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Abstract: It is critical for all students to learn algebra, including the ability to generalize, to function in our increasingly complex world. This pretest/intervention/posttest study of preservice elementary teachers (N = 63) in their math methods course assessed their knowledge of writing and applying algebraic generalizations using instructor-made rubrics along with analysis of work samples and reported insights. Initially, although most subjects could solve a specific case, they had considerable difficulty determining an algebraic rule. After a problem-solving-based teaching intervention, students improved in their ability to generalize, however, they encountered more difficulty with determining the algebraic generalization for items arranged in squares with additional single items as exemplified by x^2+1, than with multiple sets of items, as exemplified by 4x.

Keywords: algebra; generalizations; intervention; pre-service elementary teachers

Overview

It is critical for all school-aged students to learn algebra, including the ability to generalize, to function in our increasingly complex world (National Council of Teachers of Mathematics [NCTM], 2000; RAND, 2003). Preservice elementary teachers play a critical role in initiating and developing algebraic reasoning in grades K-6, however the research base of teachers' knowledge regarding algebraic instruction is rather limited (Doerr, 2004; Kieran, 2006). Many call for increased attention to algebraic reasoning in the elementary grades to ease the transition from arithmetic and build understanding of the abstract concept of variables (Kieran, 1992; Kaput, 2000). At the same time, teachers' weak conceptual understanding of essential subject-matter knowledge is well known (Ma, 1999). The transition from a procedural approach in arithmetic to a structural understanding of algebra does not come easily (Kieran, 1992). Without the prerequisite content knowledge on the part of preservice elementary teachers, meeting these objectives for students is unlikely. To meet the goals of teaching algebraic reasoning in the elementary school curriculum, we need to understand more about how preservice teachers are prepared for this undertaking.

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Algebraic reasoning at the elementary level takes many forms, including extending pictorial and number patterns, doing and undoing, understanding equivalence, solving for an unknown, and writing a generalization for a pattern (Carpenter, Franke, & Levi, 2003; Kaput, 2000; NCTM, 2000). It is this latter aspect of algebra that we will address. Because students' understanding of writing generalizations is enhanced using pictorial geometric patterns (Bishop, 1997), we investigate how writing algebraic generalizations from pictorial patterns affects *preservice teachers'* understanding. Therefore, this study examines the following questions: given a pictorial pattern, how do preservice elementary teachers interpret the pattern and write a corresponding generalization? And after completing the activities, how do preservice teachers perceive their ability to teach algebraic generalizations?

Theoretical Framework

The literature is replete with studies documenting both students' and elementary preservice teachers' difficulty with beginning algebraic reasoning and writing generalizations. MacGregor and Stacey (1997) investigated students' algebraic learning and found that students did not easily learn how to express simple relationships in algebraic notation. Students also misused algebraic symbols and syntax in relatively basic problems (allowing, for example, the letter *h* to represent height). Mac Gregor and Stacey found that misleading teaching materials reinforced the erroneous concept that a letter represents an object. Students extend patterns numerically more easily than they can generalize about them (Mac Gregor & Stacey, 1997; Zaskis & Liljedahl, 2002). Approaching algebraic expressions and equations from a contextual vantage point, Bishop (1997) asked seventh and eighth grade students to model perimeter and area problems with pattern blocks and tiles, and then generalize the relationships symbolically. Bishop found that the use of mathematical patterns promoted algebraic reasoning, but not all students were able to generalize. Gray, Loud, and Sokolowski (2005) found that students in college algebra classes and calculus classes had difficulty using variables as generalized numbers.

In contrast, students from classrooms that were a part of intensive staff development projects for in-service elementary teachers were found to be capable of algebraic reasoning. Third graders were able to generalize and formalize their mathematical thinking about even and odd numbers (Kaput & Blanton, 2000). In that study, students initially used computation to solve problems about even and odd numbers; later, they used the terms even and odd as placeholders (or variables). Although they were not at a formal symbolic level, the students in this study also perceived even numbers as multiples of two. On a state assessment, third graders in this project outscored fourth graders from a classroom not involved in the effort to improve the teachers' algebraic instruction (Kaput & Blanton, 2001).

Bishop and Stump (2000) examined preservice elementary and middle school teachers' conceptions of algebra. In a semester course, the preservice teachers engaged in college-level algebraic experiences involving generalization, problem solving, modeling, and functions. They found that many preservice teachers did not understand what distinguishes arithmetic from algebra, and of those that did make the distinction, a majority held a procedural perspective even at the end of the semester course.

Methodology

Sixty-three white undergraduate elementary preservice teachers (53F, 10M) enrolled in a mathematics methods class participated in the study. 79% of the participants completed and 11% were currently enrolled in a college level math course. All students took a pretest on the first day of class and an identical posttest nine weeks later, after the intervention had been completed. This instrument consisted of two problem sets in which drawings depicted the pattern described in the problem. The pre- and posttest consisted of two problems. The first problem set focused on writing a rule for the number of legs in sets of four-legged tables, 4n; the second problem set presented a progressively larger design that could be described as x^2+1, consisting of boxes arranged in a square with one additional box. For each problem, subjects were asked to: 1) solve for a specific case; 2) describe the generalization in words; 3) write an algebraic generalization; and 4) describe their strategy. A scoring rubric was developed by qualitatively categorizing student responses on the assessment at four levels: proficient, basic, developing, and poor.

The intervention involved two forty-minute activities conducted on different days where students worked in small groups to generate algebraic generalizations for sets of symbols (Sharp & Hoiberg, 2005). The instructional sequence was taught through problem solving. The launch of the lesson occurred as the instructor demonstrated her thinking in analyzing the first pattern set. Then, during the exploration, groups tried to solve the remaining problems cooperatively and the instructor provided hints and suggestions but not solutions. During the summary, a student from each group came to the front of the classroom, presented the group's solution to a problem, and discussed strategies. After input of ideas from other groups, the key points for each pattern were summarized.

Results

Pretest results showed that preservice teachers could continue a pattern and solve numerically for the next case. They had difficulty expressing ideas in words, writing a generalization, recognizing a pattern of square numbers, and explaining a strategy. The posttest revealed that the preservice teachers made significant growth in their understandings of algebraic generalizations as a result of the intervention activities. In addition to what they could do on the pretest, they could now express ideas in words, write a generalization, recognize a pattern of square numbers, and explain a strategy.

Our results corroborate prior research regarding the ability to generalize. Results differed for each type of question and the performance was stronger for the generalization $z=4n$ than for $z=n^2+1$. Preservice teachers were more successful at generalizing the pattern for the first problem set as shown in Table 1. Initially, 98% students could extend this pattern numerically and 89% could write a generalization. After the posttest, the percent of students able to write the generalization increased to 95%. Preservice students' ability to explain how they arrived at the answer, write a generalization, and explain strategies all improved.

Table 1.
Percent of students (N=63) scoring at each level for Z=4n

	Type of Q	Proficient (3)	Basic (2)	Developing (1)	Poor (0)
		Pretest-Posttest	Pretest-Posttest	Pretest-Posttest	Pretest-Posttest
1	Extend Pattern Numerically	98-97	0-0	0-0	2-3
2	Explain in Words	65-86	25-13	8-0	2-1
3	Generalization	89-95	5-2	3-0	3-3
4	Strategy	65-86	25-13	8-0	2-1

The second question, to extend the pattern of a number of boxes arranged in a square pattern plus one additional square proved to be more difficult for preservice elementary teachers, however, increases in ability to solve the problems occurred during the study. Initially, only 79% could extend the pattern numerically, and 41% could write a generalization. After the posttest, 97% of the students could extend the pattern numerically, and 98% could successfully write a generalization.

Table 2.
Percent of students (N=63) scoring at each level for $Z=n^2+1$

	Type of Q	Proficient (3)	Basic (2)	Developing (1)	Poor (0)
		Pretest-Posttest	Pretest-Posttest	Pretest-Posttest	Pretest-Posttest
5	Extend Pattern Numerically	79-97	8-0	0-1.5	13-1.5
6	Explain in Words	43-95	30-5	10-0	18-0
7	Generalization	41-98	6-0	11-2	41-0
8	Strategy	26-90	22-8	17-2	33-0

Preservice students were finally asked what they learned from the unit on algebra with a written survey. Responses were coded into three categories. The most frequent category of response addressed increased knowledge of techniques and strategies for writing a generalization. Students commented, "I was able to learn different strategies to show the problem," "There are many ways to solve them," and "Making up problems helped." About half of the students expressed a better understanding of the importance of teaching algebra as a result of the activities. The third category centered on improved ability to solve for a generalization. Many students commented that presenting and sharing strategies with the class helped them better understand how to arrive at a generalization. Almost half the students volunteered that they perceived an improvement in ability.

Discussion and Conclusion

Consistent with prior research, even though 79% of the students had completed a college level mathematics course, the pretest indicated that writing generalizations was difficult for many preservice students. The posttest results indicated that preservice elementary teachers' ability to write a generalization of the type y=4n and $y=n^2+1$ increased throughout the study. This is a difficult area of the curriculum for preservice elementary teachers, however, when problems were placed in a context and taught in a problem solving environment, understanding improved. As would be expected, more students were successful at the y=4n type of problem. This is the type of question that is most typically found on grades 3-6 state assessments.

Students' work and comments during the practice showed they enjoyed the work but found it challenging. Inquiry, problem solving, and critical thinking occurred as students devised algebraic equations for sets of symbols. We recommend that instruction in algebraic generalization include group inquiry following a launch, explore, and summarize sequence. We also believe that projects that are complex and require analysis of the work of others be part of project-work in mathematics for preservice teachers. Many states now have adopted NCTM recommendations for teaching more algebraic reasoning in the elementary grades. An area for continued study is to see if incoming groups of preservice teachers improve on their initial understanding of writing generalizations.

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