USING ASTROBIOLOGY AS A PLATFORM TO STUDY THE IMPACT ON THE MATHEMATICAL CONTENT KNOWLEDGE AND PEDAGOGICAL CONTENT KNOWLEDGE OF ELEMENTARY EDUCATION MAJORS

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USING ASTROBIOLOGY AS A PLATFORM TO STUDY THE IMPACT ON THE MATHEMATICAL CONTENT KNOWLEDGE AND PEDAGOGICAL CONTENT KNOWLEDGE OF ELEMENTARY EDUCATION MAJORS

By

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Dissertation

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in Interdisciplinary Studies

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Using Astrobiology as a Platform to Study the Impact on the Mathematical Content Knowledge and Pedagogical Content Knowledge of Elementary Education Majors

Chairperson: Ke Wu, Ph.D.

Early mathematical skills have long been hailed as a cornerstone and as the best predictor of later success in mathematics and literacy. This perception highlights the importance of elementary educator’s mathematical content knowledge (MCK) and pedagogical content knowledge (PCK). This study explored a novel approach to motivating and facilitating preservice elementary educators’ engagement in an interdisciplinary context. Astrobiology is a growing interdisciplinary field with extraordinary educational potential. It has the potential to provide an exciting science framework structure to mathematics for preservice educators. Due to its interdisciplinary content, astrobiology offers preservice educators an opportunity to see math content through a science lens, an approach that may appeal to students with diverse interests. Although astrobiology research has been on the rise and has contributed greatly to the science field and to society, more research on astrobiology education in schools and colleges needs to be done to understand the best pedagogical approaches to such a diverse topic that encompasses multiple disciplines. Using a quasi-experimental design, this study examines whether the implementation of astrobiology modules focused on science questions could be used as an effective platform to deliver mathematical instruction that focuses on MCK and PCK. Specifically, this dissertation investigates the impact of such modules on preservice elementary educators’ MCK and PCK, both quantitatively and qualitatively. A comprehensive analysis involving nonparametric statistics and qualitative analysis found insufficient sample evidence at the alpha level of 0.05 (\(\alpha = 0.05\)) to warrant rejection that the astrobiology based mathematical modules had no effect on the preservice teachers’ MCK or PCK. However, one test found a positive correlation between the module and an increase in astrobiology knowledge. The qualitative examination exposed a decrease in the quality of responses for the MCK and PCK areas. This affect could be attributed to the limiting factors of the study. These factors have implications for both teaching future research in the intersection of astrobiology education and MCK and PCK.
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There have been many people that have been supporters of my journey through graduate school and this dissertation. When I finished my undergraduate degree in mathematics, I never thought that I would ever return to higher academia as a student. However, after being hired as a mathematics instructor at Blackfeet Community College (BCC), I needed a higher degree. Dr. Michael Ceballos was at that time working on his Ph.D. at the University of Montana (UM) and urged me to continue my education. After the passing of my mom and talking to my family, I decided to enroll full-time as a graduate student at UM. I was among a cohort of four Native American Tribal College professors seeking our master's degrees that Dr. Michael Ceballos mentored as we started graduate school fall semester of 2015. We each had our path and mine kept me at UM.

At UM, I became one of Dr. Ke Wu's graduate students. She has been a tremendous source of both educational and emotional support. I couldn't have done this without her incredible guidance. I also must thank Dr. James (Jim) Hirstein, whom I met when I was teaching at BCC, he is indeed a role model for all math teachers and an excellent resource. I am fortunate to have been his student many times before his retirement. Dr. David Erickson has helped me gain a better sense of the discipline from the mathematics education seminars and has stretched my thinking not only in class but our discussions. My journey led me to not only work with Dr. Sandy Ross through my committee but also at UM's Graduate School, and the ease at which I could talk and relate to him was incredible. All this work would not have been possible if it weren't for Dr. Bill Holben and the committee taking a chance on me and accepting me to the NSF IGERT – Montana Ecology of Infectious Disease (MEID) Program. I learned so much and was part of a fantastic group of both professors and students. One of which was Dr. Michael Ceballos. He's not just a mentor, but I consider him my champion. I recognize and value his continued support and encouragement.

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CHAPTER I

INTRODUCTION

Many students find mathematics difficult to master (Schofield, 1982; Singh, Grandville, & Dika, 2018). Students struggle with the content as it moves from procedural to abstract, making it difficult to acquire and retain important concepts (Brizuela & Schliemann, 2004). This study explores ways of presenting mathematical content that increases student engagement and understanding via a science platform. In this introduction, I address (a) the context of the research, (b) how the framework has influenced the approach of the design in an interdisciplinary manner, and (c) how astrobiology has become a feature in the research, as well as (d) a brief examination of the development of elementary teachers’ mathematical knowledge for teaching. This section and the following rely heavily on some key terms; therefore, (e) defining those terms has been inserted before addressing the (f) theoretical framework of knowledge for teaching. The theoretical framework leads directly to the (g) mathematical content for elementary students that was the focus of the (h) problem statement and (i) research questions and, last, the (j) significance of the study.

Context

Mathematics education has been characterized as a stand-alone subject and as an essential component of science, technology, engineering, and mathematics (STEM). Mathematics education reform movements that are focused on maximizing conceptual learning have gone through several iterations, from the best practices of the Progressive Movement of the 1920s to the current Common Core standards (Ellis & Berry III, 2005;
Klein, 2003). The accepted approaches to teaching mathematics have moved back and forth between different forms of progressive thinking and a more traditional (classical) line of teaching philosophies (Bidwell & Clason, 1970; Ellis & Berry III, 2005; Klein, 2003). Progressive methods ranged from early attempts at student-centered approaches to teaching (Klein, 2003) to just focusing on the basic mathematics needed for daily life (Resnick & Ford, 2012). Traditional methods focused more on concepts to establish a hierarchy of mental habits, so students could make richer connections by developing their logic and analytical reasoning skills (Ellis & Berry III, 2005; Tate, 1995). For this study, these two approaches have been combined through the use of collaborative learning and engaging mathematical tasks to maximize the participants’ understanding of mathematics.

**Interdisciplinary Approach to Mathematics in Conceptual Design**

A current concern in mathematics education involves the use of interesting contexts or frames of reference (Brand, 2008). Science and mathematics are complementary because science relies heavily on mathematics to describe processes and outcomes (Singh, Granville, & Dika, 2002). For example, measuring wind speed, comparing weights of materials of different densities, and determining the circumference of the Earth all require units of measure and calculation (i.e., mathematics) to quantitatively describe these systems. However, mathematics is often taught as a stand-alone subject or with casual or detached references to such practical applications. If the mathematical content is delivered and mathematics is practiced in the context of a scientific problem, using a blend of both progressive and traditional methods, students
may find both subject areas (i.e., science and mathematics) more accessible, more memorable, and, perhaps, more exciting.

It has been suggested that emphasizing the connections between science and mathematics as part of an integrated approach to teaching may benefit the student (Honey, Pearson, & Schweingruber, 2014, p. 1). Teaching mathematics content in the context of real-world issues can make STEM subjects more relevant to both students and teachers (Honey et al., 2014, p. 1). Advocates of this approach believe that the integration of mathematics and science subject material motivates students’ interest and engagement, increasing achievement and promoting persistence (Honey et al., 2014, p. 1) Mathematics and science are two subjects that work in tandem quite well. The disciplines within the broad field of science, such as biology, chemistry, geology, physics, ecology, and so on, are conventional, and most students are aware of them. Many questions in these fields of science require mathematics skills to answer them. Often, mathematics is taught in general terms or so abstractly that learners have a difficult time putting the concepts to use in practical applications in other disciplines (Behr, Lesh, Post, & Silver, 1983).

**Role of Astrobiology**

Astrobiology is considered a relatively new scientific discipline, gaining recognition over the last 50 years. Astrobiology is a diverse subject that encompasses many different scientific disciplines; therefore, not just one discipline is used to convey different mathematical concepts. Using different types of science under the common theme of astrobiology helps strengthen the understanding of both science and the mathematical applications that are an essential part of the field.
Astrobiology covers a diverse scope of subjects within the STEM fields. It is a subject area that has had some projects developed for educational purposes with some being utilized for educational purposes in teaching science with some mathematical applications (Arino de la Rubia et al., 2009; Sneider & Ohadi, 1998). However astrobiology is not widely used, mostly due to teachers being unfamiliar with the many disciplines that fall within the purview of the broad subject. The focus in astrobiology is to answer three foundational questions:

1. How does life begin and evolve?
2. Is there life beyond Earth, and, if there is, how can we discover it?
3. What is the future of life on Earth and in the known universe? (Des Marais, Nuth, Allamandola, Boss, Farmer, Hoehler et al., 2008).

To answer these questions, scientists need understanding spanning many traditional disciplines, including biology, astronomy, physics, Earth science, planetary science, microbiology, evolutionary biology, cosmochemistry, and mathematics.

Expanding student awareness of the multitude of different fields within the overarching discipline of astrobiology is an excellent way to spark student interest during a time when current technology is turning science fiction into science reality.

**Development of Elementary Teachers’ Mathematical Knowledge for Teaching**

Understanding mathematics is difficult for many students. Building a solid foundation in mathematics is an essential aspect of their educational journey, and their teachers are central to their success. Many teachers in the United States have a dearth of mathematical understanding and skills (Ball, Hill, & Bass, 2005). This study investigated whether the use of interdisciplinary modules broadened preservice teachers’
understanding of two types of mathematical content knowledge (MCK): common content knowledge (CCK) and knowledge of students’ conceptual thinking.

For elementary education majors, there are usually two or three courses that address MCK. These courses are designed to cover the material that teachers will be required to teach across a range of grade levels in elementary school mathematics. These courses are critical to developing the skills of elementary education majors in CCK and specialized content knowledge (SCK), as well as general pedagogical content knowledge (PCK). This study examined the ability of interdisciplinary teaching modules to develop preservice teachers’ common and SCK and was conducted in the first of the series of two mathematical courses designed for elementary education majors.

**Definition of Terms**

- **Mathematical Knowledge for Teaching (MKT)** – The knowledge required to effectively teach mathematics (Welder, 2007).
- **Mathematical Content Knowledge (MCK)** – The knowledge of mathematics and its structure (Turnuklu & Yesildere, 2007).
- **Pedagogical Content Knowledge (PCK)** – The “particular form of content knowledge that embodies the aspect of content most germane to its teachability” (Shulman, 1986, p. 9).
- **Common Content Knowledge (CCK)** – The mathematical knowledge that a well-educated adult would possess (Ball, Thames, & Phelps, 2008).
- **Specialized Content Knowledge (SCK)** – Knowledge that exceeds the expectation of any well-educated adult but does not necessitate understanding of teaching or students (Ball, Thames, & Phelps, 2008).
• Knowledge of students’ conceptual thinking – The knowledge of how students examine and understand mathematical concepts, which includes understanding common student comprehension, misconceptions, mistakes, struggles, and general interest in mathematics (Welder, 2007).

• Knowledge of content and teaching – The knowledge that integrates knowing about teaching and knowing about mathematics (Ball, Thames, & Phelps, 2008).

• Astrobiology – Astrobiology is the study of the origin, evolution, distribution, and future of life in the universe. It includes an understanding of biological, planetary, and cosmic phenomena, as well as astronomy and astrophysics, Earth and planetary sciences, microbiology and evolutionary biology, cosmochemistry, and other relevant disciplines (Board, 2008, p. 1; Fletcher, 2014).

• STEM – Science, technology, engineering, and mathematics

• Preservice teachers – Students who are enrolled in a teacher preparation program and who are working toward a teacher certification.

• In-service teacher – An individual who is currently engaged in teaching anywhere from kindergarten to grade 12.

**Theoretical Framework of Knowledge for Teaching Mathematics**

The theoretical framework for this study has been composed from that of Shulman (1986) and Ball, Thames, and Phelps (2008) work on content and pedagogy. Shulman (1986) listed seven categories that he considered to be the most important in teacher knowledge: (i) PCK, (ii) content knowledge, (iii) general pedagogical knowledge,
(iv) curricular knowledge, (v) knowledge of learners and their characteristics, (vi) knowledge of educational settings, and (vii) knowledge of educational ends, purposes, values, and their philosophical and historical grounds. The first four categories focus on various aspects of what consisted of teacher knowledge, which were considered foundational pieces in teacher education programs. The other three categories examine the content and pedagogical-specific aspects of teaching that Shulman (1986) referred to as a “blind spot” with regard to research on teaching (p. 8). He also suggested that content knowledge is thought of in three different categories: 1) subject matter content, 2) PCK, and 3) curricular knowledge. There has been research (Ball, 1990; Ball, Bass, & Hill, 2005; Ball, Thames, & Phelps, 2008; Thompson & Thompson, 1996) on expanding particular areas and issues that are encompassed across these three categories.

Ball, Thames, and Phelps (2008) used theories of Shulman (1986) for their foundational approach of defining mathematical knowledge for teaching. They hypothesized that content knowledge and PCK (Shulman, 1986) could be divided into subdomains (see Figure 1). They contend that there are four subdomains within the two primary domains of subject matter content and PCK that could be considered the most important: CCK, SCK, knowledge of content and students (KCS), and knowledge of content and teaching.

Mathematical Content

Keystone concepts are important skills for students to master so they can become successful in subsequent mathematics courses, such as algebra, geometry, and calculus, and have been identified in research by some interested parties, including curriculum writers, textbook companies, educational researchers, and educational organizations.
Welder (2007) compiled the results of an analysis of the relevant research done by mathematics education experts that identified the prerequisite content areas that are thought to contribute to students’ ability to be successful in algebra. Welder (2007) listed nine concepts that, based on her analysis, are considered to be requirements for success in a basic first-year algebra course.

1. Numbers and numerical operations
2. Ratio/proportions
3. The order of operations
4. Equality
5. Patterning
6. Algebraic symbolism
7. Algebraic equations
8. Functions

Figure 1. Domain map for mathematical knowledge for teaching from Ball, Thames, and Phelps (2008, p. 403).
9. Graphing

Only a few of these areas were selected for examination in this study. Due to the breadth of material that makes up the cornerstones of mathematics education, not all of the concepts were investigated in this study. The items that were used include the following:

Numbers and numerical operations: Fractions – Fractions lie in the domain of number sense: “A solid grounding in fractions is a necessary pre-requisite for understanding ratios, which show up everywhere including business” (Wilson, 2009, p. 5).

Ratio and proportion – Proportional reasoning has been hailed as a cornerstone element in a student’s mathematical toolbox; it has been regarded as a fundamental concept for higher-level mathematics (Lesh, Post, & Behr, 1988).

Prealgebraic Functions: Exponents – Exponents can fall into a few different categories, such as patterning, the order of operations and algebraic expressions (algebraic symbolism), and algebraic equations and functions. Students are expected to start working with exponents as early as sixth grade, as stated in the Common Core State Standards Initiative (CCSS, 2010) for mathematics, where students use them in order of operations.

Graphing – For students to be mathematically proficient, they need to be able to explain correlations between equations, descriptions, tables, and graphs; to be able to draw diagrams of essential properties and associations; and to be able to graph the data and search for trends or patterns (CCSS, 2010).
Problem Statement

To succeed in higher-level mathematics, it is crucial that middle school students master prerequisite algebraic concepts during their K–8 education. Consequently, it is vital for preservice elementary educators to have a comprehensive understanding of both MCK and PCK. Combining mathematics with science, namely astrobiology, can further enhance this understanding of both MCK and PCK. Therefore, it is vital that preservice elementary education majors have an inclusive context for such interdisciplinary teaching to break the *silo* type instruction that often occurs. Doing so can strengthen students’ understanding of mathematical concepts in genuine applications, creating connections between academic and applied knowledge. Fortifying the teachers’ knowledge can help improve student learning.

The purpose of this study was to (a) use astrobiology as the context with which to characterize the MCK and PCK of preservice elementary educators and (b) to determine the effectiveness of astrobiology as a vehicle for learning mathematical concepts for preservice elementary educators.

Research Questions

The research questions in this study focus on an undergraduate first-semester elementary education mathematics content course. The development and implementation of the astrobiology modules were used to measure the participants’ MCK and PCK. The following questions were asked:

1. What effects does the astrobiology module (M1) have on the preservice teachers’ MCK and PCK of ratios, proportions, and fractions?
2. What effects does the astrobiology module (M2) have on the preservice teachers’ MCK and PCK of exponents and graphing?

3. What effects does the astrobiology module (M1 and M2) have on the preservice teachers’ knowledge of general astrobiology?

**Significance of the Study**

Although there are workshops and materials on astrobiology education, there is limited quantifiable data on whether these workshops or materials have increased the MCK and/or PCK of the participants.

The aims of this study were as follow:

1. teach the preservice elementary education teachers the involved important mathematical concepts in the context of a scientific background,

2. use astrobiology as a platform to enhance their MCK and PCK so they, in turn, can better prepare their future students, and

3. reduce the cascading effect of rote memorization on their students.

The goals and objectives of this study were the following:

1. **Goal 1:** Significantly increase the PCK of preservice teachers
   a. **Objective 1:** Use the modules to highlight pedagogical content and engage students in discussions on how to use pedagogy in their classrooms.
   b. **Objective 2:** Use pretest and posttest results to determine the effectiveness of the modules on pedagogical competence.

2. **Goal 2:** Significantly strengthen the MCK of preservice teachers
a. Objective 1: Use the modules to highlight mathematical content and to engage students in discussions on how to solve the subject matter.

b. Objective 2: Use the pretest and posttest results to determine the effectiveness of the modules on mathematical abilities.

3. Goal 3: Expand understanding of astrobiology content to increase science subject matter knowledge
   a. Objective 1: Use the modules to highlight astrobiology content and to engage students in discussions on how to use the subject matter in an interdisciplinary structure.
   b. Objective 2: Use the pretest and posttest results to determine the effectiveness of the modules on astrobiology knowledge.
CHAPTER II

REVIEW OF THE LITERATURE

The literature review on topics related to this study includes four areas. The first section, Background of Mathematics Education, includes an overview of mathematics education in the United States from 1920 to 2018. This background will be used to highlight the different views that have arisen in mathematics education in the United States. The second section, Challenges of Student Learning Content, examines the current mathematical standings, performance, and achievement of students, as well as the research in these areas and recommendations. The third section, PCK and MCK, looks at the research on the mathematical content areas that are relevant to the study and the rationale for addressing these content areas for preservice teachers. PCK includes not only research on the importance of this aspect for preservice teachers but the theoretical framework for the knowledge needed to teach mathematics effectively. The fourth and final section, Interdisciplinary Approach, reviews the research done on how STEM education is integrated, specifically on how astrobiology education pertains to mathematics and how it can be a useful tool to teach mathematics in context. The literature review will conclude with a synthesis of all aspects examined in this study.

**Background of Mathematics Education**

Different approaches to teaching mathematics date back as far as the 1920s with the Progressive Education Movement. During the progressive era, education explored discovery learning, a more student-centered approach to teaching (Klein, 2003). One of the influential leaders of progressive education was William Heard Kilpatrick, an
education professor at Teachers College at Columbia University and the chairman of one of the committees of the Commission on the Reorganization of Secondary Education; he was appointed by the National Educators Association Committee (Gardner, 1983; Klein, 2003). The U.S. Commissioner of Education published Kilpatrick’s report, *The Problem of Mathematics in Secondary Education*, in 1920 (Klein, 2003). In his report, Kilpatrick asserted that mathematics education should consist of only items that would be considered to have feasible value to the students in their day-to-day life. Otherwise the traditional high school curriculum should only be for a select few (Klein, 2003).

Kilpatrick’s report spurred a backlash from mathematicians whose views on education opposed Kilpatrick’s. The president of the Mathematical Association of America had already formed a committee consisting of mathematicians at universities and representatives of secondary mathematics teachers’ associations to address the forthcoming report from Kilpatrick. The committee was called the National Committee on Mathematical Requirements in 1916 (Gardner, 1983; Klein, 2003). This committee published *The Reorganization of Mathematics for Secondary Education 1923*, which is commonly referred to as the 1923 Report. The 1923 Report was comprehensive in scope and included a survey of secondary school curricula, training mathematics teachers in other countries, the psychology of learning mathematics, and even proposed curricula (Klein, 2003). The significance of the 1923 Report was to define and ultimately defend the purpose of mathematics in secondary education; it supported a more traditional approach to teaching, which was contrary to Kilpatrick’s report.

During this phase of education, mathematics was a subject of distress for government entities, schools, and other interested parties. One now very prominent and
influential group was also established in 1920: The National Council of Teachers of Mathematics (NCTM; Klein, 2003). The purpose of the NCTM is to assist in promoting the interests of mathematics in America, especially in the elementary and secondary fields by holding meetings for the presentation and discussion of papers by conducting investigations for the purpose of improving the teaching of mathematics, by the publication of papers, journals, books, and reports, thus to vitalize and coordinate the work of many local organizations of teachers of mathematics and to bring the interests of mathematics to the attention and consideration of the educational world. (NCTM Bylaws, 2014, p. 1)

The NCTM has been a leading organization in mathematics education since its establishment. It boasts a membership of approximately 60,000 mathematics education professionals and serves as an advocate for educational policies and access to lessons, regional conferences, an annual meeting, and a peer-reviewed journal, *The Journal for Research in Mathematics Education* (NCTM, 2015).

The 1930s saw educational groups advocating the critical ideas in progressivism from the 1920s, including the belief that education should focus on the whole child rather than on the content or teacher. This philosophy advocates that students should be examining ideas through active learning or experimentation (Kennedy, 1995). Although the 1923 Report garnered a lot of attention, Kilpatrick’s publication exerted more influence and led to the Activity Movement in the 1930s. The Activity Movement promoted the integration of subjects at the elementary level and disputed the separation of instruction of mathematics and other disciplines (Klein, 2003). The Activity
Movement was still going strong until the 1940s. Progressive education was alive and thriving for more than two decades.

Progressivism changed in the 1940s and was reconfigured and repackaged in the Life Adjustment Movement. At that time, the United States was embroiled in World War II, and criticism fell onto schools due to the lack of necessary arithmetic skills needed for basic bookkeeping and gunnery by the army recruits (Klein, 2003). Admiral Nimitz complained that the basic skills of the incoming military personnel should have been learned in public school; however, their mathematics were subpar and needed to be retaught (Garrett, 199; Klein, 2003). This scathing condemnation launched another undertaking in education—the Life Adjustment Movement—where schools refocused on preparing students for everyday life, such as consumer mathematics, taxes, and home budgeting, and not on more traditional mathematics courses, such as algebra, geometry, or trigonometry (Klein, 2003). This attitude fell in line with the current atmosphere of the time, resulting in unskilled or semiskilled workers for the military.

The discussion about mathematics education shifted from education professionals and mathematicians to the public after the Sputnik scare of 1957. History was changed when the Soviet Union successfully launched the first artificial satellite into space on October 4, 1957. Sputnik I was a 185-pound satellite that orbited the Earth every 98 minutes (Jolly, 2009). This launch set into motion new political and scientific developments that marked the start of the space race between the United States and the Soviet Union. The Soviet Union’s aeronautic feat sent a shockwave through the American public, prompting questions about technological superiority that led to anxiety over the thought of the potential ability of the Soviets to launch ballistic missiles that
could carry nuclear payloads to the United States. The Sputnik launch propelled the U.S. federal government into action. The creation of NASA was established through the National Aeronautics and Space Act, which Congress passed in 1958 (Garber, 2007). Sputnik, coupled with the ongoing criticism of the American education system, also led the government to pour funding into public education reforms at all levels. Hence, Congress also passed the National Defense Education Act in 1958 to focus training young STEM workers who could pull the United States to the forefront of STEM-related ventures (Jolly, 2009).

Not only did the government respond to Sputnik, but to other organizations devoted to and invested in mathematics education. The American Mathematical Society set up the School Mathematics Study Group in 1958 to develop a new mathematics curriculum for high schools. The NCTM also set up their committee, the Secondary School Curriculum Committee, which additionally came out with their own set of mathematical recommendations in 1959 (Klein, 2003).

All of these elements led the United States to shift back to a more formal and abstract line of teaching. This phase in mathematics education in the 1960s was called New Math—American Style or New Math for short (Fey, 1978). New Math-era projects were born from the collaboration of teachers, teacher educators, and mathematicians, and this change proved to be one of the most heated areas of debate in education. The curriculum in New Math was considered markedly formal; basic skill and application had little consideration (Klein, 2003). The primary emphasis of New Math was on pure mathematics versus mathematics sufficient for everyday life survival. New Math drew criticism from both mathematicians and educators. Mathematicians maintained that only
superficial aspects of their ideas were selected for use in the actual classrooms, and the teachers claimed that the mathematicians gave them an aggressively formal and too-advanced mathematics program that was not teachable to elementary or secondary students (Fey, 1978). These claims, coupled with inadequate training of educators and cosmetic changes in curricular materials by commercial texts, reflected a failure in the implementation strategies used to sell the reformation ideas. In 1973, concerns about the mathematics programs increased and subsequently resulted in Why Johnny Can’t Add: the Failure of the New Math, a bestseller book about American education (Fey, 1978).

As criticism against New Math grew, a public call for “back to basics” education arose in the early 1970s, emphasizing more traditional instructional methods (Fey, 1978). “Basic” usually refers to rote memorization, arithmetic facts, and manipulative algebra, the elements neglected in New Math. “Basic” in the past usually referred to daily life interactions, but this was not the case in this new paradigm shift, where “basic” referred to the steps leading to more complex mathematics that were overlooked in New Math. In the 1980s, two reports came out on the state of education in the United States: An Agenda for Action and A Nation at Risk (Gardner, 1983; Klein, 2003; NCTM, 1980). They embodied the two opposing viewpoints and recommendations for change that have been attributed to the rival factions of the Math Wars of the 1990s.

An Agenda for Action, published in 1980 by the NCTM as a response to the poor mathematical performance of U.S. students on studies, such as the National Assessment of Educational Progress report, recommended that problem-solving be the focus in mathematics education along with new pedagogical strategies (Klein, 2003; NCTM, 1980; Schoenfeld, 2004). Manipulatives were encouraged, where appropriate, to
demonstrate a concept or skill; the report also called for a “wider range of measures than conventional testing” (Klein, 2003). The publication had a list of eight recommendations, and each recommendation was followed by specific actions that would help with implementation.

*A Nation at Risk* (Gardner, 1983) was written by the members of the National Commission on Excellence in Education appointed by Terrell Bell, the U.S. Secretary of Education in 1983, and addressed a broad range of education issues that were focused on particular deficiencies in mathematics education and student assessment (Gardner, 1983; Klein, 2003; Schoenfeld, 2004). The report focused on what was happening in schools and gave recommendations. The Excellence in Education Commission was given specific issues to investigate in *A Nation at Risk* (Gardner, 1983). These included the following:

- assessing the quality of teaching and learning in both public and private educational institutions, ranging from K–12 to colleges to universities;
- comparing U.S. educational institutions with other advanced nations;
- investigating the association between college admissions requirements and student success in high school;
- determining which educational programs resulted in student success in college; and
- defining problems that needed to be faced and overcome if the United States wanted to be successful in its pursuit of excellence in education.

As *A Nation at Risk* captured the country’s attention, many states mobilized to compare their programs against the recommendations in the report. The initial statements in the release commanded a gut-wrenching response from readers:
Our Nation is at risk. Competitors are overtaking our once unchallenged preeminence in commerce, industry, science, and technological innovation throughout the world. This report is concerned with only one of the many causes and dimensions of the problem, but it is the one that undergirds American prosperity, security, and civility. We report to the American people that although we can take justifiable pride in what our schools and colleges have historically accomplished and contributed to the United States and the well-being of its people, the educational foundations of our society are presently being eroded by a rising tide of mediocrity that threatens our very future as a Nation and a people. What was unimaginable a generation ago has begun to occur--others are matching and surpassing our educational attainments.

If an unfriendly foreign power had attempted to impose on America the mediocre educational performance that exists today, we might well have viewed it as an act of war. As it stands, we have allowed this to happen to ourselves. We have even squandered the gains in student achievement made in the wake of the Sputnik challenge. Moreover, we have dismantled essential support systems which helped make those gains possible. We have, in effect, been committing an act of unthinking, unilateral educational disarmament. (Gardner, 1983)

The NSF funded many projects in the post-Sputnik era; however, due to the political backlash over an NSF-funded elementary school science and social science
curriculum called *Man: A Course of Study*, the NSF refused to take part in supporting anything that could have been seen as leading to a prospective national curriculum (Schoenfeld, 2004). The NSF’s refusals to play a commanding role led to an absence of leadership during this time of crisis.

The NCTM’s response was to re-create its *Agenda for Action* in the form of standards (Klein, 2003). In 1986, the NCTM established the Commission on Standards for School Mathematics (Schoenfeld, 2004). The 1989 NCTM standards, *Curriculum and Evaluation Standards for School Mathematics*, were developed and consisted of teaching “bands” that are comprised of grade levels. The focus has shifted from what was popular among teaching approaches to what students should be learning and at what grade level, giving tangible items to educators and administrators to focus on regardless of the teaching strategy. By 1997, most state governments had adopted mathematics standards that were closely aligned with the NCTM standards (Klein, 2003).

Another educational reform that arose in the late 1990s and is still in use in 2018 is commonly referred to as Common Core. Up to 2016, the CCSS (2010) was a progressive educational approach that aims to institute consistent educational standards across states. Theoretically, if a student transfers from Montana to Washington, there should be few discrepancies in what that student is learning. Many see CCSS as an attack on states’ rights to control their local education. Common Core standards were released in 2010 and examine what K–12 students should know at the end of each grade in the English language arts and mathematics (Porter, McMaken, Hwang, & Yang, 2011). Forty-two states, the District of Columbia, four territories (Guam, American Samoan Islands, U.S. Virgin Islands, and Northern Mariana Islands), and the Department of
Defense Education Activity have adopted the CCSS since their induction and until 2016 (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Recently, Common Core has come under criticism, prompting a report, *State Progress and Challenges in Implementing Common Core State Standards* (Kober & Rentmer, 2011), to be done through the Center on Education Policy. The Center on Education Policy surveyed the states that adopted CCSS (see Figure 2), and, of those who responded, they indicated that as part of their implementation strategy, most

![Figure 2](image-url)

*Figure 2.* The number of states adopting CCSS that plan to make various changes in policies and practices for K–12 education (Kober & Rentmer, 2011, p. 5).
programs, and teacher evaluation systems (Kober & Rentmer, 2011).

Common Core State Standards for Mathematics (CCSSM, 2010) has come under fire due to the change in instructional strategies that many teachers are unprepared to tackle. Professional development has become a focal point for stakeholders due to the mathematical content, and teaching standards are different from most previous state standards (Council of State School Officers, 2010). For this reason, instructional materials and the application of the new standards needed to adapt to fit the latest expectations (Bostic & Matney, 2013). In a follow-up study done by the Center of Education Policy (Rentner, 2013), researchers found that the majority of states involved in the research (a) agreed that the CCSSM are more rigorous than their previous state standards, (b) some already had curricula in place that were aligned with CCSS, and (c) most were already making changes for implementation (see Table 1). However, there have been some challenges identified with implementation.

Table 1. State challenges in implementing the CCSS (Rentner, 2013, p. 14).

<table>
<thead>
<tr>
<th>Potential challenge</th>
<th>Major challenge</th>
<th>Minor challenge</th>
<th>Not a challenge</th>
<th>Not an SEA activity</th>
<th>Not within SEA’s authority</th>
<th>Too soon to tell</th>
<th>Don’t know</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finding adequate resources to support all of the activities necessary for implementing the CCSS</td>
<td>22</td>
<td>12</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Developing educator evaluation systems that hold teachers and/or principals accountable for student mastery of the CCSS</td>
<td>21</td>
<td>11</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Identifying and/or developing the curriculum materials necessary to implement the CCSS</td>
<td>13</td>
<td>13</td>
<td>3</td>
<td>1</td>
<td>8</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table reads: Twenty-two CCSS-adopting states said it was a major challenge to find adequate resources to support all of the activities necessary to implement the CCSS, while 12 states indicated this was minor challenge. Three states did not consider finding adequate resources for CCSS implementation to be a challenge, while three other states said it was too soon to tell whether this would be a challenge.

This brief history of how mathematics education has been shaped over the last 90-plus years demonstrates the struggle between traditional (classical) and progressive
learning. What is considered essential or pertinent during the contemporary atmosphere of the nation shifts between content and knowledge (traditional) and constructivism (progressive). Student and teacher roles are very different in each approach. Within the classical and traditional ideology, students learn what the teacher teaches; they focus on skills, ideas, and factual learning. One of the strategies encompassed in progressive education is the idea that students discover what they learn; they serve as peer mentors. This progressive approach is reflected distinctly in cooperative learning that organizes the activities into academic and social learning experiences. Collaborative learning has been hailed as a tool with which to shift the educational paradigm from teaching to learning (Millis, 2012).

**Student Learning Elementary Mathematics**

**Standards**

The mathematical concepts at the elementary level are critical for future success and are of utmost importance. These concepts have been constructed in a “sequence of topics and performances,” which we know as the CCSSM (National Governors Association Center, 2010). The CCSSM outlines what students across the United States should know by the end of each completed grade level. One overarching goal of Common Core is to ensure that all students who graduate high school, irrespective of state, exit with the necessary skills and knowledge needed to succeed in either college, life, or a budding career (National Governors Association Center, 2010). Additionally, 84% of the states, the District of Columbia, four territories, and the Department of Defense Education Activity have adopted the standards that were launched in 2009.
The standards are broken into both grade levels and learning domains within mathematics. The major domains follow:

- Counting and Cardinality
- Operations and Algebraic Thinking
- Numbers and Operations in Base Ten
- Numbers and Operations – Fractions
- Measurement and Data
- Geometry
- Ratios and Proportional Relationships
- The Number System
- Expressions and Equations
- Functions
- Statistics and Probability

The standards in Common Core have been formulated through existing state standards that are considered the best, based on the knowledge and experience of teachers, based on experts in the field, and based on feedback from the public (National Governors Association Center, 2010).

CCSS (2010) have similar themes to those of other researchers; Wilson (2009) organized a set of building blocks that are the basis for all higher mathematics. He expressed these five building blocks—(a) numbers (understanding of different properties; i.e., commutative, associative, and distributive), (b) place value system (foundation for polynomials), (c) whole number operations (fluency with standard algorithms for future preparation), (d) fractions and decimals (incremental transition involving polynomials
and ratios), and (e) problem solving (word problems essential for critical thinking)—and claimed that all are the foundation on which algebra is dependent.

Common Core (Council of Chief State School Officers, 2010) and Wilson (2009) used building blocks similar to the NCTM’s *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (2006). The NCTM (2006) discussed what the team determined to be the foundations that need to be addressed for future success in mathematics, such as elements in Numbers and Operations: “students extend their knowledge of place value to numbers up to 10,000 in various context” (p. 16).

The NCTM, a professional organization that is widely considered to be the foremost authority in mathematics education within the United States, developed the Curriculum Focal Points in addition to their *Principles and Standards* (NCTM, 2000). Although the NCTM is considered the leading nongovernmental establishment with regard to mathematics education, it is a nonprofit educational association that makes recommendations for what students should be learning. This institution also publishes current research on mathematics education and related items in its journal. Although the NCTM does a tremendous amount of work, it is not responsible for the legislative recommendations to adopt mathematical standards.

Standards have been a part of the mathematical landscape since the introduction of the 1989 NCTM Standards (Klein, 2003). The standards were to serve as guide for educators to verify that they are in line with what other schools and states are teaching. They were also designed to make sure that students will be ready for abstract thinking that occurs in upper middle school and high school. At the elementary level of K–12, students are learning the fundamentals of mathematics, which is deemed by many (Bell,
1993; Canobi, 2005; Mason & Spence, 1999) to be a critical stage of development for future success. Arithmetic in K–8 and algebra in high school can separate people from mathematically related fields of study (Booth, 1988; Davis, 1995). In 2007, the Center for the Study of Mathematics Curriculum released details on each state’s—including the Department of Defense Education Activity and the District of Columbia’s—mathematical graduation requirements for public high schools (see Table 2).

Table 2. Years of High School Mathematics Required for Graduation (Reys, Dingman, Nevels & Teuscher, 2007, p. 4).

<table>
<thead>
<tr>
<th>Number of States</th>
<th>High School Mathematics Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>At least 3 years</td>
</tr>
<tr>
<td>11</td>
<td>4 years</td>
</tr>
<tr>
<td>7</td>
<td>2 years</td>
</tr>
<tr>
<td>5</td>
<td>Varies by Diploma</td>
</tr>
<tr>
<td>5</td>
<td>Specified at Local Level</td>
</tr>
</tbody>
</table>

23 states required their students to take an Algebra I course or Integrated Mathematics I, 10 states are in the process of increasing their requirements for graduation. When looking beyond high school, many colleges require students to take a mathematics course to fulfill a quantum-reasoning element of their degree, regardless of the discipline.

To facilitate a stronger competency in mathematics at the high school level and beyond, students need to build robust foundational skills at the elementary grade level and make the transition to algebra more seamless (Carraher, Schliemann, Brizuela, & Earnest, 2006). Researchers have found that there is a common foundation connecting arithmetic and algebra (Bass, 1998; Carraher, Schliemann, & Brizuela, 2000; Carraher, Schliemann, Brizuela, & Earnest, 2006; Schoenfeld, 2004).
These foundational skills can be punctuated by keystone concepts that help promote understanding of higher-order abstraction that comes in later grades (Brown & Quinn, 2006). The keystone concepts focused on in this study follow:

1. Ratio and proportions
2. Fractions
3. Exponents
4. Graphing

These concepts were selected due to their complexity of comprehension and their ability to halt mathematical progression when a lack of understanding prevails in students’ knowledge.

Selected Concepts

Rational Numbers

“Rational-number concepts are among the most complex and important mathematical ideas children encounter during their presecondary school years” (Behr, Lesh, Post, & Silver, 1983, p. 91). A rational number is any number that can be written as a ratio or, in other words, as a fraction of two integers where the denominator cannot be zero (Rosen, 2007). The importance of rational numbers has been said to take many forms, from the practical (problem-solving) to the psychological (developing mental structures) to simply mathematical (providing foundational materials for algebraic thinking) (Behr, Lesh, Post, & Silver, 1983). Students who struggle with a basic understanding of rational numbers in elementary school tend to make mistakes when more sophisticated fractional reasoning is required. Rational numbers are introduced as early as third grade as fractions and are considered one of the more difficult mathematical
concepts that middle school and junior high students encounter (Bezuk & Cramer, 1989; Common Core State Standards Initiative, 2010). The National Assessment of Education Progress has shown that students struggled when addressing rational number concepts and indicated “that most 13- and 17-year olds could successfully add fractions with like denominators, but only one-third of the 13-year olds and two-thirds of the 17-year olds could correctly add 1/2 + 1/3” (Behr, Lesh, Post, & Silver, 1983, p. 91).

**Fractions.** Understanding fractions is critical for students to compute basic arithmetic calculations for comprehending algebra (Wu, 2001). Until recently, algebra was thought to be distinct from arithmetic in the K–8 curricula. A common implementation in mathematics was to teach arithmetic first and separately, then move to the more complex elements of algebra (Carraher, Schliemann, & Brizuela, 2000). This abrupt transition from known numbers and logical operations has been changed to using unknown variables such as $x$, $y$, and $z$, as forcing students to think abstractly before they are given the opportunity to have a conceptual understanding of variables can be an unexpected transition for most children. Carraher, Schliemann, and Brizuela (2000) suggested that students’ difficulties in higher mathematics have been emphasized by the strict separation that occurs between arithmetic and algebra, and they proposed that algebra is integrated, where possible, from the start versus easing students through a transition. Kieran (1988) found that students who confused positive and negative numbers after a year of algebra extended their errors into the division of integers, indicating a lack of comprehension of fractions.

Fractions are also where students get their first introduction to the symbolism that is structural for their understanding of algebraic applications. For example, students use
symbolic algorithms to find equivalent fractions \( \frac{2}{7} = \frac{x}{14} \) however, if a student does not have a firm understanding of fractions, the algorithm has a low probability of success (Bright, Behr, Post, & Wachsmuth, 1988). Wu (2001) also stated that fractions are an important gatekeeper for an essential understanding of algebra. For example, look at how adding fractions \( \left( \frac{2}{7} + \frac{2}{7} = \frac{x}{7} \right) \) and, more substantially, how cross-multiplication

\[
\left( \frac{2}{7} = \frac{x}{14} \right) = (2 \cdot 14) = (7x)
\]

can be used to help students acquire the symbolic computational skills needed to be successful in algebra.

The inability to do basic operations with fractions causes error patterns that surface in the progression of learning algebra (Brown & Quinn, 2007). Elementary algebra relies on creating new constructs that are based on fraction concepts. In algebra, students learn how to combine like terms that are analogous to adding and subtracting fractions with the same or different denominators. Recognizing that a term that is written as \( 6xy^2 \) is the same as \( 3(2y^2x) \) is similar to finding equivalent fractions, such as how \( \frac{1}{3} \) is the same as \( \frac{2}{3} \left( \frac{1}{3} \right) = \frac{2}{6} \). Fractions evolve as students begin to learn about algebra. Even their name changes, as fractions are referred to as rational numbers when the switch to algebra from arithmetic occurs. To solve equations, such as \( 4x = 5 \), with integer coefficients, students are introduced to rational numbers in the form \( \frac{p}{q} \) with \( p \) and \( q(\neq 0) \) integers (Bass, 1998). Rational numbers become a gateway to basic algebra, which in turn is a gateway to higher mathematics. If students struggle with foundational materials, such as fractions, the effort to understand more advanced concepts becomes a
substantial roadblock to furthering students’ mathematical ability.

**Ratio and Proportions.** Rational numbers are also the foundational material behind ratio and proportions (Brown & Quinn, 2007). As with fractions, ratio and proportions concepts are also introduced at the middle school/junior high school level beginning at the sixth grade (Common Core Standards Initiative, 2010). A ratio is an association that communicates the understanding of a scalable relationship; therefore, it is more accurately deemed as a comparative index instead of a number (Behr, Lesh, Post, & Silver, 1983). A ratio compares two different quantities; it is a specific application of fractions. “The idea of ratio is at the heart of measurement” (Thompson & Saldanha, 2003, p, 15). For example, there is a class of 30 students that consists of 16 females and 14 males; the ratio of females to males is 16:14 or 16/14. Through simplification, we get 8:7 or 8/7.

Ratio and proportions have been considered by many to be a crucial element in elementary education. Lesh, Post, and Behr (1988) have stated they believe proportional reasoning to be such a critical element of mathematics that they consider it to be the capstone of elementary school arithmetic and a watershed concept of higher mathematics. Lesh, Post, and Behr state that it is such an influential concept that it bears much of the foundational support to many of the mathematical concepts that follow.

Proportional reasoning is an important concept as it is where students conceptualize measured quantities; by equating two ratios, it can be used in problem-solving settings when comparing magnitudes (Behr, Lesh, Post, & Silver, 1983). A proportion is a relationship between two quantities such that if you increase or decrease one amount by a factor $m$, then the other number must either increase or decrease,
respectively by the factor $m$ to maintain the relationship (Thompson & Saldanha, 2003). Proportionality consists of different types of reasoning tasks at different levels. At the middle school level, there are “trouble spots” in the curriculum where students often encounter problems such as (equivalent) fractions, long division, place value and percent, measurement conversion, and ratios and rates (Lesh, Post, & Behr, 1988). If students have difficulty in their proportional reasoning, they apply those weak skills in their attempts to solve problems at higher levels of mathematics, thus making the comprehension of the upper-level mathematics more difficult, not necessarily because the material is too complicated to understand, but because the students are lacking foundational skills. Difficulty in proportional reasoning confuses all the intricacies of the various problems that need unpacking. Under this rationale, proportional rationale has been hailed as a keystone concept in a student's mathematical development (Lesh, Post, & Behr, 1988).

“Two central themes are at the core of this new conception of algebraic thinking: (a) making generalizations and (b) using symbols to represent mathematical ideas and to represent and solve problems” (Carpenter & Levi, 2000, p. 5). Student interviews exhibited their understanding of the breadth of proportional reasoning far outweighed their symbolic competence (Lamon, 1993). Learning to make generalizations about mathematical concepts before symbolism has been an important feature promoted by Zoltan Dienes (Bart, 1970; Cloutheir, 2010; Hirstein, 2007) as well as Jerome Bruner (Bruner, 2009; English, 2008) in the 1960s. Allowing students to understand the fundamental concepts of proportional reasoning before introducing the symbolism that leads directly to algebraic thinking can help them understand statements as they increase
in complexity. For example: There are two sodas for every student. If $y$ is the measure of one quantity and $x$ is the measure of the other, and they are related proportionally by a factor of 2, then $y = 2x$ and as mathematical notation increases to functions $f(x) = 2x$.

**Percent.** Percents arise most often in statistical messages and put a demand on mathematical literacy. Percents express part or whole relationships and are the most common rational number used in media to convey statistical information (Gal, 2002). To understand percentages, learners need to understand rational numbers. Students start to encounter percents in the sixth grade and into the seventh grade (National Governors Association Center, 2010). To fully understand percentages, learners need to be acquainted with and comprehend rational numbers and ratios.

**Misconceptions of Rational Numbers**

Frustration with rational numbers, ratios, proportions, and percents start to cumulate when learners begin forming misconceptions about the concepts. Children start to form a framework of references from their everyday world (Vamvakoussi & Vosniadou, 2010). Rational numbers are an area in which students have serious issues learning about fractions (Ni & Zhou, 2005). With the early grades of mathematics, there is a traditional path of learning where students can make generalizations from natural numbers; however, the features that occur with natural numbers cannot be carried over to rational numbers (Durkin & Rittle-Johnson, 2015; Kieren, 1993, p. 319; McMullen, Laakkonen, Hannula-Sormunen, & Lehtinen, 2015; Steffe & Olive, 2009, p. 2).

Natural numbers are the simplest set of numbers to work with. Their magnitudes are easy to understand, they can be illustrated by one term, and their progression to the next term is considered discrete. On the other hand, rational numbers are not constrained
to the same set of characteristics (McMullen et al., 2015; Merenluoto & Lehtinen, 2004; Ni & Zhou, 2005). The understanding of natural numbers, or later with whole numbers that students learn in elementary school, is that numbers are represented discretely, and this concept hampers the formation of the rational number concept of ordered, continuous representation (Post, Cramer, Behr, Lesh, & Harel, 1993; Charalambous & Pitta-Pantazi, 2007; Ni & Zhou, 2005). Rational numbers representation is boundless with regard to the number of terms used to correspond to a fractional amount. In addition to the increase in terms, the magnitude can also be symbolized in both fractional and decimal form. For example, there are endless ways to represent:

\[
\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{25}{100} = 0.25 = 0.250 = ... \]

This fundamental change on numerical representation is often a difficult obstacle for learners to overcome (Durkin & Rittle-Johnson, 2015; Vamvakoussi & Vosniadou, 2010).

When teaching and learning rational numbers, there is a dependency to connect previous knowledge of whole and natural number reasoning in a manner that is viewing the magnitude as a ratio of two terms (McMullen et al., 2015). The prior knowledge acts as a hindrance to the mathematical understanding of the magnitudes of rational numbers. For example, in natural number reasoning, seven is larger than six as it is its successor:

\[6 < 7.\]

But that reasoning does not transfer to rational numbers where \( \frac{1}{7} \) is not the successor of \( \frac{1}{6} \). They both represent a fraction of a whole, and one piece out of six is larger than one piece out of seven: \( \frac{1}{6} > \frac{1}{7} \). McMullen et al. (2015) described these two main conceptual distinctions regarding density as “a) the sequencing of numbers and b) the presence of a successor number” (p. 15). Students start to develop these
misconceptions as synthetic concepts, a representative of intermediate knowledge that creates a connection between their initial take of the concept and the intended exact perspective, that perpetuate through their academic careers (Durkin & Rittle-Johnson, 2015; Vamvakoussi & Vosniadou, 2010).

Some errors have been identified as misapplied multiplication to mixed numbers where students simplified $\frac{2}{3}$ as three times two-thirds, $3 \cdot \frac{2}{3} = 2$ and simplified it by canceling the threes (Cangelosi, Madrid, Cooper, Olson, & Hartter, 2013). Error patterns have been discovered across all significant operations (add, subtract, multiply, and divide) with mixed numbers and fractions in both elementary students and elementary teachers (Brown & Quinn, 2006; Newton, 2008).

It has been identified from pilot investigations that many of the misconceptions that have been acknowledged in young students were also widespread among teachers (Post, Harel, Behr, & Lesh, 1988). Moss and Case (1999) suggested that some of the complications that arise with students' comprehension of rational number concepts are related to the teaching practice of explicitly overemphasizing rules over meaning.

**Pedagogy Relate to Teaching of Rational Numbers**

Educators have a broad spectrum of roles in the classroom, from a teacher of large groups to small, individual tutor, assessor, and friend. In these roles, teachers have to relate to students in certain teaching-related tasks (Post, Harel, Behr, & Lesh, 1988). Instruction showed a variety of different methods that can help students’ connection representations of rational numbers to verbal understanding as well as symbolism. Research done by the Rational Number Project (RNP) (2002) validate the use of a curriculum that includes multiple representations, specifically manipulative models, as a
method of developing students’ conceptual knowledge of mathematical ideas (Post, Behr, Lesh, & Wachsmuth, 1985; Post, Harel, Behr, & Lesh, 1988).

The RNP was funded by the National Science Foundation from 1979 to 2002 and was dedicated to the research and understanding of teaching and learning rational numbers. This cooperative, multi-university research project produced over 90 publications that have helped teachers and researchers uncover best practices and areas of concern within the rational number domain (Rational Number Project, 2002).

Cramer, Behr, Post, & Bezuk (1997) and Tzur (1999) have cited that there are many factors involved in teaching rational numbers that include using multiple physical models as well as other representations and being able to traverse between them and teacher involvement in the learning process should be a focus alongside children’s interactions with each other; in other words, cooperative learning. Subconstructs that lie under the umbrella of rational numbers identified by Kieren (1976), such as part-whole, quotient ratio number, operator, and measure, also need to be examined individually as well as their relationship to one another (Behr, Lesh, Post, & Silver, 1983; Baker, Czarnocha, Dias, Doyle, & Kennis, 2012). Teachers have to use all of the elements involved in rational numbers to help students move their learning trajectories from an operational understanding (procedural and algorithmic) to a more fundamental understanding (conceptual and abstract).

For these reasons, it is paramount for teachers to have a firm understanding of rational numbers. Teachers have broader implications than just the transmission of knowledge; they also convey attitudes and values regarding mathematics (Post, Harel, Behr, & Lesh, 1988). Students view their teachers as archetypes of proper mathematical
solving and, as such, role models. It is vital for them to display the necessary cognitive abilities to ensure success in their classrooms (Ernest, 1989).

**Exponents**

Ratio and proportional reasoning make their official appearance in the CCSSM in sixth grade and continue into seventh grade, where mathematics progresses into expressions and equations and the standards highlight students’ ability to use properties of operations to generate equivalent equations and solve real-world mathematics problems (Common Core State Initiative, 2010). Expressions and equations continue into eighth grade where the emphasis is placed on exponents and linear equations (Common Core State Initiative, 2010). Hill, Shilling, and Ball (2004) state that students learn about exponential notation in late elementary grades into middle school. As with proportional reasoning, exponents also serve to help students understand measured quantities.

Exponents are the next stepping-stone in mathematics that allow learners to engage more deeply in the concept of repeated multiplication (Confrey & Smith, 1995). They are a critical element in mathematics due to their significance in modeling population growth, radioactive decay, compound interest, earthquake magnitudes, construction, etc.

Exponential functions are another mathematical juncture where students see the complementary effects of representation and abstraction. Exponential functions have multiple forms of representation that include tree diagrams, embedded figures, and logarithmic spirals as well as more common forms of graphs, tables, and equations (Confrey & Smith, 1995). Dreyfus (1991) discusses the learning process of exponents through four stages: 1) being able to use a single representation, 2) using multiple representations in parallel, 3) making connections among parallel representations, and 4)
being able to integrate representations and transfer between various representations. An example with regard to exponents would be finding the population of grizzly bears after nine years with a growth rate of 1% per year given an initial population of 197. Stage 1 would consist of building on the concept of multiplication, wherein students can set up a discrete dynamical system and perform repeated application of a relatively simple equation, such as \( P(t) = P(t - 1) + 0.01P(t - 1) \), a population at time \( t \) = population of the previous time step + 1% growth per year, the population of the previous time step. Students can repeat this equation nine times to find the answer of 215.46 bears, rounded to a conservative 215. Teachers can use different visual representations in this step that include tables and graphs to illustrate what is happening algebraically. Stage 2 would take into consideration a different representation, for example using the formula

\[
P(t) = (1 + r)^t P(0)
\]

derived from the original discrete dynamical system formula (Stage 3).

\[
\begin{align*}
P(t) &= P(t - 1) + rP(t - 1) \\
P(t) &= 1 \cdot P(t - 1) + r \cdot P(t - 1) \\
P(t) &= (1 + r)P(t - 1) \\
P(1) &= (1 + r)P(0) \\
P(2) &= (1 + r)P(1) \\
P(2) &= (1 + r)(1 + r)P(0) \\
P(2) &= (1 + r)^2 P(0) \\
\cdots \\
P(t) &= (1 + r)^t P(0)
\end{align*}
\]
After calculating $P(9) = (1 + 0.01)^{9}197$ students see they also receive an answer of 215.46 bears, the same as the discrete dynamical system calculation. Working through the derivation is critical for students to be able to make links between the parallel representations; without it, the fourth stage (integrating and flexible switching) cannot occur seamlessly.

Another parallel representation that occurs within exponential notation is thinking in function form, substituting $f(x)$ for $y$. This step is a fundamental step in algebraic thinking and reasoning. Functional thinking is making inferences about relationships between two or more varying quantities to make generalizations. Representation and explaining the relationships can take many forms, such as using natural language, tables, graphs, and variable notation, and employing reasoning to understand and calculate functional activities (Blanton, Stephens, Knuth, Gardiner, Isler, & Kim, 2015; Gardiner & Sawrey, 2016). Presenting mathematics from algebraic functions usually progresses to plotting ordered pairs on a graph or from a data table to a graph (Leinhardt, Zaslavsky, & Stein, 1990). Functional thinking and reasoning have been considered a component of high school curricula as students enter into algebra. However, research has shown that very young learners have the capacity for functional thinking (Blanton & Kaput, 2004; 2011; Blanton et al., 2015; Gardiner & Sawrey, 2016; Leinhardt, Zaslavsky, & Stein 1990). Functional thinking requires the development of representational infrastructure and scaffolding of various forms (Blanton & Kaput, 2004).

The representational form is important at this stage of development, as there is not a sharp distinction between elementary and advanced mathematical thinking (Dreyfus, 1991).
In the same way that the ten basic skills suggested by the National Council of Supervisors of Mathematics in 1978 (problem-solving, estimation, approximation, graphical analysis, etc.) cannot be taught effectively in isolation from one another, the teaching act cannot be separated from the mathematical content which it is intended to convey nor from the psychological overtones which human beings tend to impose on cognitive schema. (Post, Harel, Behr & Lesh, 1988, p. 181-182)

Multiple representations, such as function machines, graphs, ordered pairs, and others, promote meaningful connections between key concepts (Stein, Baxter, & Leinhardt, 1990). One process to help representations materialize is visualization. Kaput (1987) theorized generating mental representations relies on the library of representational systems that can be realized by the individual.

**Misconceptions of Exponents**

Misconceptions could develop from overgeneralizing an essentially correct conception or some interference from common knowledge (Leinhardt, Zaslavsky, & Stein, 1990). More often than not, exponential notation is misconstrued as a simple multiplication of the base and exponent (Lim, 2010). For example, $3^2 = 3 \cdot 2 = 6$. If this small error made in elementary school is not corrected and explained, it could have negative long-term effects such as when students are in algebra and trying to simplify expressions such as $3a^2 + 4a^2$. Most students correctly identify the answer as $7a^2$ but then write their final answer as $14a$ (Lim, 2010). There are several rules when dealing with exponents: 1. Zero-exponent rule $a^0 = 1$; 2. Power rule ($a^m)^n$; 3. Negative Exponent Rule $a^{-n} = \frac{1}{a^n}$; 4. Product Rule $a^m \cdot a^n = a^{m+n}$; and 5. Quotient Rule $\frac{a^m}{a^n} = ...
When tested, students have the most difficulty when choosing the correct rule followed by calculation errors (Lepp, 2011).

**Pedagogy Related to Teaching Exponents**

Pedagogy is a crucial part of student development with regard to understanding and utilizing exponential functions. The terminology and algebraic rules that come second nature to teachers may be confusing to students (Lim, 2010). Students’ mathematical journeys start with the fundamental concepts of addition and subtraction of part/whole numbers, and teachers can build on these ideas by helping students move into more complex notations, such as abstraction (Leinhardt, Zaslavsky, & Stein 1990). The goal of teaching is to expand on the students’ knowledge of learned concepts. How a teacher instructs students on exponents, functions, and representation is a result of the domain as well as their knowledge of how student understanding develops in that field (Leinhardt, Zaslavsky, & Stein 1990). Many students have difficulty learning the rules of exponents without full representation, for example understanding why the zero-exponent rule $a^0 = 1$ without explaining $\frac{a^2}{a^2} = 1$ and $\frac{a^2}{a^2} = a^{2-2} = a^0$. Additionally, multiple representations are also applicable such as:

\[
\begin{align*}
2^4 &= 2 \cdot 2 \cdot 2 \cdot 2 = 16 & 2^4 &= 16 \\
2^3 &= 2 \cdot 2 \cdot 2 = 8 & 2^3 &= 16 \div 2 = 8 \\
2^2 &= 2 \cdot 2 = 4 & 2^2 &= 8 \div 2 = 4 \\
2^1 &= 2 & 2^1 &= 4 \div 2 = 2 \\
2^0 &= 1 & 2^0 &= 2 \div 2 = 1
\end{align*}
\]

There needs to be a proper process of representation of abstraction. In other words, there must be a meaning associated with notation before a symbol should be used (Dreyfus,
1991). More description of the pedagogical knowledge is located in teaching rational numbers and is included in the session on PCK.

**Theoretical Framework of Knowledge for Mathematics**

**Brief History of Pedagogical Content Knowledge**

Adequate preparation of highly qualified educators is a subject of concern not only in the United States, but throughout every country with a mission of educating children. Preparing teachers through teacher education programs can vary through institutions as well as countries; however, research has been conducted on identifying professional competencies that teachers must encompass, including cognitive abilities with regard to mathematics (Schmidt, Blömeke, Tato, Hsieh, Cogan, Houang, & Schwille, 2011). There is a movement to change teaching from what Freire (1972) called a narrative characteristic approach, where the subject (the teacher) narrates to the listening objects (the students), to a more engaging dialogue that communicates the depth of conceptual material. A shift of knowledge of the teacher is focused on content and more recently to pedagogy (Shulman, 1986). To accomplish deeper understanding by the students, a teacher must also have a sound comprehension of the subject matter and pedagogy surrounding the subject.

Shulman (1986) first introduced this aspect of organizing information with regard to teaching as three distinct categories: a) subject matter content knowledge, b) PCK, and c) curricular knowledge. He described content knowledge, as more than just knowing facts or concepts of a particular subject, it is essential for teachers to be able to define the accepted details of the subject matter at hand and be able to explain and justify the theories behind those ideas. Also, it is important to communicate why the propositions
are essential within the field of study and beyond, both in application and theory.

Shulman (1986) went on to explain PCK as a dimension of subject matter but specifically for teaching. In short, PCK is where content and pedagogy intersect and blend for a better comprehension of how features of subject matter are ordered, modified, and presented for instructional purposes (See Figure 3).

![Pedagogical Content Knowledge Diagram](image)

*Figure 3. Pedagogical content knowledge (PCK) (Mishar & Koehler, 2006, p. 1022).*

To engage PCK, educators should use the most useful types of representations, illustrations, analogies, examples, explanations, and demonstrations that make the subject matter understandable to others. Shulman (1986) affirms that PCK also encompasses the ability to understand within a discipline, what makes the learning process manageable or complicated, and what prior knowledge students of varying ages bring with them to the learning process. Information on prior knowledge is important to understanding what teaching strategies would be most effective for the learners. This idea has bloomed into what is commonly known as cognitively guided instruction (CGI), where students are understood to have some prior knowledge on subjects and are not considered blank slates upon entering the classroom (Bowman, 2003; Carpenter, Fennema, & Franke, 1996). CGI was developed through a research project in the early 1990s by Thomas Carpenter and Elisabeth Fennema (Bussinger-Ste, 2009). The third component has not gotten much
fanfare with regard to research and curricular knowledge. Curricular knowledge is associated with curriculum and materials from which educators pull information to educate students. This not only includes additional texts but alternative instructional materials, such as software, visual aids, films, laboratory demonstrations, programs, and others (Shulman, 1986).

Dr. Deborah Ball, a professor and researcher, as well as many other researchers, such as Grossman (1990), Marks (1990), Cochran, DeRuiter, and King (1993), van Driel, Verloop, and De Vos (1998), and Ma (1999) have done much in the field of PCK to further define and classify the aspects of the theory. Ball (1988) utilized theories of Shulman (1986) in her dissertation on the importance of assessing teacher’s PCK when teaching mathematics (Marks, 1990; McCray, 2008). This movement into researching and defining PCK began the issue of multiple definitions across multiple publications that lacked consistency.

Van Driel et al. (1998) constructed a table to survey the landscape of how different researchers viewed PCK (see Table 3). There is a slight departure from the Knowledge components in different conceptualizations of PCK (source p. 268). classification of PCK by Cochran, DeRuiter, and King (1993); they renamed PCK as pedagogical content knowing (PCKg) to address the qualities of knowledge development. For the purposes of this study, researchers who have examined the multiple facets of PCK with regard to mathematics will be reviewed.
Pedagogical Content Knowledge

Shulman’s theory on PCK and subject matter knowledge has been extended into many other facets within those two domains. Ball et al., (2005) go on to describe and model the different strands that are encompassed in PCK and subject matter knowledge, specifically with regard to mathematics. Figure 4 depicts the Hill et al. (2008) model of the different aspects.

**Figure 4.** Model of subject matter knowledge and pedagogical content knowledge as shared in Hill et al. (2008, p. 377).

Within-subject matter knowledge there exists:

**CCK** – Common content knowledge: “...knowledge that is used in the work of teaching in ways in common with how it is used in many other professions or occupations that also use mathematics” (Hill et al., 2008). “The mathematical
knowledge and skill expected of any well-educated adult” (Ball et al., 2005).

**SCK** – Specialized content knowledge: “… the mathematical knowledge that allows teachers to engage in particular teaching tasks, including how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems” (Hill et al., 2008). “The mathematical knowledge and skill needed by teachers in their work and beyond that expected of any well-educated adult” (Ball et al., 2005).

**Knowledge at the mathematical horizon** – “Horizon knowledge is an awareness of how mathematical topics are related over the span of mathematics included in the curriculum” (Ball, Thames & Phelps, 2008, p. 403).

Within PCK there exists:

- **KCS** – Knowledge of content and students: “…focused on teachers’ understanding of how students learn particular content” (Hill et al., 2008). “The knowledge that combines knowledge of content and students” (Ball et al., 2005).

- **KCT** – Knowledge of content and teaching – “The knowledge that combines knowledge of content and teaching” (Ball et al., 2005).

**Knowledge of curriculum** – The materials “…from which the teacher draws those tools of teaching that present or exemplify particular content and remediate or evaluate the adequacy of student accomplishments” (Shulman, 1986).

The difference between the two sides are on one; it examines the knowledge of the representation of the subject matter and the other is understanding student conceptions and what type of learning difficulties can occur. Each side of the oval has
two main compartments that are the primary focus with regard to improving the mathematical ability of teachers, which are CCK and SCK under subject matter knowledge and KCS and KCT under pedagogical content knowledge. With these different dimensions, it may be difficult to distinguish one from the other, even with the given definitions. For example, the difference between CCK and SCK would be that a teacher with a sound understanding of CCK would be able to recognize when a student answers a question incorrectly, be able to identify an incorrect definition in a textbook, be able to use mathematical notation correctly, and be able to do the work they assign their students (Ball et al., 2005; Ball, Thames, & Phelps, 2008). A teacher with a solid background of SCK would be able to not only recognize student errors but also be able to examine and assess those errors, be specific about mathematical language, and able to give accurate justifications and use numerical illustration (Ball et al., 2005). When examining the PCK side of the oval, a teacher with a significant understanding of KCS would be able to predict students’ errors and general misunderstandings, be able to infer incomplete answers, and be able to envision how students will respond to specific tasks (Ball et al., 2005). An educator with a quality comprehension of KCT would be able to sequence the lessons for teaching, be able to assess the benefit or disadvantage of varying representations and be able to respond and evaluate students’ unusual tactics to solving problems (Ball et al., 2005).

Mathematical Knowledge for Teaching

“There is growing recognition that mathematical knowledge alone does not guarantee better teaching and attempts are being made to define the various forms of knowledge needed for teaching” (Tirosh, 1999). In the field of mathematics teacher
education, researchers have been increasing the scope of what PCK entails through developing detailed descriptions of the different conceptualizations of what knowledge is needed for teaching mathematics (Silverman & Thompson, 2008). Ball (1990) and Thompson and Thompson (1996) report that to teach for comprehension of concepts, educators must also possess mathematical knowledge for teaching, which combines the mathematical knowledge that is usual for individuals working in various professions and the mathematical knowledge that is specific to teaching (Hill et al., 2008). Mathematical knowledge for teaching reflects on the quality of instruction as well as content. Teachers’ understanding of mathematical content is crucial, and often the broad field of mathematical content knowledge is often spread across the different segments, such as CCK, SCK, and KCS due to the subtlety of the lines between the types of knowledge (Ball, Thames, & Phelps, 2008).

mathematical content knowledge for teaching as being composed of two key elements: “common” knowledge of mathematics that any well-educated adult should have and mathematical knowledge that is “specialized” to the work of teaching and that only teachers need know. (Ball, Hill, & Bass, 2005, p. 22)

All teachers should be proficient with CCK, the ability to perform the work they assign their students; meaning, instructors should not waste time struggling to answer questions or complete exercises. The second piece, SCK, is the mathematical knowledge that is only needed for teaching purposes. Table 4 demonstrates the traits that entailed this piece.
Table 4. Mathematical Tasks of Teaching. Adapted from Ball, Thames, and Phelps (2008, p. 400).

<table>
<thead>
<tr>
<th>Mathematical Tasks of Teaching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Presenting mathematical ideas</td>
</tr>
<tr>
<td>Responding to students’ “why” questions</td>
</tr>
<tr>
<td>Finding an example to make a specific mathematical point</td>
</tr>
<tr>
<td>Recognizing what is involved in using a particular representation</td>
</tr>
<tr>
<td>Linking representations to underlying ideas and other representations</td>
</tr>
<tr>
<td>Connecting a topic being taught to topics from prior or future years</td>
</tr>
<tr>
<td>Explaining mathematical goals and purposes to parents</td>
</tr>
<tr>
<td>Appraising and adapting the mathematical content of textbooks</td>
</tr>
<tr>
<td>Modifying tasks to be either easier or harder</td>
</tr>
<tr>
<td>Evaluating the plausibility of students’ claims (often quickly)</td>
</tr>
<tr>
<td>Giving or evaluating mathematical explanations</td>
</tr>
<tr>
<td>Choosing and developing useable definitions</td>
</tr>
<tr>
<td>Using mathematical notation and language and critiquing its use</td>
</tr>
<tr>
<td>Asking productive mathematical questions</td>
</tr>
<tr>
<td>Selecting representations for particular purposes</td>
</tr>
<tr>
<td>Inspecting equivalencies</td>
</tr>
</tbody>
</table>

It is clear that elementary teachers need a diverse background in mathematics, as they are the foundational material that fortifies students’ knowledge moving forward. However, their understanding, too often, is marred by mistakes or a lack of comprehension (Post, Harel, Behr, & Lesh, 1988). It is not good enough to just understand the procedures for mathematics; it is vitally important to understand a broad comprehension of mathematical teaching tools.

A wide net of teaching practices has been a theme throughout research since Shulman introduced the concept. Researchers Fennema and Franke (1992) assert that mathematical knowledge for teaching is comprised of four elements: knowledge of the content, knowledge of pedagogy, knowledge of students’ perceptions, and the teachers’ viewpoint. Their conceptualization of teaching mathematics effectively is that the
knowledge is interactive, meaning that in any given situation, the knowledge of the content is connected to pedagogy and student comprehension combined with beliefs that generate a unique set of teaching practices. Central to theories of Fennema and Franke (1992), as well as other researchers such as Ball, Thames, and Phelps (2008) and Grossman (1990) to name a few, is that teachers must have a firm understanding of the content to be able to apply pedagogical knowledge, knowledge of learners, their beliefs, connections to other disciplines, and multiple representations. Many college students majoring in elementary education are unaware of complexities outside of knowing the basics of arithmetic that surround mathematics.

**Preservice Elementary Education Teachers**

Although many elementary education majors are seeking positions in the early grade levels, grades kindergarten through fifth grade, they could be entering into programs that prepare them to teach at any level between kindergarten and eighth grade. The knowledge from fifth grade to eighth grade leaves a critical gap of knowledge that preservice educators do not feel particularly comfortable with: the prealgebra stage of mathematics (Kelly & Tomhave, 1985). There are programs that train teachers for the K–5 level, however, in the last few years there has been a flood of trained K–5 elementary school teachers into the job market and a shortage in mathematics, science, and special education teachers (Beck, 2013).

Mathematics had become a gatekeeper to many different career opportunities. When a student is poorly prepared for college-level mathematics, they often have to take remedial courses to be considered *college ready* (Bryk & Treisman, 2010). This reality can be tough for elementary teachers to see the association between what they need to
teach and how they show it to their students who might eventually go into fields such as medicine, engineering, science, or architecture (Wilson, 2009). If an elementary teacher’s conceptual structures of mathematics are disjointed facts and formulas, their lessons will, in turn, likely also be disconnected facts and methods (Carpenter, Fennema, Peterson, & Carey, 1988; Post, Harel, Behr, & Lesh, 1988; Silverman & Thompson, 2008). Limited teacher knowledge about mathematics, along with their beliefs, are both strong but equally important as they both impact student learning (Fey, 1979).

Exposure to mathematics as early as in preschool and kindergarten with approximate number system has been shown to correlate with standardized mathematical achievement scores of ninth grade students (Mazzocco, Feigenson, & Halberda, 2011). The reach of teachers extends further than content or proficiency; it also extends to their general perspective of mathematics. Mazzocco (2007) asked a third grader if she was interested in doing some mathematics activities with her. Her response was, “Oh, yes! Math is my teacher’s favorite subject!” (Mazzocco, 2007). Attitudes about mathematics are a byproduct of teaching and can either be an asset or a disservice depending on the opinion of the teacher. If a teacher has a poor outlook on mathematics and conveys this unfortunate view to their students, this can have an even larger effect on children who have a mathematics learning disability or mathematics-specific anxiety (Gresham, 2007; Mazzocco, 2007). Mathematics anxiety is rooted in instruction due to having to perform mathematical operations or questions that can lead to feelings of tension, panic, helplessness, shame, nervousness, fear, and distress (Çatlioğlu, Birgin, Coştu, & Gürbüz, 2009; Gresham, 2007).
A sizable percentage of preservice teachers have been reported to experience high levels of mathematics anxiety when compared to other college groups (Çatlıoğlu, Birgin, Coştu, & Gürbüz, 2009; Gresham, 2007; Jackson & Leffingwell, 1999; Kelly, & Tomhave, 1985). When a profession requires that an individual be proficient in an area they have serious apprehensions about, there is a concern for the effectiveness of their teaching abilities (Gresham, 2007). This apprehension begs the question of how can teacher preparation programs help to reduce the nervousness that surrounds mathematics of such an influential demographic in the K-12 system? Many researchers have done work in the area of mathematical anxiety (Brady & Bowd, 2005; Gresham, 2007; Hembree, 1990; Vinson, 2001) and, depending on the purview of their studies, they have found that through restructuring of approaches, courses can help reduce any apprehension that occurs in preservice educators. One such method is the use of concrete models and engaging elementary school-level teachers in real-world applications of mathematical content, utilizing mathematics-related projects, games, and demonstrations to help develop their mathematical understanding (Brady & Bowd, 2005; Ramani & Siegler, 2008). Since educators in the early grades play a significant role in the formation of mathematical ability and attitudes of students far beyond the time in their classrooms, by helping preservice teachers develop conceptual knowledge over procedural knowledge and incorporate PCK, you can help these individuals within this group who are math-phobic individuals conquer their fears when it comes to mathematics.

How do you teach mathematics for comprehension of concepts and PCK to preservice educators? Kinach (2002) devised a five-step cognitive strategy to guide preservice teachers’ thinking with regard to PCK. Professors would need to 1) identify
the preservice teachers’ PCK for a particular topic, 2) assess the preservice teachers’ explanation, 3) open a dialog to help challenge and to transform their concepts of reasoning, 4) transform the prospective teachers’ explanation by applying the topic to a condition that is tough to explain, and 5) sustain the confusion but resolve it by applying the topic to a situation that allows for a clearer explanation and representation. This asserts how important methods courses specific to mathematics are, as they play a significant role in helping to develop PCK in preservice elementary teachers. One might argue that these budding educators should take more mathematics courses to strengthen their abilities. However, Begle (1979) reviewed the National Longitudinal Study of Mathematical Abilities and found a stronger correlation between the number of credits a teacher had in mathematics methods courses and student performance versus the number of mathematics courses. Another study (Darling-Hammond, 2000) done on students’ achievements in both mathematics and science, showed that courses in mathematics pedagogy had a greater effect on student outcomes versus if the teacher had done simply more coursework. This combined effort shows that educators in teacher preparation programs understand the importance of dissecting the concepts for students in addition to the pedagogy surrounding mathematics (Ball 1990). In addition to helping preservice teachers understand pedagogy and conceptual knowledge, studies also contend that elementary school level educators need to include authentic learning situations, in other words, real-world application to the content (Brady & Bowd, 2005; Gresham, 2007).

**Integration of Astrobiology in Mathematics Education**

Today’s classrooms may look much like they did over a hundred years ago with regard to the structure, but the move to make mathematics user-friendly is moving
forward. In most high schools throughout the United States, mathematics courses are still compartmentalized; algebra 1, geometry, algebra II, probability and statistics, precalculus, and calculus are among the most common (Reys, Dingman, Nevels, & Teuscher, 2007). Although there are a handful of states that offer integrated mathematics I, II, and III (Reys et al., 2007), mathematics has often been approached as a stand-alone subject even though it lends itself to many other disciplines. In fact, it is helpful when it is put into context to help students to make connections, thus avoiding the dreaded, “Why do I have to learn this?” question that often plagues many mathematics teachers. Real-world application of mathematics content at the elementary level has become a tool for helping students overcome mathematics-induced anxiety (Brady & Bowd, 2005).

Frykholm and Glasson (2005) stated that often students are unable to solve problems because they do not understand the context in which the questions are rooted. The integration of subjects such as mathematics and science can bring together overlapping concepts that can enhance learning in context (Furner & Kumar, 2007; Nassif & Zeller, 2006).

STEM education has the ability to go beyond traditional methods of silo type teaching into a more integrative design by nature (Breiner, Harkness, Johnson, & Koehler, 2012). In doing so, learning can create a bridge between these related disciplines. The interdisciplinary nature of astrobiology can be a great vehicle for executing many subjects (Des Marais et al., 2008; Foster & Drew, 2009; Nassif & Zeller, 2006; Quinlan, 2015; Staley, 2003). The breadth that astrobiology encompasses makes it excellent for multidisciplinary education. Its interdisciplinary facets include extreme environments, extremophiles, geographical sciences, planetary and atmospheric science,
early evolution, paleontology, engineering, planetary protection, and the search for extraterrestrial intelligence (Staley, 2003). Astrobiology is appealing due to its ability to span a variety of disciplines in the physical, biological, and social sciences as well as mathematics. Its interdisciplinary facets include, but are not limited to, extreme environments, extremophiles, geographical sciences, planetary and atmospheric science, early evolution, paleontology, engineering, planetary protection and the search for extraterrestrial intelligence (Staley, 2003). The essential question astrobiology seeks to answer is what are the origins, evolution, and distribution of life in the universe and does life exist elsewhere? Astrobiology education has large focus areas in biology and chemistry. However, “the fundamental approach to the study of biology is that chemistry is based on physics and mathematics and that biology is based on chemistry” (Blumberg, 2011, p. 510). Even though astrobiology may invoke the impression of material that could be overwhelming or considered too difficult for elementary education, due to its inherently fascinating subject matter, astrobiology is well-suited for teaching science to a wide range of learners, from kindergartners to graduate students (Staley, 2003).

Astrobiology has grown into such a significant area that the NASA initiated the NASA Astrobiology Institute in 1998 to develop the field of astrobiology and provide a scientific structure for flight missions (National Research Council, 2008). One element in their mission is to provide scientific components for education that range from kindergarten to grade 12 as well as at the collegiate levels from undergraduates to graduate students (National Research Council, 2008). The NASA Astrobiology Institute believes the education and public outreach element is a crucial conduit to relay NASA’s discoveries to the general public. Only a few of the successful education and public
outreach components include science activities and curriculum that was designed to meet educational standards, public lectures, and interactive websites. A goal of the NASA Astrobiology Institute is to motivate future scientists by using ongoing efforts that span from middle school to high school and beyond to college and hopefully into graduate school.

Astrobiology education is growing and expanding, and with this progression comes the need for a better understanding of the type of impact these educational programs are having on knowledge. Research has been conducted on the influence of perceptions of the field or progress of the learning experience (Arino de la Rubia, 2012; Arino de La Rubia et al., 2009; Foster & Drew, 2009), however, there is very little data on the increase of mathematical knowledge through astrobiology. Foster and Drew (2009) conducted pre- and post-course surveys in addition to an assessment of knowledge to evaluate the perceived and actual learning experiences of students enrolled in a pilot course in astrobiology at the University of Florida. Sneider and Ohadi (1998) tested the effectiveness of a constructivist-historical teaching approach on the ability to change students’ misconceptions about Earth’s shape and gravity at elementary and middle school grade levels. Although the study discussed mathematics, it was from a historical point of view and no calculations were actually performed. Arino de la Rubia (2012) conducted a study, *Astrobiology in the Secondary Classroom* (ASC), where modules were created that emphasized interdisciplinary connections in mathematics and science fields. ASC materials were piloted in eight U.S. locations and an analysis of the research of the high school students participating in the ASC project showed statistically significant increases in students’ perceived knowledge and science reasoning. Arino de
La Rubia et al., (2009) first evaluated the ASC curriculum with 14 teachers to determine interest levels in Earth science, engineering, space science, and general science. This program has been successful in increasing perceived knowledge and interest in astrobiology, however, there was no intersection of MCK and PCK reported with astrobiology.

Discussion/Synthesis

There has been a tremendous amount of work done on PCK in multiple subject areas, including mathematics. There are also workshops and materials on astrobiology and astrobiology education across numerous science-based disciplines. However, there has not been any quantifiable data collected on whether these workshops or materials have increased the content knowledge of the participants and furthered their mathematical expertise or pedagogical knowledge. The aims of this study are to enlighten the involved preservice elementary education teachers in essential mathematical concepts in the context of a scientific background and also serve as a platform for enhancing their PCK to serve their future students better, thus aiding in a more substantial understanding of concepts of the teachers and ceasing the cascading effect of rote memorization to their students.

The significance of this literature review is to highlight the significant areas of concern with regard to specific mathematical content areas, preservice teachers’ conceptual understanding, pedagogical approach, and context of the material. The research is directly related to the research questions involving the astrobiology modules’ effect on the participants’ MCK and PCK of ratio, proportion, exponents, and graphical representation, in addition to their knowledge of astrobiology.
CHAPTER III
METHODOLOGY

Research Questions

This study focuses on the first of two courses of an undergraduate elementary education mathematics content course. The development and implementation of a qualitative and quantitative instrument was done to measure the participants’ mathematical content knowledge and the delivery vehicle of the astrobiology modules. The research questions that guided the design and methodology of the study were:

1. What effects does the astrobiology module (M1) have on the preservice teachers’ MCK and PCK of ratio, proportions and fractions?
2. What effects does the astrobiology module (M2) have on the preservice teachers’ MCK and PCK of exponents and graphing?
3. What effects do the astrobiology modules (M1 and M2) have on the preservice teachers’ knowledge of general astrobiology?

Research Design

The research follows a nonrandomized pretest/posttest quasi-experimental design. Quasi-experimental design is similar to an experimental design but lacks random assignment. The nonrandomization of the experiment occurs from the students self-selecting (registering) into the respective two sections used for the experiment. The two sections will be referred to as section 1 and section 2. This investigation was designed to improve preservice elementary education teachers’ comprehension through the use of astrobiology modules. In addition to the content, the research also examined student’s knowledge of elements of PCK.
The pretest was used to determine participants’ MCK, PCK, and knowledge on astrobiology prior to each module. The pretest had elements of both modules present so that each group received the same test. More details on the modules follow in subsequent sections. The two modules were designed to administer one for each section of Mathematics for K-8 Teachers I (M135). One section was assigned to be the experimental group participating in an astrobiology module, whereas the other section was assigned to the control for that module. The modules are focused on different content. M1 concentrates on rational numbers, whereas M2’s application is on exponents. This design was to present a module in one section, whereas the other served as the control for that specific mathematical content (see figure 5). Since the pre- and posttest had the elements from both modules present. Module 1 (M1): Growth Curves was administered to section 1 and covered exponents and graphical representation. Section 2 served as the control for this module as they did not receive any instruction on exponents or graphical representation. Module 2 (M2): Solar System Scale Measurement was administered to section 2 and covered fractions, ratios and proportions. Section 1 served as the control for this module as they did not receive any instruction on fractions, ratios or proportions. The objective was to examine an increase on the questions, from pretest to posttest, involving exponents and graphical representation for Section 1 while section 2 responses for those questions remained steady. Similarly, for section 2 regarding questions involving fractions, ratios and proportions; section 2 responses for questions targeting those concepts would increase from pre- to posttest while section 1 responses would remain unchanged. For the third question, both sections were examined to determine if an increase occurred from pretest to posttest on astrobiology related items.
At the end of the experiment both groups were given a posttest similar to the pretest to determine outcomes of the experiment. The posttest also included four exit questions about the module using a Likert scale of 1-5.

**Course Setting**

University of Montana IRB approval for research was secured for this project (Appendix A). At the time of the study, teacher education students were required to take two semesters of mathematics for elementary education, (M135) and Mathematic for K-8 Teachers II (M136). For students to enroll in these two courses, they must fulfill the prerequisites: students must be pre-education majors and have successfully completed intermediate algebra (M095) with a C or higher or have an appropriate score on the ALEKS placement exam at a level 4. After the research was conducted, the University of Montana changed their mathematics course sequence from two courses to three.

These two courses were designed to give prospective K–8 teachers the content background to teach mathematics and to pass the national Praxis exam needed for professional licensing. These courses covered problem solving, set and logic, functions,
whole numbers, number theory, integers, rational numbers, proportional reasoning, decimals, percents, real numbers, algebraic thinking, probability and statistics, geometry, and measurement. Due to the nature of the topics chosen for the experiment and the module design, the first course in the sequence was the only class that directly addressed the subject matter (ratio and proportion, spatial reasoning, and exponents).

**MATH 135 Course Design and Instruction**

**Construction.** During spring semester 2015, M135 was a five-credit course that met over a 16-week semester. Depending on the section, students either met five days a week for 50 minutes each day or met twice a week for 2 hours and 20 minutes. During spring semester 2015, there were two sections. For simplicity the sections will be referred to as section 1 and section 2. Class 1 met Monday through Friday from 9:10am to 10:00am. Class 2 met twice a week, Monday and Wednesday from 5:10pm to 7:30pm.

**Instructors.** During spring semester 2015, a master’s level graduate student, through a teaching assistantship, taught section 2. The other section was taught by an associate professor of mathematics education: section 1. A course coordinator handled the timing and content aspects of the course and held weekly meetings with the instructors to cover course objectives, exams, and ensure the classes were aligning within the given time frame of instruction. Instructors had autonomy regarding their homework, quizzes, exams, and activities; however, final exams given in the two sections were identical.

**Course Objectives.** As stated in the common course syllabi, when students successfully complete the M135 course, they should be able:

1. To identify and solve problems in elementary mathematics.
2. To model the number systems: natural numbers, integers, rationals, and real numbers.
3. To become familiar with the use of manipulatives to enact arithmetic operations.
4. To apply basic problem-solving strategies to ratio, proportion, and percent problems.
5. To use mathematical modeling and basic algebra to approach real-world problems.
6. To solve problems using probability and statistics, including designing simulations.
7. To communicate mathematics both in oral and written form.

Course Materials. The M135 course curriculum sequentially followed chapters 1–8 of the textbook, *A Problem Solving Approach to Mathematics for Elementary School Teachers, 11th edition* (Billstein, Libeskind, & Lott, 2012). This textbook was used for the two sections of M135 and was also used in the subsequent course, M136.

Population and Sample

IRB approval (Appendix A) was obtained on December 26th, 2014 to conduct the research. The population was preservice elementary education majors in the United States enrolled in a similar first course of mathematics for elementary education. The sample population for this study were the students enrolled in the M135: Mathematics for K-8 Teachers Course I during the spring 2015 semester at the University of Montana. The study group for this research, who were consistently involved over the research period, consisted of 19 participants (n = 19) seeking their teaching degree and certification for kindergarten through eighth grade. All participants were instructed to take the pretest,
background survey, and posttest. The total number of the students from both sections was 61, but the authentic number of participants for the study was 19. This was due to attrition by students not completing all three requirements for the study: 1) attend every teaching session, 2) take the pretest, and 3) take the posttest. One student opted out of the experiment prior to conducting the research. Of the 60 students who gave consent to be participants, 55 completed the background survey, 53 completed the pretest, and 43 completed the posttest. A total of 19 students completed all three requirements of the study. The demographics for the participants who answered questions on the background survey, across both sections, were primarily first-year Caucasian females not of Hispanic descent.

**Sampling Method**

There were two sections of M135 during the spring semester of 2015; students selected the section according to their own schedule and preference. All students in those two sections were invited to participate in the project. Both groups were administered a background survey, pretest, and posttest. The background survey contained the subject information and informed consent form (Appendix B) for the intended participants. The participants for this study self-selected into their module groups based on their course section.

**Teaching Modules**

Two modules were created for this research study. These modules used space science as a platform for conveying important mathematical topics. The modules’ design used methods from the Minority Institute Astrobiology Collaborative as a framework for direction. The Minority Institute Astrobiology Collaborative is a combined effort across
minority-serving institutions to advance astrobiology research and education. A pretest and posttest were designed to gather comprehension of topics before and after the intervention, and a background survey was constructed to collect demographic information on the participants.

Classroom Setting

Both modules were taught in the same classroom in the Liberal Arts building on the University of Montana’s main campus. The classroom contained six hexagonal groups made of two 48 in. × 24 in. trapezoidal activity tables. The tables sat between 4 and 6 participants per group. Participants had unassigned seating. There was a computer, screen, and whiteboard at the front of the room.

Technology used for the modules included the computer in the classroom with the overhead projector. A PowerPoint presentation for each module was created from the original lesson plan to lead each set of participants through the activities and lessons. The M1–Growth Curve lesson plan (Appendix C) was detailed for the participants in an 18-slide PowerPoint presentation (Appendix D). The M2–Solar System Scaled Measurement lesson plan (Appendix E) was detailed for the participants in a 21-slide PowerPoint presentation (Appendix F). In addition to the electronic technology, the whiteboard and whiteboard markers were used to transcribe participant work and answers.

M1 was covered in one 75-minute class session (Tuesday). M2 took two 50-minute class sessions (Monday and Wednesday).

The researcher was treated as a guest lecture for the purposes of the study. She taught the elements of the modules in both sections for the duration of the research.
Materials for Module 1

The materials used for M1 (Growth Curve) included clear sheet protectors, tissue paper (enough for approximately 10 sheets), a flashlight, student worksheet 1 (Appendix G), and the student group worksheet (Appendix H).

Module 1

The first module, Growth Curve, was designed for participants to understand what exponential growth means for microorganisms. They examined three stages of growth dynamics: lag phase, exponential (log) phase, and stationary phase. They hypothesized what would occur to the batch culture after the stationary phase (death phase). Using this information, participants were then asked to postulate why learning about microorganisms is important not just to science but specifically astrobiology. Participants explored exponents, exponential equations, and how their graphs (representation) differed from linear equations.

Microorganisms are an important aspect of astrobiology. Scientists learn about how early these life-forms have survived, evolved, and died on Earth in extreme habitats, helping them extrapolate how this evolutionary process might occur on other planets. Examining microorganism growth curves will be a great introduction to exponential growth.

This module consisted of group work, discussion, a hands-on activity, and a brief lecture followed by another hands-on activity. Examples of common student errors were provided to help the participants use their MCK to help guide them through the PCK to dissect the errors. Participants were engaged in a discussion on what they knew about microorganisms, which led to a dialogue on where microorganisms can be found and why
extremophiles are important. Participants were already in groups according to their seating configuration, which conserved time in the completion of Student Worksheet 1. Participants were to look for an exponential pattern ($y = 2^t$) and generate a general formula for exponential growth: $y = a(1 + b)^t$. It was anticipated that the participants might be familiar with growth rate ($b$) and growth factor ($1 + b$), so the class was guided through some of the more difficult aspects of the general formula. Common student errors regarding exponents were discussed, as well as bases raised to the zero power. Participants were then led in a discussion of the differences in exponential growth versus exponential decay. Afterward, the focus shifted back to the original equation and what happens to growth in real-world versus theoretical instances (equations). A demonstration of how scientists count microorganisms was given using sheet protectors and a flashlight to simulate Optical Density ($OD_{600}$) readings in order to graph cell growth, followed by a brief lecture on the real-world example of *Sulfolobus solfataricus* and its various growth phases. Participants were then given another assignment, Student Worksheet 2, to work on based only on the exponential growth phase. They were to determine what happened to the graphs of imaginary microorganisms as various aspects of their equations changed.

**Materials for Module 2**

The materials used for M2 (Solar System Scaled Measurement) were cardboard or cardstock, markers, paper, pencils, tape, calculator, rulers (paper or regular) or meter sticks, fabric tape measure (1 for each group), a basketball, a beach ball, peppercorn, a pinhead, various round balls smaller than the basketball, and student activity sheets (Appendix I and Appendix J).

**Module 2**
The second module, Solar System Scaled Measurement, was designed to get preservice elementary educators (participants) interested in space science’s and math’s connections to real-world situations. Starting with warm-up problems involving fractions, participants started to recall information and get into the habit of thinking rationally. The aim was for participants to gain a greater understanding of the properties of proportions through the rules of fractions and the errors or misunderstandings their future students will face.

The study of our solar system is important to understand the vastness of the known universe. The module focused on the distances between the planets and our sun, strengthening students’ spatial reasoning skills and introducing them to proportions and proportional reasoning.

The module included group work, discussion, a hands-on activity, and a brief lecture followed by another hands-on activity. Examples of common student errors were provided to help the participants use their MCK to help guide them through the PCK to dissect the errors. The module led participants through rules of fractions and multiplication properties. Using fractions, participants worked out a general rule for proportions: \( \frac{a}{b} = \frac{c}{d} \). With a discussion question of where we would need to use proportions, the researcher directed them into a discussion about our solar system’s sun and planets. A short lecture on the size of the planets in our solar system compared to the sun followed the discussion. To help the participants assess their spatial reasoning, spherical objects were placed at the front of the classroom, and participants were to discuss their sizes to estimate which objects were proportional to some of the celestial bodies in our solar system (the sun, Mercury, Venus, Earth, and Mars). After all the
groups made their predictions, a discussion was directed toward the dangers of planet exploration and microorganisms. Participants were asked questions on their knowledge of microorganisms and how they reproduce or replicate, and their ideas and answers were written on the board (CGI). Next, participants were asked about the requirements for life and where could they find microorganisms. A discussion of extremophiles followed and eventually led to why studying extremophiles on Earth might be helpful in determining how life might survive in similar harsh environments on other planets. This discussion was then looped back to the size of the planets (Mercury to Mars) compared to the sun and exactly how far away the planets were from the sun and from each other. The participants, in groups, were given tape measures and one or two spherical items from the previous planet size estimation activity to measure the diameter. After the participants found the diameters, the class was directed in a discussion of how to scale the planets down in size if these various items were to be compared to a basketball, which served as a representation of the sun. Using Earth as the first example, participants were led through a proportional scaling using a basketball as the sun. This activity led to another class discussion on whether this scaling would be the same for all of the planets. After a participant came up with a general proportional formula for scaling the planets to the basketball (sun), they were given a handout worksheet and asked to calculate the remaining planets. When the participants were done, the class was directed back to their estimates from the beginning of the lesson to determine whether they were correct. Then a group discussion of distance was initiated, and participants were guided through an exercise to scale the size of Earth’s orbit based on the basketball’s size. Participants were then directed to the back of the activity sheet to complete a group activity on scaling
planets’ (Mercury to Mars) orbits. A discussion of their findings and what the scaled distances they found meant regarding the size of the scaled planets proceeded the activity.

**Data Collection**

Spring semester classes began on January 26, 2015. During the first week, the instructors of both sections of M135 informed students of the experiment involving the modules. All students were invited to participate ($n=61$) and were informed that they would be tested as part of a doctoral dissertation study. They were informed that their involvement was voluntary, and the results would be confidential. Due to this fact, no incentives were given for participation.

**Measures**

- **Background survey.** The background survey (Appendix K) was designed to collect each participant’s demographic and educational information. Basic demographics such as race, ethnicity, gender, and age range were collected. Other collected details included educational history, mathematics courses taken, science courses taken, interest in STEM areas, familiarity with astrobiology, and confidence in their current mathematics skills. This data collection helped the researcher examine any major differences between the groups, as the students self-selected the M135 section in which they were enrolled to ensure homogeneity in both sections.

- **Instrument administration.** The background survey, pretest (Appendix L), and posttest (Appendix M) were built and administered through the Qualtrics data collection program. The class instructor emailed participants a link to the consent form that served as a precursor to the pretest for each section. The email included an explanation of the research, a statement of confidentiality, and links to the background survey and pretest.
Participants were instructed to complete the background survey and pretest outside of class before February 2. The background survey should have taken approximately 20 minutes to complete, and the pretest should have taken approximately 30 minutes to complete. The instructions stated that the sections would require 45 minutes to make certain students gave themselves enough time to complete all the elements. If students declined the IRB Consent form, they were instructed to disregard the background survey. The background survey also collected information on the participants’ comfort level with astrobiology-related items in addition to mathematics.

Upon completion of the module instruction, participants were sent a follow-up email thanking them for their participation and support of the study. They were again provided with a link to the posttest in the follow-up email. The posttest should have taken approximately 30 minutes or less to complete. Again, the instructions stated 45 minutes to make certain students gave themselves enough time to complete all the questions.

**Pretest/Posttest item development.** The instrument examined three idiosyncratic aspects relating to the modules and to the course’s mathematical objectives: astrobiology, basic mathematics, and pedagogy of student errors. The researcher selected 13 test questions to make up the given modules’ material. Because the experiment aimed to discern whether astrobiology can be an effective mechanism for the delivery of mathematical concepts, it stands to reason that the participants should be evaluated on their knowledge of astrobiology-related knowledge. Included in the instrument were six questions that distinctly corresponded to astrobiology elements.

Mathematical knowledge is a major concern not only in the class but for this experiment. The participants’ mathematical knowledge was assessed in two of the
instrument’s aspects. First, participants simply solved two test items involving fractions and exponents, helping to assess the participants’ MCK. The researcher also evaluated their mathematical knowledge by looking at the pedagogy of student errors. The instrument consisted of four items where the participants examined a question that a student had solved with the student’s work and answer included. One other test item was used to assess MCK on exponents. The participant needed to determine if the student answered the question correctly and in the correct manner, describe the student’s thought process given his or her work, and relate what feedback they would give this student regarding the question and the answer, so the researcher could assess elements of the participants’ PCK.

This instrument was developed to test the participants’ MCK and PCK in the areas of rational numbers, exponents, graphing, and student errors, as well as their knowledge of astrobiology. Because the study involved knowledge of astrobiology, the understanding of mathematics, and the comprehension of students’ mathematical errors, the test items were developed to reflect all these aspects. Assessment items were established through a review of test items developed for the Minority Institute Astrobiology Collaborative and consultations with faculty members at the University of Montana.

The pretest consisted of 13 questions, some of which had multiple parts. The first section of the test inquired about the participants’ knowledge of astrobiology-related items. The second portion investigated the participants’ comprehension of mathematical items such as ratio and proportion, fractions, solving for an unknown, and problems involving exponents.
The posttest consisted of the same 13 questions as the pretest. However, four exit survey questions at the end focused on the impression the astrobiology modules had on the participants. These last 4 questions were constructed on a Likert scale: 1 = strongly disagree, 2 = disagree, 3 = not sure, 4 = agree, and 5 = strongly agree. These additional questions follow:

1. I found the astrobiology modules interesting.
2. Learning about astrobiology made me more interested in learning the mathematics involved with the module.
3. I learn mathematics more easily when it is presented in a manner in which I see connections to other topics.
4. I would like to use this module in the future when I get a teaching position.

**Targeting the assessment.** For the pretest and posttest assessments, each measure was developed to address elements that are covered in M1: Growth Curve and M2: Solar System Scaled Measurement. These elements include basic knowledge of astrobiology-related subject matter, mathematical questions, and student response information regarding mathematical questions. For the student response questions, the participants had three parts to consider when answering: 1. Look at the student’s answer. 2. Explain the student’s thinking. 3. What feedback would the participant give the student on that particular question?

**Reliability and validity.** To determine the reliability of the pretest as a measure, it was administered to students in the mathematics education graduate seminar, MATH 504, which consisted of mathematics master and doctoral students in mathematics and mathematics education. Three graduate students completed the pretest, and the data was
analyzed and discussed. Recognizing that the sample size was rather small, all three of the graduate students agreed on the answers and offered suggestions on minute changes to questions to increase clarification. In discussions of the test’s validity, a consensus arose that the questions were suitably targeted for MCK, PCK, and astrobiology material. Threats to validity were group selections (as the participants were not randomly assigned to the various sections of the course); experimental mortality; small sample size; different noncompleter totals in each section; and the effects of pretesting on posttests, where the pretest could affect the posttest scores.

**Data Analysis**

Based on the research questions, two levels of analysis were performed: descriptive and inferential. Descriptive analysis was performed to gain an understanding of the participant demographics in the two courses to ensure both sections were similar in composition. The researcher performed two levels of inferential analysis. The first was performed to determine whether students in the two sections were similar in composition and to ensure those who completed all tasks were no different from the students who failed to complete them. The second level of inferential analysis helped answer the research questions and included a qualitative piece, which was also analyzed.

**Descriptive Analysis**

The participants in each section of M135 completed the online background survey, pretest, and posttest. However, not all participants gave indicators across all three survey items. Some filled out the background survey and the pretest but chose not to participate in the posttest, some failed to fill out the background survey but participated in one of the two tests, and some refused to give any identifiers on the instruments. With
these added complications, only 19 students completed all three items (background survey, pretest, and posttest) across both sections with identifiers. See Table 5.

Table 5

*Descriptive Analysis of Participants*

<table>
<thead>
<tr>
<th></th>
<th>Completed Background Survey</th>
<th>Completed Pretest</th>
<th>Completed Posttest</th>
<th>Completed All Elements with Identifiers</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sec 1</strong></td>
<td>$n = 27$</td>
<td>$n = 25$</td>
<td>$n = 27$</td>
<td>$n = 8$</td>
</tr>
<tr>
<td><strong>Sec 2</strong></td>
<td>$n = 28$</td>
<td>$n = 28$</td>
<td>$n = 16$</td>
<td>$n = 11$</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>$n = 55$</td>
<td>$n = 53$</td>
<td>$n = 43$</td>
<td>$n = 19$</td>
</tr>
</tbody>
</table>

The demographic statistics were collected from the background survey and were compared to determine homogeneity between the study group and the noncompleter group, as well as between the two sections. Data analysis was performed on the pretest and posttest, and a preliminary examination was conducted on the two groups within each section. These groups included the finishers ($n = 19$) and the other participants who did not complete all three requirements for the study based on the initial background survey, the noncompleters. This investigation was to determine if the finishers ($n = 19$) were statistically the same as the noncompleters based on the completed background survey. The items tested were from the background survey and included race, ethnicity, sex, and educational history. See Table 6 for coding of each element.

Table 6

*Response Coding of Background Survey*

<table>
<thead>
<tr>
<th>Question</th>
<th>Selection Items</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>I strongly identify with the following race:</td>
<td>Caucasian/White</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Hispanic or Latino</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>African American/Black</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Native American/American Indian/Alaskan Native</td>
<td>4</td>
</tr>
<tr>
<td>Ethnicity:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------------------------------</td>
<td>--------</td>
<td></td>
</tr>
<tr>
<td>Asian or Pacific Islander</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>No, I am not of Hispanic descent</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Yes, I am of Hispanic descent</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sex:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>1</td>
</tr>
<tr>
<td>Female</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Educational History</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>First year student (Freshman)</td>
<td>1</td>
</tr>
<tr>
<td>Second year student (Sophomore)</td>
<td>2</td>
</tr>
<tr>
<td>Third year student (Junior)</td>
<td>3</td>
</tr>
<tr>
<td>Fourth year student (Senior)</td>
<td>4</td>
</tr>
<tr>
<td>Post baccalaureate student</td>
<td>5</td>
</tr>
<tr>
<td>Graduate student</td>
<td>6</td>
</tr>
<tr>
<td>Other: (fill in blank)</td>
<td>7</td>
</tr>
</tbody>
</table>

Parametric assumptions were not met by completers or the noncompleters, as the sample size was less than 31 participants in both sections, with the small exception of section 2, in which 28 participants completed the background survey. Additional analyses were performed on the variances, which were unequal, and the distributions were not normal; therefore, non-parametric analyses were performed on descriptive and inferential statistics.

A Kruskall–Wallis Test was performed for each section in lieu of a standard analysis of variance (ANOVA) on the background survey to determine if the participants from each section were statistically the same types of students. An additional Kruskall–Wallis test was performed to determine if the participants who completed all three elements of the study could be considered representatives of the course section.

A Mann–Whitney U-Test was performed for each section in lieu of an independent t-test on the pretest to determine whether the participants who completed all three elements of the study could be considered representatives of the course section. Furthermore, a Mann–Whitney U-Test was also performed to determine whether both sections harbored the same type of students with regard to the information on the pretest.
and posttest. At this point, only the participants who completed all of the research elements were analyzed.

Once completers were determined, the two sections were compared against each other with a Mann–Whitney U-Test to ascertain whether a difference existed between the two section scores on the pretest and posttest.

**Instrument scoring**

The researcher scored the pretest and posttest items manually. Each item was examined for familiarity and correctness using rubrics. Questions 8 and 11 were multiple choice and therefore simply yielded either a 0 for an incorrect selection or a 1 for the correct selection. Word problems 9, 10, 12, and 13 had three parts: a, b, and c. Section a was a “yes or no” question and was included in the inferential statistics.

IBM SPSS Statistics 24.0 was used to tabulate scores within and across groups and to evaluate outcomes of not only the individual questions but also the various segments of the pretest/posttest. Items examining astrobiology questions were grouped together, as were items investigating MCK and PCK elements according to the module that was targeted.

**Rubrics**

The pretests and posttests contained elements that pertained to M1, in which Section 1 was the experiment group, and elements that pertained to M2, in which Section 2 was the experiment group. Test questions 9, 10, 11, and 13 analyzed M1. Test questions 7, 8, and 12 analyzed M2. The test also examined questions 1–6, which applied to knowledge of astrobiology.
A rubric was created to ascertain knowledge gained on questions 1–6 that asked participants to give simple answers: 0 = no understanding or no answer, 1 = partial correctness or some understanding, and 2 = accurate correctness or established a very good understanding. These scores were added to the two multiple-choice questions (8 and 11). For the questions that required participants to give a yes or no answer in part a (questions 9, 10, 12, and 13), one point was assigned for a correct response or zero for an incorrect response. The point totals were then analyzed using a Wilcoxon matched pairs signed rank test to determine whether a difference existed in the ranking of the pretest and posttest on the aspects of M1 for section 1, M2 for section 2, and astrobiology for both sections. On problems 9, 10, and 13 for section 1 and problem 12 for section 2, the subsequent parts, b and c, required the participants to explain the student’s work and give feedback based on the answer. These parts, b and c, along with part a, were examined later through qualitative analysis to determine the extent of the participants’ MCK and PCK. See Table 7 for the classification of questions belonging to each module and the maximum value assigned to each problem.

Table 7

<table>
<thead>
<tr>
<th>Pretest and Posttest Problem Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
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<td>10a</td>
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<tr>
<td>11</td>
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<td>12a</td>
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</tbody>
</table>
A qualitative rubric was designed to assess the quality of the participants’ levels of MCK and PCK from the pretest to the posttest. Criteria for scoring MCK included clear, consistent, and convincing evidence in the participants’ responses that they understood the concept, an accurate identification of the correct mathematical operation for the given problem, and an accurate and fully supported solution to the question in the form of feedback to the student. Criteria for scoring PCK included clear, consistent and convincing evidence in participants’ responses that they understood the student’s work and could clearly articulate the mistakes that were made, as well as how they could help correct any misconceptions that occurred. MCK and PCK understanding were evaluated according to the following scale: Level 1 = limited to none, Level 2 = basic, Level 3 = proficient, and Level 4 = advanced.

The MCK levels were adapted from the National Board of Teaching Standards (2017) and Enhancing Professional Practice: A Framework for Teaching (2007) and are as follows:

Level 1 MCK: The response provides little or no evidence of the ability to model problem situations, employ techniques and procedures, and explain the mathematical operation depicted in the given problem. Characteristics include incomplete and inaccurate explanation of the given problem, inaccurate or missing identification of the mathematical operation that does not fit the given data (or the equation is missing), inaccurate or missing explanation of the given mathematical operation, and an incomplete or missing explanation of the relationship in the given situation. Regarding PCK, the participant displays little
or no understanding of the range of pedagogical approaches suitable for student learning of the content.

Level 2 MCK: The response provides limited evidence of the ability to model problem situations, employ proper techniques and procedures, and explain the mathematical operations depicted in the given problem. Characteristics include incomplete and/or inaccurate explanation of the given problem, inaccurate or missing identification of the mathematical operation that does not fit the given data, somewhat inaccurate and unsupported solutions to the given problem, and an incomplete explanation of the relationship in a given situation. Regarding PCK, the participant reflects a limited range of pedagogical approaches or some approaches that are not suitable to the discipline or to the students.

Level 3 MCK: The response provides clear evidence of the ability to model problem situations, employ proper mathematical techniques and procedures, and explain the mathematical operation depicted in the given problem. Characteristics include accurate explanation of the given problem, accurate identification of the mathematical operation that fits the given data, accurate solutions to the given problem although it lacks full support, and a logical explanation of the relationship in a given situation. Regarding PCK, the participant’s response reflects a familiarity with a wide range of effective pedagogical approaches in the discipline.

Level 4 MCK: The response provides clear, consistent, and convincing evidence of the ability to model problem situations, employ proper mathematical techniques and procedures, and explain the mathematical operation depicted in
the given problem. Characteristics include a complete and accurate explanation of the given problem, accurate identification of the mathematical operation, a complete explanation that fits the given problem, accurate and fully supported solutions to the given problem, complete and accurate modeling of a given situation, and appropriate identification of the mathematical operation. Regarding PCK, the participant’s response reflects familiarity with a wide range of effective pedagogical approaches in the discipline, anticipating student misconceptions.

**Interrater reliability**

For qualitative research to provide meaningful information, the collection and recording of relevant data must be accurate (Randle, 2012). To test interrater reliability, the researcher and an outside evaluator, an Assistant Professor in Mathematics, assessed questions involving MCK and PCK from the completers on the pretests and posttests independently. Three types of benchmarks were involved in evaluating how much agreement is sufficient: percentage of absolute agreement, Cohen’s kappa, and intra-class correlation (Graham, Milanowski, & Miller, 2012). Absolute agreement was set for this study at 85%. The suggested threshold for demonstrating an acceptable agreement level for the absolute agreement method is between 75% and 90% (Hartmann, 1977).

**Research question analysis**

Test items were separated into the areas that were used to answer the research questions.

**Research question 1.** What effects does the astrobiology module (M2) have on preservice teachers’ mathematical content knowledge and pedagogical content knowledge of ratio, proportion, and fractions? To examine this question, M1 items were
separated, and a Wilcoxon matched pairs signed rank test was conducted on the pretest and posttest scores of participants for section 1. Section 2 was the control for M1 items; therefore, a Mann–Whitney U-Test was performed to determine whether a significant difference in the scores existed between sections 1 and 2 on the pretest and posttest for M1. M1 targeted ratio, proportion, and fractions, and M2 targeted exponents and graphical analysis. Qualitative analysis was performed on the quality of answers the participants gave on parts b and c for questions 9, 10, and 13, which involved MCK and PCK.

Research question 2. What effects does the astrobiology module (M2) have on the preservice teachers’ MCK and PCK of exponents and graphing? To examine this question, M2 items were separated, and a Wilcoxon matched pairs signed rank test was conducted on the pretest and posttest scores of participants for section 2. Section 1 was the control for M1 items; therefore, a Mann–Whitney U-Test was performed to determine whether a significant difference existed in the scores for sections 1 and 2 on the pretest and posttest for M2. Qualitative analysis was performed on the quality of answers the participants gave on parts b and c for question 12, which involved MCK and PCK, and question 7 was analyzed for MCK.

Research Question 3. What effects does the astrobiology module have on preservice teachers’ knowledge of astrobiology? To examine this question, astrobiology items were separated, and a Wilcoxon matched pairs signed rank test was conducted on the pretest and posttest scores of participants on the astrobiology-related content items for both sections to determine whether a significant difference existed in the scores for sections 1 and 2. A Mann–Whitney U-Test was performed to determine whether a
significant difference existed in the scores between sections 1 and 2 on the pretest and posttest for the astrobiology related questions, 1–6.

**Overall pretest/posttest comparison per section.** Although it was not a direct research question, a separate Wilcoxon test analysis was implemented to look generally at the pretest results versus the posttest results to determine whether there was an effect given the overall test.
CHAPTER IV
RESULTS

Two levels of data analysis were performed on the information provided by the participants: descriptive and inferential. A descriptive analysis was performed on the information provided in the background survey by both classes to determine if the two sections, 1 and 2, were comparable in composition. Another set of analyses, descriptive and inferential, was performed in each section to determine if the individuals who completed all elements of the study were comparable in composition to the students who only finished some elements. Inferential analyses were performed to answer the three research questions.

Results of Descriptive Analyses

Preliminary Analysis of the Background Survey

Hargreaves (1974) reported three motives for the inclusion of demographic categories: 1. They enable the comparison of various studies to determine if they are similar in populations. 2. They allow for an examination of the random assignment or assist in matching subjects when randomization is not achieved. 3. They provide a foundation for identifying subgroups that may differ in success among the various treatments being compared. Demographic information was collected through the background survey, and an analysis was performed on the demographics and educational background of the participants in the two sections of M135.

Section 1. Of the 27 participants in section 1, 27 identified as Caucasian, including 23 non-Hispanics, two Hispanics, four males, and 23 females. Educational
history was a bit more diverse: 12 participants were first-year students or freshman, seven were second-year students or sophomores, four were third-year students or juniors, three were fourth-year students or seniors, and one participant identified as a graduate student. Of the completers, eight were Caucasians, eight were non-Hispanics, eight were females, six were freshmen, one was a junior, and one was a senior. See Table 8 and Table 9 for a more detailed categorization of section 1.

Table 8

<table>
<thead>
<tr>
<th>Demographics of Section 1</th>
<th>Caucasian</th>
<th>Non-Hispanic</th>
<th>Hispanic</th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>Completers</td>
<td>8</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>Unfinished</td>
<td>19</td>
<td>15</td>
<td>2</td>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>Total</td>
<td>27</td>
<td>23</td>
<td>2</td>
<td>4</td>
<td>23</td>
</tr>
</tbody>
</table>

Table 9

<table>
<thead>
<tr>
<th>Educational Backgrounds of Section 1</th>
<th>Freshman</th>
<th>Sophomores</th>
<th>Juniors</th>
<th>Seniors</th>
<th>Post-Bac</th>
<th>Graduate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Completers</td>
<td>6</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Unfinished</td>
<td>6</td>
<td>7</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>12</td>
<td>7</td>
<td>4</td>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Section 2. Of the 28 participants in section 2, 26 identified as Caucasian, one as Native-American, 26 as non-Hispanic, two as Hispanic, two as male, and 26 as female. Educational history here was also a bit more diverse: nine participants were first-year students or freshman, seven were second-year students or sophomores, four were third-year students or juniors, two were fourth-year students or seniors, two were postbaccalaureates, and one participant identified as a graduate student. Of the completers, 10 were Caucasian, two were Hispanic, nine were non-Hispanic, 11 were female, two were freshmen, three were sophomores, two were juniors, one was a senior,
one was a postbaccalaureate, and two were graduate students. See Table 10 and Table 11 for a more detailed categorization of section 2.

Table 10

Demographics of Section 2

<table>
<thead>
<tr>
<th></th>
<th>Caucasian</th>
<th>Native-American</th>
<th>Non-Hispanic</th>
<th>Hispanic</th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>Completers</td>
<td>10</td>
<td>0</td>
<td>9</td>
<td>2</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>Unfinished</td>
<td>16</td>
<td>1</td>
<td>17</td>
<td>0</td>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>Total</td>
<td>26</td>
<td>1</td>
<td>26</td>
<td>2</td>
<td>2</td>
<td>26</td>
</tr>
</tbody>
</table>

Table 11

Educational Backgrounds of Section 2

<table>
<thead>
<tr>
<th></th>
<th>Freshman</th>
<th>Sophomores</th>
<th>Juniors</th>
<th>Seniors</th>
<th>Post-Bac</th>
<th>Graduate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Completers</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Unfinished</td>
<td>7</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>9</td>
<td>7</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Analysis of Participants’ Demographic Backgrounds

A Kruskal–Wallis test was performed to determine whether any differences existed based on the participants’ demographic backgrounds in both sections. The results indicated no significant difference between the two sections. An additional Kruskal–Wallis test was performed to determine whether any differences existed between the participants who completed all features of the study and the noncompleters for each section separately. The results indicated no significant difference between the completers and the noncompleters in terms of race, ethnicity, sex, or educational history for section 1 and section 2. See Appendix N for more details.

Analysis of Completers and Noncompleters’ Pretest Scores

Because no significant difference between completers (study) and noncompleters was found regarding demographic information, a non-parametric test, the Mann–Whitney, was conducted to determine whether a difference existed in the pretest scores
between the noncompleters in all elements and the completer (study) group for each section separately. The Mann–Whitney tests indicated no significant difference in the completers’ (study) scores on the pretest and the noncompleters’ scores on the pretest for both sections. See Appendix N for more details. Future analyses were performed on only the completer (study) groups from each section.

**Effect Size Between Completers and Noncompleters**

Because no significant difference existed between completers (study) and noncompleters regarding demographic information or pretest scores, a series of non-parametric tests were conducted to determine whether a difference existed in the pretest and posttest scores between the two sections.

**Pre- and PostTest Comparison Between Sections.** Since there was shown to be no significant difference between completers (study) and unfinished with regards to demographic information or pretest scores, a Mann-Whitney U-Test was conducted to examine if there was a difference on the pre- and posttest scores between the two sections. See Table 12.

Table 12

*Mann-Whitney U-Test mean rank for the pretest and posttest for sections 1 and 2*

<table>
<thead>
<tr>
<th>Test Statistics</th>
<th>PreTest</th>
<th>PostTest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mann-Whitney U</td>
<td>23.500</td>
<td>22.500</td>
</tr>
<tr>
<td>Wilcoxon W</td>
<td>89.500</td>
<td>88.500</td>
</tr>
<tr>
<td>Z</td>
<td>-1.717</td>
<td>-1.785</td>
</tr>
<tr>
<td>Asymp. Sig (2-tailed)</td>
<td>0.086</td>
<td>0.074</td>
</tr>
<tr>
<td>Exact Sig. [2*(1-tailed Sig.)]</td>
<td>0.091b</td>
<td>0.075b</td>
</tr>
</tbody>
</table>

a. Grouping Variable: Section  
b. Not corrected for ties.

Notes. * $p < 0.05$. 

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A Mann-Whitney test indicated there was a moderate difference in the score of section 1 on the pretest ($Mdn = 62.5$) than section 2 group score on the pretest ($Mdn = 45.0$), $U = -1.717$, $p = 0.086$, $r = 0.394$. These results suggest that the section 1 group and the section 2 are relatively similar with regards to the pre-and posttest. The effect size ($r = 0.394$) suggests the difference between the two sections is moderate. Eta squared ($\eta^2$) was also calculated to determine how much variability of the ranks is accounted by section. $\eta^2 = 0.16$ indicates that a very small percent (16%) of the variability of the ranks is accounted by being in either section.

A Mann-Whitney test indicated there was a moderate difference in the score of section 1 on the posttest ($Mdn = 70.0$) than section 2 group score on the posttest ($Mdn = 77.5$), $U = -1.785$, $p = 0.074$, $r = 0.400$. These results suggest that the section 1 group and the section 2 are relatively similar with regards to the posttest. The effect size ($r = 0.400$) suggests the difference between the two sections is moderate. Eta squared ($\eta^2$) was also calculated to determine how much variability of the ranks is accounted by section. $\eta^2 = 0.18$ indicates that a very small percent (18%) of the variability of the ranks is accounted by being in either section.

**M1 Pre-PostTest Comparison Between Sections.** A Mann-Whitney U-Test was conducted to compare the pretest, posttest results with regards to questions pertaining to module1 (M1) of section 1 and section 2, see Table 13.

Table 13

<p>|Mann-Whitney U-Test mean rank for the pretest and posttest module 1 items for sections 1 and 2|</p>
<table>
<thead>
<tr>
<th>Test Statistics</th>
<th>M1Pre</th>
<th>M1Post</th>
</tr>
</thead>
</table>

A Mann-Whitney test indicated there was not a significant difference in the score of section 1 on the pretest items pertaining to module 1 ($Mdn = 75.0$) than section 2 group score on the pretest items pertaining to module 1 ($Mdn = 50.0$), $U = -1.941$, $p = 0.052$, $r = 0.164$. These results suggest that the section 1 group and the section 2 are similar with regards to the pre-and posttest items pertaining to module 1. The effect size ($r = 0.164$) suggests the difference between the two sections is small. Eta squared ($\eta^2$) was also calculated to determine how much variability of the ranks is accounted by section. $\eta^2 = 0.16$ indicates that a very small percent (16%) of the variability of the ranks is accounted by being in either section.

A Mann-Whitney test indicated there was a moderate difference in the score of section 1 on the posttest items pertaining to module 1 ($Mdn = 75$) than section 2 group score on the posttest items pertaining to module 1($Mdn = 66.7$), $U = -1.320$, $p = 0.187$, $r = 0.311$. These results suggest that the section 1 group and the section 2 are relatively similar with regards to the posttest items pertaining to module 1. The effect size ($r = 0.311$) suggests the difference between the two sections is moderate. Eta squared ($\eta^2$) was also calculated to determine how much variability of the ranks is accounted by finishing. $\eta^2 = 0.10$ indicates that a very small percent (10%) of the variability of the ranks is accounted by being in either section.

<table>
<thead>
<tr>
<th></th>
<th>18.500</th>
<th>25.500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mann-Whitney U</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wilcoxon W</td>
<td>73.500</td>
<td>80.500</td>
</tr>
<tr>
<td>Z</td>
<td>-1.941</td>
<td>-1.320</td>
</tr>
<tr>
<td>Asymp. Sig (2-tailed)</td>
<td>0.052</td>
<td>0.187</td>
</tr>
<tr>
<td>Exact Sig. [2*(1-tailed Sig.)]</td>
<td>0.055&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.203&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

a. Grouping Variable: Section  
b. Not corrected for ties.  
Notes. * $p < 0.05$.  

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M2 Pre-PostTest Comparison Between Sections. A Mann-Whitney U-Test was conducted to compare the pretest, posttest results with regards to questions pertaining to module2 (M2) of section 1 and section 2, see Table 14

Table 14

Mann-Whitney U-Test mean rank for the pretest and posttest module 2 items for sections 1 and 2

<table>
<thead>
<tr>
<th>Test Statistics</th>
<th>M1Pre</th>
<th>M1Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mann-Whitney U</td>
<td>27.000</td>
<td>24.000</td>
</tr>
<tr>
<td>Wilcoxon W</td>
<td>93.000</td>
<td>90.000</td>
</tr>
<tr>
<td>Z</td>
<td>-1.432</td>
<td>-1.674</td>
</tr>
<tr>
<td>Asymp. Sig (2-tailed)</td>
<td>0.152</td>
<td>0.094</td>
</tr>
<tr>
<td>Exact Sig. [2*(1-tailed) Sig.]]</td>
<td>0.177^b</td>
<td>0.109^b</td>
</tr>
</tbody>
</table>

a. Grouping Variable: Section
b. Not corrected for ties.

Notes. * p < 0.05.

A Mann-Whitney test indicated there was a moderate difference in the score of section 1 on the pretest items pertaining to module 2 (Md\(\text{n} = 62.5\)) than section 2 group score on the pretest items pertaining to module 2 (Md\(\text{n} = 45.0\)), \(U = -1.432\), \(p = 0.152\), \(r = -0.329\). These results suggest that the section 1 group and the section 2 are relatively similar with regards to the pre-and posttest items pertaining to module 2. The effect size \((r = -0.329)\) suggests the difference between the two sections is moderate. Eta squared \((\eta^2)\) was also calculated to determine how much variability of the ranks is accounted by finishing. \(\eta^2 = 0.11\) indicates that a very small percent (11%) of the variability of the ranks is accounted by being in either section.

A Mann-Whitney test indicated there was a moderate difference in the score of section 1 on the posttest items pertaining to module 2 (Md\(\text{n} = 77.5\)) than section 2 group score on the posttest items pertaining to module 2 (Md\(\text{n} = 70\)), \(U = -1.674\), \(p = \)
0.094, $r = -0.384$. These results suggest that the section 1 group and the section 2 are relatively similar with regards to the posttest items pertaining to module 2. The effect size $(r = -0.384)$ suggests the difference between the two sections is moderate. Eta squared ($\eta^2$) was also calculated to determine how much variability of the ranks is accounted by finishing. $\eta^2 = 0.16$ indicates that a very small percent (16%) of the variability of the ranks is accounted by being in either section.

**Astrobiology Pre-PostTest Comparison Between Sections.** A Mann-Whitney U-Test was conducted to compare the pretest, posttest results with regards to questions pertaining to astrobiology of section 1 and section 2, see Table 15.

<table>
<thead>
<tr>
<th>Test Statistics</th>
<th>Astro Pre</th>
<th>Astro Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mann-Whitney U</td>
<td>32.500</td>
<td>43.5</td>
</tr>
<tr>
<td>Wilcoxon W</td>
<td>88.500</td>
<td>109.500</td>
</tr>
<tr>
<td>Z</td>
<td>-0.962</td>
<td>-0.042</td>
</tr>
<tr>
<td>Asymp. Sig (2-tailed)</td>
<td>0.336</td>
<td>0.996</td>
</tr>
<tr>
<td>Exact Sig. [2*(1-tailed Sig.)]</td>
<td>0.351$^b$</td>
<td>0.968$^b$</td>
</tr>
</tbody>
</table>

a. Grouping Variable: Section  
b. Not corrected for ties.  
Notes. * $p < 0.05$.

A Mann-Whitney test indicated there was not a significant difference in the score of section 1 on the pretest items pertaining to astrobiology ($Mdn = 62.5$) than section 2 group score on the pretest items pertaining to astrobiology ($Mdn = 50.0$), $U = -0.962$, $p = 0.336$, $r = -0.22$. These results suggest that the section 1 group and the section 2 are similar with regards to the pre-and posttest items pertaining to astrobiology. The effect size ($r = -0.22$) suggests the difference between the two sections is small. Eta
squared ($\eta^2$) was also calculated to determine how much variability of the ranks is accounted by finishing. $\eta^2 = 0.05$ indicates that a very small percent (0%) of the variability of the ranks is accounted by being in either section.

A Mann-Whitney test indicated there was not a significant difference in the score of section 1 on the posttest items pertaining to astrobiology ($Md/n = 79.15$) than section 2 group score on the posttest items pertaining to module 2 ($Md/n = 83.3$), $U = -0.042, p = 0.996, r = -0.01$. These results suggest that the section 1 group and the section 2 are similar with regards to the posttest items pertaining to astrobiology. The effect size ($r = -0.01$) suggests the difference between the two sections is small. Eta squared ($\eta^2$) was also calculated to determine how much variability of the ranks is accounted by finishing. $\eta^2 = 0.0$ indicates that a very small percent (0.0%) of the variability of the ranks is accounted by being in either section.

### Participant Examples

Tables 16 and 17 illustrate participants’ responses on the pretest and posttest across the various rubric levels.

Table 16

<table>
<thead>
<tr>
<th>Level</th>
<th>Ratio</th>
<th>Proportion</th>
<th>Posttest</th>
<th>Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(9)</td>
<td>(13)</td>
<td>(9)</td>
<td>(13)</td>
</tr>
<tr>
<td></td>
<td>I do not know if this answer is correct</td>
<td>I do not know if the student’s work is correct</td>
<td>No, the student’s work is not correct. I cannot explain the student’s thinking. I would explain to the student that ratios involve multiplication, or repeated addition, but such that has to be done at the same rate per animal.</td>
<td>No. They multiplied straight across instead of cross multiplying. They had the right idea, just didn’t do it the correct way.</td>
</tr>
</tbody>
</table>
2
(9) No They used addition/subtraction instead of multiplication

(13) a. No
b. They multiplied bottom with bottom and then with the top.
c. I would tell them that to solve for x you must cross multiply. \(x = 17\)
while 5 times 51 then you must divide the leftovers into the whole number.

(10) No They used addition/subtraction instead of multiplication

(13) A. No
B. They’re just multilingual everything together.
C. It’s cross multiplication. So I would tell them to set up a new equation after cross multiplying and then you must divide for x.

3
(10) a. No
b. They are thinking that there are 4 parts white which is subtracted by 1 part red which equals 3 total liters of red, and then there are 6 liters of the dark pink plus the three left over parts of white that must be added to get the nine liters of white paint to get the light pink.
c. I would tell the student that 4 parts could be in that one liter, and the liters are not equal to parts.

(13) A. No
B. They subtracted 17 from 51 leaving 34 as the total from there they added 5 since it was left giving the answer \(x = 39\)
C. Set up into a ration and cross-multiply

(9) A. No
B. They subtracted 17 from 51 leaving 34 as the total from there they added 5 since it was left giving the answer \(x = 39\)
C. Set up into a ration and cross-multiply

(13) The student’s work is incorrect. The student was thinking that they could subtract 51 and 17 to get their answer. The feedback I would give the student is to cross multiply, then divide each side by 17, to get the x by itself.

4

(13) No The student thought that the relationship between fractions could be determined by finding the difference between the denominators and then adding that difference to the first numerator to find the second. The student must cross multiply to set up a solvable equation. \(5x51=17X\)
\(\Rightarrow 255=17X \Rightarrow\) divide both sides by 17 \(\Rightarrow 15=X\)
Table 17

Examples of Participant Responses on the Pretest and Posttest, Section 2

<table>
<thead>
<tr>
<th>Level</th>
<th>Pretest Exponents</th>
<th>Posttest Exponents</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>growth of something at the same rate? uh I don’t know</td>
<td>something that grows outside of something else</td>
</tr>
<tr>
<td>2</td>
<td>a. No, the students work is not correct. The student should have divided by 3 to solve for y. b. I can see that they knew that 3x9 = 27, however they did a completely different operation than was stated. 3y means 3 x y. c. I would recommend plugging in their answer for y back into the equation to see if their answer matches what the equation equaled.</td>
<td>The student’s work is incorrect, the answer should be 9. The student instead of solving for y, they just listed the answer as 27 because it’s the sum of 9+9+9. I would show the student that you have to divide the 27 by 3 so that you get y by itself, thus solving for y and getting 9.</td>
</tr>
<tr>
<td>3</td>
<td>The work is correct, but the answer is wrong. The student knows that there are three 9s to make up 27</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>a. Yes b. The student knew that 9+9+9=27. He assumed that y=9. c. I would encourage this student to check his work. Problems in the future may not always have these small of numbers. It would be best for him to realize the relationship that y has with the final answer of the equation.</td>
<td></td>
</tr>
</tbody>
</table>

Interrater Reliability

An analysis was performed on the responses to items related to MCK and PCK by the researcher and an independent evaluator, an Associate Professor in Mathematics. An absolute agreement percentage of 85% was chosen as the threshold for reliability, as it was an upper end of the range for an acceptable agreement. Three questions with parts b (MCK) and c (MCK and PCK) were examined. The percentages of agreement were 87.2%, 88.1%, and 86.5%. Following the assessment of the absolute agreement, a
discussion between raters was organized to deliberate on nonmatching items. We reached an agreement that for some items, determining whether they belong to one category or an adjacent one was difficult.

**Results from the Research Question Analyses**

**Research Question 1: M1 Pretest/Posttest Comparison for Section 1**

A Wilcoxon matched pairs signed rank test was conducted to determine whether a difference existed in the ranking of M1 knowledge in the pretest and posttest for section 1. The analysis results indicated no significant difference \( (n = 8, z = -0.378, p > 0.05 [p = 0.705]) \). The results suggest that the posttest scores were no different from the pretest scores for section 1 in M1.

Among the eight participants in section 1, a qualitative analysis was performed on their responses to questions 9b, 9c, 10b, 10c, 13b, and 13c. These questions were included to evaluate the participants’ pretest MCK. Of the eight participants for questions 9b, 9c, 10b, 10c, 13b and 13c, there were 11 category responses of 1 from the rubric, seven category responses of 2, 20 category responses of 3, and one category response of 4. The high number of 3s indicates that the overall participants’ understanding of the mathematical content was above average. A similar posttest analysis was completed and yielded the following results: 12 category responses of 1, seven category responses of 2, 19 category responses of 3, and no category response of 4. The category responses of 1 increased from the pretest to the posttest, and the category responses of 3 and 4 decreased.

Among the eight participants in section 1, a qualitative analysis was performed on their responses to questions 9c, 10c, and 13c. These questions were included to evaluate
the participants’ pretest PCK. Of the eight participants for questions 9c, 10c and 13c, there were seven category responses of 1 from the rubric, 14 category responses of 2, three category responses of 3, and no category response of 4. The high number of category responses of 2 indicate that the participants’ understanding of the pedagogy content was slightly low. A similar posttest analysis was completed and yielded the following results: six category responses of 1, 17 category responses of 2, one category response of 3, and no category response of 4. The category responses of 2 increased from the pretest to the posttest, and the category responses of 3 decreased.

**Control Results for M1.** Section 2 answers were analyzed using a Wilcoxon matched pairs signed rank test for questions 9, 10 and 13 to determine if there was any significant change from the pretest to posttest. The analysis resulted indicated no significant difference \( (n = 11, z = 0.940, p > 0.05 [p = 0.347]) \). The results suggest that the posttest scores were no different from the pretest scores for section 2 regarding M1.

**Research Question 2: M2 Pretest/Posttest Comparison for Section 2.**

A Wilcoxon matched pairs signed rank test was conducted to determine whether a difference existed in the ranking of the pretest and posttest regarding the M2 knowledge for section 2. The analysis results indicate no significant difference \( (n = 11, z = -0.844, p > 0.05 [p = 0.399]) \). The results suggest that the posttest scores were no different from the pretest scores for section 2 regarding M2.

Among the 11 participants in section 2, a qualitative analysis was performed on their responses to questions 7, 12b, and 12c. These questions were included to evaluate the participants’ pretest MCK. Of the 11 participants over the three questions, there were
12 category responses of 1 from the rubric, nine had a category response of 2, 11 had a category response of 3, and one had a category response of 4. The totals of category responses 1–3 indicates that the class had a diverse understanding of the mathematical content on the pretest. A similar posttest analysis was performed and yielded the following results: 14 participants had a category response of 1, seven had a category response of 2, 10 had a category response of 3, and two had a category response of 4. The category responses of 1 increased from the pretest to the posttest, the responses of 3 decreased, and category responses of 4 increased.

Among the 11 participants in section 2, a qualitative analysis was performed on their responses to question 12c. Question 12c was included in to evaluate participants’ pretest PCK. Of the 11 participants, six had a category response of 1 from the rubric, four had a category response of 2, one had a category response of 3, and none had a category response of 4. The high number of category responses 1 and 2 indicate that the participants’ understanding of the mathematical content was below average. A similar posttest analysis was performed and yielded the following results: eight participants had a category response of 1, one had a category response of 2, one had a category response of 3, and one had a category response of 4. The category responses of 1 increased from the pretest to the posttest, the category responses of 2 decreased, and the category responses of 3 and 4 increased.

**Control Results for M2.** Section 1 answers were analyzed using a Wilcoxon matched pairs signed rank test for questions 7, and 12 to determine if there was any significant change from the pretest to posttest. The analysis resulted indicated no significant difference ($n = 8, z = -1.394, p > 0.05 [p = 0.163]$). The results suggest
that the posttest scores were no different from the pretest scores for section 1 regarding M2.

**Research Question 3: Astrobiology Pretest/Posttest Comparison.**

A Wilcoxon matched pairs signed rank test was conducted to determine whether a difference existed in the ranking of the pretest and posttest in terms of astrobiology knowledge for section 1. The analysis results indicate no significant difference ($n = 8, z = -1.183, p > 0.05 \ [p = 0.237]$). The results suggest that the posttest scores were no different than the pretest scores for section 1 in terms of astrobiology knowledge.

A Wilcoxon matched pairs signed rank test was conducted to determine whether a difference existed in the ranking of the pretest and posttest in terms of astrobiology knowledge for section 2. Analysis results indicate a significant difference at the $\alpha = 0.05$ level ($n = 11, z = -2.403, p < 0.05 \ [p = 0.016]$). The results suggest that the posttest scores were higher than the pretest scores for section 1 in terms of astrobiology knowledge.

**Overall pretest/posttest comparison per section.** A Wilcoxon matched pairs signed rank test was conducted to determine whether a difference existed in the ranking of the complete pretest and posttest for section 1. The analysis results indicated a significant difference at the $\alpha = 0.05$ level ($n = 8, z = -2.316, p < 0.05 \ [p = 0.021]$). The results suggest that the posttest scores were higher than the pretest scores for section 1.

A Wilcoxon matched pairs signed rank test was conducted to determine whether a difference existed in the ranking of the pretest and posttest for section 2. The analysis results indicated a significant difference at the $\alpha = 0.05$ level ($n = 11, z = -2.274, p <$
The results suggest that the posttest scores were higher than the pretest scores for section 2.
CHAPTER V
DISCUSSION

This chapter begins with an overview of the purpose and methodology of this study. Research results are first summarized with regard to the three research questions and discussed in more detail with the implications and relation to the current literature. Recommendations for future research and practice are presented along with the limitations of the study. Lastly, the conclusion for this study is provided.

Overview of the Study

The need for mathematically competent elementary educators has become a paramount demand for the future, long-term success of students (Post, Harel, Behr, & Lesh, 1988). Welder (2007) suggested that one of the elements to success in gateway courses such as algebra is the mastery of prerequisite algebra concepts throughout K–8 mathematics education such as (1) numbers and numerical operations, (2) ratio/proportions, (3) order of operations, (4) equality, (5) patterning, (6) algebraic symbolism, (7) algebraic equations, (8) functions, and (9) graphing.

Student achievement in high school mathematics and beyond has been directly influenced by their K–8 teachers’ knowledge and attitudes (Fennema & Franke, 1992). Ball et al. (2005) found that many teachers in the United States are lagging in their mathematical skills and understanding. So often a lack of knowledge stems from developing inclusive context, and the real-world application of mathematical content assists in the learning process and helps individuals overcome mathematics-induced anxiety (Brady & Bowd, 2005).
The aim of this study was to investigate the effects of two astrobiology modules on preservice teachers’ astrobiology-related knowledge and MCK and PCK in different mathematical concept areas: (1) ratio, proportions, and fractions; and (2) exponents and graphing. A qualitative instrument was developed specifically to measure MCK and PCK. This instrument was administered to students enrolled in two different sections of M135: Mathematics for K–8 Teachers Course I during the spring 2015 semester at the University of Montana. The study group for this research, who were consistently involved over the research period, consisted of 19 participants \( n = 19 \) seeking their teaching degree and certification for kindergarten through eighth grade.

A quasi-experimental design was implemented to determine the outcomes for the research questions. Nineteen matched pair in the Wilcoxon Matched Pairs Signed Rank tests were analyzed and used to investigate the effects of M1 and M2 on preservice elementary educators’ MCK, PCK, and astrobiology knowledge in the areas of ratio, proportion, exponents, and graphical representation. A qualitative analysis was performed both pretest and posttest on the questions about student errors to determine the quality of responses from the participants.

Summary of the Research Results

Research Question 1

What effects does the astrobiology module have on preservice teachers’ mathematical content knowledge and pedagogical content knowledge of ratio, proportion, and fractions? M1 was used in a pretest and posttest comparison for section 1. Upon completion, there was no indicated difference in the ranking from the pretest to the
posttest on the aspect of module 1 on ratio, proportion, or rational numbers for section 1 among the eight participants.

- The Wilcoxon Matched Pairs Signed Rank Test showed no significant difference among participants from the pretest to the posttest scores on whether the participants MCK with regard to recognizing whether a student’s mathematical reasoning was correct, $z = -0.378, p > 0.05 (p = 0.705)$.

- Qualitative analysis showed no significant improvement in the participants’ responses to MCK-related items (questions 9b, 9c, 10b, 10c, 13b, and 13c) from the pretest to the posttest. Category responses, Level 1 = limited to none, Level 2 = basic, Level 3 = proficient, and Level 4 = advanced, included the following in Table 18:

Table 18

<table>
<thead>
<tr>
<th>Category Responses for MCK for Section 1</th>
<th>Rubric Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre – Test Counts</td>
<td>1 2 3 4</td>
</tr>
<tr>
<td>Post – Test Counts</td>
<td>12 7 19 0</td>
</tr>
</tbody>
</table>

- Qualitative analysis showed no significant improvement in the participants’ responses to PCK-related items (questions 9c, 10c, and 13c) from the pretest to the posttest. See Table 19.
Research Question 2

What effects does the astrobiology module have on preservice teachers’ mathematical content knowledge and pedagogical content knowledge of exponents and graphical reasoning? M2 was used in a pretest/posttest comparison for section 2. Upon completion, there was no indicated difference in the ranking from the pretest to the posttest on the aspect of module 2 of exponential or graphical reasoning for section 2 among the 11 participants.

- The Wilcoxon Matched Pairs Signed Rank Test showed no significant difference of participants from pretest to the posttest scores on whether the participants MCK with regard to recognizing whether a student’s mathematical reasoning was correct, $z = -0.844, p > 0.05$ ($p = 0.399$).

- A qualitative analysis showed no significant improvement in the participants’ responses to MCK-related items (questions 7, 12b, and 12c) from the pretest to the posttest. Category responses included the following in Table 20:

Table 20

\begin{tabular}{lcccr}
\hline
\textit{Category Responses for MCK for Section 2} & \multicolumn{4}{c}{Rubric Scores} \\
\hline
 & 1 & 2 & 3 & 4 \\
Pre – Test Counts & 12 & 9 & 11 & 1 \\
Post – Test Counts & 14 & 7 & 10 & 2 \\
\hline
\end{tabular}
• Qualitative analysis showed no significant improvement in the participants’ responses to PCK-related items (questions 7, 12b, and 12c) from the pretest to the posttest. See Table 21

Table 21

<table>
<thead>
<tr>
<th>Category Responses for PCK for Section 2</th>
<th>Rubric Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Pre – Test Counts</td>
<td>6</td>
</tr>
<tr>
<td>Post – Test Counts</td>
<td>8</td>
</tr>
</tbody>
</table>

**Research Question 3**

What effects does the astrobiology module have on preservice teachers’ knowledge of astrobiology? This question was examining if the modules had an effect on the participants’ knowledge from pretest to posttest, no control was used. Module 1 (M1) was used to determine if the subjects had an increase from pre- to posttest for section 1. M2 was used in to determine if the subjects had an increase from a pre- to posttest for section 2.

• A Wilcoxon Matched Pairs Signed Rank Test showed no significant difference in participants from the pretest to the posttest scores on whether their astrobiology knowledge increased for section 1, \( z = -1.183 \), \( p > 0.05 \) (\( p = 0.237 \)).

• A Wilcoxon Matched Pairs Signed Rank Test showed there was a significant difference of participants from the pretest to the posttest scores on whether the participant's astrobiology knowledge increased for section 2, \( z = -2.403 \), \( p < 0.05 \) (\( p = 0.016 \)).
The results of the overall pretest/posttest comparison per section were conducted to determine whether there was a difference in the ranking of the complete pretest and posttest.

- A Wilcoxon Matched Pairs Signed Rank Test for section 1 showed there was a significant difference in the ranking of the complete pretest and posttest, \( z = -2.316, p < 0.05 \) (\( p = 0.021 \)). The results indicate that the posttest scores were higher than the pretest scores for section 1.

- A Wilcoxon Matched Pairs Signed Rank Test for section 2 showed there was a significant difference in the ranking of the complete pretest and posttest, \( z = -2.274, p < 0.05 \) (\( p = 0.023 \)). The results indicate that the posttest scores were higher than the pretest scores for section 2.

**Discussion of the Results**

**Mathematical Content Knowledge**

The mastery of rational numbers such as ratio and proportion and exponents separately are critical to students’ success in future algebra-based courses (Behr, Lesh, Post, & Silver 1983; Bezuk & Cramer, 1989; Common Core Standards Initiative, 2010). This study looked at using astrobiology as a delivery system to teach important mathematical concepts and pedagogy to provide a foundation based in science to difficult content. This research was unable to show a significant increase, at \( \alpha = 0.05 \), of the participants’ MCK through the module intervention for either section.

The participants’ MCK varied on the pretest, as some subjects had low scores; however, a majority of the scores were quite high. This high pretest score made the posttest result less favorable for a positive outcome. Participants’ answers on the pretest
were usually well thought out, and they appeared to be invested in answering the questions correctly. This initially high investment resulted in a more favorable outcome on the pretest scores. The mathematical content appeared to be well established in the pretest. When the participants followed up the module intervention with the posttest, of the students who did well, it appeared many of them gave answers that were summaries of their first answer. However, they offered fewer details and performed less favorably on the posttest. This phenomenon was demonstrated in the responses these students provided on the pretest and posttest.

- The example responses of participant responses who declined in the posttest analysis include the following:

**Participant A Pretest Answer:** A. No. B. Yes. C. 3y represents 3 multiplied by some value y. To solve for y, you need to get it alone so divide 3 on each side. 3 divided by itself cancels out so you have y=27/3, which is 9. 9 multiplied 3 times is 27.

**Participant A Posttest Answer:** no, yes, you need to solve for y. so 3y=27, get y alone by dividing over the 3 so 27/3=?

**Participant B Pretest Answer:** No, Instead of multiplying 51 by 5 and dividing by 17, they subtracted and added

**Participant B Posttest Answer:** No. They used subtraction instead of multiplication

**Pedagogical Content Knowledge**
MCK alone does not guarantee better teaching, and there are various forms of knowledge that are also needed for teaching (Tirosh, 1999). Other aspects are needed to round out a capable teacher’s skillset, such as pedagogical content knowledge. PCK is a dimension of subject matter but is specific for teaching (Shulman 1986). This study looked at using astrobiology as a delivery system to teach important mathematical concepts and pedagogy to provide an exciting foundation to difficult content. This research was unable to show a significant increase, at $\alpha = 0.05$, of participants’ PCK through the module intervention for either section. Subjects demonstrated either a fairly astute grasp of PCK or lacked the fundamental understanding to give meaningful feedback. Again, the same phenomenon occurred in assessing PCK that occurred in MCK, and when the posttest followed the module intervention, it appeared many participants gave answers that were summaries of their answers from the pretest. They gave fewer details and performed less favorably on the posttest. This phenomenon is demonstrated in the response these students provided on the pretest and posttest.

- The example responses of participants who declined in the posttest analysis include the following:

**Participant C Pretest Answer:** a) no b) i think they misinterpreted the meaning of "to" c) they should try to set up a ratio and then cross multiply to find the number of alligators

**Participant C Posttest Answer:** [Left blank]

**Participant D Pretest Answer:** A. No it is incorrect. B. The student subtracted 2 from 5 equaling 3, which was the ratio. After they subtracted from 5 they also subtracted from the 15 thinking subtraction would find the correct answer. C. Set
it up into a fraction so they can visually see that the 5 correspondences with the 15 and the 3 correspondences with the unknown variable.

**Participant D Posttest Answer:** A. No B. They looked for the difference between 5 and 3 and took that number to the other known number in the equation [sic] C. Set up into ratio form

PCK has been a topic of controversy with regard to how it is handled in both research and in practice. The conceptualization of PCK has been criticized based on the lack of theoretical and empirical grounding that PCK is a distinct category in a teacher’s knowledge base (Ball, Thames, & Phelps, 2008). Some scholars have disapproved of the narrow definition of PCK, as put forth by Shulman, and they have argued that it should be expanded to encompass curriculum knowledge, beliefs, or emotions (Friedrichsen, Van Driel, & Abell, 2010; Zembylas, 2007; Grossman, 1990). It would stand to reason that the subject’s PCK would be more procedural than conceptual when explaining student errors, as a majority of preservice educators are not privy to giving feedback to actual students in either their first or second year. This would also change the instructional strategies they propose (Depaepe, Verschaffel, & Kelchtermans, 2013). Some of the preservice subjects had a difficult time giving substantive feedback on the student errors. This facet could be due to the lack of actual interaction with students and the inability to fully comprehend what or how they would respond to the questions adequately.

**Astrobiology**

Astrobiology is interdisciplinary by nature, as it encompasses many different science-based disciples (Breiner, Harkness, Johnson, & Koehler, 2012; Staley, 2003;
Sullivan & Morrison, 2008). It is also an exciting field, as it harnesses the imagination between science fiction and science reality (Cowie, 2016). The interest in astrobiology has the ability to capture an individual’s attention due to the growth of the science fiction genre over the last 80 years (Herrick, 2008, p. 22). This study looked at using astrobiology as a delivery system to teach important mathematical concepts and pedagogy so as to give an exciting foundation to difficult content. The outcomes on the astrobiology portion of the intervention were different in the two sections. Section 1 elicited no significant changes, at $\alpha = 0.05$, from the pretest to the posttest scores; however, section 2 did produce a significance increase, at $\alpha = 0.05$, from the pretest to the posttest scores on the aspect of astrobiology knowledge $p < 0.05$ ($p = 0.016$). Upon review of the differences between the two sections, M2 involved discussions on microorganisms in much more detail than M1, as it was the method used to deliver the mathematical concepts of exponents and graphical analysis. Although microorganisms were discussed in M1, they were not the primary mode of delivery.

Astrobiology can be effective in engaging students because it harnesses people’s preoccupation with space and aliens, which have been and continue to be an element in pop culture (Billings, 2012). Astrobiology can be connected to Next Generation Science Standards’ (NGSS Lead States, 2013) second dimension: being specific to each discipline as crosscutting concepts (Quinlan, 2015). Inside the second dimension, seven crosscutting concepts dovetail exceptionally well with mathematics and most notably with this experiment: patterns, scale, proportions and quantities systems, and system models. To accomplish this, directed teaching in the appropriate area must be addressed in the proper time frame of instruction. These results, although split according to the
different sections, correspond well with the literature with regard to the complimentary instructional design.

One feature of this study was to determine if there would be an increase in astrobiology knowledge. Although the results were split between the two sections, this research is unique with regard to the quantification of knowledge gained or not with regard to astrobiology. There are many studies (Arino de la Rubia, 2012; Arino de la Rubia et al., 2009; Foster & Drew, 2009; Staley, 2003) that have looked at the perceptions of participants or students of the field or how to incorporate astrobiology into education, but there is very little statistical evidence about increasing knowledge. This study was able to provide results of a direct intervention on the participants’ astrobiology and mathematical knowledge versus the perceptions.

**Limitations of the Study and Lessons Learned**

There are factors that may have impacted the results of this study, including potential aspects such as semester timing, motivation of participants, timing of the assessments, assessment delivery, curriculum, and module design. This study was done in February, approximately one month into the semester. This course has a prerequisite mathematics course that ensures the students will be able to comprehend the material. This prerequisite course could have an impact on the participants’ mathematical knowledge. Participant motivation was also impacted, as the module intervention was offered as an extra credit assignment for participants by their instructors. This option consequently made the investigation a low-stakes opportunity, as it was not for credit in the actual course. The low return on identifiers was a clear indication that the participants were not as heavily invested in the module intervention compared to a graded assignment.
for credit (Padilla-Walker, Thompson, Zamboanga, & Schmersal, 2005). This low investment could also be characterized in the vague answers given from the completers on the posttest assessment when compared to the pretest answers.

The truncated answers on the posttest assessment could also be attributed to the condensed time period that occurred between the pretest and posttest. The pretest was administered directly before the module interventions and was immediately followed up with the posttest. This compressed time period could have exerted a type of test fatigue in the participants, as many of the participants demonstrated a proper understanding of basic mathematical concepts in the pretest. This factor contributed to a small if any increase in their knowledge base. In fact, it had the opposite effect on many participants. This factor could be attributed to test fatigue, where cognition starts to wane the longer the time on task (Ackerman & Kanfer, 2009). The pretest elicited some great responses; however, as the intervention went on, some of the participants’ posttest responses were subpar in comparison. This could have been mitigated by a slight variation in the posttest items. The test subjects already invested time and effort into the pretest questions and did not feel it necessary to give full energy to the exact questionnaire items; however, this tendency could have been averted if the details on the posttest were different questions.

The assessment pieces were not given using traditional pencil and paper; instead, they were administered online to conserve class time used for the intervention. Using an online data collection program such as Qualtrics also served as a roadblock to a more robust data collection. Using the online platform excluded collecting the work done by the participants on the pretest and posttest problems. When measuring PCK, multiple points of data are preferred (Morrison & Luttenegger, 2015). Using a paper form of the
test would have been ideal; however, the instructors of M 135 were gracious enough to allow time to be taken for the module intervention in their courses and assessing outside of standard class time appeared to be the ideal approach for data collection. Online assessment in the subjects’ time outside of class could also attribute to the high scores on the pretest assessment. Although the participants were asked not to use any search tools available to them, this was done on the honor system, and there remained the possibility that the subjects were able to research the questions before providing answers.

The module design time frame was also very short in duration. The results may have been impacted differently if the research had been conducted over a longer period of time covering more mathematical, pedagogical, and astrobiology content. With the addition of more time, this would allow for the collection of more artifacts of participants’ thinking with regard to PCK that would provide more indications of the changes in their ability (Park & Oliver, 2008).

The modules were also taught in different time frames with regards to the section. Section 1 was four times per week and section 2 was twice a week. The two sections had the same number of contact hours but structured differently. The course had a course supervisor, who ensured the instructors covered the same content each week and had similar exams. The different section timing could have played a role in the differences between the two courses due to retention of content by the participants.

Effect size also could have been a contributing factor. A series of Mann-Whitney U-tests were conducted to examine the differences between the pretest scores between the two sections: the null hypothesis is that the two sections are the same and the alternative
is that they are not the same ($H_0$: section 1 = section 2, $H_a$: section 1 ≠ section 2) at the alpha level of 0.05 ($\alpha = 0.05$).

The overall pretest was analyzed and yielded a p-value of 0.086 which is greater than the alpha level, therefore the null, that the two sections are similar, cannot be rejected. Upon further analysis the effect size of $r = 0.394$ suggests there is a moderate difference on the pretest scores between the two sections. The eta squared indicated that 16% of the variability of the scores on the pretest is accounted by being in either sections, therefore the section was not a large contributing factor for the variability in the overall pretest scores.

The scores that pertained to M1 were tested and yielded a p-value of 0.052 which is slightly greater than the alpha level, therefore the null, that the two sections are similar, cannot be rejected. Upon further analysis the effect size of $r = 0.164$ suggests there is a small difference on the pretest scores between the two sections. The eta squared indicated that 16% of the variability of the scores on the pretest is accounted by being in either sections, therefore the section was not a large contributing factor for the variability in the overall pretest scores.

The scores that pertained to M2 were tested and yielded a p-value of 0.152 which is greater than the alpha level, therefore the null, that the two sections are similar, cannot be rejected. Upon further analysis the effect size of $r = 0.329$ suggests there is a moderate difference on the pretest scores between the two sections. The eta squared indicated that 11% of the variability of the scores on the pretest is accounted by being in either sections, therefore the section was not a large contributing factor for the variability in the overall pretest scores.
The scores that pertained to astrobiology were tested and yielded a p-value of 0.336 which is greater than the alpha level, therefore the null, that the two sections are similar, cannot be rejected. Upon further analysis the effect size of $r = 0.336$ suggests there is a moderate difference on the pretest scores between the two sections. The eta squared indicated that 0% of the variability of the scores on the pretest is accounted by being in either sections, therefore the section was not a contributing factor for the variability in the overall pretest scores.

While the effect size indicated that there was no difference in which section the participants were in for the intended outcomes, the sample size was very small. The small sample size impedes this study’s generalizability to a larger population of preservice elementary educators.

**Implications for Teaching**

The factors that contributed to the limitations of the study were analyzed for future recommendations for teaching. Understanding how these factors influence teaching is important for both curriculum design and implementation.

When designing a multidisciplinary unit that is based on content, the unit should integrate seamlessly with the timeline for the proposed content within the course syllabus. This forethought has a higher probability of student commitment, as there will be less disruption in the course and a lower likelihood the material will be taught twice. In determining prior student knowledge, using assessments that will influence participants’ motivation will yield better results than offering assessment pieces as extra credit.
When developing a course or modules for preservice elementary educators, a comprehensive analysis that includes both MCK and PCK before engaging in any content material should be done to evaluate the participants’ base knowledge. This initial examination would provide a better indication of the students’ understanding (Pintrich, 2002). This will inform the instructors of what content areas are of the most significant concern before employing any instruction. A follow-up examination that includes MCK and PCK elements should also be done to determine whether the students acquired the necessary knowledge for teaching (Hill, Ball, & Schilling, 2008).

In addition to content, PCK should also be a focus in courses preparing students to become educators. The lack of proper PCK training proposes a problem when preservice teachers are appointed to student teaching and into their first formative years of instruction (Botha & Reddy, 2011; McAninch, 2015). Understanding the common student mistakes is paramount in guiding students’ knowledge and subsequent work. PCK elements such as student errors should be designed into the content curriculum for aspiring educators. This understanding of student work trims time to correct mistakes before they become habits by students. The ability to efficiently discover pedagogical errors quickly and to have students adjust their conceptual understanding will ensure long-term solutions to errors (Ball, Thames, & Phelps, 2008).

**Implications for Future Research**

The factors that contributed to the limitations of the study were also analyzed for future recommendations for research. Understanding how these factors influence research is important for both research design and implementation. As mentioned earlier, implementation of any proposed new modules or curriculum will be better received when
integrated seamlessly into the existing course structure. If integration into existing classes is not possible, conducting a search within sections of courses for willing participants would be preferable. This approach would be better suited to acquiring participants who are more invested in the study. Recruiting participants to conduct the research outside of an existing course also allows more time for adequate assessment prior to and following the intervention.

It is important to use different formats for instrument implementation to determine which platform would yield more consistent results. Both online and paper modes could be tested with open-ended questions and multiple-choice methods (e.g., a multiple-choice test with a variety of correct answers with correct techniques, a correct answer with an incorrect means to solve it, wrong answers with proper methods to solve them, and incorrect answers with incorrect methods to solve them). This restructure would necessitate more complex mathematical and astrobiology-related problems to assess the participants’ knowledge level. In addition, more qualitative pieces such as interviews with participants should be taken into account. This would give researchers better data if the participants’ responses were too brief. These interviews also might provide more in-depth data for researchers to investigate. Again, this necessitates more time to conduct the overall research as interviews to seek detailed information is more labor intensive than pen and paper tests. The combination of all possible elements can be more useful in the data analysis.

Additionally, if preservice elementary teachers are going to be successful at preparing students to master keystone concepts in a multidiscipline approach, they need proper preparation that includes active engagement. In this study, enhancements could be
made with regard to taking the subjects out to a field or open area to visualize the solar system scale measurement activity. This should be more aligned with the curriculum flow and possibly with the weather restrictions.

Other areas could be explored in the cross-sectional analysis of astrobiology and mathematics at not only the elementary education level but also within high school and secondary education where mathematics can be a little more challenging yet functional in terms of designing to the subject matter. Case studies such as setting up and conducting actual astrobiology-related experiments to model exponential growth and performing logistical regression analysis could be done as a lab in an astrobiology unit at the undergraduate level in an existing science course.

Researching both MCK and science curriculum knowledge together within the confines of astrobiology could yield exciting results for both subject areas. Conducting research at a conference, although short, may result in better data as the participants select which professional development they wish to attend. Data collection could be twofold with regard to the knowledge of the teachers who participate and if they implemented any of the teaching techniques employed in the professional development unit. Although this time frame may be too short to be able to analyze PCK, MCK and science knowledge are certainly possibilities.

Lastly, given a larger sample size, comparing different demographic is a possible avenue for future research. The elementary education field is an increasingly female dominate profession (Ingersoll, Merrill, & Stuckey, 2014) and comparing males to females could yield interesting results.
Reflection

When examining stepping stones for future success in mathematics at the elementary grade levels, specific keystone concepts stand out as gatekeepers. These foundational concepts include rational numbers concepts and exponents (Bezuk & Cramer, 1989; Brown & Quinn, 2007; Common Core Standards Initiative, 2010; Confey & Smith, 1995). Students need ample instruction in these areas to be successful in subsequent algebra-based courses, therefore the focus shifts to how both teachers and preservice teachers at these grades levels also understand these supporting mathematical pieces. Mathematical knowledge of the teacher alone is not an indicator of better teaching and learning of their students–pedagogy is also essential (Tirosh, 1999). Teacher knowledge of mathematics in combination with their beliefs, impact student learning (Fey, 1979; Mazzocco, Feigenson, & Halberda, 2011). For these reasons, knowledge on the pedagogy of mathematical content is critical for success (Ball et al., 2005; Shulman, 1986; Thompson & Thompson, 1996). However, mathematics without a contextual interpretation can be troublesome for many students who struggle with concepts (Brady & Bowd, 2005). Putting mathematics into a delivery system, like astrobiology excites the mind because the name alone conjures images of science fiction. Astrobiology is also appealing in the educational sense due to its interdisciplinary nature (Des Marais et al., 2008; Foster & Drew, 2009; Quinlan, 2015); there are so many different teaching opportunities that can be investigated.

The significance of the study was probing on the development of astrobiology educational materials effect on MCK and PCK of the involved participants.

The aims of this study were as follow:
1. teach the preservice elementary education teachers the involved critical
dmathematical concepts in the context of a scientific background,
2. use astrobiology as a platform to enhance their MCK and PCK so they, in
turn, can better prepare their future students, and
3. reduce the cascading effect of rote memorization on their students.
The goals and objectives of this study were the following:
1. Goal 1: Significantly increase the PCK of preservice teachers
   a. Objective 1: Use the modules to highlight pedagogical content and
      engage students in discussions on how to use pedagogy in their
      classrooms.
   b. Objective 2: Use pretest and posttest results to determine the
effectiveness of the modules on pedagogical competence.
2. Goal 2: Significantly strengthen the MCK of preservice teachers
   a. Objective 1: Use the modules to highlight mathematical content
      and to engage students in discussions on how to solve the subject
      matter.
   b. Objective 2: Use the pretest and posttest results to determine the
effectiveness of the modules on mathematical abilities.
3. Goal 3: Expand understanding of astrobiology content to increase science
   subject matter knowledge
   a. Objective 1: Use the modules to highlight astrobiology content and
      to engage students in discussions on how to use the subject matter
      in an interdisciplinary structure.
b. Objective 2: Use the pretest and posttest results to determine the effectiveness of the modules on astrobiology knowledge.

Aims (1) and (2) of the study were met as were the objectives of each goal. Success at the objective level, unfortunately, did not directly correspond to overall success for the corresponding goals. This study examined two different teaching modules. However, there was an issue when associating the results with existing literature. Due to the non-existing research on using astrobiology as a platform to deliver mathematical content to increase MCK and PCK, the study was unable to make any comparisons. The lack of research done on this specific topic made it difficult to do any correlations as there is no existing literature in the cross-section of astrobiology education and increasing MCK and PCK. This study can serve to help fill the gap in the literature for subsequent research.

**Final Conclusions**

Preservice elementary educators’ understanding of mathematical concepts and pedagogy is fundamental to the success of their future students, and contextualizing mathematics is important for comprehension. This study examined a subset of keystone concepts presented under the canopy of astrobiology and demonstrated there needs to be more research on the implications that astrobiology can have on MCK and PCK. This study was unable to statistically indicate positive results regarding the use of astrobiology as a conduit of mathematical information; however, due to the limitations of the study, the conclusive finding should not be indicative of final inferences. There were limiting factors that, if corrected, could have produced more significant results. This investigation incorporates mathematics and astrobiology together for exploration and can serve as a
phase in developing more studies that examine the two disciplines together.

Understanding the limitations and further development of instruments can have an impact on future research and the implication of subjects working together harmoniously to achieve an end product, or increased knowledge, that is greater than either could produce separately.
REFERENCES


APPENDIX A: IRB APPROVAL

INSTITUTIONAL REVIEW BOARD
for the Protection of Human Subjects in Research
FWA 00000078
Research & Creative Scholarship
University Hall 116
University of Montana
Missoula, MT 59812
Phone 406-243-6672 | Fax 406-243-5330

Date: December 26, 2014

To: Meredith Berthelson, Interdisciplinary Studies
Dr. Ke Wu, Mathematical Sciences

From: Paula A. Baker, IRB Chair and Manager

RE: IRB #225-14: “Using Astrobiology as a Platform to Improve Mathematical Abilities of Pre-Service Elementary Education Majors”

Your IRB proposal cited above has been approved under the exempt category of review by the Institutional Review Board in accordance with the Code of Federal Regulations, Part 46, section 101. The specific paragraph which applies to your research is:

X. (b)(1) Research conducted in established or commonly accepted educational settings, involving normal educational practices, such as (i) research on regular and special education instructional strategies, or (ii) research on the effectiveness of or the comparison among instructional techniques, curricula, or classroom management methods.

Each written consent form used for this project must bear the dated and signed IRB stamp. Use the PDF sent with this approval notice as a “master” from which to make copies for the subjects.

University of Montana IRB policy does not require you to file an annual Continuation Report for exempt studies as there is no expiration date on the approval. However, you are required to notify the IRB of the following:

Amendments: Any changes to the originally-approved protocol must be reviewed and approved by the IRB before being made (unless extremely minor). Requests must be submitted using Form RA-110.

Unanticipated or Adverse Events: You are required to timely notify the IRB if any unanticipated or adverse events occur during the study, if you experience an increased risk to the participants, or if you have participants withdraw from the study or register complaints about the study. Use Form RA-111.

Please contact the IRB office with any questions at (406) 243-6672 or email irb@umontana.edu.
APPENDIX B: SUBJECT INFORMATION AND INFORMED CONSENT

SUBJECT INFORMATION AND INFORMED CONSENT

Study Title: Using Astrobiology as a Platform to Improve Mathematical Abilities of Pre-Service Elementary Education Majors

Investigators:

Meredith Berthelson
Graduate School
Lomason Center
224-Griz Central
Missoula, MT 59812
406-243-2572

Faculty Advisor:
Dr. Ke Wu
University of Montana
Math 201
Missoula, MT 59812
406-243-4818

Special Instructions:
This consent form may contain words that are new to you. If you read any words that are not clear to you, please ask the person who gave you this form to explain them to you.

Purpose:
The purpose of this research study is to determine if mathematics presented in the context of Astrobiology can improve content and pedagogical skills of students enrolled in Mathematics for Teachers I (M135) at the University of Montana during the spring semester of 2015. The results will be used for a doctoral dissertation as well as various publication submissions. You must be 18 or older to participate in this research.

Procedures:
You will be asked to participate in modules that contain mathematics in context of Astrobiology for 1-2 classes depending on the class time duration. A background survey and pretest will be done prior to the module activity. You will also be required to complete a posttest and exit survey. The study will take place in the normal classroom for M135, LA 235, and during the normal meeting time for the course. The surveys and pre-posttests will take approximately 45-60 minutes to complete outside of the scheduled class time.
Risks/Discomforts:
There is no anticipated discomfort for those contributing to this study, so risk to participants is minimal. Mild discomfort may result from learning new pedagogies and content knowledge. To minimize discomfort, the use of place-based pedagogy, cognitively-guided instruction, and a hands-on approach is going to be used as the teaching method in a collaborative, supportive environment. Answering the questions may cause you to think about feelings that make you sad or upset.

Benefits:
There is no promise that you will receive any benefit from taking part in this study.
Your participation in this study may help understand if Astrobiology can be a platform for teaching mathematics.
Although you may not benefit from taking part in this study, the Mathematical/Astrobiology modules are designed to help expand options of teaching science and math-based education. Participants will benefit from the opportunity to learn from more STEM materials.

Confidentiality:
Your records will be kept confidential and will not be released without your consent except as required by law.
Your identity will be kept private.
If the results of this study are written in a scientific journal or presented at a scientific meeting, your name will not be used.
The data will be stored in a locked file cabinet in a locked office.
Your signed consent form will be stored in a cabinet separate from the data.

Voluntary Participation/Withdrawal:
Your decision to take part in this research study is entirely voluntary.
You may refuse to take part in or you may withdraw from the study at any time without penalty or loss of benefits to which you are normally entitled.
If you decide to withdraw, you can do so without repercussion to your grade.
You may leave the study for any reason.
You may be asked to leave the study for any of the following reasons:
1. Failure to follow the Project Director’s instructions;
2. A serious adverse reaction which may require evaluation;
3. The Project Director thinks it is in the best interest of your health and welfare; or
4. The study is terminated.

Questions:
If you have any questions about the research now or during the study, contact: Meredith Berthelson at 406-243-6813 or email: meredith.berthelson@umontana.edu or Professor Wu at ke.wu@mso.umt.edu.
If you have any questions regarding your rights as a research subject, you may contact the UM Institutional Review Board (IRB) at (406) 243-6672.

**Statement of Your Consent:**
I have read the above description of this research study. I have been informed of the risks and benefits involved, and all my questions have been answered to my satisfaction. Furthermore, I have been assured that any future questions I may have will also be answered by a member of the research team. I voluntarily agree to take part in this study. I understand I will receive a copy of this consent form.

__________________
Printed Name of Subject

__________________  ____________________
Subject's Signature  Date

**Statement of Consent to be Photographed**
I understand that photographs may be taken during the study. I consent to having my photograph taken. I consent to use of my photograph in presentations related to this study. I understand that if photographs are used for presentations of any kind, names or other identifying information will not be associated with them.

__________________  ____________________
Subject's Signature  Date
APPENDIX C: M1 - GROWTH CURVE MODULE LESSON PLAN

Growth Curve
Grade level: Pre-service elementary educators
Time of Lesson:
1-2 hours
Original activity sources:
http://www.uta.edu/math/gk12/Lessons/L4-6%20Exponential%20growth%20and%20delay.pdf

Preface of Modules design – Astrobiology:
Microorganisms are an important aspect of Astrobiology. Scientists learn about how early these life forms have survived, evolved and died on Earth in extreme habitats, helping them extrapolate how this might occur on other planets. Examining a microorganism growth curves will be a great introduction into exponential growth.

Overview:
The purpose of this activity is for participants to understand what exponential growth means in terms of microorganisms. They will examine three stages of growth dynamics: Lag phase, exponential (log) phase and stationary phase. They will also hypothesize what will occur to the batch culture after the stationary phase (death phase). Using this information, participants will then be asked to postulate why learning about microorganism is important to not just science but specifically Astrobiology. Participants will explore exponents, exponential equations and how their graphs differ from linear equations.

Materials:
- Clear sheet protector or transparency sheets
- Tissue paper (enough for approximately 10 sheets)
- Flashlight
- Student Worksheet 1
- Student Worksheet 2

Preparation:
- May need to cut tissue paper down to make layers.

Activity Instructions
- Discussion/Short Lecture: (Have student write down their ideas before you ask for their answers.)
  - Ask participants what they know about microorganisms. List some of these ideas on the board. (Sample responses: too small to see with the naked eye, causes disease, cell structure can be different.)
  - Ask participants how do these microorganisms reproduce or replicate? (Sample responses: sexual, asexual, divide.)
    - State that Asexual or cell division is correct for most microorganisms and this is called binary fission. However, bacteria can transfer gene information through conjugation. Conjugation is when bacteria cells transfer their genetic information through contact via a bridge like connection.
Ask the participants where can they find microorganisms? (Responses will vary.) Ask participants if they could find them in: boiling water, ice, acid or salt?

- **Boiling water?** ANSWER: Yes, they are called thermophiles or hyperthermophiles.
  - Where would they find these? ANSWER: Hot springs, deep sea vents
- **Ice?** ANSWER: Yes, they are called psychrophiles or cryophiles
  - Where would they find these? ANSWER: Antarctica, glaciers
- **Acid?** ANSWER: yes, they are called acidophiles
  - Where would they find these? ANSWER: Sulpheric pools, geysers
- **Salt?** ANSWER: Yes, they are called halophiles
  - Where would they find these? ANSWER: Great Salt Lake, Dead Sea, evaporated ponds

State: These are examples of extremophiles. That means they thrive in extreme environments where most other organisms cannot. (-phile comes from the Greek *philia* which means “love”)

State: Scientists study extremophiles to examine how life may have begun and thrived on early Earth and how life might survive in similar environments on other planets.

Ask the participants why they think studying extremophiles would be important?

(Responses will vary.) Scientists study extremophiles to examine how life may have begun and thrived on early Earth and how life might survive in similar environments on other planets.

State: Scientist have studied many microorganisms and have found how long it takes them to divide, however, there are some that are difficult to measure because some are too small to count effectively.

- Have participants form groups of four people in a timely manner. Hand out student worksheets.
- **Group work:** If you were to graph cell replication what do you think it would look like? Let’s take an example of a cell’s replicating once every hour. [Have participants work on the student handout and walk around the groups to assist when necessary.]
- **Group Discussion Question:** [Participants may get stuck on the second page, divert the class’s attention to assess the need for assistance.] Ask participants what pattern they have observed from the table.
  - **ANSWER:** $2^0, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6$
  - How fast is this growing? [by 2 each time] Would this be considered the growth factor of the organism? [How fast something is growing/decaying.]
  - How much is the exponent increasing by each time it replicates?
  - **ANSWER:** the exponent increases by a factor of one each time: time interval. [Therefore just $t$ instead of something like $t+1$ or $t-2$.]
  - This would be considered the time interval of the growth the organism: $2^t$
  - Ask students how do they know where to start? How do we know we didn’t start at 15 or 200?
○ **ANSWER:** They started at 1, as indicated by the chart. Therefore, this is the initial value.

○ What do we know so far?
  - Growth factor: 2
  - Time interval: $t$
  - Initial Value: 1

○ The general format for the exponential growth model is:
  \[ y = (\text{initial amount}) \cdot (\text{growth factor})^{\text{time interval}} \]
  **Have students plug in numbers to verify validity.**

- **Group Discussion Question (COMMON STUDENT ERROR):** Ask the participants if they saw their students do this with the problem, how could they help them understand their mistake?
  \[ 2^1 = 2, \quad 2^2 = 4, \quad 2^3 = 6 \]

- **Group Discussion:** Ask student about hour 0. Have them return to the second page and look at what they put down in the pattern for the y-intercept.
  - $2^0 = 1$, Why

- **Group Discussion Question (COMMON STUDENT ERROR):** Ask the participants if they saw their students do this with the problem, how could they help them understand their mistake?
  \[ 2^0 = 0 \]
  **ANSWER:** Let’s look at the exponents in a backwards fashion, counting down to zero so we can look at what happens:

<table>
<thead>
<tr>
<th>Exponent</th>
<th>Base</th>
<th>Pattern</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>$2^3$</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>$2^2$</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>$2^1$</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>$2^0$</td>
<td>1</td>
</tr>
</tbody>
</table>

Exponent Rules:
\[ \frac{2^4}{2^2} = 2^{4-2} = 2^2 = 4 \quad \text{and} \quad \frac{2^4}{2^3} = 2^{4-3} = 2^1 = 2 \]
So: $\frac{2^4}{2^4} = 2^0$ but also $2^{4-4} = 2^0$ therefore $2^0 = 1$

- **Discussion:** Let’s look at a base that is less than 1 but in the normal counting up method so we can compare what a base that is greater than 1 compares to a base that is less than one:

<table>
<thead>
<tr>
<th>Exponent</th>
<th>Base</th>
<th>Pattern</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>$2^3$</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>$2^2$</td>
<td>4</td>
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<tr>
<td>1</td>
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<td>$2^1$</td>
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<td>0</td>
<td>2</td>
<td>$2^0$</td>
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<tr>
<td>Exponent</td>
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<tr>
<td>0</td>
<td>1/2</td>
<td>(1/2)^0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1/2</td>
<td>(1/2)^1</td>
<td>1/2</td>
</tr>
<tr>
<td>2</td>
<td>1/2</td>
<td>(1/2)^2</td>
<td>1/4</td>
</tr>
<tr>
<td>3</td>
<td>1/2</td>
<td>(1/2)^3</td>
<td>1/8</td>
</tr>
<tr>
<td>4</td>
<td>1/2</td>
<td>(1/2)^4</td>
<td>1/16</td>
</tr>
<tr>
<td>5</td>
<td>1/2</td>
<td>(1/2)^5</td>
<td>1/32</td>
</tr>
</tbody>
</table>

- **Group Discussion Question (COMMON STUDENT ERROR):** Ask the participants if they saw their students do this with the problem, how could they help them understand their mistake?
  \[
  \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{1}{6}
  \]

- **Discussion Question:** What is occurring in the pattern?
  **ANSWER:** The numbers are decreasing and getting really small.

- **Discussion Question:** Will the numbers ever get to zero? What would the graph look like?
  **ANSWER:** No, but really close.

- **Discussion Question:** So does this look like growth?
  **ANSWER:** No, the graph is getting smaller. This is opposite of growth, decay.

- **Discussion Question:** Why would this be important?
  **ANSWER:** May vary.

- **Discussion/Short Lecture:** Carbon Dating: What is it and why is it important?
  - Why is carbon dating or knowing decay rates important to Astrobiology?
Potential answers/exploratory prompts:
- Inoculation from asteroids, collisions on other celestial bodies.
- Why are these contaminations important?
  - Biological elements die off at some rate.
  - What is the likelihood of survival of biological elements?
  - Does distance affect death rate?
  - How could these biological elements survive such harsh environments in space? (e.g. Mars, Europa, etc.) How can we study the potential environments here on Earth?
  - Can we predict if contamination is successful?

- Discussion Question: Back to our original graph of \( y = 2^t \) that you have just created an exponential growth model but without limit. Why is this that important?
  
  **ANSWER:** (Student answers may vary.) There is a carrying capacity that limits growth. Carrying capacity of a biological species is the upper limit of the population size that the environment can sustain.

- Discussion Question: What happens to the graph as \( t \) increases?
  
  **ANSWER:** The graph gets large very fast.

- Demonstration: Scientists often use something called Optical Density, called OD\(_{600}\), as a way of measuring the cell density in a sample. They take “OD” readings in a timed manner. Depending on what organism they are studying, they may take OD readings every hour, every 30 minutes or every 2 hours. They are looking for population growth, which is the difference between “births” and “deaths” so therefore cell division (or “births”) is greater than cell death.
  - Hold up a sheet protector and shine the flashlight through to show how much light passes through. This would be a representation of very little microorganisms, as the light is not disrupted. (The cells [microorganisms] are not absorbing the light.) They might get the same or similar OD reading for quite a while.
  - Next, hold up a sheet of tissue paper behind the sheet protector. This would be a representation of the cells dividing and becoming denser in the sample. (The cells are absorbing the light.)
  - Repeat with layers of tissue paper. The cells are multiplying and the sample is “growing” in density.

- Hand out second student worksheet.

- Short Lecture: Scientists need to interpret their data. On your second worksheet you will see some data points from a species of *Sulfolobus* (sulf-oh-low-bus). *Sulfolobus* is the genus name for an extremophile microorganism that typically lives in volcanic hot springs such as in Yellowstone National Park. They thrive in not only hot temperatures but also acidic waters. They are found in one
of the kingdoms called Archaea. The Archaea domain or kingdom consists of single-celled microorganisms. These microorganisms or microbes are prokaryotes, which means they have no cell nucleus or any other membrane-bound organelles in their cells.

- **Group Activity:** Tell the participants that you want them, in their groups, to interpret the data as the scientist. On the second activity sheet, there is a table of data. Plot the points on the graph, be sure to have them label and number their axis. Instruct them to also label your graph with the growth phases:
  - **Stationary Phase:** When cell births = cell deaths and the sample ceases to grow.
  - **Exponential Growth Phase:** When cells are dividing at a constant rate and cell births > cell deaths.
  - **Lag Phase:** When the cells have been inoculated into the medium, the population remains temporarily unchanged.
  - **Death Phase:** The number of viable cells decreases as they may have exhausted their available nutrients, space or have too much waste products that

- **Discussion Questions:**
  - Ask the groups how they labeled their phases of growth?
  - How did they determine where these phases began and ended?

- **Group Activity:** On the second side of the second activity sheet is another graphing activity. Participants will graph the made up organisms and interpret the data. Tell them to be sure to do the tables before they start graphing. And for this part of the activity they don’t have to worry about the growth phases.
  - There are 4 organisms. If there are groups of four participants, each student in the group can take one organism. Tell the participants to explain to their group how they came up with their points.

- **Discussion Questions:** (Draw the graph of the Sacriophile on the board.)
  - Ask the groups if what you drew on the board is accurate for the Sacriophile?
  - Ask how the other graphs compared to the first one.

**Assessment:**
- **Formative:** Group discussions questions
  - Ask participants what they know about microorganisms.
Ask participants how do these microorganisms reproduce or replicate?
Ask the participants where can they find microorganisms? (Responses will vary.)
Ask participants if they could find them in: boiling water, ice, acid or salt?
Ask the participants why they think studying extremophiles would be important?
If you were to graph cell replication what do you think it would look like?
Ask participants what pattern they have observed from the table.
Would this be considered the growth rate of the organism?
How fast is this growing? Would this be considered the growth factor of the organism?
How much is the exponent increasing by each time it replicates?
Ask students how do they know where to start? How do we know we didn’t start at 15 or 200?
Ask the participants if they saw their students do this with the problem, how could they help them understand their mistake?
Ask student about hour 0.
$2^0 = 1$, Why
Ask the participants if they saw their students do this with the problem, how could they help them understand their mistake?
Ask the participants if they saw their students do this with the problem, how could they help them understand their mistake?
What is occurring in the pattern?
Will the numbers ever get to zero? What would the graph look like?
So does this look like growth?
Why would this be important?
Carbon Dating: What is it and why is it important?
Why is carbon dating or knowing decay rates important to Astrobiology?
Why are these contaminations important?
What is the likelihood of survival of biological elements?
Does distance affect death rate?
How could these biological elements survive such harsh environments in space? (e.g. Mars, Europa, etc.) How can we study the potential environments here on Earth?
Can we predict if contamination is successful?
Back to our original graph of $y = 2^t$ that you have just created an exponential growth model but without limit. Why is this that important?

What happens to the graph as $t$ increases?
Ask the groups how they labeled their phases of growth?
How did they determine where these phases began and ended?
Ask the groups if what you drew on the board is accurate for the Sacriophile?
Ask how the other graphs compared to the first one.

**Summative:**
- Student handout 1
- Student handout 2
Astrobiology

- Astrobiology is the study of the origin, evolution, distribution, and future of life in the universe. This multidisciplinary field encompasses the search for habitable environments in our Solar System and habitable planets outside our Solar System, the search for evidence of prebiotic chemistry and life on Mars and other bodies in our Solar System, laboratory and field research into the origins and early evolution of life on Earth, and studies of the potential for life to adapt to challenges on Earth and in space.

https://astrobiology.nasa.gov/about-astrobiology/
What are microorganisms?

- Ask your students what they know about microorganisms.
- Please write down a few things that you know about microorganisms.
- Write down what you know about microorganisms replication.

- A microorganism is a living single-celled organism of microscopic size
- Replication: binary fission but can also transfer gene information through conjugation.

Microorganisms

- Types
  - Bacteria
  - Fungus
  - Archea
  - Viruses
Microorganisms

- Where do you find them?
  - Water?
  - Hot water?
  - Ice?
  - Acid?
  - Salty places?
  - Radioactive places?

- Found:
  - Water? Perfect!
  - Hot water? Thermophiles
  - Cryophiles
  - Acidophiles
  - Halophiles
  - Polyextremophile/Radioresistant

These are examples of extremophiles, microorganisms that thrive in extreme environments where most other organisms cannot. (-phile comes from the Greek philia which means love.)

Microorganisms

- Why do you think studying extremophiles would be important?
  - Scientist study extremophiles to examine how life may have begun and thrived on early Earth. Using this information they can study how life might survive in similar environments in space.
  - How would you "examine" microorganisms? What would some of the elements you would look for?
Group Work

- Get into groups of 3-4
- If you were to graph cell replication, what do you think it would look like? For example if a cell replicates once every hour:

<table>
<thead>
<tr>
<th>Hours</th>
<th># of Cells</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>32</td>
</tr>
<tr>
<td>6</td>
<td>64</td>
</tr>
</tbody>
</table>

Slide 8

Group Work – Cell Replication

- What patterns do you see in the table?
- What pattern do you see?
- How fast is this microorganism growing?
- What’s happening to the exponent when the time increases?
- Can you predict at time t how many cells there are?
- What would the equation (or function) look like?
- *(Handout)*
Slide 9

Cell Replication

- What pattern do you see?
  - $2^0$, $2^1$, $2^2$, ...
- How fast is this microorganism growing?
  - By a factor of $2^t$
  - Growth Factor
- What's happening to the exponent when the time increases?
  - Increases by 1 each time interval
  - Time interval
- Can you predict at time $t$ how many cells there are?
  - $2^t$
- What would the equation (or function) look like?
  - $y=2^t$
- How do we know we didn't start at 15 or 200? Does it matter? Would this change the equation?

Slide 10

Cell Replication

- What do we know so far?
  - Growth factor: ____
  - Time interval: ____
  - Initial value: ____
- Our equation: $y=2^t$
- Given what we know and what we have come up with as our equation what would be a “general” format for an exponential growth model?

$$y = (\text{initial amount})(\text{growth factor})^{\text{time interval}}$$
Exponents

- COMMON STUDENT ERROR:
  \[ 2^1 = 2, \quad 2^2 = 4, \quad 2^3 = 6 \]

- Group Discussion: What about hour 0?
  \[ 2^0 = 1 \]

- Why? Why not \( = 0 \)?

\[
\frac{3^0}{2^1} = 1 \\
\frac{6^0}{2^1} = 1 \\
\frac{429^0}{2^1} = 1
\]

Exponential Rules

\[
\frac{2^4}{2^2} = 2^{4-2} = 2^2 = 4
\]

\[
\frac{2^4}{2^7} = 2^{4-7} = 2^{-3} = 2 
\]

\[
\frac{2^4}{2^1} = 1 \text{ but also...}
\]
Slide 13

Exponents

<table>
<thead>
<tr>
<th>Exponent</th>
<th>Base</th>
<th>Pattern</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>⅓</td>
<td>(1/2)^0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>⅓</td>
<td>(1/2)^1</td>
<td>⅓</td>
</tr>
<tr>
<td>2</td>
<td>⅓</td>
<td>(1/2)^2</td>
<td>⅔</td>
</tr>
<tr>
<td>3</td>
<td>⅓</td>
<td>(1/2)^3</td>
<td>⅔</td>
</tr>
<tr>
<td>4</td>
<td>⅓</td>
<td>(1/2)^4</td>
<td>1</td>
</tr>
</tbody>
</table>

- What is occurring in the pattern?
- Will the numbers ever get to zero?
- What would the graph look like?

Slide 14

Y = (1/2)^x

- So does this look like growth?
- Why would this (graph, information) be important?
- COMMON STUDENT ERROR:

\[
\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{1}{6}
\]

- How would you help them understand their mistake?
Exponents

- Back to the original equation $y=2^t$.
- This is exponential growth but without limit. Why is that important?
  - Carrying capacity
- What happens as $t$ increases?
- How do scientists measure microorganisms?
  - Optical Density 600 (OD$_{600}$) Readings. Measures a culture growing in density.

Data

- *Sulfolobus solfataricus* (Sulf-o-low-bus sole-fuh-tar-ic-us)
- Lives in volcanic hot springs.
- Extremely hot temperatures: 75-80°C (167-176°F)
- Acidic water: pH 2-3
- Domain or Kingdom:
  - Archaea
- Which type: eukaryotes or prokaryotes? (what’s the major difference?)
  - Prokaryotes
Growth Phases

- **Stationary Phase:**
  - When cell births = cell deaths and the sample ceases to grow.

- **Exponential Growth Phase:**
  - When cells are dividing at a constant rate and cell births > cell deaths.

- **Lag Phase:**
  - When the cells have been inoculated into the medium, the population remains temporarily unchanged.

- **Death Phase:**
  - The number of viable cells decreases as they may have exhausted their available nutrients, space or have too much waste products that

---

Graphing

- #4 on handout
  - a. Sacriophile
  - b. Bigilophile
  - c. Gremophile
  - d. Oiligophile
APPENDIX E: M2 - SOLAR SYSTEM MODULE LESSON PLAN

Solar System Scaled Measurement
Grade level: Pre-service Elementary Educators
Time of Lesson: 1-2 hours
Original activity sources: http://www.exploratorium.edu/ronh/solar_system/

Preface of Modules design – Astrobiology:
The study of our own solar system is important to understand the vastness of the known universe. For participants to comprehend, the distances between the planets from our sun will strengthen their spatial reasoning skills and introduce them to proportions and proportional reasoning.

Overview:
This module is designed to get pre-service elementary educators (participants) interested in space science and how mathematics relates to real-world situations. Starting with warm-up problems involving fractions, participants will start to recall information and get into the habit of thinking rationally. They then greater understand the properties of proportions through the rules of fractions and what errors or misunderstandings their potential future students will come across.

Materials:
- Two pieces of rope 8cm & 11cm long
- Cardboard/cardstock
- Markers
- Paper/Pencils
- Tape
- Calculator
- Rulers (paper or regular)/meter sticks
- Fabric tape measure – 1 for each group.
- Basketball, peppercorn, paperclip, pinhead or small bead
  (If you determine the sun to be the basketball you will need items that are approximately 2mm (Earth & Venus), and 1mm (Mercury and Mars) in diameter. You could also use Play-Doh and have students make the planets after they determine their scaled size.)
- Various other round objects that are smaller than the basketball: baseball, golf ball, etc.
- Student Activity Sheet

Preparation:
- Make sure you have a tape measure that is long enough to complete the activity. If you go with the given measurements, you will need enough for 40 meters for the orbit of Mars.
- If you have access to a long hall or outside, this would be a good choice for the space needed.
- Make sure your participants can easily get into small groups of approximately 4 people.
Activity Instructions
- **Warm up**: Have participants do a couple of warm up problems from fractions.
  
  $\frac{6}{27} \times \left( \frac{3}{3} \right) = \frac{2}{9}$

  $\frac{20}{36} \times \left( \frac{4}{4} \right) = \frac{5}{9}$

  $\frac{52}{4} \times \left( \frac{4}{4} \right) = \frac{13}{1} = 13$

- **Discussion Question**: Ask participants how they arrived at their answers and how they expect their students to arrive at their answers. [Greatest Common Factor should be among the responses.]
- Have participants form groups of four in a timely manner. Using their warm up exercise in greatest common factors have them discuss and solve a word problem:
- **Group work**: (Hand out the two pieces of rope.) If a piece of rope 8 cm long weighs 48 grams. What will an 11 cm length of the same cable weigh? (Go around to the groups to ensure they are on task and to give hints as to where they could go if they are stuck.)

Possible Answers:
**Most groups will probably set this problem up in a ratio/proportion method. Ask the participants how they anticipate their potential middle school students to answer this question without any knowledge of ratio or proportion? Ask the participants how do they know this works?**

Set up the problem with the two scenarios:

**Scenario 1**: How do you get from 8 to 48? $8 (\pm, \times, +) \left( \frac{6}{6} \right) \rightarrow 48$

$$8 (\pm, \times, +) \left( \frac{6}{6} \right) \rightarrow 48 \quad \Rightarrow \quad \frac{8}{11} \times \left( \frac{6}{6} \right) = \frac{48}{?} \quad \Rightarrow \quad \frac{8}{11} \left( \frac{6}{6} \right) = \frac{48}{?} \quad \Rightarrow$$

$$\frac{8}{11} \left( \frac{6}{6} \right) = \frac{48}{66}$$

Answer: 66 grams

**Scenario 2**: What if this set up occurred in any of the groups: $\frac{8}{48} = \frac{11}{?}$

Left side of the equation sign can be simplified: $\frac{8}{48} + \left( \frac{8}{8} \right) = \frac{1}{6}$
Using the simplification: \( \frac{1}{6} = \frac{11}{?} \) What would you multiply 1 by to get 11?

\[
\frac{1}{6} \cdot \left( \frac{11}{?} \right) = \frac{11}{?}
\]

Therefore: \( \frac{1}{6} \cdot \left( \frac{11}{11} \right) = \frac{11}{66} \) Answer: 66 grams

**Group Discussion Question:** Ask the participants about what they did and how they used the rules of fractions to solve a proportion problem (if they used fractions and have not done cross-multiplication yet). Ask them if there is another way to look at this problem and try to simplify it so there were no fractions involved.

**ANSWER:** Multiply by the Least Common Denominator (LCD), which is in this case, are the two denominators multiplied together (Let’s replace the \( ? \) with something else. Something that will represent the unknown number better: \( x \).

\[
\frac{8}{11} = \frac{48}{x} \quad \text{Now let’s use the LCD (ask students what would that look like?)}
\]

\[
11 \cdot x = 11x
\]

\[
\left( \frac{11 \cdot x}{1} \right) \cdot \left( \frac{8}{11} \right) = \left( \frac{48}{x} \right) \cdot \left( \frac{11 \cdot x}{1} \right)
\]

Since the \( \cdot \) means multiplication, so does putting two parentheses together like: (2)(4) = 8. So we can look at parentheses that are next to one another as implied multiplication.

\[
\left( \frac{x}{1} \right) \left( \frac{8}{1} \right) = \left( \frac{48}{1} \right) \left( \frac{11}{1} \right) \Rightarrow \frac{(x)(8) = (48)(11)}{8x = (48)(11)} \Rightarrow \frac{8x}{8} = \frac{528}{8} \Rightarrow \frac{x}{1} = \frac{66}{1}
\]

**Group Discussion Question (COMMON STUDENT ERROR):** Ask the participants if they saw their students do this with the problem, how could they help them understand their mistake?

\[
\frac{11}{8} \quad \frac{48}{3}
\]

\[
\begin{array}{c|c|c}
11 & 48 \\
-8 & -3 & \text{Answer: 45} \\
3 & 45
\end{array}
\]

**Group Discussion Question:** Ask the participants if demonstrating ratio/proportions in this manner gives them a better understanding of the rule, “cross, multiply and divide?” Tell them they can then instruct their students to talk in their groups about the two different sets and see if they can come up with a general rule:

\[
\frac{8}{11} = \frac{48}{x} \quad \text{and} \quad 8 \cdot x = 48 \cdot 11?
\]

General rule for proportions: \( \frac{a}{b} = \frac{c}{d} \quad \text{then} \quad a \times d = b \times c \quad \text{therefore cross multiplication works like using the least common denominator to simplify the fraction. To include algebraic skills, have the participants ask if: } 8 = 1x \text{ is the same...}
as: \( x = 8 \) (Communicative Property \( a \times b = b \times a \)). They can demonstrate this further with examples such as \( 3 \times 2 = 6 \) same as \( 6 = 3 \times 2 \)?

- **Group Discussion Question (COMMON STUDENT ERROR):** Ask the participants if they saw their students do the two following errors with the problem, how could they help them understand their mistakes?
  
  \[
  \frac{8}{11} = \frac{48}{x} \quad \Rightarrow \quad 8 \times 48 = 11x \quad \text{or} \quad \frac{8}{11} = \frac{x}{48} \quad \Rightarrow \quad 8 \times 48 = 11x
  \]

- **Discussion Question:** Ask student if they can give examples where would we need to use proportions? Lead them into a discussion about the sun and planets in the solar system.

- **Short Lecture:** Ask them about size of Earth and how it compares to the sun. Further the discussion with comparisons of the sun to the other planets like Mars. If participants are unaware of NASA’s Mars Exploration Rover Mission (http://mars.jpl.nasa.gov/mer/overview/), take a few minutes to talk with them about the two robots, Spirit and Opportunity, which have spent 10 years on the surface of Mars (http://mars.nasa.gov/mer10/). Prompt participants to discuss the time it would take to travel to Mars. How realistic is it for people to go to Mars? Talk about circular orbits vs. elliptical orbits and the differences in the two and how that affects when planets are close to one another in their orbits. Lead this into a larger discussion on travel to other planets. How feasible is it?

- ***SET UP DIFFERENT ROUND OBJECTS IN THE FRONT OF THE ROOM.***

- **Hand out Student worksheet 1:** Estimation. Have participants go to the front of the room in their small groups (one at a time) to estimate what objects they think are the Sun, Mercury, Venus, Earth and Mars. As the groups are cycling through looking at the round objects begin class discussion.

- **Class Discussion:** What are some of the dangers of going to another planet? (e.g., as a robot excursion like the Rover Missions, and as a species? What are the dangers of coming back to Earth?) Ask participants if we have ever found life outside of our planet in our known solar system. Do we know where have we gone so far?
  
  o Ask participants what they know about microorganisms. List some of these ideas on the board. (Sample responses: too small to see with the naked eye, causes disease, cell structure can be different.)
  
  o Ask participants how do these microorganisms reproduce or replicate? (Sample responses: sexual, asexual, divide.)
  
  ▪ State that Asexual or cell division is correct for most microorganisms and this is called binary fission. However, bacteria can transfer gene information through conjugation. Conjugation is when bacteria cells transfer their genetic information through contact via a bridge like connection. (Drawing pictures of this can help participants and students understand the concepts better.)
  
  o Ask the participants where can they find microorganisms? (Responses will vary.) Ask participants if they could find them in: boiling water, ice, acid or salt?
  
  ▪ Boiling water? **ANSWER:** Yes, they are called thermophiles or hyperthermophiles.

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- Where would they find these? **ANSWER:** Hot springs, deep sea vents
  - Ice? **ANSWER:** Yes, they are called psychrophiles or cryophiles
  - Where would they find these? **ANSWER:** Antarctica, glaciers
- Acid? **ANSWER:** Yes, they are called acidophiles
  - Where would they find these? **ANSWER:** Sulpheric pools, geysers
- Salt? **ANSWER:** Yes, they are called halophiles
  - Where would they find these? **ANSWER:** Great Salt Lake, Dead Sea, evaporated saline ponds

  **o State:** These are examples of extremophiles. That means they thrive in extreme environments where most other organisms cannot. (-phile comes from the Greek *philia* which means “love”)
  **o Ask the participants why they think studying extremophiles would be important?**
    - (Responses will vary.) Scientists study extremophiles to examine how life may have begun and thrived on early Earth and how life might survive in similar environments on other planets.

- **Group Work:** “Let’s start to scale our solar system into something we can represent so we can get an idea of the different sizes and distances. Let’s start with the largest object on the table, which would be…?” (Let participants answer = basketball.) [Start participants with using the largest celestial body in our solar system as the measurement by which all others will be scaled by, the sun.] First we need to determine a size we want to represent the sun. Do we know the diameter (be sure they use centimeters)? No, but we can measure the circumference. How can we find the diameter from the circumference? $C = \pi \cdot d$. (Have participants manipulate the formula to: $d = \frac{C}{\pi}$.) Measure the basketball for them so all groups have the same number to scale the planets.

- **Group Work:** Have 1 participant from each group come to the front to take 1-2 items and tape measures (fabric, paper or regular) to measure the diameter. On the overhead or whiteboard, write down the diameter of each item as the participants discover them. (GIVEN TIME CONSTRAINTS THIS STEP MAY BE GIVEN TO THE PARTICIPANTS.)

- **Discussion:** Now that the participants have found the diameters of the various items, they need to scale the planets down in size. If you use a basketball as the sun, you may have found the diameter to be 24.1cm.

  Ask the participants if the sun is 24.1cm in diameter, what diameter would the earth be? Have the participants work in their groups to set up the proportion problem and solve given the sun is 1,391,900 km; the earth is 12,742 km in diameter.

  \[
  \frac{\text{Actual size}}{\text{Sun (km)}} = \frac{\text{Scaled Size}}{\text{Sun (cm)}} \quad \frac{\text{Earth (km)}}{\text{Earth (cm)}}
  \]

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\[
\frac{\text{sun}}{\text{earth}} \Rightarrow \frac{1391900(\text{km})}{12742(\text{km})} = \frac{24.1(\text{cm})}{E(\text{cm})} \Rightarrow 1391900E = (24.1)(12742)
\]

(Divide each side:) \[
\frac{1391900E}{1391900} = \frac{307082}{1391900}
\]

\[
E = \frac{307082}{1391900} = 0.22\text{cm}
\]

So Earth would be…what? 0.22cm (or 2.2mm) if the sun was 24.1cm.

- **Class Discussion Question:** Would this scaling be the same for all of the planets? Why or why not?
  **Answer:** The planets would be scaled to their size based on what we determine the size of the Sun to be for our purposes.

- **Group Work:** Have participants come up with a general (proportional) formula for scaling the planets proportionally to the sun of 24.1cm.

**General Formula for proportion:**

\[
\frac{\text{Actual size}}{\text{Scaled size}} = \frac{\text{Sun (km)}}{\text{Sun (cm)}} = \frac{1391900(\text{km})}{24.1(\text{cm})} = \frac{\text{Planet (km)}}{\text{Planet (cm)}} = \frac{\text{planet_diameter (km)}}{\text{planet_diameter (cm)}}
\]

**Extension:** Some participants might bring it further to creating a formula (PD=Planet diameter):

\[
PD_{\text{cm}} = \frac{(24.1)(PD_{\text{km}})}{1391900}
\]

- **Student Activity:** On Student Handout Worksheet 1 and have participants scale the rest of the planets using calculators and the scaled sun as 24.1cm (or what you have determined to be the diameter of the basketball, sizes may vary slightly) or (if you have access to computers/tablets) have participants use Excel to create a spreadsheet using the data.

- **Class Discussion:** After the groups have compiled their data have them look at the scaled size using a ruler/meter stick. Some of the planets will be very small (Mercury, Venus, Earth, and Mars). Ask the participants if their estimations or predictions were correct from the beginning of class? Write the measurements on the white board. Next ask the participants about the distance from the Sun to the Earth? How many of them know how far away our planet is from the Sun? Is it a perfect circular orbit? Discuss with participants making a scaled down version of our solar system that is still proportional to actual size. Where would we start? [Sun] What about orbits? Since they are not perfect circles? [Find the average.]

- **Group Discussion Question:** How far away would the planets be using the same proportion? Would Mercury be in the same room? Let’s just examine the four closest planets to the sun. How about Venus, Earth, Mars? Let’s look at Earth’s orbit.
The radius is 149,570,000 km. Have the groups discuss how would they find a scaled orbit of Earth for a couple of minutes and have them report back on their ideas and thoughts. (This section will need some guidance from the teacher to help participants come up with a general formula to determine the scaled radius of each planet.)

**Teacher Guidance:** So if we want to scale the planets rotation around the sun to the new size do we have to use the scaled size of the sun in some way? [Yes.]

We used 24.1 cm for the sun’s diameter. What unit of measurement do we need for the planet’s distance from the sun? [Participants may answer centimeter, if they do ask them for a larger unit since the cm measurement will be quite a large number. If participants come back with kilometers, then ask them if they can see multiple kilometers. Ah, it’s not really scaled back proportionally to diameter of the sun. Have them look at metric measurement scale: mm, cm, dm, m, km.]

**HINT:**

\[
\frac{\text{Planet Actual (km)}}{\text{Planet Scaled (cm)}} = \frac{\text{Orbit Actual (km)}}{\text{Orbit Scaled (cm)}}
\]

Let’s use Earth: 149,570,000 km

\[
\frac{12742}{0.22} = \frac{149570000}{E_{\text{Orbit}}}
\]

\[
123742E_{\text{orbit}} = (0.22)(149570000)
\]

\[
E_{\text{orbit}} = \frac{(0.22)(149570000)}{123742} = 2582.4 \text{cm} \quad \text{(Now need to convert to meters:)}
\]

\[
2582.4 \text{cm} \Rightarrow 258.24 \text{dm} \Rightarrow 25.824 \text{m}
\]

\[
E_{\text{orbit}} = 25.82 \text{m}
\]

Earth’s orbit radius will be 25.82 m if the sun is 24.1 cm in diameter.

**Student Activity:** Have student turn over their Student Activity Sheet 1 and have participants scale the orbit radii of the rest of the planets using calculators and the scaled sun as 24.1 cm or (if you have access to computers/tablets) have participants use Excel to create add to their existing spreadsheet using the data. (GIVEN TIME CONSTRAINTS THIS STEP MAY BE GIVEN TO THE PARTICIPANTS.)

**Group Activity:** After the groups have compiled their data have them look at the scaled distances using a ruler/meter stick. Assign each group a planet (From Mercury to Mars). Have them make an informational board on either cardboard or cardstock about their planet with the following:

- Planet Name
- Diameter in km (Represent if they can with a dot or drawing in the right hand corner.)
- Proportional Diameter in cm (or mm)
- Radius of orbit in km
- Proportional radius of orbit in m

In a long hall or outside, have participants stretch out a tape measure that has metric measurements out to 40 meters. Have one member of the group find where their
planet is located on the tape measure and hold their informational board. Have the rest of the participants step away from the line to see the proportional distance and size they have created. Have participants take turns holding the signs so they may also see.

Assess
- **Formative:** Discussion questions
  - Ask participants how they arrived at their answers and how they expect their students to arrive at their answers.
  - Ask the participants how they anticipate their potential middle school students to answer this question without any knowledge of ratio or proportion? Ask the participants how do they know this works?
  - Ask the participants about what they did and how they used the rules of fractions to solve a proportion problem (if they used fractions and have not done cross-multiplication yet). Ask them if there is another way to look at this problem and try to simplify it so there were no fractions involved.
  - Ask the participants if they saw their students do this with the problem, how could they help them understand their mistake? Ask the participants if demonstrating ratio/proportions in this manner gives them a better understanding of the rule, “cross, multiply and divide?”
  - Ask the participants if demonstrating ratio/proportions in this manner gives them a better understanding of the rule, “cross, multiply and divide?”
  - Ask the participants if they saw their students do the two following errors with the problem, how could they help them understand their mistakes? Ask student if they can give examples where would we need to use proportions?
  - Lead them into a discussion about the sun and planets in the solar system.
  - Ask them about size of Earth and how it compares to the sun.
  - What are some of the dangers of going to another planet?
  - What are the dangers of coming back to Earth?
  - Ask participants if we have ever found life outside of our planet in our known solar system. Do we know where have we gone so far?
  - Ask participants what they know about microorganisms.
  - Ask participants how these microorganisms reproduce or replicate?
  - Ask the participants where can they find microorganisms?
  - Ask participants if they could find them in: boiling water, ice, acid or salt?
  - Ask the participants why they think studying extremophiles would be important?
  - Ask the participants if the sun is 24.1cm in diameter, what diameter would the earth be?
  - Would this scaling be the same for all of the planets? Why or why not?
  - Ask the participants if their estimations or predictions were correct from the beginning of class?
  - Ask the participants about the distance from the Sun to the Earth?
  - How many of them know how far away our planet is from the Sun?
  - Is it a perfect circular orbit?
  - Where would we start?
  - What about orbits? Since they are not perfect circles?
- **Summative**: Student worksheet 1

  - How far away would the planets be using the same proportion?
  - Would Mercury be in the same room?
  - How about Venus, Earth, Mars?
  - Have the groups discuss how would they find a scaled orbit of Earth for a couple of minutes and have them report back on their ideas and thoughts.
  - So if we want to scale the planets rotation around the sun to the new size do we have to use the scaled size of the sun in some way?
  - What unit of measurement do we need for the planet’s distance from the sun?
Astrobiology

Astrobiology is the study of the origin, evolution, distribution, and future of life in the universe. This multidisciplinary field encompasses the search for habitable environments in our Solar System and habitable planets outside our Solar System, the search for evidence of prebiotic chemistry and life on Mars and other bodies in our Solar System, laboratory and field research into the origins and early evolution of life on Earth, and studies of the potential for life to adapt to challenges on Earth and in space.

https://astrobiology.nasa.gov/about-astrobiology/
Warm Up

- Simplify: \( \frac{6}{27} = \frac{2}{9} \)
- Simplify: \( \frac{20}{36} = \frac{5}{9} \)
- Simplify: \( \frac{52}{4} = 13 \)

How did you arrive at your answers?

Greatest Common Factor (GCF)

Group Work

In your groups, discuss how to use GCF to solve this word problem:

If a piece of rope 8 cm long weighs 48 grams. What will an 11-cm length of the same cable weigh?

\[ \frac{8}{48} = \frac{11}{6} \]

\[ \frac{8}{11} = \frac{48}{66} \]

66 grams
Slide 5

**Proportion**

- How do you solve a proportion?
- Same?

\[
\frac{8}{48} = \frac{11}{x} \\
8 \cdot x = 48 \cdot 11 \\
x = \frac{528}{8} \\
x = 66
\]

**Same?**

\[
\frac{8}{11} = \frac{48}{x} \\
8x = 48 \cdot 11 \\
x = \frac{528}{8} \\
x = 66
\]

Why does this work?

---

Slide 6

**GCF & LCM**

- **GCF**: Greatest Common Factor:
  - The highest number that divides exactly into two or more numbers.
- **LCM**: Least Common Multiple:
  - The smallest number (not zero) that is a multiple of both

**GCF of 12 and 44?**
- 12: 1, 2, 3, 4, 12
- 44: 1, 2, 4, 11, 22, 44

**LCM of 12 and 44?**
- 12: 12, 24, 36, 48, 60, 72, 96, 108, 120, 132...
- 44: 44, 88, 132...

Least Common Multiple ~ Least Common Denominator

What would the LCD of our rope problem be?

\[
\frac{8}{11} = \frac{48}{x}
\]
Slide 7

LCD

- What would the LCD of our rope problem be? (TPS)

\[
\frac{8}{11} = \frac{48}{x}
\]

LCD : 11 * x

- What you are left with:

\[
\left( \frac{x}{1} \right) \left( \frac{8}{11} \right) = \left( \frac{48}{11} \right)
\]

- What you do to one side you have to do to the other.

\[
\left( \frac{11x}{1} \right) \left( \frac{8}{11} \right) = \left( \frac{11x}{1} \right) \left( \frac{11}{1} \right)
\]

8x = 48 * 11

8x = 528

x = 66

Slide 8

Errors

- COMMON STUDENT ERROR (Discuss with your group):

If a piece of rope 8 cm long weighs 48 grams. What will an 11-cm length of the same cable weigh?

11 – 8 = 3

48 – 3 = 45

Answer : 45 grams

- How could you help them?
Slide 9

Proportions

- Where do we use them?
- How about in an Astrobiology setting?
- Astrobiology is interdisciplinary and consists of:
  - Chemistry
  - Biology
  - Physics
  - Astronomy
  - Planetary Science
  - Ecology
  - Geography
  - Geology...
  - (TPS)
- Planetary Science is a good and easy start to introduce Astrobiology and Proportions. Why?
- Where have we gone robotically?
- How long did it take to get Spirit and Opportunity to Mars? 7, 6 months respectively.

Slide 10

Estimation

- In groups, go to the front and estimate what objects they think would represent Mercury, Venus, Earth and Mars
  - the Sun is represented by a basketball. (1 min max each group.)
- What are some of the dangers of visiting other planets?
- Have we ever found life outside of our planet in our known solar system?

<table>
<thead>
<tr>
<th>Ball</th>
<th>Pink</th>
<th>Green</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avengars</td>
<td>Soccer</td>
<td>Golf</td>
</tr>
<tr>
<td>Blue</td>
<td>Green Shamrock</td>
<td>Pink, Green, Orange</td>
</tr>
<tr>
<td>Green, Yellow Pin</td>
<td>Round silver Pin</td>
<td>Flat silver pin</td>
</tr>
</tbody>
</table>
Microorganisms

- Why do you think studying extremophiles would be important?
- Scientist study extremophiles to examine how life may have begun and thrived on early Earth. Using this information they can study how life might survive in similar environments in space.
- How would you “examine” microorganisms? What would some of the elements you would look for?

What are microorganisms?

- Ask your students what they know about microorganisms.
- Please write down a few things that you know about microorganisms.
- Write down what you know about microorganisms replication.
- A microorganism is a living single-celled organism of microscopic size
- Replication: binary fission but can also transfer gene information through conjugation.
Microorganisms

- Types
  - Bacteria
  - Fungus
  - Archea
  - Viruses

Where do you find them?
- Water?
- Hot water?
- Ice?
- Acid?
- Salty places?
- Radioactive places?

Found:
- Water? Perfect!
- Hot water? Thermophiles
- Cryophiles
- Acidophiles
- Halophiles
- Polyextremophile/Radioresistant

These are examples of extremophiles, microorganisms that thrive in extreme environments where most other organisms cannot. (-phile comes from the Greek *philia* which means love.)
Planets

- Diameter, do we know? How can we find it?
  \[ C = \pi d \]
  \[ \frac{C}{\pi} = d \]
  \[ \frac{C}{\pi} = \frac{d}{\pi} \]

- Let’s find the diameter in centimeters of the objects on the table.
- Each group will come up and get 2-3 objects. (Due to time constraints, this step will be skipped.)

- Now we need to scale the planets down in size given we will use a basketball to represent the Sun.

<table>
<thead>
<tr>
<th>Ball</th>
<th>C (cm)</th>
<th>D (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pink</td>
<td>53</td>
<td>16.87</td>
</tr>
<tr>
<td>Green</td>
<td>28.5</td>
<td>18.62</td>
</tr>
<tr>
<td>Avengers</td>
<td>23</td>
<td>7.3</td>
</tr>
<tr>
<td>Soccer</td>
<td>19.5</td>
<td>6.2</td>
</tr>
<tr>
<td>Golf</td>
<td>13.75</td>
<td>4.37</td>
</tr>
<tr>
<td>Blue</td>
<td>13.5</td>
<td>4.3</td>
</tr>
<tr>
<td>P. G. O</td>
<td>11</td>
<td>3.5</td>
</tr>
<tr>
<td>Green Sham</td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td>G, Y Pin</td>
<td>2mm</td>
<td></td>
</tr>
<tr>
<td>Silver</td>
<td>1&lt;2mm</td>
<td></td>
</tr>
</tbody>
</table>

- Sun

- If the Sun is the basketball and it’s diameter is 21.1cm, what would the Earth’s diameter be in cms? In your groups, come up with a proportion to solve this problem.

\[
\frac{64748 \text{ Sun (km)}}{64748 \text{ Sun (cm)}} = \frac{64748 \text{ Earth (km)}}{64748 \text{ Earth (cm)}}
\]

\[
\frac{1391900 \text{ Sun (km)}}{12742 \text{ Earth (km)}} = \frac{24.1 \text{ Earth (cm)}}{E \text{ (cm)}}
\]

\[ E = 0.22 \text{ cm} \]

- General formula?
  \[ \frac{64748 \text{ Sun (km)}}{64748 \text{ Sun (cm)}} = \frac{64748 \text{ Earth (km)}}{64748 \text{ Earth (cm)}} \]

- General formula for scaling all the planets?
  \[ \frac{1391900 \text{ Sun (km)}}{\text{planet_{suns}} \text{ (km)}} = \frac{24.1 \text{ Earth (cm)}}{\text{planet_{suns} (cm)}} \]

- Which is the “unknown” in the equation?
Estimations?!

- Are orbits perfect circles?
  - No. 😊
- We'll use the averages of the orbits for our exercise.
- Would Mercury be in the same room? Let's just examine the four closest planets to the sun. How about Venus, Earth, Mars? Let's look at Earth's orbit.

Orbits

- What would the proportion look like for orbits? Let's look at Earth's orbit. The radius is 149,570,000 km. Discuss in your groups.
- Report out...
- General formula:

\[
\frac{\text{Orbit Actual (km)}}{\text{Orbit Scaled (cm)}} = \frac{\text{Orbit Actual (km)}}{149570000}
\]

Now need to convert to meters, metric measurement scale: mm, cm, dm, m, km. Which one?

\[
258.4 \text{ cm} = 258.24 \text{ dm} = 25.824 \text{ m}
\]

Earth's orbit radius will be 25.82m if the sun is 24.1cm in diameter.
**Orbits**

- **Group Activity:** Hand out Student Activity Sheet 2 and have participants scale the orbit radii of the rest of the planets using calculators and the scaled sun as 24.1 cm or (if you have access to computers/tablets) have participants use Excel to create add to their existing spreadsheet using the data.

- After the groups have compiled their data have them look at the scaled distances using a ruler/meter stick. Assign each group a planet (From Mercury to Mars...). Have them make an informational board on either cardboard or cardstock about their planet with the following:
  - Planet Name
  - Diameter in km (Represent if they can with a dot or drawing in the right hand corner.)
  - Proportional Diameter in cm (or mm)
  - Radius of orbit in km
  - Proportional radius of orbit in m

  (Due to time constraints, this step will be skipped.)

---

**Orbits**

<table>
<thead>
<tr>
<th>Body</th>
<th>Diam (km)</th>
<th>Scaled Diameter (cm)</th>
<th>Orbit (km)</th>
<th>Scaled Orbit (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mercury</td>
<td>1391900</td>
<td>0.08</td>
<td>57850000</td>
<td>9.53</td>
</tr>
<tr>
<td>Venus</td>
<td>4866</td>
<td>0.21</td>
<td>108110000</td>
<td>18.75</td>
</tr>
<tr>
<td>Earth</td>
<td>12106</td>
<td>0.22</td>
<td>149570000</td>
<td>25.82</td>
</tr>
<tr>
<td>Mars</td>
<td>12742</td>
<td>0.12</td>
<td>227840000</td>
<td>40.45</td>
</tr>
<tr>
<td>Jupiter</td>
<td>142984</td>
<td>2.48</td>
<td>778140000</td>
<td>134.96</td>
</tr>
<tr>
<td>Saturn</td>
<td>116438</td>
<td>2.02</td>
<td>1427000000</td>
<td>247.56</td>
</tr>
<tr>
<td>Uranus</td>
<td>46940</td>
<td>0.81</td>
<td>2870300000</td>
<td>497.56</td>
</tr>
<tr>
<td>Neptune</td>
<td>45432</td>
<td>0.79</td>
<td>4498900000</td>
<td>782.47</td>
</tr>
</tbody>
</table>
## Orbits (feet)

<table>
<thead>
<tr>
<th>Body</th>
<th>Diam (km)</th>
<th>Scaled Diameter (cm)</th>
<th>Orbit (km)</th>
<th>Scaled Orbit (m)</th>
<th>Scaled Orbit (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mercury</td>
<td>1391900</td>
<td>0.08</td>
<td>57950000</td>
<td>9.53</td>
<td>31.3</td>
</tr>
<tr>
<td>Venus</td>
<td>4886</td>
<td>0.21</td>
<td>108110000</td>
<td>18.75</td>
<td>61.5</td>
</tr>
<tr>
<td>Earth</td>
<td>12106</td>
<td>0.22</td>
<td>159570000</td>
<td>25.62</td>
<td>84.7</td>
</tr>
<tr>
<td>Mars</td>
<td>12742</td>
<td>0.12</td>
<td>227940000</td>
<td>40.65</td>
<td>132.7</td>
</tr>
<tr>
<td>Jupiter</td>
<td>142984</td>
<td>2.48</td>
<td>718140000</td>
<td>134.96</td>
<td>442.8</td>
</tr>
<tr>
<td>Saturn</td>
<td>116438</td>
<td>2.02</td>
<td>1427900000</td>
<td>247.56</td>
<td>813.2</td>
</tr>
<tr>
<td>Uranus</td>
<td>48940</td>
<td>0.81</td>
<td>2870300000</td>
<td>497.55</td>
<td>1625</td>
</tr>
<tr>
<td>Neptune</td>
<td>45432</td>
<td>0.79</td>
<td>4459900000</td>
<td>782.47</td>
<td>2567.2</td>
</tr>
</tbody>
</table>
APPENDIX G: GROWTH CURVE STUDENT WORKSHEET 1

Name: __________________________________________________________

Cell Replication
If a cell replicates once every hour:

<table>
<thead>
<tr>
<th>Hour (t)</th>
<th># of cells (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

What is the independent variable: _________________________________

What is the dependent variable: _________________________________

What is the y-intercept: _________________________________

What is the function notation: _________________________________

Is there a constant value multiplied by the hour (t) where adding one to the result would give you the # of cells (y)? Fitting into a linear equation of: \( y = (? )t + 1 \) (Graph the results of your table to help answer this question. Remember to label your axes.)

Answer: ____________________________________________________
Is there a pattern involved in the column of cells?

<table>
<thead>
<tr>
<th>Hour (t)</th>
<th># of cells (y)</th>
<th>Pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
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<td>4</td>
<td></td>
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<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>t</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What would the equation (or function) look like:  

\[ y = \]  

Or:  \[ f(t) = \]
APPENDIX H: GROWTH CURVE GROUP WORKSHEET

Names:______________________________

*Sulfolobus* Replication

<table>
<thead>
<tr>
<th>Hour (t)</th>
<th>OD&lt;sub&gt;600&lt;/sub&gt;</th>
<th>Number of cells per milliliter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.08</td>
<td>$1.3 \times 10^8$</td>
</tr>
<tr>
<td>2</td>
<td>0.10</td>
<td>$1.7 \times 10^8$</td>
</tr>
<tr>
<td>3</td>
<td>0.12</td>
<td>$3 \times 10^8$</td>
</tr>
<tr>
<td>4</td>
<td>0.30</td>
<td>$5.3 \times 10^8$</td>
</tr>
<tr>
<td>5</td>
<td>0.45</td>
<td>$8 \times 10^8$</td>
</tr>
<tr>
<td>6</td>
<td>0.48</td>
<td>$8.3 \times 10^8$</td>
</tr>
<tr>
<td>7</td>
<td>0.47</td>
<td>$8.2 \times 10^8$</td>
</tr>
<tr>
<td>8</td>
<td>0.47</td>
<td>$8.2 \times 10^8$</td>
</tr>
<tr>
<td>9</td>
<td>0.46</td>
<td>$8.1 \times 10^8$</td>
</tr>
<tr>
<td>10</td>
<td>0.30</td>
<td>$5.3 \times 10^8$</td>
</tr>
</tbody>
</table>

1. What is the independent variable: ______________________

2. What is the dependent variable: ______________________

3. Label the graph with:
a. **Stationary Phase**: When cell births = cell deaths and the sample ceases to grow.

b. **Exponential Growth Phase**: When cells are dividing at a constant rate and cell births > cell deaths.

c. **Lag Phase**: When the cells have been inoculated into the medium, the population remains temporarily unchanged.

d. **Death Phase**: The number of viable cells decreases as they may have exhausted their available nutrients, space or have too much waste products that

4. Graph the following (imaginary) organisms, use the symbol in parentheses next to the name for the graph:

a. Sacriophile (●)  
   \[ f(x) = 3^x \]

b. Bigilophile (○)  
   \[ f(x) = 3^{x-1} \]

c. Gremophile (×)  
   \[ f(x) = 3^{x+1} \]

d. Oiligophile (Δ)  
   \[ f(x) = 3^x + 1 \]

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>X</th>
<th>Y</th>
<th>X</th>
<th>Y</th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>( , )</td>
<td>( , )</td>
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<td>( , )</td>
<td>( , )</td>
<td>( , )</td>
<td>( , )</td>
</tr>
</tbody>
</table>
5. What happens to the graphs of the different organisms compared to the Sacriophile?
APPENDIX I: SOLAR SYSTEM STUDENT WORKSHEET 1

Student Activity Sheet 1: Solar System

Names: ____________________________________________
<table>
<thead>
<tr>
<th>Estimated classroom object</th>
<th>True to scale classroom object</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td></td>
</tr>
<tr>
<td>Mercury</td>
<td></td>
</tr>
<tr>
<td>Venus</td>
<td></td>
</tr>
<tr>
<td>Earth</td>
<td></td>
</tr>
<tr>
<td>Mars</td>
<td></td>
</tr>
</tbody>
</table>

General formula:

<table>
<thead>
<tr>
<th>Body</th>
<th>Actual Body Diameter (km)</th>
<th>Scaled Diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>1391900</td>
<td></td>
</tr>
<tr>
<td>Mercury</td>
<td>4866</td>
<td></td>
</tr>
<tr>
<td>Venus</td>
<td>12106</td>
<td></td>
</tr>
<tr>
<td>Earth</td>
<td>12742</td>
<td></td>
</tr>
<tr>
<td>Mars</td>
<td>6760</td>
<td></td>
</tr>
<tr>
<td>Jupiter</td>
<td>142984</td>
<td></td>
</tr>
<tr>
<td>Saturn</td>
<td>116438</td>
<td></td>
</tr>
<tr>
<td>Uranus</td>
<td>46940</td>
<td></td>
</tr>
<tr>
<td>Neptune</td>
<td>45432</td>
<td></td>
</tr>
</tbody>
</table>

General formula:
### APPENDIX J: SOLAR SYSTEM GROUP WORKSHEET

Student Activity Sheet 2: Solar System Orbits

<table>
<thead>
<tr>
<th>Body</th>
<th>Body Diameter (km)</th>
<th>Scaled Diameter (mm)</th>
<th>Orbit Radius (km)</th>
<th>Scaled Orbit Radius (___)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>1391900</td>
<td>24.1</td>
<td>57950000</td>
<td></td>
</tr>
<tr>
<td>Mercury</td>
<td>4866</td>
<td></td>
<td>108110000</td>
<td></td>
</tr>
<tr>
<td>Venus</td>
<td>12106</td>
<td></td>
<td>149570000</td>
<td></td>
</tr>
<tr>
<td>Earth</td>
<td>12742</td>
<td></td>
<td>227840000</td>
<td></td>
</tr>
<tr>
<td>Mars</td>
<td>6760</td>
<td></td>
<td>778140000</td>
<td></td>
</tr>
<tr>
<td>Jupiter</td>
<td>142984</td>
<td></td>
<td>1427000000</td>
<td></td>
</tr>
<tr>
<td>Saturn</td>
<td>116438</td>
<td></td>
<td>2870300000</td>
<td></td>
</tr>
<tr>
<td>Uranus</td>
<td>46940</td>
<td></td>
<td>4499900000</td>
<td></td>
</tr>
<tr>
<td>Neptune</td>
<td>45432</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX K: BACKGROUND SURVEY

General Background Survey
Name: __________

Please check the appropriate response:

Demographics
I strongly identify with the following race:
  □ Caucasian/White
  □ Hispanic or Latino
  □ African American/Black
  □ Native American/American Indian/Alaskan Native
  □ Asian or Pacific Islander
  □ Other: ________________

Ethnicity:
  □ No, not of Hispanic descent
  □ Yes, I am of Hispanic descent

I am a:
  □ Male
  □ Female

What is your age:
  □ 18-25
  □ 26-30
  □ 31-35
  □ 36-40
  □ 40+

Educational History
Of the choices, which best describes you:
  □ First year student (freshman)
  □ Second year student (sophomore)
  □ Third year student (junior)
  □ Fourth year student or beyond (senior)
  □ Post baccalaureate student
  □ Graduate Student
  □ Other

Is this course a requirement for your major?
  □ Yes
  □ No
How many other college level mathematics/statistics courses (not including this one) have you taken? (e.g. M105, M115, etc.)

- 1
- 2
- 3
- 4
- 5
- 6+

Please list the highest-level mathematics course you have taken:

________________________________________________

How many college level science courses have you taken? (This includes: Astronomy, Biochemistry, Biology, Chemistry, Computer Science, Environmental Science, Forestry, Physics)

- 1
- 2
- 3
- 4
- 5
- 6+

Please list the highest-level science course you have taken:

________________________________________________

**Interest in STEM (Science, Technology, Engineering, Mathematics)**

Please select how interested are you in the following areas?

<table>
<thead>
<tr>
<th></th>
<th>Dislike Extremely</th>
<th>Dislike Very much</th>
<th>Neither Like nor Dislike</th>
<th>Like Very Much</th>
<th>Like Extremely</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space science/astronomy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computer Technology</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mathematics</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Engineering</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Physics</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Biology</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Designing/building models</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Earth Science</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chemistry</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computer modeling</td>
<td></td>
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</tr>
<tr>
<td>Geoscience</td>
<td></td>
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</tr>
</tbody>
</table>
**Familiarity with Astrobiology**

Please select your interest/knowledge of the following areas regarding astrobiology

<table>
<thead>
<tr>
<th>Area</th>
<th>None</th>
<th>Slight</th>
<th>Average</th>
<th>Considerable</th>
<th>Great</th>
</tr>
</thead>
<tbody>
<tr>
<td>The search for life in the universe</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>What scientists really do</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Definition of Astrobiology</td>
<td></td>
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</tr>
<tr>
<td>Exploring the solar system</td>
<td></td>
<td></td>
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<tr>
<td>Why scientists ask questions</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Technology uses in science</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>The importance of astrobiology</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Teaching astrobiology to my future students</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

**Mathematics Skills**

Please rate the following to correspond to your confidence in the following areas of mathematics teaching. My confidence in my ability to…

<table>
<thead>
<tr>
<th>Skill</th>
<th>Regretful</th>
<th>Poor</th>
<th>Good</th>
<th>Excellent</th>
<th>Delightful</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do ratio/proportions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Graph non-linear equations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recognize student errors in ratio/proportion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recognize student errors in non-linear equations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>recognize student’s ability to solve questions in a non-typical method</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX L: PRETEST

Astrobiology/ Mathematics Pretest

Name: _________________________________

1. Describe what is astrobiology?

2. What is a microorganism?

3. Where can a microorganism live?

4. How does a microorganism replicate?

5. What is an extreme environment? Describe one?

6. Name the planets of our solar system?

7. Describe exponential growth and Give an example.

8. Choose which graph best depicts exponential decay:

   a. ![Graph A]
   b. ![Graph B]
   c. ![Graph C]
   d. ![Graph D]

9. Student Response Investigation:

   **Question:** The ratio of fish to alligators in a swimming pool is 5 to 3. If there are 15 fish, how many alligators are there?

   **Student Response:**
10. Student Response Investigation:

**Question:** A painter has formed a light pink by mixing 4 parts white with 1-part red. There are 6 liters of a dark pink which is half red and half white. How much white should be added to the darker pink to convert it to the lighter pink?

**Student Response:**

a. Is this student’s work correct?

b. Can you explain the student’s thinking?

c. What feedback on their work would you give to this student?

11. Please circle the appropriate response below < , = or >

\[
\left( \frac{2}{3} \right)^2 < \quad = \quad > \quad \frac{5}{6}
\]

12. Student Response Investigation:
Question: What is $y$? $27 = 3^y$

Student Response:

\[
\begin{array}{c}
9 + 9 + 9 = 27 \\
y = 27
\end{array}
\]

a. Is this student’s work correct?

b. Can you explain the student’s thinking?

c. What feedback on their work would you give to this student?

13. Student Response Investigation:

\[
\frac{5}{17} = \frac{x}{51}
\]

Question: Solve for $x$: \[
\frac{5}{17} = \frac{x}{51}
\]

Student Response:

\[
\begin{array}{c}
\frac{5}{17} = \frac{x}{51} \\
12 \times 5.89 \\
5.89 \\
\frac{5}{17} = \frac{x}{51}
\end{array}
\]

a. Is this student’s work correct?

b. Can you explain the student’s thinking?

Cc. What feedback on their work would you give to this student?
APPENDIX M: POSTTEST

Astrobiology/ Mathematics Posttest

Name: ____________________________

1. Describe what is astrobiology?

2. What is a microorganism?

3. Where can a microorganism live?

4. How does a microorganism replicate?

5. What is an extreme environment? Describe one?

6. Name the planets of our solar system?

7. Describe exponential growth and Give an example.

8. Choose which graph best depicts exponential decay:

   a.  
   b.  
   c.  
   d.  

9. Student Response Investigation:

   Question: The ratio of fish to alligators in a swimming pool is 5 to 3. If there are 15 fish, how many alligators are there?
Student Response:

a. Is this student’s work correct?

b. Can you explain the student’s thinking?

c. What feedback on their work would you give to this student?

10. Student Response Investigation:

Question: A painter has formed a light pink by mixing 4 parts white with 1-part red. There are 6 liters of a dark pink which is half red and half white. How much white should be added to the darker pink to convert it to the lighter pink?

Student Response:

a. Is this student’s work correct?

b. Can you explain the student’s thinking?

c. What feedback on their work would you give to this student?
11. Please circle the appropriate response below < , = or >

\[ \left( \frac{2}{3} \right)^2 \quad | < \quad = \quad > | \quad \frac{5}{6} \]

12. Student Response Investigation:

**Question:** What is \( y \)? \( 27 = 3^y \)

**Student Response:**

\[ y = \frac{27}{3} = 9 \]

a. Is this student’s work correct?

b. Can you explain the student’s thinking?

c. What feedback on their work would you give to this student?

13. Student Response Investigation:

**Question:** Solve for \( x \): \( \frac{5}{17} = \frac{x}{51} \)

**Student Response:**

\[ x = \frac{15}{1} = 15 \]

a. Is this student’s work correct?

b. Can you explain the student’s thinking?

C. What feedback on their work would you give to this student?

EXIT SURVEY:
Please select the appropriate response for each question.
14. I found the Astrobiology modules interesting. 1 2 3 4 5
15. Learning about Astrobiology made me more interested in learning the mathematics involved with the module. 1 2 3 4 5
16. I learn mathematics easier when it is presented in a manner in which I see connections to other topics. 1 2 3 4 5
17. I would like to use this module in the future when I get a teaching position. 1 2 3 4 5
APPENDIX N: DETAILED RESULTS

Detailed Results on Descriptive Analyses

Background Survey. A non-parametric test was done to examine the effects of the demographic background of the participants between the two sections. See Table 22.

Table 22

*Kruskal-Wallis Test mean rank for demographics between sections*

<table>
<thead>
<tr>
<th>Test Statistics</th>
<th>Race</th>
<th>Ethnicity</th>
<th>Sex</th>
<th>Ed. History</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-Square</td>
<td>1.964</td>
<td>0.006</td>
<td>0.817</td>
<td>1.658</td>
</tr>
<tr>
<td>df</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>p-value</td>
<td>0.161</td>
<td>0.939</td>
<td>0.366</td>
<td>0.198</td>
</tr>
</tbody>
</table>

a. Kruskal Wallis Test
b. Grouping Variable: Identifier

Notes. * $p < 0.05$.

A Kruskal-Wallis test indicated there was not a significant difference in race between the two sections ($\chi^2(1) = 1.964, \ p = 0.161$), with a mean rank of 27 for section 1 and 28.96 for section 2. A Kruskal-Wallis test indicated there was not a significant difference between ethnicity by completers (study) ($\chi^2(1) = 0.006, \ p = 0.939$), with a mean rank of 27.58 for section 1 and 27.43 for section 2. A Kruskal-Wallis test indicated there was not a significant difference between sex of the two sections ($\chi^2(1) = 0.817, \ p = 0.366$), with a mean rank of 26.93 for section 1 and 29.04 for section 2. A Kruskal-Wallis test indicated there was not a significant difference between educational history between sections ($\chi^2(1) = 2.221, \ p = 0.136$), with a mean rank of 25.28 for completers and 30.63 for unfinished.
A non-parametric test was done to examine the effects of the demographic background of the participants who were unfinished in all elements and the completer (study) group for both sections separately. See Table 23.

Table 23

*Kruskal-Wallis Test mean rank for demographics for section 1*

<table>
<thead>
<tr>
<th>Test Statistics</th>
<th>Race</th>
<th>Ethnicity</th>
<th>Sex</th>
<th>Ed. History</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-Square</td>
<td>0.000</td>
<td>0.926</td>
<td>1.904</td>
<td>2.221</td>
</tr>
<tr>
<td>df</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>p-value</td>
<td>1.000</td>
<td>0.336</td>
<td>0.168</td>
<td>0.136</td>
</tr>
</tbody>
</table>

c. Kruskal Wallis Test
d. Grouping Variable: Identifier

Notes. * $p < 0.05$.

A Kruskal-Wallis test indicated there was not a significant difference between ethnicity by completers (study) ($\chi^2(1) = 0.926$, $p = 0.336$), with a mean rank of 14 for completers and 14 for unfinished. A Kruskal-Wallis test indicated there was not a significant difference between ethnicity by completers (study) ($\chi^2(1) = 1.904$, $p = 0.168$), with a mean rank of 12.5 for completers and 13.94 for unfinished. A Kruskal-Wallis test indicated there was not a significant difference between educational history by completers (study) ($\chi^2(1) = 2.221$, $p = 0.136$), with a mean rank of 10.69 for completers and 15.39 for unfinished. There was no difference in race for section 1, see Table 24.

Table 24

*Kruskal-Wallis Test mean rank for demographics for section 2*

<table>
<thead>
<tr>
<th>Test Statistics</th>
<th>Race</th>
<th>Ethnicity</th>
<th>Sex</th>
<th>Ed. History</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chi-Square</td>
<td>0.069</td>
<td>3.210</td>
<td>1.344</td>
<td>2.756</td>
</tr>
<tr>
<td>df</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

194
A Kruskal-Wallis test indicated there was not a significant difference between race by completers (study) $\chi^2(1) = 0.141, p = 0.707$), with a mean rank of 14.85 for completers and 14.31 for unfinished. A Kruskal-Wallis test indicated there was not a significant difference between ethnicity by completers (study) $\chi^2(1) = 3.738, p = 0.053$), with a mean rank of 16.30 for completers and 13.50 for unfinished. A Kruskal-Wallis test indicated there was not a significant difference between ethnicity by completers (study) $\chi^2(1) = 1.154, p = 0.283$), with a mean rank of 15.50 for completers and 13.94 for unfinished. A Kruskal-Wallis test indicated there was not a significant difference between educational history by completers (study) $\chi^2(1) = 1.973, p = 0.160$), with a mean rank of 17.35 for completers and 12.92 for unfinished.

**PreTest.** Since there was shown to be no significant difference between completers (study) and unfinished with regards to demographic information, a non-parametric test was conducted to examine if there was a difference on the pretest scores between the participants who were unfinished in all elements and the completer (study) group for both sections separately. See Table 25.

**Table 25**

*Mann-Whitney U-Test mean rank for the pretest for section 1*

<table>
<thead>
<tr>
<th>Test Statistics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mann-Whitney U</td>
<td>50.500</td>
</tr>
<tr>
<td>Wilcoxon W</td>
<td>203.500</td>
</tr>
<tr>
<td>Z</td>
<td>-1.029</td>
</tr>
</tbody>
</table>
A Mann-Whitney test indicated there was not a significant difference in the completers (study) score on the pretest ($Mdn = 70.0$) than the unfinished group score on the pretest ($Mdn = 70.0$), $U = 50.5$, $p = 0.304$, $r = -0.2058$. These results suggest that the completer (study) group and the unfinished group for section 1 are similar. The effect size ($r = -0.2058$) suggests the difference between the completers and the unfinished group is moderately small. Eta squared ($\eta^2$) was also calculated to determine how much variability of the ranks is accounted by finishing. $\eta^2 = 0.04$ indicates that a very small percent (4%) of the variability of the ranks is accounted by finishing all parts of the study. See Table 26.

Table 26

<table>
<thead>
<tr>
<th>Test Statistics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mann-Whitney U</td>
<td>43.000</td>
</tr>
<tr>
<td>Wilcoxon W</td>
<td>109.000</td>
</tr>
<tr>
<td>Z</td>
<td>-1.660</td>
</tr>
<tr>
<td>Asymp. Sig (2-tailed)</td>
<td>0.097</td>
</tr>
<tr>
<td>Exact Sig. [2*(1-tailed Sig.)]</td>
<td>0.106b</td>
</tr>
</tbody>
</table>

Notes. * $p < 0.05$. 

e. Grouping Variable: Identifier
f. Not corrected for ties.

A Mann-Whitney test indicated there was not a significant difference in the completers (study) score on the pretest ($Mdn = 45.0$) than the unfinished group score on
the pretest \((Md{\text{n}} = 70.0), \ U = 43.0, \ p = 0.097, \ r = -0.339\). These results suggest that the completer (study) group and the unfished group for section 2 are relatively similar. The effect size \((r = -0.339)\) suggests the difference between the completers and the unfinished group is moderate. Eta squared \((\eta^2)\) was also calculated to determine how much variability of the ranks is accounted by finishing. \(\eta^2 = 0.12\) indicates that a very small percent (12%) of the variability of the ranks is accounted by finishing all parts of the study.