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Abstract: In order to teach successfully, future teachers should not only be educated about students' conceptions, but also about different forms of knowledge and classroom culture. In our research, we examined whether the participation in the Internet-based challenging problem solving community CASMI contributes to the development of the aforementioned awareness and understanding in order to meet the needs of all students including the gifted ones. The results obtained enabled us to note that the pre-service teachers' perceptions of the project as a source of enrichment are mainly positive. However, analyzing schoolchildren's strategies, the participants preferred to use pre-determined criteria instead of writing personal formative comments adapted to the mathematical reasoning presented in the solution. Research shows that such comments could enrich the feedback by better reflecting the diversity of the learners' styles, thus helping them to reach their full potential. We suggest more attention needs to be given to the analyses of this diversity in pre-service teacher training and professional development in order to enable teachers to differentiate their teaching.

Key words: Online Problem Solving, Pre-Service Teacher Training, Diversity of Schoolchildren's Strategies, Asynchronous Assessment, Mathematical Enrichment

RATIONALE OF THE STUDY

What should future teachers know to teach successfully in a mathematical classroom that becomes more and more diverse (in terms of children's background and abilities) and at the same time be inclusive? Setting up an early 21st century research agenda for teacher's professional development and teacher education, Even & Tirosh (2002) base their recommendations on an important body of refereed literature that focuses on the development of mathematical awareness and understanding of student mathematics learning and thinking. According to them, this should be coordinated by three major axes: educating about student conceptions, educating about different forms of knowledge, and educating about classroom culture. A complex approach to teacher education is thus needed in order to eventually help meet educational needs of children struggling with mathematics and those of gifted ones who may get lost while not being challenged enough (Diezmann, Thornton & Watters, 2003; Diezmann & Watters, 2005; Freiman, 2006; Freiman, Manuel & Lirette-Pitre, 2007; Johnson, 2000b; Kettler & Curliss, 2005; Sheffield, 2003).

In our paper, we will examine whether participation in the Internet-based challenging problem solving community CASMI contributes to the development of the aforementioned awareness and understanding in order to meet the needs of all students including the gifted ones. During the semester, pre-service teachers enrolled in mathematics education courses in two Canadian universities were involved in the analysis of K-12 children's solutions by giving them an asynchronous feedback.

Working with a vision of the diverse and inclusive classroom, we keep in mind that gifted students, independently of how we define and identify them, may need additional resources that are not directly available in a regular classroom. Therefore, we believe that the Internet may provide teachers and their students with appropriate activities for every child. Several studies show that rich, contextual, and open-ended mathematical problems posted on a website can challenge all children and give them an opportunity to produce new mathematical knowledge in a situation when the answer is not obvious and the strategy is to be chosen or constructed by using different ways of reasoning and communicating. This situation may be potentially fertile for mathematically gifted learners, meeting their special needs for more challenge (Applebaum & Leikin, 2007; Barbeau & Taylor, 2009; Diezmann *et al*., 2003; Diezmann & Watters, 2005; Freiman, 2006; Freiman & Lirette-Pitre, 2008; Freiman *et al*., 2007; Johnson et al., 2007; Johnson, 2000b; Kettler & Curliss, 2005; Leikin, Levav-Waynberg & Applebaum, 2008; Sheffield, 2003). While the analysis of children's mathematical production by pre-service teachers has become an important part of mathematics education courses, little is known about the impact of participation of pre-service teachers in online activities with schoolchildren and even less about their capacity to guide young learner by means of asynchronous feedback.

In our mathematics education classes, with pre-service teachers, we explore a variety of solutions to mathematical problems submitted electronically by schoolchildren. We aim to help pre-service teachers appreciate the diversity of such solutions and learn how to guide schoolchildren in a personalized and caring manner, nurturing their curiosity, interest and perseverance, which are very important for all children and especially for the gifted ones.

In our previous publications, we discussed some data about pre-service teachers' perceptions of the CASMI project (Freiman, Vézina & Gandaho, 2005). In this paper, we will report on our exploratory research in which we combined the information gathered from questionnaires regarding pre-service teachers' perceptions of the project with their feedbacks on schoolchildren's solutions. More precisely, two particular goals have been set for our enquiry:

a) to look at how pre-service teachers perceive their participation in the project regarding online challenging problem solving as a source of enrichment.
b) to examine if, being faced with a multitude of problem solving strategies, pre-service teachers are able to evaluate the correctness of students' mathematical reasoning and to provide them with an adequate feedback.

We found that very few research data are available on these questions. Therefore, our study aims to contribute to a better understanding of teacher – student retroactive communication on problem solving and to identify promising paths of improvement in pre-service teachers' mathematics education, in order to enable future teachers to provide students with richer learning opportunities.

THEORETICAL PERSPECTIVE

In order to understand the value of mathematical enrichment activities supported by the virtual CASMI environment, we looked at the literature that analyzes the role of challenging problems in today's mathematics classroom and their importance for meeting the needs of gifted students. We also searched for different studies on virtual problem solving environments and formative feedback. In the next three subsections, we will briefly review the most pertinent findings and recommendations from the studies that guided us in our data collection and data analysis.

Problem solving in today's school mathematics and the needs of gifted students

In today's mathematics classrooms, problem solving is seen as an important vehicle for the enrichment of mathematical culture because it puts strong emphasis on the development of abilities to communicate and to reason mathematically (OECD Program for International Student Assessment, 2003). In Canada, more precisely, new approaches in teaching problem solving in mathematics are following common trends set up by the NCTM Standards (2000). These trends explicitly define the central role of problems in learning mathematics and the importance to use mathematics as problem solving tools in real life interdisciplinary contexts, therefore facilitating knowledge transfer (Tardif, 1999).

Whether it is in connection with problem solving or with the learning of mathematics in general, it has been established that gifted students learn differently than their peers. The scale defined by PISA (OECD Program for International Student Assessment, 2003) assesses several levels of mathematical literacy. The highest level described by this scale features many characteristics of mathematically gifted students. Among others, these students show insight in the solution of problems, develop abilities in mathematical interpretation of problems in real-world contexts (also see Krutetski's (1976) notion of mathematical cast of mind), identify relevant mathematical tools or methods in order to find solutions to problems set in unfamiliar contexts, solve problems involving several steps, reflect on results and generalize findings and use reasoning and mathematical argument to explain solutions and communicate outcomes. Moreover, they usually are quicker at grasping concepts and the depth of their understanding surpasses the one of other students (Johnson, 2000a). It is thus important to ask ourselves what can be done to differentiate instruction for gifted students. Among others, Johnson (2000a) makes these different suggestions:

- Students should be allowed to explain their reasoning (orally and in writing).
- Resources used in the classroom should be numerous and varied.
- Open-ended problems should be privileged.
- Students should be asked "why" and "what if" questions.
- Problems and activities should extend beyond the curriculum.

Furthermore, studies conducted in the past decades, including studies of mathematical giftedness, state the need for more challenging tasks for all students but also reveal a lack of opportunities of solving such problems for students in the regular classroom (Barbeau & Taylor, 2009). However, a new approach to problem solving provided by virtual environments has the potential to increase learning opportunities for students. Indeed, a growth in Internet-based learning opportunities in mathematics can be observed. The technology itself is developing towards socially friendly, flexible and dynamic environments in which many schoolchildren can access virtual resources from school or from home. They can now get an instant interactive access to more challenging mathematics, solve problems and submit their solutions using virtual tools. Moreover, these new learning environments provide learners with a variety of contents and tools, giving them the choice between multitudes of activities adapted to their particular pace and needs. "Technology can provide a tool, an inspiration, or an independent learning environment for any student, but for the gifted it is often a means to reach the appropriate depth and breadth of curriculum and advanced product opportunities" (Johnson, 2000a, p. 5). One of the elements that become important in such environments is the kind of feedbacks students receive. Indeed, within the socio-constructivist teaching and learning paradigm, teachers need to make valid references about children's strategies (Willson & Kenney, 2003). This can be done, among others, by giving high quality feedbacks about children's solutions. In our paper, we will focus on pre-service teachers involved in a mentoring task based on the analysis of schoolchildren's solutions to challenging mathematical problem solving online activities.

Virtual opportunities of challenging problem solving: assessing diversity

When students solve open-ended problems, they mobilize a multitude of resources (Schoenfeld, 1989). This mobilization of resources is recognized as the use of a set of skills (mathematical or not) by the Program for International Student Assessment (OECD Program for International Student Assessment, 2003). It is through this mobilization of a set of resources and a metacognitive reflection that students are able to elaborate not only divergent strategies for solving problems but also several different solutions (Poirier Proulx, 1999).

Open-ended and challenging problem solving is therefore seen as a process where students should be evaluated on the bases of their own ways of reasoning and communicating. According to Lesh & Doerr (2000), the challenge for teachers is to maintain and nurture the diversity of students' approaches, encouraging them to verbalize their thinking and explain their strategies. One of the possible solutions is to make teachers familiar with a "Problems of the week" model which proves to be an effective way to develop students into more independent learners (Webb, 2003).

This type of model is found in the CASMI, an Internet-based learning environment. Researchers argue that the use of such environments allows more schoolchildren to participate in mathematically rich contextual problem solving activities. Pre-service teachers can thus learn from students' solutions by analyzing their reasoning and communication abilities (Charbonneau, 2000; Renninger & Shumar, 2002) in didactic contexts that are more practice oriented (Bednarz, 2004). In such contexts, teachers play the role of a mentor by guiding students in their learning.

Guiding students with an effective formative feedback

Formative feedback is defined as "information communicated to the learner that is intended to modify the learner's thinking or behavior for the purpose of improving learning" (Shute, 2007, p. 1). Thus, the main goal of formative feedback is to help students understand their errors and further their reasoning. But is all feedback good feedback? It has been recently argued by Hattie & Temperley (2007) that feedback is "most effective when it aids in building cues and information regarding erroneous hypothesis and ideas and then leads to the development of more effective and efficient strategies for processing and understanding the material" (p. 102). According to Shute (2007), formative feedback serving as a corrective function should, at the least, indicate the correctness of students' answers and provide information about the correct answer. However, she specifies that a certain number of researchers agree that feedback, to be more effective, needs to give information pertaining to the improvement of the answer (instead of simply indicating the correctness of the work). Indeed, unspecific feedback can be considered useless or frustrating by students.

Galluzzo, Leali, and Loomis (2000) identified key elements linked to an effective feedback by resuming the works of Brophy. Among others, the authors insist that the teacher must:

- give a feedback which is specific to students;
- not strictly put his focus on the students' errors but also state the accomplishments;
- be specific in his comments (rather than global).

The authors also underline the importance of the knowledge of the discipline taught. Indeed, one cannot give a specific feedback if he or she is not comfortable with the discipline.

Shute (2007) did a review of the formative feedback literature and came up with these nine guidelines to enhance learning (p. 30):

- Focus feedback on the task, not the learner.
- Provide elaborated feedback to enhance learning.
- Present elaborated feedback in manageable units.
- Be specific and clear with feedback messages.
- Keep feedback as simple as possible but no simpler (based on learner needs and instructional constraints).
- Reduce uncertainty between performance and goals.
- Give unbiased, objective feedback, written or via computer.
- Promote a learning goal orientation via feedback.
- Provide feedback after learners have attempted a solution.

She also specifies three guidelines for high-achieving learners (p. 33):

- Consider using delayed feedback, especially for complex tasks.
- Use facilitative feedback, which aims to guide students by giving them comments and suggestions in link with the problem that needs to be solved. Telling students what to do is considered directive feedback rather than facilitative feedback.
- Verification feedback, which gives information pertaining to the correctness of the answer, may be sufficient. On the other hand, elaboration feedback gives more information to students, allowing them to correct their work.

Summarizing and projecting our literature review on our research questions, we claim that the combination of challenging problem solving in an online environment and the opportunity to analyze genuine schoolchildren’s solutions and to produce a formative feedback provides us with an insight into pre-service teachers’ ability to evaluate and to guide students based on the diversity of their strategies and solutions. In the next sections, we describe in more details how we proceeded with data collection and data analysis.

METHODOLOGY

In our exploratory study, we analyzed quantitatively pre-service teachers' experiences with the assessment of open-ended challenging problems in the online environment. According to our two goals, we wanted to learn about pre-service teachers' perceptions on the importance of such experiences and their impact on future classroom practices regarding the use of the enrichment activities with their students. We were also interested in the evaluation of the quality of the feedbacks given by pre-service teachers. We thus studied their abilities to understand children's strategies and communication styles. In this section, we will describe the virtual environment CASMI (*Communauté d'Apprentissages Scientifiques et Mathématiques*, www.umoncton.ca/casmi)[1], the mentoring activities in which the pre-service teachers were involved and how these activities have been evaluated. We will also present the samples and data collection tools.

Virtual environment

In the CASMI environment, schoolchildren are invited to solve challenging mathematical problems and submit their solutions electronically (Freiman & Lirette-Pitre, 2008). Pre-service teachers then analyze every solution and write a personal feedback. The problems of the week are grouped in four categories according to their level of difficulty and posted online. These problems present a variety of contexts to which schoolchildren are supposed to apply mathematical concepts from all domains of school mathematics (arithmetic, algebra, geometry, statistics).

Figure 1 (p. 11) presents one of the problems students had to solve in the CASMI. In this problem, "The Valentine's Day card", students had to find the original width and length of a piece of paper that had been folded. The problem contains a context familiar to French Canadian schoolchildren and is attractive. A variety of answers can be produced, since the only constraint is that the sum of the width and the length of the original piece of paper must be equal to 50 centimeters. Children with different abilities may extract different mathematical relationships

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[1] Although the research project took place when the website was called CAMI (Chantier d'Apprentissages Mathématiques Interactifs), the abbreviation CASMI will be used throughout this article in order to facilitate its reading.

representing and exploring them in many different ways. One solution submitted by a grade 6 student is presented in figure 2 (p. 11) and an extract from the personal feedback given to her by a pre-service teacher is presented in figure 3 (p. 12).

> The Valentine's Day card
>
> Valentine's day is coming and Reuben decides to make a Valentine's day card for Sophie.
>
> As you probably did before, Reuben takes a piece of red construction paper and folds it vertically in two. He then folds the piece horizontally and finally draws hearts and flowers while writing beautiful words of friendship everywhere.
>
> The perimeter of the folded card is 50 centimeters. Find the length and the width of the original piece of paper (before it was folded). Clearly explain your reasoning.

Figure 1. Mathematical problem presented in the CASMI

> The Valentine's day card
>
> If we unfold it, it's going to be twice as big, and if we unfold it again, it's going to be twice as big again.
>
> $50 \times 2=100$
> $100 \times 2= 200$
>
> Answer: 200 centimeters

Figure 2. Solution submitted electronically in the CASMI

Extract from the feedback

I believe that you tried to find the perimeter of the original paper (before it was folded). However, the problem was to find the length and the width of this piece of paper. I invite you to verify your answer. I am sure that you can solve this problem!!!

Thank you for participating. Bravo for your efforts! I wish to receive other solutions from you in the next few weeks.

Figure 3. Extract from the personal feedback written by a pre-service teacher

The first paragraph of the extract from the feedback contains various aspects mentioned in our theoretical framework. First of all, the fact that the student didn’t seem to understand exactly was she was looking for is underlined and an "appropriate interpretation” of the question is given. Moreover, the student is invited to review her work. Finally, a strong belief in the child’s capacity to correctly solve the problem is visible. The second paragraph, written in the last section of the feedback, values the student’s participation and efforts and aims to encourage her to solve more problems in the CASMI in the near future.

While all children are asked explicitly to explain their reasoning, not all of them show their work and sometimes, it is not obvious to see mathematical reasoning beyond the explanations. All this may represent important challenges to pre-service teachers who are not used to solving problems in different ways, analyzing reasoning and giving critical comments back to students. Therefore, working within the CASMI environment, they get this genuine opportunity to look at this variety of mathematics created by children.

Participants

During the 2006 winter semester, a total of 70 pre-service teachers participated in our research. Thirty-two were enrolled into the Middle School (5-8) Teacher Preparation Program and 18 were enrolled in the High School (9-12) Teacher Preparation Program at Université de Moncton. Twenty more were enrolled in the Secondary Mathematics Teacher Preparation Program (7-11) at Université Laval. The collaboration between the two researchers never aimed to make any comparisons between the two groups. There was no specific interaction between the two groups. According to our theoretical perspective, we focused on each participant's perception using a survey and we assessed the quality of randomly selected feedbacks. In this case, we can consider these two groups as one combined population (one group) rather than as two different populations.

Instruments

During the semester, feedbacks were written to schoolchildren using an electronic form built into the CASMI site (figure 4, p. 14)[2]. All pre-service teachers had to log-in individually to assess solutions randomly assigned to them. Our form was divided in three sections. The first section, *Greeting*, was situated at the beginning of the form and allowed pre-service teachers to make a first contact with students by writing comments pertaining to their participation or the efforts that were made, as well as general comments with regard to the submitted solution. The second section of our form, the rubric, contained six different components used by Math Forum to score solutions: interpretation, strategy, exactness in calculations, completeness, clarity, and quality of reflection. We developed our own pre-built set of criteria according to the specific features of each component. These criteria were presented as multiple choice items. Thus, in their formative feedback, pre-service teachers could choose one of these pre-determined criteria for each component. The chosen criterion could also be accompanied (or replaced) optionally by an open comment, which permitted the personalization of the feedback. Finally, in the last section of the electronic form called *Signature*, pre-services teachers could summarize their thoughts about the student's production and invite them to visit the CASMI again in order to

[2] An English version of this electronic form is presented in appendix 1 (p. 29).

solve more problems. So, in every section of the electronic form, pre-service teachers were capable of writing comments and thus of personalizing the feedback given to schoolchildren.

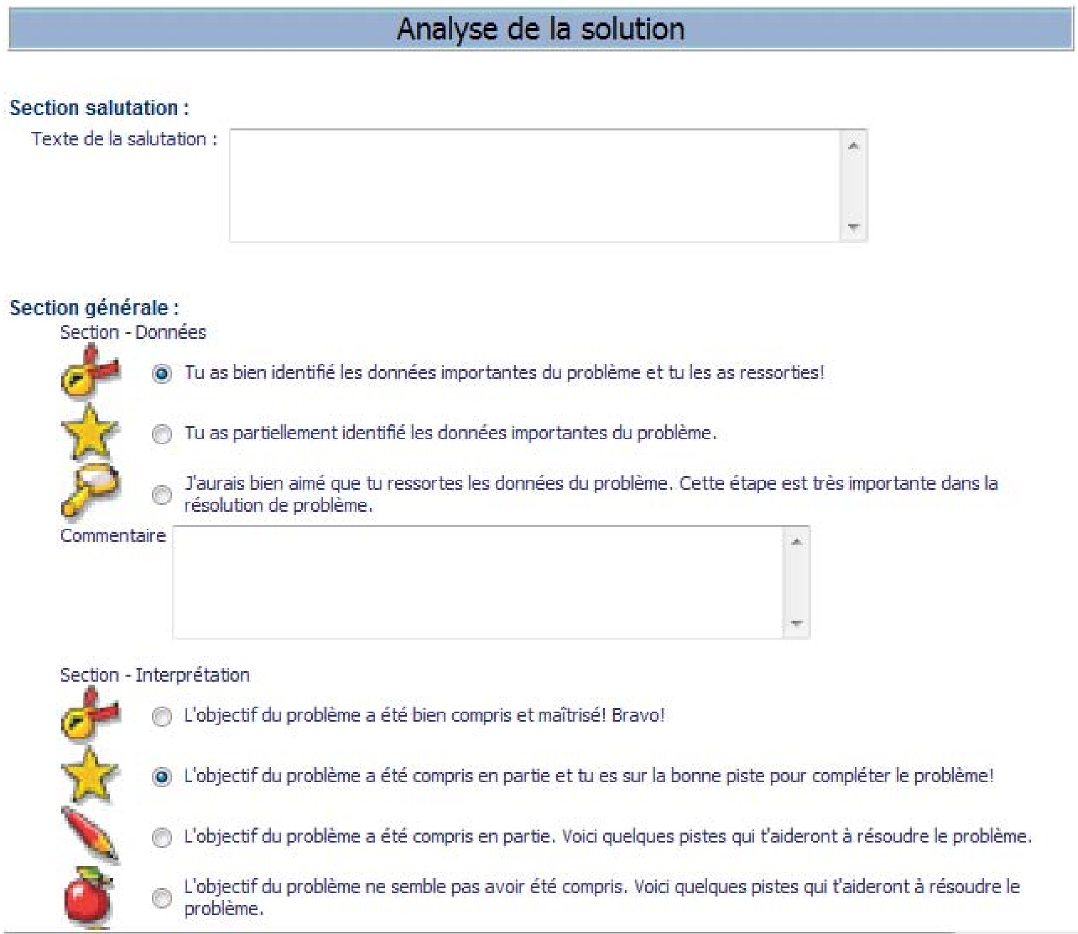


Figure 4. Electronic form in the CASMI site

At the end of the semester, a questionnaire including open-ended questions as well as multiple choice questions was distributed. The questionnaire was divided into ten sections, pertaining to different aspects of the project: 1) General information on the participants; 2) CASMI project and the didactics course; 3) CASMI project and the student doing mathematics; 4) CASMI project and teachers; 5) Appreciation of the CASMI website; 6) Accessibility of the problems; 7) Problems' content; 8) Functioning of the CASMI website; 9) Continuation of the CASMI project; and 10) Use of the site with the preservice teachers' future students. The answers to the questions as well as the comments gathered in the questionnaire permitted us to collect qualitative and quantitative data concerning pre-service teachers' perceptions pertaining to the CASMI project and teacher training as well as to the CASMI project and teaching and learning mathematics. The multiple choice questions employed a four-point Likert scale: 1 = Completely in agreement, 4 = Completely in disagreement.

Procedures

Université de Moncton.

At the Université de Moncton, pre-service teachers enrolled in the Elementary (K-8) Teacher Preparation Program must take two courses in mathematics education. Within each course, they conduct a project related to CASMI. Most of the pre-service teachers participating in our project were enrolled in their second math education course and were already familiar with the resource. While during the first course they are required to do reflective analyses of their experience and are guided by the course instructor in their assessment process, the second course requires more autonomous work and better quality of feedback. Fifty students evaluated up to ten solutions each. During the math education classes, each problem as well as different ways of solving it and communicating related strategies were discussed. Pre-service teachers thus understood the problems before having to assess schoolchildren's work.

Université Laval.

The participants at Université Laval were all enrolled into the Secondary Mathematics Teacher Preparation Program. In this program, pre-service teachers have to take three courses in mathematics education. Within the framework of our research project, twenty pre-service teachers enrolled into their third and final math education course received a brief presentation of the CASMI, which they were not familiar with. A document explaining the evaluation rubric and presenting examples of feedbacks was also given to them. In a four weeks period, each pre-service teacher evaluated a total of twelve productions submitted by students.

At the beginning of each week, before they received students' productions, pre-service teachers had to solve the four "problems of the week" presented in the CASMI. These problems were then revised in class. This revision made it possible to avoid any confusion that could be allotted to the various problems. Moreover, pre-service teachers were asked to present different strategies used when solving these problems. Therefore, they were made aware of different ways to solve one problem. It is important to note that the pre-service teachers' feedbacks were strictly evaluated on formative bases. Following each week, comments pertaining to pre-service teachers' feedbacks were emitted by the professor. These comments made it possible for the pre-service teachers to adjust their formative evaluations week after week.

Data Analysis

A total of 65 pre-service (47 from Université de Moncton and 18 from Université Laval) answered the questionnaire. A theme analyses of the qualitative data collected in the questionnaire was realized. Frequency distributions were calculated to analyze the multiple choice items.

In addition to data from the questionnaire, we analyzed formative feedbacks written by pre-service teachers. Out of a total of 924 schoolchildren's solutions submitted to ten problems posted during the semester, we randomly selected 200. We developed and validated an evaluation grid containing 53 variables[3]. These variables reflected elements reported in our theoretical framework and were divided into nine categories. The first category was *General* and it permitted us to determine the correctness of students' answers and then check if pre-service teachers had identified that answer as being correct or incorrect. The same variables were repeated for the next two categories, *Greeting* and *Signature*. We were interested to see if pre-service teachers added personalizing elements to their message (i.e. smiley, humor, etc.) and if they congratulated students on their work or thanked them for participating. Elements of feedbacks more directly in link with the mathematical aspect of the student's solution also interested us. For each of our six components, we evaluated if pre-service teachers had chosen the appropriate criterion in the pre-built set of multiple choice items specific to these components. Ideas present in the feedback examined were analyzed. For each idea, we checked, among others, if pre-service teachers underlined the correctness of the answer, the correctness of the reasoning and if they identified students' errors. Elements more linked with the quality of feedbacks, like specificity or reference to students' work, were also evaluated. In addition to that, we checked if pre-service teachers gave facilitative, verification or elaboration feedback.

[3]Some of these variables were repeated for every criterion or for every different idea present in a comment. The evaluation grid thus contains a total of 271 variables.

This analysis enabled us to gather information about the quality of submitted solutions by the students as well as the quality of the feedback provided by the pre-service teachers.

Resume and Analyses of the Most Important Findings

a) How do pre-service teachers perceive their participation in the project regarding their future work on challenging problem solving in the mathematics classroom?

The participation in the CASMI project allowed pre-service teachers to analyze concrete solutions of real schoolchildren. The first part of our analyses concerned pre-service teachers' perceptions of the CASMI project and according to the previously described elements of our questionnaire, we found that 84.6 % of participants agreed or strongly agreed that their feedbacks were important for schoolchildren. Eighty percent found that the project helped them better understand schoolchildren's reasoning and 67.7 % found that it helped them better understand the problem-solving process in mathematics. Moreover, 83.1 % affirmed that they had learned more about formative feedbacks, 66.1 % say that the project gave them the chance to review mathematical concepts, and 78.5 % of pre-service teachers said that the project gave them ideas for teaching. Finally, 81.5 % of them agree or strongly agree that the CASMI project not only enables teachers to differentiate their teaching but also enriches the mathematics curriculum. The complete results on pre-service teachers' perceptions of the CASMI project are presented in table 1 (next page).

Table 1
Pre-service teachers' perceptions of the CASMI project

	Completely in agreement		In agreement		In disagreement		Completely in disagreement		I don't know	
	Frequency	%	Frequency	%	Frequency	%	Frequency	%	Frequency	%
Your feedback is important for the student	40	61.5	15	23.1	2	3.1	1	1.5	6	9.2
The content of the problems enriches the math curriculum	26	40.0	27	41.5	7	10.8	3	4.6	0	0.0
The project…										
helped me to understand the student's reasoning	7	10.8	45	69.2	10	15.4	3	4.6	0	0.0
helped me better understand the problem-solving process in math	10	15.4	34	52.3	15	23.1	5	7.7	1	1.5
allowed me to perfect my techniques in formative evaluation in math	18	27.7	36	55.4	6	9.2	3	4.6	2	3.1
allowed me to review math concepts	19	29.2	24	36.9	15	23.1	5	7.7	1	1.5
gives teachers ideas for math courses	28	43.1	23	35.4	11	16.9	1	1.5	2	3.1
allows teachers to differentiate their teaching	19	29.2	34	52.3	7	10.8	1	1.5	4	6.2

The results obtained from the analyses enable us to note that the pre-service teachers' perceptions of the CASMI project as a source of enrichment are mainly positive in all aspects of the questionnaire. Those results are consistent with our previous data (Freiman *et al.*, 2005). However, in this study, we decided to conduct an in depth analyses of the quality of the feedbacks given by pre-service teachers in order to track their abilities to assess students' solutions and to guide them, in the process, towards better problem solving strategies. The second part of our analyses, concerning the types of feedbacks given by pre-service teachers, is presented in the next section.

b) Being faced with a multitude of problem solving strategies, are pre-service teachers able to evaluate the correctness of students' work and to provide students with an adequate feedback in order to guide them and to help them improve their problem solving skills?

Our methodological framework defined certain aspects that are important when giving a feedback. Among those aspects, pre-service teachers need to be able to assess if the solution submitted by a student is correct. It is also important for schoolchildren to be guided and to get feedback which is directly linked with the work they have done. Keeping this in mind, we analyzed 200 feedbacks given by pre-service teachers in order to study their ability to evaluate students' work and to give a quality feedback. We found that in 78.5 % of cases, pre-service teachers were able to correctly identify if students' answers were correct. They made an incorrect evaluation 10.5 % of the time (i.e. indicating to a student that his answer was correct when it wasn't and vice versa) (table 2, p. 21). Moreover, for each component of the evaluation rubric, pre-service teachers were invited to choose a criterion specific to the component and linked with the student's work (table 3, p. 21). They chose the appropriate criterion 70.0 % of the time for the component Interpretation, 72.5 % of the time for the component Strategy and 72.0 % of the time for the component Clarity. This percentage goes up to 79.0 % in the case of the component Correctness, 77.5 % for the component Completeness, and 80.0 % for the component Quality of reflection.

Table 2
Choice of the criterion in order to identify if the student's answer was correct

	Frequency	Percent
No criterion selected	3	1.5
Incorrect choice of criterion	21	10.5
Partially correct choice of criterion	19	9.5
Correct choice of criterion	157	78.5
Total	200	100.0

Table 3
Choice of the criterion specific to the component

	No criterion selected		Incorrect choice of criterion		Partially correct choice of criterion		Correct choice of criterion	
Component	Frequency	Percent	Frequency	Percent	Frequency	Percent	Frequency	Percent
Interpretation	7	3.5	11	5.5	42	21.0	140	70.0
Strategy	9	4.5	29	14.5	17	8.5	145	72.5
Clarity	19	9.5	13	6.5	22	11.0	144	72.0
Exactness in calculations	10	5.0	20	10.0	12	6.0	158	79.0
Completeness	14	7.0	13	6.5	16	8.0	155	77.5
Quality of reflection	14	7.0	14	7.0	10	5.0	160	80.0

Since every feedback could be personalized by writing a comment, we then asked ourselves which kind of analysis and recommendations were present in the individual comments that were written. The analyses of the 200 feedbacks given by pre-service teachers shows that 70.5 % of these feedbacks place little or no importance on the successes of students and tend to strictly focus on their errors or on challenges for them to overtake (table 4, p. 22). Moreover, although the majority of comments do refer implicitly to schoolchildren's work, 60.5 % of them are general and lack in precision (table 5, p. 22).

Table 4
Feedback in the form of positive feedback or focusing on the student's errors

	Frequency	Percent
Positive feedback	161	29
Focusing on student's errors or on challenges	391	70.5
Total	552	99.5

Table 5
General or specific comment

	Frequency	Percent
General comment	336	60.5
Specific comment	219	39.5
Total	555	100.0

Thus, the problem may not reside as much in the criteria-based assessment of students' answers as in the (informal) feedback they give (or do not give). Among the 200 solutions that were analyzed, 100 contained some incorrect reasoning or calculation mistakes (table 6, p. 23). Our findings show that for 81.0 % of these solutions, at least one comment, directly linked to one of the components in the rubric, was made by pre-service teachers (table 7, p. 23). However, even though 81.0 % of the incorrect solutions were commented on at least once, in several cases (i.e. for several components), pre-service teachers seemed to be satisfied by choosing one of the pre-determined criteria and didn't write any comments in order to enrich their feedback. We do not know why they did not take the time to write more comments. In our future work, we will need to conduct interviews with the participants in order to learn more about their reasons for choosing a particular criterion over another.

Table 6
Correctness of the student's answer

	Frequency	Percent
No answer	1	0.5
Incorrect answer	100	50.0
Partially correct answer	36	18.0
Correct answer	63	31.5
Total	200	100.0

Table 7
Feedbacks given to students whose answers contained some incorrect reasoning or calculation mistakes

	Frequency	Percent
No feedback	7	7.0
Feedback directly linked to one of the six components of the rubric	81	81.0
Feedback given in the sections *Greeting* or *Signature*	12	12.0
Total	100	100.0

Moreover, they do not seem to fully appreciate the diversity of students' approaches which, according to Lesh & Doerr (2000), is a challenge for teachers. It is important for them to maintain and nurture that diversity. The pre-service teachers that participated in our study were not in a guiding mode and did not encourage schoolchildren to further their reflection. Indeed, instead of being built on students' work, comments that were written strove students' thinking towards pre-determined answers which is contrary to current tendencies in mathematics education (Astolfi, 2006).

CONCLUSION

This study draws its originality from the fact that it focuses not only on pre-service teachers' perceptions but on the link existing between these perceptions and the quality of their formative asynchronous feedback. Linking pre-service teachers' perceptions of what an Internet resource on problem solving can bring to improve mathematics teaching and learning to their ability to analyze children's thinking, we aimed to develop practical recommendations on how to build more solid assessment competences in pre-service teachers.

Participation in the online project allowed pre-service teachers to experience new mathematical problem solving approaches which stress the use of a multitude of strategies and communication means by schoolchildren. They perceived their experience as valuable since it permitted them to better understand the problem solving process and, in particular, children's ways of communicating their reasoning. They observed that some problems allow different data interpretation, different solving strategies and sometimes different answers.

Some strategies may be plausible, even ingenious. Others may contain misinterpretations, misconceptions, or alternative views. In order to be able to guide children through their learning, pre-service teachers have to become competent not only in mathematics but also in feedback pedagogy, which sometimes work in the counter direction of the traditional didactical contract (Brousseau, 1986, 1988, 1998; Poirier, 2001). When communicating with schoolchildren about problem solving, our pre-service teachers get the chance to work on contextual open-ended problems revising their own views of problem solving and its role in mathematics learning. They also reinforce their own conceptual understanding of mathematics and develop a better understanding of how children think and explain their thinking. While writing feedbacks, pre-service teachers put in practice their ability to understand the problem itself and to guide children towards better problem solving strategies (Freiman *et al.*, 2005; Metallidou, 2009).

It’s not easy to understand a child’s reasoning when it is expressed out loud. Asynchronous assessment is even more challenging because there is no opportunity to give feedbacks in another way than written comments. But our data shows a lack of such personal comments. The comments’ general character may be a result of the pre-service teachers’ lack of mathematical background as well as lack of time. If the first two issues can be address by better teacher training strategies, the last one may raise a concern. Indeed, when schoolchildren are allowed to use a variety of strategies and communication means, teachers must give feedback to every one of them. If pre-service teachers don’t have the time to do it with 10 students, how will they find the time to do it with 30 students, and possibly 30 different strategies? Are changes necessary to the school system or to the working ethics of pre-service teachers?

REFERENCES

Applebaum, M. & Leikin, R. (2007). *Teachers' Conceptions of Mathematical Challenge in School Mathematics.* Paper presented at the PME 31 (The International Group for the Psychology of Mathematics Education), Seoul, Korea.

Astolfi, J.-P. (2006). *Chercheurs et enseignants : Repères pour enseigner aujourd'hui*. Paris: Institut national de recherche pédagogique.

Barbeau, E. J. & Taylor, P. J. (2009). *Challenging Mathematics In and Beyond the Classroom*.Boston, MA: Springer.

Bednarz, N. (2004). Former les futurs enseignants à la didactique ou par la didactique. In Y. Lenoir (Ed.), *Les enseignants du primaire entre disciplinarité et interdisciplinarité : quelle formation didactique?* (pp. 91-109). Laval, Qc: Les presses de l'Université Laval.

Brousseau, G. (1986). Fondements et méthodes de la didactique des mathématiques. *Recherches en didactique des mathématiques, 7*(2), 33-115.

Brousseau, G. (1988). Le contrat didactique : le milieu. *Recherches en didactique des mathématiques, 9*(3), 309-336.

Brousseau, G. (1998). *Théorie des situations didactiques*. Grenoble: La pensée sauvage éditions.

Charbonneau, L. (2000, November). *Solving Mathematical Problems within a Virtual Environment.* Paper presented at the XVIII ICIE International Conference, Québec.

Diezmann, C. M., Thornton, C. & Watters, J. J. (2003). Addressing the needs of exceptional students through problem solving. In F. Lester & R. Charles (Eds.), *Teaching mathematics through problem solving* (pp. 169-182). Reston, VA: National Council of Teachers of Mathematics.

Diezmann, C. M. & Watters, J. J. (2005). Catering for mathematically gifted elementary students: Learning from challenging tasks. In S. K. Johnson & J. Kendrick (Eds.), *Math Education for gifted students* (pp. 33-46). Waco, Texas: Prufrock Press.

Even, R. & Tirosh, D. (2002). Teacher Knowledge and Understanding of Student's Mathematical Learning. In L. D. English (Ed.), *Handbook of International Research in Mathematics Education* (pp. 219-240): Lawrence Erlbaum.

Freiman, V. (2006). Problems to discover and to boost mathematical talent in early grades: a challenging situations approach. *The Montana Mathematics Enthusiast, 3*(1), 51-75.

Freiman, V. & Lirette-Pitre, N. (2008). Building a virtual learning community of problem solvers: example of CASMI community *The International Journal on Mathematics Education, 41*(1-2), 245-256.

Freiman, V., Manuel, D. & Lirette-Pitre, N. (2007). CASMI: A virtual learning collaborative environment for enrichment. *Understanding of our gifted*.

Freiman, V., Vézina, N. & Gandaho, I. (2005). New Brunswick pre-service teachers communicate with schoolchildren about mathematical problems: CAMI project. *ZentralblattfuerDidaktik der Mathematik, 37*(3), p. 178-190.

Galuzzo, G. R., Leali, S. A. & Loomis, D. (2000, November 17-21). *Do We Have To Give Standardized Tests of Teacher Content Knowledge?* Paper presented at the de la 21st Annual Meeting of the National Council of States, Miami, Florida.

Hattie, J. & Temperley, H. (2007).The Power of Feedback. *Review of Educational Research, 77*(1), 81-112.

Johnson, D. T. (2000a). Teaching Mathematics to Gifted Students in a Mixed-Ability Classroom. *ERIC Digest E594*.

Johnson, D. T. (2000b). Teaching Mathematics to Gifted Students in a Mixed-Ability Classroom. *ERIC Digest E594*.

Kettler, T. & Curliss, M. (2005). Mathematical Acceleration in a Mixed-Ability Classroom: Applying a Tiered-Objectives Model. In S. K. Johnsen & J. Kendrick (Eds.), *Math Education for Gifted Students* (pp. 83-92). Waco, Texas: Prufrock Press, Inc.

Krutetski, V. A. (1976). *The Psychology of Mathematical Abilities in School Children*. Chicago: The University of Chicago Press.

Leikin, R., Levav-Waynberg, A. & Applebaum, M. (2008). *Multiple Solution Connecting Tasks*. Paper presented at the International Research Workshop of the Israel Science Foundation, Haifa, Israel.

Lesh, R. & Doerr, H. M. (2000). Symbolizing, Communicating, and Mathematizing: Key Components of Models and Modelling. In P. Cobb, E. Yackel & K. McClain (Eds.), *Symbolizing and Communicating in Mathematics Classrooms: Perspectives on Discourse, Tools and Instructional Design*. Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Metallidou, P. (2009). Pre-service and in-service teachers' metacognitive knowledge about problem-solving strategies.*Teaching and Teacher Education, 25*(1), 76-82.

NCTM. (2000). *Principles and Standards for School Mathematics*: National Council of Teachers of Mathematics.

OECD Program for International Student Assessment.(2003). The PISA 2003 Assessment Framework - Mathematics, Reading, Science and Problem Solving Knowledge and Skills. Retrieved July 4, 2005, from http://www.pisa.oecd.org/document/29/0,2340,en_32252351_32236173_33694301_1_1_1_1,00.html

Poirier, L. (2001). *Enseigner les maths au primaire : notes didactiques*. Ville St-Laurent, Qc: Éditions du Renouveau pédagogique.

Poirier Proulx, L. (1999). La résolution de problèmes en enseignement : Cadre référentiel et outils de formation. Paris: De Boeck université.

Renninger, K. & Shumar, W. (2002). Community Building with and for Teachers at the Math Forum. In K. Renninger & W. Shumar (Eds.), *Building Virtual Communities: Learning and Change in Cyberspace* (pp. 60-95). New York, NY: Cambridge University Press.

Schoenfeld, A. H. (1989). Teaching mathematical thinking and problem solving. In L. B. Resnick & L. E. Klopfer (Eds.), *Toward the thinking curriculum: Current cognitive research* (pp. 83-103). Alexandria, VA: Association for Supervision and Curriculum Development.

Sheffield, L. J. (2003). Extending the Challenge in Mathematics: Developing Mathematical Promise in K-8 Students. Thousand Oaks, California: Corwin Press.

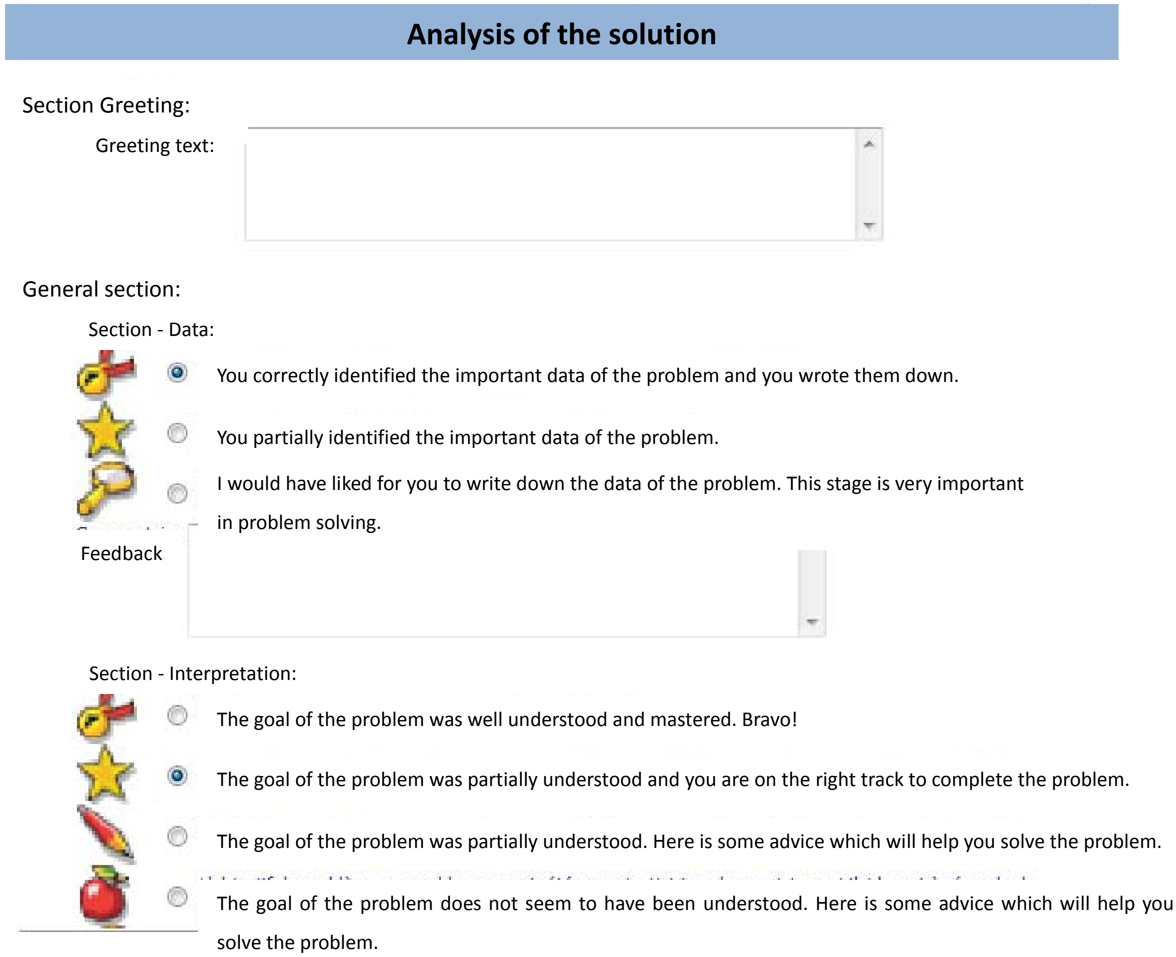
Shute, V. J. (2007). *Focus on formative feedback* (No. RR-07-11). Princeton, NJ: Educational Testing Service.

Tardif, J. (1999). *Le transfert des apprentissages*. Montréal: Les Éditions logiques.

Webb, N. (2003). The Impact of the Interactive Mathematics Program on Student Learning. In S. Senk & D. Thompson (Eds.), *Standard-Based School Mathematics Curricula: What are they? What do Students Learn?* (pp. 375-398). Mahwah, NJ: Lawrence Erlbaum.

Willson, L. & Kenney, P. (2003). Classroom and Large-Scale Assessment. In J. Kilpatrick, W. Martin & D. Schifter (Eds.), *A Research Companion to Principles and Standards for School Mathematics* (pp. 53-67). Reston, VA: National Council of Teachers of Mathematics.

Appendix 1. English version of the electronic form in the CASMI site



Analysis of the solution

Section Greeting:

Greeting text:

General section:

Section - Data:

- (selected) You correctly identified the important data of the problem and you wrote them down.
- You partially identified the important data of the problem.
- I would have liked for you to write down the data of the problem. This stage is very important in problem solving.

Feedback

Section - Interpretation:

- The goal of the problem was well understood and mastered. Bravo!
- (selected) The goal of the problem was partially understood and you are on the right track to complete the problem.
- The goal of the problem was partially understood. Here is some advice which will help you solve the problem.
- The goal of the problem does not seem to have been understood. Here is some advice which will help you solve the problem.