The Mathematics Enthusiast

Volume 7 | Number 1 Article 2

1-2010

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Abstract: *In this study, we document and analyse problem-solving approaches that high school teachers exhibited as a result of using dynamic software (Cabri-Geometry) to construct and examine geometric configurations. What type of questions do teachers pose and pursue while representing and exploring mathematical tasks or objects dynamically? To what extent their initial problem solving strategies are enhanced with the use of the tools? Results indicate that the use of the tool offered the participants the opportunity of constructing geometric configuration (formed by simple mathematical objects) that led them to identify and explore key mathematical relations.*

Keywords: CAS; Dynamic geometry; mathematical relations; problem solving; teaching and learning of geometry; problem posing; conics

Introduction

The explosive development and availability of computational tools (Spreadsheets, Computer Algebra Systems –CAS- dynamic software (Cabri-Geometry, Geometry Inventor, Geometer's Sketchpad), and graphic calculators) have notably influenced not only the ways how mathematics is developed, but also how the discipline can be learned or constructed by teachers and students. In particular, tasks or problems' approaches based on the use of the tools offer teachers and students the opportunity of representing and examining the tasks from perspectives that involve visual, numeric, geometric, and algebraic reasoning. Thus, it is important to document the impact and types of

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transformations that the use of those tools brings into school mathematics. As Artigue (2005) mentioned "school, as is the case every time that it faces an evolution of scientific and/or social practices, can neither stand apart from this evolution, nor ignore the new needs it generates" (p. 232).

What type of mathematical competencies becomes relevant for teachers to promote during their instructional practices that enhance the use of computational tools? What hypothetical students' learning trajectories can be identified while examining the tasks through the use of computational tools? What features of mathematical thinking can be enhanced with the use of particular tools? These are important questions that need to be discussed in order to shed light on the relevance, for teachers, to systematically use diverse computational tools in their problem solving approaches. We recognize that different tools may offer various opportunities for teachers and students to identify, represent, and explore relationships embedded in mathematical problems.

> Regardless of the particular tools that are used, they are likely to shape the way we think. Mathematical activity requires the use of tools, and the tools we use influence the way we think about the activity…[Understanding] is made up of many connections or relationships. Some tools help students make certain connections; other tools encourage different connection (Hiebert, et al, 1997, p.10).

The use of dynamic software seems to offer teachers and students the possibility of constructing and analysing mathematical relationships in terms of loci that result from moving elements within the representation of the problem (Santos-Trigo, 2008). In general, the use of the tools can help teachers identify and explore potential instructional trajectories to frame the development of their lessons. In this perspective, it is relevant for teachers to use computational tools to document the type of mathematical thinking that can emerge in students' problem solving approaches.

Conceptual Framework

Fundamental principles associated with problem solving approaches and the use of computational tools were used to organize and structure the development of this study, namely:

(i) The recognition that teachers need to think of their mathematical instruction as a problem solving activity in which contents, problems or phenomena are seen as dilemmas that need to be examined, explained, and solved in terms of formulating and pursuing relevant questions or inquiry methods (Santos-Trigo, 2007). As Postman and Weingartner (1969) stated:

> Knowledge is produced in response to questions…Once you have learned how to ask questions –relevant and appropriate and substantial questions- you have learned how to learn and no one can keep you from learning whatever you want or need to know, (pp.23).

In similar vein, Romberg and Kaput (1999) recognize that teachers should provide proper learning conditions for their students to appreciate, value and develop a mathematical thinking consistent with the practice of the discipline. In particular, they need to participate in genuine mathematical inquiry.

> By genuine inquiry, we mean the process of raising and evaluating questions ground in experience, proposing and developing alternative explanations, marshaling evidence from various sources, representing and presenting that information to a larger community, and debating the persuasive power of that information with respect to various claims (p. 11).

Roschelle, Kaput, and Stroup (2000) recognize that the use of technology plays an important role in mediating the process of inquiry: "Inquiry allows incremental, continual growth of understanding from child's experience to the core subject matter concepts" (p. 50). That is, teachers need to problematize the content they teach by formulating and discussing questions that lead them to identify difficulties that might arise while their students use computational tools in their learning experiences. In this process, they should constantly reflect on ways to articulate and structure their lesson and problem solving activities.

> Articulation requires reflection in that it involves lifting out the critical ideas of an activity so that the essence of the activity can be communicated. In the process, the activity becomes an object of thought. In other words, in order to articulate our ideas, we must reflect on them in order to identify and describe critical elements (Carpenter & Lehrer, 1999, p.22).

Jaworski (2006) discusses the relation between the notion of inquiry and cognitive and social perspectives.

> Inquiry, or investigative methods in mathematics teaching are seen to fit with constructivist view of knowledge and learning: they demand activity, offer challenges to stimulate mathematical thinking and create opportunities for critical reflection on mathematical understanding (Jaworski, 2006, p. 199).

Further, she also mentioned that:

> While inquiry tools might offer developmental possibilities for individuals within social settings, the prevalence of social norms and processes of social enculturation will be more powerful influences on learning than will cognitive stimulus central in constructivist theory (Jaworski, 2006, p. 200).

(ii) The importance for teachers and students to think of distinct ways to represent, explore or solve mathematics problems. Here, the use of technology might provide a means to examine problems representations from distinct perspectives. Guin and Trouche (2002) recognize the importance of the process, for people, to transform an artefact (a material object) into an instrument or problem-solving tool. This process involves aspects related to both the actual design of the artefact and the cognitive process shown by the user during the appropriation of the tool. Teachers' direct participation in designing mathematical tasks that involves the use of computational tools becomes important not only, for teachers, to recognize and discuss ways to employ the software in problem solving activities; but also to identify and analyse theoretical instructional trajectories that might help their students to transform the tool into a problem solving instrument.

> Students' engagement with, and ownership of, abstract mathematical ideas can be fostered through technology. Technology enriches the range and quality of investigations by providing a means of viewing mathematical ideas from multiple perspectives (The NCTM, 2000, p. 25).

(iii) Learning takes place within a community that promotes that active participation of all members. Thus, it is important to provide an environment in which each member values not only the need to express his/her own ideas; but also to listen and understand other's ideas. As Wells (2001, pp. 179-180) stated:

> …the way in which an activity unfolds depends upon the specific participants involved, their potential contribution, and the extent to which the actualisation of this potential is enabled by the interpersonal relationships between participants and the mediating artifacts at hand…Knowledge is constructed and reconstructed between participants in specific situations, using the cultural resources at their disposal, as they work toward the collaborative achievement of goals that emerge in the course of the activity.

Thus, a reflective community promotes activities in which members have the opportunity of posing questions, making observations, using computational tools to identify, represent, explore mathematical relations, and communicate results.

> A fundamental aspect of a community of learners is communication. Effective communication requires a foundation of respect and trust among individuals. The ability to engage in the presentation of evidence, reasoned argument, and explanation comes from practice (NRC, 1999, p.50).

The Research Design, the Participants, and General Procedures

Eight high school teachers, all volunteers, participated in three hour weekly problem-solving seminar during one semester. The aim of the sessions was to select, design and work on a set of problems that could eventually be used in regular instruction. In this process, all the participants had opportunities to reveal and discuss their mathematical ideas openly and use both dynamic software and hand-held calculators to solve the problems. The idea was that the participants became familiar with the use of the software by representing themselves directly the mathematical objects dynamically. In addition, they also explore possible hypothetical learning trajectories that their students could follow during their instructional activities. While solving and discussing all the problems, themes related to curriculum, students' learning, and the evaluation of students' mathematical competences were also addressed. For example, the didactical sequence to study regular contents that appear in courses like analytic geometry was questioned since the use of dynamic software allows the study of the various conic sections dynamically at the same time. That is, one particular task could lead to the discussion of all conic section.

During the development of the sessions, the participants worked on the problems individually and in pairs. And later, they presented their work to the whole group. In general, the instructional activities were organized around a particular pedagogic approach in which the participants were encouraged to use an inquiry process to deal with the problems. As Jaworski (2006) indicates:

> It [the instructional approach] is a social process in the sense that a participant is a member of a community (e.g., of teachers, or of students learning mathematics) with its own practices and dynamics of practice which go through social metamorphoses as inquiry takes place. It is an individual process in that individuals are encouraged to look critically at their own practice and to modify these through their own learning practice (p. 2002).

Data used to analyse the work shown by the participants come from electronic files that they handed in at the end of the sessions and videotapes of the pair work and plenary sessions. In addition, each participant, at the end of the semester, presented the collection of problems and results that they had worked throughout the sessions. Here, it is important to mention that, in this study, we are interested in identifying and discussing problem solving approaches that emerge while high school teachers use computational artefacts rather than analysing in detail individual or small group performances.

Presentation of Results and Discussion

To present the results, we focus on analyzing what the participants exhibited while working on one task that involves the use of Cabri-Geometry software. The participants relied on the use of the software directly to construct a geometric configuration that led them to identify and explore all conic sections. The participants were given two points A & B as the initial objects to construct a geometric configuration to identify mathematical relations. What can you do with two points, A & B situated on the plane? This was the initial question that led the participants to construct dynamic configurations that eventually helped them to identify and explore particular mathematical results.

We focus on presenting initially the work of a pair of teachers to show the type of mathematical activity that they engaged while working on this task. Later, they presented their work to the whole group and received comments and responded to some questions. At this stage, the initial pair's work became the group's work since all participants contributed to the search of both conjectures and arguments around the initial pair's report.

A pair, Martin and Peter, considered drawing line AB, the perpendicular bisector of segment AB and perpendicular lines to line AB (L_1 and L_2) passing by point A and B respectively.

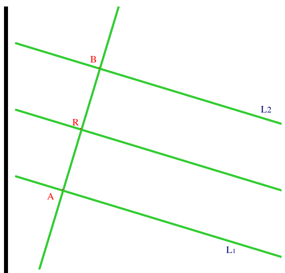

Figure 1: Drawing line AB, the perpendicular bisector of segment AB and perpendicular lines.

Martin and Peter were aware that Cabri Geometry software could be used to draw a conic that passes by five given points. Thus, their initial purpose was to situate five points in order to draw a conic passing by those points. Where should those points be situated? To this end, they drew two points P, Q on line L_2 and reflected those point with respect to the perpendicular bisector of AB to obtain points P' and Q' (figure 2). This arrangement was based on the consideration of the symmetry of an ellipse, for example (as possible candidate). Based on this information they drew (using the software) the conic that passes by the five points.

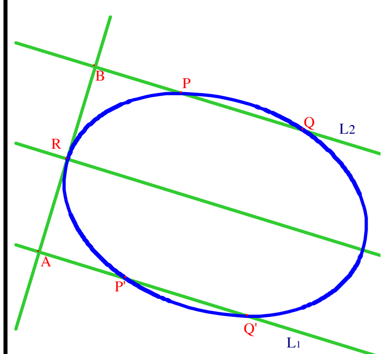

Figure 2: The figure that passes by points P, Q, R, P', and Q' seems to be an ellipse.

This pair recognized that when they decided to draw a conic (using the software command "conic") they thought of situating the five points on lines, so they can move them and observe the behavior of that conic. That is, they intended to construct a dynamic representation of a conic in which they could move certain points within the configuration and observe the behavior of the figure. Indeed, a crucial step to transform the use of the software into a problem-solving tool is to think of mathematical objects in terms of dynamic representations.

From visual to formal arguments

It is important to mention that when this construction was presented (by the two participants) to the whole seminar, then all agreed that the figure generated by the software represented visually an ellipse (visual recognition). However, some participants stated that it was necessary to present an argument to justify that such figure held properties associated with the ellipse. How do we know that the figure satisfy the definition of the conic? Where are the foci located? Where is its center? These were important questions to answer in order to show that, for that figure, the sum of the distance from the foci to any point on the figure is a constant number (definition of ellipse). At this stage, all members of the seminar began to think of ways to argue that the figure was an ellipse.

During the development of the plenary session, it was recognized that a problem solving strategy to identify and examine properties of the figure was to use the Cartesian system. Thus, without losing generality, the participants decided to take the perpendicular bisector of segment AB as the x-axis. Similarly, they took the perpendicular bisector of segment AB as the ellipse's focal axis, R and R' the points where the focal axis crosses the ellipse, that is, the ellipse's vertices and point C the center of the ellipse as the midpoint of segment RR' (figure 3).

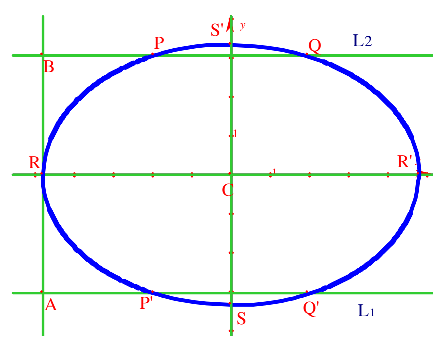

Figure 3: Identifying elements of the ellipse.

It is important to mention that the use of the software allows moving and changing the position of the objects easily. So, Figure 2 could easily be transformed into figure 3.

At this stage, the coordinator of the session (the researcher) asked: What type of equation represents an ellipse where its center is the origin of the Cartesian system? All agreed that the equation of the ellipse could be expressed as

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where $b^2=a^2-c^2$

Based on this information, they argued that to identify the ellipse's foci it was sufficient to draw a circle with center at point S' and radius the length of segment CR', since the intersection points of that circle with the focal axis will determine the ellipse's foci (figure 4).

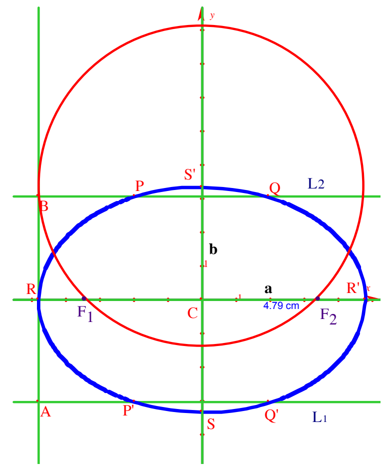

Figure 4: Drawing the ellipse's foci.

To verify that any point on the figure held the definition of the ellipse, Maria suggested to use the software to situate any point M on the figure and observed that by moving point M along the ellipse the sum of distances F_1M+F_2M remains constant (figure 5).

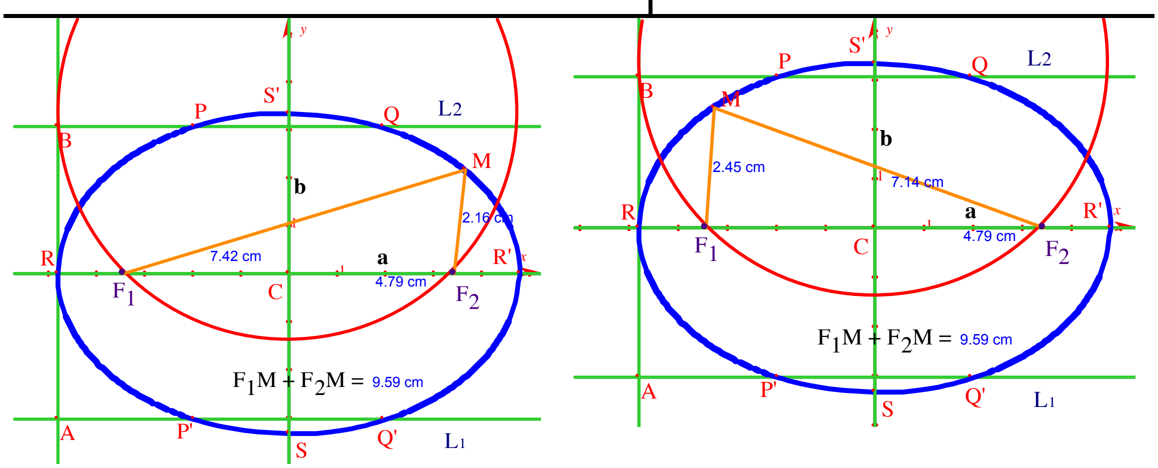

Figure 5: Verifying empirically the definition of the ellipse by moving point M along the ellipse.

Thus, they identified the equation of the above ellipse as $\frac{x^2}{(4.79)^2} + \frac{y^2}{(3.34)^2} = 1$ (Figure 5a).

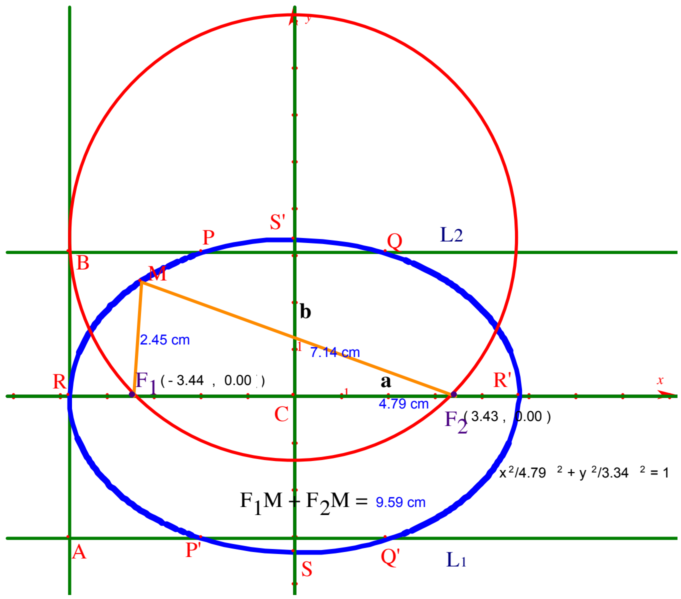

Figure 5a: Finding the equation of the ellipse.

Comment: The use of the tool helped the pair (Martin & Peter) situate five points on the plane to generate a conic section by using the five points software's command. To situate the points, Martin and & Peter relied on the idea of symmetry (P & P' and Q & Q' are symmetric with respect to the perpendicular to line AB that passes through point R) and a line on which points P & Q could be moved freely. Indeed, during the pair's presentation to the group other possibilities to situate the points on the plane were examined. For example, Robert proposed to situate P and Q on a circle with center on line L_2 (Figure 5b).

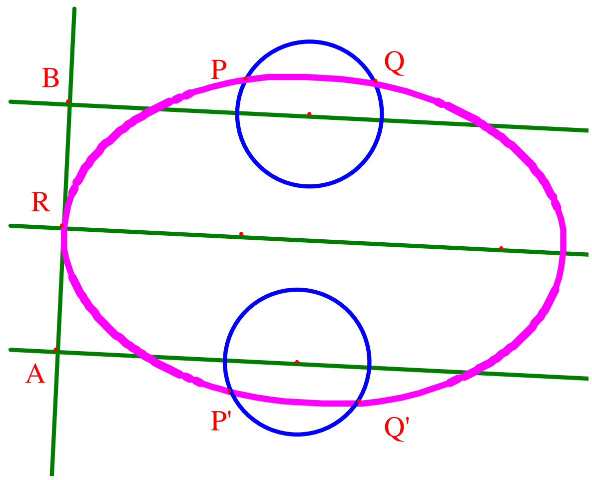

Figure 5b: Drawing a conic section by situating points P & Q on a circle.

It was observed that during the pair's presentation, the other members of the group not only proposed other ways to generate the conic section; but also explored properties attached to that figure. That is, the group became an inquiry community during the pairs' presentations and its members not only posed questions to examine properties of the figure; but also identified possible learning trajectories to frame their students' learning experiences.

Looking for other mathematical relations:

At this stage, it was natural for the participants to start moving points within the representation to explore the behavior of the figure. Points P and Q are candidates to be moved since P' and Q' depend on the position of P and Q respectively.

What does it happen to the ellipse when points P is moved along line L_2? The software allowed the participants to explore this type of questions since geometric properties of the mathematical objects (perpendicular bisector, symmetric points, etc) are maintained within the construction. For example, when point P is moved along line L_2, they found that at certain position of P the ellipse became a hyperbola (figure 6).

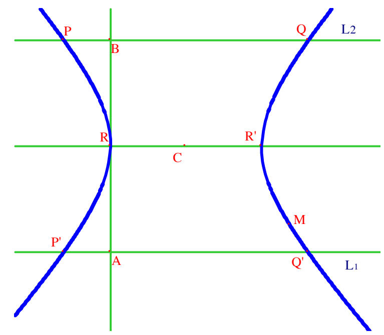

Figure 6: Moving P along line to generate a hyperbola.

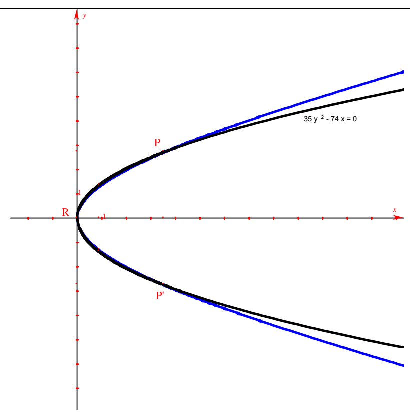

Figure 7: When point P is at infinity, a parabola will show.

Indeed, it could noticed that by moving either point P or Q along line other conics could be generated and their goal was to identify positions of P or Q in which a particular conic appears and also to verify the corresponding properties. In particular, they observed that when point Q goes to infinity a parabola is shown (figure 7).

Based on the idea of drawing symmetric points within a dynamic configuration, the participants proposed other isomorphic ways to draw some conics. For example, David's configuration involves drawing segment AB and lines L_1 and L_2 passing by point B and A respectively. These lines get intersected at point M. Line L_3 is the bisector of angle BMA. On line L_1, he situated points P and Q and reflect them with respect line L_3 to obtain P' and Q'. Using the software, he drew the conic that passes by point P, Q, P', Q' and R (intersection of segment AB and line L_3) to get figure 8. Again by moving point P along L_1 it was observed that at different positions of P other conics appear (figure 9).

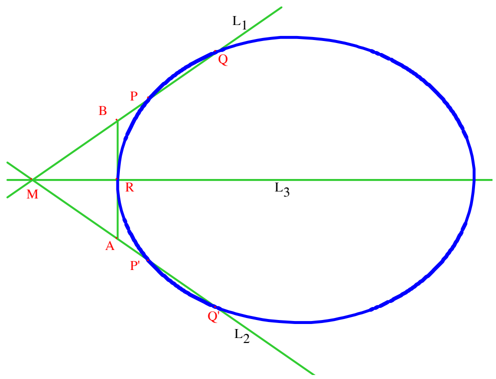

Figure 8: An isomorphic construction to generate all conics.

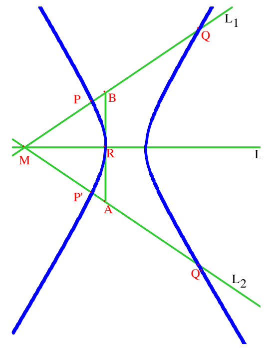

Figure 9: By moving point P along line L_1 other conic sections appear

Thus, the work presented by the Martin and Peter inspired the other participants to think of similar construction to generate the conic sections. That is, the community identified the essential elements associated with Martin & Peter's idea to develop a tool (geometric configuration) to identify and discuss properties associated with the conic sections. In addition, working on this type of tasks helped them recognize the importance of visualizing relations, analyzing particular cases, measuring attributes, and looking for analytical arguments. Thus, there is evidence that the participants have identified a set of strategies that are useful to represent and examine dynamic configurations. It is clear the use of the software facilitates they ways in which teachers and students can apply those problem-solving strategies. As the NCTM (2000) suggests "…no strategy is learned once and for all; strategies are learned over time, are applied in particular contexts, and become more refined, elaborate, and flexible as they are used in increasingly complex problem situations" (p. 54).

Reflections: The participants were surprised that the use of the software helped them identify and explore conic sections properties based on constructing a particular dynamic configuration. All recognized that the use of the software not only can offer their students the possibility of exploring visually and empirically the behavior of mathematical objects, but it can also be useful to make sense of results expressed algebraically. In addition, the use of the software may help students participate in the process of reconstructing some mathematical results and help them transit from visual and empirical explanations to formal approaches to the problems. Here, all the participants were aware that the use of the software provides a different path to study the conics, compared with traditional approaches based on paper and pencil, and also to develop new results. They also recognized that the use of the software offers the potential of constructing dynamic configurations in which they can identify interesting mathematical relations. In particular, some teachers were surprised to observe that all the conics, that they teach in an entire analytic geometry course, could be generated by moving points within a representation that involves points, perpendicular lines and a perpendicular bisector. In addition, they observed that the software functions as a vehicle to explore relations among objects that are difficult to think of or identify using only paper and pencil approach. Furthermore, the facility to quantify lengths of segments allows the problem solver to identify and explore the plausibility of particular mathematical relations. Thus, the participants conceptualized learning as a process in which their ideas and approaches to tasks are refined as a result of examining openly not only what they think of the problem but also discussing and criticizing the ideas and approaches of other participants. In particular, the participants recognized that initial incoherent attempts to solve the problems could be transformed into robust approaches when the learning environment values and promotes the active participation of the learners. In general, they recognized that the use of the tool not only helped them visualize and explore mathematical relations but also to recognize potential learning trajectories that their students could follow.

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Note: We acknowledge the support received by Conacyt (reference 47850).