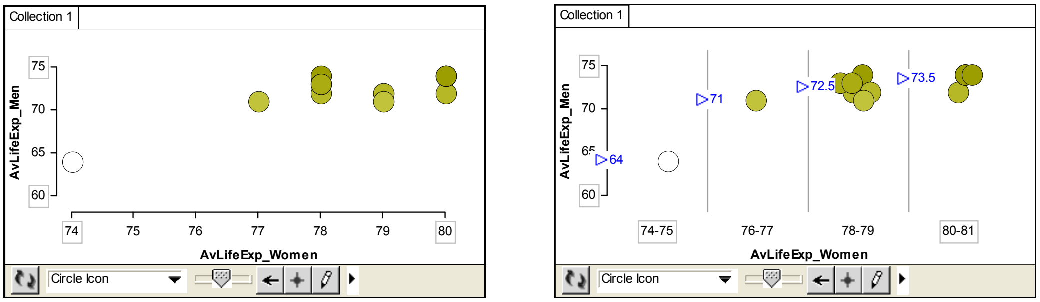

Figure 5: Life expectancy of men vs. life expectancy of women

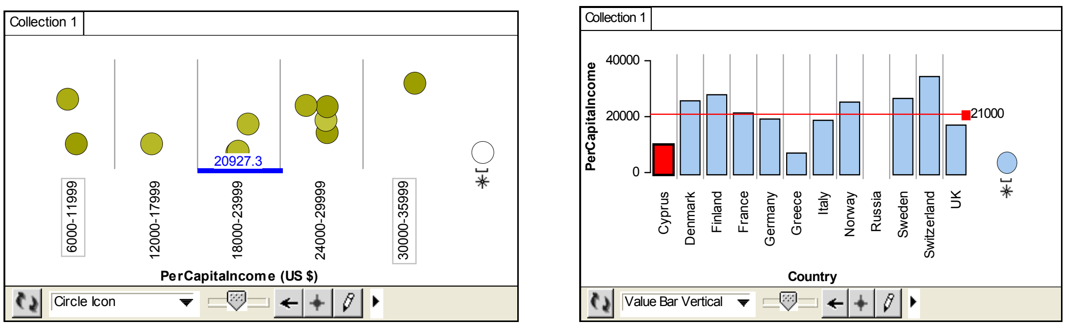

Figure 6: Dotplot and value bar graph of per capita income

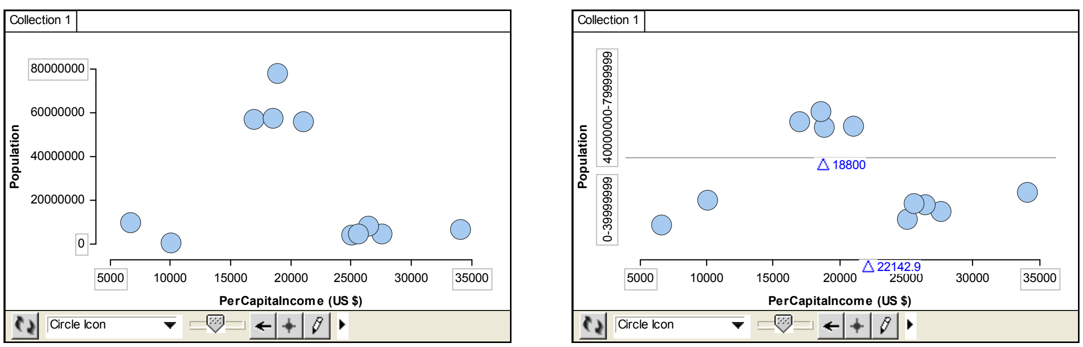

Figure 7: Per capita income vs. population size

teachers to explore the relation between the two variables (Konold, 2002; Noss, Pozzi, & Hoyles, 1999) and to further support their conjecture regarding the positive relationship between life expectancy for women and life expectancy for men.

Anna's and Sophia's approach to the second question, where they had to compare the overall mean per capita income to the per capita income for Cyprus was similar. Using various graphical displays, they concluded that not only the per capita income in Cyprus was much lower than in most of European countries, but they also observed that "*per capita income clusters around 1800-24000…only three countries make less than 18000*" (Figure 6-left). Similarly, by displaying the data as value bars (in which the length of each bar represents the per capita income for the corresponding country), and then introducing a cut point to divide the per capita income into two groups – one below the mean per capita income and one above it – they concluded that, *"with the exception of Greece, the other countries are close to or well above the* *overall mean*" (see Figure 6-right).

Anna's and Sophia's exploration of the data gave rise to new questions in relation to per capita

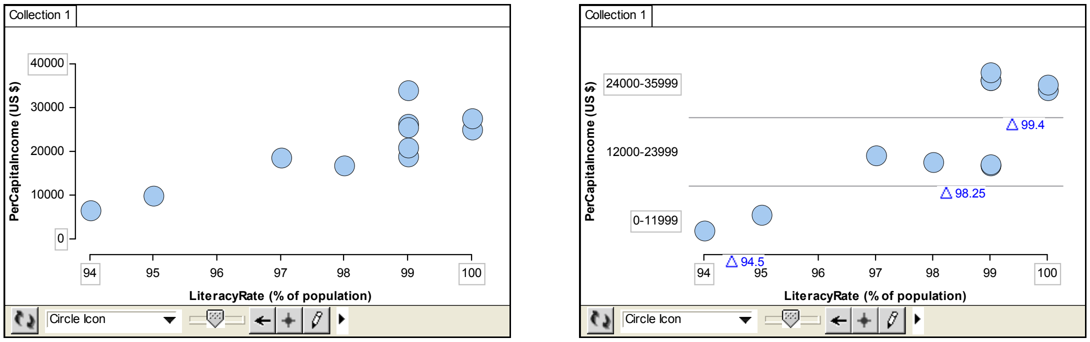

Figure 8: Literacy rate vs. per capita income

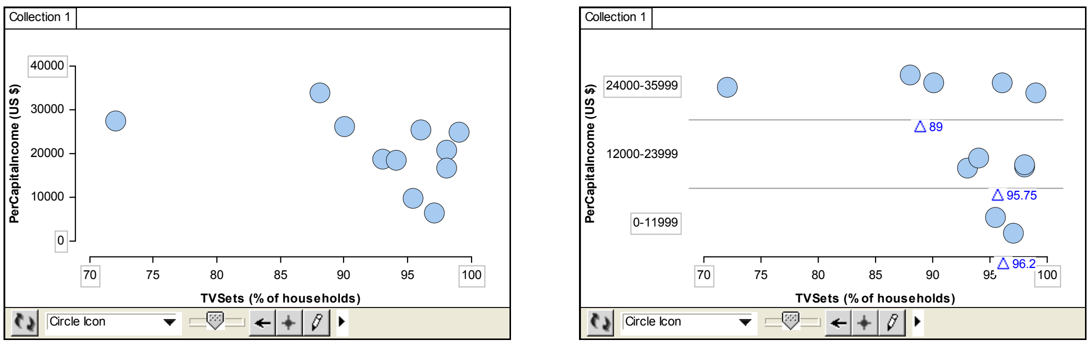

Figure 9: Percentage of households with TV sets vs. per capita income

income. For example, they investigated the relationship between per capita income and population size (see Figure 7). Looking at Figure 7-left, they concluded that, with the exception of Greece and Cyprus– the two countries on the lower far left of the plot – countries with a smaller population tended to have higher per capita income. They proceeded to investigate the correctness of their hypothesis by dividing the countries into two groups, those with population of at least 40 million people and those with population smaller than 40 million. The graph they got (Figure 7-right) supported their argument that per capita income tends to be smaller for countries with a larger population.

Anna and Sophia also explored the relation between per capita income and literacy level. Based on their graphs (Figure 8), they concluded that there seems to be a positive relationship between per capita income and literacy level.

Subsequently, they investigated whether there is any relationship between per capita income and percentage of households with a TV set. Before looking at the data, they conjectured that there would be a positive relationship between the two variables. However, their graphs (see Figure 9) did not support this conjecture. In particular, Anna made the important observation that Finland and Switzerland, the two countries with the smallest percentage of households with a TV set were at the same time the two countries with the highest per capita income.

The two teachers also explored the relationship of per capita income to other variables including life expectancy, number of patients per doctor, number of students per teacher, etc. They made important observations, which however will not be discussed here.

In the last question, asking teachers to compare the overall mean of two variables of their choice to the corresponding values for Cyprus, the two computer partners' explorations again went much beyond what the question required. One variable they investigated was total area. By plotting a dotplot of the area of the twelve countries (Figure 10-left) they noted that, with the exception of Russia, all the other European nations appear to have about the same size. *"This is very misleading",* Sophia pointed out: *"Cyprus does not have the same area as England or France!"*

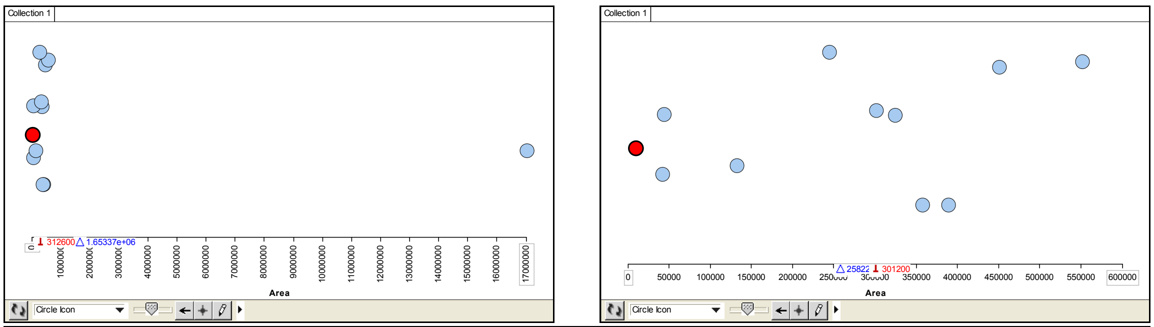

Figure 10: Dotplots of European countries' total area, before and after omitting Russia

Looking more closely, Anna and Sophia realized that the reason the dotplot looked misleading was Russia's huge area, which makes the rest of the countries appear close together on the graph and it also makes the mean area go up a lot. They noticed that the mean area (the point marked with a triangle in Figure 10-left) has a value exceeding the area of all countries in the data set other than Russia and concluded that, in this specific situation, the mean is not a very good summary of the "typical" area of a European country and that the median (the point marked with a line) is a better summary. They decided to delete Russia and see the effect of their action on the values of the mean and the median. Deleting the data point corresponding to Russia changed the scale and shape of the graph, while the mean value went down from 1 653 370 km^2 to 260 000 km^2 (Figure 10-right), and got close to the median, the value of which decreased only slightly (from 312 600 km^2 to 301 200 km^2). The two teachers concluded that, in this context, deleting the case corresponding to Russia makes the mean a more informative measure of the center of a distribution.

Discussion on Pedagogy**:** The group activity was followed by a whole class discussion. During the discussion, teachers stressed the advantages of using Tinkerplots® to approach important statistical concepts like the mean. They noted that when teaching statistics using only traditional means of instruction, most students *"learn the different concepts as a set of techniques that they do not really understand and they cannot apply in real world settings*". When for example being taught about the mean without using technology, most students can easily learn the procedure for calculating it, they do not however understand its meaning and how it can be used as a representative value of a set of data values. Dynamic statistics software, on the other hand, the teachers pointed out, offer tools that may aid even elementary school children, who have little statistical background, build understanding of some of the subtle aspects of the mean, such as its sensitivity to extreme values.

Teachers expressed once again their enthusiasm regarding Tinkerplots® and its capabilities compared to more conventional technological tools. One of the teachers, for example, said that he regularly uses calculators, and occasionally also uses the software Excel in his mathematics classes. When using Excel, he noted, his students can draw standard graphical representations and can easily get numerical summaries of data like the mean and the median. However, he added, through use of Tinkerplots®, students can do much more. They can make and test hypotheses; they can investigate relationships among different variables by easily constructing and manipulating their own representations of data:

> Students can investigate numerous relationships. For example, they can easily provide answers to questions like: "Is there a relationship between life expectancy of men or women with the mean number of patients per doctor?" "What is the relationship between a country's area, population, and population density?" "How are per capita income and percentage of households with TV sets related ?" I can't imagine these relationships being explored as easily with other software. Use of Tinkerplots® allows us to ask questions, make and test conjectures, and discover relationships that we would not have even imagined without technology or when using some other type of software. This is one of only a handful of educational software I have worked with that has encouraged me to work using divergent reasoning.

The remaining teachers' comments were in the same spirit:

> Tinkerplots® allows students to quickly draw graphical representations to explore the relationships they are interested in. Without spending their time on meaningless procedures, they can focus on analyzing and interpreting data, and on drawing conclusions based on data.

> Using this software, one can very easily and quickly make many comparisons among different variables and draw a lot of useful conclusions.

Several teachers pointed out that use of software like Tinkerplots® encourages students to generate their own questions, which can go much beyond what is required by the textbook problems. Additionally, when using technology students can easily have access to more recent data than what is available in the textbook. In particular, one teacher noted that the data which students are asked to analyse in the "Population in Europe" task is *"quite outdated since the textbooks were published several years ago"*. Other teachers agreed, pointing out that, for example, per capita income for Cyprus has doubled, life expectancy of Cypriot women has now reached 82 years and of Cypriot men 79 years, and the literacy rate has gone up to 98 percent. They stressed that, when using the software, the teacher could import from the Internet and provide his students with more recent statistics for both Cyprus and the other European countries. Additionally, students could have access to data for all European countries, not only the twelve in the table provided in the "Population in Europe task". *"Increasing the number of countries would be extremely useful",* a teacher noted, since *"here we had only twelve data values, and possibly some of the conclusions we drew might not hold for a larger number of countries."*

During the discussion, we took advantage of the comment made by one of the teachers that a possible relation that the students could investigate is the relationship between a country's area, population, and population density, to give teachers ideas as to how they could use Tinkerplots to help their students overcome one of the main difficulties in moving from elementary to secondary school – the transition from arithmetical to algebraic thinking. We stressed that the fact that software like Tinkerplots, which combine dynamic capabilities with the ability for the learner to enter formulas or commands, could engage elementary school students in constructing symbolic relationships, and thus help them build bridges to algebraic reasoning. When having students work on the activity in Figure 1, the teacher could, through class discussion, help them see that population density describes the crowdedness of a country and that, to find out how crowded a country is, one needs to consider both area and population. Students could use the dynamic statistics software to figure out the relationship between population, area, and population density (i.e. *population density=area/population*). Subsequently, the teacher could encourage them to generalize their thinking by giving them the population and area of a country not in the dataset and asking them to find the population density of this country. This way, students would experience the advantages afforded by the power of generalization.

5. CONCLUSIONS

Data literacy has become a fundamental skill for living in an information era where important decisions are made based on available data. In order for students to develop a data-oriented mindset and robust data literacy skills, there ought to be significant changes to the instructional methods and tools typically employed in the classroom to teach statistical concepts. In particular, technology should assume a much more central role in statistics teaching and learning. Recognizing this need, the current study investigated the potential of the dynamic statistics software Tinkerplots®, an educational package specifically designed to support statistics instruction in early grades. We explored the perceptions, actions and interactions that a group of Cypriot teachers had with this technological tool during professional development seminars introducing them to the software.

Findings of the study are very encouraging. They suggest that exposure, during the professional development seminars, to the dynamic statistics software Tinkerplots® brought about important changes in the participating teachers' approaches to statistics and its instruction. The presence of the dynamic software increased teachers' interest in statistical investigation, it gave them the opportunity to explore data in ways that had not been possible for them before (Hammerman & Rubin, 2003) The data analysis tools offered by the software provided the means for teachers to focus on statistical conceptual understanding and problem-solving, rather than on recipes and computational procedures. We witnessed the emergence of a community of highly motivated educators, enthusiastic about the affordances the software offers for delving deeply into the data. Being convinced that instructional use of Tinkerplots® could lead students to the construction of much more powerful understandings of statistical concepts, these teachers were eager to employ the software in their own classroom.

The qualitative methodology employed in this case study, the small scale of the study, and its limited geographical nature, means that generalizations to cases that are not very similar should be done cautiously as the specific group of teachers investigated might not be representative of all elementary school teachers. However, the study findings do suggest that use of dynamic statistics software does indeed have the potential to enhance statistics instruction. We do believe, and there are strong indications in this study to support our belief, that use of such software in the statistics classroom can promote active knowledge construction by encouraging students to build, refine, and reorganize their prior understandings and intuitions about statistics. Use of Tinkerplots® in particular – a software with a design based on the way young children learn statistics – can provide an inquiry-based learning environment through which genuine endeavors with data can start at a very young age. In combination with appropriate curricula and other supporting material, use of Tinkerplots® can help students develop a strong conceptual base on which to later build a more formal study of statistics.

The research literature in the area of statistics education indicates very poor statistical intuitions among most college-level students and adults. Our firm belief is that people are capable of statistical reasoning and that the difficulties they face in reasoning about statistical phenomena are similar to other failures in mathematical understanding – they are primarily the result of deficient learning environments and of reliance on "brittle" formal methods (Wilensky, 1997). As researchers such as Pratt (1998), Paparistodemou et al. (2002), and Lehrer and Schauble (2004) have illustrated, when given the chance to participate in appropriate instructional settings that support active knowledge construction, even very young children can exhibit well-established intuitions for fundamental statistical concepts. Innovative educational software such as Tinkerplots®, which are aligned with constructive views of learning, allow children to explore ideas in contexts that are both rich and meaningful to them. They afford young learners with tools they can use to construct their own conceptual understanding of statistical concepts – tools for not only data display and visualizations, but also thinking and problem solving. Use of such software, in combination with appropriate curricula and other supporting material, can help students develop a strong conceptual base on which to later build a more formal study of statistics.

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