The Mathematics Enthusiast

Volume 8
Number 1 *Numbers 1 & 2*

Article 10

1-2011

Historical perspectives on a program for mathematically talented students

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Abstract: The University of Minnesota Talented Youth Mathematics Program (UMTYMP) is a highly accelerated program for students who are extremely talented in mathematics. This paper describes our experiences running UMTYMP since its inception thirty years ago, the challenges in implementing such a program, and how changes in the student body have necessitated changes in the program over three decades.

Key words: accelerated, high school, mathematics program

1. Introduction

There is a wide variety of research into the many and multifaceted issues in providing opportunities for mathematically talented or gifted students, ranging from the identification of students to the best methods of instruction for the population. While these issues can separately be excellent sources for further discussion, the development and implementation of a large ongoing program involves addressing most of these concerns in a specific contextual and highly integrated fashion. This paper examines the evolution, success, and challenges of the 30-year old University of Minnesota Talented Youth Mathematics Program (UMTYMP), which continues to operate as the leading accelerated mathematics program in Minnesota. To our knowledge the program is unique in terms of number of students, scope of the curriculum, and the granting of college-level honors credit to students in middle or high school.

UMTYMP was started in order to provide Minnesota's most mathematically talented students with an alternative educational experience. Each year approximately 400 students in grades 6-12 take their mathematics courses through UMTYMP instead of their own schools. During their first two years of the program, students cover four years of standard high school curriculum: algebra I and II, geometry, and pre-calculus. The final three years of the program are comprised of honors-level collegiate courses in calculus, linear algebra, and vector analysis. Along the way, students must develop a strong work ethic and problem-solving

The Montana Mathematics Enthusiast, ISSN 1551-3440, Vol. 8, nos.1&2, pp.189- 206

skills. Many continue on to upper-division and graduate level mathematics courses before finishing high school.

Our significant historical perspective allows us to identify and discuss practices and issues which have remained unchanged, and those which appear to be quite different than just ten to fifteen years ago. Because any discussion of our program requires knowledge of the context, Section 2 gives a brief overview of UMTYMP's design and goals; a more extensive description is given in [3]. In Section 3 we discuss the specific issues of student selection, retention, characteristics, and the evolution of the program over the past fifteen years. This is followed by a brief look in Section 4 at statistical data to see how well UMTYMP has achieved its goals. Section 5 describes how our internal assessments have resulted in changes to the UMTYMP structure, and some of the ways UMTYMP could be used in educational research. Finally, Section 6 discusses the inherent challenges in expanding or duplicating the program.

2. Program Description

2.1. Origins of UMTYMP. The idea of a mathematics program for talented students in Minnesota originated in the mid-1970s. Several faculty members, including one of the authors, had attended district-wide high schools for academically talented students during their childhoods and felt that their experiences were very positive influences on their mathematical success. In addition, a new faculty member in educational psychology who had actively participated in Julian Stanley's Study of Mathematically Precocious Youth (SMPY) at Johns Hopkins University was a strong proponent of accelerated courses for gifted students, and advocated for an accelerated mathematics program in Minnesota. (See [4] and [5] for summaries of the SMPY findings on acceleration for students similar to those in UMTYMP.)

While the formation of a new high school was not feasible for those faculty members, these ideas led the development in 1976 of a two-year program, located at Macalester College in St. Paul, Minnesota, which provided supplemental mathematics for talented students in the Minneapolis/St. Paul metropolitan area. It covered essentially the same material as the first two years of the current UMTYMP curriculum. While this program was quite successful, the issue of providing additional advanced coursework in calculus for students entering ninth or tenth grade became evident.

The head of the mathematics department at the University of Minnesota at the time was very sympathetic to the idea of providing high level courses for young students, and the department agreed to develop a calculus course for the Macalester program graduates. Very shortly thereafter, the Macalester program lost its funding and the mathematics departments at the University of Minnesota agreed to fund and administer the entire sequence of courses. In the fall semester of 1980, the first UMTYMP high school and calculus level courses were offered at the University.

2.2. *Program Overview.* The overall goal of UMTYMP is to provide a challenging, stimulating and nurturing academic program for students who are exceptionally talented in mathematics.[1] In their home schools these students often face the stigma of being good at math; we provide them with a chance to immerse themselves in a culture of mathematics and meet other students with the same talents. We also emphasize how mathematics in general and UMTYMP in particular can increase their future opportunities. Family interest in college achievements of UMTYMP graduates seems to be a major factor in their support of the program; parents seem drawn to the fact that the schools most attended by our alumni include prestigious institutions such as MIT, Stanford, Harvard, the University of Chicago and Caltech. (See Table 2, Section 4.) Unfortunately, we sometimes have uninterested and poorly performing students who were pushed to participate in UMTYMP because their parents (incorrectly) see our program as a way to get their children in their "dream school," regardless of student performance.

Many details of the actual implementation of the program are dictated by logistic and administrative constraints. Classes meet for a two-hour session once or twice per week after the regular school day, totaling about 35 sessions from September through May. This highly compressed schedule makes every moment of class time valuable and has a profound impact on the curriculum, teaching styles, and even the screening process to get into the program; see Section 3 for details.

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[1] See [7] for an overview of effective learning environments for mathematically talented students.

The first two years of UMTYMP comprise the "high school component," and the content is aligned with Minnesota's state standards for high school mathematics.[1] In two years UMTYMP students cover four years' worth of high school curriculum in algebra, geometry and pre-calculus. A single hour of our class time corresponds to about one week's worth of material in a high school classroom. Our instructors are therefore forced to cover only the central ideas and techniques, leaving students to learn the computational details on their own while working on extensive homework assignments. This course structure led to initial concerns that high schools would not count UMTYMP courses towards their graduation requirements, but a state law passed in 1984 requires schools to grant high school credit on their transcripts for students who have completed our courses.

After completing pre-calculus, students move on to the three year "calculus component," which consists of honors level courses in single variable calculus, linear algebra, differential equations, multivariable calculus and vector analysis. The courses are more theoretical, and cover more topics, than the standard calculus sequence in our mathematics department. Students in this component receive honors level Institute of Technology credit for the courses on a University of Minnesota transcript; if they later choose to attend the University they will have already satisfied nearly half the requirements for a mathematics degree. If they enroll at a different undergraduate institution, the credits will either transfer or earn them placement into higher level courses there. Our intent is that no UMTYMP student would ever have to retake a course in the calculus sequences at any other institution and can proceed directly to post-calculus classes.

Based on our years of observations, we take the approach that understanding can be challenging and fun, but that learning computation skills, algorithms and how to use conceptual reasoning takes serious effort. Lecturing on the main ideas and then handing the students a textbook to learn the material on their own is not enough [6]. Thus a central feature of UMTYMP is the broad, deep support system, designed to enable virtually all interested and motivated students to be successful; see Section 3.2 for details. Significant emphasis is placed

[1] This helps minimize difficulties for students at their own high schools. A Minnesota state law stipulates that if a student covers typical high school mathematics as part of an accelerated mathematics program at a college or university, the student's high school must recognize the courses as fulfilling the mathematics graduation requirements.

on developing effective work habits and individual problem solving skills, so that students learn that these abilities are as important as performance on classroom examinations. To stress the importance of clearly communicating mathematical ideas, each calculus homework assignment includes one problem whose solution should be written in a “professional” manner, roughly comparable to an example problem in a textbook. Students quickly learn that they must organize their work and write coherent explanations if they wish to earn full credit.

3. Specific Issues and Observations

This section describes our approaches to specific programmatic issues such as selection, recruitment and retention of students, and our teaching approach, all of which have remained largely unchanged during the last decade. We also discuss the defining characteristics (or lack thereof) of our mathematically talented students, and examine historical trends in our program.

3.1. *Student Selection and Recruitment.* Students who wish to enter either the high school or calculus components of UMTYMP must achieve satisfactory scores on entrance exams developed by our academic staff. In the past, we relied heavily on local schools to identify potential students. Before taking our entrance exam, a student would have to score in the 95th percentile or above on any national standardized mathematics examination and be recommended for their program by a teacher. In hindsight, however, this method of recruiting was too restrictive; in particular we discovered that teachers, whether through intentional or unintentional bias, tended to recommend male students over females of equal ability. In the interest of fairness our entrance exams are now open to any interested student in the appropriate grade levels.

The qualifying exams measure computational ability, but they stress critical thinking and speed. The qualifying exam for the high school component, for example, has 50 multiple choice questions to be answered in only 20 minutes. In each question students are given two quantities and asked to determine if one is larger than the other, if they are equal, or if there is not enough information to decide; see Figure 1 for examples. A high score indicates a solid command of pre-algebra skills and the ability to process mathematics quickly. This has proven to be an effective way to find computationally strong students who can also handle the rapid pace of the courses. This process is far from perfect; it probably excludes students who are quite talented but work slower. However, budget and staffing issues require us to offer only accelerated courses, and hence UMTYMP is (unfortunately) not appropriate for those students.

While students who pass the entrance exams are invited to enroll in the program, we still ask parents and students to discuss the commitment with each other before accepting the offer.

Since all students entering UMTYMP have mathematical talent, the best predictors for student success are enthusiasm about mathematics in general (i.e. beyond algebraic computation), and the willingness to put in the effort to comprehend the ideas. These observations are consistent with recent studies on success in school and beyond [1]. Many of our underperforming students have time conflicts in their busy schedules or are simply not interesting in thinking deeply about mathematics.

We have found from earlier equity efforts that running mathematical enrichment programs throughout the academic year is a wonderful recruitment tool, since it introduces students to the type of mathematical thinking used in UMTYMP. Our Saturday morning classes are open to any students in grades 4-7 and cover subjects ranging from explorations of area and volume to spherical geometry and topology. Students who participate in the enrichment program are frequently eager to join UMTYMP, have better qualifying scores than general students, and are more successful in the program if they enroll.

As with many mathematics programs, we have difficulty consistently attracting females and students from traditionally under-represented minority groups. At times we have launched major initiatives to increase their numbers, such as the Bush Foundation Initiative described in [3], which succeeded in raising our female enrollment to over 40%. These gains are difficult to sustain once initiative funding ends. While the percentage of female students for UMTYMP still remains high for comparable programs, it has decreased to between 20% and 30%. In recent years we have started a new enrichment program which currently serves a diverse population of over 250 girls in grades 4-6, with the hope that a good number will eventually qualify for and enroll in UMTYMP

(1) x and y are positive numbers and x < (x+y)/3.
  (a) x
  (b) y

(2) The sum of the remainders when each of these numbers is divided by 3:
  (a) 3, 10, 12, 19
  (b) 6, 11, 25, 27

Figure 1. Practice questions for the UMTYMP High School Component Entrance Exam. Students must determine the size relationship between the two quantities in each question.

3.2. *Retention*. A major challenge in UMTYMP is the retention of students once they enter the program, since the work and learning expectations both in the classroom and at home are typically very different from anything encountered in their K-12 education. For example, the conceptual approach and work expectations in UMTYMP Calculus significantly exceed those of Advanced Placement or International Baccalaureate courses. Moreover, the once-a-week format requires students to focus more intensely on the classroom lecture and activities. Note-taking skills are often nonexistent, causing difficulty when we cover concepts not in the textbook. Outside the classroom, students must learn to start their homework sufficiently early or risk turning in sloppy, half-finished assignments.

To counteract these problems we have created an extensive support system for students. Although class time is precious, we spend time in Calculus I discussing note-taking, study habits, and other tips for success in a college level course. Each semester we have ten optional study sessions to help students with their homework or to prepare for exams. We monitor exam and homework scores and quickly notify students (and their parents) when their work is below expectations, and frequently require them to attend study sessions or make other special arrangements. Because our students are not generally on the University campus, our instructors hold “virtual office hours” in which they answer questions from students via phone and email. As a general rule, nearly every student in UMTYMP who is interested in mathematics and is motivated to work is able to complete the entire five year program at a satisfactory level.

Preventing UMTYMP students, especially females, from feeling isolated in class is another major retention concern. Our most successful strategy with females has been placing them in workshops which are 30%-50% female. When possible we also place them with female instructors or classroom assistants who are strong role models. This strategy has proven highly effective, and in recent years our retention rate with female students has been greater than or equal to the rate with male students.

3.3. *Student Characteristics*. Having dealt with thousands of mathematically talented students, we can make one fascinating observation: beyond a shared affinity for mathematics, there are no particular characteristics which set UMTYMP students apart from their peers.

In fact, our typical class is a microcosm of any American high school. We have musicians, athletes, and self-professed "computer geeks." Many of our students are introverted and awkward around other people, whereas others are extroverted, charismatic, and revel in being the center of attention. Some UMTYMP students are highly gifted and could be successful in any subject, while others have no particular interest or ability outside of mathematics. Each group has its own set of challenges. The gifted students are often involved in so many advanced courses that putting enough emphasis on UMTYMP to meet the heavy work demands can be an issue. Many of the students who are focused only on mathematics have not distinguished themselves academically, and it can be challenging for them to understand the high quality of writing and organization we expect in their work.

3.4. *Classroom Instruction*. UMTYMP can be a very challenging teaching assignment. In the high school component, for example, we cover a full year's worth of high school mathematics in about 30 hours, including 6 hours of testing. This would be impossible except for the fact that the students can learn routine topics–which comprise a large portion of the curriculum on their own without any formal instruction. Teachers must focus on the central ideas and most significant types of problems and trust their students to develop computational skills on their own.

The range of students in our courses also has an effect on instructional practices. Sixty to seventy percent of the students in a typical algebra class are seventh graders, with the rest in sixth or eighth grade. Although they have an aptitude for mathematics, there can be a tremendous difference in learning styles and focus; anybody who has taught students at this level knows, for example, that an eighth grade female can be far more emotionally mature than a sixth grade male. Some of our students prefer to learn concepts through self-instruction. The end result is that the teacher does not always have the full attention of every student. Some may be working individually on a problem, or discussing it with their neighbor. Our instructors have to learn to deal with this apparent lack of focus, so long as it does not bother the other students in the classroom. The reader can find a full discussion of the challenges of teaching in UMTYMP in [3].

3.5. *Historical Stability and Changes*. The most notable stable feature of UMTYMP is the continuing high interest of extremely talented students and families to participate in a program of this scope and magnitude. This need has intensified in the current public school environment, in large part because of the recent emphasis on high-stakes standardized tests. Schools have been forced to focus their resources on the mainstream curriculum, with many fewer opportunities for mathematically talented students. As a result, students are drawn to UMTYMP but come with little mathematical exposure beyond routine (if excellent) calculation skills. Overall, the problem solving, study, and communication skills of incoming students are weaker than ten to fifteen years ago.

Student attitudes have also changed significantly in the last decade. Students nowadays are generally much more involved in extracurricular activities, and try to squeeze UMTYMP into a packed lifestyle. Many of the supportive features of our program go unused by students, not for a lack of interest or need, but rather a lack of time. Despite repeated warnings from the program to students and parents about this issue, the unfortunate consequence is that we occasionally lose good students who could succeed with more time and focus.

Although our students are busier, they are increasingly younger and younger. Through the year 2000, the median grade level of our Calculus I students was tenth grade. Now the majority are in ninth grade, with a large contingent of eighth graders who started the high school UMTYMP program in sixth grade. This shift comes with a corresponding decrease in the overall maturity level of our students. Furthermore, our younger students are simply incapable of sitting still and staying focused for a long lecture. Whether this is a physiological fact or a byproduct of our cultural environment, we have had to adapt. Our class time has morphed from a rather traditional two hour lecture and workshop presentation to a more student-centered environment, with a blend of content presentation and student activities in small groups. The lectures focus on big ideas and central computations, while the workshops have group work specifically designed for UMTYMP students. This format remains effective even with the more mature students in their fourth or fifth year of the program.

One recent difficulty is the wish to include formal proofs and reasoning as part of the conceptual work. There is a dearth of textbooks that can meet our needs: rigorous treatment of a broad number of topics, but readable by a high school student. Overall we spend considerably more time searching for suitable textbooks now than in the past; currently all three years of the calculus program are undergoing a textbook search.

The high school program has some special issues. The time constraints and fast pace are more difficult for students in grades 6-8, and the difference in expectations between UMTYMP and their regular school work is more pronounced. The textbooks available at this level have also suffered more prominently in conceptual material and presentation. The so-called "college algebra" text used today is clearly less challenging than the high school text used twenty years ago. The geometry textbooks are even more problematic; although we strongly believe in group work and constructivist learning, we do not have enough class time for the exploration/conjecture model which is common in today's books. Our current text has evolved so far in this direction that it is now unsuitable for UMTYMP, and we are in the process of switching to a more traditional but well written book. However, this requires that the high-school instructors teach an UMTYMP course which will be significantly different than the courses and their own schools, causing an extra burden for them which was not present in 1995.

Field of Study	UMTYMP Alumni %	National %
Engineering	25.92%	5.80%
Mathematics	18.18%	1.05%
Physical Sciences	15.28%	1.50%
Computer Science	11.61%	2.75%
Biological	7.74%	5.35%

Table 1. Comparison of self-reported UMTYMP alumni degrees to national totals of all earned Bachelor's degrees (using an average of 1996-97 and 2001-02 data from [2].)

4. Outcomes

By their very nature, UMTYMP students are highly intelligent, so it comes as no surprise that many of them go on to be extraordinarily successful in their undergraduate studies and subsequent careers. This makes it difficult to measure the effect UMTYMP has had on our alumni and their success, particularly given the lack of any control group; it is not feasible for us to tell parents, "Your child has qualified for UMTYMP, but we would like to keep her out of the program and track her future progress."

We are initiating a large scale project to contact a thousand or more of our alumni in an effort to evaluate the long term effects UMTYMP has had on their undergraduate studies and subsequent careers.

In the meantime, although the absence of a control group makes it is difficult to make direct measurements of the program's impact on participants, it is possible to use our alumni database to make some broad observations which indicate a deep influence on students. Anecdotal data from surveys and other information from our alumni strongly support these observations.

One measure of the program's influence is which majors and careers are chosen by our alumni. A significantly higher percentage choosing paths in science, technology, engineering and mathematics (STEM) related fields may indicate that UMTYMP motivates and encourages students to use their mathematical talents in their careers. Table 1 compares the self-reported degrees earned by UMTYMP alumni to the national averages of all Bachelor's degrees. Even assuming our students' natural preference for mathematical and scientific subjects, they are earning degrees in STEM fields at a phenomenal rate. Given their aptitude and the number of math credits earned in UMTYMP, it should come as no surprise that mathematics is the most common degree. However, the percentage is striking: 18 times the national average. Other areas are also impressive: physical sciences are 10 times the national average, engineering 5 times the national average, and computer sciences 4 times the national average. Nearly 39% of our alumni also go on to earn master's degrees, most commonly in mathematics, medicine, computer science and electrical engineering. About 19% of our alumni have earned doctorates in a wide variety of fields; including at least 18 Ph.D.'s in mathematics.

The remaining degrees not included in Table 1 are distributed between the humanities, social sciences, and various technical/professional fields. We do not view it as a programmatic failure when students finish UMTYMP and continue to a non-STEM field, since breadth and scope of education is the cornerstone of the liberal arts philosophy common at colleges and universities throughout the United States. Our anecdotal evidence also indicates that the intellectual demands and conceptually heavy content of UMTYMP encourage students to use these skills in future careers. Alumni pursuing careers in fields as varied as law to music performance state that, although they may never need to compute the value of a flux integral, the work habits, qualitative reasoning and problem solving skills developed in UMTYMP are invaluable to their future careers.

The program has had a profound impact at the University of Minnesota. A number of UMTYMP alumni have received graduate degrees in mathematics at the University of Minnesota, and two more are currently enrolled in the Ph.D. program. Our alumni permeate the department's faculty and staff as well, including: a highly respected Full Professor who was a graduate of the very first UMTYMP class; a member of the advisory committee for our Masters in Financial Mathematics program; and the director of our computer systems administration staff. At the undergraduate level, UMTYMP has been responsible for a large number of very high quality students enrolling at the University of Minnesota. The school has made attractive accommodations with credit, placement and scholarships in an effort to recruit our students. As Table 2 shows, we have been very successful at retaining these students who might have otherwise attended one of the other prestigious schools on the list.

Institution	Current	Historical
1.	University of Minnesota	University of Minnesota
2.	MIT	MIT
3.	Stanford	Stanford
4.	Berkeley	University of Wisconsin
5.	Harvard	Harvard
6.	Northwestern	Berkeley
7.	University of Wisconsin	Caltech
8.	Yale	University of Chicago
9.	Columbia	Northwestern
10.	Caltech	Carleton College
11.	Carleton College	Yale

Table 2. The fifteen most attended undergraduate institutions among our alumni, both current and historical.

5. Assessment and Research Perspectives

A key component in attracting students and their families to participate in UMTYMP is the presentation of current data on student admissions to colleges and universities and subsequent successful careers. This requires maintaining and regularly updating a robust alumni database. Our statistical database was originally created in the late 1980's to study the effect of certain programs aimed at increasing female participation in UMTYMP. It has since been updated to provide extended data about the undergraduate studies and career choices of our alumni.

Maintaining the database requires real effort, but the current and potential usage far outweighs the costs. The database has been an extremely valuable resource for grant proposal data as well as an impressive statistical history of UMTYMP students' achievements, including the data used in this paper. It has also provided evidence to help UMTYMP make effective programmatic decisions to better serve certain subgroups of our student populations. For example, careful analysis of female applicants who passed the high school entrance exam (see Section 3.1) showed that school teachers were doing a poor job of identifying quality female candidates for the program. This led us to develop new approaches to attract and retain female students, changes which have had a lasting effect on UMTYMP.

UMTYMP regularly provides parents and the University with data on alumni degrees, college admissions, schools attended, majors achieved, career directions and related data. In addition, several questionnaires have been collection from alumni concerning the usefulness of UMTYMP coursework in college majors, the role of UMTYMP's conceptual approach in college mathematics and science courses, and other similar questions. We are currently working to improve our data collection procedures and boost our response rate, so we can continue to perform detailed and accurate longitudinal analyses.

Current UMTYMP students are also generally quite willing to be involved in qualitative studies on various issues. The program has informally gathered information on the work and study expectations of UMTYMP compared to their regular school work, on social and scheduling issues to be involved and successful in UMTYMP, and on parental pressure. These informal studies could be made more formal and handled in more traditional ways.

In addition to these passive analyses, UMTYMP provides a relatively self-contained environment for researching pedagogical techniques or other educational issues. The courses, curriculum and examinations themselves allow interesting and longitudinal studies on understanding of important topics in single and multivariable calculus and linear algebra. For example, most exams are (covertly) broken into conceptual and computational components, and the sub-scores provide global pictures of student understanding as well as information to help specific students improve performance. In a more formal approach, a current study of UMTYMP student understanding and misconceptions about series and sequences has provided some interesting initial results which could influence future instruction. This analysis will be continued for several years, and assess these concepts with the same students as they progress through the calculus program. More studies of this nature in the specific setting of UMTYMP students could be quite useful for other undergraduate issues in mathematics.

6. Issues Concerning Expansion and Duplication

The success of UMTYMP begs the obvious question: why has the program not been duplicated? To our knowledge nobody has ever tried to start a similar full-scale program at another location, although we have had modest success in expanding our own program to other sites throughout the state of Minnesota. This section describes those efforts and describes some of the challenges which would be faced by anybody interested in starting a similar program. Key aspects include long-term individual and institutional commitments along with the acceptance and support of local K-12 educational systems.

6.1. *Expansion within Minnesota*. Because UMTYMP receives crucial financial support from state government funds, UMTYMP has always been expected to make efforts to serve students throughout Minnesota. In the past, portions of UMTYMP have been offered at various “satellite” sites in cities throughout Minnesota such as Rochester, Saint Cloud and Duluth. Yet all of these cities have struggled with maintaining a full program. Several major issues appear to be common to all of these sites.

The demographics and geographic distribution of the population in Minnesota play a key role. About 65% (roughly 3.2 million) of Minnesotans live in the Minneapolis-Saint Paul (Twin Cities) Metropolitan Area. The satellite UMTYMP sites are all regional population centers, ranging from about 70,000 to 180,000 residents, which are surrounded by sparsely populated rural areas. The satellite programs have always begun with large classes – although still an order of magnitude smaller than the Twin Cities site – including some extremely talented students. As times passes, however, they inevitably experience lower and inconsistent enrollments. The smaller local populations force the sites to rely mostly on one major school district to provide the bulk of their students. When that district’s interests change or administrative support for such a program wanes, there is a very significant effect on

UMTYMP participation. This issue is avoided in the Twin Cities site, which draws from dozens of large school districts. Decreased interest in one district is usually counterbalanced by increased interest in another.

The other major difficulty is finding high quality instructors who are both capable and willing to teach UMTYMP students. Because of the scope of the program and its curriculum, it requires both a dedicated high school teacher to handle the first component, and a college professor to teach the calculus courses.

While the high school component is important, the program is even more dependent on the availability of quality college faculty. This is clearly reflected at the Rochester site, where a highly technological and well-educated population base[3] ensures parent and student demand for the program; unfortunately, Rochester has no four-year college or university and hence no local mathematics faculty. The site has only been successful when University instructors travel from the Twin Cities to Rochester on a weekly basis to teach the UMTYMP courses. In contrast, St. Cloud and Duluth have large universities with faculty members who were once involved with UMTYMP in the Twin Cities and are enthusiastic about teaching the courses. However, the local economies and school districts in those cities are not producing enough students to sustain the sites indefinitely.

6.2. *Duplication outside of Minnesota.* While there are several outstanding summer programs[4] and a few programs which provide an accelerated academic year program of high school mathematics, UMTYMP is the only program we know of which systematically provides a five year program including honors level college courses. Anybody wishing to start a similar program would face all of the issues involved in expanding UMTYMP within Minnesota, magnified by the lack of the central office providing administrative and curricular support. A complete analysis of all the requirements for a successful program would be beyond the scope of this article, but some key components are:

• A long-term commitment of college mathematics faculty to create and teach the college-level courses. This also requires a commitment from the department chair and school administrators to support and professionally reward faculty for these efforts.

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[3] Rochester is home to both the internationally renowned Mayo Clinic and a major IBM facility.

[4] For example, see the programs supported by the American Mathematical Society's Epsilon Awards.

• College faculty knowledge of K-12 students and school curricula. This is essential in designing high school courses which simultaneously satisfy school criteria and provide suitable preparation for honors calculus courses.

• Prior experiences with K-12 schools and teachers in order to obtain their trust and confidence that the program will help bright students learn more mathematics, and not harm their own schools' mathematics program by "removing" their best students.

• An administrative office to handle complex issues such as qualifying exams, tuition and student fees, student transportation, and communication with students and parents. These issues cannot be effectively handled in an informal way, and can seriously undermine an otherwise intellectually exciting program.

• The enthusiastic support of the students who attend the program and their parents. Attending such a program is a deep and fairly expensive commitment at several levels. Being able to provide accurate and compelling data on the value of this effort is absolutely critical to maintaining the program.

There are a myriad of other issues which need to be addressed to run a successful program beyond the main items above. Yet UMTYMP has demonstrated that all of these issues can be successfully navigated and provide a unique experience for large numbers of mathematically talented students. The personal and academic pleasure of teaching students with these mathematical interests and capabilities is exceptional, with many instructors regarding these classes as highlights in their teaching careers. The sense of satisfaction of seeing these students grow mathematically and move onward to significant careers is comparable to watching one's undergraduate and graduate advisees succeed. These are among the best reasons for urging other mathematicians to become involved in similar programs. Anybody interested in developing a similar program is invited to contact us directly for more information.

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