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Cognition and Affect in Mathematics Problem Solving with Prospective Teachers[1]

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Abstract: Recent studies relating the affective domain with the teaching and learning of mathematics, and more specifically with mathematics problem solving, have focused on teacher education. The authors of these studies have been ever more insistently pointing to the need to design educational programs that take an integrated cognitive and affective approach to mathematics education. Given this context, we have designed and implemented a program of intervention on mathematics problem solving for prospective primary teachers. We here describe some results of that program.

Keywords: Mathematics Teaching, Problem Solving, and the Affective Domain

Problem solving (PS) has always been regarded as a focal point of mathematics, and in the last 30 years its presence in curricula has increased notably (Castro, 2008; Santos, 2007). It is regarded as the methodological backbone to approach mathematics content since it both requires and helps develop skills in analysis, comprehension, reasoning, and

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application. At the same time, it is now being proposed as an item of curricular content in its own right as a core competence that students need to acquire. Castro (2008) and Santos (2008) recognize that attempts to teach students general PS strategies have been unsuccessful. Also, it seems important to emphasize the lack of attention in textbooks to learning heuristic problem solving strategies (Schoenfeld, 2007; Pino & Blanco, 2008).

The results of the Programme for International students Assessment (PISA) of 2003, 2006, and 2009 have highlighted the importance of mathematics problem solving (MPS) in compulsory education. One of the aspects tacitly accepted in the curricula at this educational level is the influence of affect on the teaching and learning of mathematics in general, and of MPS in particular. Already in the 1980s, Charles & Lester (1982) were observing that: "*The problem solver must have sufficient motivation and lack of stress and/or anxiety to allow progress towards a solution*" (p.10). In their work, they recognized that factors involving cognition, experience, and affect influence the MPS process. Among the affective factors that they explicitly noted were interest, motivation, pressure, anxiety, stress, perseverance, and resistance to premature closure. It is currently accepted that the cognitive processes involved in PS are susceptible to the influence of the affective domain in its three fundamental areas: beliefs, attitudes, and emotions (Sriraman, 2003).

Initial Primary Teacher Education, the Affective Domain (Beliefs, Attitudes, and Emotions), and Problem Solving

Research on the affective domain has also expanded to the field of initial teacher education and the professional development of in-service teachers. It is considered that, in their actions in the classroom, teachers cannot dissociate affect from content when faced with a specific activity for pupils at a specific level.

Influence of beliefs

> "*When prospective primary teachers enter an Initial Education Centre they bring with them the educational baggage of many years in school. They thus naturally have conceptions and beliefs concerning Mathematics and the teaching/learning of Mathematics that derive from their own learning experience*" (Blanco, 2004, p.40). Furthermore: "*Few apparent changes in their beliefs were affected as a result of traditional mathematics method courses*" (Chapman, 2000, p.188).

It is important to distinguish the beliefs of prospective primary teachers (PPTs) about mathematics as an object – about its teaching and learning, beliefs which depend on affect – and their beliefs about themselves as learners – beliefs related to their self-concept, self-confidence, expectations of control, etc.

Beliefs about mathematics and problem solving

According to Llinares and Sánchez (1996), prospective teachers acquire a technical school culture that conditions their approach to mathematics tasks and learning as future teachers. For them mathematics teaching is the transmission of specific information and mathematics learning is done through repetition. The teacher's role consists of presenting the content in a way that is clear and concise, and the learner's role consists of listening and repeating.

According to Szydlik, Szydlik & Benson (2003), research has shown that prospective teachers tend to "*see mathematics as an authoritarian discipline, and that they believe that doing mathematics means applying memorized formulas and procedures to do textbook exercises*" (Szydlik, Szydlik & Benson, 2003, p.254).

Prospective primary teachers have a very traditional idea of mathematics problems that are quite different from the suggestions of current curricular proposals (Blanco, 2004; Johnson, 2008). This leads to "*a contradiction between their personal experience, which they judge as having been negative and monotonous, and their conception of mathematics as linked to reasoning and rigour*" (Blanco, 2004, p.42). Furthermore, these beliefs constitute a kind of lens or filter through which they interpret their own personal learning processes and orient their teaching experiences and behaviours (Chapman, 2000), thus limiting their possibilities for action and understanding (Barrantes & Blanco, 2006). Moreover, "*these beliefs are [internally] consistent*" (Blanco, 2004, p.41).

For Schoenfeld (1992), beliefs form a particular view of the world of mathematics, setting the perspective from which each person approaches that world, and they can determine how a problem will be tackled, the procedures that will be used or avoided, and the time and intensity of the effort that will be put into the task. Consequently, these beliefs need to be taken into account in teacher education, which, if necessary, will have to try to promote their change and the generation of new beliefs (Blanco, 2004).

Prospective primary teachers regard MPS as a rote mechanical process, have few resources with which to represent and analyse problems, never look for alternative strategies or methods for their solution, and make no use of the different guidelines and hints they might be given to help them towards a solution (Blanco, 2004; Córcoles & Valls, 2006; NCTM, 2003), thereby generating a vision of themselves as incompetent problem solvers (NCTM, 2003).

The beliefs that most influence motivation and achievement in mathematics are *students' perceptions about themselves in relation to mathematics* (Kloosterman, 2002; Skaalvik & Skaalvil, 2011). Hernández, Palarea & Socas (2001) and Blanco et al. (2010) note that PPTs generally do not see themselves as capable or skilled as problem solvers, with most of them experiencing feelings of uncertainty, despair, and anxiety which block their approach to the task – in sum, most of them consider themselves to be incompetent at PS. A major difference between successful and unsuccessful problem solvers lies in their beliefs about MPS, about themselves as solvers, and about how to approach the task (NCTM, 2003).

Influence of attitudes

What students think about mathematics influences the feelings that surface towards the subject and their predisposition to act in consequence. That is, if students have negative beliefs about mathematics or its teaching, they will tend to show adverse feelings towards related tasks, in particular presenting avoidance behaviour or simple rejection of those tasks. This predisposition towards certain personal intentions and behaviours is what one calls attitudes.

One can distinguish between mathematics attitudes themselves and attitudes towards mathematics as a subject. Mathematics attitudes have a marked cognitive component, and relate to general cognitive skills that are important in mathematics tasks.

Studies in Spain have shown that PPTs have few mathematics attitudes in this sense on aspects related to PS (Blanco, 2004; Corcoles & Valls, 2006).

In attitudes towards mathematics, the affective component predominates. It is manifest in interest, satisfaction, and curiosity, or, on the contrary, in rejection, denial, frustration, and avoidance of mathematics tasks. Positive interest and attitudes towards mathematics seem to decline with age, especially during secondary education (Hidalgo, Maroto, Ortega & Palacios, 2008).

Influence of emotions

The emotions aroused in students by a mathematics task are affective responses characterized by high intensity and physiological arousal reflecting the charge of positive or negative meaning that a task has for them Studies of emotion have focused on the role of anxiety and frustration and their impact on achievement in mathematics, noting that one of the difficulties of mathematics education is seeing its teaching as essentially cognitive, and detached from the field of emotions (De Bellis & Goldin, 2006).

Emotions appear in response to an internal or external event which has a charge of positive or negative meaning for the person. Thus, in facing a mathematical task a pupil may encounter difficulties which lead to the frustration of their personal expectations, causing the appearance of essentially negative valuations of the subject. Various authors agree that anxiety interacts negatively with cognitive and motivational processes, and therefore with the pupil's overall performance (De Bellis & Golding, 2006; Zakaria & Nordim, 2008). In this regard, Hidalgo, Maroto, Ortega & Palacios

(2008) found a strong negative correlation between pupils' levels of anxiety towards mathematics and their final marks at the end of the course. This correlation is also present when comparing the levels of anxiety and positive attitudes towards mathematics. The relationship between anxiety and mathematics education has also been transferred to the case of prospective teachers, for which there is already a substantial literature (Peker, 2009).

Recent work has established relationships between anxiety and self-confidence. Thus, pupils with more anxiety towards mathematics have less confidence in their mathematical abilities and as learners of mathematics (Gil, Blanco & Guerrero, 2006; Isiksal, Curran, Koc & Askun 2009). "*Many of the negative emotional attitudes towards mathematics are associated with anxiety and fear. Anxiety to be able to complete a task, fear of failure, of making mistakes, etc., generate emotional blockages of affective origin that have a repercussion on the students' mathematics activity*" (Socas, 1997, p.135). Zevenbergen, (2004) notes that PPTs show "*low levels of mathematics knowledge as well as considerable anxiety towards the subject*" (Zevenbergen, 2004, p.5).

With respect to mathematics teaching and learning, there are various moments at which the relationship between emotions and cognitive processes becomes visible: when, following the proposal of a mathematics task, the structure of the activity is understood or relevant information is retrieved; when problem-solving strategies are being designed, including the recall of formulas or mechanical procedures; and when the PPTs are involved in the process of the control and regulation of their own learning coupled with a clear methodological approach to teaching the mathematics which they had come to reject.

It therefore seems appropriate to consider studying the beliefs, attitudes, and emotions of prospective teachers when they are dealing with PS. The lack of reflection on these issues is one reason for the persistence of PPTs' inappropriate conceptions and attitudes. In their passage through initial teacher education, they have not been led to re-conceptualize their role as primary teachers. Authors such as Mellado, Blanco & Ruiz (1998), Chapman (2000), Uusimaki & Nason (2004), and Malinsky, Ross, Pannells & McJunkin (2006) suggest that the origin of the negative beliefs of prospective teachers in their initial teacher education could be attributed to the influence of their own experiences as learners of mathematics, i.e., to their experiences when they themselves were being taught mathematics in school and to their teachers at that time, and to the mathematics courses in their teacher education programs.

A Research Project with Prospective Primary Teachers on Cognition and Affect in Problem Solving

The above references clearly show that the cognitive and the affective are closely related, that beliefs, attitudes, and emotions influence knowledge, and that knowledge in turn affects those same three aspects.

The study of this relationship between affect and cognition has also been explored with teachers. Teachers' concepts and values determine the image of mathematics in the classroom, and condition the type of teacher-pupil relationship. Conceptions influence attitudes, and both of them influence the teacher's behaviour and the pupils' learning (Ernest, 2000). In order to foster change in our prospective teachers' views of teaching, we shall have to incorporate conceptions and attitudes as part of a process of discussion and reflection in our initial teacher education programs (Mellado, Blanco & Ruiz, 1998; Stacey, Brownlee, Thorpe & Reeves, 2005; Johnson, 2008). There thus seems to be a clear interest in studying the affective and emotional factors involved in the mathematics education of PPTs since, as future teachers, their beliefs and emotions towards mathematics will influence both the level of achievement and the beliefs and attitudes towards the subject of their pupils.

De Bellis & Goldin (2006) and Furinghetti & Morselli (2009) note that studies of students' performance and problem solving have traditionally concentrated primarily on cognition, less on affect, and still less on cognitive-affective interactions. However, a growing number of studies recognize the importance of integrating the affective and cognitive dimensions into the teaching and learning of mathematics (Amato, 2004; Zan, Bronw, Evans & Hannula, 2006; Furinghetti & Morselli, 2009; Blanco et al., 2010).

Some authors, such as Furinghetti & Morselli (2009), specifically note the need to simultaneously develop cognitive and affective factors in teacher education programs. In this regard: "*The role of teacher education is to develop beginning teachers into confident and competent consumers and users of mathematics in order that they are better able to teach mathematics*" (Zevenbergen, 2004, p.4).

In this context, we considered that there was a need to undertake a research project on MPS in initial primary teacher education with the consideration of its cognitive and affective aspects. Initial teacher education is conceived of as being just one part of a continuous and permanent process in a teacher's professional life in which emotional education is an indispensable complement to cognitive development. Indeed, cognitive and affective aspects are essential elements in the development of teaching as a profession.

We believe that gaining the capacity to solve mathematics problems should be an achievable goal in an educational environment in which students are allowed to generate their own PS strategies and compare them with other alternatives. In particular, we believe that the way in which PS is approached is highly personal. Each student will have to be helped to discover their own particular style – their own capabilities and limitations. We must avoid conveying to our students only heuristic rules or methods, but instead be sure to help them develop positive attitudes and emotions towards MPS based on their own past and present experiences.

Objective of the research project

In our research study, we set ourselves the following general objective: to design, develop, and evaluate an intervention program to enhance the performance of PPTs in MPS, and to lay the foundations for them to learn to teach MPS at the primary school level, integrating in a single model cognitive aspects of PS and emotional education (Annex 1).

Additionally, we set different specific objectives relating to the study population, two of which were:

- To describe the participating prospective teachers' conceptions about MPS.
- To describe and analyse their attitudes, beliefs, and emotions related to MPS, and in particular their expectations of control.

In addition, two specific objectives relating to the teacher education program were pursued:

- To evaluate the development of the program with respect to the PPTs' levels of anxiety.
- To describe the aspects of the program which they found to be most significant.

During the 2007-08 academic year, we conducted a pilot study that served to fine-tune the program. We performed the actual field-work during 2008-09.

Data collection and analysis

The nature of the research problem and the data collection led us to use a combination of qualitative and quantitative methods to relate, compare, and contrast the different types of evidence. The implementation of the program followed an action research approach since the ultimate goal is to help the participants develop their thinking, modify their attitudes, and seek ways to overcome their difficulties in MPS.

Annex I presents the plan of the 13 sessions comprising the program, specifying the objectives of each session, the instruments used to obtain information (open and closed questionnaires, diaries, and forums), the nature of the data, and the corresponding type of analysis. The participants in the program were a core group of 55 PPTs in the Education Faculty of the University of Extremadura (Spain) in the third year of their course.

All the program sessions lasted two hours, and were audio and video recorded, accompanied by field notes. The Moodle Virtual Platform was used as support for the program's documents, information, and forums, as also is indicated in Annex I.

Apart from the open and closed questionnaires specifically indicated in Annex I, the following research tools were employed:

- *Observation of the behaviour in the classroom* of both teacher and students, video recorded with two cameras, with subsequent transcription and analysis.
- The *Moodle platform* (Universidad de Extremadura) is a useful tool for the presentation of information and communication. It allows information to be stored for later analysis (both qualitative and quantitative), with the date and time and the subject contributing the information being reliably logged. It allows one to evaluate the participation, and to see whether the students have attained specific learning objectives, providing feedback as well as motivation to the students. A reference to the use of this platform in the present research can be found in Caballero, Blanco & Guerrero (2010).
- *Diaries* (Nichols, Tippins & Wieseman, 1997), kept on the Moodle virtual platform. These allow the collection of observations, sensations, reactions, interpretations, anecdotes, introspective remarks about feelings, attitudes, motives, conclusions, etc.
- *A forum*, also via the Moodle virtual platform, on some of the specific content or situations arising in class.

For the data analysis, we used the program packages SPSS 15.00 for the quantitative analysis of the questionnaires that we are given in Annex I. For the qualitative analysis, we followed the recommendations of Goetz & LeCompte (1984) and Wittrock (1986), establishing a process similar to that described in Barrantes & Blanco (2006) based on units of analysis (Goetz & LeCompte, 1984) and the categories noted in the instruments.

Analysis of Results and Discussion

The breadth of the research study and the characteristics of this present communication only allow us to present some partial results. In particular, we shall refer to some of the results on the PPTs' conceptions about PS, on certain aspects related to the affective domain, particularly those concerning the students' expectations of control, and on some general aspects of the program's evaluation.

What do the prospective teachers understand by a mathematics problem?

Our analysis of the questionnaires showed the prospective teachers to hold very traditional conceptions about mathematics problems. Thus, they referred to them as closed statements which explicitly or implicitly indicate the procedure to follow for their solution. The responses to the items of the questionnaire on "*What do I understand by a mathematics problem?*" (Annex II) reflect the classifications noted by some authors in the literature. In this sense, their formulation of a problem is in the form of a text which gives all the information to be resolved, which Borasi (1986, p.135) calls a "*word problem*"; and the method of solution explicitly or implicitly suggested in the text involves a translation of the words of the problem to a mathematical expression, which Charles & Lester (1982, p.6) call a "*simple or complex translation problem*"; and the solution of this expression involves using a known algorithm, which Butts (1980, p.24) refers to as an "*application problem*".

The basic referents of their problems are arithmetic operations, algebraic algorithms, or, to a lesser extent, calculations of areas. It was interesting to note that the contexts they describe are those that have been traditional in mathematics textbook problems since the late nineteenth century. Thus, in both years of the study, there are references to problems of taps, the number of heads and legs of farm animals, trains and distances, and the comparison of ages. It stood out that in no case was there any reference to specific situations of their or their potential pupils' immediate environment, or to such everyday resources as mobile phones or personal hobbies. This result, which we did not find in the literature we reviewed, seems especially important because it is necessary to link problem solving with the pupils' interests and relate the problems to their immediate environment.

Of a total of 178 problems, 126 (70.8%) were arithmetic with a structure involving addition or multiplication, representing elementary shopping or business situations[2]. Another 31 (17.4%) were questions of arithmetic proportionality[3]. There were 7 problems (3.9%) involving equations in which the situations were related to ages, taps, speeds of trains or cars, and farm animals[4]. Geometry problems accounted for 5.7%, and were very basic, referring to the calculation of areas[5].

In analysing this PS situation in class with the students (4-XI-2008), we thought that it was convenient to focus on the following question:

- *Do you think there are other types of problems? If so, write down two examples.*

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[2] There are 47 apples in an apple tree. Mary has picked 37 apples. How many apples are left in the tree? In a fruit shop, 1 kg of apples costs 1.75 euros. If Laura buys two kilos, how much money has she spent altogether?

[3] We know that Juan has eaten 2/3 of a cake, and his brother Sergio 5/6 of the rest. How much of the cake is left? Three friends have 40 euros to spend. The first spent 2/5 of the total, and the second 2/3 of what the first spent. How much did the third spend?

[4] On a farm there are horses and chickens. In total there are 74 feet and 12 beaks. How many horses and how many chickens are there on the farm?

[5] Calculate the area of a square whose side is 2 cm.

Observation of the recordings and the analysis of this last question brought out the difficulty they were having in establishing mathematics activities that were different from those they had proposed, and which had been analysed previously in that same program session.

Thus, 14 participants answered directly that there are no different types of problem. Two examples of these responses are the following:

- "*I think not, because throughout my school life I always had problems of the same type.*"
- "*The truth is that I have no idea. The maths problems that I know are those of always.*"

Another 34.5% again insisted on the same kinds of problem noted above, but involving situations concerned with other mathematics content such as statistics or probability that had not specifically appeared previously.

The question prompted some students to guess that there really must exist other types of mathematics problems, but they found themselves unable to give any examples:

- "*After what we have seen today, there must be other types, but right now I can't think of any.*"
- "*Clearly there must be, but I am unable to find any examples.*"

This conception of MPS contrasts with what is imagined in today's curricula which consider a much broader view of problems, different perspectives (in terms of content, application, and methods), and in the usual classifications such as those we presented in the program which show a variety of different possibilities. Consequently, initial teacher education programs should intensify the attention given to these issues.

The PPTs' expectations as problem solvers themselves

For the 5th session, we adapted the Battery of Scales of Generalized Expectations of Control, BEEGC-20 (Palenzuela et al., 1997), to the context of PS. This adaptation consisted of a closed questionnaire, with responses on a scale of 1 to 10, targeted at determining the students' expectations of control when faced with MPS. We wanted to examine whether they believed their success or failure in PS would be a true reflection of their actions, or rather be simply at the mercy of luck or chance. We also wanted to determine their expectations of self-efficacy, i.e., to what extent they felt themselves capable of solving mathematics problems. This was the second specific objective that we indicated above in Sec. 3, and whose partial results we shall consider in the following paragraphs.

The results showed the participating students had a high expectation of contingency on their actions (perseverance, effort, commitment, ability), and a low external locus-of-control reflecting luck or chance.

Thus, their responses to Item 1 ("*My success in solving mathematics problems will have much to do with the effort I put into it*"), with a mean score of 6.69, showed that they see effort as being crucial for success in MPS. The result was similar for Item 15 ("*If I try hard and work, I will be able to solve successfully the mathematics problems that I am set*") which was directly related to the dependence of success in problem solving on effort and application. Additionally, 54.9% indicated on Item 11 ("*In general, success or* *failure in solving a mathematics problem will depend on my actions*") that success would depend on their own actions.

Rinaudo, Chiecher & Donolo (2003) and Martínez (2009) also refer to high levels of control, and the subjects studied by Orozco-Moret & Diaz (2009) and Yara (2009) attributed success in MPS to ability and effort. However, many prospective teachers become blocked when faced with these mathematics tasks, and in many cases end up by abandoning the effort, as was shown in the observations of their behaviours in the fourth and eighth sessions. This reflected a certain contradiction between what they expressly stated and their actions in class in dealing with these mathematics tasks. These observations also revealed their lack of knowledge of procedures and heuristics with which to tackle mathematics problems.

With respect to their expectations of self-efficacy, these prospective teachers showed little confidence in themselves and their abilities when solving quite normal problems of mathematics. Thus, 70.58% said they had "*thoughts of insecurity when doing MPS*" (Item 14) and 64.7% "*had doubts about their ability to solve mathematics problems*" (Item 2). In this regard, Caballero, Guerrero & Blanco (2008) and Hernández, Palarea & Socas (2001) also note that PPTs in initial teacher education do not see themselves as capable or skilled in mathematics.

That the PPTs mainly attributed their success or failure in solving mathematics problems to their own actions and not to helplessness or luck means that they are largely attributing success to internal, unstable, and controllable factors. This is beneficial for their future learning situations. On the contrary, their low expectations of self-efficacy, i.e., their lack of confidence in their capacity to solve the mathematics problems they will be set, would seem to be prejudicial for the future learning.

Their high expectations of contingency together with low expectations of self-efficacy foster the development of negative attitudes towards solving mathematics problems, leading the PPTs to consider that failure in this activity is due to a lack of ability rather than to any lack of effort. As suggested by Martínez (2009), the result is to severely lower their expectations of success, and to encourage them to abandon any persistence in trying to learn how to solve mathematics problems. Similarly, their low expectations of self-efficacy and their not very high expectations of achievement are suggestive of an algorithmic approach to learning.

Some results of the program of MPS and emotional education

To evaluate the program, one of the instruments we used was the State/Trait Anxiety Index (STAI) self-assessment questionnaire adapted from Spielberger (1982). We presented this questionnaire on three occasions – at the beginning of the program, on its completion, and four months after its completion. In the present communication, we shall compare the results of four of its items:

- *When I am solving mathematics problems I feel calm.* (calm)
- *When I am solving mathematics problems I feel secure.* (security)
- *I feel comfortable when I am solving a mathematics problem.* (comfort)
- *I feel nervous when I am faced with a mathematics problem.* (nervousness)

The results indicated a positive trend in the period covered by the program, with a decrease in anxiety about MPS continuing four months after the program. Even though there was a slight regression relative to the actual moment of completion of the program, the data were better than those obtained at the beginning. This reflects a major advance with respect to the control of anxiety following participation in the intervention program.

The participants also declared a change in attitude: "*The program has changed our attitude to MPS, even though the content we have acquired throughout our lives is impossible to change in just 13 sessions*" (10 FS 1). Other evidence shows their desire to integrate cognitive with affective aspects: "*This program has also been useful in that, when we are teaching, we will know to take into account not only what our pupils know, but also their attitudes and emotions, which, by my own experience, I know have a great influence both positive and negative*" (7 DP 4).

The debate that took place in the evaluation session and in the forums showed important reflections and concerns which we interpreted as an attempt to approach PS in a systematic and orderly fashion, as a result of the procedures and heuristics worked on in the general model during the program. Thus, in the evaluation session (Session 13), one student notes that: "*The execution of the steps [in a problem solving strategy] helps me to concentrate, to analyse the problem, and to understand it better*" (18 MV 2). Another student says: "*In my case, it helped me to be more orderly in presenting problems*" (10 FS 2), and expresses a desire to apply it in her future work as a teacher: "*We dealt with aspects that we shall subsequently transmit to our own pupils, and with methods that we will use such as the steps to follow for problem solving and relaxation methods*" (18 MV 3).

Conclusions

The present work has confirmed the importance of considering in an integrated form the cognitive and affective aspects of mathematics teaching and learning at different levels, especially in initial teacher education. This can help foster the change in our prospective teachers' beliefs and attitudes along the line laid out in current curricular proposals.

As one of the students stated, it is difficult in just 13 class sessions for our PPTs to learn both MPS for themselves and how to teach it to their future primary pupils. But it is gratifying to note that a change in attitude was initiated, and that they themselves valued positively their first-person experience in the program, and that the content of the program fell within their expectations as future teachers.

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Annex I. Intervention Program

The program ran from October to December 2008 except for the evaluation session which took place in April 2009, four months after completion of the program.

Session 1. (27 and 28 October) Presentation of the program to students. 55 PPTs.

27-X. The students were provided with extensive information about the workshop, working methods, and objectives.

27-X. Initial questionnaire - Commitment to the workshop. Objective: To determine the participants' self-perception as problem solvers, and their degree of commitment to the workshop. (Open questionnaire; qualitative analysis.)

28-X. Conceptions and knowledge of MPS. What do I understand by a problem? Objective: To analyse students' conceptions and knowledge about mathematics problems. (Open questionnaire; qualitative analysis.)

28-X. Affective domain in MPS. Objective: To examine the students' affective factors (beliefs, emotions, and attitudes) that influence their development in MPS. Adaptation to MPS of the questionnaires of Gil, Blanco & Guerrero (2006), and Caballero, Guerrero & Blanco (2008) on the affective domain in mathematics. (Closed questionnaire; quantitative analysis.)

Session 2. (4 November) Conceptions and affective aspects of MPS. 53 PPTs.

Presentation and discussion of the results of the previous questionnaires. We analyse the PPTs' conceptions of MPS, comparing them with the perspectives outlined in the curriculum (as specific content and as method) and with those described by different authors. Likewise, the results of the questionnaire on affect are discussed, expanding them with other previous results (Gil, Blanco & Guerrero, 2006; Caballero, Guerrero & Blanco, 2008).

Session 3. (7 November) Problems vs exercises. 55 PPTs.

Differentiation of exercises and problems, and presentation of other types of problems based on different classifications (Borassi, 1986; Butts, 1980; Charles & Lester, 1982; etc.).

Forum on the Moodle platform concerning the content of Sessions 2 and 3.

Session 4. (11 November) How do I approach MPS? Before, during, and after. 55 PPTs.

Pre-test.
Objective: To evaluate the participants' own impressions that arise at different moments of MPS. Two problems proposed for solution which will allow us to observe their knowledge and affects at different stages of the PS process at this early stage of the program. They will be asked to respond to the same open questionnaire on three occasions – before seeing the problem, while they are solving it, and after having had to deal with the activity. (Open questionnaire; qualitative analysis.)

Adaptation of the STAI (state / anxiety) to MPS – pretest. Objective: To determine the level of anxiety that MPS provokes in the students. Adaptation of the STAI (Spielberger, 1982) to MPS. (Closed questionnaire; quantitative analysis.)

Session 5. (14 November) Personal involvement. Causal attributions, and behaviour and stress. 54 PPTs.

BEEGC-20 adapted to MPS. Objective: To examine the causal attributions relating to MPS (expectations of success and of the locus-of-control, of helplessness, and of self-efficacy). Adaptation of the BEEGC-20 Questionnaire of Palenzuela et al. (1997) to MPS. (Closed questionnaire; quantitative analysis.)

Initiation of a discussion in class on the content of the questionnaire, which will be followed by a specific forum on the Moodle virtual platform.

Session 6. (18 November) Emotional coping: relaxation, breathing, and self-instruction. 55 PPTs.

Presentation of results of the previous questionnaire, and analysis of the interventions in the forum.

Information and explanation of different aspects of emotional education and its relationship with PS. Mainly the topics covered in the questionnaire.

Session 7. (21 November) Overview of the Integrated Model[6] of MPS I. 55 PPTs

Sessions 8–10. (25 November, 2 December, 5 December) Application of the Integrated Model, with specific problems.

Session 11. (9 December) Specific activities of the Integrated Model for primary pupils. 55 PPTs.

In this session, we present specific activities adapted to the primary level that allow the PPTs to work with pupils aged 6 to 12 at different stages of the general model – basically, the comprehension and analysis of problems, and the design of strategies.

Session 12. (12 December) General model of MPS. 55 PPTs.

The PPTs work specifically on problems in a complete and continuous application of the Integrated Model.

STAI adapted to MPS (state / anxiety). Post-test I. Objective: To determine the level of anxiety that MPS provokes in the students

[6] Blanco et al. (2010) proposed a theoretical model based on general models of PS (Bransford & Stein, 1987; Polya, 1957; Santos, 2007), on the cognitive-behavioural models of Zurilla & Goldfried (1971) and on the systemic model of De Shazer and the Milwaukee group (De Shazer, 1985). It consists of a process of experimentation and reflection based on the general model, and structured into five steps: (i) accommodation / analysis / understanding / familiarization with the situation; (ii) search for and design of one or more PS strategies; (iii) execution of the strategy or strategies; (iv) analysis of the process and the solution; and (v) How do I feel? What have I learnt?

after the workshop. (One problem.) Adaptation of the STAI of Spielberger (1982) to MPS.
(Closed questionnaire; quantitative analysis.)

Session 13. (16 December) Evaluation of the PPTs and the workshop. 55 PPTs.

Evaluation: Proposal of a problem for the PPTs to solve by following the Integrated Model, in order to evaluate the knowledge they have acquired about the General Model.

Workshop evaluation. How have I managed my resources? Objective: To determine the strengths and weaknesses of the workshop, and the progress the participants have made in MPS.
(Open questionnaire; qualitative analysis.)

Classroom discussion about how the workshop has functioned in relation to its proposed objectives, and about the participants' individual goals and commitments. Audio and video recordings and field notes, and opening a forum on the Moodle virtual platform.

Session to evaluate the research. (April 2009) 34 PPTs.

STAI (state / anxiety). Post-test II. Objective: To determine the level of anxiety that MPS provokes in the students four months after completion of the program.

Annex II

"*What do I understand by a mathematics problem?*"

a. Full name ____

1. Formulate the statement of three mathematics problems.
2. Indicate why mathematics problem solving is important in compulsory education.
3. Write down some personal reflections about your own experience in mathematics problem solving in primary and secondary school.
4. What consideration does mathematics problem solving merit on your part?
5. Add something that you find significant, and have not written.