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Abstract: *The paper describes design and outcome of a 3-year action research program on the teaching mathematics to gifted students in grades 1-9 in mixed ability classes in Denmark 2003-2006. The intention was to combine ideas and experience of many teachers with theories and suggestions of researchers to test and develop useful recommendations for future teaching.*

Key words: Action research; mathematically gifted; proficiency; differentiation.

Introduction

Different ability of students has been an accepted challenge to schools and debate on teaching for years. Recently the discussion in Denmark has been extended to challenging the extent and possible handling of differentiation to gifted students.

2003-06 the Municipal School Authority of Aarhus, Denmark in cooperation with VIA University College of Teacher Education initiated an action research program, where I was the researcher and also acted as the project manager. During this period we developed and tried out ideas on teaching of clever students in mathematics. Experience from this work and a sample of findings made in other countries was a platform to an extension from 5 teachers and 3 schools in the first year to 35 teachers at 13 schools in term 2004-05 and 18 teachers at 8 schools in term 2005-06. Almost all teachers and schools were changed every year.

Aim, target group and a proposed yearly schedule were sent with an invitation for taking part to all 52 primary & lower secondary schools in the municipal area. Almost every school has grades 1-9.

Aim of the program

The aim was to contribute to increased attention on the proficiency challenge in math teaching, and to develop and try out approaches, which first and foremost supports the

mathematically able. The assumption was that this can be done in an ordinary mixed ability classes and show profitable to all students.

The target group was schools with desire to optimize conditions to students with proficient qualifications – and teachers with a proficient background for math teaching (this was not meant to be a course on mathematics).

Yearly schedule and research design

The research-design involved close connection to actual teaching practice. Five mutual meetings during the school year were mainly informative to, from and among the teachers, and combined with my research between the meetings. The meetings thus provided information, collected findings and kept everyone informed on progress.

① August	*Start-up-meeting* with presentation of earlier results, appointments for try-outs and reporting.

The purpose of this first meeting was to ensure a common background to the collegial talks in the group. Second and third year of the project the teachers were shown two short Danish movies on gifted students and heard one of my taped interviews with a gifted student from the former year. A mathematical inlay was about the winning strategy in playing NIM. The outcomes were also these appointments and memos to participating teachers:

1. Prepare information to students and their parents on the developmental work.
2. Make appointment with coordinator, who will supervise 1-2 lectures. The purpose of my visit was to offer a concentrated collegial sparring on the routines or way teachers try to meet the mathematical challenging (gifted) students in their math teaching. Thus I visited all classrooms for at least one 45 minutes each and had a short talk afterwards with the every single math teacher on their strategies to the gifted students in their classes! Beforehand, the teachers were asked to point out the two (or some of the) most gifted students. I also suggested the teachers to be clarified on how to show the intended attention to these students when teaching them.
3. Read the report/book (on results from former year) before coordinator visits at schools.

② October	*Full-swing-meeting* with supplemental ideas and support.

At this meeting I presented the group to an overview on different routines noted during my visits to classes. The project teachers were asked to comment and justify these, e.g.:

Program of work of the lecture (day) on blackboard in class

Mental math routines

Connections to other subjects like P.E. and science

Explanations for only part of class

Teacher: ”I don’t expect everybody to do all problems”, hard extra assignments to some

Mutual project with one number-able group among five (following the ideas of Howard Gardner)

Number-stories, focus on oral presentation

Guided discovery using concrete materials

Confidence on students organizing own investigation

IT as an extra possibility for differentiation

As *competitions* might be a suitable challenge to students with extra time and efforts, the teachers were also informed of some national and international possibilities. The Nordic KappAbel competition www.kappabel.comin all Nordic languages takes place every year and is meant for grade 8. The Kangaroo competition: http://www.mathkangaroo.org is not in Danish language, but suited for many more grade levels.

Every teacher was also asked to *prepare an answer* to one of these questions for the next meeting of the group:

1. **Thoughts on goal setting**

   How do you make gifted students aware and conscious on own goals?

2. **Thoughts on student's pre-understanding**

   How best to catch the special qualifications and experience of gifted students in a concrete area (eventually before a certain teaching sequence)?

3. **Thoughts on planning**

   How can the gifted students take part?

   How do I meet the expectations of these students?

   How do these students become co-responsible for planning?

4. **Thoughts on way of organization**

   Experience with gifted students in whole class teaching, group work and individual work?

   When does an organizational form work and when not?

5. **Thoughts on differentiation of teaching**

   What have you been changing and done differently to different students? Tasks, texts-formulations, materials, ...?

   Bring an example of something, you consider very successful and try to explain why?

6. **Thoughts on assessing with the students**

   How do you carry out a (mutual) evaluation, which also gives room to the gifted student?

   Give an example of a good method.

Every teacher was asked to arrange to *visit a colleague* (at another school), *and have a visit by a colleague* (preferably another). Appointments were made at this meeting.

③ December	*Mid-term-meeting* with evaluation so far and communication of new/more ideas.

The meeting was about:

- Impressions and considerations after the mutual sparring with colleague teacher

- Midway evaluation of the developmental work
- Best practice, ideas and strategies for mutual inspiration (every teacher was asked to bring at least one)
- Synopsis to the yearly report, year 3 this became a collection of recommended problems
- Separation in ”writing-groups” – with responsibility for different grade levels
- Presentation of my interview-guide and actual appointments on interviews.

My interview-guide for interviews with 2 gifted students in every class was this:

1. Are you good at mathematics? How do you know?
2. When do you feel, you learn the most in math lessons?
3. Give an example of a task, you find especially fruitful. Why do you find this task so good?
4. Are you working especially well with others in your class? Who for example?
5. How often do you talk with your math teacher about difficult tasks?
6. Do you think your math-teacher is demanding enough of you? Or too much?
7. Do you have a good advice to teachers with talented students in their class?
8. Eventually?

④ March	*Almost-done-meeting* with mutual orientation and a frame for reporting. The participant’s contribution to joint report on experiences and recommendations sent to coordinator for compilation.

I made interviews with 2 gifted students from every class in the project: 10 students in year 1, 69 students in year 2, 36 students year 3. All interviews were transcribed and a copy given to the teacher. At the March meeting I presented patterns and similarities from the student interviews:

- The students are very different. All are gifted, but remarkably many are also good on quite different fields as sport, music, ...
- Quite many have (also time consuming) other interests
- Some students are rather special, but do get along well in classes. In any case nobody was interested in jumping past a grade in school, when I (jokingly) asked for that
- Some, but far from everybody, are able to "explain" their interest in mathematics. Many consider it caused by parents (counting cars, some parents are even teachers themselves, etc.) and some by other reasons (a certain math teacher, a book-present including a calculator etc.)
- Almost all gifted students were happy to be challenged more than most students in class! And some are not at all.

Following this presentation of findings we had a round in groups on coordinator findings. Every teacher had transcription of own two student's interview and was asked to select an essential statement (e.g. only 10 lines) from one of them. E.g. some statements about the teachers' handling of the proficiency challenge in mathematics teaching. The excerpts were shown and discussed in groups in order to find recommendations to the teacher or to the school(s).

Report / book

Before the yearly final meeting of the group I wrote the report/book on theoretical findings, contributions and recommendations from teachers, excerpts of student interviews and suggestions for new routines and strategies. Year 1 this was an internal report, year 2 this became a "real" book (more than 100 pages) and in year 3 the report became a problem book. The books were printed with support from the local authority (year 2) and the Ministry of Education (year 3), so they were sent for free to all 1.000 math teachers in the city of Aarhus. The rest can still be bought at printing cost (Mogensen, 2005).

⑤ April	*Final* meeting with publication of concluding report (and eventually a press release).

The final meeting presented some up-to-date resources, which teachers might want to draw on in their teaching, e.g. a digital math encyclopedia (in Danish) and interactive (electronic) blackboards. Speeches were held and everyone had the newly printed report/book.

In the following section I will present more of the overview and findings from this Danish action research project. These are also published 2008 in a report from the European Comenius 2.1 project: *Meeting in Mathematics* (Meeting, 2008).

What does it mean to be gifted?

All teachers in this project had students, they considered especially gifted or especially challenging. But how can a teacher know who they might be?

This decision was left up to the individual teacher. Some teachers based their choice on regular assessment through written tasks or tests. Some teachers had known the students for several years, some had just been appointed to the class. In each case the choice was not made until the action research program was three months underway.

Seen this way, the gifted students numbered two out of a typical total of 25 students in each class, or 8%. However, in intelligence research you will often meet the expression, "students with special qualifications". These students are approximately 2% of the total number by IQ-test, and might very well be among the gifted students mentioned above.

There was a large variation in teachers' perception of gifted students. The following characteristic may be a support for parents and teachers, who are in doubt. The table is provided by the Mensa organization (www.mensa.org). Although the two columns are not alternatives, Mensa members suggest the right column to present characteristics of the 2% most intelligent children.

Gifted student	Student with special qualifications
Is interested	Is extremely inquisitive
Has good ideas	Has wild crazy ideas
Ironical	Sarcastic
Answers questions	Poses questions to the answers
In the top of the class	Ahead of the class
Learns easily	Knows already
Popular among peers	Prefers adults
Remembers well	Makes informed guesses
Accepts information	Adapts information
Likes to go to school	Likes to learn
Fond of structured learning	Gets on with complexity
Has a talent	Has many talents
Becomes happy	Becomes ecstatic
Becomes angry	Becomes furious

Gifted students therefore do not necessarily constitute a homogeneous group, as they would fit in both columns of the table above. But they always challenge the teacher in matters regarding form and content in teaching. The challenge may not be noisy or obtrusive. Some of these students can be silent, pleased by a strong structure or "keeping their heads down", to be almost invisible in the classroom. Others may be seen as clumsy, anti-social or arrogant – and anyhow extremely visible in the classroom.

In any case they are mathematically challenging to the teacher. And one should consider various approaches when meeting these students. Some teachers said: I don't think I have any really gifted students – although I have some who are smart. Perhaps you should see ability or giftedness as a wide spectrum and support the student differently.

Numerous attempts to uncover the competence of students have been made; this is reflected in many publications. The Russian psychologist Krutetskii (Krutetskii, 1976) suggested that mathematically gifted students were good at

- Reasoning quickly

- Generalizing
- Manipulating abstract concepts
- Recognizing and using mathematical structures seen before
- Remembering rules, patterns and solutions seen before
- Finding shortcuts, which means thinking "economical".

Krutetskii (1976) also mentions two significant norms of behavior of gifted students. Firstly working with mathematics does not tire them; they can keep on for hours. Secondly they have an ability to see cross-curricular problems through mathematical eyes.

In 1995 a report was published by the group: "Task Force on the Mathematically Promising" (NCTM, the American National Council of Teachers of Mathematics) prompted by the requirement to increase attention to talented math students in the USA. In the report Sheffield (Sheffield, 1999) describes mathematical promise *as a function of ability, motivation, belief and experience or opportunity. None of these variables are considered to be fixed, but rather are areas that need to be developed, so mathematical success might be maximized for an increasing number of promising students.*

The assumption that abilities **can** be enhanced and developed is supported by knowledge from brain research, where it is understood that experience results in changes in the brain.Together with the NCTM-report, this suggests that motivation should be affected and treated seriously when a school culture makes students keep low profiles to avoid being labeled as *nerds*. Self-confidence and good role models amongst classmates and teachers are decisive for students' attitude to the subject.

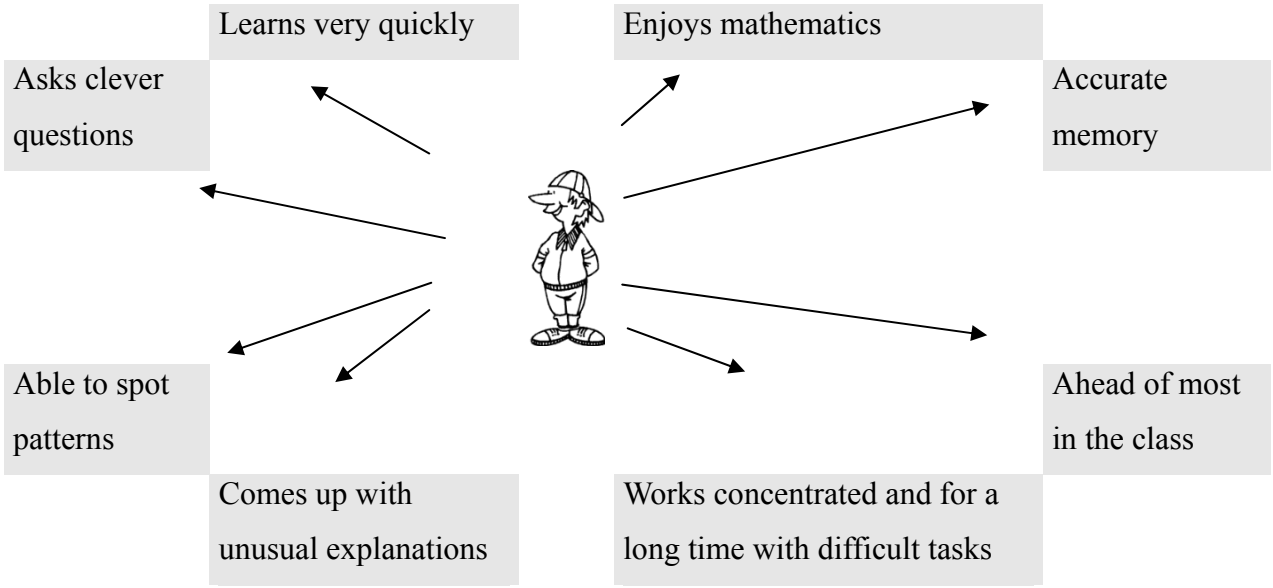
Sheffield suggests these characteristics of mathematically gifted students:

- Early and persistent attention, curiosity and good understanding of "quantitative" information.
- Ability to grasp, imagine and generalize patterns and connections.
- Ability of analytic, deductive and inductive reasoning.
- Ability to shift a chain of reasoning as well as the method.
- Ability of easy, flexible and creative handling of mathematical concepts.
- Energy and perseverance in problem solving.
- Ability to transform learning to a new situation.
- Tendency to formulate mathematical problems – not just solving them.

- Ability to organize and ponder information in many ways and sort out irrelevant data.

Please notice that this list does not include the ability of calculating fast and correctly! Of course many of them are capable of doing that – but Sheffield insists this it is neither a necessary nor a sufficient condition for being a mathematically gifted student. A lot of these students are impatient with details and reluctant to use time on computations.

Koshy offers the characteristics below partly based on work with British teachers (Koshy, 2001):



Risk

It is tempting to combine such suggested lists, so as to build a single checklist suited to estimate mathematical potential. However, there is a risk in using such a simplified list for the following reasons:

- Gifted students show their special talent only if there are stimulating opportunities for this.
- Some students play down their scope of abilities to avoid extra homework.
- Some students conceal their abilities in order not to be different – and be bullied.
- Multilingual students may have language problems.
- Some students have social problems or lack of self-confidence – e.g. no support from home.
- Other outside factors may also affect and provide ability, motivation, attitude and opportunities.

Of course teachers spot capable students more easily when there are challenging contexts of teaching and learning, i.e. these students get an opportunity to show their special abilities. This may take place in talks with classmates, elderly students or siblings, parents, teachers or school counselors. Observing how students approach and solve relevant tasks in and out of school may also help teachers to notice gifted students.

Parents' role

Some children show particular abilities before their start in school, and one could imagine a talk about this to take place with parents at the enrolment of kids in school.To make sure it happens, a line with focus on this should be included in the application form.

Parents' ambitions may also result in inquiries to the school about special consideration for their children. On the other hand there may be a total lack of support from home. Some countries are better than others at breaking the social heritage.

The role of parents regarding support and challenge was emphasized in interviews with some of the students and teachers in the action research project. Here is a typical statement by a Danish teacher in the action research project:

"The condition/A prerequisite to go further in teaching and learning than normally requested at a certain grade level is to explain at the first parents meeting how you intend to teach the students:
By keeping a focus on challenge also for the gifted students
By offering all students suitable and challenging opportunities
By assuring parents that nobody will be lost, the scope is to amass successes rather than defeats.

At a parent-teacher meeting, the teacher gives some examples of oral communication in teaching, e.g. the teacher could go through a teaching unit, and give the parents the same sort of tasks, which the teacher later would introduce to their children.
Ask the parents to reply, comment on the answers and tell them what teachers would expect, including creative remarks, add that these are welcome.
Concerning homework (or in periods the lack of same), it is likewise necessary to clarify that it is not volume, but quality that counts. The students must be able to explain their line of thought."

The statement suggests, that the role of the parents should be supportive, not demanding or a transfer of unfulfilled parents' ambitions.

Test

The qualifications or learning outcome of students can partly be assessed by a test. If written tests are used for all students, it is important to remember the limitations. Teachers may ask themselves:

- Will the test results tell something new about the individual student?
- Does the test contribute to planning of better teaching?
- Is the test also suited to the gifted students?
- Does the test method enable creative thinking?
- Is there a risk of losing surprising solutions or comments?
- Does the test fit the grade level and the curricular goals?

A test may be so easy that it either does not provide an optimal challenge or misleads some students to believeit to be more difficult than it actually is. The tests used by Krutetskii were not diagnostic but purely research tests. Each series reveals only one or few aspects and manifestations of the mathematical abilities being studied. And the 72 tests are of four basic categories, where three *"correspond to the three basic steps in solving a mathematical problem (gathering the information needed to solve the problem, processing this information while solving the problem and retaining in one's memory the results and consequences of the solution). The fourth category concerns the investigation of types of mathematical ability. (p.98)"*

This may be a reminder: Any cleverly designed test will map only some aspects of what might characterize mathematical giftedness.

Experience and strengths

How does a teacher use the experience and strengths of gifted students?

To make every teaching effective, you should start from recognizing the backgrounds of the students. But each of the strengths is accompanied by disadvantage, when teaching in a multilevel classroom. The following table makes use of some of the characteristics, Krutetskii, Sheffield and Koshy pointed out. Several tables like this one below appear in various publications (Baltzer, Kyed, Nissen & Voigt, 2006), and the description in the two columns is often found to explain the social challenge of some gifted students:

The strength	**The disadvantage**
Is curious Thinking critical	Poses questions, that may embarrass others Critical and intolerant towards others
Works alone Remembers earlier rules and solutions	Seems superior and obstinate Opposes exercises
Does abstract thinking Has high expectations	Rejects details, looks for simple solutions Perfectionist
Shows energy and patience in problem solving Works goal-oriented	Loses interest, when things do not develop as intended Is impatient with the slowness of others
Generalizes patterns and connections Transfers learning to another situation	Does not like routines, will easily be bored Formulates complicated rules and systems
Finds shortcuts Thinks ”economically”	Gets frustrated by inactivity Interrupts and seems hyperactive

Goal

Are there especially good opportunities to make gifted and motivated students aware of and conscious about setting their own goals?

Yes, we can suppose so. And it may very well be a necessary step in order to meet the particular experience and strengths of these students. Well aware that cultures and settings may differ between schools and countries, I would like to mention that the following viewpoints are based on Danish experience.

When working with very capable students such common goals for a class may be too modest. The gifted student can aim higher than other students in the group. In the Danish action research scheme I interviewed 115 gifted students. Only very few felt too loaded by tasks and expectations from their mathematics teacher, who even had them in focus as especially gifted. On the contrary, to many students it was the other way around, i.e. most were eager to have at least a few more challenging tasks.

So three questions may be asked:

- Would it help to make goals more visible and involve the students in matters of organization and evaluation?
- How do teacher expectations affect the attitude and work of gifted students?
- Should teachers be ambitious on behalf of their students?

I will offer an answer to these questions below.

Planning

Can capable students co-operate in planning their math work? Yes, action research confirmed this. But it implies expectation, initiative and support by the math teacher. Learning a subject such as mathematics is an individual process, taking place in a social context. Co-operation is part of the learning process; in Denmark it is even included as an aim in the subject curriculum:

Danish Mathematics Curriculum grades 1-10 (Aims, section 2).

Teaching shall be organized so that students build up mathematical knowledge and proficiency on the basis of their own prerequisites. Students shall, independently and together, experience that mathematics is both a tool for problem-solving and a creative subject. The teaching shall give students a vivid insight and further their imagination and curiosity.

The curriculum is a common condition for all students, and it stipulates sharing responsibility in setting goals and choosing contents.However, the curriculum was not addressed to young students, i.e. it was not formulated in a language well-suited for young students, and it

is a majorchallenge for math teachers to pass it on and interpret the demands for the class. Nevertheless, teachers *ought* to do that.

As is the case in many countries, the Danish curriculum of mathematics is imbuedwith a constructivist view on learning, i.e. Knowledge and insight cannot just be fed from teacher to student, but have to be constructed by each student with the assistance of a teacher and in interplay with classmates. The learning process takes placed in a social setting where students can develop meta-cognitive abilities to monitor and direct their own learning and performance.

This means students share some responsibility in an active learning process. Here it is fundamental to success that the students practice self- and peer-assisted-evaluation. It is possibly the best argument for portfolios as tools of reflection and documentation in school.

It is certainly an important idea for the teacher to invite capable students to *think ahead*; having their own ideas, aiming further than the common goal in class, but still in correspondence with the math curriculum. In younger grades the teacher could encourage capable students to learn each their own tables way ahead of the rest of the class, or ”tempt” them by mentioning prime numbers and square root. In lower secondary or middle school, capable students could be prompted to work with reduction or trigonometry at high school level. Teachers could encourage the capable students to go deeper or ahead.

Perhaps math teachers should take regular *developmental talks* with capable students individually or in groups – or might differentiation of goal and plan be handled in whole-class discussion? Many teachers in the action research project were considering advantages and disadvantages of various forms of organization. In every class students are different: they show different interests, intelligence and professional proficiency. Hence, when teacherswant to present the individual student with learning situations, which correspond to the student’s background, they need to differentiate the teaching.

There are plenty of ways to differentiate:

1. ***Short introduction to new content/tasks***

   You can make an arrangement with the class, setting students to work independently after a common introduction. The capable students are quick to catch the point and may on that account sooner than the rest continue their individually work. Students needing further

assistance can thereafter go through more examples. The capable students work individually or together with the tasks.

This form relies on teachers to discuss teaching organization with their students. One should not emphasize teaching of the able at the expense of weaker students. Through participation in meta-discussions, students will become conscious about learning in various ways, some are quick and pick up matters easily; while others are slow, having to struggle more with the issue at hand.

2. ***Grouping by academic criteria***

   This is when the capable students are put together in morepermanent groups, where they challenge each other.

   In a group of academically capable students you could expect more independent work, but the group should continue to have the attention of the teacher. It must not become a suit-yourself group. When the students are grouped at levels, it is easier for the teacher to pose challenging questions and tasks and give further inspiration to the gifted as well as to the weaker ones. The grouping should be fixed for a period and made by the teacher based on joint decisions by teacher and students, possibly backed up by tests.

   When a school has more classes at the same or close-age levels, the grouping could also be done by "setting". This means more teachers can cooperate to find and compose material suited to various levels and thus prepare a more goal-oriented teaching of the various groups.

3. ***Amount of content/time***

   Let students solve the same tasks at different levels – or differentiate in time. The more capable students can handle more tasks or the same tasks in shorter time. It is crucial that capable students are being challenged and develop a culture, which makes it attractive to get as far as they can. This means, you must have a stock of extra tasks, preferably different tasks. It may also imply that capable students must do more extensive work on tasks, for instance open-ended tasks, solvable at different levels.

4. ***Different tasks***

   Working within a content area, you may present tasks in various degrees of difficulty, which the student elects/gets handed. Likewise you could differentiate by materials, e.g. let capable

students use a 10-sided "dice" instead of a regular 6-sided one, use other basic arithmetical operations, etc.

Based on my experience and action research, I recommend the following variety of tools to teachers when it comes to differentiation:

Difference in demands	You do not have to be equally tolerant of the quality or the quantity of the individual work of the individual student. You should also be able to: • create interest around a topic • choose/produce good introductions • form teams or groups for collaboration • give the students sufficient time • promote the "mathematical discourse" • create rigorous discipline combined with a pleasant atmosphere.
Difference in time	The time, given to the individual students for one and the same task may differ. It is likewise important to make time to talk with a group or with individual students. On that account: • Fit out the classroom to enable students to be autonomous, e.g. in getting paper, scissors, glue, extra tasks, mathematical games, computer programs, calculators, etc. • Establish structure, e.g. giving your students a sense of propriety. • Arrange to have consecutive math lessons! Eventually this must be a collective decision at school.
Difference in assistance	• Prioritize your use of time for different students. • Make use of students helping each other.
Difference in topics	• Give students frequent opportunities to work with different topics depending on need, interest, and inclination.

Difference in way of teaching	Vary your approach, of course adjusted to the different students. I recommend all these forms in a sensible balance: • Exposition by the teacher (of new content or homework). • Discussions between the teacher and the students and among students themselves. • Appropriate practical work. • Consolidation and practice of fundamental skills and routines • Problem solving, including the application of mathematics to everyday situations • Investigations and experiments.
Difference in educational resources	Textbooks are controlling! However, very few teachers will teach completely without textbooks. Apply also: • Supplementary written material. There is a lot: booklets, timetables, statistics, advertisements, news, etc. (Usually such material must undergo a certain adaptation). • Own introductory presentation (eventually with the assistance of colleagues) of activities of limited duration and specific goals or thematic work for longer time. • Student surroundings in a wide sense (TV, sport, preferences, opinions, experiences). • Observations of students and their work. • Calculators and computers are wonderful teaching tools also to increase variation in content and teaching style.

Difference in goals	Taking-off in continuous assessment the students will set for different goals. But the final goal of school and mathematics teaching must be the same to all! You may apply "untraditional" methods to obtain knowledge about the students' outcome of mathematics teaching, e.g.: • grade 6 students can tell all the class (and teacher) about the cost of a hobby • grade 7 students can write a report about quadrangles instead of a ordinary homework • grade 8 students can write in a log book once every other week about their mathematical findings.

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