7-2010

Rehearsal or Reorganization: Two Patterns of Literacy Strategy Use in Secondary Mathematics Classes

Anne Adams

Follow this and additional works at: <https://scholarworks.umt.edu/tme>

Part of the [Mathematics Commons](#)

Let us know how access to this document benefits you.

Recommended Citation

Adams, Anne (2010) "Rehearsal or Reorganization: Two Patterns of Literacy Strategy Use in Secondary Mathematics Classes," *The Mathematics Enthusiast*. Vol. 7 : No. 2 , Article 14.

DOI: <https://doi.org/10.54870/1551-3440.1195>

Available at: <https://scholarworks.umt.edu/tme/vol7/iss2/14>

This Article is brought to you for free and open access by ScholarWorks at University of Montana. It has been accepted for inclusion in The Mathematics Enthusiast by an authorized editor of ScholarWorks at University of Montana. For more information, please contact scholarworks@mso.umt.edu.

Rehearsal or Reorganization Two Patterns of Literacy Strategy Use in Secondary Mathematics Classes

Anne Adams¹
University of Idaho

Abstract: This study presents two critical cases illustrating distinct patterns in teachers' use of literacy strategies in secondary mathematics classes. The cases are part of a professional development project designed to enhance teachers' pedagogical skills by developing content literacy strategies for use in secondary mathematics and science classrooms. Teachers' beliefs about teaching mathematics, their uses of writing and vocabulary development strategies, and goals for student learning were examined via interviews, classroom observations, reflections on teaching, and teacher posts to an on-line discussion forum. Results show patterns of literacy strategy use were related to teachers' views of pedagogy and of mathematics. Ned, who held a procedural approach to teaching mathematics, used strategies as a rehearsal tool to support remembering correct ideas, fact, and procedures. Christine, with a conceptual approach to teaching, used literacy strategies as a tool to support deepening and reorganizing student understanding of mathematical concepts and relationships.

Keywords: literacy strategies; mathematics teacher education; mathematics and writing; teacher professional development

Language is the medium of human interaction as well as much of human thought. As such, learning mathematics is as much about learning language as about mathematical objects and relationships. The language of mathematics both describes concepts and helps to shape them (Usiskin, 1996). "Words are tools for thinking—in mathematics as well as in other disciplines" (Countryman, 1992), p. 57). Mathematicians and students use language to make sense of new information, develop new ideas, and organize their understanding of the relationships among these, as well as communicate their understanding. Essentially, the use of language is integrally involved in the development of concepts and relationships and in our understanding of the world around us (Vygotsky, 1962). Without language we would not have mathematics.

¹College of Education; Department of Curriculum & Instruction
P.O. Box 443082
Moscow, Idaho 83844-3082
aeadams@uidaho.edu

Literacy: Tools for Thinking about Mathematics

Effective use of literacy skills underlies effective use of mathematics. “Language arts provide the tools for teachers and students to read and understand problems, to write and draw their way toward understanding, and to communicate effectively” (Fogelberg, Skalinder, Satz, Hiller, Bernstein, & Vitantonio, 2008, p. 4). One aim of reform based mathematics education is to create classrooms where mathematical understanding is a reality for students. Infusing literacy strategies into instruction may provide a key. The National Council of Teachers of Mathematics (NCTM) argues the importance of giving students “experiences that help them appreciate the power and precision of mathematical language” (NCTM, 2000, p. 63). When writing to learn, different from writing to demonstrate learning, ideas occur as one writes. This practice offers opportunities to practice a variety of mathematical and thinking processes including observing patterns and relationships, making generalizations and conjectures, inferring, predicting, communicating, summarizing, interpreting, organizing, explaining, representing mathematical ideas, reflecting, and justifying one’s thinking. These are components of NCTM’s process standards for learning mathematics described in *Principles and Standards for School Mathematics* (NCTM, 2000). Students should be able to communicate their mathematical thinking coherently and clearly, analyze and evaluate the mathematical thinking strategies of others, and use mathematical language to express ideas precisely (NCTM, 2000).

With this information in mind, the Literacy Instruction for Secondary Mathematics and Science Teachers (LIMSST) project was created. The goal of this professional development project was to enhance the pedagogical skills of secondary mathematics and science teachers by infusing additional literacy skill instruction into their curriculum. The project set out to encourage and support them in developing content literacy strategies for use in their classrooms. The aim was to teach a selection of research based literacy strategies and to support the teachers over the course of a school year as they learned to integrate these strategies into their content area courses. Although the literature suggests many potential benefits of integrating literacy strategies into mathematics classes (Countryman, 1992; Forget, 2004; Murray, 2004), there has been little research showing how such strategies are actually being used. This study examines the literacy strategy practices of mathematics teachers in the project.

Literacy Tools for Learning

Writing in mathematics can play a positive role in students’ construction of knowledge during mathematics learning activities. Writing serves as a means of helping students organize, analyze, interpret, and communicate mathematical ideas, leading to a deeper understanding of content concepts (Burns, 2004; Holliday, Yore, & Alverman, 1994). “Writing in mathematics can also help students consolidate their thinking because it requires them to reflect on their work and clarify their thoughts about the ideas” (NCTM, 2000, p. 61). The writing process can play a vital role in developing mathematical literacy and understanding. Research has shown that writing increases understanding, achievement, and problem solving skill (Bangert-Drowns, Murley, & Wilkinson, 2004; Borasi & Rose, 1989; Clarke, Waywood, & Stephens, 1993; Herrick, 2005; Steele, 2005, 2007)

The language of mathematics is both abstract and complex. In order to help students work through the complexities, they need experiences connecting everyday language to mathematics, distinguishing the various meanings and contexts of mathematical vocabulary terms, and connecting new mathematical knowledge with prior knowledge (Harmon, Hedrick, & Wood, 2005; Rubenstein, 2007). Frequent opportunities to use mathematics in context serve to create connections between mathematical ideas and how they are used, just as foreign languages are learned through use in context.

Beneficial learning opportunities include higher order processing skills such as analysis, synthesis, and evaluation of concepts and definitions (Adams, Thangata, & King, 2005; Monroe, 1997). In general, students need support in learning to use specific text features and in reading comprehension strategies as they interact with text and interpret the meaning on the page (Carter, & Dean, 2006; Kenney, Hancewicz, Heuer, Metsisto, & Tuttle, 2005; Vacaretu, 2008). Through this process, students of mathematics develop skill in reading mathematical text and construct meaning of mathematical ideas. Reading and writing can serve as tools for learning and thinking about the language and concepts of mathematics. Use of writing as a meaning making tool in mathematics class can support students both in learning mathematics and developing higher order thinking skills.

Theoretical Framework

The meaning that one constructs for mathematical ideas is interlaced with the language with which one learned to reason about those concepts, words, and symbols (Pimm, 1995). Effective vocabulary instruction supports students in their own sense making by connecting words and concepts to students' prior knowledge, involving students in higher order thinking as they develop meaning for words, and providing frequent opportunities for them to use mathematical vocabulary and language in meaningful ways (Adams, 2003; Adams, Thangata, & King, 2005; Harmon, Hedrick, & Wood, 2005; Monroe, 1997; Thompson & Rubenstein, 2000). Constructing visual representations of concepts and vocabulary not only serves to deepen understanding, but to improve ability to recall the knowledge for later use (Marzano, Pickering, & Pollock, 2001; Rubenstein & Thompson, 2002).

Noting students' difficulties in understanding formal definitions for mathematical concepts, (Tall & Vinner, 1981) distinguished between the constructs of concept image and concept definition in explaining individuals' conceptions of concepts. They termed the concept definition as the formal words used to define a concept. In contrast, the concept image consists of all the various mental images, processes, and any associated properties brought to mind by an individual in considering a given concept. This concept image denotes an individual's conceptual structure of a concept (Tall, 1988; Tall & Vinner, 1981). When thinking of a term, it is the concept image that comes to mind, not the concept definition. The various components of an individual's concept image need not be coherent or consistent. A learner's concept images may also be in conflict with the formal concept definition accepted by the mathematics community. Concept images are based on an individual's experiences and may embody many facets not included in the concept definition. For example, the concept *slope of a line* is defined as rise/run, yet the term *slope* may evoke in an individual the various images of a graph of a line and its associated steepness, a table of values indicating the relative change in y for a one unit change in x, a ski slope or a road (which may have a changing slope), a formula for calculating the slope of a line, or the coefficient of x in a linear equation. Tall (1988) contends that students cannot use

concept definitions alone. When a student is simply given a concept definition, it forms a weak concept image. Since humans rely primarily on their concept images, students need opportunities to develop strong concept images that are in accordance with the desired concept definition.

Two Views of Vocabulary Development

Two broad views of vocabulary development are presented. Based on constructs identified by Tall and Vinner (1981), I have termed these the concept definition view and the concept image view.

Learn the Concept Definition. This view holds that each vocabulary term has a precise and static verbal definition that must be learned. Such definitions are found in mathematics textbooks or in dictionaries and represent the true meaning of the term. Meanings are fixed and external to students, who all need to learn the same meaning for each term. Knowledge of the correct definition should provide an individual with a correct concept image. Useful learning activities make terms and definitions memorable or allow one to review the terms so they are not easily forgotten. Common approaches include presenting formal definitions for terms or asking students to find these in textbooks or dictionaries.

Develop Concept Images. An alternative to the view above is that of developing concept images. In this view, students make sense of each term in their own personal way. Experiences with a concept lead learners to associate various mental images, processes, and characteristics with that concept. These various associations comprise the learners' images of the concept. Learners construct concept definitions from these concept images. While the terms that label concepts were created by other humans and carry commonly understood meanings, the sense that individual students make of these meanings is highly personal (Tall, 1988; Tall & Vinner, 1981). Such meanings take time to develop and are best approached in multiple ways, exploring critical attributes of concepts and examining relationships between terms and ways to connect meanings to other aspects of one's life or of mathematics. Through a variety of appropriate experiences, concept images become more refined as understanding of the concept deepens, and moves closer to the formal concept definition (Fogelberg et al., 2008; Marzano, 2004; Tall, 1988).

Two Views of Writing

Two distinct views of writing, a product centered view and a process centered "writing to learn" view, have emerged in education. Within the product centered view of writing, the purpose of writing is to create a polished piece of work that demonstrates what the writer knows. This work should be factually and grammatically correct (National Writing Project & Nagin, 2003). Such writing records what has been learned and can be viewed as "writing to record." The process view of writing takes a different approach. In this view, students learn through the process of writing. Such writing can support thinking and learning as the writer analyzes, interprets, and synthesizes ideas, constructing new understandings (Emig, 1977; Forget, 2004), and can be viewed as writing to learn. The resulting writing may be personal and unpolished, but presents a record of the learner's thinking while attempting to understanding new concepts and relationships or to solve problems.

Writing to record. Teaching within the product view of writing focuses on recording accurate, factually correct content. This content may have been memorized as accuracy is critical and texts and teachers are likely to have more accurate information than learners. The polished, final product of such writing serves as a record of what has been learned and may be assessed for errors.

Writing to learn. The process or "writing to learn" view focuses on the process of developing and explaining ideas. Within this view, writing is learned through the process of writing and is fundamental to learning in all content areas. In writing, students analyze, synthesize, and interpret content, thereby constructing new knowledge (Emig, 1977; National Writing Project & Nagin, 2003). Writing is a powerful reasoning tool itself and serves to make thinking visible.

To illustrate, if a teacher asks students to write the steps for finding a per cent equivalent to a fraction, the teacher is exhibiting a "writing to record" view of writing. A teacher who asks students to write about connections between fractions and percents, or asks them to write about the similarities and differences between a fraction and its equivalent per cent is demonstrating a "writing to learn" view. Students are asked to think about a new idea or connection and write about it as they think. This view is also illustrated in asking students to explain why a procedure works or justify a solution to a problem.

Literacy Instruction in Math and Science for Secondary Teachers (LIMSST)

The LIMSST project was developed and implemented by three faculty members in the College of Education at a public university, one in science education, one in language arts education, and myself in mathematics education. Funding was awarded to the university and six partner rural school districts (five of them identified as high-need districts) in a northwestern state for the academic year 2007-2008 by a Federal Eligible Partnership Subgrant. Key project activities included a one week summer workshop to develop literacy strategy use. On-going support throughout the year was provided via three follow-on workshops, three classroom visits to observe and support teachers in literacy strategy use, and a learning community linked via required participation in an on-line discussion forum. Throughout the teacher workshops, project staff actively used the same literacy strategies they were teaching participants to use. Participants worked and learned collaboratively, focusing deeply on how learners use language to make meaning of content and on using strategies of reading, writing, and oral discussion to do so.

The following broad themes were developed on multiple levels throughout the year across all project activities:

- Learning involves making meaning of information and one's experiences.
- Literacy tools for learning and thinking involve reading, writing, oral discussion (speaking and listening), and thinking (reflection).
- Literacy strategies can be integrated into instruction as meaning making tools.

Project staff presence at the schools for observations and conversations was important both to serve as a reminder to integrate literacy strategies and to demonstrate our interest in helping teachers learn how to do this effectively. During the visits, staff were able to use the observations as starting points for conversations about how each teacher could modify existing practices to support the development of student thinking about mathematics using literacy strategies.

Method

The purpose of this study was to understand how mathematics teachers infused the literacy strategies developed through participation in a professional development project into their secondary mathematics classes. This qualitative case study used a constructivist perspective to examine the nature of literacy strategy use by 12 mathematics teachers who participated in the Literacy Instruction for Secondary Math and Science Teachers (LIMSST) professional

development project. The study examined two questions: 1) What literacy strategies did the LIMSST teachers use in their secondary mathematics classes and how were they using and/or modifying literacy strategies? and 2) What influenced teachers' use of literacy strategies?

Participants

The LIMSST project participants consisted of a group of 15 secondary teachers (12 mathematics, 3 science) who shared the intent to learn and infuse literacy strategies into their classes over the course of an academic year. Because research has shown that few mathematics teachers use content literacy strategies (Fisher & Ivey, 2005; Lesley, 2005; Moje, 2006), these teachers provided an opportunity to observe how secondary teachers use literacy strategies in mathematics instruction. This study is limited to the 12 mathematics teachers in the project.

The project teachers were from small, often isolated, rural communities. These teachers had the benefit of small classes, ranging from 4 to 20 students. However, in some cases this benefit was offset by the need to teach as many as six different courses or subjects in a day and by a lack of mathematics teaching colleagues at their grade level (or at any grade level) with whom to collaborate. Many of these districts have found meeting state mandated goals challenging due to limited funding and a variety of social problems facing the communities. The 12 mathematics teacher participants had a variety of backgrounds, perspectives, and teaching histories. All had volunteered for the project and had some level of administrative support for participating; some had stronger administrative encouragement to participate. They were paid a small stipend, primarily for their time in preparing documents for the project. Nine of the teachers taught mathematics exclusively, one taught both mathematics and science, and two taught all subjects in self-contained classrooms, one in 6th grade and the other in a multi-grade class with eight students spanning grades five through eight. Three were junior high teachers, four were high school teachers, and three taught both junior and senior high school classes.

Data Collection

The findings of this study are based on data gathered in the LIMSST professional development project over the course of the 2007-2008 academic year. Data sources include formal interviews with participant teachers, classroom observations and observation debriefs, teacher posts to an on-line discussion forum, planning and participation notes from professional development workshops, and artifacts such as lesson plans using literacy strategies, teachers' reflections on teaching these lessons, and examples of student work. Participants completed the Teacher Belief Inventory (Luft & Roehrig, 2007), a seven item protocol designed to elicit teachers' beliefs about teaching, learning, and students in mathematics and science classes. In addition, teachers answered four questions probing their beliefs about content literacy strategy use in mathematics and science classes and their purposes in using reading, writing, vocabulary development and discussion strategies in their content classes.

Data Analysis

Data were analyzed and interpreted through the lens of the role of literacy in developing mathematical understanding. The data analysis process began with ongoing preliminary analysis (Ely, Vinz, Downing, & Anzul, 1997), in which frequent reading and categorizing of data served to inform further data collection, to shape categories, and to consider potential themes. Once data collection was completed, summative analysis was conducted to identify essential themes and features of the data (Ely et al. 1997). This involved further consolidation and interpretation of the data.

Data were analyzed from three perspectives: by teacher, by literacy strategy, and by the analytic themes that emerged from preliminary analysis. The themes included which strategies teachers implemented, how teachers used strategies, teachers' purpose in using a strategy, and how teachers think about teaching mathematics. The intent was not to describe use of specific strategies by individual teachers, but to determine the variety of ways in which literacy strategies might be used across mathematics teaching and to investigate factors influencing such use.

In looking for patterns of strategy use and the meanings teachers brought to strategy use, increasing attention was paid to how teachers introduced and framed each literacy strategy for students, clues about teachers' purpose in using a particular strategy, and what a teacher hoped students would gain from use of the strategy. Periodic discussion with project staff and ongoing reading of related literature additionally continued to shape understanding of the data and the relationships and influences on teachers' use of literacy strategies.

Upon further examination of the data, it became evident that teachers' patterns of literacy strategy use cut across writing, reading and vocabulary development strategies and were aligned with teachers' learning goals for their students. An analytic framework was developed from the data and supported by literature. This framework presents two general patterns of literacy strategy use. I have termed these patterns the Rehearsal pattern and the Reorganization pattern.

Trustworthiness was addressed via multiples sources of data, thick description, and a lengthy time in the field. Data collection took place over a period of more than a year, beginning in May 2007 and ending in June 2008. During this time, a large amount of data was collected from multiple sources. Teachers submitted 48 lesson plans and accompanying reflections, and posted more than 120 entries on the project's interactive website. In addition, 34 observations and 67 interviews were conducted. Member checking occurred at multiple points over the year as teachers met with me and other project staff, participated in workshops, and communicated via email and the interactive website. Additional checking took place in frequent discussions with other project staff.

Findings: Two General Patterns of Literacy Strategy Use

Examination of literacy strategy use by all 12 teachers indicated two distinct patterns of use, rehearsal and reorganization. These patterns of use were exhibited primarily across the broad categories of vocabulary development and writing. The more prevalent pattern among project teachers was the rehearsal pattern of use. While there also appeared to be differences in the ways teachers used reading strategies, in general, reading was used minimally in mathematics class. As a result, the present discussion of strategy use will focus on vocabulary development and writing. After a general description of the Rehearsal pattern and the Reorganization pattern, illustration will be provided with two critical cases. Table 1 presents a summary of the two patterns.

Rehearsal

Teachers exhibiting the rehearsal pattern of literacy strategy used these strategies as tools to provide students with multiple opportunities to revisit, review, and rehearse facts, concepts, and procedures that had been formally taught. In the realm of vocabulary development, these teachers expected their students to learn and remember formal concept definitions for important terms. Writing was used as an additional opportunity to reexamine information. Students might be asked to review notes, class presentations, or sections of the textbook and summarize important ideas or procedures in their own words. If reading was assigned, it was typically

confined to reading word problems or to reviewing a textbook presentation of material that often had been previously taught in class.

Reorganization

With the reorganization pattern of literacy strategy use, teachers used strategies as thinking tools to support students in developing conceptual understanding of mathematical ideas and procedures. Vocabulary development strategies were used to help students develop concept images that they could use in making meaning of important terms and ideas. Writing was used as a thinking tool, to help students form and become aware of their own ideas about the mathematics concepts and procedures they were learning. Reading, while seldom used, was most frequently seen as a tool to help in interpreting word problems that students needed to solve.

Table 1: Two Patterns of Literacy Strategy Use

	Rehearsal	Reorganization
Vocabulary Development	Learn Concept Definition	Develop Concept Images
Writing	Record Procedures, Facts, Rules	Form Thoughts Raise Awareness of Own Ideas

Ned and Christine: Critical Cases Exemplifying Two Ways of using Literacy Strategies

Ned and Christine were the least experienced teachers in the LIMSST project. These two teachers shared many similarities in their backgrounds and in the classes they chose to target in developing literacy strategy use. Their background characteristics are summarized in Table 2. Although Ned was a generation older than Christine, both were reasonably new teachers, each with two years experience teaching high school mathematics. Both had earned master's degrees prior to teaching high school and had taught mathematics to older students before earning a teaching certificate. Christine had earned a standard secondary mathematics teaching certificate, while Ned had followed an alternative route to certification.

Table 2: Background characteristics of Ned and Christine

	Ned – Rehearsal	Christine - Reorganization
Age	early 50's	early 30's
Education	M.S.	M.A.T.
K-12 Teaching Experience	2 years high school teaching, mathematics	2 years high school teaching, mathematics and Spanish
Target Class	Algebra 1 Lower achieving students	Integrated Math (topics in Algebra 1 and Geometry) Lower achieving students
Previous Teaching Experience	Taught math to adults night school – 1 year	Taught math at community college – 2 years Tutored math at university center – 3 years
Prior Background	Engineering/military	Accounting
Certification	Alternative Secondary Mathematics Certificate	Standard Secondary Mathematics Certificate

Despite their many similarities, these two teachers exhibited distinctly different ways of using literacy strategies and had very different purposes in their uses of these strategies. The Rehearsal pattern of use was exemplified by Ned and the Reorganization pattern of use was shown by Christine. These teachers' patterns of use are compared in the following section.

Two ways to Use Vocabulary Development Strategies

The LIMSST project workshops had introduced a variety of vocabulary development strategies that could be used to support students in their own sense making of concepts and terms. These strategies included the Frayer Model, word sort (Barton and Heidema 2002), and word wall (Fogelberg et al., 2008). Project teachers themselves later introduced the Visual Verbal Word Association (VVWA), a strategy similar to the Frayer Model. In general, project teachers chose a few vocabulary development strategies to implement and used these in ways consistent with their views of vocabulary and concept development and with their previous teaching practice. All teachers tended to use the same few vocabulary development strategies; however, their views of vocabulary development led them to use the strategies in two different ways. Hence, despite using the same vocabulary strategies, students in these classes were offered different types of opportunities to learn the language of mathematics, based on their teacher's view of vocabulary development.

Teachers who viewed vocabulary as a set of concept definitions to be learned provided learning experiences intended to enhance memory of terms and their definitions. Teachers who focused on supporting students in developing rich concept images used the same strategies as the other teachers, but used them to help students make meaning of concepts, develop multiple rich concept images, refine these images, and examine relationships between concepts.

The Frayer Model (Barton & Heidema, 2002; Billmeyer & Barton, 1998; Roe, Stoodt, & Burns, 2001) proved to be a favorite tool among project teachers. This tool uses a graphic format to help students develop their understanding of concepts and conceptual relationships and understand what the concept is and is not. Students complete a diagram with a definition for the term in their own words, determine the term's essential attributes, and refine their understanding by selecting examples and non-examples of the concept from their own experiences. Using this strategy, students essentially explain their own understanding of a concept. An example of the Frayer model is presented in Figure 1.

The Verbal Visual Word Association (VVWA) is a similar strategy commonly used by project teachers. It is also organized within a graphic format and includes a definition, a visual representation of the term, and characteristics or some personal association with the term (Barton & Heidema, 2002). VVWA is useful for concepts that have a visual component or are more concrete, such as geometric figures, or that show a relationship, such as the slope of a line which relates the rise to the run on a graph. Figure 2 presents an example of VVWA from Christine's class.

Vocabulary Development Strategies Used for Rehearsal: Ned

Ned's approach to vocabulary development was to emphasize learning and remembering the formal concept definition. His classroom activities used vocabulary development strategies in ways that made formal definitions visible and provided opportunities to practice and review these. Ned looked for evidence that students knew the formal definition of vocabulary terms. "Make sure that they have the definition. Make sure that they have, in their notebooks, they write down the vocabulary....The Frayer Model is a good one to relate those—example and definition

and diagram" (Ned, TBI Interview, 5/21/08). In Ned's view, each of these aspects serves to make the true meaning apparent to the learner and, once recorded, is available for reference and review.

Definition (in your own words)	Facts/Characteristics
A solution of a compound inequality joined by 'and' and makes both inequality true	$a < x < b$
"2 inequalities joined together with and"	$x > a \text{ and } x < b$

Compound Inequalities that are joined with the word AND.	
Examples	Non-Examples
$-3 > x \text{ and } x < 2$	$x < -3 \text{ and } x > 4$
$-3 < x < 2$

Figure 1: Frayer Model – Student work from Ned's class – 11-07-07

In using the Frayer Model (Figure 1) as a tool to teach a concept definition, students must understand the definitions they are given and revisit these often, trying to memorize the words. They may not have had opportunity to connect the definition with any personal meaning, with concept images, or with other mathematical ideas. Hence, the only way to learn and remember these is to revisit and rehearse the information often. The Frayer Model provides a convenient way to do this, because it organizes useful information about the term.

It was important to Ned that students have the correct information, and he did not expect them to figure it out for themselves. "We worked on Frayer Models for the same vocabulary words but they had to use the definitions from their notes" (Ned, Web Post, 12/12/08). In using the Frayer Model, Ned provided students with all the information he wanted them to include. At times, he allowed students to record information directly onto the Frayer model form as he lectured. "I give an answer for each of the four sections of the Frayer Model during my lecture. I have a stack of Frayer Model blanks available. I normally pass them out when Vocab words are coming up. Sometimes I wait until after the lecture so they have to transcribe from their notes" (Ned, Web Post, 12/07/08).

During one observed lesson, Ned used the Frayer Model as a review before a test. He provided multiple blank Frayer Model forms. Each form named a concept related to inequalities or absolute value equations and provided an entire page on which to write. Ned's goal was for students to reread information in the text or in student notes, providing another opportunity to learn the material they had been given. Ned wrote that the activity "was a restatement and summary of notes they should have had in their math notebooks...[It] required the students either

to read the section in the book again or the review pages. Some of the students actually found information in their notebooks” (Ned, Reflection, 11/07/07).

For Ned’s students the task was to find and learn the concept definition, as opposed to generating their own. When used in this way, the Frayer Model is transformed from a tool for student use in developing and refining their concept images into a device for recording the concept definition and illustrative examples.

Vocabulary Development Strategies Used for Reorganization: Christine

Christine designed learning activities to help her students develop concept images, the various pictures and images one draws on when thinking of a concept or term. Her use of the Frayer model illustrates its application as a tool to support students in development of their concept images. “I actually conducted a dice throwing experiment to show the difference between [theoretical and experimental probability]....After we showed this, I just let them do the Frayer model based on what they learned from the modeling” (Christine, Web Post, 11/20/07). Completing the Frayer model helped students organize their thinking about the concept. Christine described ideas for scaffolding students in finding and thinking about relevant information to use in completing a Frayer model. “You could ask them what they know about a particular topic, write down anything they say on the board and then they can use that list to help them to fill out the chart [Frayer model diagram]” (Christine, Web Post, 11/20/07).

Christine tried using the VVWA strategy to help students differentiate perimeter, area, and volume: “They drew pictures for them. On the VVWA's, for the characteristics of the shapes, they wrote down the formulas for area. I tried to avoid giving them formulas for perimeter, since they really aren't necessary.” When her students continued to confuse the terms *area* and *perimeter* she “had them daily write definitions in some format for area and perimeter, until it was clear they could tell the difference” (Christine, Web Post, 11/29/07).

Practice using terms has a role in a meaning making approach, but this role is different than in a concept definition approach. While the meaning of a concept or term must be constructed, the label must be learned and connected to that meaning. In Christine's classroom, what students were practicing was making meaning of these terms, not a formal definition. Practice was involved in creating these meaningful connections, but it was characterized by engagement in a variety of related experiences, rather than rehearsal of information.

In February Christine again tried the VVWA strategy, after engaging students in an activity to develop meaning for midpoint of a line segment. Reflecting on the lesson, she wrote:

The VVWA went well. They developed their own meaning, and it was correct, of the word and were able to reinforce this using the VVWA. I have taught this lesson before but instead of allowing them to discover the meaning of midpoint, I just told them about it. They really seemed to grasp the idea of midpoint much more than students in previous years did. (Christine, Reflection, 2/27/08)

VWA	
<u>Vocabulary Term</u>	<u>Visual Representation</u>
Midpoint
<u>Definition</u>	Personal association or characteristic (could be examples)
a point that divides the points $\overline{AC}$ into two congruent segments	Congruent points equal

Figure 2: Visual Verbal Word Association: Student work from Christine's class: 2-27-08

She later elaborated on her approach:

I guide them quite a bit, but we usually do some sort of activity where they "discover" the concept. For example, with area, they have to cover squares and rectangles with centimeter grid paper, count the number of unit squares that cover the surface (to get the idea of area), and then from there they develop the formula for area of rectangles and squares. But by the end, most can tell me in their own words what the definition of area is. Then we fill out the VWA, but I ask them to tell me what a good definition would be. (Christine, Web Post, 12/08/08)

Christine believed that ideas about a concept must be developed. Words can help with this, but are not enough. Learners also need experiences and examples of concept in order to make meaning of them. Christine's students were encouraged to share ideas and use reference materials as they made meaning of a concept. She assessed their current understanding frequently and continued to provide new learning activities, giving them time to revisit concepts as they refined their understanding and continued to create additional elements of concept images. Time and a long list of topics to teach limited Christine in this. She could not provide such time for every concept, so chose the central concepts that would receive emphasis. Her goal was to support students in making meaning.

In summary, Ned and Christine used the same vocabulary development strategies, but had different purposes in using them. While Ned gave his students frequent opportunities to learn and rehearse formal concept definitions for terms, Christine's students engaged in multiple activities intended to make meaning of concepts, develop multiple rich concept images, refine these images, and examine relationships between concepts.

Two Ways to Use Writing

LIMSST teachers found frequent opportunities to engage their students in writing about mathematics. But in this area too, strategies were used in different ways. Some teachers described the writing process as helpful in remembering information, while others used writing to help students think or to help them connect their current knowledge to new ideas.

Writing as Rehearsal: Ned

Ned used lengthy writing assignments that required students to supply rules and examples for various procedures he had taught. The purpose of these assignments was two-fold: to review and remember procedures and definitions, and to focus students on topics for review. Ned said, "I think that being able to describe what you are doing mathematically in words is helping you retain it more" (Ned, TBI Interview, 5/21/08).

For example, in a lesson to review methods of solving inequalities, Ned first asked students to read a review section in the text. He then asked them to use their own words to explain how to solve inequalities by adding or subtracting, and to provide an example. Students were also asked to write how to solve inequalities by multiplying or dividing by a positive number and by a negative number as well as to provide related definitions and examples (Ned, Observation, 11/07/07). Samples of student writing from this lesson (Figure 3) are focused on steps of procedures and some sections are incomplete. A number of student papers show the same phrases or examples, suggesting that these appeared in the text or in class presentations.

Student C		Student D	
Explain in your own words how you ...	Give an Example	Explain in your own words how you ...	Give an Example
Solve Inequalities by adding or subtracting. on the variable side! If adding subtract that # from both sides If subtracting, add that # to both sides	$\begin{array}{r} x-3 > 11 \\ +3 \quad +3 \\ \hline x > 14 \end{array}$ $\begin{array}{r} x+5 < 9 \\ -5 \quad -5 \\ \hline x < 4 \end{array}$	Solve Inequalities by adding or subtracting. add or subtract on <u>both</u> sides	12 < x-1 6 < x-1 2 < x-1 $\begin{array}{r} 2 < x-1 \\ +1 \quad +1 \\ \hline 3 < x \end{array}$
Solve Inequalities by Multiplying or Dividing by a Positive Number. on the variable side! If mult, divide that # on both sides If div, mult by that # on both sides	$\begin{array}{r} 3m > 9 \\ \div 3 \quad \div 3 \\ \hline m > 3 \end{array}$ $\begin{array}{r} 4, \frac{m}{4} < 16 \\ \times 4 \quad \times 4 \\ \hline m < 64 \end{array}$	Solve Inequalities by Multiplying or Dividing by a Positive Number. You would multiply or divide to both sides.	12 < x-1
Solve Inequalities by Multiplying or Dividing by a Negative Number. Same as with a positive number, except change the inequality sign and the answer to a negative or positive	$\begin{array}{r} -2m > 10 \\ \div -2 \quad \div -2 \\ \hline m < -5 \end{array}$ $\begin{array}{r} \frac{m}{5} < -3 \\ \times 5 \quad \times 5 \\ \hline m < -15 \end{array}$	Solve Inequalities by Multiplying or Dividing by a Negative Number. multi. or divide by the negative # on both sides. Remember to change your sides.	

Figure 3: Sample of student writing from Ned's review of methods of solving inequalities

Writing as Reorganization: Christine

In contrast to Ned, Christine used short writing activities that were integrated throughout a learning activity with many related components. Her purpose in choosing these activities was to support student thinking and meaning making, with a goal of students developing or reorganizing a set of related understandings.

In Christine's view, writing about one's process helps both with remembering and with awareness, bringing one's learning more clearly into focus. But it also serves as a reflection tool as students examine and explain their own thinking. She asked students to "write to make meaning of terms and concepts. Using kids' meaning makes much more sense to them" (Christine, Interview, 2/27/08). Other writing focused students thinking on their own background knowledge in preparation for connecting it to new learning. She explained:

I use writing quite a bit more than I use reading....I wanted them to think about what it was they were doing and explain....I had them explain their steps to problems. I had them write journals about things that they learned or everything they know about a particular topic. Sometimes I would have them write about something before we talked about it. "What's your prior knowledge of this topic?" (Christine, TBI Interview, 5/29/08)

When her students wrote about a given topic, Christine did not expect correct descriptions of information that had been taught. She was instead looking for descriptions and explanations of students' current understandings of the topic and of its relationships with various mathematics concepts. For example, in her lesson about line segments and midpoints, Christine began by asking students to engage in an activity to help develop meaning of *midpoint* before they completed the VVWA shown previously. Ensuing work engaged students in using the term in several short problems and writing their thoughts as they proceeded. For example, they were given the statement, "B is the midpoint of AC" and asked to "write what this tells you about the segments and about the lengths of the segments." This exercise was followed by a similar one: "AB = 2. Write what other information you can conclude about these segments." (Christine, Observation, 2/27/08) The lesson concluded with students making a brief open-ended journal entry in which students are asked to write what they have learned about segments and their lengths (Figure 4). Writing samples from Christine's class vary widely, both in ideas presented and in words and examples used to portray these ideas. Her students write in personal ways describing the sense they are making of ideas.

Student A:

i have learned that the capital letters AB
w/out a line over them mean the length
of the segment $\overline{AB}$. I have also learned
that the midpoint, for example if you
have $P \quad Q \quad R$ then Q is the
midpoint if it divides

Student B

LL#3 : 2-27-08
What you have learned about segments and their lengths.
I have learned that a segment is a line consisting
of two end points. It also can contain some points
in the middle.

Figure 4: Samples of student writing from Christine's midpoint lesson.

Two Purposes of Writing in Math

While Ned and Christine both engaged students in mathematical writing, their uses of writing served very different purposes. Ned used writing as a memory tool. In writing, his students created a personal reference tool that summarized material that had been taught in class. Such writing focused on recording what students had learned, such as facts, definitions, rules, or procedures. It might also record steps students used in their own execution of a procedure. Writing served to raise student awareness of what they had learned and resulted in a written record that could be used for review. Ned could also use this record as an assessment tool to determine what students have remembered.

In contrast, Christine used writing as a student thinking tool to help students construct their own understanding of concepts and relationships or to solve problems. She encouraged students to describe their current understanding of concepts, relationships, or strategies, or to analyze patterns and relationships. She also encouraged them to make connections between their current knowledge and new ideas. Such writing could also be used for assessment of the nature of student understanding. Reading student writing allowed Christine to make purposeful instructional decisions and to design learning activities that improved or deepened student understanding.

Teachers' View of Learning Mathematics

In addition to differences in how literacy strategies were used, Ned and Christine differed in their views of their role as a teacher. Ned had a procedural focus to teaching mathematics, and

he viewed his role as a teacher as one of “guiding the students through a process so that they can obtain those skills” (Ned, TBI Interview, July 2007). His view of mathematics teaching and learning is in line with a traditional procedural view of mathematics.

Teachers with a traditional orientation view mathematics as having an existence independent from human existence, and learners passively receive this mathematical knowledge by listening to or watching knowledgeable others (Philipp, 2007; Simon & Tzur, 1999) and practicing procedures they have seen demonstrated (Smith, 1996). There is a fixed body of information to be learned and the teacher’s role is to transmit this information and ensure that students have received it (Barkatsas & Malone, 2005). This view is associated with a focus on mathematical procedures and correct answers, with the role of problem contexts being minimized (Thompson, 1992). It is also referred to as a procedural orientation.

In contrast, Christine's views were consistent with a conceptual orientation toward teaching and learning math. “I think my role is to help guide students to learn and not just to teach and tell, teach by telling, but trying to find ways to help them discover for themselves (Christine, TBI Interview, July, 2007). Within a constructivist or conceptual orientation teachers view knowledge of mathematics concepts, relationships and procedures as constructed by the student through intellectual engagement in mathematical exploration and problem solving, analyzing patterns and relationships, and justifying their mathematical reasoning. Learning is a process of meaning making on the part of the learner, and teaching is a process of facilitating students’ meaning making (Jones, 1997). Teachers need to identify current student understandings and provide learning opportunities to support students in building on and extending these (Barkatsas & Malone, 2005).

Both Ned and Christine used literacy strategies in ways consistent with their views of teaching and learning mathematics. Ned provided opportunities for his students to receive, review, and rehearse important information. He believed students need to learn and remember correct ideas, facts, and procedures. Christine thought a teacher's role is to provide opportunities for students to make their own meaning and develop conceptual understanding of the structure of mathematics, and she supported her students in doing so. Despite their different viewpoints, each found literacy strategies helpful in pursuing their learning goals for their students.

Discussion of Results

Teachers fit new teaching ideas into their current understanding of teaching and learning. Their views about learning and understanding serve as frames, the “underlying structures of belief, perception, and appreciation” (Schon, 1995, p. 23) and shape all aspects of their teaching. In this study, teachers' views of learning and understanding mathematics proved a powerful influence both on which literacy strategies they chose to use and on how they used these strategies. These frames determined what teachers noticed and how they interpreted what was noticed. Additionally, the frames determined what it meant to know and understand mathematics as well as what should be known in mathematics: a set of procedures to be executed or a complex and rich network of meanings and relationships between concepts and a variety of ways to use these in thinking about mathematics and solving problems. These views influenced the nature of goals set for learning, how teachers thought about understanding mathematics, the nature of learning experiences offered to students, and what it means to be a successful student.

This study demonstrated that when students are asked to use literacy strategies as tools for rehearsal, the emphasis is on revisiting and understanding the ideas of others. Students may review or restate these ideas, but they remain another's ideas and the student's task is simply to

learn, remember, and use these. However, when literacy strategies are presented as tools for meaning making, learners are encouraged to interpret their experiences in meaningful ways. They have the opportunity to interact with a variety of ideas: some their own, some proposed by others. From this interaction there is the potential to integrate ideas and reorganize them into a new and meaningful structure. This process is at the heart of sense making. While not all students will engage with ideas in such meaningful ways, the opportunity and support for doing so exists, and literacy strategies support such opportunities.

Purely providing teachers with literacy strategies is not enough to allow them to use the strategies as intended. In general, the same vocabulary development strategies were used by LIMSST project teachers of both views. Teachers implemented a small number of vocabulary development strategies and the nature of this implementation was consistent with their views of vocabulary and concept development. Teachers' views of vocabulary development led them to use the strategies in two different ways. Students with teachers holding a concept definition view had multiple opportunities to learn and practice concept definitions, while students with teachers holding a concept image view were engaged in learning experiences to develop a variety of concept images related to the concept at hand. Hence, despite using the same vocabulary strategies, students in these classes were offered different types of opportunities to learn the language of mathematics, based on their teacher's view of vocabulary development.

Similarly, the type of learning that was supported in the use of writing was related to teachers' views of writing and their goals for learning mathematics. Teachers who viewed writing as a tool for practicing or demonstrating what has been learned asked students to write accounts of rules and procedures or of applications of these rules and procedures. They also supported students in ensuring that these accounts were accurate and were responsible for selecting and shaping the information students recorded.

In contrast, teachers who viewed writing as a tool for thinking provided a variety of opportunities for students to write about their observations and understandings, and to form connections about mathematical ideas. They supported students in deepening personal understanding through the use of writing and through integrating writing with other learning activities. These teachers used writing as a tool for thinking in three ways. It was initially used to engage students in thinking about prior knowledge, making current knowledge available for use as a starting point in considering new ideas. Writing was also used in subsequent learning activities as students explored and analyzed mathematical ideas, observed relationships, and wrote their thoughts to clarify, organize, and revisit them. In a final use of writing, students were asked to make sense of and explain their observations and analyses, justifying their reasoning. In these classes, students were supported in exploring ideas that did not always match those of the teacher. Writing was used as a thinking tool for reflection and interaction with ideas.

Implications and Conclusion

Literacy strategies can enhance learning in two ways. They can be used to increase student opportunities to focus on and practice procedures, increasing awareness of these, and providing additional opportunities to rehearse material to be learned. Alternatively, the strategies can support students in making observations, identifying patterns and relationships, clarifying thinking, supporting higher order thinking skills such as reasoning, justification, synthesizing ideas, and constructing new meaning. While both types of learning can benefit students, each affords a different benefit.

We are just beginning to understand how teachers can and do make use of literacy strategies in mathematics classes. Future research is needed to further examine factors that affect how teachers use literacy strategies. Such research would also provide better understanding of how to teach and support effective use of literacy strategies, for use in both teacher training and professional development. Additional research examining the relationship between a teacher's views of teaching learning and their use of literacy strategies would help us understand this complex relationship and provide tools to better support teachers in effective use of literacy strategies. Limited research suggests that literacy strategies are powerful learning tools for mathematics as well as other content areas. Research into the relationship between student achievement and literacy strategy use would strengthen arguments that these tools are effective across content areas.

References

- Adams, T. L. (2003). Reading mathematics: More than words can say. *The Reading Teacher*, 56(8), 786-795.
- Adams, T. L., Thangata, F., & King, C. (2005). "Weigh" to go! *Mathematics Teaching in the Middle School*, 10(9), 444-448.
- Bangert-Drowns, R. L., Murley, M. M., & Wilkinson, B. (2004). The effects of school-based writing-to-learn interventions on academic achievement: A meta-analysis. *Review of Educational Research*, 74(1), 29-58.
- Barton, M. L., & Heidema, C. (2002). *Teaching reading in mathematics* (2nd ed.). Aurora, CO: Mid-Continent Research for Education and Learning.
- Billmeyer, R., & Barton, M. L. (1998). *Teaching reading in the content areas: If not me, then who?* Denver, CO: Mid-Continent Research for Education and Learning.
- Borasi, R., & Rose, B. J. (1989). Journal writing and mathematics instruction. *Educational Studies in Mathematics*, 20, 347-365.
- Burns, M. (2004). *Writing in math class*. Sausalito, CA: Math Solutions Publications.
- Carter, T. A., & Dean, E. O. (2006). Mathematics intervention for grades 5-11: Teaching mathematics, reading, or both? *Reading Psychology*, 27, 127-146.
- Clarke, D. J., Waywood, A., & Stephens, M. (1993). Probing the structure of mathematical writing. *Educational Studies in Mathematics*, 25(3), 235-250.
- Countryman, J. (1992). *Writing to learn mathematics: Strategies that work*. Portsmouth, NH: Heinemann.
- Ely, M., Vinz, R., Downing, M., & Anzul, M. (1997). *On writing qualitative research*. London: Routledge Falmer.
- Emig, J. (1977). Writing as a mode of learning. *College Composition and Communication*, 28(2), 122-128.
- Fang, Z. (1996). A review of research on teacher beliefs and practices. *Educational Researcher*, 38, 47-65.
- Fisher, D., & Ivey, G. (2005). Literacy and language as learning in content area classes: A departure from "every teacher a teacher of reading". *Action in Teacher Education*, 27(2), 3-11.
- Fogelberg, E., Skalinder, C., Satz, P., Hiller, B., Bernstein, L., & Vitantonio, S. (2008). *Integrating literacy and math: Strategies for K-6 teachers*. New York: The Guildford Press.

- Forget, M. A. (2004). *Max teaching with reading and writing: Classroom activities for helping students learn new subject matter while acquiring literacy skills*. Victoria, BC: Trafford.
- Harmon, J. M., Hedrick, W. B., & Wood, K. (2005). Research on vocabulary instruction in the content areas: Implications for struggling readers. *Reading & Writing Quarterly*, 21, 261-280.
- Herrick, C. J. (2005). *Writing in the secondary mathematics classroom: Research and resources*. State University of New York College at Cortland, Cortland, NY.
- Holliday, W. G., Yore, L. D., & Alverman, D. E. (1994). The reading-science-learning-writing connection: Breakthroughs, barriers, and promises. *Journal of Research in Science Teaching*, 31, 877-893.
- Kenney, J. M., Hancewicz, E., Heuer, L., Metsisto, D., & Tuttle, C. L. (2005). *Literacy strategies for improving mathematics instruction*. Alexandria, VA: Association for Supervision and Curriculum Development.
- Lawson, A. E. (1989). A theory of instruction: Using the learning cycle to teach science concepts and thinking skills. *National Association for Research in Science Teaching Monograph*, 1.
- Lesley, M. (2005). Looking for critical literacy with postbaccalaureate content area literacy students. *Journal of Adolescent & Adult Literacy*, 48(4), 320-334.
- Lortie, D. (1975). *Schoolteacher*. Chicago: University of Chicago Press.
- Luft, J. A., & Roehrig, G. H. (2007). Capturing science teachers' epistemological beliefs: The development of the teacher beliefs interview. *Electronic Journal of Science Education*, 11(2), 38-63.
- Marzano, R. J. (2004). *Building background knowledge for academic achievement*. Alexandria, VA: Association for Supervision and Curriculum Development.
- Marzano, R. J., Pickering, D. J., & Pollock, J. E. (2001). *Classroom instruction that works*. Upper Saddle River, NJ: Pearson.
- Moje, E., B. (2006). Integrating literacy into the secondary school content areas: An enduring problem in enduring institutions [Electronic Version]. www.umich.edu/~govrel/adoles_lit/moje.pdf-. Retrieved August 20, 2008.
- Monroe, E. E. (1997). Effects of mathematical vocabulary instruction on fourth grade students. *Reading Improvement*, 34, 120-132.
- Murray, M. (2004). *Teaching mathematics vocabulary in context*. Portsmouth, NH: Heinemann.
- National Writing Project, & Nagin, C. (2003). *Because writing matters: Improving student writing in our schools*. San Francisco: Jossey-Bass.
- NCTM. (2000). *Principals and standards for school mathematics*. Reston, VA: Author.
- Pajares, M. F. (1992). Teachers beliefs and educational research: Cleaning up a messy construct. *Review of educational research*, 62, 307-332.
- Pimm, D. (1995). *Symbols and meanings in school mathematics*. London: Routledge.
- Roe, B. D., Stoodt, B. D., & Burns, P. C. (2001). *Secondary school literacy instruction: The content areas*. Boston: Houghton Mifflin Company.
- Rubenstein, R. N. (2007). Focused strategies for middle-grades mathematics vocabulary development. *Mathematics Teaching in the Middle School*, 13(4), 200-207.
- Rubenstein, R. N., & Thompson, D. R. (2002). Understanding and supporting children's mathematical vocabulary development. *Teaching Children Mathematics*, 9, 107-112.
- Schon, D. A. (1995). *Frame reflection: Toward the resolution of intractable policy controversies*. New York: Basic Books.

- Steele, D. F. (2005). Using writing to access students' schemata knowledge for algebraic thinking. *School Science and Mathematics*, 105(3), 142-154.
- Steele, D. F. (2007). Understanding students' problem-solving knowledge through their writing. *Mathematics Teaching in the Middle School*, 13(2), 102-109.
- Tall, D. O. (1988). Concept image and concept definition. In J. d. Lange & M. Doorman (Eds.), *Senior secondary mathematics education* (pp. 37-41.). Utrecht: OW&OC.
- Tall, D. O., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, 12(2), 151-169.
- Thompson, D. R., & Rubenstein, R. N. (2000). Learning mathematics vocabulary: Potential pitfalls and instructional strategies. *Mathematics Teacher*, 93(7), 568-574.
- Usiskin, Z. (1996). Mathematics as a language. In P. Elliott, C. & M. Kenney, J. (Eds.), *Communication in mathematics, K-12 and beyond, NCTM 1996 Yearbook* (pp. 231-243). Reston, VA: National Council for Teachers of Mathematics.
- Vacaretu, A. S. P. (2008). Reading texts & writing problems to improve problem solving. *Mathematics Teacher*, 101(6), 451-455.
- Vygotsky, L. S. (1962). *Thought and language*. Cambridge, MA: The MIT Press.
- Zinsser, W. (1988). *Writing to learn*. New York: Harper & Row.