The Mathematics Enthusiast

Volume 9
Number 1 *Numbers 1 & 2*

Article 5

1-2012

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Frode Olav Haara

Kari Smith

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A case-study examination of the influence of a value-based intervention on two teachers' use of practical activities in mathematics teaching

Frode Olav Haara[1]
Sogn og Fjordane University College, Norway

Kari Smith
University of Bergen, Norway

Abstract: *This study sets out to examine the influence of a value-based intervention on two elementary school teachers' use of practical activities in mathematics teaching. The intervention was a "Values and Knowledge Education" (VaKE)-based in-service course that introduced the two teachers to a value-based approach to mathematics teaching. The introduction included examples that were supported by use of practical activities. Interviews prior to the intervention made the teachers aware of an inconsistency between the desired and actual practice of their own teaching. The intervention provided them with a possibility of narrowing the gap between vision and practice by changing practice. Qualitative data show how the VaKE approach offered an alternative that opened up for increased use of practical activities in the teaching of mathematics, but also showed how good intentions of changing practice might be restrained or hindered by beliefs and previous experience.*

Keywords: Mathematics teaching; Beliefs; Practice; Practical activities; Values and Knowledge Education.

[1] frode.olav.haara@hisf.no

***The Mathematics Enthusiast*, ISSN 1551-3440, Vol. 9, nos.1&2**, pp.77-110

Today the educational policy in Norway (KD, 2006) encourages the use of practical approaches to mathematics teaching.[2] Using practical activities[3] is one way of doing this. However, Norwegian research shows that teachers find it difficult to change existing practice (Kjærnsli, Lie, Olsen, Roe & Turmo, 2004; Klette, 2003) and that teachers of mathematics do not necessarily acknowledge the theoretical consensus supporting practical activities (Alseth, Breiteig & Brekke, 2003; Haara & Smith, 2009). If a teacher is going to use more practical activities, the teacher has to believe that such an approach supports student learning.

Values and Knowledge Education (*VaKE*)

VaKE is a teaching approach that emphasizes developing students' moral and ethical values[4] through the acquisition of new disciplinary knowledge within a constructive learning environment (Patry, Weyringer & Weinberger, 2007). Based on a constructive theory of learning with a foothold in both sociocultural learning theory and radical constructivism, and influenced by Kohlberg's theory on moral development through social interaction (Kohlberg, 1976), the teacher who wants to follow the *VaKE* paradigm teaches through the introduction of a moral dilemma. This implies that the students have to choose between two possible decisions. Two factions of students are then formed, based on the students' decisions. This is followed by a moral viability check through discussion, first within each faction and then

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[2] From a mathematical didactical perspective, traditional teacher-dominated teaching has been challenged by the influence of theories of teaching and learning, ethno-mathematics and realistic mathematics education. In addition, the development of mathematics teaching in Norway is influenced by societal factors. Norwegian society needs to increase the numbers of students entering higher education in mathematics and science, a realization which has given extra weight to the political and societal demands for the development of additional, or even change of, working methods in the teaching of mathematics in elementary school. This is a longitudinal and manifold process that has brought about an increased focus on the practical relevance and use of practical activities in school mathematics as one domain of development. For a more thorough introduction to the background of changes in mathematics teaching for educational policy reasons, see Haara, Stedøy-Johansen, Smith and Kirfel (2009).

[3] In Haara and Smith (2009), we define a practical activity to include all forms of engagement where the pupil uses physical objects while carrying out the activity at hand. That means including the opportunity for physical activity, and not just the use of artefacts or material found in nature.

[4] The term *Values* in *VaKE* refers to the emphasis given to moral and ethical aspects through the use of dilemmas that challenge the students' opinion of right and wrong. Hence, in *VaKE* there is no explicit element of value regarding the application of mathematics (Skovsmose, 2002).

between the two factions. The need for new disciplinary knowledge to better illuminate different aspects of the topic and provide more coherent arguments through the collecting of new knowledge, is revealed. Rounds of discussion, and content viability checks on arguments are then possible, until both factions are ready to present their conclusions as the final moral and content viability checks.[5] The teacher and the class close the sequence by capitalizing on the whole process. Accordingly, the teaching aims to develop students' critical thinking, basic values and ethical principles.

Research Question

In this article we examine the influence of the introduction to a value-based intervention on two teachers' use of practical activities in mathematics teaching, based on the following two assumptions. First, elements of value and viability with regard to the application of mathematics are not commonly used to increase the use of practical activities in school mathematics. It might therefore offer a new approach to the use of practical activities in mathematics teaching and initiate reflective processes regarding beliefs (Lerman, 2002) about using practical activities in mathematics teaching. Experience with a different setting for practical activities might stimulate reflection on one's own beliefs, which is essential for a lasting change of practice (Wilson & Cooney, 2002). Second, the introduction of new mathematical content in a *VaKE*-based learning environment entails a sociocultural approach. In sociocultural learning theory, the construction of knowledge takes place through interaction or activities of a social and cultural kind (Dysthe, 2001). Conversation and joint activities are crucial to learning, and each individual's development is recognized by changed participation in the practical situation. Communities of practice are important for the development of knowledge, and social factors become more than a frame surrounding the learning situation

[5] See Patry, Weyringer and Weinberger (2007) for a detailed review of each step of the *VaKE* methodology.

(Wenger, 1998). Such features characterize an encouraging environment for practical activity-based teaching (Bell, 1993; Meira, 1995; Wæge, 2007). Therefore, an unmodified application of the *VaKE* method can be applied when introducing new mathematical content, supported by practical activities, in an attempt to influence the teacher's use of practical activities. Based on the described prevailing situation, and the assumptions presented, our research question is as follows.

How does the introduction to a VaKE-based teaching approach, supported by practical activities, influence two elementary school mathematics teachers' use of practical activities in mathematics teaching?

Theoretical Background

Beliefs

The *Teachers Matter* report (McKenzie, Santiago, Sliwka & Hiroyuki, 2005) confirms the important role teachers play in students' learning. According to the work of Shulman (1987) and Handal and Lauvås (1987), teachers' professional knowledge, which combines disciplinary knowledge, didactical knowledge and beliefs, is regarded as the most fundamental impact factor on teachers' professional choices. Furthermore, beliefs, values and attitudes can be seen as part of an individual belief system, where the conviction about an issue or task often develops into "values, which house the evaluative, comparative, and judgemental functions of beliefs and replaces predispositions with an imperative to action" (Rokeach, as cited in Pajares, 1992, p. 314). Such views imply that teachers' beliefs are fundamental factors influencing teachers' practice, and that they influence disciplinary and didactical choices made by each teacher. Factors that make an impact on teachers' professional knowledge are dynamic features (e.g. Korthagen & Vasalos, 2005), but a teacher's beliefs are seen as an impact factor that have been found to be difficult to challenge

and to change (Borasi, Fonzi, Smith & Rose, 1999; Chin, Leu & Lin, 2001; Pehkonen, 2003; Philipp, 2007; Thompson, 1992; Wilson & Cooney, 2002). Furthermore, the change of all other impact factors is more or less regarded as superficial and temporary if they are not in accordance with the teacher's prevailing beliefs (Day, 2004; Lloyd, 1999; Pehkonen, 2003). It seems that if teachers are to make a sustainable change in teaching practice, their beliefs need to be challenged (Wilson & Cooney, 2002).

Rokeach (1968) and Pehkonen (2003) look at different degrees of knowledge as subsumed in personal beliefs. Beliefs that are in accordance with an objective coherence in the surroundings are established as knowledge. Beliefs that remain as subjective knowledge are disputable, and therefore susceptible to be influenced by feelings (Grelland, 2005) and personal evaluation of good or bad consequences (values) when transformed into action. In a review of research on teachers' beliefs, Pajares (1992) identifies several commonalities concerning beliefs, summed up by Beijaard, Verloop, Wubbels and Feiman-Nemser (2000, p. 262) who suggest three common features of beliefs:

1. They are highly individual, deeply personal, and seem to persist.
2. They are formed by past experiences.
3. They represent an individual's understanding of reality enough to guide thought and behavior and to influence learning.

The understanding of beliefs as subjective knowledge influenced by feelings materialized through actions, and thereby defined as values, seems to be recognized as the way beliefs are visualized (Bishop, 2001). Moreover, through the fundamental influence that beliefs have on the interpretation of impressions and new knowledge, Pajares (1992) ascribes to beliefs a filtrating effect on new impulses. This is in accordance with the fundamental position of beliefs emphasized in the research literature on beliefs in mathematics teaching

(Pehkonen, 2003). Beliefs are influenced by new impulses and make an impact on how impulses are interpreted (Feiman-Nemser & Remillard, 1996).

Changing Practice and Changing Beliefs

According to Kerem Karaağac and Threlfall (2004, p. 137), with reference to Lerman (2002), the assumption within research on teachers' beliefs about mathematics teaching and learning has been "that awareness of a difference between beliefs and practice would result in some attempt to change". Within this area of study, however, there is a growing body of research that reports cases where the teacher either does not try to change even though he/she is aware of a difference between beliefs and practice (Kerem Karaağac & Threlfall, 2004) or simply does not become aware of such a discrepancy (Raymond, 1997). Hence, a discrepancy between beliefs and practice does not always initiate an attempt to change.

However, a change in beliefs increases the possibility of developing practical knowledge (Beijaard et al., 2000), but because of the presence of feelings, beliefs are found to be resistant to change. Independently of the content of presented arguments or experiences, efforts are made to interpret the impressions so as to support prevailing beliefs. Should that prove impossible, the arguments or experiences are ignored or rejected as a result of the influence of feelings, such as irritation or even anger (Pehkonen, 2003). Pehkonen (2003) further states that if a person's beliefs are supposed to change, it is a long process demanding personal engagement. Based on Shaw, Davis and McCarty (1991), Pehkonen (2003) suggests that the teacher must accept being challenged with a problem, doubt or an inconsistency between attitude and practice, and feel responsible to do something about it. The teacher must also have a vision of how teaching ought to be and prepare a plan for how the vision may be realized.

Shulman (1987) and Handal and Lauvås (1987) see the development of teaching practice as a cyclic process based on the impression that all impact factors are dynamic.

According to Kolb (1984), teachers' practical experiences generate observation and reflection and are based on general notions that are tested and developed in new situations. This provides the teacher with experiences at a higher level. The developmental process (*experiential learning*) is cyclic (Kolb, 1984), in the form of a helix. The process alternates between reflection and action (Korthagen & Wubbels, 2001). Korthagen and Vasalos (2005) develop this further by focusing specifically on teachers' reflections and actions attached to fundamental beliefs and views (*core reflections*). If one is supposed to change practice, both beliefs and actions must be changed. Such an impression about change of beliefs is also presented by Handal and Lauvås (1987, p. 12): "we experience our own practical efforts very much in the light of structures, concepts and theories transmitted to us in such a way that this may even lead us to change our values and beliefs to some extent". Teachers' *pedagogical content knowledge* (Shulman, 1987) and teachers' professional development are influenced during and by practice.

In the essay "The Logical Categories of Learning and Communication", Bateson (1972) links *learning* to the element of *change*. According to Bateson, a logical hierarchy of learning and communication can be identified and applied to suggest what priorities are relevant for change of teacher practice. The hierarchy consists of different levels of influence, with the levels of the hierarchy labeled 0, 1, 2 and so forth. With regard to change of practice and beliefs, level 0 in the hierarchy is about receiving and developing actions (here, practice) based on internal or external signals received by the teacher. Level 1 relates to how the teacher acts to change actions in accordance with responses to experienced practice. Level 2 focuses on the teacher's internal responses to the experiences at level 1. Level 2 then relates to changes of beliefs based on experiences initiated by practice (level 0) and change of practice (level 1). Hence, existing beliefs need to be challenged to create a permanent change of practice.

Independently of the chicken-and-egg discussion about what comes first, practice or beliefs, we agree with Pehkonen (2003) and Shaw, Davis and McCarty (1991) that the impact must stem from an experienced inconsistency between vision and practice. Transferred to the mathematics classroom, this means that teachers must be given the opportunity to initiate change in teaching practice if change of beliefs is to be facilitated.

Methods

In this article we examine the influence of the introduction to a value-based intervention on two teachers' use of practical activities in mathematics teaching. Since we wanted to focus on this particular excerpt of what might influence teachers' use of practical activities, we decided to apply a "two-case" comparative case study (Flick, 2006; Yin, 2003) to collect qualitative data. This approach was chosen because of its appropriateness when investigating "a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident" (Yin, 2003, p. 13). We find that "case studies of teachers can be used intentionally to prompt teachers to reflect upon and examine their own beliefs and practices" (Thompson, 1992, p. 143).

The data were collected from two teachers over a period of about 18 months. Data collection instruments were multiple: interviews, video-recorded observations of teaching together with the teachers' own reactions and impressions about the content of the recorded lessons, log-writing and a questionnaire based on open-ended questions. This is in accordance with Yin (2003, p. 14), who states that "the case study relies on multiple sources of evidence, with data needing to converge in a triangulating fashion". The importance of multiple sources of evidence offered by a case-study approach is also emphasized by research reviews on the change of mathematics teachers' beliefs about mathematics and mathematics teaching (Philipp, 2007; Thompson, 1992; Wilson & Cooney, 2002).

The two teachers, *Vivian* and *Walter* (pseudonyms), were recruited to the study by their respective principals upon our request for a teacher from their respective schools. We contacted these two schools because they were supposed to participate in an EU-FP7 project that aimed to try out *VaKE* in science teaching, but which did not make it to the final stage in competing for an EU-FP7 grant.[6] We asked the school principals to find a teacher recognized as an acknowledged teacher by the work environment (Haara & Smith, 2009),[7] and who was interested in developing his/her teaching of mathematics. Vivian has been teaching mathematics and other subjects in the Norwegian upper primary school (students 9 to 13 years old) system for 10 years, and Walter has been teaching mathematics and other subjects in the Norwegian lower secondary school (students 13 to 16 years old) system for 5 years. They are about the same age, and both have 30 ETCS (*European Credit Transfer and Accumulation System*) in mathematics from their Norwegian teacher education.

An intervention was designed for the case studies (Lane, Weisenbach, Little, Phillips & Wehby, 2006). The intervention was a 20-hour-long in-service course in *VaKE* held by one of the two researchers responsible for the research project, focusing on applying *VaKE* when teaching mathematics. The course consisted of two gatherings of two five-hour-long course days each, and focused on *VaKE*, areas on which *VaKE* is based (constructivism, value education, moral dilemmas in teaching), and on the professional development of teachers. In between the two gatherings the course participants prepared suggestions for themes and dilemmas for mathematics lessons based on the *VaKE*-method and how practical activities could be included in the mathematics lessons. The first gathering consisted of lectures presenting the course literature, and there was an emphasis on practical examples allowing for

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[6] EU-FP7 is EU's 7th framework program for research and technological development, and the *VaKE* projec*t* was one of the eight finalists for the grant (Patry et al., 2007).

[7] In Haara and Smith (2009), acknowledged teachers of mathematics are defined to be "teachers who are viewed as competent mathematics teachers by the principal and earn respect from colleagues, pupils and other groups of relevance within the working environment".

teaching of mathematics through moral dilemma supported by a practical activity. An example of this focused on airline overbooking policies or salary payments for completed work. The second gathering focused on change of practice using themes and practical activities suggested by the two participating teachers, for instance, on choosing between refurbishing the playground at the school and expanding the computer facilities for the students of one class, or on delivering a tender for a house building contract.

The data collection period started when Vivian and Walter were interviewed about 6 months prior to the intervention. The interviews focused on their opinions on mathematics and school mathematics in general and their present and future teaching practice. Each semistructured interview lasted for approximately 75 minutes and was recorded and transcribed. Essences of meaning were extracted from the transcriptions (Kvale, 2006) and interpreted through a hermeneutical approach. The interpretation process contributed to the planning of the forthcoming intervention since it offered impressions of how beliefs about mathematics and teaching in general, and more specifically about practical activities in mathematics teaching, were part of Vivian's and Walter's visions of teaching. These impressions also served as references for comparison in the analysis of data produced after the intervention.

Vivian and Walter were observed and filmed in 3 mathematics lessons each. The observations took place within a two-week period starting about a month after the intervention. Observational data were collected when Vivian taught mathematics in 4^{th} grade (students 9 to 10 years old), and Walter taught mathematics in 8^{th} grade (students 13 to 14 years old). Respectively, the first lesson was typical for the kind of mathematics teaching that Vivian and Walter traditionally practiced, and the other two were based on the introduction of new mathematical content in a *VaKE*-based environment supported by a practical activity opportunity. Immediately after each lesson the teacher and the researcher who video-recorded

the lesson, watched it together. During these sessions, Vivian and Walter were free to comment on what they saw (Jacobs & Morita, 2002). This gave access to Vivian's and Walter's reflections and observations on the recent teaching experience. Comments and evolving discussions were recorded and transcribed.

The transcribed comments from the video sessions were coded. From the comments made by the teachers we created units (Grønmo, 2004) that were then categorized as "positive", "negative" or "neutral" (Jacobs & Morita, 2002). Units including discussion of practical activities, isolated or within the progress of the *VaKE*-methodological structure, were divided into five subcategories and given an interpretation according to the teacher's comments: "positive – unconscious", "positive – conscious", "neutral", "negative – conscious", "negative – unconscious". This is in accordance with how people are conscious about some reactions and prevented from being conscious about other exhibited reactions. Unconscious reactions are difficult to explain. In other words, the observing teachers' reactions could be separated similar to the distinction between conscious and unconscious values (Bishop, 2001; Grelland, 2005).

Vivian and Walter wrote personal logs. They started on the day they received the in-service course information and reading list. The logs cover the last approximately 12 months of personal impressions about mathematics teaching, the in-service course, and experiences in accordance with both observed and independently conducted *VaKE* lessons. The same categorizing system as with the video sessions was used in the analysis of the two logs, but based on systematic extraction of meaning of sequential content organized in a matrix (Grønmo, 2004), structured by a timeline, and the participants.

Exactly 12 months after the intervention started, Vivian and Walter responded to an open-ended questionnaire focusing on beliefs regarding factors with influence on their use of practical activities in mathematics teaching. The questionnaire was validated by 3 researchers

and 3 mathematics teachers in elementary school, who commented on the relevance and clarity of the questions. The questions did not focus on *VaKE*, but were developed based on interpretations stemming from the analysis of the preintervention interviews, observations and video sessions. The collected data were analyzed in the same way as the logs, but the matrix was structured by the questions and participants.

Based on the analysis of the logs and questionnaires and in accordance with the interpretations of the prequestionnaire analysis, Vivian and Walter were interviewed once more at the end of the project, about 1 month after responding to the questionnaire. The logs and questionnaires served as data-producing devices in a triangulation quest for points of refutation and confirmation of prequestionnaire interpretations. The interviews were structured, and the interview guide was divided into three main parts.

- The teacher's beliefs about mathematics and practical activities in mathematics.
- The teacher's response to the value-based intervention.
- The influence of the intervention on the teacher's teaching of mathematics.

From a hermeneutical perspective, our interpretations in the analysis have probably been affected by our unconscious prejudices, although the triangulation process and validation by Vivian's and Walter's interpretations strengthened the viability of our conjectural suggestions and the subsequent discussion of how the intervention influenced the teachers' use of practical activities. Hence, in the analysis we used both a phenomenological approach and a hermeneutical approach (Grønmo, 2004). The phenomenological approach is recognized in the use of Vivian's and Walter's experience with the intervention program as a basis for the analysis. The hermeneutical approach is reflected in the comparison of the influence of the intervention with the preintervention situation, as well as similarities and discrepancies between the two teachers' beliefs about the teaching of mathematics.

Findings

The findings are reported through a description of beliefs Vivian and Walter had about mathematics and practical activities in mathematics, their response to the value-based intervention and the use of *VaKE* supported by practical activities in teaching. This follows the pattern of four phases for teacher change, reported by Shaw, Davis and McCarty (1991) and Pehkonen (2003):

- experiencing personal inconsistency
- feeling responsible for doing something about the inconsistency
- developing a vision of how teaching ought to be
- making a plan for how the vision can be realized

Vivian

Experiencing personal inconsistency. Vivian was fairly open about her own lack of understanding of generalized mathematics in the preintervention interview and indicated that she did not always see the application of theoretical dimensions to real-life situations. She was more focused on mathematics in a strictly real-life context, with an emphasis on the practical application of mathematics. Furthermore, she was an active teacher, who enjoyed being the focus of attention and explaining the mathematical content at hand, as she explained during the preintervention interview:

> Vivian: I think I am very present … and very active. In a mathematics lesson which could actually be boring, I still feel that I am creative, and I feel … I think that my problem maybe is that I am too … ehh … active. So what happens … especially in mathematics … what happens when I am about to explain something … then it is like Oh yes! (changes her voice), and then I like to use things which they know. Imagine!

> (changes her voice again) … and then I tell a little story about something ….

Vivian used narratives and relied on the students' imagination when using examples in teaching. In her opinion, the teacher had to explain the mathematical content to the students, and then the students had to do quite a lot of exercises to internalize the content. Kuhs and Ball (1986) refer to this "as content-focused with emphasis on conceptual understanding". The students' understanding of ideas and processes is emphasized through the instruction of the mathematical content, and the lessons might vary considerably from lesson to lesson. It was important to Vivian that the students both have fun and learn, and that they are offered some exiting experiences when learning mathematics. In accordance with Ernest's (1989) recognized pattern for an *Explainer's* use of curricular materials, this meant to Vivian that the textbook approach was enriched through her introducing additional examples, problems and activities of real-life relevance.

The preintervention interview revealed that Vivian was confident that her students learned mathematics, but she was not satisfied with her own organizing priorities. She felt that the lessons ought to be more varied, and she wanted to be more attuned to what Kuhs and Ball (1986) refer to as "learner-focused", in the sense of focusing the teaching more on the students' active involvement. She therefore experienced an inconsistency between her teaching and her beliefs about how mathematics ought to be taught.

Feeling responsible for doing something about the inconsistency. Vivian was clear about her bad conscience for what she experienced as a lack of variation in her teaching. In her opinion, the content-based teaching of mathematics for which she had been an exponent, with emphasis on the progress and approaches suggested by the textbook, ought to be supported by an expanded organizational repertoire, as she stated during the final interview in the project:

Vivian: My mathematics teaching ought to consist of exercises which the students master, exercises which challenge the students, use of the textbook, use of different tools and props, collaboration among the students, individual work, work through theoretical approaches, work through practical approaches, and so forth. I would like my teaching to be varied.

Developing a vision of how teaching ought to be. Vivian wanted her teaching to be more varied and student focused. She also wanted to make her instructive *Explainer* role less dominant. The introduction of practical activities supported by a *VaKE*-based approach provided her with an opportunity to change her practice, as she concluded during the observation of one of the video-recorded *VaKE*-based lessons:

Vivian: I have missed such an approach in mathematics …. I have needed something to change my teaching of mathematics with, and this is what I have been missing!

Making a plan for how the vision can be realized. On 2 occasions, 3 weeks after the in-service course, Vivian used dilemmas, which she found relevant to the students' real-life interests. The first dilemma depended on, in terms of mathematics, economics calculations related to choosing between computer accessories for the students involved and a new climbing frame area for all students in school. The second dilemma involved economics and volume calculations related to choosing between a party for the entire school to celebrate the new climbing frame area, and refurbishing the school entrance. The students had access to props. In the first lesson it was fake money, and in the second lesson it was drinking glasses, deciliter and liter measures and free access to water. The dilemmas required the students to work with the four arithmetical operations, money values, estimation, measuring and

geometrical figures. The props made it possible to systematize information practically and carry out operations that initiated, simplified and confirmed or refuted the students' calculations.

Vivian was conscious of her neutrality while applying the *VaKE* approach, but she was very focused on setting "a conflict zone". The competitive organization appealed to her. She reorganized the classroom before the lessons, initially grouping the students on the floor. Vivian clarified the moral dilemma and each student made a written, initial decision on the dilemma. Based on the students' decisions she then divided them into two groups, separated by a front line. "It is you against them!", she said several times to each group, referring to the students in the other group.

When observing the video recording of her own teaching, she reported that she could see that the *VaKE* approach introduced a new organizational possibility to her mathematics lessons:

> Vivian: And that is just what this math builds on. That you actually do not only sit and work on some numbers, you actually go into yourself a bit … because when you start to tear at something inside yourself, you automatically become more motivated, and then you approach the problem in another way than you would do if you just sat there.

At the same time she claimed that the new method occasionally resembled her regular approach:

> Vivian … and I have got something of a revelation by entering this project, and I now feel that one of my strengths is that I have motivated students … and that the reason for that maybe is because I challenge them in relation to themselves to some extent ….

She was familiar with challenging the students and pitting them against each other, but not in such a planned and structured way. This was supported by the video recordings, which showed that she was comfortable with the organizational demands of the *VaKE* method and that she was able to let the students and the method set the pace of the lesson.

In the interview at the end of the project, Vivian revealed that she believed that her teaching of mathematics and the use of practical activities in the teaching had changed:

Researcher: Did your use of practical activities change after you were introduced to VaKE?

Vivian: Yes, it is much more … it is no longer so structured. Now I start trying to make the students curious, investigative and uncertain for a while. I give them a challenge which involves them, and then … they can get a feeling of solving, and I can focus on challenges which occur. So it is a bit different now.

Walter

Experiencing personal inconsistency. Whereas Vivian was content with focusing on practical applications, Walter found in the preintervention interview that is was important to emphasize both the theoretical dimension and the practical applications of theoretically based results:

Walter: Well, it is a theoretical subject, but at the same time one can approach it in a practical way, and I feel that is very important.

Furthermore, Walter and Vivian held different views about how mathematics ought to be taught. In the preintervention interview he emphasized, as did Vivian, that the teacher should explain the mathematical content and that this should be followed by the students'

work on exercises. However, the observation of lesson 1 showed that Walter taught in a more traditional way than Vivian. He explained the new content and examples to the students before they worked on exercises. Finally, Walter gave a summary of the lesson. Whereas Vivian focused on motivating the students, Walter to a larger extent wanted mathematics as a subject to be self-motivating, as he reveals through his description of his mathematics lessons in the preintervention interview:

> Walter: … and traditionally school mathematics is kind of a mix between a theoretical review, usually using the blackboard, and a conversation with the students, and then this is combined with solving exercises in the textbook. That is in a way how I have experienced mathematics myself through my own schooling, and how I to a large extent teach myself … although I sometimes perhaps would have wished that I could vary my teaching more.

Walter's teaching seems to be in accordance with a "content-focused view with emphasis on conceptual understanding" (Kuhs & Ball, 1986), but it is, to a larger extent than Vivian's teaching, "content-focused with emphasis on performance" (Kuhs & Ball, 1986) . In this approach it is assumed that acquiring the content motivates further studies and practical applications.

In the preintervention interview, Walter expressed beliefs about mathematics as a general education subject:

> Walter: Everybody needs mathematics. That is, a certain basic mathematical knowledge … in order to make reasonable, good choices. And one will be confronted with it no matter what … regardless of profession … if

not with pure, formal mathematics, then certainly with a mathematical way of thinking.

Researcher: Are you thinking about the terms which you used earlier [in the interview], like problem solving, logical reasoning, and structuring …?

Walter: Yes! Because I think that mathematics is an educational subject which structures one's thoughts … which I often miss among the students. If they are given some kind of problem or exercise or something, they are not able to see logical flaws, and in my opinion that has to do with mathematical thinking ….

In Walter's opinion, the educational subject dimension of mathematics seems to vanish as an argument for maintaining interest in learning mathematics when compared with the legitimacy of the general education dimension in mathematics that he remembered from his own time as a student. He sees mathematics as an educational subject based on concepts such as curiosity, logic and persistence, but mathematics proves not to be as self-motivating to the students as he would expect it to be. In fact, he reveals that he always has a bad conscience for his lack of practical activity-based teaching. A more varied lesson structure would hopefully increase the students' interest in mathematics, as he reveals in this sequence from the preintervention interview:

Walter: I do have to say … I have always had an ambition to use practical activities in mathematics because I think it is a very useful approach if you can combine it … with another kind of mathematics teaching, so that the students are given a balance towards … well, solving of exercises and such. And I must admit that I have always had a bad conscience for my lack of practical activity-based teaching.

Walter seemed to be influenced by both a "content-focused view with emphasis on conceptual understanding" and a "learner-focused view" (Kuhs & Ball, 1986), but suppressed the influence because of bewilderment about how to change his teaching, which becomes apparent during the preintervention interview:

> Walter: It is a bit about … that I am not used to using it, and I spend much more time in preparing such activities. And, obviously, you did not get trained in such teaching during teacher education. And that … and that puts you … and the textbooks do not emphasise such teaching either, and that leaves you to … to your own … oh, what is the word I am looking for? … That is, my own … you have to rethink, maybe be a bit creative, and that … is maybe a bit time consuming in a … well, in the hectic school day.

Walter's traditional teaching is a compromise between his beliefs about how mathematics ought to be taught and his awareness of the advantage of emphasizing structure, performance and textbook applications when teaching mathematics, a phenomenon previously shown by, for instance, Cooney (1985), Lloyd (1999) and Raymond (1997). Hence, Walter experienced a personal inconsistency between his beliefs about mathematics teaching and his actual teaching, since his teaching lacked variety and did not prioritize practical activities in the way he wanted.

Feeling responsible for doing something about the inconsistency. As with Vivian, Walter expressed a kind of guilt for lacking variety in his teaching. Moreover, in the preintervention interview he was not entirely willing to accept the students' prevailing opinion, who saw mathematics from a utility perspective only:

> Walter: For instance, I remember compared to my own schooling, I thought it was really funny to get some practical … the daily puzzle or things like that to work on. But when I try such problems with students … they do not seem to see any point in it … Well, what is this then? Are we supposed to wo … (changes his voice). Often they do not understand the problem at all. They are not used to think in a … in a mathematical way.

He therefore felt that instead of the rather traditional teaching, he should teach more in accordance with a "learner-focused view" (Kuhs & Ball, 1986), and include more practical activities in his teaching.

Developing a vision of how teaching ought to be. Walter did not have the same starting point regarding the *VaKE* approach as Vivian, and based on the organization of his regular teaching, Walter's vision implied a more radical change of practice. The intervention introduced Walter to an approach that he believed could challenge the present suppression of his mathematics teaching beliefs, as seen on separate occasions in his log during the in-service course:

> Walter: Making teaching more realistic is a massive challenge, especially when compared to one's own view about what teaching is, and ought to be. I believe the VaKE project to be useful in this respect.

> Walter: I especially approve of using such a methodology as an approach to teaching mathematical content, and then later on concentrate on the theoretical approach to the mathematical topic at hand. I believe that the students are more easily able to see that what we are supposed to learn

> is relevant to learn, that this is something which they actually may find useful.

Making a plan for how the vision can be realized. As with Vivian, Walter on 2 occasions about 3 weeks after completing the in-service course, taught by introducing 2 dilemmas that he found relevant to the students' real-life interests. The first dilemma depended on economics calculations and the calculation of an area of a planned house where a compound area consisting of different geometrical shapes represented the new mathematical content. In terms of mathematics, the students worked on calculating construction costs. The second dilemma was about a nonregular pyramid-shaped box of chocolate pudding and a lack of coherence between the quantity of pudding stated on the package and the measured quantity of pudding in the package. The new mathematical content was represented by a pyramid-shaped polyhedron, the theorem of Pythagoras, and the connection between cubic centimeter and deciliter. In the first lesson the equipment for the practical activity consisted of the traditional compass, protractor and ruler, but in the second lesson, these were accompanied by an actual package of the chocolate pudding polyhedron.

The observations of the lessons show that Walter experienced some challenges. He struggled to find his position in the context, and the students were not sure what was expected of them. They seemed curious and interested at first, but the lessons did not work out the way Walter had planned. The dilemma discussions did not develop as planned for two reasons. One of the discussion groups was outnumbered in both *VaKE*-based lessons, and Walter did not succeed in pushing the two groups to find arguments in favor of the group's point of view. In the end Walter found the lessons to be rather boring and worthless, an impression that he states explicitly in his log after both *VaKE*-based lessons:

Walter: I had the first VaKE-session today, and it was done pretty much the way I had planned. I would perhaps have hoped for more engagement from the students, but it turned out to be rather boring.

Walter: I had the chocolate pudding session today, and I have to say that the lesson was not a success. I felt that the dilemma at hand engaged only a few of the students.

As the observations of the video recordings proceeded, Walter expressed doubts about his loyalty to the *VaKE* approach in mathematics teaching. In his opinion, he did not seem to be able to make the students aware of the moral aspects of the dilemmas. In fact, he changed his view on the *VaKE* approach as he gained more experience with it. When observing himself and the class on video in the second lesson he stated that the *VaKE* approach would be appropriate to use after the mathematical content had been introduced in another way, instead of combining the introduction of mathematical content and the value emphasis:

Walter: In general I think that such approaches … VaKE-approaches in relation to mathematics, would be best to have when you have finished a mathematical topic. Because then you can use the knowledge, put it into a setting which in a way creates engagement and shows that you need mathematics in daily life.… Because … if you do it when you are introducing a mathematical topic, I believe … that the students will find it difficult to do the necessary calculations, and then the foundations disappear for some of the arguments which they may put forward ….

In his opinion, the calculations that the students would need to do in order for the dilemma discussion to become active, were too complicated, a situation that is also described by Lloyd (1999). It would therefore be better to revisit the mathematics they had learned in a

traditional manner by applying it in a *VaKE*-based context. In the end, Walter argued for his usual teaching to be a way to make the students better prepared or disciplinary-skilled enough in a mathematical theme before relying on mathematical arguments in discussions focusing on moral dilemmas, as shown in this concluding comment from the same video observation session:

> Walter: As a way of teaching it obviously brings along more noise, and it becomes a bit more difficult to see what each student actually does. If they are seated at separate desks it gives me a much better overview … what each student does, if he is disturbing others or not …. Students who work in groups often make teaching more complicated than when students work individually.

Discussion

The preintervention interviews revealed that both Vivian and Walter claimed that they believed in using practical activities in mathematics teaching, and that they were interested in changing their practice in order to increase the use of practical activities. They did not find their current teaching to be in accordance with personal visions, and they struggled to find personal acceptance for increased reliance on practical activities in mathematics teaching. A change of practice towards an increased use of practical activities would therefore only be temporary or superficial unless the change made an impact on their beliefs and didactic knowledge (Bateson, 1972; Wilson & Cooney, 2002).

Vivian was enthusiastic about the theoretically supported approach to mathematics teaching provided by the value-based intervention. It acknowledged elements of her previous teaching, and she referred both to how she was influenced and how she experienced excitement among the students. “I have probably never seen the students this engaged!”, she

said during the observation of the first *VaKE*-based lesson. Vivian became more aware of her own role and about making the students engaged without her direct involvement and guidance. Hence, her role as a facilitator became more important (Ernest, 1989), and she personally felt that she had experienced a kind of revelation by participating in the study. Finally, her impression of the students' work with mathematical content was also influenced. She experienced the group work as coherent with her opinion that students were active learners, an opinion which she reported she had not been able to include in mathematics teaching in the same way as she had done when teaching other subjects.

Walter was also enthusiastic at first, but developed a resistance towards the thought of introducing new mathematical content through the *VaKE* approach supported by practical activities, as the experience with the approach increased. Walter experienced that the positive expectations that followed the in-service course disintegrated when he applied the *VaKE* approach in his own teaching. This feeling was reinforced by watching video recordings of his lessons, all of which proved a setback regarding his vision of how to change practice. Making his suppressed beliefs about how mathematics ought to be taught explicit once more seemed to capitulate to the prevailing and familiar way of teaching mathematics. Similar situations are described by Kerem Karaağac and Threlfall (2004) and Raymond (1997), but the case of Walter refers to a situation where the teacher actually attempted to change practice. He experienced constraints that prevented him from further consideration of the new approach as a possible way to learn mathematics through a new perspective and increase the use of practical activities. His rather modest level of didactic knowledge of mathematics, revealed in the preintervention interview through his bewilderment about how to arrange for appropriate use of practical activities, and the response from the students to his new approach to teaching strengthened this impression. Hence, he withdrew to the established form of teaching familiar to himself and the students.

Vivian and Walter experienced the *VaKE* approach in different ways, which led to different outcomes. Vivian maintained her enthusiasm about a value-based approach supported by practical activities. Walter did not. The main reason for this, in our opinion, is found in the different starting points of the two teachers. Vivian's beliefs were not challenged to the same extent as Walter's beliefs were. Her vision of teaching proved to be within an approachable reach. The discrepancy between Walter's beliefs and experiences of constraints given by his teaching practice of mathematics and the actions used in the value-based approach was too wide, and in a way he "broke" the cycle of reflection and action necessary to change practice and beliefs (Kolb, 1984; Korthagen & Vasalos, 2005). In the in-service course, a community of learning was created for Vivian and Walter (Wenger, 1998). Vivian entered a productive moderation process since her beliefs were not severely challenged and her students did not meet a teaching approach that was totally different from what they had experienced before. Vivian and her students were able to explore the new approach together. Walter's beliefs were deeply challenged and his students met a teaching approach that was quite alien to them. Walter therefore lacked the moderation process from which Vivian so successfully benefitted. Having said this, though, professional growth can take the form of maintaining present beliefs after having had the courage to challenge them. Walter tried to change his practice and had the courage to challenge his beliefs about using practical activities for teaching mathematics, but this did not lead to change because of the influence from what he experienced as restraining constraints.

Conclusions

Changing beliefs about the teaching of mathematics is an extensive and longitudinal process (e.g. Pehkonen, 2003; Wilson & Cooney, 2002). Change of beliefs and change of practice can be independent of each other, i.e. they are not synonymous. However, change of beliefs and change of practice are often tangled in such a way that when one is changed it will

cause change to the other. In this study we aimed to examine the influence of the introduction to a value-based intervention on two teachers' use of practical activities in mathematics teaching. The two teachers, Vivian and Walter, were introduced to a value-based approached to teaching mathematics that opened up practical activity support opportunities, which implied a change of practice for them both. From this study, we can note that Vivian approved of the alternative practice, both as a teaching approach, and as a possibility to increase the use of practical activities, while Walter did not. A more thorough examination of the study reveals, however, that the change of practice challenged both Vivian's and Walter's beliefs about how to teach mathematics and the possibilities for using practical activities. It is a common impression that beliefs have a filtrating effect on new impulses (Beijaard et al., 2000; Pajares, 1992; Philipp, 2007), and since the applied change of practice was not too controversial in relation to Vivian's prevailing beliefs, her positive attitude towards an increased use of practical activities and student involvement was strengthened. Walter found the change of practice to be too controversial in relation to his prevailing beliefs, and instead of maintaining the positive attitude towards increasing the use of practical activities through applying the value-based approach nurtured by the offered intervention, he returned to the previously established teaching practice as the preferred way of teaching mathematics.

Regardless of the tangled question of whether a change of practice implies change of beliefs, or if change of beliefs implies change of practice (e.g. Bateson, 1972; Kolb, 1984), we are left with the impression that Vivian managed to offer the new teaching approach to the students in a way that appealed to them, whereas Walter did not. There might be several encouraging or restraining constraints that paved the way for such a course of events, and the impact from different constraints are not necessarily similar for Vivian and Walter. Nevertheless, we find that three constraints on this occasion need to be mentioned on behalf of both teachers. First, we would like to mention the two teachers' beliefs about teaching

mathematics and their didactic knowledge as crucial impact factors. Second, the impact of the intervention in which Vivian and Walter participated must be acknowledged. Third, the students' response to the new teaching approach probably also played a role in forming Vivian's and Walter's acceptance of the use of a value-based teaching approach supported by practical activities. To maintain a changed practice, it seems that the changed practice must also lead to a change of beliefs. If not, practice will eventually drift back to its initial pattern or to something less radical than the alternative practice. The isolated findings in this study show that Vivian entered a process that might lead to increased use of practical activities in her future teaching, whereas Walter in the end found his traditional way of teaching to suit him better. For Walter, this return implied staying faithful to the explicit practice he upheld when entering this study, as his professional conscience did not allow for increased use of practical activities.

In this article, we base our cautious suggestions on interpretations of data stemming from the cases of two teachers' experiences with the introduction to a value-based approach to changed practice in mathematics teaching. The interpretations have been validated by the 2 teachers through a triangulating process. Our temporary interpretations were tested and reformulated in the light of their logs and responses to an open questionnaire. Finally, the interpretations were validated by conducting individual interviews with the 2 respondents, which allowed for their personal interpretations. Hence, the contextual interpretations are based on multiple data sources and we believe the interpretations to be well justified, despite the limitations of basing a study on a relatively small and narrow empirical source (Yin, 2003).

We want to conclude that the in-service course, which emphasized the use of practical activities in mathematics teaching through a value-based approach to new mathematical content, influenced one of the participating teacher's beliefs about teaching mathematics and

increased the space given to practical activities in her teaching. Furthermore, we add another study to the body of research that confirms that awareness of a difference between beliefs and practice will result in some attempt to change (Kerem Karaağac & Threlfall, 2004; Lerman, 2002). However, the case of Walter shows that the influence from restraining constraints might result in an aborted attempt to change. We hope that Vivian's and Walter's reported struggles and challenges with the correspondence between beliefs and practice will bring about further research on persistent change of teachers' practice.

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Acknowledgment

The authors would like to thank Dr. Christoph Kirfel at the University of Bergen for his comments during the writing process.