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Mathematical Problem Solving in Training Elementary Teachers from a Semiotic Logical Approach

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Abstract: The aim of this article is to consider the professional knowledge and competences of mathematics teachers in compulsory education, and to propose basic tasks and activities in an initial training programme in the framework of a global proposal for “Immersion” in the curriculum of the educational phase which the trainee teacher would go on to work in. Problem-solving, in this context, is considered as being an inherent part of mathematics and this is described in terms of problem-solving, establishing connections between concepts, operations and implicit processes in the mathematical activity (conceptual field) and their relationships problem-solving; and it is assumed that the learning of problem-solving is an integrated part of learning in mathematics.

Keywords: Problem Solving, Teacher Training, Didactical Analysis, Semiotic Logical Approach (SLA).

Introduction

The analysis of the results obtained, in recent years, in different national (in Spain) and international assessments shows that the knowledge of mathematics (Problem Solving) of students in compulsory education (K-9 Grades) is insufficient in terms of the desired curriculum. What needs to be done to improve the learning and teaching of mathematics and, in particular problem solving in this educational stage? This question is addressed here by reflecting on the role played by teachers in primary and secondary education in the pursuit of an effective learning of mathematics and problem solving. At present, the initial training of teachers in primary and secondary education takes place

within the European Higher Education Area, where primary school teachers need four years training and secondary school teachers are required to have completed a mandatory Professional Masters degree. This initial teacher training has a great opportunity for improvement.

Problem solving in mathematics education

Problem solving has always been regarded as a basic component in the construction of mathematical culture. However, when mathematical knowledge is presented in its final state, what prevails is the conceptual organization of the objects of such knowledge in which problem solving appears again as a core of relevant mathematical knowledge. In the early eighties, in view of the primacy of the concepts and their properties as well as their algorithmic use, problem solving was vindicated as a key activity in the learning of mathematics, which has led to the development of an emerging theoretical and practical body of research in mathematical education, and a notable increase of its presence in the curriculum, either as a further block of contents or as cross content but specific to mathematics at the corresponding level (Santos-Trigo, 2007, Castro, 2008). The follow-up research on problem solving clearly shows that, despite all amount of effort, there are no significant data on the improvement in this on the part of the students and different questions arise ranging from the need to establish relationships and existing connections between the development of the understanding of mathematical contents and problem solving skills, to the need of having theoretical bases to guide problem solving (Lester and Kehle, 2003).

Some authors such as Lesh and Zawojewski (2007), suggest that the rise of research in problem solving was very important between 1980 and 1990, and that some trends are presently aimed at putting an emphasis on critical thinking, technology and mathematical problem solving, and analysis of how mathematics is used in other sciences and professions that does not match the way mathematics is taught in school, or the development of problem solving in other settings or contexts such as situated cognition, communities of practice or representational fluency. These directions and perspectives in solving mathematical problems are, at the present, promising lines of research.

The knowledge and professional skills of a mathematics teacher

The concern, from the point of view of mathematics education, regarding teacher's knowledge and professional skills has been and is, a constant research topic, and is based on the following conjecture: The knowledge and professional skills of the mathematics teacher must be acquired through different scientific domains: mathematics, mathematical didactics and educational sciences. The initial teacher training should enable the trainee teachers to increase their knowledge about mathematics and mathematical didactics as a specific field of professional competence (mathematics education) and a field of research, along with other issues arising from educational sciences.

Shulman pointed out in 1986, for the first time, the importance of the specific subject to teach in teacher training. This author identified three categories of professional knowledge of teachers: *Knowledge of the specific subject*, *pedagogical content knowledge* (PCK) or in the context here: the *didactical content knowledge* (DCK) and *curricular knowledge*. Subsequently, Bromme (1988, 1994) described the qualitative

characteristics of the major areas of professional knowledge: Knowledge of mathematics as a discipline, knowledge of mathematics as school subject, philosophy of mathematics schools, pedagogical knowledge and specific pedagogical knowledge of mathematics. The author proposed that the teachers' professional knowledge is not simply a conglomeration of these domains of knowledge, "but an integration of the same", which occurs during teaching practice or during professional teaching experience.

Semiotic Logical Approach (SLA). Assumptions

Semiotic Logical Approach (SLA) (Socas 2001a and b, 2007), when understood as a theoretical-practical proposal (formal-experimental), aims to provide tools for the analysis, description and management of problematic situations or phenomena of a mathematical didactical nature from a perspective based on semiotics, logic and competence models (semiosis), and takes one of the great problems of mathematics education, the study of difficulties and errors of students in learning mathematics as a reference (Freudenthal, 1981). Logical and Semiotic Aspects of SLA uses Peirce's Phenomenology (1987) as a reference. Peirce, starting from the logic conceived of as a science of language, describes the development of a science of signs and meanings called semiotics which can be used to analyse, within the semiotic constructs, different phenomena of logic, mathematics, physics and even psychology, which is why this phenomenology is used here. Semiotics is a theory of reality and knowledge that one can have of phenomena through signs which are the only means available. Semiotic inference emerges in sign analysis where what is analysed are the trademarks or observable and overt expressions of inference, which Pierce organized as a logical theory (semiotics) that has three references closely linked to one another. Therefore, if the aim is to study any

phenomenon (problem situation), which is the starting point in SLA, this will always be analysed from a given context and by means of three references organized as first, second and third, which is defined as primary or basic semiotic function which is determined by the sign, object and meaning references (Socas, 2001b). This can be used to determine the notion of representation as the semiosis determining such references. Therefore, the representation is a sign that:

- (1) has certain characteristics that are proper (context)
- (2) sets a dyadic relationship with the meaning
- (3) establishes a triadic relationship with the meaning via the object, this triadic relationship being such that it determines the sign of a dyadic relationship with the object and the object to a dyadic relationship with the meaning (Hernández, Noda, Palarea and Socas, 2004).

As far as the Educational System is concerned, SLA uses the Begle's diagram of school mathematics as a reference, which shows the mutual relationships between the different components in the training process and defends the need to set multiple perspectives and procedures in the field of the teaching / learning of school mathematics (cited in Romberg, 1992). To do this, two different parts must be distinguished: the "educational macro system", where both disciplinary knowledge and the institutions or persons involved intervene in the education system, and, the "educational micro system", which is made up of three references or basic elements: mathematical knowledge (mathematics), students and teachers, and their relationships in a context determined by the following components: social, cultural and institutional, which is shown in the figure below:

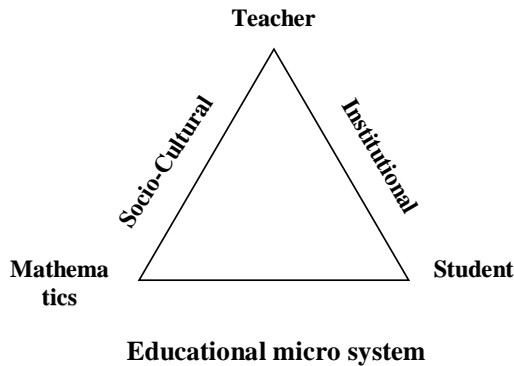


Figure 1: Elements of educational micro system

The three essential relationships are:

Relationship1: Between the mathematical knowledge and the student, which is called "school mathematics learning as a conceptual change".

Relationship 2: Between the mathematical knowledge and the teacher, called: "adapting the curricular mathematics content to be taught".

Relationship 3: Between the mathematical knowledge and the teacher via the student which is called: "interactions".

Thus, the three elements and the three essential relationships contextualized in the three components of the context determine the teaching / learning process in the regulated systems, thereby characterizing the six core contents that are a part of the mathematics teacher's professional knowledge, in addition to those derived from the three previous mentioned relationships: mathematical knowledge in a disciplinary sense, the curricular mathematics knowledge and the mathematics curriculum of an educational stage. It is in this framework that the difficulties, obstacles and errors that students have or make in the construction of mathematical knowledge are examined. SLA organizes three models of competence: Formal Mathematical Competence (FMC), Cognitive Competence (CC) and Teaching Competence (TC), which constitute the references that define the General

Semiosis which plans and manages research in the educational micro system (Socas, 2010a, 2001b, 2007 and 2010).

This General Semiosis can be used to plan and manage both the problems of teaching and learning in the educational micro system, and the didactical mathematical problems to be studied.

The Formal Mathematics Competence Model (FMC) can be used to describe the conceptual field of the mathematical object in the thematic level in which both their functions and phenomenology are being considered.

The Cognitive Competence Model (CC) is the second reference and takes into account the above mentioned Formal Mathematic Competence Model, it refers to the specific cognitive functions of students when they use the mathematical objects in question and structural aspects of learning.

The Teaching Competence Model (TC) is the third reference, and it also considers the above mentioned aspects (formal mathematical competence and cognitive competence) and describes the actions of the subjects involved, the communication processes, the mediators, the situations, the contexts, which occur in education.

Three basic assumptions of SLA are now proposed here: Mathematical Content Analysis, Didactical Analysis and the Curricular Organization.

The didactical analysis and the curricular organization are the concepts that SLA uses to characterize the knowledge of mathematical content from the professional point of view. The didactical analysis allows the comprehension of the professional problem, while the organization curricular plans his development.

Mathematical Content Analysis

The Formal Mathematics Competence Model (FMC) sets out the conception of mathematical literacy and the different relationships between the elements characterizing it. The FMC is organized by means of the semiosis that characterizes and relates the conceptual, phenomenological and functional aspects of mathematical content involved in the problematic situation to be addressed in the educational micro system, and would appear as below:

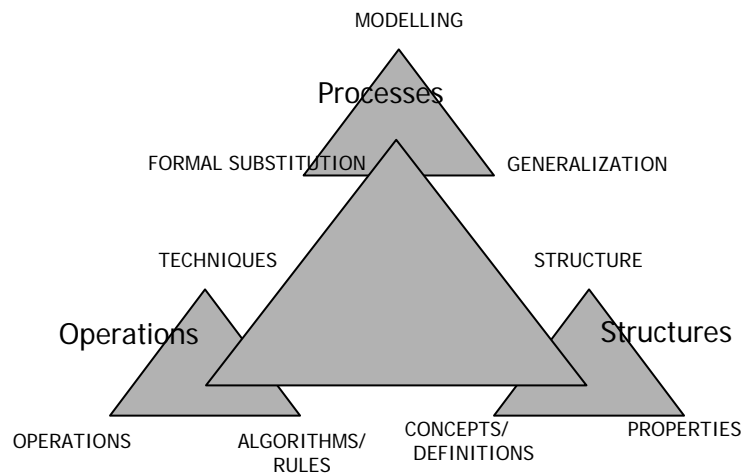


Figure 2: Domains of mathematical activity

The different domains of mathematical activity are expressed within this model in relation to the conceptual field from a formal perspective and its different relationships, i.e., it describes the duality of mathematical objects in relation to conceptual/procedural mathematical knowledge of the field in question. Any activity is described in relation to the three components: operations, structures and processes, and relationships, which we explain later. Each component, in turn, is determined by three others that describe a new semiosis: 1) The operations component for the semiosis: operations, algorithms (rules) and techniques, 2) the structures component for: concepts (definitions), properties and

structure, and 3) the processes component for: formal substitutions, generalization and modelling. This organization of the conceptual fields is contextualized in the problematic situations that are addressed in the language (representations) and in the arguments (reasoning) that are used in developing it.

The three context components are similarly determined by the respective semiosis. In the case of problematic situations: identification, approach and resolution; in representations (language): recognition, transformation (conversion) and elaboration (production), and in arguments: description, justification and reasoning. This organization of mathematics by the FMC can be used to consider problem solving as an inherent part of mathematics and to describe it in terms of problem solving. Hence, the following aspects characterize mathematical culture in SLA:

1. Mathematics is a multifaceted discipline
2. Mathematical culture emerges and develops as a human activity of problem solving
3. The problems have one common feature: the search for regularities (identification, approach and resolution). Modelling is the mathematical process par excellence
4. Mathematical culture creates a system of signs able to express regular behaviour
5. The set of regularities is organized into conceptual fields
6. The conceptual elements of these fields are mathematical objects

The Formal Mathematical Competence Model can also be used to establish the connections between concepts, operations and processes involved in mathematical

activity and their relationship to problem solving, which is, generally speaking, relevant for mathematics education, and particularly for problem solving.

Didactical Analysis in the Semiotic Logical Approach (SLA)

Semiosis can be used to identify and understand the didactical mathematical problem, whose reference framework is comprised of the curriculum organizers (Rico, 1997) and the initial notion that Freudenthal (1983) put forward for didactic analysis as follows: "the analysis of the curricular content of mathematics is performed to serve the organization of its teaching in educational systems".

Didactic analysis is organized according to the following triad: formally described curricular mathematics, semiotic representations and difficulties, obstacles and errors, it also facilitates the identification and understanding of the didactic problem to be addressed.

Didactic analysis implies, in relation to the curricular component, a review of the curricular contents from the formal perspective: operational, structural and processual (using processes), but also implies a necessary relationship with the students linked to their interests and motivation.

The semiotic representations component involves a review of the curricular content in relation to different forms of representation of the objects in question, as well as the presentation of information to students. The following states of the historical development of the mathematical object are considered in this section: semiotic, structural and autonomous, that also implies a necessary relationship with the students linked to the coordination between the forms of expression and representation and the interests and motivation of the students. The component difficulties, obstacles and errors,

require a review of the curricular content in relation to these three aspects, with a dual aim of prevention and remedy, making it possible, for example, from the perspective of prevention, to set the levels or cognitive skills required of students in relation to the mathematical object in question. The identification of the errors generated by the students needs analytical tools which can get into the complexity of learning difficulties in mathematics. One way to address this would be, as reported by Socas (1997), to take the three directions of analysis into consideration, like three coordinated axes which would more accurately identify the origins of the error and would enable the teachers to devise more effective procedures and remedies. These three axes would be determined by their origin: i) in an obstacle, ii) in the absence of meaning; iii) in affective and emotional attitudes.

Curricular Organization

Not only do mathematics teachers need knowledge about the discipline of mathematics and the curriculum, but they also require didactical mathematical knowledge (DMK) in order to organize the mathematical content for teaching.

This is professional knowledge that includes the appropriate elements of analysis to understand, plan and do a professional job. The teacher needs to expand and connect different perspectives on the curricular mathematics content, in such a way that its consideration is not only from the internal logic of the discipline, which may emerge as being too restrictive, formal and technical, but from the curricular dimension, a more open perspective and one which integrates the teaching of mathematical knowledge more, and this is not possible to put into practice from only the theoretical consideration of knowledge about the discipline of mathematics and the curriculum, to convert this into

the mathematical knowledge to be taught. This professional knowledge develops in the subject Didactic of the Mathematics for teachers, structured according to the didactical analysis and the curricular organization.

The curricular organization emerges from the organizers of the curriculum (Rico, 1997), and must be understood as those teaching skills that can be used to plan mathematical content for teaching, i.e., planning and evaluating mathematics classroom schedules, which is determined by the following triad: context, teaching/learning and assessment.

In so far as the context reference is determined by the semiosis described by the problem situation, which refers to the environments in which the activities take place, the contextualization, which is determined by the specific goals, specific skills and teaching content involved in the activity, and the levels, referring to the complexity of mathematical tasks: reproduction, connection and reflection, skills demanded by the same, taken from the PISA Project (Rico and Lupiáñez, 2008), or to stages of development: semiotics, structural and autonomous, taken from SLA (Socas, 1997).

Proposal for training mathematics teachers

The different areas of knowledge (mathematical and didactical mathematical) that can be used to support the training proposal have been described here in general terms. But before going on, it is necessary to make a few comments about the trainee teachers who this training proposal is aimed at. Several studies conducted at the University of La Laguna (Spain), in which students from several other Spanish universities have also participated, show that the students who start teacher training courses in primary

education teaching have huge gaps in basic mathematical knowledge. As regards problem solving, the situation is that 18 year old students, with more than 12 years of studying mathematics in the educational system and learning to solve problems, still tend to concentrate on the data of the problem as their general cognitive strategy, without demonstrating a clear understanding of the problem and without identifying operational, structural (conceptual) or processual relationships, given in the data, often providing solutions that cannot be valid for the conditions of the problem, which furthermore clearly shows a lack of cognitive strategies (heuristic methods) and a lack of critical thinking (Palarea, Hernández and Socas, 2001; Hernández, Noda, Palarea and Socas, 2002 and 2003). Subsequent studies show no improvement on the previous results, finding that students show a predominance of operational rather than structural and processual thinking, and it is this thinking that is mostly behind the solution to any mathematical task, which many times is unsuccessful, even when the applied operational knowledge is correct. This suggests that the emphasis that the teaching of mathematics puts on operational knowledge may be creating difficulties and obstacles for the student to apply, for example, heuristics and strategies to solving problems that are more associated with structural and even processual thinking, which creates difficulties in achieving mathematical competence (Socas et al., 2009).

As regards trainee teachers of mathematics in secondary education, the starting assumptions were that the design of the plan should take two essential aspects into account, on the one hand the mathematical training of future teachers (graduates in mathematics) and, on the other hand, the lack of a specific didactic training for professional work (teacher), except for that formed by existing knowledge, implicit

theories, values and beliefs that had come from their experiences as students of mathematics during their schooling, and which are, in many cases, an obstacle to properly channelling many aspects of professional thinking. The analysis of educational reforms leads one to believe that such reforms require the teacher to be able to take on the new curriculum changes which actually means confronting new tasks. The latter necessarily implies significant changes in training mathematics teachers which can be summarized in the following points:

- Scientific and educational training tailored to this new curricular change.
- Training to work with students who have a high degree of heterogeneity in basic skills, interests and needs.
- A change in attitudes among teachers so that they can develop the educational aspects of teaching, adopt flexible approaches and delve into a more interdisciplinary vision of culture.
- A conception of the curriculum as a research tool that can be used to develop concrete methods and strategies of consolidation and adaptation.
- Assessment and exercising of teamwork as well as the development of a strong professional autonomy (Camacho, Hernández and Socas, 1998).

Fundamentals of the Proposal

The analysis of the knowledge and skills that a maths teacher must have in compulsory education, shows that two essential questions need to be answered: What are the basic tasks and activities in an initial training plan for maths teachers in compulsory

education? And whether the theory and practice dichotomy is enough to provide a response to the tasks and basic activities of teacher training?

Llinares (2004, 2009 and 2011) proposes the articulation of three systems of activities or tasks to develop the knowledge and skills of a mathematics teacher: "Organise the mathematical content to teach it", "Analyze and interpret the production of the students" and "Manage the mathematical content in the classroom". A reflection and analysis of the two questions leads one to consider that the three aforementioned activities systems are at least necessary. These are the activities that also emerge as necessary and essential in all three relationships in the Semiotic Logical Approach (Socas, 2001a and 2007). As for the second question, one can see the need to make progress in the dichotomy between theory and practice with knowledge to develop the professional skills to design and manage teaching practice in mathematics. The general aspects of the basic proposal take the following as a reference: the analysis of mathematical content, the didactic analysis of curricular content and organization. This is a comprehensive proposal for the training of mathematics teachers, which aims to facilitate a reconciliation between disciplinary mathematical knowledge (DMK) to curricular mathematical knowledge (CMK), to pedagogical mathematical knowledge (PMK) and knowledge of educational practice (KEP). This can be achieved by means of a proposal that ranges from the general comprehensiveness of the curriculum and of the disciplinary mathematical knowledge, to the organized totality of curricular content as content to be taught. The situation is depicted in the graph below, which expresses the cyclical nature of the proposal.

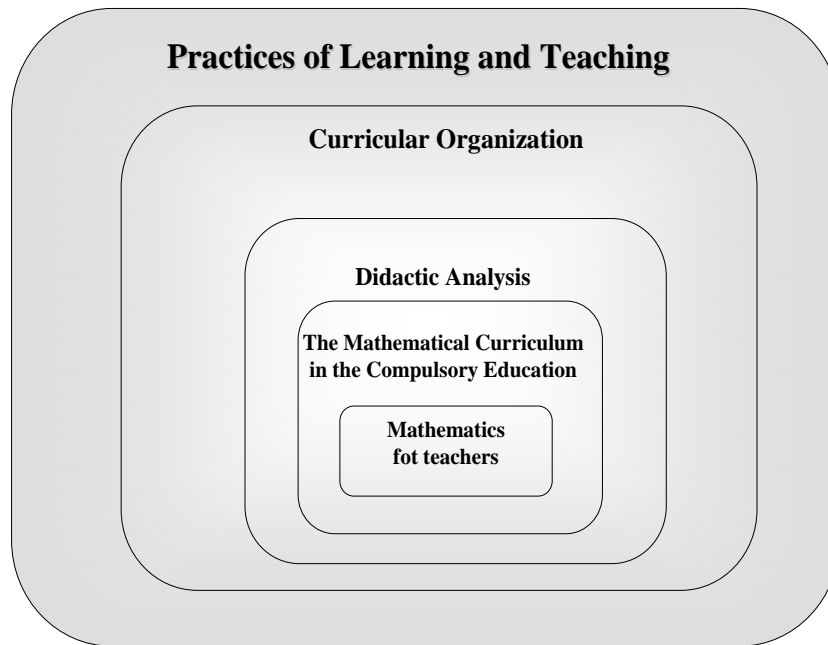


Figure 3: Proposal for the training of mathematics teachers

The analysis of the mathematical content plays a role, in this proposal, in the re-teachers' conceptualization of mathematics, and together with the didactic analysis of curricular content and organization, in the development of the school subjects of the didactics of mathematics and teaching practice of mathematics, where the three previously mentioned professional activities have a place.

Professional activity will be considered first, "organizing the mathematical content to teach it". This deal with solving a professional problem that requires analysis, understanding and planning, and can be represented by the following semiosis: curricular mathematical content, disciplinary mathematical content, and mathematical content for teaching.

First, the teacher needs to organize the curricular mathematical content (CMC), the desired mathematical content that is definable in the domain of the disciplinary mathematical content, although it is not organized under that logic. This CMC is

extracted via precise and precise mechanisms and organizations from the disciplinary content and is inserted in the curriculum. Once these actions have been performed by different elements of the educational system, curricular mathematical content knowledge is intrinsically different to the disciplinary knowledge, at least in its epistemological aspect, and supports interpretations from different perspectives, for example functional, as part of a common basic culture (Rico and Lupiáñez, 2008), the second derives from the discipline itself, scholarly mathematical knowledge, which we call disciplinary mathematical content (DMK) or formal mathematical knowledge (Socas, 2010a) and the third is the mathematical content for teaching (MCT), which includes both the taught and the mathematical content assessed (Hernández et al., 2010). The three components are interrelated in a process called transposition or adaptation of mathematical content, but have their own independent organization. The organization of curricular mathematical content comes from a pedagogical order implicit in the curriculum designers, and is associated with basic mathematical competence as part of a common culture. The organization of the mathematical content for teaching is compiled using the didactic order as a starting point, and is associated with the subjects' competence in didactical mathematical knowledge (DMK) and determines the sequence and level of the mathematical content in the teaching proposal with regard to basic mathematical skills and the other basic skills.

The professional task of organizing the mathematical content for teaching involves competence in the three areas of mathematical content. The question is now what happens to our students and how does one involve them in professional tasks that enable them to be competent professionals who can identify, analyze, understand and

plan for these three areas of mathematical content? As has been shown, students who begin teacher training for primary school have huge gaps in basic math skills which is why they need a revision of the discipline in terms of some "mathematics" to train them professionally, to improve not only their knowledge but their beliefs about the ends of this knowledge in compulsory education (Socas et al., 2009).

Mathematics for teachers in compulsory education

Teacher training programs have generally been designed to include in subjects, like mathematics, mathematical content as disciplinary knowledge, which is developed by explicating the different conceptual fields, and by considering mathematics as a fundamentally instructive tool that is organized primarily from the point of view of its internal logic, which means characterizing mathematical knowledge by using an organization based on its key concepts and on an introduction using a logical sequencing, i.e. the material is organized in the way a mathematicians would. On the other hand, the mathematical content of the curriculum that the teacher must impart has been determined by various agents of the educational macro system via a process that is generally unknown to the future teacher. The curriculum is organized by a list of contents that are related to the skills and competencies to be developed, the same happens with the evaluation process, and is immersed in a particular conception of understanding teaching and learning. Therefore, the curricular organization of the mathematical content, the object of education in a stage of education, needs to be seen as a systematic organization, which considers mathematical content as a fundamentally cultural and basic element, which is organized from an epistemological and phenomenological perspective capable of developing basic mathematical skills, and is introduced by means of an educational

organization as well as criteria for assessing the acquired knowledge and skills. The subject: mathematics for teachers in compulsory education would deal with revising different aspects of curricular mathematical content relevant to the stage of education in which the teachers have to exercise their profession from the disciplinary mathematical perspective, facilitating the teachers with a re-conceptualization of curricular mathematical content. This is a process of immersing the trainee teacher in curricular mathematical content which they will have to organize for teaching afterwards. This is, ultimately, a proposal for basic training in a closed curricular structure, which is approached from formal mathematical competence and basic mathematical competence, i.e. the analysis and understanding of curricular mathematical content in disciplinary terms with epistemological, phenomenological and applicability references, in which students complete their basic training related to such issues at the level of conceptual systems involved: operations, structures and processes in problem-solving situations, using the reasoning and the appropriate language for the thematic level in question by means of tasks and activities of differing natures but necessary for linking the school tasks and activities.

The didactic of mathematics for teachers in compulsory education

The next item to be considered is the second group of activities and tasks to be developed by the trainee teacher: "Analyzing and interpreting students' production" which refers to the knowledge and ability to mobilize different resources: analogical and digital mathematical representations, difficulties, obstacles and errors associated with the object of teaching mathematical content. Take, for example, the role of the difficulties, obstacles and errors of students in this analysis and interpretation. It is known that

learning mathematics creates many difficulties for the students and that these differ in nature. Some difficulties originate in the educational macro system, but generally speaking, they originate in the educational micro system: student, subject, teacher and educational institution. These difficulties are connected and reinforced in complex networks that, in practice, materialize in the form of obstacles and are manifested by the students in the form of errors. The error will have different roots, but will always be considered as the existence of an inadequate cognitive schema in the student, and not only because of a specific lack of knowledge or an oversight. The difficulties may be grouped into five major categories associated to 1) the complexity of the objects of mathematics, 2) mathematical thinking processes, 3) the teaching processes developed for the learning of mathematics, 4) cognitive development processes of students and 5) affective and emotional attitudes toward mathematics (Socas, 1997). In addition to the curricular and disciplinary mathematical knowledge, the trainee teacher of mathematics requires didactical mathematical knowledge (DMK) to be able to organize the mathematical content for teaching. This is specific professional knowledge that has to be provided by the subjects belonging to the didactics of mathematics, which includes the elements of analysis for adequately understanding, planning and conducting professional work. This knowledge is developed under the two constructs discussed above, didactic analysis and curricular organization.

Best Practices

The proposed teacher training should focus on the organization and development of best practices for the attainment of the skills required, these have to be developed within the framework of problem solving of a professional nature and associated with the

knowledge and resources that the teacher must mobilize to obtain the solution to the problem.

Thus, mathematical problems emerge from the situations developed in the curriculum and are addressed from the FMC in the subject of mathematics for the teachers in terms of the language and reasoning involved in the conceptual field in question. This immersion of the student continues because the problem solving must be organized for teaching, usually in the context of a classroom program, which must be considered from the training analysis. It involves incorporating the consideration of the difficulties, obstacles and errors of students in the different domains of mathematical activity. The trainee teachers of mathematics perform different activities and tasks of application, related to the various mathematical fields, and conclude in all situations with the elaboration of a map of the mathematical knowledge being dealt with, organized in terms of the six disciplinary mathematical content areas according to the FMC model, i.e., operations, structures, processes, representations, problems and reasoning. Certain tasks developed by the trainee teachers of mathematics in the course in a report format, all of which are from a questionnaire, are presented below:

Task 1: Report on numeration systems and decimal system

(For example, the first questionnaire has questions about the relations between the different numerical systems, the description of the numerical systems from the decimal representation and the representation in the number line of the different numbers).

- Analysis of the errors made and of the blank responses, as well as determination of their cause or origin.

- To characterize the D numeration system (Decimals) as is clear from the answers to the questions.
- Analysis of the representational procedures on the number line of the numbers proposed in the questionnaire.
- Decimal numbers in the curriculum of compulsory primary or secondary education.
- To elaborate a map of the numbers in the primary or secondary education.
- To elaborate a map of the procedures for representing numbers in primary or secondary education.

Task 2: Report on operational, structural and processual knowledge in mathematics

- Analysis and evaluation of the mathematical discipline according to SLA.
- Analysis of unanswered questions and the mistakes made in the questionnaire, determining the source of errors.
- Analysis of operational, structural and processual knowledge used in the questionnaire responses, both correct and incorrect.
- Self-evaluation of the type of knowledge used in the answers.
- Analysis of the mathematics curriculum in primary or secondary education. Choosing a course and a content block about numbers, algebra and functions, and analysing them from an operational, structural and processual perspective, identifying the systems they use for representing mathematical objects, the problems they give rise to and the reasoning they propose, with special emphasis on identifying the heuristic content.

- Analysis of a mathematics textbook in compulsory secondary education. Choosing two consecutive themes on numbers, algebra and functions, and then analyzing them from the aforementioned perspective.

Task 3: Report on mathematical problem solving

- Solving the problems correctly in various sessions.
- Analysis of the difficulties and errors made in the different sessions in solving the problems of the questionnaires.
 - a) To identify the following phases in each problem: acceptance, blockage and exploration
 - b) To determine the source of the difficulties and errors.
- To identify the different reasoning (and different heuristics) used in the given questionnaire responses.
- To analyze the map of the contents involved in solving the mathematical problems proposed in the questionnaire, paying special attention to the mathematical tools and reasoning (heuristics) used.
- To develop a new map of knowledge involved in the correct resolution of the proposed problems.

Final considerations

A proposal is suggested here, for training student teachers in primary and secondary mathematics to improve the learning and teaching of mathematics in these education stages because as Sowder said (2007), many of the difficulties that mathematics students have are to do with the teaching they receive, but what does preparing a trainee maths teacher competently really involve? This proposal opts to

develop three systems of basic activity that can determine the knowledge and skills of the teacher, presented as professional tasks from a global perspective in the context of problem solving in the case of their profession.

The three systems of professional activities categorize teachers according to different skills, for example, in the case of the activity: organizing the mathematical content to teach it, puts students in these skills areas: knowledge of the contents of mathematics from a global perspective in which the resolution of problems is an inherent part of the mathematical culture that should be taught and the ability to translate this into learning expectations, and the design and planning of learning sequences. In the case of the activity of analyzing and interpreting the students' mathematical production places students in the skills area regarding understanding and working based on the students' representations including their idiosyncrasies, and knowing and working with the difficulties, obstacles and errors of the students.

As regards the activity of knowing how to manage mathematical content in the classroom, this places students in the skills area of designing and controlling problematic situations appropriate to the different levels and possibilities of the students, and observing and assessing students in learning situations. The case of training maths teachers leads one to consider the basic situations of meaningful and effective work and how these should be dealt with by a professional comprehensive approach. The comprehensive approach is set in the context of trainee teachers, and it articulates and connects different subjects in a global proposal which seeks to ensure a comprehensive and inclusive vision of mathematics and of teaching and learning mathematics,

encouraging the active participation of students, which shows how we get closer to understanding reality through mathematical culture and how it is perceived by them.

Research has shown that when mathematics programs, from the disciplinary approach, are used with trainee teachers as a finished product, they are insufficient. Providing trainee teachers with an epistemological and phenomenological analysis of mathematical objects of teaching involves not only knowing the conceptual systems involved, their languages and problems, but also the usefulness of mathematical objects and their use, which could be successfully used to deal with the interpretation of the aims of the mathematics curriculum in this educational stage and confidently take on the didactical mathematical knowledge. The organization of mathematical knowledge using the phenomenology / epistemology pairing involves paying special attention to the use, management and function that this knowledge can have at a given time, without losing sight of its internal logic. Finally, it is important to emphasize that this global proposal for training mathematics teachers by "immersion" in the curriculum of the educational stage where they will work in the future, will allow them to develop, in this environment, the knowledge and skills needed in their professional work.

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