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CALCULATING DEPENDENT PROBABILITIES

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In the 2004 European soccer competition France were one of the favourites to win the World Cup and Thierry Henry, their star forward, was one of the favourites to be top goal scorer. Bookkeepers were offering odds of 4 : 1 on France winning the competition and odds of 8: 1 on Thierry Henry being the top scorer. A large number of punters went into betting shops in the United Kingdom and made a single bet that France would win the competition **and** that Thierry Henry would be top scorer. The counter clerks in the betting shops accepted the bets and punters making the bets believed that a £1 stake would bring a return of £42. (*A £1 stake on France winning the competition at odds of 4 : 1 gives £5 (=£4 plus return of the £1 stake). The £5 then being bet on Thierry Henry being top scorer at odds of 8 : 1 gives £45(= £40 plus £5.) In general if a bet is made on two outcomes and the odds of each outcome are $m : 1$ and $n : 1$ then the return on a £1 stake is $£(m + 1)(n + 1)$).*

This calculation is only valid, however, if the two events are independent. In this case the events are clearly not independent since if France do win the competition they will have played more games and are likely to have scored more goals. Since Thierry Henry is their most likely goal scorer it follows that he is more likely to be the top goal scorer overall. The example below shows how the probabilities should be worked out.

In November 2004 England played Spain in a friendly soccer match. The tables below show some of the odds being offered by the bookmakers William Hill.

Spain to win 2 – 0	16:1
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First player to score	
Raul (Spain)	2:1
W Rooney(England)	3:1
Morientes (Spain)	7:2
M Owen	9:2
Another Spanish player	6:1
Another English player	7:1

Spain to win 2 – 0 and Raul to score first	25:1
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The bookmaker has acknowledged that Spain winning 2 – 0 and Raul scoring first are not independent events since a £1 bet would receive only £26 and not £(16 + 1)(2 + 1) = £51! Clearly the events are not independent since if Spain win 2 – 0 a Spanish player must have scored first!

Are the odds of 25 :1 consistent with the other odds offered?

Consider the odds offered against the player to score first. We first change these odds to probabilities(see ‘Odds that don’t add up’ Teaching Mathematics and its Applications 1994)

First player to score			
	Bookmaker’s odds	‘Adjusted’ probabilities	True Probabilities
Raul	2:1	0.3333	
W Rooney	3:1	0.2500	
Morientes	7:2	0.2222	
M Owen	9:2	0.1818	
Another Spanish	6:1	0.1429	
Another English	7:1	0.1250	
		Total 1.2552	1.000

(Note that in this context the ‘True Probabilities’ merely reflect the amount of money staked by the punters on each player. They do not measure the real probability a player will score first – if indeed such a probability exists!)

If Spain win 2 – 0 an English player could not have scored first. The conditional probabilities of each of the Spanish players scoring first are shown below

First Spanish player to score	Conditional probabilities given that a Spanish player scores first
	Total = 1.000

In order to make the same percentage profit as before the bookmaker adjusts these conditional probabilities by multiplying by 1.2552. The table below shows the adjusted probabilities and the associated odds.

First Spanish player to score	Probabilities	Adjusted probabilities	Bookmaker’s Odds
		Total = 1.213	

(for explanation of changing probabilities to odds see ‘Odds that don’t add up’ Teaching Statistics 1994)

The bookmaker should therefore, to be consistent, be offering the following odds.

Luis Figo to score first given that Portugal won 2 - 0	3.202:1
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A punter who bets £1 should, therefore, receive $£(20 + 1)(3.202 + 1) = £88.24$ if Portugal win 2 – 0 and Luis Figo scores first. In practice he or she would receive only £41.

Readers are invited to submit their answer to the following.

On the same match the bookmaker also quoted the following odds.

England to win 3 – 1	16:1
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England to win 3 – 1 and Emile Heskey to score the first goal	66:1
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To be consistent what odds should be offered on England winning 3 – 1 and Emile Heskey scoring the first goal?

(Assume that the odds of 16:1 against England winning 3 – 1 and 9:2 against Heskey being first player to score a goal are sensible odds. i.e. they reflect the amount of money bet by punters.)

Fletcher