IF MATHEMATICS IS A LANGUAGE, HOW DO YOU SWEAR IN IT?

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Swears are words that are considered rude or offensive. Like most other words, they are arbitrary symbols that index meaning: there is nothing inherently wrong with the letters that spell a swear word, but strung together they conjure strong meaning. This reminds us that language has power. This is true in mathematics classrooms too, where language practices structure the way participants understand mathematics and where teachers and students can use language powerfully to shape their own mathematical experience and the experiences of others.

When people swear they are either ignoring cultural norms or tromping on them for some kind of effect. In any language and culture there are ways of speaking and acting that are considered unacceptable. Though there is a need for classroom norms, there are some good reasons for encouraging alternatives to normal behavior and communication. In this sense, I want my mathematics students to swear regularly, creatively and with gusto. To illustrate, I give four responses to the question: If mathematics is a language, how do you swear in it?

Response #1: To swear is to say something non-permissible.

I’ve asked the question about swearing in various discussions amongst mathematics teachers. The first time I did this, we thought together about what swearing is and agreed that it is the expression of the forbidden or taboo. With this in mind, someone wrote $\sqrt{-1}$ on the board.
whiteboard and giggled with delight. Another teacher reveled in the sinful pleasure of scrawling % as if it were graffiti. These were mathematical swears.

Though I can recall myself as a teacher repeating “we can’t have a negative radicand” and “we can’t have a zero denominator”, considering the possibility of such things helps me understand real numbers and expressions. For example, when the radicand in the quadratic formula is negative \((b^2 - 4ac < 0)\), I know the quadratic has no roots. And when graphing rational expressions, I even sketch in the non-permissible values to help me sketch the actual curve. Considering the forbidden has even more value than this.

Though it is usually forbidden to have a negative radicand or a zero denominator, significant mathematics has emerged when mathematicians have challenged the forbidden. Imaginary numbers opened up significant real-world applications, and calculus rests on imagining denominators that approach zero. This history ought to remind us to listen to students who say things that we think are wrong, and to listen to students who say things in ways we think are wrong (which relates to response #3 in this article). We can ask them to explain their reasoning or to explain why they are representing ideas in a unique way.

Knowing what mathematical expressions are not permitted helps us understand the ones that are permitted. Furthermore, pursuing the non-permissible opens up new realities.

Response #2: Wait a minute. Let’s look at our assumptions. Is mathematics really a language?

Good mathematics remains cognizant of the assumptions behind any generalization or exploration. Thus, in this exploration of mathematical swearing, it is worth questioning how mathematics is a language, if it is at all.
It is often said that mathematics is the universal language. For example, Keith Devlin has written a wonderful book called “The Language of Mathematics”, which is a history of mathematics that draws attention to the prevalence of pattern in the natural world. The book is not about language in the sense that it is about words and the way people use them. Its connection to language is more implicit. Humans across cultures can understand each other’s mathematics because we share common experiences of patterns in the world and of trying to make sense of these patterns. We can understand each other. Understanding is an aspect of language. There are other ways in which mathematics can be taken as a language, and there can be value in treating it as a language, as demonstrated by Usiskin (1996).

However, it would not be so easy to find a linguist who calls mathematics a language. Linguists use the expression ‘mathematics register’ (e.g. Halliday, 1978) to describe the peculiarities of a mainstream language used in a mathematical context. David Pimm (1987) writes extensively about aspects of this register. It is still English, but a special kind of English. For example, a ‘radical expression’ in mathematics (e.g. “$3\sqrt{2} + \sqrt{5}$”) is different from a ‘radical expression’ over coffee (e.g. “To achieve security, we have to make ourselves vulnerable.”) because they appear in different contexts, different registers.

Multiple meanings for the same word in different contexts are not uncommon. Another example significantly related to this article is the word ‘discourse’, which has emerged as a buzzword in mathematics teaching circles since reforms led by the National Council of Teachers of Mathematics in North America. The word is often used as a synonym for ‘talking’ (the practice of language in any situation) and also to describe the structure and history of mathematics classroom communication (the discipline of mathematics in general), which, of course, has a powerful influence on the practice of language in the classroom. Both meanings
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have validity, so it is up to the people in a conversation to find out what their conversation partners are thinking about when they use the word ‘discourse’. It is the same for the word ‘language’.

Who has the right to say mathematics is a language, or mathematics is not a language? Language belongs to all the people who use it. Dictionaries describe meanings typically associated with words more than they prescribe meaning. By contrast, students in school often learn definitions and prescribed meanings – especially in mathematics classes. This is significantly different from the way children learn language for fluency.

When we are doing our own mathematics – noticing patterns, describing our observations, making and justifying conjectures – language is alive and we use it creatively. When we make real contributions to a conversation it is often a struggle to represent our ideas and to find words and diagrams that will work for our audience. For example, I have shown some excerpts from students’ mathematical explorations in Wagner (2003). The students who worked on the given task developed some new expressions to refer to their new ideas and in the article I adopted some of these forms, calling squares ‘5-squares’ and ‘45-squares’ (expressions that have no conventional meaning). When we are doing our own mathematics we try various words to shape meaning. By contrast, mathematical exercises – doing someone else’s mathematics repeatedly – are an exercise in conformity and rigidity.

One role of a mathematics teacher is to engage students in solving real problems that require mathematical ingenuity, which also requires ingenuity in communication because students have to communicate ideas that are new to them. Once the students have had a chance to explore mathematically, the teacher has another role – to draw their attention to each other’s mathematics. When students compare their mathematical ideas to those of their peers and to
historical or conventional mathematical practices, there is a need to standardize word-choice so people can understand each other’s ideas. In this sense, the mathematics register is a significant language phenomenon worth attending to. However, there is also value in deviating from it with awareness. Teachers who resist the strong tradition of pre-reform mathematics teaching are swearing, in a way, by deviating from tradition.

Response #3: Swear words remind us of the relationship between language and action.

There are connections between inappropriate words (swearing) and inappropriate actions. For example, it is inappropriate to use swear words publicly to refer to our bodies’ private parts, but it is even less appropriate to show these private parts in public. It is taboo.

This connection between action and words exists for appropriate as well as inappropriate action. Yackel and Cobb (1996) describe the routines of mathematics class communication as ‘sociomathematical norms.’ These norms significantly influence students’ understanding of what mathematics is. Because teachers use language and gesture to guide the development of these norms, this language practice relates to conceptions of what mathematics is and does. Thus, I suggest that there is value in drawing students’ attention to the way words are used in mathematics class, to help them understand the nature of their mathematical action. This goes beyond the common and necessary practice of helping students mimic the conventions of the mathematics register. Students can be encouraged to investigate some of the peculiarities of the register, and to find a range of ways to participate in this register.

For example, we might note that our mathematics textbook does not use the personal pronouns ‘I’ and ‘we’ and then ask students whether (or when) they should use these pronouns in mathematics class. When I asked this question of a class I was co-teaching for a research project,
most of the students said personal pronouns were not appropriate because mathematics is supposed to be independent of personal particularities, yet these same students continued to use personal pronouns when they were constructing their new mathematical ideas. A student who said, “Personally, I think you shouldn’t use ‘I’, ‘you’, or ‘we’ or ‘me’ or whatever” also said later “I’m always thinking in the ‘I’ form when I’m doing my math. I don’t know why. It’s just, I’ve always thought that way. Because I’m always doing something.” (The research that this is part of is elaborated in Wagner, 2007.) The tension between students’ personal agency in mathematical action and their sense of how mathematics ought to appear is central to what mathematics is.

Mathematical writing tends to obscure the decisions of the people doing the mathematics. Students are accustomed to word problems like this: “The given equation represents the height of a football in relation to time…”. The reality that equations come from people acting in particular contexts is glossed over by the structure of the sentence. Where did the equation come from? The perennial student question, “Why are we doing this?”, may seem like a swear itself as it seems to challenge the authority of classroom practice. However, it is the most important question students can ask because even their so-called applications of mathematics typically suggest that equations exist without human involvement.

Though I find it somewhat disturbing when mathematicians and others ignore human particularities, it is important to recognize that this loss is central to the nature of mathematics. Generalization and abstraction are features of mathematical thinking, and they have their place in thoughtful human problem solving. There is value in asking what is always true regardless of context. There is also value in prompting mathematics students to realize how mathematics obscures context and to discuss the appropriateness of this obfuscation. Mathematics students
should make unique contributions (using the word ‘I’) and find ways of generalizing (losing the word ‘I’).

This connection between agency-masking language form and mathematics’ characteristic generalization, is merely one example of the way language and action are connected. Whenever we read research on discourse in mathematics classrooms we can consider the connections between mathematics and the aspects of discourse described in the research. As with the example given here, talking about these connections with students can help them understand both the nature of mathematics and the peculiarities of the mathematics register. A good way of starting such a conversation is to notice the times when students break the normal discourse rules – the times that they ‘swear’. We can take their mathematical swears as an opportunity to discuss different possible ways of structuring mathematical conversations.

Response #4: I’m not sure how to swear mathematically, but I know when I swear in mathematics class!

The connection between human intention and mathematics reminds me of one student’s work on the above-described investigation that had ‘5-squares’ and ’45-squares’. For the research, there was a tape recorder at each group’s table. Ryan’s group was working on the task, which is described in the same article (Wagner, 2003). Ryan had made a conjecture and was testing it with various cases. Listening later, I heard his quiet work punctuated with muffled grunts of affirmation for each example that verified his conjecture, until he exclaimed a loud and clear expletive, uttered when he proved his conjecture false.

Linguistic analysis of swearing practices shows how it marks a sense of attachment (Wajnryb, 2005). Ryan swore because he cared. He cared about his mathematics. He cared about
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his conjecture and wanted to know whether it was generalizable. His feverish work and his frustrated expletive made this clear. I want my students to have this kind of attachment to the tasks I give them, even if it gets them swearing in frustration or wonder (though I’d rather have them express their frustration and wonder in other ways). The root of their frustration is also behind their sense of satisfaction when they develop their own ways of understanding. As is often the case with refuted conjectures, finding a counterexample helped Ryan refine the conjecture into one he could justify.

To help my students develop a sense of attachment to their mathematics, I need to give them mathematical investigations that present them with real problems. They may swear in frustration but they will also find satisfaction and pleasure.

Reflection

Swearing is about bucking the norm. The history of mathematics is rich with examples of the value of people doing things that others say should not be done. Thus there is a tension facing mathematics teachers who want both a disciplined class and one that explores new ideas.

Though my own experiences as a mathematics student were strictly discipline-oriented, I try to provide for my students a different kind of discourse – a classroom that encourages creativity. I want my students to swear mathematically for at least four reasons. 1) Understanding the non-permissible helps us understand normal practice and to open up new forms of practice. 2) Creative expression casts them as participants in the long and diverse history of mathematical understanding, which is sometimes called the universal language of mathematics. 3) Attention to the relationship between language and action can help students understand both. 4) The student
who swears cares: the student who chooses a unique path is showing engagement in the discipline.

References


