The Mathematics Enthusiast

Volume 10
Number 1 *Numbers 1 & 2*

Article 4

1-2013

Problem Solving in the Primary School (K-2)

Richard Lesh

Lyn English

Chanda Riggs

Serife Sevis

Follow this and additional works at: <https://scholarworks.umt.edu/tme>

Part of the [Mathematics Commons](#)

Let us know how access to this document benefits you.

Recommended Citation

Lesh, Richard; English, Lyn; Riggs, Chanda; and Sevis, Serife (2013) "Problem Solving in the Primary School (K-2)," *The Mathematics Enthusiast*. Vol. 10 : No. 1 , Article 4.

DOI: <https://doi.org/10.54870/1551-3440.1259>

Available at: <https://scholarworks.umt.edu/tme/vol10/iss1/4>

This Article is brought to you for free and open access by ScholarWorks at University of Montana. It has been accepted for inclusion in The Mathematics Enthusiast by an authorized editor of ScholarWorks at University of Montana. For more information, please contact scholarworks@mso.umt.edu.

Problem Solving in the Primary School (K-2)

Richard Lesh¹

Indiana University

Lyn English

Queensland University of Technology

Chanda Riggs

Sharon Elementary School

Serife Sevis

Indiana University

Lesh: “Do you really think your children can do this?”

Riggs: “So far, nobody has taught them yet about what they can’t do.”

Abstract: This article focuses on problem solving activities in a first grade classroom in a typical small community and school in Indiana. But, the teacher and the activities in this class were not at all typical of what goes on in most comparable classrooms; and, the issues that will be addressed are relevant and important for students from kindergarten through college. Can children really solve problems that involve concepts (or skills) that they have not yet been taught? Can children really create important mathematical concepts on their own – without a lot of guidance from teachers? What is the relationship between problem solving abilities and the mastery of skills that are widely regarded as being “prerequisites” to such tasks? Can primary school children (whose toolkits of skills are limited) engage productively in authentic simulations of “real life” problem solving situations? Can three-person teams of primary school children really work together collaboratively, and remain intensely engaged, on problem solving activities that require more than an hour to complete? Are the kinds of learning and problem solving experiences that are recommended (for example) in the USA’s Common Core State Curriculum Standards really representative of the kind that even young children encounter beyond school in the 21st century? ... This article offers an existence proof showing why our answers to these questions are: Yes. Yes. Yes. Yes. Yes. Yes. And: No. ... Even though the evidence we present is only intended to demonstrate what’s possible, not what’s likely to occur under any circumstances, there is no reason to expect that the things that our children accomplished could not be accomplished by average ability children in other schools and classrooms.

Keywords: Common core standards; elementary mathematics education; problem solving in elementary school;

¹ ralesh@me.com

Can Children Solve Problems involving Concepts they have not been Taught?

Most people's ordinary experiences are sufficient to convince them about the truth of two important assumptions about learning and problem solving.

- First, the kinds of things that students can learn, and the kinds of problems that they can solve, tend to be strongly influenced by the things they already know and are able to do. So, the accompanying “common sense assumption” is that these prerequisites must be mastered before students are expected to learn relevant new ideas, or solve relevant new types of problems. And consequently, learning is viewed as a long step-by-step process in which prerequisites are checked off one at a time.
- Second, concepts and abilities do not go from unknown to mastered in a single step. They develop! And, so do associated abilities. In fact, especially for the most important “big ideas” in the K-12 curriculum, development typically occurs over time periods of several years, and along a variety of dimensions – such as concrete-abstract, intuition-formalization, situated-decontextualized, specific-general, or increasing representational fluency, or increasing connectedness to other important concepts or abilities. So, in situations which are meaningful and familiar to students, rapid developments often occur for clusters of related concepts and abilities. And, in these contexts, students' ways of thinking often integrate ideas and abilities associated with a variety of textbook topic areas – so that the resulting knowledge and abilities are organized around experiences as much as around abstractions.

For readers who are familiar with Vygotsky's zones of proximal development, the title of this section poses a question that is clearly naïve. Learning does not occur in this all-or-nothing manner. For example, in a series of projects known collectively as The Rational Number Project (RNP, 2011), it is well known that the "difficulty level" of a given task can be changed by years – simply by changing the context or the representational media in which problems are posed (e.g., written symbols, written language, diagrams or graphs, concrete models, or experience-based metaphors). Consequently, when students encounter a problem in which some type of mathematical thinking is needed, all of the relevant concepts and abilities can be expected to be at some intermediate stages of development – not completely unknown, yet not completely understood – regardless of whether these concepts or abilities have been formally taught.

In fact, for researchers who have investigated what it means to "understand" the most powerful and important ideas in the elementary school curriculum, it has become clear that most of the "big ideas" that underlie the K-12 curriculum begin to develop in early years— in topic areas ranging from rational numbers and proportional reasoning (RNP, 2011), to measurement and geometry (e.g., Krutetskii, 1976), to statistics and probability (e.g., Zieffler, Garfield, delMas, & Reading, 2008), to early ideas in algebra (English, in press; Thompson, 1996) or calculus. In fact, in each of these domains of mathematical thinking, many important understandings typically begin to develop even in the primary grades (K-2). Such observations are reminiscent of Bruner's claim, long ago, that: *Any child can be taught any concept at any time – if the concept is presented in a form that is developmentally appropriate* (Bruner, 1960). Of course, the "if clause" in this quote is very significant. That is, in order for remarkable developments to occur,

relevant concept and abilities need to be accessible in the forms that are developmentally appropriate.

For the problems that will be described in this chapter, the two primary tests of developmentally appropriateness are: (a) Do the children try to make sense of the problem using their own “real life” experiences – instead of simply trying to do what they believe that some authority (such as the teacher) considers to be correct (even if it doesn’t make sense to them)? (b) When the children are aware of several different ways of thinking about a given problem, are they themselves able to assess the strengths and weaknesses of these alternatives – without asking their teacher or some other authority? When these two criteria are satisfied, children are able to go from “first-draft of thinking” to “Nth-draft of thinking” without interventions from an outside authority.

When referring to “real life” sense-making abilities, it is important to emphasize that we are not assuming that a first grader’s “real life” interpretations of experiences are the same as an adult’s one. For example, for first graders, children’s stories often engage their sense making abilities more than situations that an adult might consider to be a “real life” situation. So, for the problems that we’ll describe in this article, the tasks were presented in the context of stories such as *Two Headed Stickbugs*, *The Proper Hop* (for *Beauregard the Frog*), *Fussy Rug Bugs*, *Isabelle Talks*, *The Royal Scepters*, or *Tubby the Train* (see Figure 1) – most of which appeared first in Scott Foresman’s longest running kindergarten book - written by Lesh & Nibbelink (1978).

For our purposes in this article, some other important of “real life” characteristics that we tried to build into our problems include the following. (a) The product that the children are challenged to produce often is not just a “short answer” to a pre-

mathematized question. It is a sharable and reusable tool or artifact that needs to be powerful (in the given situations) and also sharable (with other people) and reusable (in situations beyond the given situation). (b) The children need to know who needs their product - and why? Otherwise, they won't be able to assess the strengths and weaknesses of alternative products; and, they won't be able to judge: is a 3-second answer sufficient; or, is a 3-minute or 30-minute needed? (c) Product development is likely to require several cycles of testing and revising – similar to first, second, or third drafts of written descriptions of drawn pictures. (d) Products will often need to integrate ideas and procedures drawn from a variety of textbook topic areas. One reason for that this is true is because “real life” problem often involves partly conflicting constraints and trade-offs (e.g., low cost but high quality, simple but powerful).

<h3>Fussy Rugbugs</h3> <p>Ten-footed rugbugs live on little rugs. Each bug has its very own little rug. Rugbugs like to make towns. To make a town, several bugs push their rugs close together. Inside a town, the bugs run over the rugs quite a lot to visit each other. With ten feet, they run very fast. But they also track lots of dirt.</p> <p>Rugbugs are fussy about their towns. They don't like spaces between rugs. Spaces make them trip and fall. But, they don't like rugs sticking over other rugs either.</p>	<h3>Isabel Talks</h3> <p>John Harvey Hopkins ran a rabbit ranch. He ran his ranch about the same way a cowboy would run a cattle ranch. Of course, his lasso was quite a bit smaller. His pony, Isabele wasn't very big either. The way John Harvey figured, a big horse was fine for chasing big cows. But, a little horse was better for chasing little rabbits.</p> <p>There were fences. A ranch wouldn't be a ranch without fences.</p>
<h3>The Royal Scepters</h3> <p>King Albert and Queen Alberta rules the peaceful kingdom of Kino. The Kingdom was so peaceful tha the king and queen had absolutely nothing to do. They had the most boring jobs in the entire kingdom. Every day they just sat there, holding their royal scepters and twiddling their royal thumbs. ... They were miserable.</p> <p><i>My scepter is longer than yours, said the King Albert.</i></p> <p><i>It is not, snapped the queen. Let's compare.</i></p>	<h3>New Tracks for Tubby</h3> <p>Tubby the Train works at the Zoo.</p> <p>Tubby loves his job. He takes children on wonderful rides to see the animals. Big animals - like Edna the elephant. And small animals - like Mike the monkey and his friends.</p> <p>Round and round he goes; and he stops when the animals put on a show.</p> <p>Only one thing is wrong! Tubby's tracks are old and worn. They go <u>bump, bump, bump!</u> And, <u>screech, screech, screech!</u> The tracks no longer <u>fit together smoothly</u>. So, the clanging and banging frighten the animals. And, they run away to hide where the children can't see them.</p>

Figure 1. The First Pages from Six Stories

Figure 1 shows the six contexts that were used for the problems which will be described in this article. Then, Figure 2 briefly describes the tasks that accompany each of these stories. For each task, the children worked in groups of three; the work spaces

usually were at least the size of large poster boards; and, the products usually needed to include letters to “someone else” describing how they could do and what our students did. This letter-writing aspect of the tasks was emphasized partly because the school in which we worked was focusing on writing abilities. But, it also was used because, to be consistent with our published principles for designing *model-eliciting activities* (Lesh, et. al., 2000), it is important for the products to be more than single solutions to isolated problems. Solutions to our problems also needed to be sharable and reusable. So, one straightforward way to achieve this goal was to ask children to write letters describing tools or artifacts that can be used by others.

Two-Headed Stickbugs	The Proper Hop
<p>Two different sized “stickbugs” were used – one made with popsicle sticks, and the other made with meter-long strips of wood. In a warm-up activity, the children work in groups of three, and use meter-long “stickbug” to measure as many distances as possible in the playground of their school. (note: On boring playgrounds, markers can be placed using highway markers or signs.) The teacher should record these distances by drawing arrows and points on a poster-sized photograph (or drawing) of the school yard. Then, the next day, children should again work in groups of three, and use the teacher’s notes and popsicle-sized stickbugs to create miniature scale-models of their playground.</p>	<p>The green dots shown below are lily pads. The lines between dots indicate “proper hops” (which must be horizontal or vertical hops to adjacent lily pads). The red, yellow, and orange dots are where three of Beauregard’s friends live. And, the children’s task is to find a place where Beauregard (x) should live so that the sum of the distances to his three friends’ houses is as small as possible.</p>
Fussy Rugbugs	Isabel Talks
<p>What is the largest number of rugbugs that can live totally inside the area shown below? (Give each group enough post-it stickies to cover the area; and, give approximately one-third of the stickies to each child. Either square or round stickies can be used. But, in either case, the stickies should (a) be completely inside the area indicated, (b) not overlap, and (c) fit together as closely as possible.</p>	<p>The “trees” shown below should be unevenly distributed. They represent apple trees, with big juicy apples that Isabel loves to eat while standing in their shade. The goal is to build a closed fence which encloses the largest number of apple trees. The fence is a loop of soda straws or coffee stirrers – strung on a closed loop of string.</p>

Figure 2. Tasks for Six of the Stories

Can Children Create Important Mathematical Concepts on Their Own?

If we examine the title of this section, two opposing answers to this question seem to be equally obvious - depending on how the question is interpreted. If the question is intended to ask: Can children invent the most fundamental concepts in algebra, geometry,

or calculus?, then the answer clearly must be: No! It took years for some of history's most brilliant mathematicians to invent these concepts. So, average ability children cannot be expected to do such things during single class period? But, if the question is asking: Can children use numbers to describe mathematically interesting situations in the mathematical "objects" involve more than simple counts of discrete objects (i.e., cardinal numbers), then one of the main points of this paper is that the answer to this question is: Yes! For example, the six problems that we describe in this article involve using numbers to describe locations (coordinates, or ordinal numbers), lengths or distances (or other types of measurable quantities), signed quantities (negative numbers), directed quantities (vectors), actions (operators, transformations, functions), changing quantities (rates or intensive quantities), or accumulating quantities (calculus). In particular, for the six stories described here:

- Children's responses to the Stickbug Problem often use numbers to describe lengths, distances, and sometimes even coordinates – if the "map" is thought of as a simple kind of grid.
- Children's responses to the Proper Hop Problem often use numbers to describe locations, actions (hops), number patterns, or quantities that have both a magnitude and a direction.
- Children's responses to the Fussy Rugbugs Problem often use numbers to describe areas or dimensions (concerning how "rugs" are aligned within shapes).
- Children's responses to the Isabel Talks Problem often use numbers to describe relationships between areas and perimeters, and even negative

numbers (because when borders are rearranged to include some new “trees” and other “trees” tend to be lost).

- Children’s responses to the Royal Scepters Problem often use numbers to describe scaling-up, proportions, ratios, lengths, distances, shapes (e.g., rectangles, triangles), and sometimes angles or areas .
- Children’s responses to the Tubby the Train Problem often use numbers to describe lengths, angles, and negative quantities (which occur pieces of tracks are inserted or deleted in order to eliminate dead ends, or in order to enlarge or shrink enclosed areas).

Of course, from a child-eye view, the preceding situations are not about ordinal numbers, coordinates, signed numbers, vectors, operators – or areas, volumes, or densities. To the children, they are simply contexts in which numbers are used to describe things such as: hops, measuring sticks, sticky post-it notes, straws, or paths. Nonetheless, because the tasks require children to externalize their thinking in forms that are visible to the students themselves (as well as teachers and researchers), the seeds are apparent for many of the most important “big ideas” that span the entire K-12 mathematics curriculum.

In general, what research based on models & modeling perspectives (Lesh & Doerr, 2003) shows that, if children clearly recognize the need for a specific kind of mathematical description, diagram, artifact, or tool, and if the children themselves are able to assess strengths and weaknesses of alternative ways of thinking, then remarkably young children are often able to produce impressively powerful, reusable, and shareable tools and artifacts in which the mathematical “objects” being described involve far more

than simple counts. However, even though children are able to generate such descriptions without guidance from adults, this claim does not imply that there is no role for teachers. For example, even if children succeed in developing a powerful, sharable, and reusable artifact or tool in response to a problem, they usually lack powerful ways to visualize underlying constructs, and they are not often aware of strengths and weaknesses of alternative ways of thinking. Furthermore, because their results often integrate concepts and procedures drawn from a variety of textbook topic areas, they usually have not unpacked these ideas-or, expressed them using elegant language and notations.

Can Teams of Primary School Children Work Collaboratively, and Remain Intensely Engaged, on Problem Solving Activities that Require an Hour to Complete?

Lesh: How long do you think primary school children are able to work on these kinds of tasks? And, what is it about such activities that stimulate sustained work from children?

Riggs: In general, the children worked on one modeling activity for two or three consecutive days for an hour or more each day. The fourth day was reserved for sharing explanations of their modeling to their classmates. Due to the cooperative nature of the activities, complemented by children's engagement in problem solving, the children were highly motivated and often requested additional time to devote to the task. Through sharing, children learned to appreciate diversity in problem solving. I believe that introducing concepts through interesting children's stories gives the children a purpose for their learning; this purpose is what stimulates them to complete the task no matter the amount of time required or how challenging it seemed. The children viewed learning as

something they wanted to do instead of something they were required to do; modeling activities provide that motivation. The activities were designed to open and close within a week. One reason for this policy was because class time is precious. These stories served as “chunks” that children could use to organize ideas and skills related to a central “big idea”. If these “chunks” got too large, the children would lose sight of the “big idea”. Memorable stories also help children remember what they have learned. The children continued to think about the “big idea” after class - and after we moved to other topics. Weeks after they had finished activities directly associated with one of our stories, they often referred back – saying: This is like Stickbugs, or Beauregard, or Tubby the Train. Then, they would use concepts and abilities that they had developed during those tasks. ... So again, several smaller stories are better than one big story.

Lesh: How much and what kind of guidance did you need to provide in order for children to be successful for these tasks?

Riggs: When the children work in groups, they tend to persevere when they otherwise might have given up. But also, in every one of our activities, children worked together to build some concrete tools or artifacts – such as pathways, fences, villages, maps, or scaled-up houses. So, as long as they clearly understood what was needed and why, and as long as they were able to test their thinking without asking me “Am I done?” or “Is this right?”, they were able to move from first-draft thinking to second and third-draft thinking without much guidance from me.

Self-assessment is important because, in complex activities, if children need to wait for their teacher’s approval at each step, then things move too slowly, and young

children lose interest. Also, when children express their thinking in forms that are visible, it's easier for teachers to wisely pinpoint when and what kind of help is needed. Usually, the kind of help that is most effective involves reflecting more than guiding. For example: interventions that tend to be most helpful are questions like: Who needs this? Why do they need it? What do they need to do? Does everybody agree? ... They are not commands like: Think of a similar problem that you have solved. Or: Everybody listens to Alice. ... The goal is for children to iteratively express, test, and revise their own ways of thinking. The goal is not for them to superficially adopt the teacher's ways of thinking.

Figure 3. The Proper Hops Problem

Figure 4. The Rugbug Problem

As the “letter” in the Figure 4 shows, 1st graders’ letters cannot be expected communicate objectively to another person. But, in this particular study, the school where we were working had made a school-wide commitment to focus on writing. So, the opportunity was too good to miss. For our purposes, the main point of trying to craft letters was not to press the children for writing excellence. The main purpose was (a) for the children to understand that “someone else” wanted to use the information, tool, or artifact that they produced, and (b) to emphasize the tools and artifacts needed a be useful to other people to use. In other words, we wanted to make it as clear as possible to the children that their procedure (or tool) need to be sharable and reusable. ... On the one

hand, the solutions that children produce in such situations are highly situated forms of knowledge. On the other hand, sharable and reusable solutions also are transferrable.

Figure 5. Apples for Isabelle Problem

Figure 6. Measuring with Royal Scepters & Laying Tracks for Tubby the Train

What is the Relationship between Problem Solving & the Development of “Prerequisite” Concepts & Skills?

Lesh: This project was not an experiment that treated your children like guinea pigs in a laboratory. It was simply a joint effort that you and our research team decided to provide the best kind of learning experiences for your children. Yet you, like most teachers, administrators, and schools on these days, are being held accountable for learning gains which are measured by standardized tests which (I believe) don't measure much beyond low-level skills. So, even though we didn't have any experimental “control group”, how do you think your students will perform, compared to others, on standardized tests that are relevant to you and others in your school?

Riggs: I believe that my students will perform as well, if not better, on standardized assessments after using the model eliciting activities. Given that the children learn to problem solve in ways that make sense to them, and they can see their results from the models created, the model eliciting activities provided a knowledge base where information can be retrieved and applied as needed. The students' ability to apply what they had learned became evident when they would remember the "big idea" weeks after we had finished the activity, and when they would apply it to situations in their own lives. One example: Three weeks after completing The Proper Hop, a student stated that living in an apartment complex is like living in Sugar Swamp - there are a lot of lily pads. After helping Beauregard to find the best lily pad closest to his friends, this student understood why her Mom didn't want her to walk all the way over to the other side of the complex to visit a friend. It was too far away; it was like Beauregard hopping 20 hops. She said that her Mom allowed her to go next door to visit a friend; for Beauregard, it would only

be one or two hops. This student also wished she could pick the location of her apartment to be close to her friends - just like she helped Beauregard find his home in The Proper Hop.

Ever since the seminal work of William Brownell (1970), it has been known that, even if we only care about skill-level knowledge, “varied practice” is far more effective than “routine practice” (or drills that are repeated again and again). Brownell identified three kinds of varied practice. The first type involves mixed activities in which attention shifts among several skills – rather than emphasizing just one. This is effective partly because “understanding” involves more than knowing how to do something; it also involves knowing when to do it. The second type of varied practice involves practicing skills in a full range of situations in which they are intended to be useful. This is effective partly because useful skills need to be flexible, not rigid. And, the third type of varied practice involves using skills during complex activities – similar to the way excellent chefs not only know how to use each of the tools sold in chef’s catalogues, but they also know how to orchestrate the use of these tools during the development of complex meals.

Can Primary School Children Engage Productively in Authentic Simulations of “Real Life” Problem Solving Situations?

According to the models & modeling perspectives that underlie our work (Lesh & Doerr, 2003), we reject the notion that children learn, or learn to be effective problem solvers, by first learning concepts and skills, and then learning to use them in meaningful “real life situations.” By far the most important characteristic of the models & modeling

perspectives that distinguish our work from traditional research on problem solving is the recognition that – regardless of whether investigations focus on decision making by medical doctors, business managers, chess players, or others in real life decision-makers - in virtually every field where learning scientists have investigated differences between ordinary and exceptionally productive people, it has become clear that exceptionally productive people not only do things differently, but they also see (or interpret) things differently. Furthermore, when problem solvers interpret situations they don't simply engage models that are completely mathematical or logical in nature. Their interpretations also tend to include feelings, values, dispositions, and a variety of metacognitive functions. But, instead of mastering these other higher-order functions separately, and then attaching them to mathematical models, research on models and modeling shows that they develop as integral parts of the relevant interpretation systems (Lesh, Carmona & Moore, 2010).

- Traditionally, problem solving has been characterized as a process of (a) getting from givens to goals when the path is not obvious, and (b) putting together previously learned concepts, facts, and skills in some new (to the problem solver) way to solve problems at hand. But, when attention shifts toward models & modeling, problematic situations are goal directed activities in which adaptations need to be made in existing ways of thinking about givens, goals, and possible solution steps. So, modeling is treated as a way of creating mathematics (Lesh & Caylor, 2007); and, modeling and concept development are expected to be highly interdependent and mutually supportive activities – especially for young children.

- Traditionally, problem solving strategies and metacognitive functions have been specified as lists of condition-action rules – and have been thought of as providing answers to the question: What should I do when I’m stuck (i.e. when I am not aware of any productive ways of thinking about the problem at hand). But, when attention shifts toward models & modeling, the goal of metacognitive processes is to help problem solvers develop beyond their current ways of interpreting the situations, rather than helping them identify “next steps” within current ways of thinking.
- Traditionally, problem solving in mathematics education has focused on individual students working without tools on textbook word problems. But, because research on models and modeling tends to focus on simulations of “real life” situations, problem solvers often are diverse teams of students each of whom are likely to have access to a variety of specialized technical tools and resources. So, capabilities that become important include: modularization, communication, explanation, and documentation - as well as planning, monitoring, and assessment – all of which tend to be overlooked in the traditional mathematics education problem solving literature; and, all of which emphasize modern socio-cultural perspectives on learning.

Because model development activities are, above all, research sites for directly observing the development of interpretation systems that involve some of the most important aspects of what it means to “understand” many of the most important concepts and “big ideas” in mathematics education, research on models and modeling has led to new views about: (a) how the modeling cycles that students go through during one 60-

minutes model-eliciting activity often are remarkably similar to developmental sequences that Piagetian psychologists have identified during timespans of several years based on normal everyday experiences, (b) how average ability students often develop (locally) through several Piagetian stages during single 60-minutes problem solving episodes, (c) how students' final-draft solutions often embody mathematical thinking that is far more sophisticated than traditional curriculum materials ever dared to suggest they could be taught, (d) how student solutions which are expressed in the form of sharable and reusable tools often enable students to exhibit extraordinary abilities to remember and transfer their tools to new situations, (e) how the processes that enable students to move from one model to another seldom look anything like currently touted "learning trajectories" which describe learning and problem solving using the metaphor of a point moving along a path, (f) how the tools and underlying models which students produce in "real life" model development often integrate concepts and abilities associated with a variety of textbook topic areas, (g) how students' early interpretations often involved collections of partial interpretations – which tend to be both poorly differentiated and poorly integrated, (h) how later interpretations tend to notice patterns of information, rather than the kind of pieces of information that tend to dominate earlier interpretations, (i) how model development tends to involve gradually sorting out and integrating several earlier interpretations, (j) how model development often occurs along a variety of interacting dimensions – such as concrete-abstract, intuition-formalization, specific-general, global-analytic, and so on, (k) how the origins for final interpretations often can be traced back to several conceptual grandparents, and (l) how final models tend to include not only systems of logical/mathematical "objects", relations, operations, and

patterns, but they also usually included dispositions, feelings, and a variety of relevant metacognitive functions.

Are the Learning & Problem Solving Experiences Recommended (for example) in the USA's Common Core State Curriculum Standards Representative of Those Children Encounter beyond School in the 21st Century?

For mathematics in the primary school (K-2), the main themes of the CCSC Standards are clear. One of its laudable overall goals is to focus on deeper “conceptual” treatments of fewer standards. Another is to emphasize research-based learning progressions about how students’ mathematical knowledge, skill, and understanding develop over time. And, another is to treat mathematical understanding and procedural skill as being equally important.

- What do the CCSC Standards mean by focusing on deep treatments of a small number of “big ideas”? They say: *Mathematics experiences in early childhood settings should concentrate on (1) number (which includes whole number, operations, and relations) and (2) geometry, spatial relations, and measurement, with more mathematics learning time devoted to number than to other topics.*
- What does mathematical understanding look like? They say: *One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from.*

- Modeling with mathematics is mentioned in only one small paragraph in these standards. And, what do the CCSC Standards mean by “modeling with mathematics”? They say: *Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.*
- The goal of describing and comparing measurable attributes is mentioned in precisely one sentence in the CCSC Standards for the primary grades. But, this sentence is overwhelmed with statements and examples focusing on number operations, and on counts of discrete objects in sets.

The preceding prejudiced portray of a view of mathematics, learning, and modeling that is extremely different than the one described briefly in this article. The CCSC preoccupation with counts is not focused. It is narrow. And, it is not at all consistent with the kinds of situations that even young children encounter where numbers and arithmetic outside their school classrooms. Similarly, the CCSC’s notion of what it means to “understand” important concepts and processes completely overlooks the development of powerful sense-making systems - that is, models for describing (quantifying, dimensionalizing, coordinatizing, or in general: mathematizing) situations in forms so that the concepts and procedures that they profess to emphasize will be useful beyond mathematics classrooms (Lesh & Sriraman, 2010; Lesh, Sriraman & English, 2013).

Similarly, the notion of modeling in the CCSC as “applying mathematics that they know to solve problems arising in everyday life” is not at all what we have described in this paper – where 1st grade children learned to actively develop impressively

sophisticated descriptions of meaningful situations – similar to those that occur beyond school classrooms. And finally, the CCSC’s notion of “research progressions” completely ignores the large literature on situated cognition – where knowledge is recognized as being organized around mathematically rich experiences (like our stories) as much as around the kind of decontextualized abstractions that the CCSC Standards continues to emphasize in the examples and detailed descriptions of curriculum goals that are given. Why is this oversight so important? One reason is because most “learning progressions” of the type that the CCSC appears to have in mind envision long strings of prerequisites as being necessary to “master” before children can proceed to more important milestones. So, learning is thought of as a long and arduous process – which looks nothing like the rapid local developments that we describe in this article.

Certainly “real life” situations where number and arithmetic concepts are useful involve many kinds of mathematical “objects” including beyond counts. Examples include locations, actions, weights, likelihoods, and so on. But, unlike the word problems that fill K-12 textbooks, which can be characterized as situations described by a single rule (or function) going in one direction. “Real life” situations often involve several “actors” or several functions – so that feedback loops and 2nd-order effects are important, and where issues such as maximization, minimization, or stabilization occur regularly. For example, in the story-based problems that we have emphasized here, most of them involved several interacting arithmetic operations, as well as issues such as minimization or maximization.

Most of all, this article is intended to portray mathematical model development as an important aspect of mathematical “understanding” that is unabashedly optimistic about

the level of mathematical thinking that is accessible – even to primary school children, and to students of average-ability as measured on standardized achievement tests.

References

- Brownell, W. A. (1970). Psychological considerations in the learning and teaching of arithmetic. In J. K. Bidwell & R. G. Clason (Eds.), *Readings in the history of mathematics education* (pp. 504–530). Washington DC: National Council of Teachers of Mathematics. (Reprinted from Tenth yearbook of the National Council of Teachers of Mathematics: The teaching of arithmetic, pp. 1–31, 1935. New York: Bureau of Publication, Teachers College, Columbia University.)
- Bruner, J. (1960). *The process of education*. New York: Vantage Books.
- English, L. D. (In press). Reconceptualising statistical learning in the early years through data modeling. In L. D. English & J. Mulligan (Eds.), *Reconceptualising early mathematics learning*. Dordrecht: Springer.
- Krutetskii, V. (1976). *The psychology of mathematical abilities in school children*. Chicago, IL: University of Chicago Press.
- Lesh, R. & Doerr, H. (2003) *Beyond constructivism: A Models & Modeling Perspective on Mathematics Teaching, Learning, and Problems Solving*. Hillsdale, NJ: Lawrence Erlbaum Associates.
- Lesh, R., & Caylor, B. (2007). Introduction to the special issue: Modeling as application versus modeling as a way to create mathematics. *International Journal of computers for mathematical Learning*, 12(3), 173-194.
- Lesh, R., & Nibbelink, W. (1978). *Mathematics around us: Kindergarten*. Glenview, IL: Scott Foresman & Co.

- Lesh, R., Carmona, G. & Moore, T. (2010) Six sigma learning gains and long term retention of understanding and attitudes related to models & modeling. *Mediterranean Journal for Research in Mathematics Education*, 9(1), 19-54.
- Lesh, R., Kelly, A., Hoover, M., Hole, B., & Post, T. (2000) Principles for developing thought-revealing activities. In A. Kelly & R. Lesh (Eds.) (2000) *The Handbook of Research Design in Mathematics and Science Education*. Hillsdale, NJ: Lawrence Erlbaum Associates.
- Lesh, R., & Sriraman, B. (2010). Reconceptualizing mathematics education as a design science. In B.Sriraman L. English (Eds). *Theories of Mathematics Education: Seeking New Frontiers*. Monograph 1 of Advances in Mathematics Education, Springer Science, Berlin/Heidelberg, pp. 123-146.
- Lesh, R., Sriraman, V., & English, L. (2013). Theories of learning mathematics. In *Encyclopedia of Mathematics Education*, Springer Reference Works. Retrieved from <http://www.springerreference.com/docs/html/chapterdbid/313336.html>.
- The Rational Number Project. (2011). A Five Volume Series containing more than 100 publications from the Rational Number Project – spanning the years from 1979-2010. University of Minnesota Press.
- Thompson, P. W. (1996). Imagery and the development of mathematical reasoning. In L. P. Steffe, P. Nesher, P. Cobb, G. A. Goldin, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 267-285). Mahwah, NJ: Erlbaum.
- Zieffler, A., Garfield, J., delMas, R., and Reading, C. (2008). A framework to support research on informal inferential reasoning. *Statistics Education Research Journal*, 7(2), 40-58.