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The role of theory when studying epistemological characterizations of mathematics lecture(r)s

Magnus Österholm¹

Introduction and background

The study presented in this paper is a contribution to the scientific discussion about the role and use of theory in mathematics education research. In particular, focus is here on the use of and comparison between different types of theories and frameworks, which is discussed primarily through the example of an empirical study examining what types of messages about mathematics are conveyed in lectures. The main purpose of this paper is to examine how different types of theories and frameworks can affect different parts of the research process.

The role of theory in research

In research, the use of theory is an important and frequently discussed issue. When publishing research reports, it is most often a demand that you should relate to a theory. This centrality of theory is also evident from how different researchers describe the relationship between research and theory. Silver and Herbst (2005) discuss a general function of theory when they place it in the centre of the scholarship triangle, which consists of research, problems, and practice, where theory functions as a connection between all three parts of the triangle. Lester (2005, p. 458) describes four general purposes of using a research framework (later I discuss relationships between similar notions such as theory and framework); to give structure to a research study, that a framework is always needed for

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data to make sense, to come further than common sense, and in order not to be limited to finding answers to local problems. Also Niss (2006, 2007) describes different general purposes and functions of a theory, for example to predict or explain phenomena, to organize observations and interpretations into a coherent whole, and to give a methodology for empirical studies.

The descriptions above are from a general perspective, but if we think more specifically about the different parts of a research study it could be of interest to, as Bergsten (2008, p. 190) does, ask how “a theoretical basis adopted for a study influence the nature of the purpose, questions, methods, evidence, conclusions, and implications of the study”. This highlights the potential of a theory to affect all parts of a research study. However, for a specific study it might be that some choices in some parts of the study are not based on a given theory and there are also “numerous cases where [...] the research is carried out without really involving the theory which is being invoked” (Niss, 2006, p. 9).

The diversity of theories

If theory does, or should, play a central role in all or many parts of a research study, how should we handle the situation when different studies that focus on the same phenomenon use different types of theories? One problem can then be to build new research on previous research if different theories are used, since:

Taking isolated research results at face value, without relating them to the conditions and constraints of the research processes behind them, provides no criteria or bases for relating them to other seemingly contradictory or similar results. (Bergsten, 2008, p. 189)

To relate results to the theory that has been used in the process of producing these results is therefore an important aspect. However, it is not necessarily the case that research based on different theories, which can even be partially contradictory, produces results that are impossible to combine in some way, for example if the contradictory parts of the theories are not relevant for the studies in question. As previously mentioned, it can also be that the parts of specific research studies that are compared are not dependant on the given theory. This relationship is also highlighted by Prediger’s (2008, pp. 284-

285) conclusions from a comparison between researchers' use of different theories: "the theoretical base alone did not completely predetermine their conceptualizations", showing that "research practices and theoretical bases of course are strongly connected, but it would be a misleading simplification to propose a direct causal or deterministic connection". It is therefore not enough to focus on the comparison of theories in themselves, whether (partially) contradictory or (fully) compatible theories are used, but it is important to become more specific regarding if and how theory affects different parts of the research process. This issue is examined in the present study.

Regarding how to relate different theories to each other, Bikner-Ahsbals and Prediger (2006) describe four different ways to do this; unify, integrate, network, and compete/compare. To unify refers to the creation of one theory from several theories that deal with the same phenomenon, while to integrate refers to the combination of different, but compatible perspectives. Networking refers to a strategy that includes for example both comparing and integrating theories, and demands cooperation between different research groups that represent different research cultures. The present study focuses mainly on compete/compare, "by treating the same set of data from the basis of different theories, similarities and differences of theories can be specified although the diversity is respected" (p. 55). There are several examples of studies that have compared different theories regarding how they affect some part(s) of the research process; Prediger (2008) examines the effect on the formulation of research questions, while several other studies examine the analysis and interpretation of a given set of data from different theoretical perspectives (Bergsten, 2008; Bergsten & Jablonka, 2009; Even & Schwarz, 2003; Gellert, 2008), often highlighting the effect of theory on several aspects/parts of the research process. All these comparative studies show clear differences in the descriptions and interpretations that are done based on different theories. The authors also discuss more detailed relationships between theory and some part of the research process.

Bergsten and Jablonka (2009) note that from observing the same situation, described through transcripts of student dialogues, the analyses based on two different theories have actually not used

the same data, partly because focus is on different parts of the transcripts and partly because one theory demands more data than what is present in the transcripts. Although much of the method, regarding the collection and presentation of data, was given beforehand as a common starting point for both theories, this example shows an effect of theory on methodology in a more intricate manner. However, to say that the theory is *affecting* the methodology might not be suitable, but perhaps methodology should be seen as *a part of* the theory, which for example Radford (2008) is suggesting. More discussion about what constitutes a theory is given later.

When Gellert (2008, p. 221) compares the use of two different theories, he finds that interaction patterns are described as “emerging” in one theory and as “reflections of macro-social structures” in the other theory, and he remarks that “emergence and structure are quite opposite concepts”. Despite this apparent contradiction between the theories, Gellert suggests a way of coordinating the results originating from the use of these two theories. He creates a combination of the two perspectives that could be seen as having “improved the usefulness and accuracy of the interpretation” (p. 223), but leaves it up to the reader to judge if this is actually the case. In line with the work of Gellert, one way to look at the combining of studies that focus on the same phenomenon but use different theories could then be to view the use of different theories as looking at the same thing from different angles or perspectives. The use of different theories is then not seen as a problem of having or creating potential contradictions, but that the use of different theories will complement each other. This is in line with Bikner-Ashbahr and Prediger (2006, p. 54) who do not see diversity of theories as a defect but as richness and as showing the character of the subject regarding its complexity; “we should not aim to reduce this complexity. Instead, the richness gained should be better exploited”. In similar fashion, Even and Schwarz (2003, p. 309) note that a classroom situation is too complex to be understood from only one perspective, and that a more complete understanding requires the use of different perspectives.

Above, the potential benefits of seeing the same thing from different perspectives are highlighted, but is it actually “the same thing” that is being examined so that different theories could be said to

complement each other? Similarly, are two theories contradictory when they include or produce contradictory statements about “the same thing”? For example, just because two theories claim to be about learning, are they then comparable in the sense that you can see them as potentially contradictory, or are the theories actually focusing on different things and just using the same word for these things? This problem is discussed by Rodriguez, Bosch, and Gascón (2008), regarding the difficulty to say that theories deal with the same questions since they see each theory as defining and formulating their own problems without there being an easy way to “translate” between theories. In their study they examine the notion of metacognition, which is a concept created in one theory and they see a difficulty in translating it to another theory. Instead of focusing directly on the concept, they examine the original practical problem that created the need for the notion of metacognition in the theory in question, and then focus on this problem when trying to translate to another theory. Based on this example, the authors argue that all comparisons between theories need to be done via the type of problematic situations or issues that were the origin to the specific content of a theory. Perhaps this could be seen as looking for a least common denominator for theories, regarding the type of event, situation or phenomenon that can actually, to some extent, be seen as “the same thing” seen from different perspectives. This idea shifts focus from looking at differences to looking at potential commonalities regarding relationships between different theories. In the present study, both similarities and differences between theories are discussed in relation to empirical data.

The concept of theory

Theory has so far been discussed without giving an explicit definition of this notion, which sometimes seems possible to do, thus basing the discussion on some type of common understanding of this notion, perhaps created through common properties of examples of constructs labeled as theory. However, the existing “absence of clear definitions of ‘theory’ and ‘theoretical’ in many publications that invoke these terms” (Niss, 2007, p. 1308) becomes problematic if we wish to go deeper in our awareness of the use

of theory in research. Therefore, we also need to discuss what we mean by 'theory' and relate this meaning to comparative studies of theories in use.

There are many concepts in use that seem to relate to some aspect of theory in research; "Model. Construct. Theory. Paradigm. Framework. To some, these words have vastly distinct meanings, while to others they are separated by shades of grey" (Mewborn, 2005, p. 1). Words like approach and perspective can also be added to the list, and also that some of the words can be combined, for example in 'theoretical framework'. It is neither possible nor necessary to discuss and define all these words here. Instead, my discussion is limited to the two notions that seem most commonly referred to in discussions about theory in research; theory and framework. The works of Niss (2006) and Radford (2008) are utilized regarding the notion of theory while the work of Lester (2005) is utilized regarding the notion of framework and to some extent also regarding the notion of theory.

Niss (2006) defines a theory as consisting of an organized network of *concepts* and *claims*, where the concepts are linked in a connected hierarchy and where claims are either fundamental (i.e. of axiom type) or derived from the fundamental claims. Radford (2008) defines theory as consisting of a system of *basic principles* (P), a *methodology* (M), and a set of *paradigmatic research questions* (Q). There are many similarities and overlaps between these two definitions. *Basic principles* can be regarded as *claims* of a certain kind, which also *methodology* can since Niss describes that theories can function as methodology. *Concepts* are not mentioned as one separate part of a theory by Radford, but P is "characterized by its hierarchical structure and the ensuing meaning of its key concepts" (Radford, 2008, p. 320), highlighting the similarity with the definition by Niss.

The set of *paradigmatic research questions* is not found in the definition of theory by Niss (2006), and he does not discuss the relationship between research questions and theory. Radford (2008) describes Q as "templates or schemas that generate specific questions" (p. 320) and not as a fixed set of questions. His reason for including Q in the definition of theory is that "because they [theories] emerge

as responses to particular problems, they bear the imprint of the initial questions that they sought to answer” (p. 321).

In this paper, I choose to define theory in accordance with Radford (2008). This choice is made because Radford describes a structure that is easier, compared to the one given by Niss, to relate to when comparing the use of different theories, which is also the focus of the article by Radford.

Mason and Waywood (1996) describe two aspects of the use of theory; as foreground or background theory, which can be related to that it is not only theory that is explicitly referred to that affects research. Both these aspects can be labeled as theory also when using the definition by Radford (2008, p. 320) since the system of basic principles “includes implicit views and explicit statements”. The notion of approach, defined by Bergsten (2008, p. 192) as “a more informal inclination by the researcher to interpret an observed commonsensical problem”, can therefore also be included in the notion of theory as defined by Radford.

The relationship between research questions and theory was highlighted through the definition of theory by Radford, and is also an important issue in Lester’s (2005) discussions about different types of frameworks, in particular regarding theoretical and conceptual frameworks. For a theoretical framework, there is reliance on a formal theory and research questions “would be rephrased in terms of the formal theory that has been chosen” (p. 458), while:

A conceptual framework is an argument that the concepts chosen for investigation, and any anticipated relationships among them, will be appropriate and useful given the research problem under investigation (Lester, 2005, p. 460).

The use of theory is not limited to the theoretical framework, since conceptual frameworks “may be based on different theories” (Lester, 2005, p. 460). The difference between the two types of frameworks seems to be the ordering of theory and research questions; in a theoretical framework the theory decides what types of questions can be asked while in a conceptual framework the questions

affect the choice of theory. Frameworks in general could then be described as different ways of doing research, in particular regarding different ways of using theory, while theory (as defined by Niss and Radford) can be seen as a type of structure of concepts and statements. This view of frameworks is somewhat different than what is presented by Bergsten (2008), when he discusses the role of different types of frameworks in the process of analyzing data, also referring to Lester when utilizing the notion of framework. Bergsten focuses on the multitude of sources/theories as a central aspect of conceptual framework, for example when discussing one specific study; “the research framework [is] conceptual, since the study uses theoretical concepts from various sources rather than one overarching theory” (p. 193). I do not see the number of sources/theories as the main difference between the two types of frameworks because it becomes problematic what constitutes *one* theory or *several* theories and, as argued above, I interpret Lester’s descriptions of different frameworks as primarily be about different ways of relating theory and research questions.

Lester does not define the notion of theory, and the general definition of framework seems quite similar to the definition of theory used in this paper; “a research framework is a basic structure of the ideas (i.e., abstractions and relationships) that serve as the basis for a phenomenon that is to be investigated” (Lester, 2005, p. 458). The relationship between framework and theory is therefore not easy to sort out. A more in-depth analysis of this relationship is not done here, but I choose to characterize the different types of frameworks based on the different relationships between research questions and theory as described above.

Purpose and structure of the present paper

The main purpose of this paper is to examine how different types of theories and frameworks can affect different parts of the research process, when using the notions of framework and theory as described earlier. More specifically, the discussions in this paper focus on two aspects:

- The utilization of different types of frameworks and different types of theories.
- Relationships between different parts of the research process and different parts of theories.

The aim is not to discuss the use of theories only from a “theoretical perspective” but the discussions also focus on the content and structure of a specific empirical study; about epistemological characterizations of mathematics lectures (this topic is discussed more in the next section). In relation to this specific research topic, a purpose is also to examine if/how it seems possible to unify or integrate different theories used when studying this topic, and not only to compare them. I have examined some aspects of both comparison and integration of theories related to this research topic before (Österholm, 2009), but then without relating to a specific empirical study and without a more in-depth view of the notion of theory.

Thus, in order to discuss the use of theory in research, an empirical study is used as a starting point. This empirical study is described in the next section, as a study in itself including descriptions of background, purpose, method, analysis of results, and conclusions, and thereby becoming somewhat separated from the overarching purpose of the present paper. However, the description of the empirical study is not as elaborate as could be expected if the study was reported independently, partly in order not to make this paper as a whole too long and partly in order to create a better overview of the empirical study for the discussion of the use of theory. Some parts of the empirical study is therefore affected by and adjusted to the overarching purpose of discussing the role of theory in research. The empirical study can therefore be seen as somewhat “artificial”. The same empirical study has been described elsewhere (Österholm, 2010), where some parts of it are more elaborately described, but also where the focus is different; on exploring and discussing the methodology of the study.

Similarly as Prediger (2008), a type of “classroom problem” or “issue of classroom practice” is taken as the starting point for the discussions of the use of theory in research. In contrast to the study of Prediger, research questions are then not discussed from the perspective of some given theories, but the first step is here the use of a conceptual framework in relation to the given problem. Thereafter, the method of the empirical study is described, followed by the results and conclusions. The

conclusions are the end of the separate empirical study, and thereafter the focus is redirected to discussing the issue of use of theory in different ways in relation to the presented empirical study.

Firstly, in relation to the given conclusions from the empirical study, the need to “add more theory” is discussed, as a means to be able to interpret the conclusions in a broader perspective. At this point, the use of two different theories is discussed, in particular regarding a comparison between them based on the definition of theory by Radford (2008) and in relation to different parts of the research process.

Finally, the focus is directed to the differences between conceptual and theoretical frameworks by analyzing if/how some part of the empirical study would have been different if a conceptual framework had not been the starting point, but if one of the two theories introduced in relation to the interpretation of the conclusions had been used together with the given classroom problem as a starting point for the whole empirical study, that is, if a theoretical framework had been used. Besides being able to compare the use of different frameworks, this utilization of different theories also allows for an analysis of relationships between parts of the research process and parts of theories, in particular since these theories are introduced at a late stage in the process but then also related to other parts of the research process.

The empirical study

Introduction

The teaching activities in a mathematics classroom can be very different, depending on the content, the teacher, the students, available materials, the classroom environment, etc. The subject of mathematics can have very different character depending on these properties of teaching activities, that is, that properties of the teaching situation can produce different pictures or characterizations of mathematics. The focus in this empirical study is on this type of issue of classroom practice, regarding how certain properties of teaching can give different pictures of mathematics.

Others have studied this type of issue, for example regarding what properties of mathematics “is conveyed by mathematics textbooks” (Raman, 2004, p. 389) where a comparison between different

textbooks showed “conflicting messages regarding the status and purpose of mathematical definitions”. Shield (1998, p. 516) also examines textbooks, in order to “identify the types of messages about mathematics [...] inherent in their presentations”, where messages in textbooks is shown to differ compared to “the intent of recent reports and syllabuses” (p. 521).

Ikonomou, Kladrimidou, Sakonidis, and Tzekaki (1999, p. 170) focus their analysis on properties of mathematics lessons regarding “epistemological features” of the lessons, by examining how the teacher handles “the nature, meaning and definition of concepts, the validation procedures and the functionality of theorems”. Their conclusions focus on a comparison with mathematics as a discipline, and they see large differences, for example that activities in the classroom are reduced to procedures and that central notions are not separated (such as definition and theorem).

Although using different notions, I see the given examples of previous studies as focusing on the same kind of phenomenon, regarding a characterization of some properties of teaching that can be interpreted as messages about mathematics. Similar to the examples of other studies, the purpose of the present empirical study is to examine the types of messages about mathematics inherent in, or conveyed by, properties of teaching.

In order to be able to focus on this issue in a research study, we need to discuss in more detail what can be meant by pictures of, or messages about, mathematics, and also to limit the study to focus on some aspect/property of teaching. In addition, it is also needed to discuss how to examine certain properties of teaching as messages about mathematics, in order to plan and carry out an empirical study focusing on this issue. Thus, there is a need to discuss these central aspects and concepts, which can be done through a conceptual framework, in order to have a structure for conceptualizing and designing the research study (Lester, 2005).

A conceptual framework

Messages about mathematics can be related to the basic philosophical issues of epistemology and ontology, that is, the nature of knowledge and the existence/location of (mathematical) entities respectively. The discussions are here limited to epistemological aspects, through two questions regarding the nature of knowledge; what knowledge is and how knowledge is acquired, which are shortly referred to as questions about properties of *knowledge* and *knowing* respectively.

There are many aspects of epistemology, and many types of epistemologies, in relation to mathematics and mathematics education (Sierpiska & Lerman, 1996), so there is a need to limit the focus of the present study, which is done by focusing on one central aspect in relation to each of the two mentioned questions about knowledge and knowing. In relation to *knowledge*, I relate to a central distinction in mathematics education regarding two different types of knowledge; conceptual and procedural knowledge (Hiebert, 1986). In relation to *knowing*, I relate to properties of argumentation (Toulmin, 1958), regarding the process of drawing conclusions.

A next step is to relate these two aspects of epistemology to properties of teaching. Among the examples of similar studies done before, which examine messages about mathematics inherent in some properties of the teaching situation, some have focused on epistemological aspects; regarding properties of textbooks (Raman, 2004) and regarding properties of classroom activities (Ikonomou et al., 1999). For the present empirical study, I choose to focus on properties of oral communication, because it is a very central aspect of teaching and it is more focused than to examine classroom activities more generally. I also choose to limit the study to focus on one-way communication; lectures, in order to reduce the complexity somewhat by not including aspects of dialogue or discussions among several persons.

Thus, the focus of the present empirical study is properties of oral communication that could be said to convey something about epistemological aspects of mathematics. The communication can include explicit statements about knowledge and knowing in mathematics, but also other properties of

communication can represent certain aspects of knowledge or knowing. Since the most common activity in a lecture probably is to present mathematics and not to talk *about* mathematics (from an epistemological perspective), the present empirical study focuses not on explicit statements about epistemology.

The *types of statements* used in a lecture could highlight the epistemological aspect regarding the nature of *knowledge*. In relation to conceptual and procedural knowledge, I therefore separate two main types of statements; statements about the use of mathematical objects, labeled *use-statements* (related to procedural knowledge) and statements about properties of mathematical objects, labeled *object-statements* (related to conceptual knowledge). Here I choose to use the more general notion 'mathematical object', which can refer to concepts as well as procedures. The difference between the two types of statements is therefore that they describe either properties of objects or the use of such objects, which is seen as a central aspect regarding the difference between conceptual and procedural knowledge. For example, the statement "The derivative of $\ln x$ is one over x " is an object-statement while the statement "When you take the derivative of $\ln x$ you get one over x " is a use-statement.

As mentioned before, the *types of argumentations* could highlight the epistemological aspect regarding the nature of *knowing*. For the limited purpose of this empirical study, I choose to use a simplified version of the elaborate structure of argumentation presented by Toulmin (1958). This simplified structure consists of a conclusion that is drawn (or a *claim*, using Toulmin's vocabulary) together with statement(s) used as argument for this conclusion (or *data*, using Toulmin's vocabulary). The words or wordings used to make explicit the argumentative relationship between statements (e.g. words such as 'therefore' and 'since') are here labeled *connect-words*. These words or wordings are of great importance when locating arguments, and focus can be put on these types of words in the analysis of types of argumentations.

When focusing on the *types* of statements and the *types* of argumentations, focus is not on the mathematical content of the statements or the argumentations, and the purpose of the analysis in this empirical study is not didactical, in the sense that the focus is not on aspects of teaching and learning the mathematical content nor on the teaching and learning of argumentation or proving. Instead, the analysis of types of statements and types of argumentations in lectures is used in order to draw conclusions about what is conveyed *about* mathematics, in particular regarding epistemological aspects.

In summary, the conceptual framework used in this empirical study focuses on epistemological properties of oral communication in mathematics lectures by discussing the following concepts and relationships between concepts:

- Aspects of epistemology, regarding knowledge and knowing.
- How epistemological aspects can be conveyed through properties of oral communication, either explicitly (through direct statements about epistemological aspects) or more implicitly.
- Aspects of knowledge, regarding conceptual and procedural knowledge, in relation to the types of statements used in oral communication.
- Aspects of knowing, regarding argumentation, in relation to the use of connect-words.

Purpose

As noted in the introduction, the overarching purpose of this empirical study is to examine the types of messages about mathematics inherent in, or conveyed by, properties in teaching. This purpose has now been further specified; to examine properties of oral communication that could be said to convey something about epistemological aspects of mathematics. That is, the purpose is to perform a type of epistemological characterization of mathematics lectures. Through the conceptual framework, this purpose has also been specified regarding different aspects of epistemology (the four points at the end of the last section).

The length of this paper is limited by not including results and analysis of the epistemological aspect of knowing, thus only including the aspect of knowledge, regarding the different types of statements; use- and object-statements. However, since the aspects of knowledge and knowing are integrated in the analysis of data, both aspects are related to in the description of method.

Thus, the purpose of the present empirical study is to compare different lectures regarding how they, through properties of statements in the oral presentations, convey something about epistemological aspects of mathematics. In relation to this purpose, the following research questions are in focus:

- Is there a tendency to use different types of statements in different lectures?
- How can potential differences between the lectures be characterized?

Method

Two mathematics lectures at university level are analyzed in this empirical study. The lectures have different lecturers and different mathematical content. One lecture is part of a course in calculus and this particular lecture is about improper integrals, while the other lecture is part of a course in statistics for natural scientists and this particular lecture is about some examples of discrete probability distributions. Both lectures are approximately 45 minutes long. Only the lecturers' activity is analyzed, in order to focus on one type of discourse; the one used in lecturing, and not in for example dialogue. The lectures were recorded with audio and video, but students' statements are not audible in the recordings and the camera is always focusing on the lecturer's activity at the whiteboard.

The analysis in this paper focuses on the lecturers' auditory communication, and the lectures were transcribed from the audio recording, but using the video recording in case of doubt in the process of transcription and in case of unclear references in the lecturers' statements (e.g. referring to "this" or "that" when pointing to something on the whiteboard).

In order to create a clear structure in the analysis, statements from the lectures are analyzed in several steps. The first step in the analysis is to mark use- and object-statements, and also connect-words in the transcription. Each coherent section of the transcription is then extracted from the transcription, for further analysis. A coherent section refers to a set of statements that are connected through the use of connect-words. Such a section can for example be only one conclusion together with an argument, as in the following example from the lecture in calculus, where the connect-words are in italics: “The derivative of $\ln x$ is one over x , which is larger than zero, *which means that* it grows all the time”. Note that there is actually a linguistic ambiguity about exactly what the word ‘which’ in ‘which means’ refers to, but logically all information given before the conclusion is needed, and this full statement is therefore regarded as the argument. A section can also include several argumentations, as in the following example from the lecture in statistics, where the connect-words are in italics:

And you can show that the expected value is one over p . This can be seen as. Yes if we imagine that for example p is zero point two. Then this *means that* we will succeed on average every fifth time. And this also *means that* the expected value then becomes one over zero point two, which is five. *So* it is exactly that we will have to do on average five tries in this case.

A next step in the analysis, which is mostly relevant for sections that do not consist of a single argumentation, is to extract the relevant statements from the excerpt and arrange them in a structured manner, which for the latest example can be done in the following way, where the connect-words are in italics:

1. You can show that the expected value is one over p .
2. p is zero point two.
3. *Means that*: We will succeed on average every fifth time.
4. *Means that*: The expected value is one over zero point two, which is five.
5. *So*: We have to do on average five tries.

From this structure it is easier to analyze how the statements are related according to the connect-words, although the analysis has to include some considerations to what is reasonable, as was done in

the previous example about the derivative, since it is not always clear exactly what is referred to as being the argument for the conclusion. In such situations, the logically necessary statements previously stated are listed as included in the argument. In the example above, we see that line 3 is only based on line 2 as an argument, while line 4 cannot only be based on the previous line, although the exact same type of connect-words are used. The result of this type of analysis is then summarized in a three column table with a conclusion, the argument(s) for this conclusion, and the connect-words used in the argumentation. From the example above, one line in the table thus becomes:

We will succeed on average every fifth time	P is zero point two	Means that
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Based on the content of the produced tables, each lecture can then be characterized through examining the different types of statements (use- and object-statements); quantitatively regarding which type of statement is most commonly used and qualitatively regarding the formulation of statements of different kinds.

Results

Through utilization of the described method, the produced table for each lecture consists of 39 lines for the calculus lecture and 43 lines for the statistics lecture. Each line in the table corresponds to one argumentation, which consists of a conclusion, the statement(s) used as argument(s) for the conclusion, and the connect-words.

When studying how common the different types of statements are in the two lectures, a clear difference between these lectures appears: In the calculus lecture, use-statements appear as a conclusion on four lines in the table (10 %) and as an argument on two lines (5 %), while in the statistics lecture, use-statements appear as a conclusion on 22 lines (51 %) and as an argument on 18 lines (42 %). Thus, in the calculus lecture object-statements are most common, while in the statistics lecture the two types of statements are about equally common.

Besides this quantitative characterization of the use of different types of statements, it can also be noted that there are many examples of statements of one type that can easily be reformulated in order to turn it into a statement of the other type. For example, in the statistics lecture there is the use-statement “if you add a constant to all values of the function, this will not change the variation”, which can be reformulated into a statement saying that a property (the variation) of two functions is the same (i.e. an object-statement). An example of the opposite type of reformulation is taken from the calculus lecture, where there is the object-statement “(the graph of) one over x has a similar appearance (as the graph of one over x squared)”, which can be reformulated into “if we sketch the graph of one over x , the result is similar as when we sketch the graph of one over x squared” (i.e. a use-statement).

Conclusions

The study of use- and object-statements shows a potential difference between the two lectures regarding some epistemological aspects. However, since many statements seem easy to reformulate into the other type of statement, there is some arbitrariness, but not necessarily randomness, regarding what type of statement is used. These observations highlight the questions if or how these properties of communication can be seen as primarily tied to the individual, to the mathematical content, to the type of course, or to other aspects of the situation. These questions seem possible to examine in more detail using the type of method for data analysis presented in this empirical study, but for a larger set of data. Thus, it would be interesting to compare epistemological characterizations of different settings (not only lectures) for the same person and of communication of different persons (in particular to include also students) within one setting.

The use of different theories

Let us now return to the overarching purpose of this paper and discuss properties of the empirical study in relation to the use of theory. In the conclusions in the empirical study it is questioned whether properties in the discourse could be seen as tied to the individual, to the mathematical content, to the type of course, or to other aspects of the situation. A possible way of examining this question is also described, as a continuation of the use of the conceptual framework, through more extensive empirical

studies regarding variations in properties of communication for different persons and different situations. However, you could also see it as necessary to relate to a theory for the interpretation of the results from the data analysis, for example in order to make it possible to relate to other kind of studies that are examining similar kind of data or phenomena. This aspect of *interpreting* the results through a theory can also be seen as a way of *explaining* the results by relating to a theory that describes more general aspects of the observed situation or phenomenon. Two different theories are here introduced as examples of the use of theory for interpreting and explaining the empirical results.

Introducing theory as a means to explain empirical results

It is here not possible to give full descriptions of two theories. Therefore, I relate to two theories that are believed to be familiar enough for the reader to make the description and discussion comprehensible. The two theories are here labeled as *cognitive theory* and *discourse analytic theory*. From the short description of these theories given here it is not self-evident that these can actually be labeled as theories, according to the definition by Radford (2008). The label of each of these two theories does not refer to a specific theory in the sense that I can refer to a specific article or book that describes the theory, but refers to something that is common in several different specific theories. Perhaps each of the two theories could be better characterized as an *approach* (Bergsten, 2008) or as a *perspective* (Lester, 2005) – I return to the discussion of this issue.

A very short description of the foundation of each theory is given below, together with an interpretation of the conclusions in the empirical study. Thereafter, the use of these two theories is discussed, including a comparison based on Radford's (2008) defining components of a theory.

The cognitive theory focuses on mental objects and processes of individuals. Regarding epistemological properties of communication, focus can then be put on individuals' epistemological beliefs. In addition, a lecture can be described using the notions of sender and receiver. The lecturer's beliefs can be seen as a cause for how s/he presents the mathematics, for example that epistemological beliefs are a basis for how it is argued that one knows something. How the lecturer presents the mathematics is then

influencing how the students think about mathematics, including epistemological aspects. In this theory, focus is on cause and effect, where the study of properties of communication in the lecture can be relevant both as a sign of the lecturer's beliefs and also as a potential cause for students' beliefs.

This focus on cause and effect seems common in educational research about beliefs, for example when studying teachers' *attributed* beliefs (Speer, 2005) or when *explaining* students' differences in performance through differences in beliefs (Schommer, 1990).

Based on the cognitive theory, the empirical study can be said to characterize *the lecturer*, since focus is on properties of individuals. The observed differences regarding properties of communication can then be explained by that the lecturers have different epistemological beliefs, or at least that different beliefs are active in the two lectures.

The discourse analytic theory focuses on the discourse in itself as a central process, and not seeing the discourse as a product of other, more basic, (mental) processes. A lecture can then be described using the notions of participation and enculturation in a discourse community. The lecturer's statements are not seen as a reflection of some cognitive structure, but as being constitutive themselves, as a part of the social situation (Edwards, 1993). The study of properties of communication in the lecture can then be relevant both for characterizing the discourse community and also by seeing the lecture as a part of students' enculturation (including becoming familiar with the discourse somehow related to epistemological aspects of mathematics).

Based on the discourse analytic theory, the empirical study can be said to characterize *the lecture*, since focus is on properties of the situation. The observed differences regarding properties of discourse can then be explained by that different discourse communities are observed in the two lectures.

Looking back at the empirical study and the use of the two theories, the following summary can be given.

1. Issue of classroom practice: Different messages about mathematics inherent in, or conveyed by, properties of teaching.

2. Purpose of research study: Examine epistemological characterizations of oral communication in mathematics lectures.
3. Conclusion: Difference between two lectures regarding the epistemological characterization.
4. Interpretation/explanation:
 - (a) Alternative 1, cognitive theory: Properties of the lecturer, difference regarding beliefs.
 - (b) Alternative 2, discourse analytic theory: Properties of the lecture, difference regarding discourse communities.

Above, the use of the two theories seems only to affect point 4, since these theories were introduced at that stage. However, we need also to examine if and how other parts of the research process need to be changed or reinterpreted in some way due to the introduction of a new theory.

The effect of theory on different parts of the research process

As a starting point, the two theories are compared based on the definition of theory by Radford (2008), which relates to aspects of the research process, and here focus is on specific aspects of the presented empirical study.

Regarding *basic principles*, both theories include the concepts of mental object/process and communication/discourse, but a main difference between them is regarding where and how these concepts are located in the hierarchical structure of the system. While the cognitive theory regards properties of communication as an effect of mental processes, the discourse analytic theory begins “not by questioning the existence or ontology of underlying cognitive representations but by questioning their epistemological basis in discourse” (Edwards, 1993, p. 218).

Regarding *methodology*, the same empirical data, analyses and results in the empirical study are relevant for both theories. This observation is specific for the present empirical study and does not

mean that the same observation would always be made when using these two theories. In the next section, relationships between theory and methodology are discussed more.

Regarding *research questions*, the same purpose and questions are used, and are of interest, for both theories. However, in the process of interpreting/explaining the results it became clear that 'epistemological characterization' refers to different things in the two theories, whether characterizing the lecturer or the lecture. This difference regarding research questions can be seen as stemming from the differences regarding basic principles, which also Radford (2008, p. 322) highlights when noting that "research questions must be clearly stated within the conceptual apparatus of the theory".

When applying the two theories to the empirical study, some differences between the two theories have been observed, in particular regarding basic principles that create different interpretations/explanations of results and also a reinterpretation of the purpose. However, there are also many similarities in the use of these two theories, which is evident in particular since it was possible to introduce the theories at such a late stage of the research process, when the theories could utilize the same empirical data, analyses and results. Thus, although some basic differences have been noted between the theories, the same type of empirical study is relevant for both theories.

Another similarity in the use of the two theories is that, as noted when introducing them, both function as explanation of the observed differences between lectures. That is, the question of what is "causing" these differences is not an empirical question but the question is answered within each theory.

However, this question is only answered at a general level, and both theories have a common interest in a type of follow-up question that is not answered within the theories but that demands empirical research; regarding the "limits" or "borders" for variations in the properties of communication. For the cognitive theory, this issue can be described as dealing with how general or context dependent epistemological beliefs are (see Limón, 2006), for example if it is reasonable to talk about epistemological beliefs for mathematics in general or if the beliefs are different for different areas of mathematics. For the discourse analytic theory, this issue can be described as dealing with the borders

and overlaps between different communities (see Skott, 2009), for example if it is reasonable to talk about a discourse for mathematics in general, regarding epistemology, or if different areas of mathematics create their own discourse communities about epistemological aspects.

The “follow-up question” discussed above is quite similar to the question asked at the end of the empirical study, before the two theories were introduced, regarding what aspects of the observed situation can be tied to the different properties of communication. Although the empirical study described here is somewhat artificial and partly adjusted to suit the overarching purpose of the present paper, this similarity of questions of interest at least shows a potential for such a similarity to exist. Based on these similarities between the uses of the two theories it is suggested that, for the specific topic of study in the present empirical study, the main difference between the two theories is a choice of wording when describing the observed phenomenon. It seems possible to “translate” descriptions within one theory into descriptions within the other theory, by noting what type of issue or phenomenon is being referred to in the languages of the theories – similarly as suggested by Rodriguez et al. (2008) in their comparison of theories. It should be noted, again, that this discussion focuses on the use of theories in relation to one specific empirical study, and does not deal with a general comparison of theories regarding their differences, similarities or compatibility. Therefore, it is not necessarily the case that the same relationship between these theories would be evident in relation to some other empirical study. This fact is also one important point I try to make; that it is more fruitful, and even necessary, to discuss the use of theories in relation to specific empirical studies in order to be able to examine if and how different theories, and different parts of theories, can affect different parts of the research process.

The process of choosing a theory to use

If the similarities between the uses of the two theories in the present empirical study are so great that the main difference is a choice of words, do we need to use a theory at all, or if we do, on what basis do we choose a theory?

Regarding the choice between the cognitive theory and the discourse analytic theory, this could be based on a personal conviction about how to view the more general issue of the relationship between mental processes and discourse. In this case, a *choice* between theories is perhaps not made, but the specific theory used serves as a means to make explicit the personal conviction. Even if not made explicit, these types of personal convictions can be labeled as theory. However, it can sometimes be of interest to separate these more general types of theories when discussing the role of theory in research, for example by labeling “a more informal inclination by the researcher to interpret an observed commonsensical problem” as an approach (Bergsten, 2008, p. 192) or by labeling “the viewpoint the researcher chooses to use to conceptualize and conduct the research” as a perspective (Lester, 2005, p. 458).

There is also the option of not choosing any of the two (or other similar) theories in relation to the empirical study. As previously described, questions triggered by the empirical results could be pursued without adding a new theory. As mentioned, the same types of questions are also of interest within the two theories, which might even more question whether it is relevant to add a new theory. Another alternative could then be to use the results from continued empirical studies as a basis for choosing a theory, regarding which theory seems more *useful*. For example, if further studies show that the epistemological characterizations are more tied to the individual than to the situation; that the variation is larger between individuals in the same situation than between situations for the same individuals, a theory locating the source of variation at the individual level could be more suitable. In this way, a theory is not seen as something generally applicable but the use of theory is adjusted according to specific needs and circumstances, which is in line with the notion of conceptual framework (Lester, 2005).

So far, after using a conceptual framework in the empirical study, two theories were introduced late in the research process. It was then examined how the use of these theories can affect different parts of the research process, in particular the interpretation/explanation of empirical results, the methodology, and different aspects of research questions (including reinterpretation of existing ones and possible

future questions of interest). These discussions about the use of theories in research can now be expanded and deepened by also discussing the use of theoretical frameworks, which is done in the next section by examining the possibility to introduce the two theories at an earlier stage in the research process.

The use of conceptual and theoretical frameworks

As discussed in the introduction of this paper, in relation to Lester's (2005) description of different frameworks, a main difference between theoretical and conceptual frameworks is regarding the relationship between research questions and theory. A simplified way of describing this relationship for a theoretical framework is that the theory comes first and that the research questions follows from, or are to some extent included in, the theory. For a conceptual framework on the other hand, the research questions "decide" what type of theory to use.

Based on the specific empirical study described in the present paper, when using a conceptual framework, it can be noted that it is not as simple as to say that research questions and theory are "separated" from each other and that one "affects" the other. In the empirical study, the starting point was a certain question, and based on this question there was a need to relate to and use certain theories. However, the original question was then also made more specific through the use of these theories. More generally, a research question cannot exist in a theoretical vacuum, but we could perhaps distinguish different types of questions based on how closely connected they are to a theory. For example, what Prediger (2008) labels as issues of classroom practice and Bergsten (2008) labels as observed commonsensical problems could be seen as questions formulated based on more intuitive grounds. However, these types of questions are not entirely non-theoretical; in relation to the issue of classroom practice given by Prediger, Radford (2008) states that the described issue would be difficult to relate to if using a certain theory, and Bergsten notes the use of an approach (i.e. a more informal, implicit type of theory) when interpreting the observed commonsensical problem. The types of questions that are more intuitively formulated need to be made more precise in order to function as

research questions, in particular that central concepts need to be defined and relationships between them need to be described, which are central parts of a theory.

Thus, for a conceptual framework, there is a kind of dual relationship between research questions and theory, but where, as Lester (2005) highlights, focus is always on arguments for choosing relevant theories in relation to the topic of study. However, looking at the empirical study described in this paper, it can be noted that the arguments for using a certain theory focus on the need for a certain *type* of theory, for example a theory dealing with aspects of epistemology in mathematics or a theory dealing with possible explanations of observed results. The choice of a specific theory of the needed kind was then perhaps affected more by, for example, personal viewpoints or convictions (as discussed also in the previous section). Thus, even if focus is on arguments for the relevance of theories used in relation to the topic of study, there can be a complex relationship between theories (including the more informal, implicit types) and research questions.

Instead of using a conceptual framework in the empirical study presented in this paper, what could the effect be of using a theoretical framework? That is, we decide to use one theory beforehand, and let this theory affect all aspects of the research process, in relation to a topic of study. In the following, a comparison is made between the use of the cognitive theory and the discourse analytic theory, when using them as part of a theoretical framework in relation to the same issue of classroom practice used before; to examine the types of messages about mathematics inherent in, or conveyed by, properties in teaching, including the limitation to focus on aspects of communication in teaching and epistemological aspects of messages about mathematics. Of course, this thought experiment is rather artificial, in particular since the described topic of study might not be equally relevant for both theories, but this limitation is seen as relevant in order to highlight some aspects of the potential use of these theories. Since I do not present any new empirical studies using these theories as part of theoretical frameworks, I limit the discussion to research questions and methodology.

Regarding the formulation of research questions in relation to the topic of study, it has already been noted that the questions given in the empirical study are relevant to both theories but that they were interpreted somewhat differently. Therefore, the arguments for studying these questions would be different when using the two theories and the specific wording of the questions might also be different. For example, the cognitive theory could formulate the questions more specifically about relationships between epistemological beliefs and properties of discourse, while the discourse analytic theory could formulate the questions more specifically about examining potential similarities and differences between different communities.

Regarding methodology, the discourse analytic theory focuses on discourse in itself in communities, and therefore focuses on the use of data from genuine situations in these communities, and not on creating “artificial” situations for data collection. The recording of lectures represents data of such genuine situations. The cognitive theory does not focus on discourse, but can be said to regard recordings as the method for being able to observe, indirectly, aspects of mental objects and processes. Therefore, for this theory there is openness for different types of data that could make it possible to give an, indirect, picture of the mental. To only rely on recordings of lectures might then also be seen as a limitation, since you do not know if it is epistemological beliefs that are the main cause of observed differences. Instead, the cognitive theory could utilize a study with two parts; something similar as has been done in the present empirical study together with a questionnaire or interview that in a more direct manner could examine epistemological beliefs. These two parts of data could then be related to each other in order to examine if/how epistemological beliefs affect the properties of discourse.

Thus, studies using the two theories as part of theoretical frameworks could have been quite different, although starting with the same topic of study. These differences could make it difficult to notice the potential similarities between the studies that have been noticed when starting with a conceptual framework and applying the two theories later in the research process. Therefore, one argument *against* using theoretical frameworks can be that potential similarities between studies focusing on the

same kind of phenomenon or topic become more hidden. However, you could also argue in similar manner *for* using theoretical frameworks. If several studies use the same theory, possibly examining somewhat different phenomena, this could simplify the process of relating results from these studies to each other and to see the results from one study in a broader perspective. For continued studies about the role of theory in research it then becomes important to be aware of these different aspects of the use of theory, in particular that it is central to not only compare different theories but to compare the uses of different theories in specific empirical studies.

Conclusions

In relation to the given purpose of this paper, some central results of the discussions are here summarized regarding four aspects of the use of theory in research: (1) The utilization of different types of frameworks, (2) The utilization of different types of theories, (3) Relationships between different parts of the research process and different parts of theories, and (4) Potential unification/integration of theories in relation to the specific research topic of the empirical study.

Regarding the different types of frameworks, they seem to have different pros and cons in relation to if/how we can relate results from different studies to each other in a fruitful manner. Theoretical frameworks can simplify the comparison of results from studies that have used the same theory while conceptual frameworks can simplify the comparison of studies using (partly) different theories.

Some more specific properties of conceptual frameworks have here been problemized, in particular regarding the primacy of research questions when choosing theories to use, where focus is on the relevance of the theories for a specific study. A complexity has been highlighted in the discussions of the issue regarding relationships between research questions and theories. In particular, different types of questions have been discussed, from more informal/intuitive interpretations of observations to more specified research questions, and that the development of research questions occur in a dual interaction with theories, where also different types of theories are active.

Thus, different types of theories have been discussed based on how they are used; some theories are of a more informal type that are perhaps most often not referred to explicitly as theories while other theories are explicitly referred to and used in a study. Regarding the more informal types of theories, authors label these types of theories in different ways, for example as approaches (Bergsten, 2008) or perspectives (Lester, 2005), which both are described as something not part of a framework. Other authors distinguish between background and foreground theories (Mason & Waywood, 1996). In this paper, the relationships between these types of theories have not been discussed in depth, but there is a need to create a more coherent structure of these types of theories, including utilization of explicit definitions of central notions such as theory and framework. Such a structure would be beneficial for the continued work on comparing the use of different (types of) theories in order to increase our awareness of the role of theory in research.

Important when comparing different theories is to not only focus on how they are used but also how they are chosen. The argumentation for choosing suitable theories is central for conceptual frameworks, which creates a greater potential to also make explicit how the theories are used. For theoretical frameworks, there might be a greater risk that focus is on stating what theory is used and neither on why or exactly how. However, regardless of the kind of framework used, it can never be taken for granted that all parts of a theory have affected all parts of the research process.

Several examples have been given showing that it is more fruitful and also necessary to discuss the use of theories in specific empirical studies and not only to compare the descriptions of different theories. The example in the present paper shows that there can be empirical studies that are very similar although based on theories that can be seen as very different, and partially contradictory. This way of comparing theories seems to become more common in mathematics education research, which is shown by several studies referred to in this paper.

Finally, I turn to the use of theories in relation to the specific topic of epistemological characterizations of mathematical discourse. Based on a conceptual framework, using “smaller” theories, the addition of two “larger” theories at the end of the research process did not add much to the empirical study. For each theory a type of explanation of the empirical results was added, and there was also a reinterpretation of the original purpose. These additions and changes were mostly a matter of wording and suggestions for further studies were similar for both theories and also similar to the suggestions given before the theories were introduced. The need to add any of these two theories is therefore challenged. However, when instead using these two theories as part of a theoretical framework, the empirical studies based on the two theories could have been more different, although then perhaps hiding potential similarities between the uses of these theories and possibly creating results of more limited usefulness. Thus, for continued studies of this topic it is suggested to continue using the conceptual framework and continue the critical examination of possible future needs to add other theories, of any size or type.

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