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Mark Applebaum

Viktor Freiman

Roza Leikin

# *Prospective teachers' conceptions about teaching mathematically talented students: Comparative examples from Canada and Israel*

**Mark Applebaum[1], Viktor Freiman[2] and Roza Leikin[3]**
**[1]Kaye Academic College of Education, Beer Sheva, Israel**
**[2]Université de Moncton, Canada**
**[3]University of Haifa, Haifa, Israel**

**Abstract:** In this paper we analyze prospective mathematics teachers' conceptions about teaching mathematically talented students. Forty-two Israeli participants learning at mathematics education courses for getting their teaching certificates, and fifty-four Canadian pre-service (K-8) teachers participating in mathematics didactics course were asked to solve a challenging mathematical task. We performed comparative analysis of problem-solving strategies, solution results and participants' success. Based on the discussion with 25 Israeli participants we composed an attitude questionnaire, in which prospective teachers were asked to express their degree of agreement with statements expressing different beliefs about education of mathematically talented students. The questionnaire was presented to 56 Canadian and 28 Israeli prospective elementary and middle school teachers. We describe similarities and differences between the attitudes of the two populations and suggest their possible explanations. Based on the results of this study we make several suggestions for teacher education programs.

**Key words:** Challenging task, teacher preparation, mathematically promising students

## INTRODUCTION

Teacher preparation is a crucial factor in creating opportunities for mathematically promising students to realize their abilities by means of challenging mathematical tasks (Even et al., 2009, Sheffield, 1995). To what extent are teachers ready to work with mathematically promising students when they finish teacher education programs? We conducted an exploratory study in two different cultural contexts: in an Education College in the southern part of Israel and in French-language Canadian University in the south of New Brunswick. We asked prospective mathematics teachers enrolled in mathematics education courses to solve a challenging task and to answer a questionnaire that examined their beliefs about teaching mathematically promising students.

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We start with review of literature related to the characteristics and educational needs of mathematically promising and mathematically talented students. We also discuss the role of teachers in the education of such students. We then describe the study structure, the results of the study and finish with some questions that remain open for future investigation.

## MATHEMATICALLY PROMISING STUDENTS HAVE SPECIAL NEEDS

The NCTM Standards (2000) stressed that school mathematics has to provide all students, independently of their ability level, with equal opportunities in learning mathematics. Equal opportunities mean matching of the mathematics education to the mathematical potential of learners. NCTM (1995) set up a task force that defined the notion of mathematical promise as a function of four key factors: ability, motivation, belief, and experience. Wertheimer (1999) claimed that taking care of mathematically promising students is an essential educational issue because these students have the potential to become leaders and problem solvers in the future.

Sharing an inclusive view on the education of children with high ability in mathematics, we consider that both mathematically talented students and those that have potential to move beyond standard skills and are highly motivated are part of this group. Therefore, mathematically promising students may possess several characteristics known from the literature on mathematical giftedness such as excellent selective memory and faster progress in their learning (Ponamorev, 1986; Krutetskii, 1976). They also have strong motivation, increased concentration, intuition, originality, stability and flexibility (Goldin, 2009; Yurkevich, 1977; Ponamorev, 1986; Subotnik, Pillmeier & Jarvin, 2009). Krutetskii (1976) pointed at such high abilities in mathematics as formalization, abstraction, finding short solutions, inversion in thinking process and generalization. Mathematically talented students stand out for their ability to work systematically and quickly, getting an insight into the problem's mathematical structure (cf. Heintz, 2005). The ways they solve problems, usually differ from those of regular students (Krutetskii, 1976). Finally, many of these children are prominent in their higher ability to verbalize and explain symbolically their solutions (Freiman, 2006).

Several authors stress that mathematically promising students have to be provided with multiple opportunities that would foster their mathematical understanding, creativity, curiosity, thoroughness and imagination (Ervynck, 1991; Piirto, 1999; Silver, 1997; Sheffield, 2003) and mathematical tasks for the mathematically promising students should be especially challenging (Applebaum&Leikin, 2007; Sheffield 2003; Freiman, 2006). Based on Polya (1973), Schoenfeld (1985), and Charles & Lester (1982), Leikin (2004) suggested that mathematically challenging task should (a) be motivating; (b) not include readily available procedures; (c) require an attempt; and (d) have several approaches to the solution. "Obviously, these criteria are relative and subjective with respect to a person's problem-solving expertise in a particular field, i.e. the task that is cognitively demanding for one person may be trivial (or vice versa) for another" (Leikin, 2004, p. 209).

Following Brousseau (1997) we acknowledge importance of teachers' role in "devolution of a good task" to any student and claim that this role is critical in creating suitable learning environment for mathematically promising students. In order to create such an environment a teacher should be mathematically educated, be able to assess students' potential and fit mathematical challenge to their abilities and needs. In this context, our exploratory study was aimed at analyzing (a) teachers' strategies when coping with challenging mathematical tasks (b) teachers conceptions about mathematically promising students and their education.

## Teachers' knowledge associated with teaching mathematically promising students

Approaches implemented in each particular classroom and the mathematics employed depend on teachers' knowledge and beliefs. Research stresses the importance of teachers' knowledge (Shulman, 1986) and beliefs (Cooney, 2001, Thompson, 1992) for decision making in the process of teaching. Teachers' knowledge and beliefs are interrelated and have a very complex structure (see, for example, Leikin, 2006). In this study our focus is on the types of knowledge characterized by Shulman (1986) as composed of teachers' subject-matter knowledge i.e. knowledge of mathematics, and teachers' pedagogical content knowledge which includes the knowledge of how the students cope with mathematical tasks, and the knowledge of how to create an appropriate learning environment. We also differentiate between beliefs about the nature of mathematics and beliefs about teaching mathematics with special attention to mathematics and mathematics teaching related to mathematically promising students.

The need of mathematically promising students' in especially challenging tasks may be negatively perceived by their teachers. The negative views depend on their previous experiences and the lack of mathematical and pedagogical readiness to deal with challenging tasks. They sometimes reflect teachers' skepticism about the possibility of increasing mathematical challenge in their classroom (Leikin, 2003). There is a lack of research evidence on how teachers deal with challenging investigative mathematical tasks intended for mathematically talented students and on their readiness to work with these students. Our paper is therefore focused on deepening our knowledge about the two above mentioned components: teachers' capacity to solve challenging tasks and their views on mathematics education of mathematically promising students.

Nowadays, mathematically talented students often study in heterogeneous classes and do not get special treatment, since teachers in these classes lack knowledge and skills to take care of them. Teachers often lack of instructional materials they may use with the students in the heterogeneous environment, and even when they have the appropriate material available, they do not know how to use it. Moreover, teachers are not always aware of the mathematical potential of their students, and consider as promising only those who get high grades and/or behave well. Besides when students do not follow all the prescriptions, choose their own ways of solving problems, perform their tasks quickly and misbehave when bored during the lesson, they are perceived by the teachers mainly as trouble-makers. Additionally, teachers themselves do not always understand students' original solutions and do not know why and how to encourage students' critical and independent thinking and creativity.

Considering specific learning needs of mathematically talented students we stress the special skills and knowledge the teachers need for organization of an appropriate teaching process. Is there a need for special preparation for teachers and if yes, what kind of preparation it should be? Different countries e.g., Australia, USA, Israel, Korea, Japan, Russia and others (Leikin, 2005) have different approaches in this matter.

## Education of mathematically promising students, and their teachers in Israel and Canada

Research literature in the field of teacher education (Stigler & Hiebert, 1998) stresses that teaching is a culture-based activity. The authors of this paper have rich intercultural experience in mathematically promising students' education due to their personal histories. All three come from the former Soviet Union educational system, where school education of mathematically talented students was an important element. We studied in mathematical classes or special mathematical schools (e.g. Mathematical School #30- http://www.school30.spb.ru/), and attended mathematical summer camps. During our school years, we met a very special kind of teachers who were usually professional mathematicians who were themselves mathematically gifted and often graduated from similar special programs. Those teachers were very enthusiastic and committed to the concept of special educational programs for the gifted and talented (Evered & Karp, 2000; Freiman & Volkov, 2004; Karp, 2007). The later experience of the authors is based on the realities of Israeli and Canadian education. The present study has been conducted in two different countries, Canada and Israel.

In Canada, each province governs its own educational system. The issue of teaching mathematically talented students is viewed and resolved in different ways. In New Brunswick, there is strong emphasis on inclusive teaching and learning; all children should be involved in all activities. However, as result of recent study of inclusion in schools (MacKay, 2006), the government has started to develop and implement new policies that should better respond to the need of students with special needs. Gifted students are explicitly mentioned as part of this group (GNB, 2007).

Changes are already being made in many schools and some of them begin to take care of mathematically talented students (Freiman, 2008). At the Université de Moncton, prospective teachers work with challenging mathematical problems posted on the CASMI site (www.umoncton.ca/casmi, see the paper of LeBlanc & Freiman in this issue). The site allows prospective teachers to evaluate authentic students' solutions and may be used as resource in their future work. The problem that we use in this study was originally posted on this site and our preliminary analysis of submitted solutions allowed us to construct our investigation with Israeli university students. Working with challenging tasks on the CASMI site, as well as some other projects we develop future teachers' awareness of the special needs of mathematically promising students. However, more is to be done in order to ensure their better preparation.

In Israel, during the past decade, awareness of the importance of promotion of high ability students has been growing. Education of talented children and adolescents is considered to be "the springboard for the development of democratic society strong in its scientific advancements, industry, high technologies, humanities, and arts". (Rachmel & Leikin, 2009, p. 6). A steering committee of the Division of Gifted Education in the Ministry of Education (Nevo, 2004) devised recommendations for the advancement of education of talented schoolchildren. Educational programs for students who are highly able in mathematics are coordinated by the Ministry of Education or by some non-profit organizations. Israeli Universities are also involved in promoting mathematics education of high ability students. Schools organize special mathematic classes, special mathematic groups (mainly starting in the 7$^{th}$ grade), mathematic circles, and competitions. Additionally, various out-of-school activities are developed for such students. Among those activities are mathematical clubs, Mathematical Olympiads, students' conferences and integration of school students in university courses (more details can be found in Rachmel, 2007; Rachmel & Leikin, 2009).

The Division of Gifted Education of Israeli Ministry of Education encourages teachers to get special education, though there is still a shortage of corresponding programs. In the last six years there were open five special teaching certification programs (in three teacher training colleges and two Universities) and the first M.A. program (in Haifa University) devoted to the education of gifted students. These programs are mainly interdisciplinary and are not focused on specific school subjects.

Mathematically promising students also get special treatment both through the efforts of the Ministry of Education (e.g., Epitomizing and Excellence in Mathematics Program, Zaslavsky & Linchevski, 2007) and those of different non-profit organizations (e.g., Excellence-2000 Association, MOFET Association, and more, e.g. Applebaum, Schneiderman & Leikin, 2006; Schneiderman, Applebaum & Leikin, 2006). Teachers of mathematics working in these programs have to participate in seminars devoted to enrichment in secondary school mathematics. Unfortunately, there are still not enough courses specifically devoted to education of mathematically talented students.

As presented above the education of mathematically promising students and their teachers differs meaningfully in the two countries and thus we wondered whether the differences in the policy affected prospective teachers' conceptions associated with this issue. That is why, in our study, we ask participants from both countries about their beliefs regarding their own educational needs in preparation as professional teachers able to work with mathematically talented students.

**THE STUDY**

## The purpose and the questions

The main purpose of the study presented in this paper was exploring prospective teachers' conceptions about teaching mathematically promising students. To examine teachers' mathematical knowledge we ask: How do teachers themselves cope with an investigation task intended for mathematically promising students? What problem-solving strategies do they use? In order to explore teachers' pedagogical conceptions associated with teaching mathematically talented students we ask: How do teachers define mathematically talented students? What do they think about mathematical tasks suitable for mathematically talented students? What are their views on the preparation of teachers for mathematically talented students? We compared the responses of participants from Israel and Canada.

## Population and procedure

The study was conducted in two stages.

## Stage A

Forty two Israeli college students enrolled in mathematics education courses as part of teaching certification program took part in solving a challenging task. Then 25 of these students participated in a follow-up discussion about the task they solved, the needs of mathematically promising students and the knowledge and skills of teachers of the gifted. The preliminary analysis of this stage of the study has been presented at the ICME-11 Congress (Applebaum, Freiman & Leikin, 2008). We performed qualitative analysis of the collected data: categorized problem solving strategies used by the prospective teachers and performed content analysis of the discussion conducted by the first author of this paper. Categories derived from the analysis of the discussion were used in the attitude questionnaire at the second stage of the research.

## Stage B

Fifty-four (New Brunswick) Canadian prospective elementary (K-8) teachers enrolled in mathematics didactics course and 28 Israeli prospective elementary school teachers enrolled in mathematics education course as part of their mathematics teacher training (Grades K-8) were asked to answer the questionnaire. All Canadian participants were asked additionally to solve the task that Israeli teachers had solved at Stage A of the study.

## The tools: data collection and data analysis

### The problem

The teachers were asked to solve the following problem:

*Represent number 666 as a sum of consecutive natural numbers. Find as many different presentations as possible.*

In accordance with above discussed theoretical views on needs of mathematically promising students and the role of challenging tasks in their education we proposed this problem to the participants of our study since: (a) this problem has more than one solution, (b) the problem allows different problem-solving strategies[1], (c) it is an inquiry-based problem and a solver can create his/her own strategy; (d) this problem does not demand any extracurricular knowledge. The challenging nature of this task for mathematically promising students was validated by the three authors.

The problem was solved individually by each participant during one 45-minute-long session. The analysis of teachers' problem-solving performance was done qualitatively. All teachers' problem solving strategies were described. We analyzed the effectiveness of

[1] We differentiate between solution and solution strategy as follows: solution is the result obtained for the problem by the implementation of a solution strategy. The answer for a problem can contain a number of solutions since the problem considered herein is open-ended and has 5 different solutions on the set of natural numbers.

the strategies and the relationship between the strategies and the solutions. To summarize this analysis we quantified the results (presented later in Figure 1).

### Discussion with the teachers

We supposed that teachers may have some knowledge about mathematically promising students despite the fact that their program did not include special courses devoted to this issue. The discussion allowed us to learn the participants' ideas about teaching such schoolchildren, their characteristics and needs. Twenty-five teachers participated in this whole group discussion. The discussion was recorded by an assistant. The content analysis of the discussion allowed us to reveal the main categories in teachers' responses. Later teachers' responses were used in the attitude questionnaire to compare beliefs of Israeli and Canadian participants.

### Attitude questionnaire

According to research literature and the analysis of the discussion with 25 Israeli participants we composed an attitude questionnaire that allowed teachers to express their level of agreement with different beliefs related to the education of students with high abilities in mathematics. The questionnaire includes 3 main parts:

Part A: Characteristics of students that have high ability in mathematics,

Part B: Characteristics of tasks suitable for these students,

Part C: Preparation of teachers for teaching mathematically talented students.

Each part included statements that reflected Israeli teachers' beliefs expressed during the discussion. For each statement there were six ranks from which the teachers were asked to select the most appropriate level of agreement (form 1 - fully disagree to 6 - fully agree).

The validation of the questionnaire was performed both for the content validity and internal consistency of each questionnaire part. Content validity was examined in the course of the discussion of the three authors of this paper. All the clauses about which there was any kind of disagreement were changed. The reliability (internal consistency) of the questionnaire was checked for each of the three parts using Cronbach’s alpha.

The reliability was found to be satisfactory to permit the use of this instrument: $\alpha$ =.91 for Part A, $\alpha$ =.73 for Part B, and $\alpha$ =.83 for Part C of the questionnaire. We analyzed responses of teachers in Israel and Canada and compared them. T-test was applied to analyze whether the means of two groups are statistically different from each other for each part of the questionnaire and for each one of the questionnaire statements.

## RESULTS

In the first part of this section, we discuss the strategies used by teachers when solving the problem as well as different resulting solutions. In the second part, we analyze several issues related to mathematically promising students raised during the follow-up discussion. In the third part we analyze the results of the attitude questionnaire.

### Solving the problem: Strategies and solutions

Overall the teachers used five different strategies when solving this problem. Figure 1 presents the number of teachers who employed each strategy. As follows from the table, five different solutions were found by teachers and five different strategies were used. The table also shows which strategies led to particular solutions.

In the following section of this paper we provide in-depth analysis of the strategies and solutions. We describe different strategies used by the teachers and discuss the complexity of the solutions according to the level of mathematical knowledge and connectedness required in order to apply the strategy correctly and find as many solutions as possible. Then we analyse differences and similarities between Canadian and Israeli teachers in the use of different strategies.

| No of solutions | | Strategies used / Solutions (results) found | | | | | Trial and error | | Dividing 666 and surrounding a median number | | Using properties of arithmetic sequence explicitly | | Writing equations and using patterns | | '6' pattern | | Total No | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 221+222+223 | 165+166+167+168 | 70+71+…+77+78 | 50+51+…+60+61 | 1+2+…+35+36 | **Israel** | **Canada** | **Israel** | **Canada** | **Israel** | **Canada** | **Israel** | **Canada** | **Israel** | **Canada** | **Israel** | **Canada** |
| Total No | | | | | | | 16 38% | 39 72% | 15 36% | 14 26% | 2 5% | **0** | 5 12% | 1 2% | 4 10% | **0** | **42** | **54** |
| No of solutions according to a strategy and a result | **One sol.** | V | | | | | 9 | 11 | 1 | | | | | | 2 | | **12 29%** | **13 24%** |
| | | | | | | V | | 2 | | | | | | | | | | |
| | **Two sol.** | V | V | | | | 2 | 8 | 1 | 2 | | | 1 | | | | **9 21%** | **20 37%** |
| | | V | | | V | | 1 | | | | | | 1 | | | | | |
| | | V | | V | | | | 2 | 3 | 2 | | | | | | | | |
| | | V | | | | V | | 4 | | 2 | | | | | | | | |
| | **Three sol.** | V | V | V | | | 4 | 1 | 4 | 1 | | | | | 2 | | **14 33%** | **8 15%** |
| | | V | V | | V | | | 1 | 1 | | | | | | | | | |
| | | V | V | | | V | | 2 | | 2 | | | | | | | | |
| | | V | | | V | V | | 1 | | | | | | | | | | |
| | | V | | V | V | | | | | | 1 | | | | | | | |
| | | V | | V | | V | | | 2 | | | | | | | | | |
| | **Four sol.** | V | V | | V | V | | 5 | | 2 | | | | | | | **3 7%** | **13 24%** |
| | | V | V | V | V | | | 1 | | | 1 | | 2 | 1 | | | | |
| | | V | | V | V | V | | | | 1 | | | | | | | | |
| | | | V | V | V | V | | | | 2 | | | | | | | | |
| | | V | V | V | | V | | 1 | | | | | | | | | | |
| | **Five sol.** | V | V | V | V | V | | | 3 | | | | 1 | | | | **4 10%** | **0** |
| No of solutions in which a result is attained by a particular strategy | | 221+222+223 | | | | | 16 | 32 | 15 | 17 | 2 | | 5 | 1 | 4 | | **42** | **50** |
| | | 165+166+167+168 | | | | | 3 | 19 | 12 | 9 | 1 | | 5 | 1 | 1 | | **22** | **29** |
| | | 70+71+…+77+78 | | | | | 4 | 6 | 13 | 5 | 2 | | 3 | 1 | 2 | | **24** | **12** |
| | | 50+51+…+60+61 | | | | | 1 | 9 | 4 | 4 | 1 | | 4 | 1 | | | **10** | **14** |
| | | 1+2+…+35+36 | | | | | | 23 | 5 | | | | 1 | | | | **6** | **23** |

Figure 1: Distribution of prospective teachers' solutions according to use of different solution strategies and different results

## Trial and error strategy

Trial and error strategy was used by participants most frequently. Fifty-five of ninety-six (16 of 42 from Israel – 38% and 39 of 54 from Canada – 72%) teachers checked different combinations of numbers, some of which matched the problem conditions and some of which did not. Overall by using trial and error strategy solution 666=221+222+223 was found by 48 teachers, solution 666=165+166+167+168 was found by 22 teachers, solution 666=70+71+72+…+78 by 10 teachers, solution 666=50+51+52+…+61 was found by 10 teachers and solution 666=1+2+3+…+36 was found by 14 teachers (see Table 1).

All the teachers who used the "trial and error strategy" figured out that the solution can not contain only two addends. Twenty teachers (from both countries) found only 1 solution: 221+222+223=666. Additionally, there were 2 teachers from Canada who found only 1 solution using this strategy: 1+2+3+…+36=666. They just did the addition starting with 1 and adding other numbers until they got 666. Seventeen teachers (3 from Israel and 14 from Canada) managed to find 2 solutions with this strategy. Nine teachers (4 from Israel and 5 from Canada found 3 solutions. Seven teachers (all of them from Canada) found 4 solutions.

None of the participants tried to analyse whether their solution includes the complete set of the solutions to the problem. Obviously, some teachers, when using trial and error strategy, could do it in more systematic way than others. Those who succeeded in finding more than one solution manifested higher level of flexibility; however they did not conduct an in-depth investigation of the problem applying more advanced mathematical methods (formulas, theorems, etc) as it was the case in other strategies we discuss below.

## Dividing 666 and surrounding a median number

Twenty nine teachers (15 in Israel and 14 in Canada) divided 666 by different factors then putting the addends "symmetrically" and consequently around the quotient. Table 1 shows that almost all (3 of 4) teachers that found all five solutions used "dividing 666" strategy. There were a few solutions obtained with this strategy.

Some teachers divided 666 by 3. They found the following solution: $666:3=222$, and then they obtained three consecutive numbers by adding and subtracting 1: $222-1=221$, $222+1=223$, therefore, $221+222+223=666$.

Other teachers divided 666 by 4 and received a non-integer number: 666:4=166.5, so they had to add and subtract 0.5 and 1.5 to obtain four natural addends: $166.5-0.5=166$, $166.5+0.5=167$, $166.5-1.5=165$, $166.5+1.5=168$, so $165+166+167+168=666$.

The teachers that divided 666 by 9 found the following solution: $666:9=74$, then the sum of 9 addends was: $70+71+72+73+74+75+76+77+78=666$. Teachers that divided 666 to 12 found another solution: $666:12=55.5$, then the sum of 12 addends is $50+51+52+...+61=666$.Finally, the last solution was: $666:36=18.5$ leading to the discovery of the sum of first 36 natural numbers $1+2+3+...+36=666$ or $666:37=18$ and then the sum is $0+1+2+3+...+36=666$, that presents the same solution (if you decide that 0 can also be used).

Clearly, when implementing this strategy, teachers used the fact that consequent natural numbers form an arithmetic sequence. Dividing 666 by a particular number they searched for a median of a sequence which either belonged or did not belong to the sequence. Furthermore they used the property of an average of arithmetic sequence: The median member of an arithmetic sequence is a mean of all its terms. Thus the sum of all terms of an arithmetic sequence is: $x_1+x_2+...+x_n=n\cdot median\{x\}$. This strategy also allows to prove that there exactly 5 solutions to this problem.

A. Thirty-six is the maximal number of addends: Since for 36 terms of the sum the minimal term is 1 then for bigger number of addends a sum must include addends smaller than 1, thus not natural. E.g., $666:37=18$, this leads to the following sum of consecutive numbers $0+1+2+3+...+36=666$ which includes 0 which is not natural.

B. In order to be able to form a set of natural numbers being symmetrically distributed around a median number; it (the median) must be a natural number or (natural+0.5). The median number of *n* consecutive numbers, when *n* is odd, belongs to the sequence (see sums of 3 and 9 terms earlier). All the numbers in the sequence are obtained by adding $\pm 1\cdot \mathrm{k}$ for natural $\mathrm{k}$. The median number of *n* consecutive numbers, when *n* is even, does not belong to the sequence. For example $666:4=166.5$, then 4 natural numbers around 166.5 are obtained by adding $\pm 0.5, \pm 1.5$ to 166.5. Similar results we obtain for the sums that include 12 and 36 terms. Since among numbers smaller than 37 only 3, and 9 are odd divisors of 666 and only 4, 12 and 36 are even numbers that divide 666 with reminder 0.5 there are no other solutions for the problem on the set of natural numbers.

Since none of the teachers provided these explanations explicitly we may claim that the teachers applied this strategy by intuitively using number sense and properties of arithmetic sequence.

Interestingly, this strategy was the most frequently used in finding solutions $165+166+167+168=666$ (used by 7 teachers out of 14 who found this solution), $70+71+72+...+78=666$ (used by 5 out of 15 teachers), $1+2+3+...+36=666$ (used by 4 out of 5 teachers who found this solution).

## Using properties of arithmetic sequence explicitly

Two teachers (both from Israel) used formula of the sum of n first terms of arithmetic sequence. This led to the equation in two variables.

$$\left.\begin{array}{l} a_1 = m, \quad m \in N \\ d = 1 \\ N = n, \quad n \in N \\ S_n = 666 \end{array}\right\} \Rightarrow S_n = \frac{(2a_1 + d(n-1))n}{2} \Rightarrow 666 = \frac{(2m + 1\cdot(n-1))n}{2} \Rightarrow 1332 = (2m+n-1)n$$

Then this equation was divided into a series of systems of two equations with two variables

$$\begin{cases} 2m+n-1=1332 \\ n=1 \end{cases}, \quad \begin{cases} 2m+n-1=666 \\ n=2 \end{cases}, \quad \begin{cases} 2m+n-1=444 \\ n=3 \end{cases}, \ldots, \quad \begin{cases} 2m+n-1=1 \\ n=1332 \end{cases}.$$

One these teachers found 3 solutions for n=3,4 and 9:

$221+222+223=666$, $50+51+52+...+61=666$ and $70+71+72+...+78=666$.

The other teacher found 4 solutions: $221+222+223=666$, $50+51+52+...+61=666$, $70+71+72+...+78=666$ and $165+166+167+168=666$.

## Using equations

Six teachers (five from Israel and one from Canada) used this strategy. Two teachers found 2 solutions by solving different equations that represented sums of consecutive numbers.

Three teachers found 4 solutions by constructing 11 equations and solving them:

$$x+(x+1)=666,\quad x+(x+1)+(x+2)=666, \ldots, x+(x+1)+(x+2)+\ldots+(x+11)=666$$

One teacher found all 5 solutions by solving all the equations:

$$x+(x+1)=666,\quad x+(x+1)+(x+2)=666, \ldots, x+(x+1)+(x+2)+\ldots+(x+35)=666$$

It seems that this strategy may arise from a routine procedure that students are used to applying in school.

## Last number is 6

Four (all from Israel) teachers based their solution strategy on the fact that the last digit of the sum of consecutive numbers has to be 6.

Two teachers saw that $1+2+3=6$, and $(666-6):3=220$. They found only one solution: $221+222+223=666$

Two other teachers found 3 solutions by using sums of 3, 4 and 9 consecutive numbers whose sum ends in 6:

$221, 222, 223-$ as described above with $5+6+7+8=26$ and $(666-26):4=160$, thus the numbers are $165, 166, 167, 168$.

Finally $0+1+2+3+4+5+6+7+8=36$ and $666-36:9=70$, thus the numbers are $70, 71, 72, 73, 74, 75, 76, 77, 78$.

## Summary

From the analysis of the solutions produced by the participants from two different countries we learn about similarities and differences between these two groups of population.

Both in Israel and in Canada most of the participants used two main strategies: trial and error strategy and "dividing 666" strategy (98% of Canadian participants and 74% of Israeli participants). None of the teachers used more than 1 strategy when solving the problem. Israeli teachers used 5 different strategies while their Canadian colleagues only

3 strategies (see Table 1). Israeli participants also used properties of arithmetic sequence (5%) and using '6' pattern (10%).

More than one solution was found by 71% of the participants in Israel and by 76% of the participants in Canada. We found that 50% of Israeli participants and 39% of Canadian participants found 3, 4 or 5 different solutions. Only 4 (10%) Israeli teachers and none of Canadian participants found all five solutions.

Most of the participants in our study – prospective mathematics teachers in Israel and Canada - did not attempt to find the whole set of solutions for the given problem. This finding is disappointing. We assume that mathematics classroom for the mathematically promising students should be based on mathematics culture that encourages students to find the complete set of solutions for any problem. We consider that programs for mathematics teachers should include tasks of this kind and stress the importance of problems with multiple solution strategies and multiple results in education of mathematically promising students. Proving that a problem does not have an additional answer (besides those found) and examining the problem for additional results through implementation of different strategies should be a part of routine in teachers' courses as well as in school mathematics classrooms especially when dealing with the education of mathematically talented students.

## TEACHERS' BELIEFS RELATED TO MATHEMATICALLY PROMISING STUDENTS AND THEIR EDUCATION

This section of the paper presents the results of the discussion with 25 Israeli participants about their conceptions about mathematically promising students and their education. The discussion was organized with focus on three main questions: (1) Who are they, the mathematically promising students and what are their needs? (2) What tasks are necessary to meet those needs, and (3) Do teachers feel ready to work with mathematically promising students in their classroom and what kind of education they need for that?

### Who are they,-"Mathematically Promising Students"

When answering this question, the teachers addressed a wide range of characteristics of mathematically promising students. We organized the answers by the following categories: mathematically promising students have advanced mathematical reasoning, they solve problems differently from other students, they work at a higher pace. Note that practically none of the teachers mentioned personal characteristics of the mathematically promising students such as motivation, beliefs, sensitivity. Only one of them (Tair) said that mathematically promising students are "thirsty for knowledge".

### Mathematical reasoning

In this category we included teachers' replies that referred to students' mathematical reasoning. These teachers stressed that mathematically promising students may be characterized by different qualities of mathematical thinking, by advanced level of their logical reasoning and abstraction they perform. Michal and Inbal clearly expressed this opinion.

Michal: *The student who has developed **logical thinking**, **abstract thinking**, enjoys mathematics*

Inbal: ***Reasoning** is a very important component that proves that the student understood this material.*

### Problem solving:

Some teachers stressed that mathematically promising students solve problems differently from other students, they find original problem-solving strategies, can solve unconventional problems, and can cope with many different tasks:

Yosuf: *The student who has high thinking skills and can solve problems from real life.*

Inbal: *The student who uses **original strategies** that he did not study at school and can apply them to new material. He can find **connections between different topics** in mathematics.*

Ruti: *The student who can **solve non-standard** and inquiry based problems.*

Hani: *I have 2 students in the 6-th grade who, from my point of view, are very promising, since they **solve all the tasks that I give them**. However, they cannot explain their solutions, but this is not a necessary characteristic of promising students.*

## Pace of learning and thinking:

Many teachers believe that mathematically promising students are quicker than others when performing mathematical tasks:

Suad: *The student who can solve problem in **less time** than other students.*

Hani: *The student who understands the material **quickly** and could study with students of higher grade.*

## Do Mathematically Promising Students have a different approach to learning Mathematics? If they do, what are those approaches?

When discussing this question, teachers referred to the main needs of the students: "deepening" (of their knowledge) (Yosuf, Hani, Ruti, Michal, and Aved), enrichment (Inbal and Aved) and acceleration (Inbal).

### Different types of tasks

When reasoning about teaching approaches suitable for mathematically promising students many teachers focused on special mathematical tasks. Their ideas in this respect related to the abovementioned "deepening" or enrichment approaches.

Yosuf: I always prepare several special tasks aimed at mathematical thinking development for 3 students in my class who always complete their class work before other students.

Inbal: I bring ***extra curriculum tasks*** and the tasks for "deeper" learning to my class. The tasks can enrich students' comprehension in mathematics.

Hani: Such students usually complain when we are solving problems slowly, so I give them tasks on the ***same topic but more complex***.

Ruti: At our school we have Mathematical Laboratory. Mathematically promising students attend it once a week and work there with ***inquiry based problems***.

Michal: One of the ways is to ask the mathematically promising student what s/he prefers. What kind of tasks does s/he want to solve? He should be able to choose ***between inquiry based, open–ended or other types of complex problems.***

Aved: Challenging the mathematically promising students ***with the tasks from various Mathematical Olympiads***. The tasks that contain ***"deepening"*** and ***enrichment***.

## Social interactions of different kinds

Some teachers mentioned that mathematically promising students require different learning environments with regard to the social interactions in which they are involved. Yosuf thought that these students should help others which may be useful for themselves. Ayad expressed an opinion that learning in homogeneous classes may better suit the needs of talented students.

Yosuf: I ask the students who have completed their work to help other students. This helps them to organize their own thinking. And I discovered that students usually understand the explanations of the mathematically promising classmates better than mine.

Ayad: Promoting mathematically promising students will become more effective if they study in ***homogenous groups***.

## Acceleration

Some teachers think that mathematically promising students should be taught at a higher pace in order to realize their mathematical potential

Inbal: Another way is to ***move the student to a different class where s/he can learn with students of the same mental level.***

### Is the task you solved suitable for mathematically promising students?

Despite the complexity of the task the participants appreciated the importance of the incorporation of such tasks in teacher education programs. During the whole group discussion 25 Israeli participants agreed that this problem was challenging and suitable for mathematically promising students. Their arguments were: the problem has different solutions, there is more than one answer, it was inquiry based problem and so on.

Inbal: *The task **does not demand extensive knowle**dge, but rather higher order thinking skills. So this task can be a **challenge for the students** of various grades from the 3rd up to the 12th .*

Michal: *[The task that requires] not only search of solutions but **hypothesizing or developing some theory** may be very challenging for students.*

Inbal: *The **beauty of this question** is that unless you found the correct approach **you never know if you have all the solutions**. So you are in some conflict with yourself.*

Tair: *This task **has different solutions** unlike almost all the usual tasks in a primary school. More than that, there are different ways for finding these solutions…*

### Do teachers need special preparation for teaching "mathematically promising students"?

When this question was discussed, all teachers expressed their disappointment about not having at least one course in their Teachers' Training Program that focuses on special approaches to teaching mathematically promising students and their needs.

Michal: *One of the courses has to cover the topic: "**various needs of students**".*

Ruti: *In teaching mathematics the problem with the mathematically promising students is very complicated. **In addition to a special course, teachers need to get experience**.*

Yosuf: *I feel that during all my years in the Pedagogical College I learned nothing about work with the mathematically promising students.*

Raya: *I think that we **need the course** that will instruct how to choose problems, what kind of problems are preferable for each age, what are the materials and ways for teaching mathematically promising students and so on…*

Tair: *In my opinion, there must be a special separate course during teachers' training that has to touch upon the problems we have talked about.*

Based on opinions expressed by this group of participants, we developed an attitude questionnaire presented below.

## Attitude questionnaire

As we described in the methodological section, the attitude questionnaire was based on the analysis of the beliefs expressed by the Israeli prospective mathematics teachers that participated in the whole group discussion at stage A of our study. At stage B, the questionnaire was given to one group of Israeli prospective teachers (N=28) and two groups of Canadian prospective teachers (N=56). As presented in the methodology section the three parts of the questionnaire were composed by combining teachers' statements during the discussion and the beliefs described in the literature

There were three parts in the questionnaire:

Part A: Characteristics of mathematically promising students,

Part B: Types of mathematical tasks suitable for advancement of high ability students,

Part C: Education of mathematics teachers for teaching talented students.

As presented in the methodology section, all three parts of the questionnaire had high internal consistency that allowed quantitative analysis of the data. We compared the responses provided by the Israeli participants to the responses of Canadian participants (using T-test). Figures 2A, 2B and 2C show the results of the analysis of the three parts of the attitude questionnaire and compares results received for Israeli and Canadian participants. Figures 3A, 3B and 3C present percentage of the teachers who agreed or strongly agreed with the statements in the questionnaire.

## Characteristics of mathematically promising students

In general Israeli teachers' agreement with statements about special characteristics of mathematically talented students was stronger than that of Canadian participants (Figure 2A). The average score for Part A of the questionnaire in the Canadian group of teachers was neutral (M=3.89 with SD=0.85; between 3 – slightly disagree and 4 – slightly agree), whereas for Israeli teachers it was positive (M=4.56 with SD=0.46; between 4 – slightly agree and 5 – agree). Both in Israel and in Canada none of the items in Part A of the questionnaire received a score higher than 5. Mean scores between 4 (slightly agree) and 5 (agree) were found for 11 of 13 statements (all except 1.4 and 1.7 in Fig 2A) for the

Israeli group of teachers and for 5 of 13 statements (1.2, 1.3, 1.6, 1.8, 1.9 in Fig 2A) for the Canadian group of participants.

The three highest mean scores for Canadian group of prospective teachers were obtained for the following categories: 1.9 – Mathematically talented students can solve new problems – those that were not solved in the classroom previously (M=4.376 SD=1.03); 1.6 – Mathematically talented students enjoy solving mathematical problems (M=4.30, SD=1.10), 1.3 – Mathematically talented students can understand more abstract mathematics than usual students (M=4.16, SD = 1.21). The Israeli group of teachers chose as most correct the following statements: 1.3 – Mathematically talented students can understand more abstract mathematics than usual students (M=4.96, SD=0.88), 1.9 – Mathematically talented students can solve new problems – those that were not solved in the classroom previously (M= 4.93, SD=0.72), 1.1 – Mathematically talented students solve mathematical tasks quicker than other students (M=4.89, SD = 0.96).

In spite of the fact that both Canadian and Israeli teachers chose statements 1.3 and 1.9 among the three most acceptable there were significant differences in their responses. As mentioned earlier, Israeli teachers provided higher agreement scores to almost all the statements in the questionnaire. Thus the highest mean score in the Canadian group (M=4.37; SD=1.03) is smaller than ninth mean score in the Israeli group (M=4.57, SD=0.69). This observation possibly explains significant differences that we found between the attitudes of Israeli and Canadian teachers to 10 of 13 items in Part A of the questionnaire.

The most significant differences between the attitudes of the two groups are observed for statements 1.1 (Mathematically talented students solve mathematical tasks quicker than other students) and 1.5 (Mathematically talented students participate in mathematics lessons more enthusiastically than other students). Whereas Israeli teachers agreed with these statements ($M_{1.1}$=4.89, $SD_{1.1}$=0.96; $M_{1.5}$=4.79, $SD_{1.5}$=0.88), Canadian teachers' mean agreement score for the same statements was lower than "slightly agree" ($M_{1.1}$=3.72, $SD_{1.1}$=1.62; $M_{1.5}$=3.82, $SD_{1.5}$=1.01). Interestingly these differences relate to cognitive (1.1) and affective (1.5) characteristics of mathematically talented students.

The characteristics that got the lowest agreement score in both countries were 1.4 (Mathematically talented students like helping other students), 1.7 (Mathematically

| | | M (SD) | | t |
|---|---|---|---|---|
| | | Canada N=56 | Israel N=28 | |
| **Part 1:** | **Characteristics of mathematically talented students** | **3.89 (0.85)** | **4.56 (0.46)** | **4.69*** ** |
| 1.1 | Mathematically talented students solve mathematical tasks quicker than other students | 3.72 (1.62) | 4.89 (0.96) | 4.19*** |
| 1.2 | Mathematically talented students prefer learning with students who are good in mathematics | 4.02 (1.21) | 4.32 (1.31) | 1.04 |
| 1.3 | Mathematically talented students can understand more abstract mathematics than usual students | 4.16 (1.21) | 4.96 (0.88) | 3.51** |
| 1.4 | Mathematically talented students like helping other students | 3.63 (1.04) | 3.96 (1.35) | 1.17 |
| 1.5 | Mathematically talented students participate in mathematics lessons more enthusiastically than other students | 3.82 (1.10) | 4.79 (0.88) | 4.36*** |
| 1.6 | Mathematically talented students enjoy solving mathematical problems | 4.30 (1.10) | 4.75 (0.80) | 2.15* |
| 1.7 | Mathematically talented students like to work in small groups with students of different levels of knowledge in mathematics | 3.38 (0.97) | 3.75 (1.14) | 1.46 |
| 1.8 | Mathematically talented students can solve problems in original ways | 4.12 (1.14) | 4.79 (0.88) | 2.97** |
| 1.9 | Mathematically talented students can solve new problems – those that were not solved in the classroom previously | 4.37 (1.03) | 4.93 (0.72) | 2.92** |
| 1.10 | Mathematically talented students know many facts in mathematics | 3.89 (1.06) | 4.57 (0.69) | 3.53** |
| 1.11 | Mathematically talented students remember any mathematical statement they ever learned | 3.53 (1.04) | 4.29 (0.85) | 3.58** |
| 1.12 | Mathematically talented students ask many questions unpredicted by the teacher | 3.91 (0.98) | 4.57 (1.14) | 2.63* |
| 1.13 | Mathematically talented students like to participate in mathematical competitions | 3.95 (1.14) | 4.68 (0.91) | 3.2** |
| *p<0.05; **p<0.01; ***p<0.001 | | | | |

Figure 2A: Attitudes towards characteristics of mathematically talented students.

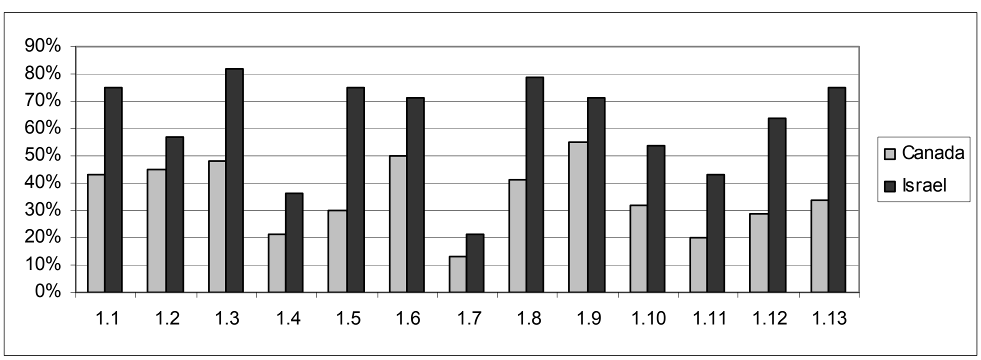


Figure 3A: Percentage of strongly positive attitudes (6–completely agree, 5–agree) to characteristics of mathematically talented students talented students like to work in small groups with students of different levels of knowledge in mathematics), 1.11 (Mathematically talented students remember any mathematical statement the students ever learned). In both groups mean agreement scores for these characteristics were close to the middle of the scale.

Statements 1.4 and 1.7 belong to the group of 3 of 13 statements in Part A for which no significant difference between the responses of Israeli and Canadian participants was found. Additional statement on which no significant difference was obtained is statement 1.2 – Mathematically talented students prefer learning with students who are good in mathematics. All three statements belong to the group of social characteristics of mathematically talented students and prove that teachers characterize them as learners who prefer working with students of the same ability level (if at all).

Additional evidence of the significance of differences between the attitudes of Canadian and Israeli teachers can be seen in Figure 3a that shows percentage of the participants in each country who marked middle and high level of agreement for different statements. Figure 3a demonstrates that whereas more than 50% of Israeli participants agreed or strongly agreed with 10 of 13 the statements in Part A of the questionnaire (except 1.4, 1.7 and 1.11), less than 50% of Canadian participants chose these levels of agreement for 12 of 13 statements.

## Tasks suitable for the mathematically talented students

In this section we present comparative analysis of the attitudes of Israeli and Canadian prospective mathematics teachers demonstrated in Part B of the questionnaire.

Part B of the questionnaire reveals additional differences between beliefs of Canadian and Israeli teachers about mathematical tasks suitable for mathematically talented students. The level of agreement with the statements in Part B of the questionnaire expressed by Israeli participants is higher than that expressed by Canadian participants (Figure 2B) for five of six statements. The average score for Part B both groups was between "slightly agree" and "agree" levels (For Canadian group: M=4.10, SD=0.69; for Israeli group: M=4.61, SD=0.58).

Unlike Part A of the questionnaire where in both groups no statement received a score higher than 5, in Part B in the Israeli group the mean score of at least 5 was obtained for 2 of 6 statements: 2.5 – Problem form mathematical Olympiads are suitable for mathematically talented students (M=5.00, SD=0.77) and 2.6 – New problems – those that were not solved in the classroom previously – are suitable for mathematically talented students (M=5.25, SD=0.80). We found significant differences between the attitudes of the two groups to these two statements. The most serious difference related to suitability of Olympiad problems for students with high abilities in mathematics.

| | M (SD) | | t |
|---|---|---|---|
| | Canada N=56 | Israel N=28 | |
| **Part 2: Mathematical tasks suitable for mathematically talented students** | **4.10 (0.69)** | **4.61 (0.58)** | **3.52**** |
| 2.1 Difficult problems that regular students cannot solve are suitable for mathematically talented students | 3.73 (1.04) | 4.11 (1.23) | 1.39 |
| 2.2 Problems from extra-curricular topics are suitable for mathematically talented students | 4.32 (0.99) | 4.18 (1.12) | -0.57 |
| 2.3 Regular problems that all students solve are suitable for mathematically talented students | 3.80 (1.20) | 4.29 (1.05) | 1.90 |
| 2.4 Investigation problems that require discovery of new facts and their proof or refutation as suitable for mathematically talented students | 4.38 (0.95) | 4.82 (1.02) | 1.90 |
| 2.5 Problems from mathematical Olympiads are suitable for mathematically talented students | 3.86 (1.07) | 5.00 (0.77) | 5.61*** |
| 2.6 New problems -– those that were not solved in the classroom previously -– are suitable for mathematically talented students | 4.53 (0.94) | 5.25 (0.80) | 3.67** |
| *p<0.05; **p<0.01; ***p<0.001 | | | |

Figure 2B: Attitudes towards the types of tasks suitable for mathematically talented students

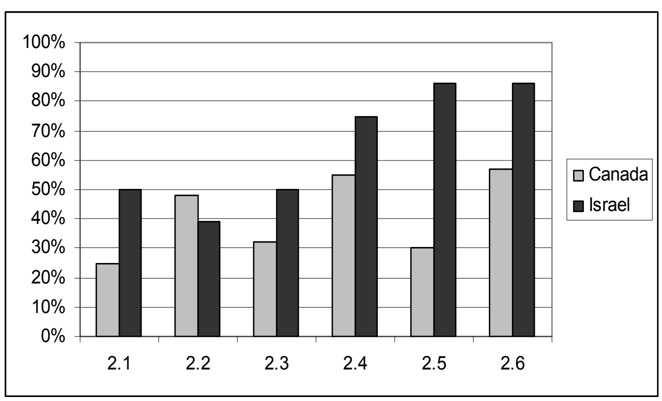


Figure 3B: Percentage of strongly positive attitudes (6–completely agree, 5–agree) towards the types of tasks suitable for mathematically talented students

Both Israeli and Canadian participants chose "New problems – those that were not solved in the classroom previously" as most suitable for the education of mathematically talented students, though these scores were significantly different (see Figure 2B).

The mean scores showing attitudes of Israeli teachers to four remaining statements in Part B of the questionnaire were all between 4 (slightly agree) and 5 (agree). This way Israeli teachers demonstrated positive attitudes to all the statements included in this part. In contrast attitudes of the teachers in Canadian group were neutral, from 3 (slightly disagree) to 4 (slightly agree) for 3 of 6 statements. The two highest mean scores for Canadian teachers were obtained for statements: 2.4 – Investigation problems that require discovery of new facts and their proof or refutation are suitable for mathematically talented students (M=4.38 SD=0.95) and 2.6 – New problems – those that were not solved in the classroom previously – are suitable for mathematically talented students (M=4.53, SD=0.94).

Statement 2.2 – Problems from extra–curricular topics are suitable for mathematically talented students – was scored by Canadian teachers (M=4.32, SD=0.99) slightly higher than by Israeli teachers (M=4.18, SD=1.12), though the difference was not significant. Statement 2.1 – Difficult problems that regular students cannot solve are suitable for mathematically talented students received the lowest agreement score in both countries (see Figure 2B).

Additional evidence of the differences between the attitudes of Canadian and Israeli teachers towards tasks suitable for mathematically talented students can be seen in Figure 3.B that shows percentage of the participants in each country who marked neutral and high level of agreement for different statements. Figure 3b demonstrates that *more than* 70% of Israeli participants agreed or strongly agreed with 3 of 6 the statements in Part B of the questionnaire (2.4, 2.5 and 2.6), and additionally about 50% of participants agreed or strongly agreed with 2 other statements (2.1 and 2.3). More *than* 50% of Canadian participants chose these levels of agreement only for 2 of these 3 statements. Statement 2.5 received an even lower level of agreement – only 30%.

## Education of teachers of mathematically promising students

| | M (SD) Canada N=56 | Israel N=28 | t |
|---|---|---|---|
| **Part 3: Education of mathematics teachers for work with talented students** | **4.01 (0.90)** | **4.85 (0.57)** | **5.23*****|
| 3.1 To teach talented students teachers have to learn more mathematics than other teachers | 3.50 (1.21) | 4.79 (1.03) | 5.08*** |
| 3.2 To teach talented students teachers have to study special classroom settings | 4.20 (1.29) | 5.25 (0.65) | 5.00*** |
| 3.3 To teach talented students teachers have to learn ways for identification of high abilities students | 4.41 (1.17) | 5.25 (0.65) | 4.23*** |
| 3.4 To teach talented students teachers have to learn how to solve investigation problems | 4.50 (1.18) | 5.54 (0.51) | 5.63*** |
| 3.5 To teach talented students teachers have to know their special psychological characteristics | 4.70 (1.11) | 4.64 (1.42) | -0.19 |
| 3.6 To teach talented students teachers have to be gifted in mathematics | 2.73 (1.24) | 3.64 (1.34) | 3.00** |
| *p<0.05; **p<0.01; ***p<0.001 | | | |

Figure 2C: Attitudes towards characteristics of the education of teachers of mathematically talented students.

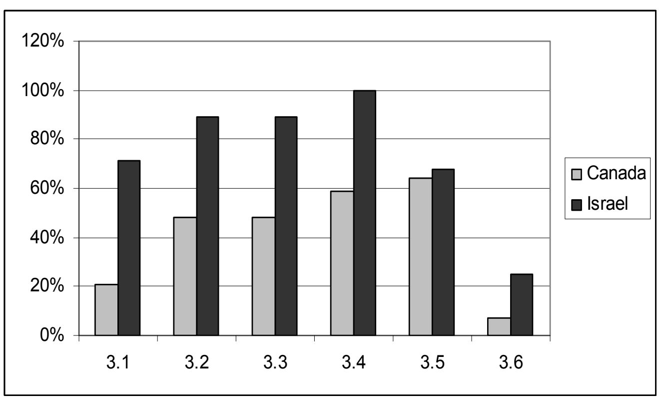


Figure 3C: Percentage of strongly positive attitudes (6–completely agree, 5–agree) towards the education of teachers of mathematically talented students.

Results from Part C of the questionnaire reveal highly significant differences between Canadian and Israeli teachers' attitudes. Similar to parts A and B, the level of agreement of Israeli participants with the statements in Part C is significantly higher than that of

Canadian participants (Figure 2C). The average score for Part C in the Canadian group of teachers was slightly positive (M=4.01, SD=0.90), whereas for Israeli teachers it was close to the middle of the scale (M=4.85, SD=0.51). Similar to Parts A and B, mean scores for all the statements in Part C for Canadian participants were below 5 (agree). Looking at the attitudes of Israeli participants, we learn that 3 of 6 scores were above 5 (agree), whereas those of Canadian group were between 4 (slightly agree) and 5 (agree) for 4 of 6 statements.

The two highest mean scores for Canadian group of students were obtained for the following categories: 3.5 To teach talented students teachers have to know their special psychological characteristics (M=4.70, SD=1.11) and 3.4 To teach talented students teachers have to learn how to solve investigation problems (M=4.50, SD=1.18).

The teachers from Israeli group demonstrated the highest agreement (between "agree" and "completely agree" levels) with the following three statements: 3.4 Teachers have to learn how to solve investigation problems (M=5.54, SD=0.51), 3.2 Teachers have to study special classroom settings (M=5.25, SD=0.65), and 3.3 Teachers have to learn ways for identification of high abilities (M=5.25, SD=0.65).

The statement that got the lowest score in both countries was statement 3.6 To teach talented students teachers have to be gifted in mathematics (see Figure 2C). The mean agreement score of Israeli teachers was almost neutral (M= 3.64; CD=1.34), and Canadian teachers' attitudes were even negative (M=2.73, CD=1.24). From this observation, it becomes clear that our participants do not think that being gifted is a necessary condition for teachers working with gifted students.

Significant differences between the attitudes of Canadian and Israeli teachers can be seen in Figure 3C that shows percentage of the participants in each country who marked middle and high level of agreement for different statements. Figure 3C demonstrates, for example, that whereas 100% of Israeli participants agreed or strongly agreed that teachers need to learn how to solve investigation problems, only 60% of their Canadian colleagues seem to share this point of view at the same level of agreement.

The most striking difference can be observed in the statement affirming that talented students' teachers have to learn more mathematics than other teachers (more than 70% of Israeli teachers vs. 20% of Canadian peers agreed or strongly agreed with this statement). Similar percentages of participants from both countries seem to agree equally only with the statement that teachers need to learn more about psychological characteristics of gifted and talented students.

## Additional comparison

To finish this report we provide additional information in Figure 4 that demonstrates the percentage of teachers who express positive attitude (at slightly agree, agree, and strongly agree levels) to the statements in the questionnaire.

Figure 4 demonstrates that Israeli participants were more positive in all the three parts of the questionnaire. More than 80% of Israeli teachers agree (at different levels) with 11 of 13 statements in Part A of the questionnaire and with 5 of 6 statements in Parts B and C. Among Canadian participants less than 80% agreed with 10 of 13 statements in Part A, and with 5 of 6 statements in Parts B and C.

Figure 4 demonstrates that both Israeli and Canadian participants were the most positive with respect to statements 1.6, 1.9, 2.4, 2.6, 3.4, and 3.5. At the same time the difference in the attitudes of Israeli and Canadian participants is very clear in case of statements 1.1, 1.11, 2.3, 2.5, 3.1, 3.2 and 3.6. The least popular statements among the participants from both countries were 1.7, and 3.6.

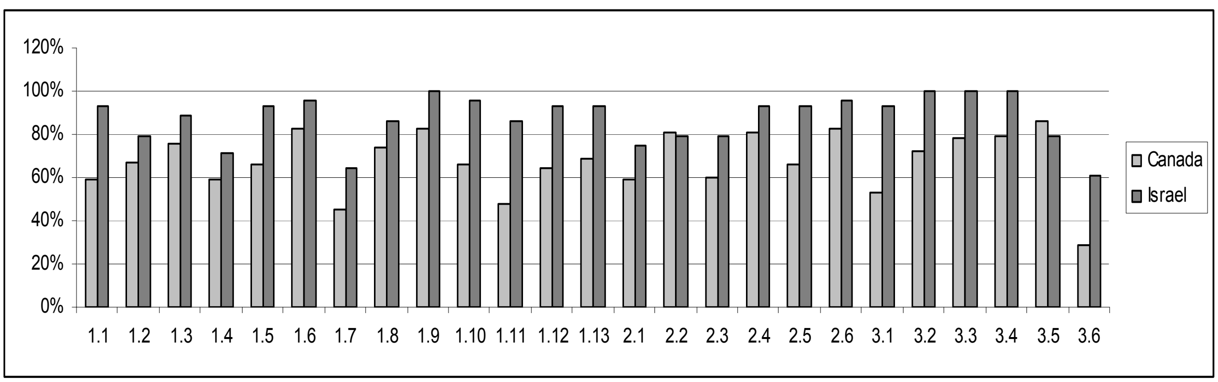


Figure 4: Percentage of positive attitudes of teachers to the beliefs statements in the questionnaire.

In conclusion, dissimilarities in the views of the representatives of two countries speak of interesting and meaningful differences in the education of prospective mathematics teachers as related to the issue of mathematically talented students. These differences were reflected in the participants' attitudes revealed in our questionnaire. We suppose that a detailed qualitative investigation of teacher training in different countries can explain many of the findings of this study.

**CONCLUDING REMARKS**

The results of our data analysis, can lead to several conclusions about teaching mathematically promising students.

First, our findings show that teachers cope with challenging tasks with varying levels of success. The majority of teachers used 'non-systematic' strategies, without analysis of the efficiency of the strategies. Indeed, these results suggest that teachers need better mathematical preparation in terms of solving open-ended challenging tasks that would enable them not to limit the problem solving process with finding of one suitable solution. On the contrary, teachers should be encouraged to perform in-depth investigation, assess of strategies' efficiency, search for different ways to solve problems, and for possible generalizations in terms of developing mathematical theories. Acquiring such cognitive and meta-cognitive skills will help teachers in guiding their students on the way to deeper and more meaningful mathematical knowledge.

Comparing solutions and strategies of Israeli and Canadian participants we were not able to draw far going conclusions. However, we can state that Israeli teachers used both non-systematic strategies and systematic ones (that they have previously learned in a different context), whereas most Canadian prospective teachers used only non-systematic strategies. Comparative analysis of school mathematical curricula and of the teacher educational programs in the two countries may shed more light on the findings of this study. We assume that continuation of the study that will employ different types of challenging tasks (Applebaum & Leikin, 2007) also can contribute to our understanding of the discovered phenomena.

Our findings from the discussion with Israeli participants suggest that they were aware of the qualities of mathematically promising students in mathematics classrooms. While the list of characteristics of these students given collectively lacks some important features, teachers recognize special learning needs of mathematically promising students and value investigation and challenging tasks as important for the mathematical development of these students. Namely, the task they were asked to deal with was characterized by teachers as potentially rich in terms of higher order thinking, theory building, and leading to the development of appropriate strategies.

According to the teachers participating in the discussion, special needs of mathematically promising students can be met with particularly challenging, open-ended and investigative tasks of higher difficulty level and increasing complexity. However, the teachers saw such tasks as rather exceptional for today's mathematics classroom and rarely used by teachers This confirms the need for a more challenging curriculum for mathematically promising students already mentioned by several researchers working with the mathematically promising students (e.g., Sheffield, 2003, Freiman, 2006). In spite of the afore-mentioned opinion expressed by the teachers in the discussion they themselves did not feel prepared for dealing with such tasks in their classroom. Their feeling was consistent with the data obtained in the first part of our analysis that shows that only few teachers were able to find (almost) all solutions to the problem. Their mathematical background should be, therefore, reinforced by mathematically challenging tasks and investigations.

Regarding the social aspects of teaching mathematically promising students, the teachers' opinions vary meaningfully. Some teachers speak about the benefits of homogeneous learning environment, while others consider that mathematically promising students will benefit more while helping less capable students in heterogeneous classes.

The results of the questionnaire analysis can deepen our knowledge of beliefs of prospective teachers regarding the definition of mathematically promising students, their particular educational needs and teachers' readiness to meet those needs in the process of education.

Why did Israeli prospective teachers agree with questionnaire statements (23 of 25) more than their Canadian colleagues? One plausible explanation can be that the statements were built according to the results of a discussion in which only Israeli teachers took part. Discussion with Canadian prospective teachers could reveal other statements and lead to a different distribution of answers. However, using our data, we can investigate further whether the level of mathematical preparation can be a factor reinforcing Israeli teachers' perception of the necessity of stronger mathematical background for work with mathematically promising students. Finally, our data from questionnaires suggest that there was a wider variety of opinion about social than about cognitive issues related to mathematically promising students. The fact that Israeli participants agreed less about psychological aspects needs further investigations.

Our study is an exploratory small-scale study. It would be interesting to use our instruments with larger and culturally more diverse prospective teachers' populations. There is also a need for more rigorous study of the preparation of mathematics teachers for the education of mathematically promising students. While more rigorous studies would be needed to get into the situation details, some recommendations can be made regarding teachers' training and professional development associated with teaching mathematically talented students. Teacher education programs should:

- Expose teachers to the complexities of teaching mathematically promising students.
- Develop in teachers stronger higher-order thinking skills and their abilities to investigate challenging tasks by proposing such tasks during their training.
- Amplify teachers' didactical inventory of teaching strategies to allow identification and fostering of mathematically promising students' abilities using inquiry- based, challenging and investigative tasks.

At the next stage we intend to investigate how mathematically promising students deal with the mathematical problem used in this study, what are their own views on their needs, and compare teachers' beliefs and expectations with the real situation in their classes.

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