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Elijah Bodish

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Cubism and the Fourth Dimension

Elijah Bodish¹
The University of Montana

Pablo Picasso. "Portrait of Daniel-Henry Kahnweiler." Oil on canvas, 1910. (Cabanne 138).

¹ E-mail: elijah.bodish@gmail.com

When one looks into the subject of geometries that attempt to explain fourth-dimensional space, it is inevitable that one encounters references to Cubism. The purpose of this paper is to find what the similarities between this mathematical concept and cubism are. There are many historical arguments as to how the cubists encountered literature about the fourth-dimension, and whether they were exposed to it at all, which I will for the most part omit and instead let the art speak for itself. It is important to see how two fields are interrelated in order to gain a better understanding of both fields, in this case art and geometry. In addition, visualizing things that the human eye cannot immediately perceive, that must be left up to the mind is important to people who want to gain a better understanding of their reality.

Leone Batista Alberti, in 1435, wrote the first book that discussed central projection and section, the process in which an artist would transfer an object onto a canvas by imagining that the image is traced onto a window, parallel to the artist's eye, which is looking out onto the subject.

“Tracing on Glass, After Nature”, from Frederic Goupil, *La perspective expérimentale, artistique, methodique et attruyante ou l'orthographie des formes* (1860) (Henderson, plate 41).

When one considers the space around oneself, as only perceived visually, all lines appear to converge away from the observer. This notion is adapted into the technique of perspective drawing, an attempt to render an image that visually makes sense on a canvas but spatially is inaccurate in its representation. In a one-point perspective drawing, there is a horizontal horizon line, which lies at infinity. On the horizon line there is a vanishing point in which all lines parallel to the z-axis intersect.

When a cube is projected onto a two-dimensional surface using a perspective technique (I) there is much less confusion as what the image is representing in space. However, if the cube is drawn on a two-dimensional surface and is not distorted in any way (II), all vertices are of equal length and no lines intersect at the horizon, in this case it is much more difficult to determine what the image represents in space.

In Plato's "Allegory of the Cave", he discusses with Socrates a hypothetical world where people are born chained in a cave where they would only see the shadows of reality. Then at a certain time, they would be unchained and upon leaving the dark cave and approaching the light, the former prisoners would initially be blind to reality. Now imagine that humans have been similarly "chained" in the fourth-dimension so that they can only see the shadows cast into a third dimension and are blind to the fourth-dimension. This hypothetical idea is part of what created theories and geometries concerning the fourth-dimension, and is part of what made it popular since a better understanding of extra dimensions would bring a more enlightened understanding of reality.

The fourth-dimension is built from the similarities found in the geometry we are accustomed to visualizing. Beginning with a zero-dimensional point, and then by moving that point in any direction for any length creates a line (and an x-axis). Moving the line perpendicular to the x-axis creates a plane (and an x and y-axis). Then moving the plane perpendicular to both the x and y-axis creates a space (and

an x, y, and z-axis). This is the space we are accustomed to with a left-right, forward back, and up-down, it is easy to grasp what images of objects, in three-dimensional space, represent, regardless of how distorted they are because it is intuitive to us. By analogy of the previous transformations, moving a space perpendicular to the x, y, and z-axis creates a fourth dimension. However, due to our being stuck in three-dimensional space, we cannot visualize a fourth-dimensional coordinate system, or what an object in the fourth-dimension would look like. Two main methods of representing four dimensional objects, the slicing method and the projection method, have developed in an attempt to make the unseen seen.

The slicing technique may go as far back as 1846, when Gustav Theodor Fechner, in his book *Vier Paradoxa*, “may have published the first discussion of two-dimensional beings being unaware of the third dimension that surrounds them” (Henderson 18). The technique was popularized mainly by Edwin Abbotts book *Flatland*, which “E Jouffret discusses...in his 1903 *Traite elementaire de Geometry a quatre dimensions*, a book known to Duchamp and certain to his cubist friends” (Henderson 25). In *Flatland*, a two-dimensional being known as A. Square is visited by a sphere from three-dimensional spaceland. A. Square then proposes to the sphere that maybe spacelanders could be unaware of a surrounding fourth dimension. The sphere is infuriated by the idea of higher dimensions, but Abbot gets across the message that the ideas he proposes are not impossible to grasp.

When A. Square first encounters the sphere, from A. Square’s perspective it is a series of circles, starting with a point, increasing in size, then reducing in size back to a point, and finally disappearing, in the same fashion it appeared.

A. Square is observing the sphere passing through flatland. (Abbott 143).

The slicing model of visualizing the fourth dimension stems from these notions in that a fourth dimensional being passing through spaceland would appear to be a three-dimensional object gradually increasing and then decreasing in size. If a four-dimensional sphere were to enter spaceland, then it would look like a regular sphere of three-dimensions that appears from seemingly nowhere, increases and decreases in volume, and then disappears. Due to the regularity of a sphere, this

is not a very compelling example, since it doesn't provide much information as to how the sphere would look in the fourth dimension. A more interesting example is the hypercube (a fourth-dimensional cube) passing through the third dimension at right angles to the main diagonal of the hypercube. According to Ian Stewart this is Charles Hinton's, a late nineteenth century British physicist and mathematician, favored method of viewing a hypercube (Abbot 175).

Slices of a hypercube perpendicular to its main diagonal (Abbot 175)

Slices of a cube perpendicular to its main diagonal (Abbot 166)

In order to get a better understanding of the fourth dimension, these slices of our perception of the object must be viewed separately but considered as a whole.

The projection method of visualizing the fourth-dimension utilizes projective geometry, a product of perspective drawing. A cube can be drawn on a two-dimensional surface using projection techniques to appear as a square within a square, in which the cubes vertices are distorted in their actual length and the location of the smaller square on the z-axis is not readily discernible unless it is known that the object is a cube.

A projection of a cube onto a plane

Analogously, a hypercube can be projected into three-dimensional space as an object containing eight cubes, including a surrounding cube. However, similar to the projected image of the cube the projection of the hypercube distorts the lengths of the vertices and the location of the eight cubes, in the fourth dimension, in respect to each other.

A drawing of a projection of a hypercube into space

Cubism was born out of the paintings made by two friends, Georges Braque and Pablo Picasso, in France during the early twentieth century. They were both attempting to move in a direction that opposed traditional perspective drawings of the world around them. Guillaume Apollinaire, an art critic and poet, wrote that Braque and Picasso were “moving toward an entirely new art which will stand, with respect to painting as envisaged theretofore as music stands to literature. It will be pure painting as music is pure literature” (Stokstad 1077). Picasso suggested, “the viewer should approach the painting the way one would a musical composition...by analyzing it but not asking what it represents” (Stokstad 1077). “Pure Painting” may be a representation of the fourth dimension, a more complete way of looking at space, but also a way of seeing it that cannot be understood completely until each part of it is analyzed one by one. This method of viewing cubist paintings is similar to the method employed when one considers the slicing model of the fourth dimension.

Georges Braque. “Man with a Guitar.” Oil and sawdust, 1914. (Cabanne 50).

Despite the obvious similarity between Picasso and Appolinaire's statements about cubism's relations to music and the subject matter of Braque's painting being a guitar, this piece also provides evidence for the cubist's influence by the ideas of

fourth dimensional geometry. The painting is made up of various slices of space reduced to a two-dimensional image, and then represented together to imply their relation to one another. The head at the top of the painting is shaped like a cube and floats separately from the rest of the figure, adding another element to the fragmentation of the painting. Braque's painting could be seen as a time lapse of a higher dimensional figure passing through a lower dimension.

Marcel Duchamp, an important member of two early nineteenth century art movements Dada and Cubism, seems to have the most technical knowledge of fourth dimensional geometry compared to the rest of the artists involved in the cubist movement. Using his painting from 1912, "The Bride," as a starting point Duchamp set out to create a pictorial representation of the fourth dimension that surpassed all his earlier attempts. From 1915 to 1923, Duchamp worked on this piece, which ended up being titled "The Bride Stripped Bare by Her Bachelors, Even (The Large Glass)."

Marcel Duchamp. "The Bride Stripped Bare by Her Bachelors, Even (The Large Glass)." Oil, varnish, lead foil, lead wire, and dust on glass panels encased in glass, 1915-1923. (Stokstad 1103).

In his notes for the painting, Duchamp stressed the, "distinction between the 'fluidity' of the bride's domain (top panel), and the strictly measured three-dimensional perspective of the bachelor apparatus (bottom panel)" (Henderson 134). Part of Duchamp's understanding of the fourth dimension is related to the idea that fourth-dimensional objects, due to their fluidity, are immeasurable, which is illustrated by the cloudy abstraction that makes up part of the bride. Duchamp

seemed to relate to the notion of fourth-dimensional objects being the more complete image of the object, and that the part of the object we are accustomed to seeing is only a small piece in a larger body. Similar to the projection technique of viewing fourth-dimensional objects, Duchamp wrote in his notes on the painting that in order to grasp the fourth dimension one must “construct all the three-dimensional states of the four –dimensional figure the same way one determines all the planes or sides of a three dimensional figure” (Henderson 140). “A Large Glass” was painted on a glass pane because Duchamp believed that in order to “permit an imaginative reconstruction of the numerous four-dimensional bodies” (Henderson 141), the viewer must be able to see the images from multiple perspectives. By painting on glass, it allows the viewer to wander around the piece and get a better understanding of a higher dimensional object. Similar to how a sphere appears as a circle from only one perspective and it is not apparent that the object is a sphere until it is observed as a circle from all perspectives by wandering around it. Another interesting reason why Duchamp may have used glass as a canvas is its similarity to the sphere looking into flatland. The observer sees on the bottom panel a three-dimensional image made into a two-dimensional slice of the observer’s space.

Fourth-dimensions of space are often misinterpreted as time being the fourth dimension. Although both theories are represented in cubism, they are distinctly separate. Time has been described as a fourth dimension since Joseph Lagrange’s *Theories des fonction analytiques*, from 1797. However, Charles Hinton “repeatedly turned to the notion that time could be defined as a fourth spatial dimension of geometry, not simply another number necessary to describe a place at a certain time” (Robbin 25). Hinton often used the example of a spiral being pulled through a plane, which from the point of view of a Flatlander appears to be a point moving around in a circle. “According to Hinton the spiral is the complete static model of the events...it has a greater philosophical reality than the moving point, and thus it should be the object of our consideration” (Robbin 25). The event is as dependant upon space as it is upon time, and could therefore be said to be an invariant model of reality. This notion of time as an extra dimension came before the current status-quo understanding of space-time dictated by Einstein’s theory of special relativity and Minkowski’s geometry that goes along with it.

Special Relativity is derived from two principles. Both are experimental facts boldly assumed to hold universally. The first says that physical laws are the same for all observers. The second states that it is a law that light travels at 300,000 km/sec [c] (Kennedy 17).

From these assumed principles, one can infer that the time and space of an object vary depending upon the velocity of that object. This occurs in Einstein’s mind because if a ship is traveling at 200,000 km/sec, 100,000km/sec slower than (c) and another ship is traveling in the same direction at 100,000 km/sec, 200,000 km/sec slower than (c), and light as observed from both ships travels at a constant

speed of (c) then both ships must be measuring the speed of light with different perceptions of time and space (in this case each ship would have a different set interval for what a second and a kilometer looked like).

It would be less surprising to the astronauts in both ships if ship (I) measured (c) to be 100,000 km/sec and ship (II) measured (c) to be 200,000 km/sec since the velocity of an object moving alongside another object would logically be observed to change, depending upon the velocity of the measuring object. However, since both ships actually measure (c) to be the same, then it can be assumed that on ship (I) time passes slower and distances are smaller than ship (II) which has longer time intervals and shorter special measurements than a ship at rest. For clarification, the shortening in space of lengths “contracts only in the direction of travel, and its diameter remains the same” (Kennedy 19).

Minkowski referred to his four-dimensional space-time formulation of the special theory of relativity as entailing that “space by itself and time by itself are doomed to fade away into mere shadows, and only a kind of unity of the two will preserve an independent reality” (Joseph 426).

Since three-dimensional geometry only deals with space, it is not able to explain the implications of special relativity. In order to graph four dimensions, which is what Minkowski and Einstein believed our universe was, on a Cartesian plane the z and y-axis must be removed, it is much easier to look at the two variables that are subject to change, x and t. Since time is always flowing through our universe, and length contractions only affect the x-axis, the z and y-axis can be taken out of immediate consideration. Using Minkowski’s geometry a graph of something at rest would look like a vertical line, since time continues to pass and the objects length is constant at a single speed. The faster something moves through space the steeper the slope of the graph becomes. In Minkowski’s mind, since time and space are both subject to change in the form of either dilations or contractions respectively, the only way to understand the true nature of an event is to combine the two variable things into an invariable space-time interval. The space-time interval is constant because according to the Lorentz transformation length and time are inversely related. The smaller the length of something is the larger the time is. As a time interval increases the length decreases, and as a length increases the time interval decreases, but the space-time interval stays the same, since the ratio between the two is an inverse relation.

Although it is likely that the ambiguity of both topics of the fourth dimension caused them to be interrelated and subsequently used synonymously with each other by many people, including some Cubist painters. The idea of a more complete understanding of the surrounding world still is prevalent in both space-time geometry, and the geometry of four spatial dimensions. Many cubist paintings contain more resemblances to the slice and projection models of the fourth-dimension. However, one major piece, Marcel Duchamp’s “Nude

Descending the Staircase,” displays his views of the importance of time in its relation to space.

Partially inspired by early photographs of objects in motion, Duchamp, instead of producing a rendering of a static image captures, the motion of the object, and paints an event instead. Regardless of the distortions of the subject, since the painting takes the fourth-dimension of time into account it is in a way a more true to life rendering of reality than a static image, which is known to be distorted on a canvas as well even if the artist tries to paint the subject as it is seen.

Marcel Duchamp. “Nude Descending a Staircase #2.” Oil on canvas, 1912. (Cabanne 85).

The search for what is really real and how it can be represented honestly by an artist seems to be a major driving force for the cubist's interest in the fourth-dimension. Whether it is the invariance of space-time or the complete representation of an object in a fourth spatial dimension as opposed to the three-dimensional slice of reality we see from day to day. Many people tend to interpret cubism as the artists fragmented interpretation of the world, a representation of the fragmentation of Europe after WWI. However, it may actually be a much more optimistic art movement than people realize, while it is a rejection of the past, it also may show hope for humans' ability to find the true nature of the surrounding world. In 1908 Henry Poincare wrote, in his book *Science et Methode*, "One who devoted his life to it could perhaps eventually be able to picture the fourth dimension" (Krauss 85). Similar to the optimism of Einstein and other scientists during the early nineteenth century, cubism also put a great amount of faith into there being an order to the universe that humans can understand. It is also important that people don't view mathematics as a dull unchanging subject that is only of help to engineers and scientists. When someone brings up art and mathematics in the same sentence usually it leads to a detracting statement towards one of the topics, or M.C. Escher enters the conversation. Cubism cannot be understood completely unless it is looked at through an artist's, mathematician's, and historian's perspective. Combining these different slices of the whole interpretation is the best way to look at not just the nature of space but also the nature of people and the surrounding world.

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