The Mathematics Enthusiast

Volume 9
Number 1 *Numbers 1 & 2*

Article 10

1-2012

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Introduction

Two of the great ancient civilizations were those of the Greeks and the Chinese. Many great works of art, architecture, philosophy and literature have been produced by both of these civilizations. When it comes to mathematics in the Western World, the Greeks have also been credited with many contributions to the field, especially geometry. Anyone who has completed a standard high school mathematical curriculum has been introduced to the names of Pythagoras, Euclid and Archimedes and their methods. Not as widely credited in the Western world are the mathematical contributions of the ancient Chinese civilization. An examination into Chinese mathematics reveals their deep understanding of mathematics, in some areas at a level greater than that of the Greeks. The Chinese civilization had their own mathematical greats, Liu Hui and Zu Chongzhi who were every bit as genius as their Greek counterparts. The purpose for mathematics and the techniques utilized by the two civilizations may differ, but the knowledge of the two civilizations is remarkably similar. This paper provides the reader a summary of the two civilizations works and mathematical philosophies and a comparison of the techniques used to determine π, proof of the right triangle theory, and the famous works of each civilization and the application of the civilizations mathematical knowledge in the science of land surveying.

Chinese Mathematical History

The origins of Chinese mathematics are steeped in legend. It is said that the Yellow Emperor, who ruled sometime around 2698-2598 BC charged one of his subjects named Li Shou to create arithmetic. (Li, 1987) The creation of a mathematical system cannot be credited to one individual, but the legend does give evidence of a mathematical system present in the 26th century BC in China. Evidence from the Shang Dynasty gives the first physical evidence of mathematical application in Chinese culture. The Shang Dynasty was a well developed agricultural society from the 16th to 11th century BC. Remains of large cylindrical grain bins have been discovered, along with bronze coins, evidence of a monetary system. Additional bone artifacts, known as plastrons, have the first known writing system of China. Including in the writing are numerals of a sexagesimal system (Li, 1987).

The *Zhoubi suanjing* is the oldest known Chinese mathematical work. The *Zhoubi suanjing* is also known as *Chou Pei Suan Ching* or *Chou Pi Suan Ching* in some historical references (Dauben 2007). The author of the compilation is unknown, though it is believed to be composed sometime between 100 BC and 100 AD. However, the contents were likely from a much earlier period (Li 1987). The *Zhoubi* is a work focused on astronomy, but the mathematics and measuring methods discussed were also applicable to land surveying and construction. The *Zhoubi* was divided into two sections, the first dealing with mathematics and the second part with astronomy. The mathematics in section one is necessary for the explanations of astronomy in the second portion.

The Mathematics Enthusiast, **ISSN 1551-3440, Vol. 9, nos.1&2**, pp.207-220

Particularly of interest is its discussion of the right triangle theory in section one. While commonly referred to as the Pythagoras theorem in the Western World, it is known as the *gou-gu* theorem in ancient Chinese literature. The text concerning the *gou-gu* theorem is written as a dialogue between a teacher Chen Zi and a student Rong Fang, both of whom nothing is known historically(Dauben, 2007). After being asked be Rong Fang, Chen Zi explains how using the shadow cast by the sun at midday it is possible to determine the distance between the sun and the Earth. In this explanation first appears a statement of the *gou-gu* theorem. Commentary added to the work by subsequent authors has been argued to be proofs of the theorem, though again it is difficult to determine the exact time period the commentary was added (Dauben, 2007).

The *Zhoubi* references the Emperor Yu being able to rule the country because of the Gougu theorem. Emperor Yu, or Yu the Great, ruled China in 2070 BC (Yu the Great, 2011). He was responsible for controlling an epic flood by engineering dredges and new flood channels to control the water. In depictions of his likeness, he is often seen holding a set-square, a device similar to a carpenter's square. This is evidence the Chinese understood and were applying the principles of the right angle theory as early as 2070 BC. The *Zhoubi suanjing* not only describes problems for calculating the distances of heavenly bodies and distances on Earth, but also a description of the tools of the trade. The primary tool used is the gnomon, an L-shaped metal or wood measuring device comparable to today's set square. In the *Zhoubi suanjing*, instructions for the use of the gnomon are again given by ways of a conversation between two scholars (Li, 1987).

The *Jiu Zhang suanshu*, translated to The Nine Chapters on Mathematical Art (Dauben, 2007), is the next and perhaps most significant mathematical work in Chinese history. The *Jiu Zhang suanshu*, also known as the *Chua Chang Suan Shu*, has been referred to as the Chinese equivalent to Euclid's Elements (Joseph, 2000). It is one of the oldest mathematical texts in the world, with problems more varied and richer than in any Egyptian text (Joseph, 2000). It is of interest that no English translation of the works has been made to date, despite the fact it is perhaps the most important compilation of ancient Chinese mathematics. Regardless, the *Jiu Zhang* is a comprehensive collection of the accomplishments of the Zhou, Qin and Han dynasties, approximately 11th century BC to 220 AD (Swetz, 1992). It contains the mathematical knowledge formulated over several centuries and through circumstantial evidence has been dated to sometime around the first century AD (Li, 1987). The *Jiu Zhang* is composed of 246 mathematical problems in a problem and solution format, divided into nine chapters and covering a range of math topics (Li, 1987). The text is written as a dialogue between an anonymous teacher and student, similar to the *Zhoubi suanjing*. The problems can generally be generally assigned to one of four categories: problems with applications to everyday real world situations, pseudo-real problems, which are of the same form as the real problems but focus on impractical or implausible situations, recreational problems and finally purely mathematical problems. The real world application problems include problems dealing with topics such as field measurements, bartering, exchanging goods and taxation. Pseudo-problems are utilized to show the principles of the mathematical knowledge that is to demonstrate methods for reframing difficult problems in a simpler manner. Pseudo-problems demonstrate that with mathematics it was possible to measure the immeasurable. Recreational problems include the riddles and counting rhymes, likely included for pedagogical purposes. Purely mathematical problems included methods for calculating π. A breakdown of the *Jiu Zhang* follows: Chapter 1 lays out methods for determining area of cultivated farm land. Chapter 2 and 3 focuses on proportions. Chapter 4 covers the methods for finding square and cube roots. Chapter 5 shows techniques for determining volumes of various solid shapes related to construction. Chapter 6 details calculations for distribution of goods such as grain and labor. Chapter 7 introduces the use of the method of false position. Chapter 8 discusses problems on simultaneous linear equations and the concept of positive and negative numbers. Chapter 9 discusses the Gougu theorem and general methods for solving quadratic equations (Li, 1987). The individual responsible for gathering together the works that compose the *Jiu Zhang* is unknown.

While the material of the *Jiu Zhang* is in and of itself important, the commentary and extension of the works provided by scholars studying the text over the subsequent centuries is what truly makes the *Jiu Zhang* the most important document in ancient Chinese mathematics. One of those scholars was Liu Hui, who in 263 AD wrote an addendum where he supplied theoretical verification of the procedures and extended some theories. The collection of nine additional problems Lui wrote to extend theories on the right triangle became known as the *Haidao suanjing*, translated as The Sea Island Mathematical Manual. Lui Hui is perhaps the most important mathematician in Chinese mathematics, and his contributions will be discussed more in depth later. Zu Chongzhi was another important scholar who studied the *Jiu Zhang* and extended its works. Zu lived from 429 to 500 AD and spent his early years studying the text of the *Jiu Zhang* and the works of other mathematicians such as Lui, correcting several errors of the mathematicians (Li, 1987). He is best known for his calculation of π, which will be discussed later. Chongzhi also produced a method for determining the volume of a sphere by comparing the areas of a cross-section (Li, 1987).

Chinese mathematics continued to advance into the modern era with works such as The Ten Books of Mathematical Classics, but the importance of the *Jiu Zhang* and *Zhoubi suanjing* to the Chinese civilization would continue for the next millennium.

Around the time of the completion of the two works, the imperial civil service in China began to grow in size and stature under the Eastern Han dynasty around 25-220 BC (Lloyd, 2003). Civil servants were the government workers of their day. It has been said that the Chinese creation of civil servant examination system has had far greater impact on the world than their invention of gunpowder, paper or the compass (Crozier, 2002). The civil servant examination system allowed any man to secure a respected position in the civil service; whereas most civilizations at this time held those positions for the ruler's relatives, friends and supporters. This gave great stability to the Chinese civilization because the civil service was not dependent upon whoever was in charge (Crozier, 2002). This civil service system spread throughout the world and can be seen presently in the United States, where a change in presidency or governorship does not trigger a turnover of all civil servants.

To ensure that those entering the prestigious civil service were competent and held the values of the culture, a rigorous examination was required of all applicants. The examination included a grueling essay portion and also an oral session, where students had to exactly complete a sentence chosen at random from the text once they were given the first portion of the sentence. A canon of texts, including works by Confucius, medicinal books and other philosophical works required complete memorization. Included in the canon were mathematical texts, including the *Jiu Zhang* and *Zhoubi suanjing*. Texts of a total of over 400,000 characters had to be completely memorized if a candidate was to have any hope of obtaining a civil service position, even at the entry level, and the pass rate was only 1 or 2% (Crozier, 2002). Mathematical texts were included to ensure those collecting taxes and performing land surveys had the mathematical knowledge to complete the tasks. The civil service examination continued to exist in one form or another up to the late 19th century (Li 1987).

Greek Mathematical Understanding

The Greeks saw mathematics as a philosophical pursuit, using mathematics to prove with certainty the truths of the world around them. It was Plato who stated: "Now logistic and arithmetic treat of the whole of number. Yes. And apparently, they lead us towards truth. They do, indeed." (Thomas, 1939). Considering that mindset, it is understandable why the Greeks considered geometry above all the type of mathematics. Ancient Greek mathematics focused on logic and is known for the great detail and thoroughness used to prove propositions. One only has to examine the depth of a proof in Euclid's Elements, such as the proof of Pythagoras's Theorem (Thomas, 1939) to see this.

The history of Greek Mathematical is marked with a who's who of famous names in all of mathematics. The facts of individual's works before Euclid's time are difficult to verify because of the lack of existing documents. Scholars are in agreement that Greek Mathematics began with Thale's Ionian school, established around 600 BC (Fauvel, 1987). Thales of Miletus was a philosopher, businessman and traveler, who had spent time in Egypt and Babylon, studying their mathematics before returning to Iona in Asia Minor and establishing a school (Struik, 1987). Thales understood the relationship of similar right triangles and used this knowledge to calculate the height of pyramids and the distance of ships from shore. Thales is also credited with four propositions, though it is difficult to verify: 1. A circle is bisected by its diameter. 2. The angles at the base of an isosceles triangle are equal 3. Two intersecting straight lines form two pairs of equal angles. 4. An angle inscribed in a semicircle is a right angle (Dilke, 1987).

Pythagoras was the next in the line of important Greek mathematicians. In 531 BC, Pythagoras moved to Croton in present day Southern Italy and opened a school to study among other things mathematics. The students of Pythagoras, known as the Pythagoreans, considered the universe to be ordered by means of the counting numbers and the person who fully understood the harmony of numerical ratios would become divine and immortal (Groza 1968). Pythagoras's most famous contributions were the proof that $\sqrt{2}$ is an irrational number and a proof of the relationship of the sides of a right triangle. The Babylonians and Egyptians had understood this relationship, but Pythagoras was the first to provide a proof. Pythagoras's proof that $\sqrt{2}$ is irrational was of great concern to the Greeks, because of their belief in the harmony of numbers. Pythagoras's greatest contribution to the world may be his discovery of the mathematical representation of the musical scale.

The schools of Socrates (400 BC), Plato (380 BC) and Aristotle (340 BC) produced students who continued to develop the discipline of mathematics and methods of logic and proof. Eudoxes, a student of Plato, made several significant contributions, including the development of the method of exhaustion and theory of proportions. The method of exhaustion was built upon the work of Antiphon (Groza, 1968). The Greeks knew how to calculate the area of a polygon and by inscribing a progression of n-sided polygons within a shape of unknown area; they could calculate a close approximation of the shape's area. As the number of sides of the polygon increased, the difference between the area of the polygon and the shape decreased. As the difference became arbitrarily small, the possible values for the size of the shape were "exhausted". The method of exhaustion helped Archimedes to determine the value of π, as will be discussed later in the paper. Eudoxes work with proportions advanced the methods of the

Pythagoreans, which dealt only with rational, or measurable, numbers. Eudoxes's work presented proportions in a geometrical sense, so as to show that being able to exactly measure the elements of the proportion was not required. This theory quelled the concern Greek mathematician's held over the irrationality of $\sqrt{2}$ and helped spur other advancements (Struik, 1987).

In 300 BC Euclid produced *The Elements*, a comprehensive collection of the works of all Greek mathematical work to that time. Little is known about Euclid the man, other than he lived in Alexandria around 300 BC. *The Elements* became *the* book on mathematics in the western world and its works continue to be studied and applied even today. Outside of the Christian Bible, *The Elements* has been printed and studied more than any other book in history (Groza, 1968). *The Elements* are divided into 13 books following a logical order. Euclid used a standardized form when presenting the material. A declaration would be made, he would then state a description of the problem (ex: let ABC and DEF be triangles with right angles…), the proof would follow, typically proof by contradiction, and a conclusion would be shown, restating the declaration followed by "which was to be proved". Books 1 through 4 contain definitions, postulates, common notions, and algebraic representation of planar geometry. Books 5 through 10 cover proportions and ratios, number theory, geometric sequences, and the method of exhaustion. Books 11 through 13 deal with special geometry, covering the ideas of books 1 through 4 and extending them to three dimensional shapes. Book 13 describes the five platonic solids and defines their relationships to a sphere. The description of the design and material of Euclid's *The Elements* is drawn from the 1939 translation by Ivor Thomas (Thomas, 1939) and Chapter 3 of Fuavel and Gray's *The History of Mathematics* (Fauevel, 1987).

Archimedes was the next prominent figure to arrive on the scene of Greek mathematics. He lived from 287 to 212 BC in Syracuse and is regarded as one of the greatest mathematicians of all times (Groza, 1968). There are more of the writings of Archimedes than of any other mathematician of antiquity surviving to today (Fauvel, 1987). Though Archimedes made many great contributions to mechanics, his true passion lied with the theoretical proof of mathematics. Archimedes most famous work was the calculation of the area of a circle and the determination of π. His methods involved inscribed and circumscribed polygons about the circle. The method will be explained in more detail later. Archimedes also invented a method for discussing large numbers. He did this by attempting to define the number of sand particles that could be contained in the universe, known as the sand-reckoner problem. The system he laid out to define large numbers is similar to the use of powers today ($10^8 = 10{,}000{,}000$), which was remarkable because the Greeks had a very simplistic way of writing numbers, where powers could not be incorporated (Fauvel, 1987).

One great contribution of Archimedes, known as Archimedes's Palimpsest, was thought to be lost forever until it was discovered in 1906. The Palimpsest included copies of his treatises "Equilibrium of Planes", "Spiral Lines", "Measurement of a Circle", "On the Sphere and Cylinder", "On Floating Bodies", "Stomachion" and "The Method of Mechanical Theorems". The Method of Mechanical Theorems (The Method) discovered is the only known copy, and is an important document in understanding Greek mathematics (Heath, 2002). The Method was a letter written by Archimedes to Eratosthenes. In the letter, Archimedes discusses the manner in which he makes his discoveries of the theorems that he then went on to provide proofs. Archimedes is careful to tell Eratosthenes that discovering a postulate through experimentation and proving the discovery through proof were two separate tasks. Archimedes lays out for Eratosthenes his genius method for determining the volume of solids, such as the cone and pyramid or areas, such as the parabola. Archimedes reasoned that figures could be broken down into an infinitesimal number of elements and these individual elements could be weighed and balanced on a lever opposite a figure of known weight and center of gravity. In physics and engineering terminology, he was calculating the moments of the figures around the fulcrum of the lever. This is the first sign of the idea of an integral in mathematics (Heath 2002). In addition to the actual content of the letter, it is the fact that Archimedes reveals his methods of discovery that is so important. The Greeks only published the proofs of their theorems; leaving historians to wonder how they discovered some of the properties they were so skilled at proving. The Method is a look inside Archimedes's office.

Archimedes other important works include his theorems on areas of plane figures and on volumes of solid bodies. These can be considered the precursors to integral calculus. In his book *On the Sphere and Cylinder*, he developed expressions for the area and volume of a sphere and cylinder. In his book *Quadrature of the Parabola*, Archimedes provides an expression for the area of a parabolic segment and proves the expression true (Struik, 1987). Archimedes had many more contributions in the fields of mathematics, mechanics and hydraulics and was a celebrity in his time.

Archimedes death marks the end of golden age for Greek Mathematics. Archimedes died at the hands of conquering Romans, who imposed a slave economy, more intent on making profits that pursuing mathematical knowledge. However, despite the conquering Romans, new contributions were made to mathematics, albeit less frequently. Around 230 BC, mathematician Eratosthenes proposed a method for calculating the circumference of the Earth (Thomas, 1939). Diophantos, around 250 AD, developed a form of algebra used to solve indeterminate equations. He also made contributions to the field of number theory, such as the theorem that states if each of two integers is the sum of two squares, then their product can be resolved in two ways into two squares (Struik, 1987). Heron (or Hero) of Alexandria was one of the last great mathematicians of ancient Greece, although it is disputed during which time period he lived, most likely it was in the first or third century AD (Thomas, 1939). While his Greek predecessors were more concerned with math theory, Heron was interested in practical uses of mathematics, demonstrated by his many inventions. Heron is credited with a large volume of work covering mechanics of machines. One of his works on mathematics is *Metrica*, a book on how to calculate the area and volume of different objects. Heron is known today for Hero's formula, where the area of a triangle can be calculated if the side lengths of the triangle are known.

Even during the time of Eratosthenes, Diophantus and Heron, there was the influence of the Roman Empire. While the Greeks were more concerned with more noble pursuits, Roman saw mathematics as a tool to be used for economic gains.

Comparisons

Now that a brief history of both cultures mathematical works and understandings have been laid out, a comparison of their works and techniques for determining π and the right angle theorem will be examined, along with a discussion on land surveying techniques

Euclid's Elements vs. *Jiu Zhang suahshu*

Now would be a good time for a more in depth comparison of the form and content of the most famous works of the cultures, Euclid's *Elements* and the *Jiu Zhang suahshu*. Both works are compilations of each culture's mathematical knowledge up to that given point in time. The books, in effect, provided a means for social advancement, for those who could master them were elevated to privileged elite. However, there are significant differences in the structure and content of each book.

The Elements follows a logical progression of material. The early chapters are filled with definitions, postulates and less complex theorems. Progressively, new material is built upon earlier material. The *Jiu Zhang suahshu*'s content does not follow a sequential order. Material presented in later chapters of the book is independent of earlier material.

The books have a different structure in the way they present ideas. The *Jiu Zhang suahshu* announces the problem and then gives a solution, with little or no explanation of the methods used to get the answer. There was also no attempt to provide a proof of the assertions of mathematical properties and theorems. Most of the description of the methods to solving the problems and proofs of the theorems was provided in the commentary, by mathematicians such as Lui Hui. The purpose of the texts was not to prove beyond argument the material but to guide the reader (student) on the principles so that they could master the methods for themselves. The authors wanted to the student to work through the problem themselves. The *Jiu Zhang suahshu* gave the reader the tools to solve problems and did not concern itself with proving the authenticity of the tools.

The Elements was nearly the opposite, as its chief concern was giving rigorous proof of ideas presented. *The Elements* presents a conjecture or idea and then painstakingly goes through reasoning step by step to come to an incontrovertible conclusion that the conjecture is true. *The Elements* was not concerned with having the student think for themselves, but wanted to show the students irrefutable proof of the conclusion. It has been theorized that the Greek's infatuation with proof comes from the system of law and politics practiced by the civilization. The law system provided for jury trials, where each side presented their arguments on the merits of their case. It was the goal of each side to give a sound and uncontroversial argument (Lloyd, 2003). While this may be difficult to near impossible in the courtroom, it was an achievable pursuit in the field of mathematics.

Though the structure of the two civilizations texts would lead one to conclude the Chinese were more open than the Greeks to discovering and critically analyzing mathematics, this was not the case**.** As detailed before, the Chinese were interested in the rote memorization of their mathematical works. Before any analysis or interpretation of the material could be conducted, one first needed to memorize the entire work. The Chinese also held the creators of the works in high regard and were hesitant to question any of the assertion or methods within the work. Chinese mathematical work during this time served the utilitarian purpose of supporting the monarchy (Swetz 1977). The study of mathematics was also considered a mostly solo venture in Chinese culture. Though the Greeks had a rigid structure in their mathematical text, they were more open to oral group discussions of mathematical theory. Because of the nature of proofs being uncontroversial, it was not considered a faux paus to question an aspect of a theorem or proof.

π

It has been said: "In history, the accuracy of the ratio of the circumference to the diameter (pi or π) calculated in a country can serve as a measure of the level of scientific development of that country at that time. " (Li, 1987). Both Greek and Chinese mathematicians had innovative ways of determining the accuracy of π.

In the beginning of Chinese mathematics the ratio of the circumference of a circle to the diameter was 3:1. This was an accurate enough approximation for the tasks at hand. Other ancient Chinese mathematicians began using closer approximations of π, such as 3.1547 and 3.16 ($\sim\sqrt{10}$), but none had developed a method for accurately calculating the true value of π. In 263 AD, in his commentary on Jiu Zhang suanshu, Liu Hui proposed the "method of circle division" to calculate the true value of π. In his method, Liu inscribed regular polygons inside a circle. Liu Hui began by showing that using π = 3 gives the area of a regular dodecagon (12 sided), not that of a circle. By continually doubling the number of sides of the regular polygon, the polygon approaches the shape of the circle. Therefore Area of polygon = Area of circle, and the estimation of π becomes more accurate. Liu understood that if one knows the side length of a hexagon, one can calculate the side length of a dodecagon. Given a polygon with 2n sides and l_{2n} = side length of *2n*- polygon, then the side length of polygon with *4n* sides can be determined, applying the Gougu theorem, such that:

$$l_{4n} = \sqrt{\left\{r - \sqrt{r^2 - \left(\frac{l_{2n}}{2}\right)^2}\right\}^2 + \left(\frac{l_{2n}}{2}\right)^2}$$

Liu understood that this process could be repeated to obtain a more and more accurate value of π. Liu calculated the area of a 192-sided polygon to get $\pi = 3.14 + \frac{64}{625}$. Around 480 AD, Chinese mathematician Zu Chongzhi calculated the value of π to be in the range of $3.1415926 < \pi < 3.1415927$, that is to say he calculated the accuracy of π to seven decimal places (Li, 1987). Chongzhi did not leave evidence of his methods of calculations, but the only known method was Liu's method of circle division. If Chongzhi went this route, he would have run the recursive algorithm for a 24,576-sided polygon. Chongzhi's calculation of π was the most accurate in the known world until the 15th century, meaning his calculations were not bested for over 900 years (Liu, 1987).

The history of the value of π in Greek mathematics before Archimedes is not known. Some scholars have proposed that the Greeks used the Egyptian approximation, 3.16 or the Babylonian estimate of 3.125. However, the methods by which theses civilizations calculated these values had little to do with the ratio of the diameter and circumference of a circle (Rossi, 2004). Around 400 BC, Antiphon, in his attempt to "square the circle" developed the method of inscribing triangles inside a circle (Thomas, 1939). Although he failed to determine a technique to square the circle, his methods were picked up and extended by Archimedes.

Archimedes used a similar iterative method for determining the area of a circle, and thus the value of π. Archimedes laid out his method in his book *Measurement of a Circle.* Proposition 3 of the book states: The ratio of the circumference of any circle to its diameter is less than $3\frac{1}{7}$ but greater than $3\frac{10}{71}$ (Heath, 2002). Archimedes goes about proving this by calculating the perimeter of a circumscribed polygon, where the perimeter is greater than the circumference of the circle, and an inscribed polygon, where the perimeter is less than the circumference of the circle. Rather than actually calculate the perimeters, Archimedes used Euclid's theorem on similar triangles to produce ratios of segments. His proof demonstrated the ratio of a 96-sided polygon circumscribed. He then had an inequality where $\pi < 3\frac{1}{7}$.

Archimedes then repeated the process with a 96-sided polygon inscribed to get the inequality of $\pi > 3\frac{10}{71}$ (Heath, 2002).

Archimedes work was four hundred years prior to the work of Liu, although there is no evidence of Liu having knowledge of Archimedes work. Both the Greek and Chinese methods utilized an iterative process increasing the number of sides of a polygon. It is of interest that Archimedes method is an arithmetical process and not a geometrical process, as was the norm of Greek mathematics at that time. The Chinese were well versed in arithmetical processes.

Right Triangle Theory

Both cultures had an understanding of the right triangle. That is to say, for a right triangle with side leg lengths *a* and *b* and hypotenuse of length *c* has the relationship $a^2 + b^2 = c^2$. They understood the power of this relationship for surveying. The way each civilization approached a proof of the relationship differs.

The Zhoubi suanjing contained a specific example of the right triangle theory, specifically the 3-4-5 right triangle, known as the Gougu theorem. A proof of how to demonstrate the $a^2 + b^2 = c^2$ relationship was known as the "piling up of rectangles" (Joseph, 2000). The original diagram accompanying the explanation is shown in figure 1 below.

The explanation starts with a triangle of sides 3, 4 and diagonal of length 5. A square is to be drawn on the diagonal side then three additional half rectangles of sides 3 and 4 are to be drawn to circumscribe the square and make a plate of 7x7. The four circumscribed half rectangles form two complete rectangles of area 24. When this area is subtracted from the plate area of 49, the remainder is 25, the area of the diagonal square (Joseph, 2000).

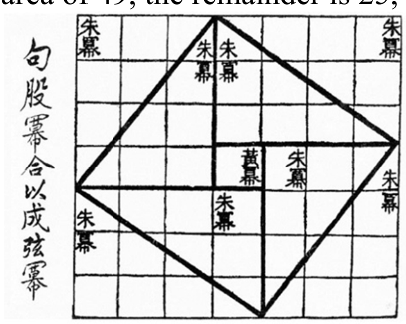

Figure 1

Lui Hui provided an extension of this proof in his commentary on the Zhoubi suanjing, known as the out-in complimentary principle (Joseph, 2000). The proof relied on the assumptions that the area of a figure remained the same under translation to another location and if a figure is divided into several smaller pieces, the summation of the area of those pieces is equal to the area of the original figure. Though a visual figure of Hui's explanation has been lost, there have been several replacements proposed by scholars. The jest of the proof is that taking the squares of the sides of a triangle and transforming the squares by cutting and rotation, a square of the diagonal can be produced (Joseph, 2000).

The Greeks understood the property of $a^2 + b^2 = c^2$ from the Babylonians, although they did not have a proof of the property (Groza, 1968). Pythagoras is credited with providing the first proof of the property, although historians have been unable to verify it was actually Pythagoras who performed the proof. Pythagoras's proof was documented in Euclid's *The*

Elements in book 1, proposition 47 (Thomas, 1939). The proof is quite rigorous and involves geometric properties of similar triangles, parallel lines and rectangles.

As was common for Greek proofs, Pythagoras's proof requires a sound knowledge of the geometric properties of triangles, whereas the understanding of the simplistic Chinese proof requires little background knowledge due to its visual nature. The written proof of the Chinese proof came some three hundred years after Euclid's *Elements* were published. The significant differences in the methods of proof demonstrate the unique origination of each civilization's proof.

Surveying

Both civilizations were expert land surveyors. The science of surveying was based primarily on the right triangle theory properties and geometric properties of similar triangles.

One of the great Greek examples of advanced surveying capabilities is the tunnel of Eupalinus at Samos, completed around 530 BC. The 1,036 meter aqueduct tunnel was excavated from both sides of a mountain and took 10 years to complete. The tunnel was built with great precision; the vertical difference at the middle meeting point is only 60 centimeters or less than one-eighth of one percent of the excavated distance (Apostol, 2004). However no evidence remains on how exactly the surveyors were able to lay out the correct orientation. One theory is that they used a succession of right triangles to traverse around the mountain. This method was later explained by Heron, not specifically for the Samos tunnel, but as a general method for tunnel construction (Dilke, 1987).

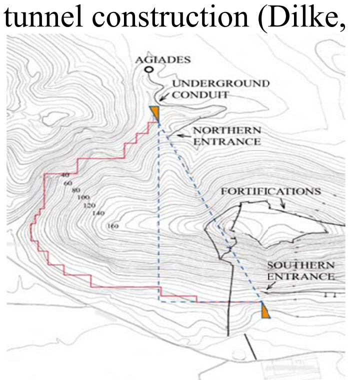

Figure 2 (Reference: Apostol 2004)

As shown in figure 2 above, the surveyor would mark one end of the proposed tunnel and then measure out a succession of right angles around the obstruction at the same elevation to locate the second tunnel entrance. Additional right triangles could be laid out at the tunnel entrances to provide a reference for a line of site to check work during construction (Apostol, 2004)

The understanding of ancient Chinese surveying comes from the *Haidao suanjing* or the Sea Island Mathematical Manual, written by Liu Hui in 263 AD. The Sea Island Manual contains nine problems dealing with the double difference method (also known as *chong cha*) of determining heights and distances. While the use of one simple ratio of a right triangle to determine unknown heights had been used for centuries, Liu developed the double difference method, which enabled surveyors to determine the both the height and distance to an inaccessible point.

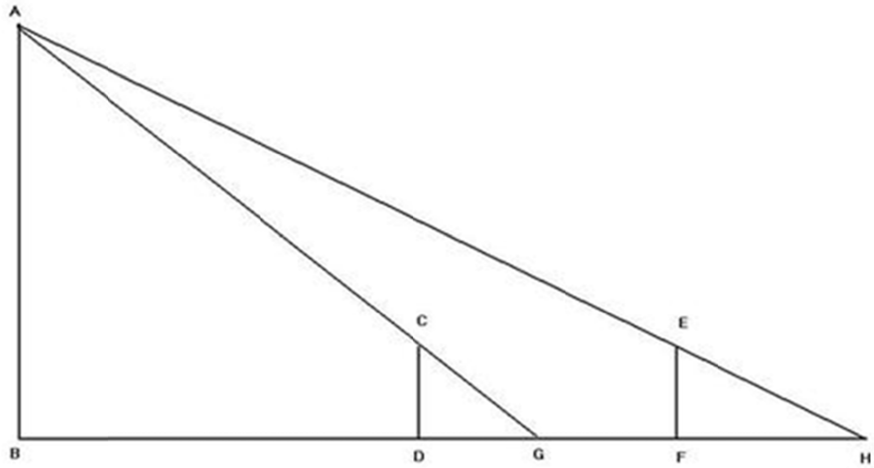

Figure 3

The following is an explanation of Lui's double difference method, referencing Figure 3 above. If point A and B were inaccessible, it would still be possible to measure the distance and height from point D. CD and EF would represent observation poles with known lengths. Stepping back from CD to the point G where A is visible and likewise to point H from EF gives the first required data. The distances DF, DG and FH are all know at this time. Using the properties of similar triangles, $\frac{DF}{FH-DG} = \frac{EA}{EH} = \frac{AB-EF}{EF}$. From these ratios, AB = $\frac{DF}{FH-DG}$ * EF + EF and using the same process of similar triangles, it can be calculated that BD = $\frac{DF}{FH-DG}$ * DG. The unknown distances of AB and BD are in terms of known distances (Li, 1987). By moving from survey observations utilizing a simple ratio and one observation point to problems with multiple observation points and ratio computations, Lui was taking a radical mathematical step (Swetz, 1992).

In a reversal of the overarching mindsets of each respective civilization, the Greeks were more concerned with the practical application of surveying techniques, while the Chinese focused on theoretical explanations (Swetz, 1992). Greeks were more proficient in the technical art of surveying while the Chinese excelled in the application of mathematics to surveying situations. It has been said that in a comparison of the knowledge of mathematical surveying techniques, China was one thousand years in advance of the Western World (Swetz, 1992).

Was there communication between the two civilizations?

It has been well documented that European Jesuit missionaries began entering China by the early 17th century AD (Li 1987). Because China was under the strong central rule of the Ming Dynasty at this time, it was impractical for the missionaries to impart their faith on the Chinese through force. Instead, the missionaries needed to entice the Chinese with science and technology (Li 1987). The Jesuits brought with them the mathematical knowledge and advancement of their respective culture and over time, the distinction between European and Chinese mathematics was gone. There is little debate on the history at this time period, but stepping back further in time, the connection between European (Greek) and Chinese cultures becomes much more unclear and speculative.

When historians examine the historical documents and study specific themes, it is difficult not to speculate there was some link between the two cultures. One such theme examined is Pythagoreans theorem. At the surface, the Pythagorean theorem and the *gou-gu* theorem seem like independent discoveries of both nations. Pythagoras lived in 6^{th} century BC and is credited with the proof and the *gou-gu* theorem was first presented in the *Zhoubi suanjing* which has been dated to sometime around 100 BC to 100 AD. However, a closer examination might lead to a different conclusion. Accounts and facts of Greek work during this time tilt more on the side of legend than fact. No proof has been uncovered that Pythagoras was the one who actually proved the theorem; it has only been attributed to him by others. Historians have speculated that all work done at the Pythagorean School was credited to Pythagorean. Because no proof exists it is difficult to determine the methods Pythagoras used if he did indeed prove the theorem. Although the *Zhoubi suanjing* is dated around 100 BC, it is generally accepted that the work contained within the compilation comes from earlier times. Using contextual evidence, the portion of the work containing the *gou-gu* theorem geometrical proof can be likely dated to the time of Confucius, around 600 BC.

A paper by Liu Dun explores the history of the method of double-false-position, which provides more speculative data to the theory that a link existed between the two civilizations (Dun, 2002). The method of the double-false-was first described in *Jiu Zhang suanshu*, indicating the method was known in China around 50 AD. Subsequently, the method was transferred to Muslim mathematicians at some point and then on to Europeans sometime during the Middle Ages. It then was brought back to China by the Jesuit missionaries, who claimed the method as their own. The paper leaves the question of how the method got to the Muslim mathematicians and at what period in time did this occur?

A similar paper was presented by Kurt Vogel, tracking the survey problem found in the *Haidao suanjing* about the sea island survey across a waterway. The method (explained in the survey section above) was discovered by Liu Hui in China around 260 AD. Two hundred years later, the method and an altered example problem appeared (Vogel, 2002) in Indian mathematics as part of the Aryahbhata text. The Indian knowledge was then transferred to the Arabs through the translation into Sanskrit. The problem, with references to Indian mathematicians, appears in Islamic mathematician al-Biruni in his work *The Exhaustive Treaties on Shadows* around 1000 AD. (Vogel, 2002). The path from the Arab world to Europe is unknown, but the method appears in the *geometria incerti auctoris* in Italy around 1000 AD.

These research papers point to the fact that information was flowing between the two civilizations, however it appears to be moving at such a slow pace as to be practically isolated. While speculation about the origins of Pythagorean's theorem provides an engaging topic, the lack of concrete evidence either way bounds the theory to stay in the conspiracy arena.

Conclusion

Due to the geographical isolation of China in ancient times, the accepted history of mathematics in the western world largely ignores their contributions. By examining the history of Chinese mathematics, one can see they had an understanding of mathematics similar to that of the western world, and Greeks in particular. In cases such as the calculation of π and the proof of the right angle theory, both Chinese and Greeks made astounding discoveries. The most prominent Greek mathematicians, such as Pythagoras and Archimedes are well known, though Chinese mathematicians with equal genius and comparable contributions to mathematics such as Lui Hui and Zu Chongzhi are pushed to the margins of fame.

References:

Apostal, Tom. "The Tunnel of Samos." *Engineering & Science*. California Institute of Technology, 2004. Web. 24 Mar 2011. <http://eands.caltech.edu/articles/LXVII1/Apostol%20Feature%20%28Samos%29.pdf>.

Crozier, J. (2002, Summer). A unique experiment. *China in Focus*, (12).

Dauben, J.W. (2007). Chinese mathematics. In V. Katz (Ed.), *the mathematics of Egypt, Mesopotamia, china, India and Islam* (pp. 187-384). Princeton, New Jersey: Princeton University Press.

Dilke, O.A.W. (1987). *Mathematics and Measurement.* London: British Museum Press

Dun, L. (2002). A homecoming stranger: transmission of the method of double-false-position and the story of heiro's crown. In Y Dold-Samplonius (Ed.), *From china to paris: 2000 years transmission of mathematical ideas* (pp. 157-166). Stuttgart, Germany: Franz Steiner Verlag.

Fauvel, John, and Jeremy Gray. *The History of Mathematics: A Reader*. 1st. Houndsmills, England: Macmillan Press, 1987. Print.

Groza, Vivian. *A Survey of Mathematics*. San Francisco: Holt, Rinehart and Winston, 1968. Print.

Heath, T.L. (2002). *The works of Archimedes*. Mineola, New York: Dover Publications, Inc.

Joseph, G.G. (2000). *The Crest of the Peacock*. Princeton, NJ: Princeton University Press.

Li, Y, & Du, S. (1987). *Chinese mathematics a concise history*. Oxford, NY: Oxford University Press.

Lloyd, G. (2003). Literacy in greek and chinese science. In H. Yunis (Ed.), *Written texts and the rise of literate culture in ancient greece* (pp. 122-138). Cambridge: Cambridge University Press.

Rossi, Corinna. (2004). *Architecture and Mathematics in Ancient Egypt.*

Cambridge: University Press.

Struik, Dirk. *A Concise History of Mathematics*. 4th Rev. ed. New York: Dover Publications, 1987. Print.

Swetz, F.J. (1992). *The sea island mathematical manual: surveying and mathematics in ancient china*. University Park, PA: Pennsylvania State University Press.

Thomas, Ivor. *Selections Illustrating the History of Greek Mathematics*. 1 & 2. Cambridge: Harvard University Press, 1939. Print.

Vogel, K. (2002). *A surveying problem travels from China to Paris*. In Y. Dold-Samplonius (Ed.), *From china to paris: 2000 years transmission of mathematical ideas* (pp. 1-8). Stuttgart, Germany: Franz Steiner Verlag.

Yu the Great. (2011). *Wikipedia*. Retrieved March 17, 2011, from http://en.wikipedia.org/wiki/Yu_the_Great