2019

# Encouraging Discourse in High School Mathematics

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ENCOURAGING DISCOURSE IN HIGH SCHOOL MATHEMATICS

By

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Professional Paper

presented in partial fulfillment of the requirements
for the degree of

Master of Arts
in Teaching School Mathematics

The University of Montana
Missoula, MT

Fall 2019

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Piper, Sarah, M.A., Summer 2019 Teaching School Mathematics

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The Common Core State Standards in Mathematics increased the emphasis on the skills of reasoning, justifying, and critiquing. Mathematics educators have struggled to meet this requirement, as it requires a distinct shift in classroom culture that both students and teachers find challenging. Visibly random groupings have been suggested as a way to effect a change in classroom norms, as students are asked to work with all members of the class. They thus learn to share their ideas as well as offer feedback on the thinking of other classroom participants. Students in three high school geometry classrooms were observed in an action research study focusing on how implementation of visibly random groupings affected discourse in the classroom. Data triangulation incorporated mathematical attitude surveys, open-ended questionnaires, teacher field notes, and student work samples. Different grouping methods were explored, and tasks and assessments were changed to accommodate the alterations that arose in the classroom culture. Pedagogical strategies also changed in response to the shifting norms. At the end of the research period, student attitudes had stayed relatively consistent, but their willingness to trust in their own reasoning as a valid source of knowledge had increased. Despite tensions around control of learning and the balance of content and process, teacher records indicate that a positive change had occurred in the classroom culture and expectations.

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# Chapter 1

## Discourse in Mathematics

One of the most common student complaints in high school mathematics is a lack of relevance. High school hallways across the country ring with groans of "when are we going to *use* this?" as students are taught traditional mathematics. The content is focused around skill building and information retrieval, and often uses a model of lecture followed by practice. This complaint is frustrating to many instructors, who have often personally experienced math in a more active and engaging way. Yet this rich experience of thinking critically and constructing new knowledge frequently has been restricted to college-level courses. In the high school curriculum, content standards are typically prioritized over the arguably more valuable mathematical practice standards (Table 1) contained within the Common Core State Standards for Mathematics (CCSS-M). This all too often leads students to see math as algorithms and recitation, rather than as a living creation of new understandings that can be applied to their world.

Table 1: CCSS-M Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

There are many possible solutions to this unfortunate situation that attempt to refocus teaching and learning to incorporate more authentic inquiry. In order to explore, it is necessary for students to engage in the process of conjecturing, explaining, and critiquing. These are all communication as well as thinking skills, which indicate that student interaction is a key method that could be leveraged to change mathematics instruction. Classroom norms traditionally lean towards direct instruction, followed by some guided whole-group practice and then independent practice time, which often rolls into homework. Very little in the way of peer interaction is explicitly encouraged, much less required. This study intends to explore an alternative classroom dynamic. What happens to students' ability to build mathematical arguments and critique those proposed by others when the classroom norm shifts to include visibly random grouping?

This will be explored through action research, in which the teacher responds to changes in the classroom as they occur, in successive cycles of planning, acting, and reflecting on the results. The nature of this type of research is that there are unexpected shifts as findings lead into avenues of exploration that were not initially anticipated. In the case of this study, the research question shifted as building a culture of inquiry became increasingly central. Upon reflection after the conclusion of the study, the original research question broadened from the initial focus on visibly random groupings. What classroom conditions support a shift to a sociomathematical norm of inquiry, and what effects does this have on students' attitudes towards mathematics?

# Chapter 2

# Review of Relevant Literature

## Why Should Math Education Change?

In order to justify a change to the status quo, it is important that a change be warranted. In current mathematics instruction, the need for a change is well-established. Calls for improvement are ubiquitous throughout history, but in recent years the need has been seen as increasingly acute. Performance on international assessments show that American high school students are slipping, especially on problems that require higher-level reasoning skills (Schmidt, Houang, Cogan, 2002). This is exacerbated by huge discrepancies between achievement when scores are adjusted for geographic region and socio-economic status (Noddings, 1994; Treisman, 2013). A lack of a coherent national curriculum that establishes a high degree of rigor and allows for nation-wide planning and professional development is negatively impacting student outcomes.

To address this perceived deficit, the CCSS-M were developed (Coleman & Zimba, 2007). The CCSS-M include both content standards broken down by course title at the high school level, and practice standards which span all grade levels (National Governors Association Center for Best Practices, 2010). These practice standards have led to many of the largest observed changes in mathematics in response to the CCSS-M, as teachers have chosen pedagogical strategies to enhance areas of instruction, but these strategies are not explicitly indicated by the standards (Bay-Williams, Duffett, & Griffith, 2016). This has led to pushback in many states as mathematics students are asked to offer explanations and multiple methods for answers, causing frustrations for parents, students, and teachers. However, there is evidence that increasing student communication of strategies and solutions contributes to mathematical success

(Rasmussen & Marrongelle, 2006; Sowder & Harel, 1998). These findings show a need for further exploration into the pedagogical strategies that best serve to increase classroom mathematical discourse, as student success in mathematics is currently inconsistent at the national level and attempts to incorporate the practice standards have had mixed success.

## How Can Increasing Discourse Help?

The expectations of teachers and students that regulate discourse and argumentation vary widely from one classroom to another. Sociomathematical norms provide a framework for investigating how students develop mathematical beliefs and values, and how they become intellectually autonomous through this process (Yackel & Cobb, 1996). Students generally move through three phases in their mathematical growth: looking for mathematical reasons rather than appealing to social status for justification; basing explanations on mathematical objects that are in their shared experience; and lastly, judging the quality and clarity of the explanation itself as the criteria for acceptance (Yackel & Cobb, 1996). As a classroom of learners progresses, shared expectations of what evidence is accepted as proof shift and are therefore specific to the particular community.

In order to improve mathematics instruction, possible changes to the sociomathematical norms must be considered. One such alteration is to remove the teacher as the source of knowledge, and instead instill the shared value of students as the creators and evaluators of mathematical knowledge. This requires a change to the existing contract between teacher and students about the trading of mathematical knowledge and the division of labor (Cox, 2004; Herbst, 2006). In one study of this unspoken contract, students who had previously experienced more traditional instructional methods were asked to create conjectures, search for justification,

present their reasoning, and evaluate whether such claims were adequately explained. This led to positive student attitudes towards proof, stronger written proofs, and increased communication skills (Cox, 2004). The disruption of the sociomathematical norm encouraged communication between students, rather than teacher-student discourse, which can improve student performance on reasoning and evaluating tasks.

This communication can also be useful for eliciting misconceptions. In an analysis of student reasoning, “approximately three fourths of the flawed arguments did not reflect faulty reasoning. Instead, they resulted from a failure to conduct a sufficiently elaborate analysis by, say, considering various counterexamples. Further, most subjects could produce counterexamples to their own arguments when asked to do so” (Cobb, 1986, p. 3). Increasing classroom discourse allows teachers more opportunities to identify the true depth of student understanding. In addition, mathematical misunderstandings can be corrected more quickly and effectively when communication between students is increased (Bossé & Adu-Gyamfi, 2011). As the role of the teacher shifts away from being the holder of knowledge and evaluator of student thinking, however, it is important for teachers to avoid the temptation to respond to student ideas that are presented with subtle cues as to the legitimacy of the argument (Rasmussen & Marrongelle, 2006). This tension is a difficult transition both for teachers and students, but ultimately smooths out as discourse becomes a sociomathematical norm in the classroom community.

## How Can Discourse in Mathematics Classrooms be Increased?

Two major themes continue to emerge in connection to mathematical discourse. The first is student reluctance to accept knowledge that is not verified by the authority of the teacher. The

second theme is a perception of mathematical reasoning as a formalized system of thinking and communicating that is disconnected from practical reasoning. Both of these must be addressed in order to effect meaningful shifts in discourse within the classroom.

In regard to the disconnect between mathematical reasoning and practical explanation, increased opportunities to explore and create conjectures is indicated as a solution (Battista & Clements, 1995; Cirillo, 2009; Rasmussen & Marrongelle, 2006). This is particularly effective when combined with investigations using dynamic geometry software (DGS) such as *The Geometer's Sketchpad* or *Geogebra*, as students are able to use the dragging functionality of these programs to rapidly explore possible conjectures (Battista & Clements, 1995; Bossé & Adu-Gyamfi, 2011; Izen, 1998; Leikin & Grossman, 2013). These experiences have been shown to deepen student understanding both of the role of proof as well as the underlying mathematical concepts. Attitudes regarding the role of proof also improve. Students who experience instruction incorporating conjecture opportunities are more likely to apply mathematical reasoning to non-routine situations, as well as to formal proof problems (Battista & Clements, 1995; Cobb, 1986). These conjectures increase communication and lead to improvements in reasoning and explanation.

Student attitudes towards authority in mathematics classrooms are influenced by the perception that mathematics is different from daily experience, and also a power imbalance between teacher and student (Cobb, 1986). This leads to reluctance to take risks and share ideas freely, as students either seek to please or avoid the teacher rather than build mathematical knowledge (Cobb, 1986). As a solution, visibly random groupings (VRG) used over a period of time have been shown to be effective at breaking down social stratification within the classroom (Liljedahl, 2014). This method of grouping prescribes that the student pairings be randomized in

full view of the class, and that groups change daily. No specific technique of randomizing is required, as long as the students are able to see that it is truly random and that all students are equally likely to be grouped together. Frequent grouping changes help facilitate the transfer of knowledge between students, increases confidence in co-constructed knowledge, and decreases student reliance on the teacher for answer verification (Liljedahl, 2014). Using VRGs also correlates with improved student attitudes towards mathematics. Additionally, allowing students to debate and resolve disagreements amongst themselves leads to increased confidence and further collaboration (Harel & Rabin, 2010; Sowder & Harel, 1998).

## Implications of Relevant Literature

A review of the literature indicates that 1) mathematics education is in need of a change if all students are to find success, 2) an increase in discourse in the classroom improves student outcomes, and 3) possible options to increase discourse exist. Specifically, considering the appeal from the CCSS-M for students to “construct viable arguments and critique the reasoning of others,” this review led to the present research study into strategies to increase mathematical discourse in high school mathematics. How does the implementation of VRG in the mathematics classroom affect student discourse and argumentation?

# Chapter 3

# Methodology

## Research Subjects

Individuals for study will be selected at a small high school in rural Montana, roughly thirty minutes from a larger metropolitan area. Approximately half of students in the high school intend to pursue post-secondary education. Three classes of high school mathematics students will be selected for consideration in this study: first-period Honors Geometry ($n = 17$), second-period Geometry ($n = 16$), and sixth-period Geometry ($n = 24$). Students are placed based on recommendations from their teacher from the previous academic year. The demographics of the classes are shown in Table 2.

Table 2: Gender and Grade by Class Period

| | Male | Female | 8th | 9th | 10th | 11th | 12th | *Total* |
|---|---|---|---|---|---|---|---|---|
| Per. 1 | 12 | 5 | 5 | 9 | 3 | 0 | 0 | *17* |
| Per. 2 | 5 | 11 | 0 | 4 | 9 | 3 | 0 | *16* |
| Per. 6 | 9 | 15 | 0 | 3 | 17 | 1 | 3 | *24* |
| *Total* | *26* | *31* | *5* | *16* | *29* | *4* | *3* | *57* |

The high school is located on a K-12 campus, which allows for middle school students to come to the high school for advanced coursework. There is a relatively high rate of student transfers into and out of district throughout the course of each school year. In the course of this

study, three students left from sixth period, and two were added to second period (one of whom had been enrolled in first semester, left during third quarter, and then returned in the fourth quarter). Much of this movement is due to the rural location and low socioeconomic status of the area. There is also an impact of seasonal farming and ranching work which affects the consistency of student enrollment and attendance.

The sixth period class includes two special-education students with a paraprofessional assigned to work with them, as well as a foreign exchange student who often receives assistance from the paraprofessional as well. All three of these students are seniors taking a fourth year of math (three years are required for graduation).

## Initial Classroom Culture

Classroom norms have been constructed throughout the school year to encourage collaboration, communication, and reasoning. To begin the year, students completed the Week of Inspirational Mathematics, including developing norms of behavior unique to their class (Boaler, n.d.). These activities were intended to establish a classroom culture of group work with groups that shift fluidly. Visibly random groupings (VRGs) had been used off and on throughout the year, with students lining up according to random facts such as number of siblings, favorite color of the rainbow, etc. and then counting off into groups.

The physical arrangement of the classroom is not conducive to grouping. Thirty desks face the front of the room, where the SmartBoard is framed by two whiteboards. The desks are the type where the chair attaches to the desk surface and are arranged in six rows of five desks. When students were told to “turn to a partner” as a quick check for understanding during a lecture, they would rotate in their seat to the nearest person rather than moving to a different spot

in the room or moving their desks. The major exception to this instructional arrangement was when VRGs were used. This led more often to students spinning desks to face towards each other. The other exception occurred during the occasional ‘speed dating’ activity. These were typically reviews and consisted of turning desks towards each other in long rows. Students would then move from one desk to the next down the row as they switched partners frequently during review cycles. Both these types of groups generally required teacher direction to move the desks, as opposed to students spontaneously moving the furniture.

## Planned Cycles of Research

Several separate phases are planned for this action research research project. Each phase will follow the “dialectic action research cycle” consisting of four steps: 1) identify an area of focus, 2) collect data, 3) analyze and interpret the data, and 4) plan subsequent actions (Gay, Mills, & Airasian, 2015, p. 456). While action research does not have the control group and randomly assigned treatments that allow for the generalizability of experimental research, it is far more appropriate for this type of research question. Action research will allow information to be gathered regarding the way these specific students learn in their mathematics classroom. Additionally, the teacher researcher will be able to develop solutions to the problems of practice that are faced in the individual school and class (Gay, Mills, & Airasian, 2015). As with any action research, the planned cycles that are described below will need to be adapted as the study unfolds in response to changes in the classroom environment and the reactions of the students. The preliminary work to establish a classroom norm of communication has laid the foundations for these research cycles to be implemented within the three classrooms selected for study.

Cycle 1: Implement VRGs

While VRGs have been used previously, they have not been part of the typical classroom day. In order to maximize the visible randomness, groupings will become a daily part of the classroom procedure. This will require a release of control on the part of the instructor, as seating charts have previously been set for each quarter and the expectation is that students remain in their assigned seat. The goal of these seating charts has been classroom management. Various methods of randomizing will be attempted until one is selected that works well. These will include handing out playing cards and grouping on face value, lining up and counting off, popsicle sticks with names, and random list generators displayed on the SmartBoard. The emphasis of all strategies will be to quickly randomize students into groups in order to maximize instructional time, while also maintaining the visibility of the random nature of the assignment. Based on the results, a method will be selected to use throughout the rest of the research. This will start during the spring semester of 2019 and continue throughout the remainder of the school year. The method of randomization will periodically be reassessed to ensure that it is meeting the goal of efficiently creating a visibly random system of placing students into groupings.

Cycle 2: Group Sizes

As VRGs become the classroom norm, group size must also be considered. Various group sizes will be attempted, from pairs through quartets, in order to see if there is an optimal number of students. This will be in response to the instructional goal of the lesson, as well as possible restrictions on physical materials. As various group sizes are attempted, the resulting richness of discussion, engagement of students, and learning outcomes will be analyzed. This will inform future group sizes. This will require monitoring throughout the research cycle as

attitudes and behaviors may shift as VRGs become part of the classroom culture. Based on the results of this cycle, VRG sizes will be adjusted in order to maximize communication and learning for all students in the classes studied.

Cycle 3: Types of Tasks

After implementing Cycles 1 and 2, it became apparent that for VRGs to be effective, the task type may need to be adapted. As Cycle 1 & 2 were implemented, student reactions to working in groups was monitored. In a typical classroom period currently, students review the previous day's homework, the teacher delivers direct instruction while students take guided notes and occasionally have "think-pair-share" time, and then students are given time to work independently. This can be summarized as the "I-We-You" model, where I (the teacher) do a problem, We (the class) work together on a similar problem, and then You (the student) practices the skill. There is little need for a grouping, visibly random or otherwise, in such an instructional model. This instructional model was found to be impeding the attempt to implement VRGs within the classroom in order to encourage discourse, which suggests the third cycle. Pedagogical strategies will be adapted, most likely reflecting the types of tasks students have previously experienced in the Week of Inspirational Math, in alignment with the "You-We-I" approach to mathematical instruction which encourages exploration and collaboration prior to direct instruction (Romano, 2018). Tasks for this cycle will focus on student exploration and interaction. Task cards and scavenger hunts, adapted from Gina Wilson's All Things Algebra: Geometry curriculum, will be utilized to encourage student interaction, rather than the typical direct instructional style followed by independent work time.

Based on the results of these changes, decisions will be made regarding further shifts that are possibly implicated. VRGs at this stage should be an expected part of the classroom routine, and students should be beginning to communicate more mathematically with each other. However, these changes may not be enough to increase the quality of reasoning and justification. Observations at this point will help to inform the need for further cycles.

### Cycle 4: Pedagogical Shifts

If there has not been a sufficient increase in reasoning and providing critical feedback through the previous cycles, it indicates that a larger change is called for in the classroom structure. This will be loosely inspired by the quadrilateral exploration described by Cox (2004). In this unit, students turn in their textbooks and use dynamic geometry software (DGS) to create their own conjectures regarding quadrilaterals. The original plan includes seven distinct cycles of exploration: parallelograms, proving quadrilaterals are parallelograms, rhombi, rectangles, squares, isosceles trapezoids, and kites. Cox has students pass through the same steps in each part: creating conjectures, writing a proof for each conjecture, critiquing the proofs as a class, and accepting or rejecting the proof (2004, p. 50). Incorporating this type of unit will possibly help to establish a classroom culture of shared knowledge creation and will represent a radical departure from the traditional daily activities of the classroom up until that point. The instructor will need to carefully monitor student responses and reactions in order to respond to concerns as they arise. These could include problems of shared effort by all group members, power dynamics that affect acceptance of accurate work, and methods of assessing learning. This cycle would be the largest change from current instructional practice, but it may become necessary to increase

discourse that focuses on reasoning and justification by enabling students to see themselves as creators and owners of mathematical knowledge.

Following these cycles, student performance, learning, and attitudes will be used to inform what further changes may be called for in order to continue improving mathematical discourse. This will take the better part of the semester, allowing ample time in which VRGs can be utilized consistently and changes studied.

## Data Collection

Prior to the beginning of the study, students in all sections will complete the Mathematics Attitude and Perceptions Survey (MAPS) which scores student attitudes regarding mathematics in the following categories: Growth Mindset, Real World, Confidence, Interest, Persistence, Sense Making, Answers (Code et.al., 2016). Additional questions will be added regarding Verification of Results, in keeping with the focus of this study on discourse, reasoning, and critical analysis. Students are also given a MAPS score on the dimension of Expertise, analyzed by comparing the direction of student responses on each question to that of an expert mindset. The full survey is included in Appendix A. Following the completion of the research study, this survey will again be administered.

Teacher field notes will be a major source of data, providing evidence of daily events within the classroom environment as well as teacher perception and reflections. This will be triangulated along with student-generated artifacts including initial conjectures, rough draft presentations of proofs with student discussion recordings, and performance on task cards and assessment tasks.

## Data Analysis

MAPS survey responses (pre- and post-implementation) will be analyzed for differences in student attitudes towards mathematics, specifically focusing on communication in mathematics. In addition, student attitudinal changes will be examined on multiple factors to determine if changes have occurred. Teacher field notes will be coded to allow grouping of themes related to classroom discourse and reasoning. These will then be compared to student artifacts as indicators of those themes. Student performance on tasks will be used to verify content learning. By examining multiple data sources in relation to each other, it will be possible to determine how reasoning and discourse within the classrooms has been impacted by the incorporation of VRGs as a classroom norm.

# Chapter 4

# Research Results

Both quantitative and qualitative data collection was conducted to allow for triangulation of results. Quantitative data focused on the modified Mathematics Attitudes Preassessment Survey. Qualitative data included teacher field notes and student sample work. These results are discussed in the following sections.

## Quantitative Data

Student data was collected using the Mathematics Attitudes Preassessment Survey (MAPS) on February 21, 2019 and again on May 7, 2019 for period 1 (Honors Geometry), period 2, and period 6 (both regular Geometry sections). In addition to the 32 questions included on the original survey, six additional Likert scale questions were included to examine students' attitudes regarding their confidence in explaining their thinking, and what sources they viewed as trustworthy. The full survey is included in Appendix A. Two additional open-ended questions were also included: "What are typical activities that students are expected to do in math classes?" and "What does it take to convince you a math result is accurate?"

MAPS question responses are organized into seven attitudinal categories for scoring: Confidence, Answers, Sense-Making, Growth Mindset, Persistence, Interest, and Real-World. An eighth category, Verification, was added with the additional six questions. Each of these dimensions was compared pre- to post-intervention by calculating the average of student scores within the category. Surveys that were not fully completed, or that were not correctly answered on the filter question, were discarded. Three such surveys were identified out of the entire data

set. Sample sizes differ from pre- to post-assessment due to absences and movement of students into and out of the district or course section. Results are summarized in Table 3. The complete data is presented in Appendix B. There was little in the way of meaningful increases from the beginning to the end of the study in any category, with the possible exception of “Interest” in period 1 which increased over a full point. However, there were also no significant declines, contradicting the findings of Code et. al. (2016).

Table 3. MAPS Data by Category, Pre- vs. Post-Intervention

| | Pre | | Post | | |
|---|---|---|---|---|---|
| | Mean | Standard Deviation | Mean | Standard Deviation | Difference in Means (Post-Pre) |
| Confidence | 0.654 | 0.143 | 0.696 | 0.116 | 0.042 |
| Answers | 0.553 | 0.094 | 0.543 | 0.120 | -0.010 |
| Sense-Making | 0.605 | 0.087 | 0.604 | 0.078 | -0.001 |
| Growth Mindset | 0.540 | 0.080 | 0.549 | 0.082 | 0.009 |
| Persistence | 0.579 | 0.121 | 0.566 | 0.145 | -0.013 |
| Interest | 0.592 | 0.095 | 0.610 | 0.095 | 0.018 |
| Real World | 0.648 | 0.085 | 0.639 | 0.098 | -0.009 |
| Verification | 0.661 | 0.108 | 0.660 | 0.102 | -0.001 |

Each student was also given an ‘expert consensus’ score in accordance with the MAPS scoring system (Code et. al., 2016). One point was given per question if the student’s answer was in the same direction (either agree or disagree) as the expert consensus. If the student responded in the opposite direction of the expert consensus, or a neutral response was given, zero points were given for the question. This score captures the degree to which a student displays attitudes

that are correlated with mathematical proficiency. The total expertise index is calculated by combining the scores for all questions except 19 (the filter statement), 22, and 31. The additional questions added in this research study (questions 33-38) were not scored for expertise. Scores are therefore out of a possible 29 points, where a high score indicates that a student has a more positive and productive mathematical mindset. The results are summarized in Table 4A and 4B.

Table 4A: Aggregated MAPS Expertise Scores, Pre- vs. Post-Intervention

| | Pre | | Post | | |
|---|---|---|---|---|---|
| | Mean | Standard Deviation | Mean | Standard Deviation | Difference in Means (Post-Pre) |
| Expertise Score (0-29) | 12.595 | 4.643 | 13.189 | 5.162 | 0.594 |

Table 4B: MAPS Expertise Scores by Class

| | Period 1 | | Period 2 | | Period 6 | |
|---|---|---|---|---|---|---|
| | Pre | Post | Pre | Post | Pre | Post |
| Average | 15.67 | 15.13 | 11.53333 | 12.25 | 11.55 | 12.54545 |
| Standard Deviation | 4.972 | 5.524 | 3.36386 | 3.75 | 4.443816 | 5.433292 |
| Sample Size | 12 | 15 | 15 | 16 | 20 | 22 |

While no significant difference in means was present, medians and quartiles did shift. These differences are shown in Figures 1-3. Worth noting is the dramatic increase in the range of expertise scores in period 1, along with the downward shift of median score. This contrast with the increase in the top 50% of scores for both periods 2 and 6. Even with this decline, however, both mean and median scores for period 1 remained higher than in the other two classes.

Figure 1: Period 1 Expertise Index, Pre- and Post-Intervention

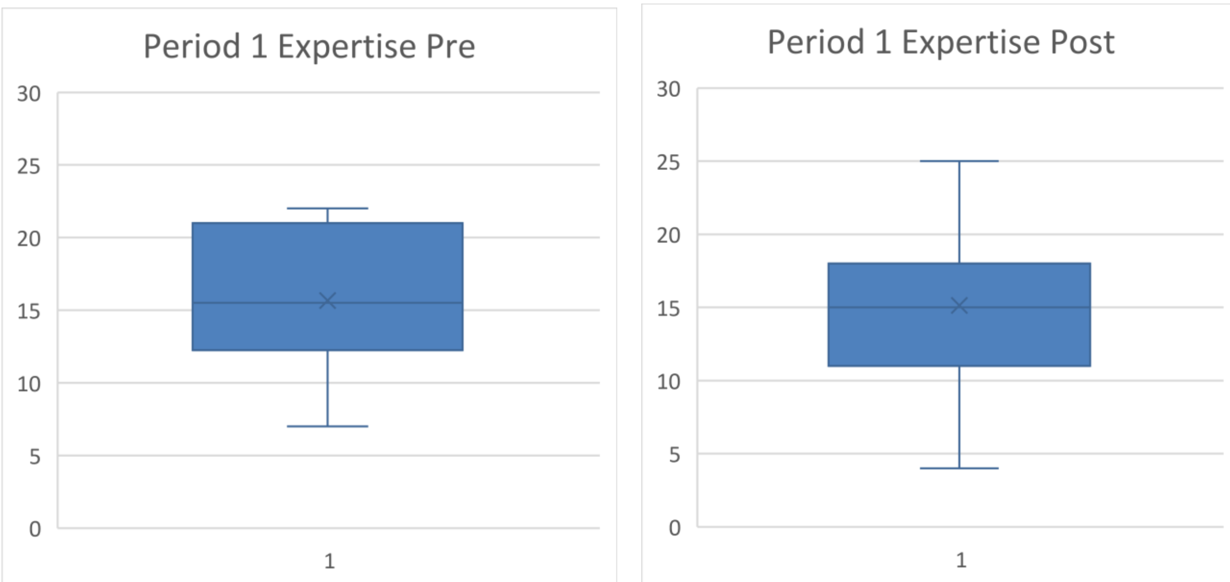



Figure 2: Period 2 Expertise Index, Pre- and Post-Intervention

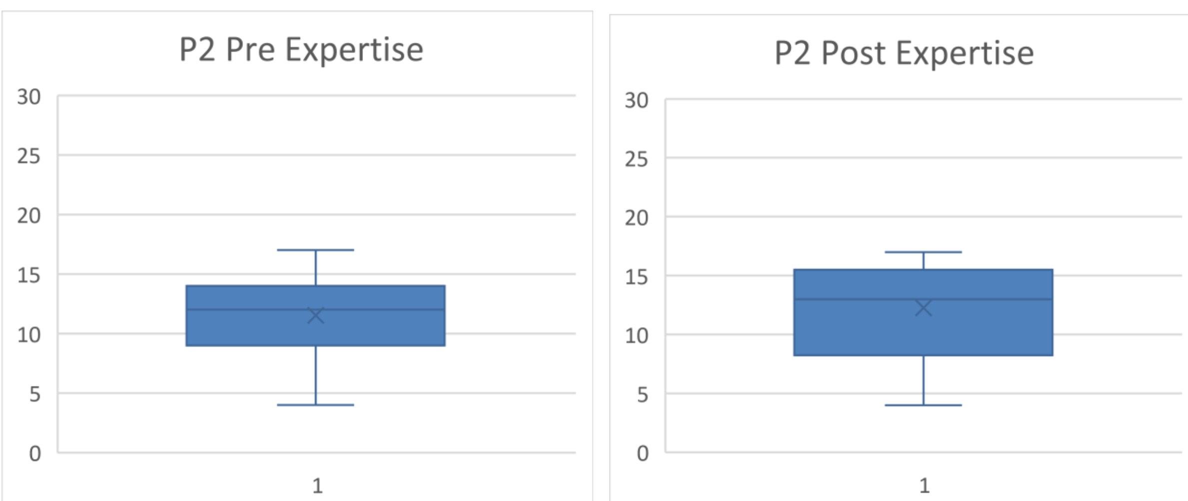
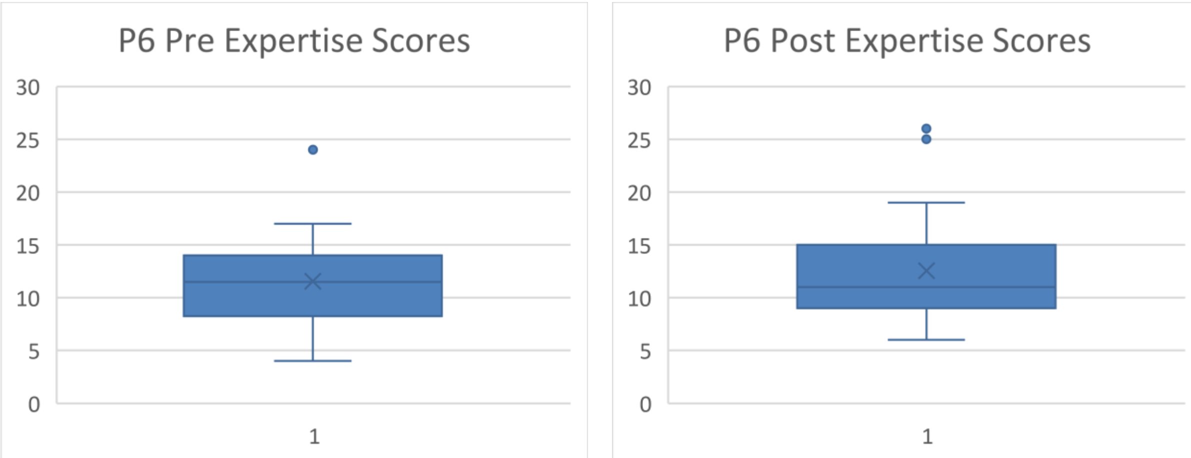


Figure 3: Period 6 Expertise Index, Pre- and Post-Intervention




Student responses to the open-ended question regarding typical activities in the mathematics classroom were coded after an initial analysis of answer categories was conducted. The following answer categories were identified for coding: use of algorithms, solving, applying, working together, learning, doing homework, behaving, and explaining/asking. After coding all student answers, the incidence of each response was tabulated, separated by course section and pre- or post-implementation. The percent of students including that response was then calculated. Results are shown below; the full data set can be found in Appendix B.

There was a 66% increase in student responses of “working together”, with roughly 30% increases in “solve” and “explain/ask” responses. Periods 1 and 2 had a higher index for “solve” while Period 6 showed a decline in this category. Both periods 2 and 6 contained more responses related to meeting behavioral expectations at the conclusion of the study than at the start, while that trend was not evident in period 1. Explaining/asking did not appear in period 1 responses at any point in the study, while period 2 and 6 both included this several times, with a strong increase in such answers in period 2 by the end of the research time.

Table 5A: Aggregated Percentage of Student Responses to ‘What are typical activities that students are expected to do in math classes?’

| | Pre | Post | Difference (Post-Pre) |
|---|---|---|---|
| Algorithm | 6.7 | 30.8 | 24.1% |
| Solve | 55 | 86.2 | 31.2% |
| Apply | 5 | 11.2 | 6.2% |
| Together | 35 | 101 | 66% |
| Learn | 31.7 | 35.4 | 3.7% |
| Homework | 161.7 | 181.9 | 20.2% |
| Behave | 6.7 | 30.8 | 24.1% |
| Explain/Ask | 55 | 86.2 | 31.2% |
| Sample Size | 47 | 53 | |

Table 5B: Percentage of Student Responses by Class Period to ‘What are typical activities that students are expected to do in math classes?’

| | Period 1 | | Period 2 | | Period 6 | |
|---|---|---|---|---|---|---|
| | Pre | Post | Pre | Post | Pre | Post |
| Algorithm | 0.0 | 20.0 | 6.7 | 6.3 | 0.0 | 4.5 |
| Solve | 8.3 | 33.3 | 26.7 | 43.8 | 20.0 | 9.1 |
| Apply | 0.0 | 6.7 | 0.0 | 0.0 | 5.0 | 4.5 |
| Together | 8.3 | 33.3 | 6.7 | 31.3 | 20.0 | 36.4 |
| Learn | 25.0 | 20.0 | 6.7 | 6.3 | 0.0 | 9.1 |
| Homework | 66.7 | 46.7 | 40.0 | 62.5 | 55.0 | 72.7 |
| Behave | 33.3 | 26.7 | 13.3 | 25.0 | 20.0 | 31.8 |
| Explain/Ask | 0.0 | 0.0 | 13.3 | 37.5 | 30.0 | 27.3 |
| Sample Size | 12 | 15 | 15 | 16 | 20 | 22 |

The other open-ended question regarding valid sources of proof was similarly coded and indexed, using the following categories that were identified in the initial review: self, proof,

checking work (plugging in results or reviewing steps), asking others, numeric/non-decimal answers, or asking the teacher.

Table 6A: Aggregated Percentage of Student Responses to "What does it take to convince you a math result is accurate?"

| | Pre | Post | Difference (Post-Pre) |
|---|---|---|---|
| Self | 21.7 | 81.9 | 60.2% |
| Proof | 41.6 | 106.5 | 64.9% |
| Check | 153.4 | 126 | -27.4% |
| Others | 103.4 | 104.3 | 0.9% |
| Numeric | 10 | 15.8 | 5.8% |
| Teacher | 18.3 | 52.8 | 34.5% |
| | | | |
| Sample Size | 47 | 53 | |

Table 6B: Percentage of Student Responses by Class Period to "What does it take to convince you a math result is accurate?"

| | Period 1 | | Period 2 | | Period 6 | |
|---|---|---|---|---|---|---|
| | Pre | Post | Pre | Post | Pre | Post |
| Self | 0.0 | 26.7 | 6.7 | 18.8 | 15.0 | 36.4 |
| Proof | 8.3 | 40.0 | 13.3 | 43.8 | 20.0 | 22.7 |
| Check | 66.7 | 33.3 | 46.7 | 56.3 | 40.0 | 36.4 |
| Others | 41.7 | 40.0 | 26.7 | 18.8 | 35.0 | 45.5 |
| Numeric | 0.0 | 6.7 | 0.0 | 0.0 | 10.0 | 9.1 |
| Teacher | 0.0 | 26.7 | 13.3 | 12.5 | 5.0 | 13.6 |
| | | | | | | |
| Sample Size | 12 | 15 | 15 | 16 | 20 | 22 |

At the end of the study, students were more likely to respond that they trust themselves, proof, and the teacher, with checking answers being a less common answer after the action

research cycles. All groups show a marked increase in student responses regarding trust in their own reasoning, as well as proof as a valid form of verification, while teacher was actually slightly decreased in second period. Trust in proof was slightly less pronounced in period 6. This group also had a higher post-intervention score for seeking others' approval than in the beginning of the study, while period 2 decreased the frequency of such responses over the intervention period. Period 1 responded less frequently that they relied on checking answers by plugging in the result or reviewing the steps of the process at the end of the study, while period 2 increased this response type slightly and period 6 remained roughly the same.

## Qualitative Data

Field notes were kept throughout the period of time spanning the two MAPS administrations, beginning February 19, 2019 and continuing through April 28, 2019. Several themes emerge upon review of notes. First, there are issues of classroom management: how to assess productivity, what to do with students who are chronically absent, and how to keep students evenly engaged in the material. Second, concerns over pacing arise, alongside worries over curricular decisions. Last, the theme of control over learning runs throughout the notes. Each of these contributes to the overall story of the changing classroom norms.

Regarding management issues, there initially are frequent mentions in the notes of frustration with how to encourage students to begin. This includes physical movement, as initially students were not moving their desks together when paired up but merely working in adjacent areas, which limited interaction. Additionally, students at the start of the study were clearly uncomfortable with the change to the existing classroom norm of taking notes and then working on problems. Attempts to create dialogue and interaction during what was viewed as

"quiet note-taking" time were a source of frustration for the teacher. In response to the teacher's question to the whole class of "What should we do next?" on a problem on the board in early March, one student responded, "We wait for you [the teacher] to do it." This frustrated the teacher but is also indicative that students were beginning to feel more comfortable expressing their thoughts. This interaction led to the realization by the teacher that 'notes' time needed to be altered to reach the goal of true student engagement in thinking critically. Shortly thereafter, guided notes were largely left behind in the classroom as typical activities evolved.

Difficulties in assessing the quality of group interaction, however, continued throughout the process. Some groups naturally worked well together, while others either ignored each other or fell into clear leader-follower dynamics rather than partnerships. Such comments regarding concerns over group dynamics in the field notes diminished as time progressed.

Issues also arose with increasing regularity, and particularly in Cycle 4 of the action research, regarding the time being spent to develop reasoning in a specific content area versus the pressure to cover all the intended curriculum. In addition to the shift towards longer time needed during notetaking in order to encourage discussion, records indicate that the teacher felt worried throughout the project with how to balance providing the time for exploration with the perceived need to move forward in the curriculum. For instance, in period 1 a conjecture list was provided by the teacher after the initial student conjectures and proofs about quadrilaterals had been presented. Student responses were positive, and they stated that this list sped up the process of conjecturing greatly. The possible reduction in cognitive demand was worrisome for the teacher, however. This conjecture list was presented later in the process to periods 2 and 6, with an intermediate step of a 'questions to explore' list presented instead (see Appendix A). However, this meant that these students were less successful at reaching the intended theorems.

Several notes throughout the research study, specifically in periods 2 and 6, describe problems of how to help students generalize their thinking. There is a pervasive concern that students in these classes appear to be tied to the specifics of an object, rather than making conceptual leaps to a more general case. Additionally, student frustration was perceived by the teacher was higher in these class periods. One group, in fact, ended up with one member having a panic attack and all three going to the counselor after presenting their first conjecture. This led to a productive conversation after class between the students and the teacher about how to separate a sense of self from an idea in order to handle feedback. The group proceeded to present other conjectures successfully. However, the combination of extended time being spent on open exploration, coupled with concerns about the depth of learning and the levels of student frustrations, continue throughout the recorded notes. Tensions of encouraging intellectual risks while still being supportive and providing scaffolds were often mentioned as a source of stress for the teacher.

This leads to the most important theme that emerged from the field notes: releasing control of the classroom learning environment. Several times, the teacher had to step in and ask the paraprofessional to stop handing students completed proofs, as the purpose of the activity was for students to practice reasoning, not to present fully formed ‘academic’ explanations. However, this left students seeking for their own solutions, which was a very new experience for them in a mathematics classroom. Several times, students expressed frustration with the assignment of exploring and conjecturing. However, by the end of the unit, students were also very excited about some of the unique results that they and their peers were finding. Control of their own learning was being deliberately shifted from the external authority of teacher to an internal locus, which led to discomfort for students. This was especially noted during

presentation times, where classes often looked to the teacher for subtle cues. Beyond obvious nods or verbalizations, students were also attuned to any minor shifts of facial features or even notetaking on the part of the teacher as possible indicators of accuracy. The teacher field notes reflect a conscious effort to be as inscrutable as possible while students were presenting their reasoning to the class in an attempt to shift the verification burden to the class. The decision to accept a proof, seek counterexamples, or ask for further explanation was left to the classroom community to decide.

This led to unintended results as verification was at times at least partly based on the perceived mathematical status of the student presenting. The teacher's desire to correct incorrect math as it was presented was confounded by the simultaneous desire to keep the burden of thinking critically on the students. This frustration over how to encourage deeper questioning when students were accepting inaccurate information without returning to the traditional role of "holder of knowledge" remained present throughout the research period. Tension over how to aid students without stepping in and removing their autonomy and intellectual ownership appear regularly through the notes.

Student work from this period shows a shift in what was expected in class. The use of scavenger hunts and task cards was deliberately increased to supplement the use of VRGs during the research period. These types of activities are accompanied by positive remarks in the field notes regarding student communication and progress. An unexpected positive consequence of such activities is also often mentioned, as student groups were able to progress at different speeds and students who were absent caught up at a separate pace from the rest of the group. Differentiation was occurring more frequently than in the more traditional lesson organization that was common before this research period began.

Additionally, student responses during this time started to show greater evidence of creative thinking than in previous work. In the types of proofs being offered for conjectures regarding quadrilaterals, students referenced reflection, symmetry, triangles, and many other topics that had not been discussed recently, without any prompting from the teacher. Also, when given back their textbooks as a resource, some students continued to use their own phrasing and conjecture sheets rather than those given by the textbook (see Appendix C for conjecture lists established by each class as proven statements). This was most evident in period 1, which also proved substantially more statements than period 6 did. When presented with task cards that used quadrilateral properties, students were able to identify gaps in their class's list of proven theorems, and successfully applied properties without direct instruction. This is a shift from previous instructional methods that had followed the "I-We-You" approach, to one in which "You-We-I" had become a more common investigatory mindset.

## Data Summary

Quantitative MAPS results by attitude category are inconclusive regarding possible shifts over the research period. Expertise index scores show a slight upward trend, though averages remained fairly static. Student responses about what they do in mathematics class, and where they look for authority, show shifts towards greater collaboration and self-confidence. This is supported by teacher field notes and student work, which show that students are more comfortable challenging each other's thinking, looking for explanations beyond the teacher, and communicating their ideas and observations to small groups. Tensions remain over how to manage timing, how to equitably assess thinking and learning, and the continued attempt to shift the responsibility for thinking and judging from the teacher to the student.

# Chapter 5

## Research Findings

There is not conclusive quantitative evidence that the incorporation of VRGs into the classroom culture caused a significant difference in student mathematical attitudes. This could be due to several possible factors. A major confounding factor is that classroom culture had been shifting prior to the start of the formal research period. This included activities beginning the first week of school and continuing throughout the first half of the year that had been selected to encourage reasoning, sense-making, and justification skills. Expertise index scores may have already been inflated over what they otherwise would have been as a result of these activities. It is unfortunate that the survey was not administered at the start of the academic year, rather than halfway through. Student responses regarding the types of activities they expect to engage in within the mathematics classroom show some changes, however, as working together and collaborating were reported far more often by the end of the research period. This indicates that though attitudes may not have changed within the relatively short timeframe of the study, expectations regarding what it means to be in a mathematical community were shifting.

This is supported by the records found in the teacher field notes. As VRG became more commonly accepted by the students, interaction and mathematical communication increased naturally. Importantly, classroom norms shifted to accommodate conversation related to mathematics, without coinciding with an increase of behavioral issues. In fact, the use of a set number of quarterly hall passes vanished entirely by the end of the year, as students were no longer seeking to leave the room. Engagement bell-to-bell stayed relatively consistent over the course of the research project. Students did not universally respond well to all the changes in the

classroom culture, but the incorporation of VRGs created minimal resistance in and of themselves.

Pairs and trios were by far the most commonly utilized groupings throughout this study. This emerged in a relatively organic way, as different pairings were attempted. The use of popsicle sticks with student names was selected early in the study as the most efficient method, as it allowed for students who were absent to be quickly removed while also creating a visibly random grouping system. Pairs were useful as two duos could then be easily grouped into a foursome to allow for comparison and further discussion. However, field notes show that groups of three seemed to work better during exploratory activities, possibly because students could support or argue over ideas more comfortably when it wasn't a direct confrontation with a single partner. As VRGs became a classroom norm, students initially needed reminders to turn desks together into groups. As time progressed, these reminders vanished, and students began asking if they were working with other people for the day. When given options to work independently or together, roughly two-thirds of students were selecting to team up into pairs or trios by the end of the research time frame. Classroom dynamics had also improved, as students were comfortable working with many different partners and were thus more willing to share ideas outside of their friend groups. However, some students that were perceived as difficult partners frequently remained isolated.  There was little outward reluctance to work in VRGs with these individuals, however, when paired by random methods.

# Chapter 6

# Conclusion

There is no one-size-fits-all approach that will improve every issue that exists in education. The purpose of this research study was to investigate what happens to students' ability to build mathematical arguments and critique those proposed by others when the classroom norm shifts to include visibly random grouping. For the students in the classrooms that were studied, much has changed yet much remains the same. Certainly, students seem to see math as including more collaboration and communication than they previously had expected. Yet their overall attitudes towards who is in control of verifying their results has not shifted clearly for all students. Encouragingly, there is a marked increase in student ownership of their work. All groups of students that were studied were much more likely to place their trust in their own reasoning by the end of the study, which is likely to be at least partially a result of having had so many experiences explaining their thinking to others by that point. However, whether such results would generalize to other groups of students remains to be seen. Also, it is possible that these changes are due to factors outside of the VRG structure in the classroom, as many factors changed throughout the course of the academic year.

In terms of ownership, students seem to have developed greater feelings of responsibility for thinking critically. This is apparent in the fact that all three classrooms, independently of each other, felt the need to create their own category for possible proofs – a space in between "accepted as proven" and "suspected to be false." Each grouping called this space something different ('proof needs improving', 'needs more explanation/information', and 'common sense but needs more proof' for period 1, 2, & 6 respectively) but all classes felt strongly enough to

advocate for the creation of this third option. This did not solve every issue, however, as classes often agreed to accept incomplete or downright incorrect justifications as mathematically accurate. Ownership of knowledge is a positive step but comes with its own dangers as control of assessing validity is handed over to the students.

While it is difficult to clearly delineate the effect VRGs had on the students, it is abundantly clear that the incorporation of VRGs into the classroom had a marked impact on the instructional practices of the teacher. In response to the goals of the action research being conducted, the instructional style shifted in a decisive way. No longer was the typical day structured around teacher-led notes and then mostly independent practice time; instead, students were expected to engage in exploratory tasks, communicate with each other, and respond to repeated iterations of the question “But why?” with mathematically justified arguments. The classroom environment itself shifted. Instead of rows of student desks, faced towards the front of the classroom, students now frequently move the desks into groups without being directed to do so. The physical space is much more fluid and adapts within a single class period several times, as students shift groups, combine and redivide, and swivel to look at the board for short periods of time. Pedagogical decisions now revolve around encouraging reasoning rather than answer-getting, as evidenced by changes in assessment types, classroom management strategies, and curriculum materials being selected.

This transition has not been smooth or easy. Students exhibited frustration, at times at high levels. The teacher also experienced frustration with the changes. Changing a culture, even one as small in scale as a classroom, is not an easy task to accomplish. There were days when things shifted back towards the traditional culture, and days when huge leaps forward occurred. There were days students asked great questions and engaged deeply with mathematical ideas,

and there were days when students were silent, sulky, checked out. And there were days when it seemed no learning at all took place, and other days when it was abundantly evident how much progress had occurred. This is attributable in part to VRGs, but also to so much more: a commitment to improving student success as critical thinkers that underpinned every decision that was made. Not all the decisions were successful, but on balance, students who experienced this instructional practice came away equipped to handle being challenged to explain their thinking, seeking rational explanations for arguments that are presented to them, and with content knowledge equivalent to what would have been gained through more traditional “sit-and-get” lessons. For these classes, VRGs were the catalyst inspiring many other cultural shifts and leading to profound changes in how both teacher and students view mathematical work.

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# Appendix A:

# Materials

MAPS questionnaire

Mathematics Attitudes & Perceptions Survey

This is a survey of your attitudes and perceptions about math; these statements all have the response choices "Strongly Agree", "Agree", "Neutral", "Disagree", and "Strongly Disagree. Please choose the response that matches *your* opinion, not what you think an instructor might say or want to hear.

| | Strongly Agree | Agree | Neutral | Disagree | Strongly Disagree |
|---|---|---|---|---|---|
| 1. After I study a topic in math and feel that I understand it, I have difficulty solving problems on the same topic. | | | | | |
| 2. There is usually only one correct approach to solving a math problem. | | | | | |
| 3. I'm satisfied if I can do the exercises for a math topic, even if I don't understand how everything works. | | | | | |
| 4. I do not expect formulas to help my understanding of mathematical ideas, they are just for doing calculations. | | | | | |
| 5. Math ability is something about a person that cannot be changed very much. | | | | | |
| 6. Nearly everyone is capable of understanding math if they work at it. | | | | | |
| 7. Understanding math means being able to recall something you've read or been shown. | | | | | |
| 8. If I am stuck on a math problem for more than ten minutes, I give up or get help from someone else. | | | | | |
| 9. I expect the answers to math problems to be numbers. | | | | | |
| 10. If I don't remember a particular formula needed to solve a problem on a math exam, there's nothing much I can do to come up with it. | | | | | |
| 11. In math, it is important for me to make sense out of formulas and procedures before I use them. | | | | | |
| 12. I enjoy solving math problems. | | | | | |
| 13. Learning math changes my ideas about how the world works. | | | | | |
| 14. I often have difficulty organizing my thoughts during a math test. | | | | | |
| 15. Reasoning skills used to understand math can be helpful to me in my everyday life. | | | | | |
| 16. To learn math, the best approach for me is to memorize solutions to sample problems. | | | | | |
| 17. No matter how much I prepare, I am still not confident when taking math tests. | | | | | |
| 18. It is a waste of time to understand where math formulas come from. | | | | | |
| 19. We use this statement to discard the survey of people who are not reading the questions. Please select Agree (not Strongly Agree) for this question. | | | | | |

| | | | | | |
|---|---|---|---|---|---|
| 20. I can usually figure out a way to solve math problems. | | | | | |
| 21. School mathematics has little to do with what I experience in the real world. | | | | | |
| 22. Being good at math requires natural (i.e., innate, inborn) intelligence in math. | | | | | |
| 23. When I am solving a math problem, if I can see a formula that applies then I don't worry about the underlying concepts. | | | | | |
| 24. If I get stuck on a math problem, there is no chance that I will figure it out on my own. | | | | | |
| 25. When learning something new in math, I relate it to what I already know rather than just memorizing it the way it is presented. | | | | | |
| 26. I avoid solving math problems when possible. | | | | | |
| 27. I think it is unfair to expect me to solve a math problem that is not similar to any example given in class or the textbook, even if the topic has been covered in the course. | | | | | |
| 28. All I need to solve a math problem is to have the necessary formulas. | | | | | |
| 29. I get upset easily when I am stuck on a math problem. | | | | | |
| 30. Showing intermediate steps for a math problem is not important as long as I can find the correct answer. | | | | | |
| 31. For each person, there are math concepts that they would never be able to understand, even if they tried. | | | | | |
| 32. I only learn math when it is required. | | | | | |
| 33. I trust answers when they are told to me by the teacher. | | | | | |
| 34. I trust answers when they are told to me by another student. | | | | | |
| 35. I trust answers when I find them myself. | | | | | |
| 36. I feel confident explaining my answers, even if others disagree. | | | | | |
| 37. I feel confident explaining why someone else's answer is incomplete. | | | | | |
| 38. Talking through an answer with other people helps me find ways to improve my explanations. | | | | | |

39. What are typical activities that students are expected to do in math classes?

40. What does it take to convince you a math result is accurate?

Conjecture List Provided to Period 1

If parallelogram, then opposite sides congruent

If parallelogram, then opposite angle congruent

If parallelogram, then consec angles are suppl

If parallelogram has one right angle, then all right angles

If parallelogram, then diagonals bisect

If parallelogram, then diagonal creates two congruent triangles

If both sets opposite sides congruent, then parallelogram

If both sets opposite angles congruent, then parallelogram

If both diagonals bisect, then parallelogram

If one set opposite sides parallelogramllel and congruent, then parallelogram

If rect, then diagonals are congruent

If diagonals in a parallelogram congruent, then rect

If rhombus, then diagonals perpendicular

If rhombus, then diagonals bisect opposite angles

If diagonals in a parallelogram perpendicular, then rhombus

If one diagonal of parallelogram bisects opposite angles, then rhombus

If one set consec sides in a parallelogram are congruent, then rhombus

If trap isos, then two sets of base angles are congruent

If trap has congruent base angles, then isos

If trap isos, diagonals congruent

If diags congruent in a trap, then isos

If kite, then diagonals perpendicular

If kite, one pair of opposite angles congruent and one pair not

Conjecture List Provided to Period 2 (Following the questions to explore)

Period 2 Possible Conjectures

If parallelogram, then diagonals bisect.

If both sets opposite sides congruent, then parallelogram

If both sets opposite angles congruent, then parallelogram

If both diagonals bisect, then parallelogram

If one set opposite sides parallelogram and congruent, then parallelogram

If rectangle, then diagonals are congruent

If diagonals in a parallelogram congruent, then rectangle

If rhombus, then diagonals perpendicular

If rhombus, then diagonals bisect opposite angles

If diagonals in a parallelogram perpendicular, then rhombus

If one diagonal of parallelogram bisects opposite angles, then rhombus

If one set consecutive sides in a parallelogram are congruent, then rhombus

If trap isosceles, then two sets of base angles are congruent

If trap has congruent base angles, then isosceles

If trap isosceles, diagonals congruent

If diagonals congruent in a trap, then isosceles

If kite, then diagonals perpendicular

If kite, one pair of opposite angles congruent and one pair not

Conjecture List Provided to Period 6 (following the questions to explore)

Period 6 Possible Conjectures

If parallelogram, then opposite angle congruent

If parallelogram has one right angle, then all right angles

If parallelogram, then diagonals bisect

If parallelogram, then diagonal creates two congruent triangles

If both sets opposite sides congruent, then parallelogram

If both sets opposite angles congruent, then parallelogram

If both diagonals bisect, then parallelogram

If one set opposite sides parallelogram and congruent, then parallelogram

If rect, then diagonals are congruent

If diagonals in a parallelogram congruent, then rect

If rhombus, then diagonals perpendicular

If rhombus, then diagonals bisect opposite angles

If diagonals in a parallelogram perpendicular, then rhombus

If one diagonal of parallelogram bisects opposite angles, then rhombus

If one set consec sides in a parallelogram are congruent, then rhombus

If trap isos, then two sets of base angles are congruent

If trap has congruent base angles, then isos

If trap isos, diagonals congruent

If diags congruent in a trap, then isos

If kite, then diagonals perpendicular

If kite, one pair of opposite angles congruent and one pair not

Questions to Explore

Period 2 Questions (Spring 2019)

Are opposite sides of a parallelogram congruent?

Are opposite angle of a parallelogram congruent?

If a parallelogram has one right angle, what are the other angles?

Do the diagonals in a parallelogram cut each other in half?

Does a diagonal in a parallelogram create two congruent triangles?

Does the shape HAVE to be a parallelogram if

… Both sets of opposite sides are congruent?

… Both sets of opposite angles are congruent?

… Both diagonals bisect each other?

… ONE set of opposite sides are both parallelogramllel AND congruent?

In what type of quadrilaterals are the diagonals congruent?

In what type of quadrilaterals are the diagonals perpendicular to each other?

In what type of quadrilaterals do the diagonals bisect the angles?

If one set of consecutive sides in a parallelogram are congruent, what happens?

What properties does an isosceles trapezoid have?

Period 6 Questions (Spring 2019)

If a parallelogram has one right angle, what are the other angles?

Do the diagonals in a parallelogram cut each other in half?

Does the shape HAVE to be a parallelogram if

… Both sets of opposite sides are congruent?

… Both sets of opposite angles are congruent?

… Both diagonals bisect each other?

… ONE set of opposite sides are both parallelogramllel AND congruent?

In what type of quadrilaterals are the diagonals congruent?

In what type of quadrilaterals are the diagonals perpendicular to each other?

In what type of quadrilaterals do the diagonals bisect the angles?

If one set of consecutive sides in a parallelogram are congruent, what happens?

What properties does an isosceles trapezoid have?

Are there congruent angles in a kite?

Are there congruent triangles formed by diagonals in shapes besides parallelograms?

Appendix B:

Data

MAPS Category Data Mean & Standard Deviation, By Period

| Confidence | | | | | | |
|---|---|---|---|---|---|---|
| | Period 1 | | Period 2 | | Period 6 | |
| | Pre | Post | Pre | Post | Pre | Post |
| Average | 10.833 | 11.933 | 13.6 | 13.8438 | 14.05 | 14.3182 |
| Standard Deviation | 1.8328 | 2.3795 | 2.67831 | 2.11925 | 2.74727 | 2.4005 |

| Answers | | | | | | |
|---|---|---|---|---|---|---|
| | Period 1 | | Period 2 | | Period 6 | |
| | Pre | Post | Pre | Post | Pre | Post |
| Average | 16.16667 | 16.73333 | 16.2 | 16.625 | 17.15 | 15.75 |
| Standard Deviation | 2.640497 | 3.065217 | 2.66333 | 3.62069 | 2.97111 | 3.55397 |

| Sense Making | | | | | | |
|---|---|---|---|---|---|---|
| | Period 1 | | Period 2 | | Period 6 | |
| | Pre | Post | Pre | Post | Pre | Post |
| Average | 14.0833 | 14.3333 | 15.0667 | 14.625 | 15.8 | 15.5227 |
| Standard Deviation | 2.05987 | 2.087 | 2.20504 | 1.40868 | 1.96469 | 1.96837 |

| Growth Mindset | | | | | | |
|---|---|---|---|---|---|---|
| | Period 1 | | Period 2 | | Period 6 | |
| | Pre | Post | Pre | Post | Pre | Post |
| Average | 10.9167 | 11.1333 | 10.6 | 11.0625 | 10.9 | 10.81818 |
| Standard Deviation | 1.25554 | 1.35974 | 1.624808 | 1.675886 | 1.75784 | 1.585054 |

| Persistence | | | | | | |
|---|---|---|---|---|---|---|
| | Period 1 | | Period 2 | | Period 6 | |
| | Pre | Post | Pre | Post | Pre | Post |
| Average | 10.5833 | 11.3333 | 11.93333 | 11.5 | 11.9 | 11.3636 |
| Standard Deviation | 1.65622 | 3.13404 | 2.59401 | 2 | 2.50799 | 3.1268 |
| Sample Size | | | | | | |
| Interest | | | | | | |
| | Period 1 | | Period 2 | | Period 6 | |
| | Pre | Post | Pre | Post | Pre | Post |
| Average | 8.25 | 9.2667 | 9.1 | 9.375 | 9.1 | 9.04545 |
| Standard Deviation | 0.9242 | 1.436 | 1.485485 | 0.992157 | 1.51327 | 1.55146 |

| Real World | | | | | | |
|---|---|---|---|---|---|---|
| | Period 1 | | Period 2 | | Period 6 | |
| | Pre | Post | Pre | Post | Pre | Post |
| Average | 13.66667 | 13.13333 | 12.46667 | 12.8125 | 12.9 | 12.9318 |
| Standard Deviation | 1.840894 | 2.028683 | 1.024153 | 1.943539 | 1.86815 | 1.90869 |

| Verification | | | | | | |
|---|---|---|---|---|---|---|
| | Period 1 | | Period 2 | | Period 6 | |
| | Pre | Post | Pre | Post | Pre | Post |
| Average | 21.16667 | 20.7 | 20.56667 | 20.5625 | 18.45 | 19.2955 |
| Standard Deviation | 2.733537 | 2.803569 | 3.270406 | 3.061224 | 2.94066 | 3.12853 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Sample Size | 12 | 15 | 15 | 16 | 20 | 22 |

MAPS Data – By Category

Period 1:

| | Confidence | | | | | Answers | | | | | | | Sense Making | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Pre** | **1** | **14** | **17** | **20** | | **2** | **7** | **9** | **16** | **28** | **30** | | **3** | **4** | **11** | **18** | **23** | |
| | 2 | 2 | 2 | 4 | 10 | 1 | 2 | 3 | 1 | 2 | 4 | 13 | 4 | 3 | 4 | 2 | 2 | 15 |
| | 3 | 2 | 2 | 4 | 11 | 1 | 3 | 2 | 3 | 3 | 4 | 16 | 2 | 3 | 4 | 3 | 3 | 15 |
| | 4 | 2 | 2 | 4 | 12 | 2 | 4 | 2 | 4 | 2 | 2 | 16 | 2 | 3 | 4 | 3 | 3 | 15 |
| | 4 | 1 | 2 | 4 | 11 | 1 | 3 | 2 | 3 | 3 | 3 | 15 | 2 | 2 | 3 | 2 | 2 | 11 |
| | 2 | 2 | 2 | 4 | 10 | 1 | 3 | 2 | 2 | 4 | 5 | 17 | 1 | 3 | 4 | 1 | 3 | 12 |
| | 2 | 2 | 2 | 4 | 10 | 5 | 2 | 3 | 1 | 2 | 5 | 18 | 2 | 2 | 4 | 1 | 1 | 10 |
| | 2 | 2 | 1 | 4 | 9 | 3 | 3 | 2 | 2 | 3 | 1 | 14 | 2 | 3 | 4 | 1 | 3 | 13 |
| | 3 | 2 | 1 | 4 | 10 | 4 | 4 | 3 | 2 | 4 | 4 | 21 | 4 | 2 | 4 | 3 | 3 | 16 |
| | 3 | 3 | 3 | 3 | 12 | 3 | 4 | 4 | 2 | 3 | 4 | 20 | 3 | 2 | 4 | 2 | 3 | 14 |
| | 4 | 4 | 4 | 4 | 16 | 2 | 2 | 4 | 4 | 2 | 4 | 18 | 2 | 3 | 5 | 3 | 3 | 16 |
| | 1 | 1 | 1 | 5 | 8 | 2 | 2 | 1 | 2 | 2 | 3 | 12 | 4 | 2 | 4 | 2 | 3 | 15 |
| | 3 | 2 | 2 | 4 | 11 | 2 | 2 | 3 | 2 | 2 | 3 | 14 | 4 | 4 | 3 | 3 | 3 | 17 |
| | **3** | **2** | **2** | **4** | **10.83** | **2** | **3** | **3** | **2** | **3** | **4** | **16.1667** | **3** | **3** | **4** | **2** | **3** | **14.0833** |
| | **1** | **1** | **1** | **0** | **1.833** | **1** | **1** | **1** | **1** | **1** | **1** | **2.6405** | **1** | **1** | **0** | **1** | **1** | **2.05987** |
| | | | | | | | | | | | | | | | | | | |
| **Pos** | **1** | **14** | **17** | **20** | | **2** | **7** | **9** | **16** | **28** | **30** | | **3** | **4** | **11** | **18** | **23** | |
| | 2 | 2 | 1 | 4 | 9 | 3 | 4 | 4 | 2 | 2 | 5 | 20 | 2 | 2 | 5 | 1 | 4 | 14 |
| | 2 | 4 | 2 | 4 | 12 | 2 | 4 | 2 | 4 | 2 | 2 | 16 | 4 | 2 | 4 | 2 | 2 | 14 |
| | 2 | 2 | 2 | 3 | 9 | 1 | 3 | 4 | 2 | 3 | 1 | 14 | 2 | 4 | 3 | 2 | 2 | 13 |
| | 2 | 3 | 2 | 4 | 11 | 2 | 3 | 2 | 2 | 4 | 5 | 18 | 3 | 3 | 4 | 2 | 3 | 15 |
| | 2 | 1 | 1 | 5 | 9 | 2 | 1 | 2 | 3 | 3 | 2 | 13 | 2 | 1 | 4 | 1 | 4 | 12 |
| | 1 | 2 | 3 | 5 | 11 | 2 | 3 | 2 | 2 | 3 | 5 | 17 | 2 | 4 | 4 | 1 | 2 | 13 |
| | 2 | 4 | 4 | 4 | 14 | 1 | 3 | 3 | 3 | 2 | 2 | 14 | 3 | 2 | 3 | 2 | 4 | 14 |
| | 3 | 2 | 4 | 4 | 13 | 2 | 3 | 2 | 3 | 2 | 2 | 14 | 3 | 3 | 4 | 2 | 2 | 14 |
| | 2 | 2 | 2 | 4 | 10 | 2 | 2 | 3 | 3 | 2 | 2 | 14 | 4 | 2 | 4 | 2 | 3 | 15 |
| | 4 | 4 | 5 | 4 | 17 | 4 | 4 | 5 | 3 | 4 | 4 | 24 | 2 | 4 | 4 | 5 | 4 | 19 |
| | 3 | 2 | 4 | 4 | 13 | 2 | 2 | 3 | 4 | 4 | 4 | 19 | 3 | 2 | 4 | 2 | 3 | 14 |
| | 2 | 2 | 2 | 4 | 10 | 1 | 3 | 4 | 3 | 3 | 4 | 18 | 3 | 3 | 4 | 4 | 4 | 18 |
| | 4 | 2 | 3 | 4 | 13 | 2 | 3 | 3 | 3 | 3 | 3 | 17 | 3 | 3 | 3 | 3 | 3 | 15 |
| | 5 | 4 | 3 | 4 | 16 | 3 | 4 | 4 | 3 | 4 | 2 | 20 | 1 | 3 | 5 | 4 | 2 | 15 |
| | 5 | 1 | 1 | 5 | 12 | 1 | 2 | 1 | 3 | 3 | 3 | 13 | 2 | 1 | 4 | 1 | 2 | 10 |
| | **3** | **2** | **3** | **4** | **11.93** | **2** | **3** | **3** | **3** | **3** | **3** | **16.7333** | **3** | **3** | **4** | **2** | **3** | **14.3333** |
| | **1** | **1** | **1** | **0** | **2.38** | **1** | **1** | **1** | **1** | **1** | **1** | **3.06522** | **1** | **1** | **1** | **1** | **1** | **2.087** |
| | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | pvalue 0.7583 | | | | |
| | | | | | | | | | | | | | | no sig diff | | | | |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Growth Mindset | | | | | | Persistence | | | | | | Interest | | | |
| 5 | 6 | 22 | 31 | | | 8 | 10 | 24 | 29 | | | 12 | 26 | 32 | |
| 2 | 4 | 2 | 2 | 10 | | 3 | 3 | 2 | 2 | 10 | | 4 | 2 | 1 | 7 |
| 2 | 4 | 2 | 3 | 11 | | 3 | 2 | 3 | 4 | 12 | | 3 | 2 | 3 | 8 |
| 3 | 4 | 3 | 1 | 11 | | 4 | 3 | 2 | 5 | 14 | | 3 | 4 | 2 | 9 |
| 1 | 4 | 2 | 3 | 10 | | 3 | 2 | 2 | 2 | 9 | | 3 | 2 | 2 | 7 |
| 2 | 5 | 3 | 2 | 12 | | 2 | 3 | 2 | 3 | 10 | | 3 | 3 | 3 | 9 |
| 1 | 4 | 2 | 1 | 8 | | 4 | 2 | 1 | 4 | 11 | | 4 | 2 | 2 | 8 |
| 1 | 5 | 4 | 1 | 11 | | 2 | 3 | 1 | 2 | 8 | | 5 | 2 | 1 | 8 |
| 2 | 5 | 2 | 2 | 11 | | 3 | 2 | 2 | 3 | 10 | | 3 | 2 | 2 | 7 |
| 3 | 4 | 3 | 3 | 13 | | 3 | 3 | 2 | 2 | 10 | | 3 | 2 | 3 | 8 |
| 3 | 3 | 4 | 2 | 12 | | 4 | 4 | 3 | 2 | 13 | | 2 | 4 | 4 | 10 |
| 2 | 3 | 3 | 2 | 10 | | 3 | 2 | 2 | 2 | 9 | | 5 | 2 | 2 | 9 |
| 3 | 4 | 2 | 3 | 12 | | 3 | 3 | 3 | 2 | 11 | | 3 | 2 | 4 | 9 |
| **2** | **4** | **3** | **2** | **10.9167** | | **3** | **3** | **2** | **3** | **10.583** | | **3** | **2** | **2** | **8.25** |
| **1** | **1** | **1** | **1** | **1.25554** | | **1** | **1** | **1** | **1** | **1.6562** | | **1** | **1** | **1** | **0.9242** |
| | | | | | | | | | | | | | | | |
| 5 | 6 | 22 | 31 | | | 8 | 10 | 24 | 29 | | | 12 | 26 | 32 | |
| 2 | 4 | 3 | 2 | 11 | | 4 | 4 | 2 | 3 | 13 | | 4 | 4 | 3 | 11 |
| 2 | 3 | 3 | 2 | 10 | | 4 | 2 | 2 | 5 | 13 | | 3 | 3 | 4 | 10 |
| 1 | 5 | 2 | 2 | 10 | | 4 | 4 | 2 | 4 | 14 | | 2 | 3 | 4 | 9 |
| 2 | 4 | 2 | 4 | 12 | | 4 | 3 | 3 | 4 | 14 | | 3 | 3 | 4 | 10 |
| 3 | 5 | 3 | 1 | 12 | | 3 | 2 | 1 | 2 | 8 | | 5 | 2 | 2 | 9 |
| 3 | 4 | 2 | 2 | 11 | | 3 | 3 | 2 | 2 | 10 | | 4 | 3 | 2 | 9 |
| 2 | 4 | 2 | 4 | 12 | | 3 | 2 | 2 | 2 | 9 | | 1 | 4 | 4 | 9 |
| 2 | 3 | 2 | 2 | 9 | | 3 | 3 | 2 | 3 | 11 | | 3 | 2 | 2 | 7 |
| 2 | 4 | 3 | 3 | 12 | | 3 | 2 | 2 | 2 | 9 | | 4 | 2 | 3 | 9 |
| 3 | 4 | 3 | 4 | 14 | | 5 | 5 | 4 | 5 | 19 | | 3 | 5 | 5 | 13 |
| 2 | 4 | 2 | 2 | 10 | | 3 | 3 | 3 | 2 | 11 | | 4 | 3 | 2 | 9 |
| 2 | 4 | 2 | 3 | 11 | | 3 | 4 | 3 | 3 | 13 | | 1 | 4 | 4 | 9 |
| 2 | 4 | 2 | 3 | 11 | | 3 | 2 | 2 | 3 | 10 | | 3 | 3 | 4 | 10 |
| 2 | 4 | 3 | 4 | 13 | | 2 | 4 | 3 | 2 | 11 | | 2 | 2 | 4 | 8 |
| 2 | 4 | 2 | 1 | 9 | | 2 | 1 | 1 | 1 | 5 | | 5 | 1 | 1 | 7 |
| **2** | **4** | **2** | **3** | **11.1333** | | **3** | **3** | **2** | **3** | **11.333** | | **3** | **3** | **3** | **9.2667** |
| **0** | **1** | **0** | **1** | **1.35974** | | **1** | **1** | **1** | **1** | **3.134** | | **1** | **1** | **1** | **1.436** |
| | | | | | | | | | | | | | | | |
| | pvalue 0.6741 | | | | | pvalue .4616 | | | | | | | pvalue 0.0439 | | |
| | no sig diff | | | | | no sig diff | | | | | | | SIG DIFF?!? | | |

| Real World | | | | | | Verification | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **13** | **15** | **21** | **25** | | | **33** | **34** | **35** | **36** | **37** | **38** | |
| 4 | 4 | 3 | 4 | 15 | | 4 | 4 | 4 | 3 | 4 | 3 | 22 |
| 3 | 3 | 2 | 4 | 12 | | 3 | 3 | 4 | 4 | 4 | 4 | 22 |
| 3 | 4 | 4 | 4 | 15 | | 4 | 3 | 3 | 2 | 3 | 5 | 20 |
| 2 | 3 | 3 | 4 | 12 | | 4 | 3 | 3 | 3 | 3 | 4 | 20 |
| 3 | 5 | 3 | 4 | 15 | | 2 | 2 | 3 | 3 | 3 | 3 | 16 |
| 4 | 4 | 3 | 5 | 16 | | 4 | 3 | 4 | 4 | 5 | 5 | 25 |
| 3 | 4 | 3 | 5 | 15 | | 5 | 3 | 4 | 4 | 3 | 3 | 22 |
| 3 | 3 | 3 | 2 | 11 | | 3 | 3 | 3 | 3 | 2 | 4 | 18 |
| 2 | 2 | 4 | 2 | 10 | | 4 | 3 | 4 | 3 | 4 | 4 | 22 |
| 3 | 3 | 5 | 3 | 14 | | 3 | 3 | 2 | 2 | 3 | 5 | 18 |
| 4 | 4 | 3 | 4 | 15 | | 3 | 3 | 5 | 5 | 4 | 5 | 25 |
| 3 | 4 | 3 | 4 | 14 | | 4 | 4 | 4 | 4 | 4 | 4 | 24 |
| **3** | **4** | **3** | **4** | **13.66667** | | **4** | **3** | **4** | **3** | **4** | **4** | **21.1667** |
| **1** | **1** | **1** | **1** | **1.840894** | | **1** | **0** | **1** | **1** | **1** | **1** | **2.73354** |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **13** | **15** | **21** | **25** | | | **33** | **34** | **35** | **36** | **37** | **38** | |
| 4 | 4 | 3 | 5 | 16 | | 4 | 4 | 5 | 5 | 4 | 5 | 27 |
| 4 | 4 | 2 | 4 | 14 | | 4 | 3 | 3 | 3 | 4 | 4 | 21 |
| 4 | 4 | 2 | 4 | 14 | | 4 | 4 | 4 | 4 | 4 | 4 | 23.5 |
| 3 | 4 | 3 | 3 | 13 | | 3 | 2 | 4 | 4 | 4 | 4 | 21 |
| 4 | 4 | 2 | 3 | 13 | | 4 | 3 | 4 | 3 | 4 | 4 | 22 |
| 4 | 4 | 4 | 4 | 16 | | 3 | 3 | 3 | 3 | 3 | 4 | 19 |
| 3 | 4 | 3 | 4 | 14 | | 4 | 2 | 2 | 2 | 3 | 3 | 16 |
| 3 | 4 | 3 | 4 | 14 | | 4 | 3 | 3 | 3 | 3 | 4 | 20 |
| 3 | 4 | 2 | 3 | 12 | | 3 | 2 | 3 | 3 | 2 | 4 | 17 |
| 1 | 1 | 5 | 2 | 9 | | 5 | 5 | 5 | 1 | 2 | 3 | 21 |
| 3 | 4 | 2 | 2 | 11 | | 3 | 3 | 3 | 2 | 3 | 3 | 17 |
| 1 | 3 | 3 | 2 | 9 | | 4 | 3 | 3 | 4 | 3 | 4 | 21 |
| 3 | 4 | 3 | 4 | 14 | | 4 | 4 | 4 | 4 | 4 | 4 | 24 |
| 3 | 4 | 3 | 4 | 14 | | 3 | 3 | 2 | 2 | 4 | 5 | 19 |
| 4 | 4 | 2 | 4 | 14 | | 3 | 2 | 5 | 5 | 4 | 3 | 22 |
| **3** | **4** | **3** | **3** | **13.13333** | | **4** | **3** | **4** | **3** | **3** | **4** | **20.7** |
| **1** | **1** | **1** | **1** | **2.028683** | | **1** | **1** | **1** | **1** | **1** | **1** | **2.80357** |
| | | | | | | | | | | | | |
| | pvalue 0.4862 | | | | | | pvalue 0.6676 | | | | | |
| | no sig diff | | | | | | no sig diff | | | | | |

Period 2;

| | Confidence | | | | |
|---|---|---|---|---|---|
| Pre | 1 | 14 | 17 | 20 | |
| | 4 | 3 | 2 | 4 | 13 |
| | 4 | 4 | 3 | 4 | 15 |
| | 3 | 3 | 5 | 4 | 15 |
| | 3 | 3 | 5 | 4 | 15 |
| | 2 | 2 | 1 | 5 | 10 |
| | 2 | 2 | 3 | 3 | 10 |
| | 3 | 4 | 4 | 3 | 14 |
| | 5 | 5 | 5 | 4 | 19 |
| | 3 | 4 | 3 | 3 | 13 |
| | 3 | 3 | 3 | 3 | 12 |
| | 5 | 2 | 3 | 3 | 13 |
| | 3 | 3 | 1 | 4 | 11 |
| | 2 | 2 | 2 | 4 | 10 |
| | 4 | 5 | 5 | 3 | 17 |
| | 4 | 5 | 5 | 3 | 17 |
| | **3** | **3** | **3** | **4** | **13.6** |
| | **1** | **1** | **1** | **1** | **2.67831** |
| Pos | 1 | 14 | 17 | 20 | |
| | 2 | 4 | 5 | 3 | 14 |
| | 3 | 3 | 3 | 3 | 12 |
| | 2 | 4 | 3 | 3 | 12 |
| | 2 | 2 | 2 | 4 | 10 |
| | 3 | 3 | 5 | 3 | 13.5 |
| | 2 | 5 | 5 | 4 | 16 |
| | 4 | 5 | 5 | 2 | 16 |
| | 4 | 5 | 5 | 3 | 17 |
| | 3 | 4 | 4 | 3 | 14 |
| | 2 | 4 | 4 | 5 | 15 |
| | 3 | 2 | 2 | 4 | 11 |
| | 3 | 5 | 4 | 3 | 15 |
| | 4 | 5 | 4 | 4 | 17 |
| | 4 | 4 | 3 | 4 | 15 |
| | 2 | 3 | 2 | 4 | 11 |
| | 3 | 3 | 4 | 3 | 13 |
| | **3** | **4** | **4** | **3** | **13.8438** |
| | **1** | **1** | **1** | **1** | **2.11925** |
| | pval 0.78 | | | | |
| | no sig diff | | | | |

| | Answers | | | | | | |
|---|---|---|---|---|---|---|---|
| Pre | 2 | 7 | 9 | 16 | 28 | 30 | |
| | 2 | 2 | 3 | 2 | 3 | 4 | 16 |
| | 2 | 4 | 2 | 3 | 3 | 3 | 17 |
| | 1 | 2 | 2 | 4 | 2 | 2 | 13 |
| | 1 | 4 | 4 | 1 | 4 | 2 | 16 |
| | 1 | 1 | 2 | 1 | 4 | 5 | 14 |
| | 2 | 2 | 4 | 3 | 3 | 5 | 19 |
| | 1 | 1 | 4 | 3 | 3 | 2 | 14 |
| | 4 | 3 | 3 | 3 | 3 | 3 | 19 |
| | 2 | 4 | 2 | 3 | 4 | 4 | 19 |
| | 2 | 2 | 2 | 3 | 3 | 2 | 14 |
| | 3 | 2 | 2 | 2 | 1 | 2 | 12 |
| | 2 | 3 | 2 | 4 | 5 | 4 | 20 |
| | 2 | 4 | 2 | 3 | 3 | 3 | 17 |
| | 4 | 1 | 3 | 3 | 4 | 5 | 20 |
| | 2 | 2 | 2 | 2 | 3 | 2 | 13 |
| | **2** | **2** | **3** | **3** | **3** | **3** | **16.2** |
| | **1** | **1** | **1** | **1** | **1** | **1** | **2.66333** |
| Pos | 2 | 7 | 9 | 16 | 28 | 30 | |
| | 1 | 2 | 3 | 5 | 5 | 5 | 21 |
| | 2 | 3 | 2 | 2 | 3 | 4 | 16 |
| | 3 | 1 | 2 | 2 | 1 | 1 | 10 |
| | 2 | 4 | 2 | 2 | 4 | 4 | 18 |
| | 2 | 4 | 2 | 2 | 4 | 3 | 17 |
| | 3 | 3 | 3 | 3 | 3 | 2 | 17 |
| | 1 | 2 | 2 | 2 | 2 | 2 | 11 |
| | 2 | 4 | 4 | 5 | 2 | 4 | 21 |
| | 4 | 4 | 4 | 4 | 3 | 4 | 23 |
| | 1 | 3 | 2 | 4 | 2 | 2 | 14 |
| | 1 | 4 | 4 | 4 | 4 | 5 | 22 |
| | 1 | 2 | 5 | 3 | 3 | 1 | 15 |
| | 2 | 4 | 3 | 3 | 3 | 1 | 16 |
| | 1 | 2 | 4 | 2 | 4 | 3 | 16 |
| | 1 | 1 | 2 | 1 | 3 | 5 | 13 |
| | 2 | 2 | 3 | 3 | 4 | 2 | 16 |
| | **2** | **3** | **3** | **3** | **3** | **3** | **16.625** |
| | **1** | **1** | **1** | **1** | **1** | **1** | **3.62069** |
| | pval 0.7139 | | | | | | |
| | no sig diff | | | | | | |

| | Sense Making | | | | | |
|---|---|---|---|---|---|---|
| Pre | 3 | 4 | 11 | 18 | 23 | |
| | 3 | 4 | 4 | 4 | 2 | 17 |
| | 3 | 3 | 3 | 2 | 2 | 13 |
| | 3 | 2 | 4 | 3 | 3 | 15 |
| | 3 | 2 | 4 | 3 | 2 | 14 |
| | 4 | 5 | 3 | 3 | 4 | 19 |
| | 4 | 2 | 5 | 2 | 4 | 17 |
| | 4 | 3 | 3 | 2 | 2 | 14 |
| | 2 | 5 | 5 | 5 | 3 | 20 |
| | 3 | 2 | 3 | 3 | 2 | 13 |
| | 2 | 1 | 4 | 2 | 3 | 12 |
| | 3 | 2 | 5 | 2 | 2 | 14 |
| | 3 | 3 | 3 | 2 | 4 | 15 |
| | 3 | 2 | 4 | 3 | 3 | 15 |
| | 1 | 3 | 5 | 2 | 2 | 13 |
| | 4 | 1 | 5 | 1 | 4 | 15 |
| | **3** | **3** | **4** | **3** | **3** | **15.0667** |
| | **1** | **1** | **1** | **1** | **1** | **2.20504** |
| Pos | 3 | 4 | 11 | 18 | 23 | |
| | 4 | 3 | 3 | 1 | 5 | 16 |
| | 4 | 2 | 4 | 3 | 3 | 16 |
| | 1 | 2 | 5 | 1 | 2 | 11 |
| | 3 | 2 | 3 | 3 | 3 | 14 |
| | 3 | 2 | 4 | 3 | 3 | 15 |
| | 2 | 2 | 5 | 2 | 3 | 14 |
| | 2 | 2 | 3 | 2 | 4 | 13 |
| | 4 | 3 | 2 | 2 | 4 | 15 |
| | 2 | 3 | 3 | 4 | 3 | 15 |
| | 3 | 2 | 3 | 3 | 3 | 14 |
| | 4 | 2 | 4 | 2 | 4 | 16 |
| | 3 | 2 | 4 | 2 | 3 | 14 |
| | 3 | 3 | 4 | 2 | 2 | 14 |
| | 4 | 3 | 3 | 3 | 4 | 17 |
| | 4 | 2 | 4 | 3 | 3 | 16 |
| | 3 | 2 | 4 | 2 | 3 | 14 |
| | **3** | **2** | **4** | **2** | **3** | **14.625** |
| | **1** | **0** | **1** | **1** | **1** | **1.40868** |
| | pval 0.5087 | | | | | |
| | no sig diff | | | | | |

| Growth Mindset | | | | | | Persistence | | | | | | Interest | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **5** | **6** | **22** | **31** | | | **8** | **10** | **24** | **29** | | | **12** | **26** | **32** | |
| 2 | 4 | 2 | 2 | 10 | | 3 | 2 | 2 | 3 | 10 | | 1 | 4 | 4 | 9 |
| 2 | 4 | 2 | 3 | 11 | | 3 | 1 | 2 | 3 | 9 | | 1 | 4 | 3 | 8 |
| 3 | 4 | 2 | 3 | 12 | | 4 | 3 | 2 | 4 | 13 | | 3 | 3 | 3 | 9 |
| 2 | 4 | 2 | 5 | 13 | | 4 | 3 | 3 | 5 | 15 | | 2 | 2 | 3 | 7 |
| 1 | 5 | 1 | 1 | 8 | | 3 | 3 | 2 | 1 | 9 | | 5 | 1 | 1 | 7 |
| 2 | 4 | 4 | 2 | 12 | | 4 | 2 | 2 | 3 | 11 | | 3 | 4 | 5 | 12 |
| 3 | 4 | 2 | 4 | 13 | | 5 | 3 | 3 | 4 | 15 | | 2 | 4 | 4 | 10 |
| 1 | 2 | 3 | 2 | 8 | | 4 | 4 | 5 | 5 | 18 | | 1 | 5 | 5 | 11 |
| 2 | 4 | 2 | 2 | 10 | | 4 | 2 | 2 | 2 | 10 | | 3 | 2 | 3 | 8 |
| 2 | 3 | 2 | 3 | 10 | | 3 | 3 | 2 | 2 | 10 | | 3 | 2 | 3 | 7.5 |
| 2 | 3 | 2 | 3 | 10 | | 4 | 2 | 2 | 3 | 11 | | 1 | 3 | 4 | 8 |
| 1 | 4 | 2 | 2 | 9 | | 3 | 4 | 1 | 4 | 12 | | 2 | 3 | 5 | 10 |
| 2 | 4 | 2 | 2 | 10 | | 3 | 2 | 2 | 2 | 9 | | 3 | 4 | 4 | 11 |
| 5 | 5 | 2 | 1 | 13 | | 4 | 4 | 2 | 4 | 14 | | 3 | 3 | 3 | 9 |
| 2 | 4 | 2 | 2 | 10 | | 4 | 2 | 2 | 5 | 13 | | 1 | 5 | 4 | 10 |
| **2** | **4** | **2** | **2** | **10.6** | | **4** | **3** | **2** | **3** | **11.93333** | | **2** | **3** | **4** | **9.1** |
| **1** | **1** | **1** | **1** | **1.624808** | | **1** | **1** | **1** | **1** | **2.59401** | | **1** | **1** | **1** | **1.485485** |
| | | | | | | | | | | | | | | | |
| **5** | **6** | **22** | **31** | | | **8** | **10** | **24** | **29** | | | **12** | **26** | **32** | |
| 2 | 4 | 3 | 3 | 12 | | 5 | 4 | 2 | 3 | 14 | | 1 | 5 | 5 | 11 |
| 2 | 4 | 2 | 2 | 10 | | 4 | 2 | 2 | 4 | 12 | | 3 | 2 | 3 | 8 |
| 3 | 3 | 2 | 3 | 11 | | 4 | 3 | 2 | 2 | 11 | | 1 | 4 | 3 | 8 |
| 1 | 5 | 2 | 3 | 11 | | 3 | 2 | 2 | 2 | 9 | | 3 | 3 | 3 | 9 |
| 3 | 4 | 3 | 4 | 14 | | 3 | 4 | 2 | 2 | 11 | | 2 | 5 | 4 | 10 |
| 1 | 5 | 2 | 2 | 10 | | 3 | 2 | 2 | 3 | 10 | | 4 | 3 | 2 | 9 |
| 2 | 4 | 3 | 2 | 11 | | 4 | 4 | 2 | 4 | 14 | | 2 | 4 | 4 | 10 |
| 2 | 5 | 4 | 4 | 15 | | 4 | 3 | 2 | 5 | 14 | | 1 | 5 | 5 | 11 |
| 2 | 4 | 4 | 2 | 12 | | 4 | 4 | 2 | 3 | 13 | | 2 | 4 | 4 | 10 |
| 2 | 4 | 2 | 2 | 10 | | 3 | 2 | 1 | 2 | 8 | | 4 | 3 | 3 | 10 |
| 2 | 4 | 4 | 2 | 12 | | 2 | 2 | 2 | 3 | 9 | | 3 | 3 | 4 | 10 |
| 2 | 3 | 1 | 5 | 11 | | 5 | 3 | 3 | 4 | 15 | | 2 | 4 | 4 | 10 |
| 1 | 5 | 3 | 2 | 11 | | 3 | 2 | 2 | 5 | 12 | | 2 | 3 | 4 | 9 |
| 2 | 4 | 1 | 2 | 9 | | 3 | 2 | 2 | 4 | 11 | | 2 | 4 | 3 | 9 |
| 1 | 5 | 1 | 1 | 8 | | 3 | 3 | 2 | 2 | 10 | | 4 | 2 | 2 | 8 |
| 2 | 4 | 2 | 2 | 10 | | 4 | 2 | 2 | 3 | 11 | | 2 | 3 | 3 | 8 |
| **2** | **4** | **2** | **3** | **11.0625** | | **4** | **3** | **2** | **3** | **11.5** | | **2** | **4** | **4** | **9.375** |
| **1** | **1** | **1** | **1** | **1.675886** | | **1** | **1** | **0** | **1** | **2** | | **1** | **1** | **1** | **0.992157** |
| | | | | | | | | | | | | | | | |
| | pval 0.4421 | | | | | | pval 0.6050 | | | | | | pval 0.5467 | | |
| | no sig diff | | | | | | no sig diff | | | | | | no sig diff | | |

| Real World | | | | | | Verification | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **13** | **15** | **21** | **25** | | | **33** | **34** | **35** | **36** | **37** | **38** | |
| 3 | 3 | 3 | 4 | 13 | | 5 | 4 | 4 | 3 | 4 | 5 | 25 |
| 3 | 4 | 3 | 4 | 14 | | 3 | 3 | 4 | 4 | 4 | 5 | 23 |
| 3 | 4 | 3 | 2 | 12 | | 4 | 3 | 2 | 2 | 4 | 5 | 20 |
| 3 | 3 | 4 | 4 | 14 | | 3 | 3 | 3 | 3 | 2 | 4 | 18 |
| 1 | 1 | 5 | 5 | 12 | | 1 | 1 | 4 | 5 | 5 | 3 | 19 |
| 2 | 4 | 5 | 2 | 13 | | 5 | 3 | 4 | 4 | 4 | 4 | 24 |
| 3 | 4 | 3 | 2 | 12 | | 3 | 3 | 3 | 1 | 5 | 4 | 19 |
| 1 | 1 | 5 | 3 | 10 | | 5 | 3 | 3 | 2 | 2 | 3 | 18 |
| 3 | 4 | 2 | 3 | 12 | | 4 | 3 | 3 | 3 | 3 | 4 | 20 |
| 3 | 3 | 3 | 3 | 11 | | 3 | 2 | 3 | 3 | 3 | 3 | 15.5 |
| 3 | 4 | 2 | 4 | 13 | | 4 | 3 | 3 | 3 | 2 | 1 | 16 |
| 2 | 4 | 4 | 3 | 13 | | 4 | 4 | 5 | 5 | 5 | 5 | 28 |
| 2 | 3 | 3 | 4 | 12 | | 4 | 3 | 4 | 3 | 4 | 4 | 22 |
| 3 | 3 | 3 | 4 | 13 | | 5 | 3 | 3 | 3 | 3 | 4 | 21 |
| 1 | 4 | 4 | 4 | 13 | | 4 | 4 | 4 | 2 | 2 | 4 | 20 |
| **2** | **3** | **3** | **3** | **12.46667** | | **4** | **3** | **3** | **3** | **3** | **4** | **20.56667** |
| **1** | **1** | **1** | **1** | **1.024153** | | **1** | **1** | **1** | **1** | **1** | **1** | **3.270406** |
| | | | | | | | | | | | | |
| **13** | **15** | **21** | **25** | | | **33** | **34** | **35** | **36** | **37** | **38** | |
| 1 | 5 | 2 | 2 | 10 | | 4 | 4 | 5 | 5 | 5 | 5 | 28 |
| 2 | 4 | 2 | 3 | 11 | | 4 | 3 | 3 | 3 | 4 | 4 | 21 |
| 3 | 4 | 2 | 4 | 13 | | 3 | 2 | 2 | 3 | 3 | 4 | 17 |
| 2 | 3 | 2 | 3 | 10 | | 4 | 3 | 4 | 4 | 4 | 4 | 23 |
| 3 | 3 | 3 | 3 | 12 | | 4 | 3 | 3 | 3 | 3 | 4 | 20 |
| 5 | 5 | 2 | 4 | 16 | | 3 | 2 | 4 | 4 | 4 | 4 | 21 |
| 2 | 3 | 4 | 4 | 13 | | 3 | 4 | 2 | 2 | 3 | 5 | 19 |
| 3 | 4 | 4 | 2 | 13 | | 5 | 4 | 2 | 2 | 2 | 2 | 17 |
| 3 | 2 | 5 | 2 | 12 | | 4 | 4 | 4 | 2 | 2 | 4 | 20 |
| 5 | 5 | 3 | 4 | 17 | | 3 | 2 | 3 | 4 | 4 | 4 | 20 |
| 4 | 4 | 3 | 3 | 14 | | 3 | 3 | 3 | 4 | 4 | 4 | 21 |
| 3 | 4 | 3 | 2 | 12 | | 3 | 3 | 3 | 2 | 1 | 4 | 16 |
| 4 | 4 | 2 | 4 | 14 | | 3 | 3 | 3 | 4 | 4 | 5 | 22 |
| 4 | 4 | 2 | 5 | 15 | | 5 | 4 | 4 | 4 | 4 | 5 | 26 |
| 2 | 2 | 4 | 4 | 12 | | 2 | 1 | 4 | 5 | 5 | 3 | 20 |
| 2 | 2 | 3 | 4 | 11 | | 3 | 2 | 3 | 3 | 4 | 3 | 18 |
| **3** | **4** | **3** | **3** | **12.8125** | | **4** | **3** | **3** | **3** | **4** | **4** | **20.5625** |
| **1** | **1** | **1** | **1** | **1.943539** | | **1** | **1** | **1** | **1** | **1** | **1** | **3.061224** |
| | | | | | | | | | | | | |
| | pval 0.5443 | | | | | | | | | | | |
| | no sig diff | | | | | | | | | | | |

Period 6:

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Confidence | | | | | | | Answers | | | | | | | Sense Making | | | | |
| Pr | 1 | # | # | # | Total | | 2 | 7 | 9 | # | # | # | | | 3 | 4 | 11 | # | # | |
| | 3 | 4 | 4 | 4 | 15 | | 2 | 3 | 2 | 5 | 3 | 2 | 17 | | 1 | 4 | 4 | 4 | 4 | 17 |
| | 2 | 2 | 4 | 4 | 12 | | 2 | 4 | 3 | 2 | 3 | 4 | 18 | | 4 | 2 | 3 | 3 | 3 | 15 |
| | 4 | 5 | 5 | 2 | 16 | | 2 | 2 | 3 | 2 | 4 | 4 | 17 | | 3 | 4 | 4 | 3 | 4 | 18 |
| | 4 | 5 | 5 | 3 | 17 | | 4 | 3 | 1 | 3 | 3 | 3 | 17 | | 3 | 5 | 5 | 5 | 1 | 19 |
| | 3 | 5 | 4 | 4 | 16 | | 3 | 4 | 4 | 4 | 3 | 4 | 22 | | 2 | 2 | 4 | 2 | 3 | 13 |
| | 1 | 3 | 2 | 4 | 10 | | 1 | 4 | 2 | 2 | 2 | 4 | 15 | | 2 | 2 | 4 | 4 | 2 | 14 |
| | 3 | 2 | 4 | 4 | 13 | | 2 | 3 | 2 | 3 | 2 | 2 | 14 | | 4 | 2 | 4 | 3 | 4 | 17 |
| | 2 | 3 | 3 | 4 | 12 | | 2 | 2 | 4 | 2 | 4 | 3 | 17 | | 3 | 2 | 3 | 2 | 4 | 14 |
| | 3 | 3 | 4 | 5 | 15 | | 1 | 4 | 3 | 2 | 5 | 4 | 19 | | 4 | 3 | 5 | 1 | 4 | 17 |
| | 3 | 2 | 4 | 4 | 13 | | 2 | 4 | 2 | 3 | 5 | 3 | 19 | | 3 | 2 | 3 | 4 | 2 | 14 |
| | 4 | 5 | 5 | 3 | 17 | | 1 | 2 | 3 | 3 | 2 | 4 | 15 | | 3 | 3 | 4 | 3 | 2 | 15 |
| | 5 | 5 | 5 | 2 | 17 | | 2 | 1 | 2 | 2 | 2 | 2 | 11 | | 1 | 4 | 4 | 4 | 4 | 17 |
| | 4 | 4 | 4 | 4 | 16 | | 2 | 4 | 4 | 2 | 2 | 2 | 16 | | 3 | 2 | 4 | 4 | 2 | 15 |
| | 2 | 1 | 1 | 4 | 8 | | 1 | 3 | 5 | 3 | 2 | 4 | 18 | | 4 | 1 | 2 | 1 | 4 | 12 |
| | 4 | 3 | 4 | 4 | 15 | | 2 | 4 | 4 | 5 | 4 | 4 | 23 | | 5 | 4 | 4 | 3 | 4 | 20 |
| | 3 | 5 | 5 | 4 | 17 | | 2 | 3 | 2 | 2 | 2 | 5 | 16 | | 2 | 4 | 5 | 2 | 3 | 16 |
| | 4 | 2 | 3 | 3 | 12 | | 3 | 3 | 2 | 4 | 2 | 3 | 17 | | 3 | 4 | 4 | 2 | 3 | 16 |
| | 3 | 4 | 3 | 4 | 14 | | 2 | 4 | 5 | 3 | 4 | 4 | 22 | | 2 | 3 | 4 | 3 | 4 | 16 |
| | 5 | 5 | 5 | 2 | 17 | | 3 | 3 | 4 | 2 | 3 | 3 | 18 | | 2 | 2 | 3 | 3 | 4 | 14 |
| | 1 | 2 | 1 | 5 | 9 | | 1 | 2 | 1 | 4 | 2 | 2 | 12 | | 4 | 2 | 5 | 1 | 5 | 17 |
| | **3** | **4** | **4** | **4** | **14.05** | | **2** | **3** | **3** | **3** | **3** | **3** | **17.15** | | **3** | **3** | **4** | **3** | **3** | **15.8** |
| | **1** | **1** | **1** | **1** | **2.747** | | **1** | **1** | **1** | **1** | **1** | **1** | **2.971** | | **1** | **1** | **1** | **1** | **1** | **1.965** |
| | | | | | | | | | | | | | | | | | | | | |
| Po | 1 | # | # | # | | | 2 | 7 | 9 | # | # | # | | | 3 | 4 | 11 | # | # | |
| | 4 | 5 | 5 | 2 | 16 | | 2 | 4 | 3 | 2 | 2 | 3 | 16 | | 2 | 3 | 3 | 4 | 3 | 15 |
| | 3 | 5 | 5 | 3 | 16 | | 2 | 3 | 2 | 2 | 3 | 3 | 15 | | 2 | 3 | 4 | 3 | 4 | 16 |
| | 4 | 4 | 3 | 2 | 13 | | 2 | 2 | 2 | 2 | 2 | 3 | 13 | | 4 | 3 | 4 | 2 | 4 | 17 |
| | 4 | 5 | 5 | 4 | 18 | | 2 | 2 | 3 | 2 | 2 | 3 | 13.5 | | 3 | 3 | 4 | 4 | 2 | 15.5 |
| | 5 | 5 | 5 | 2 | 17 | | 1 | 1 | 3 | 2 | 4 | 3 | 14 | | 2 | 4 | 5 | 5 | 3 | 19 |
| | 3 | 4 | 4 | 5 | 16 | | 1 | 4 | 2 | 4 | 5 | 4 | 20 | | 4 | 4 | 5 | 2 | 5 | 20 |
| | 4 | 4 | 5 | 4 | 17 | | 3 | 4 | 2 | 2 | 2 | 3 | 16 | | 2 | 3 | 4 | 3 | 3 | 15 |
| | 3 | 4 | 4 | 3 | 14 | | 2 | 3 | 3 | 3 | 3 | 3 | 17 | | 2 | 2 | 4 | 2 | 3 | 13 |
| | 3 | 3 | 4 | 4 | 14 | | 1 | 2 | 3 | 2 | 2 | 2 | 12 | | 3 | 3 | 4 | 2 | 3 | 15 |
| | 1 | 3 | 1 | 5 | 10 | | 1 | 2 | 1 | 3 | 1 | 2 | 10 | | 2 | 1 | 5 | 1 | 3 | 12 |
| | 2 | 5 | 2 | 4 | 13 | | 1 | 4 | 4 | 3 | 4 | 4 | 20 | | 4 | 1 | 4 | 2 | 2 | 13 |
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| | 2 | 4 | 4 | 4 | 14 | | 1 | 3 | 3 | 2 | 3 | 3 | 15 | | 4 | 3 | 3 | 2 | 4 | 16 |
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| | 4 | 3 | 3 | 3 | 13 | | 1 | 1 | 3 | 1 | 3 | 3 | 12 | | 2 | 4 | 4 | 4 | 4 | 18 |
| | 1 | 1 | 2 | 4 | 8 | | 2 | 4 | 3 | 2 | 4 | 4 | 19 | | 4 | 1 | 2 | 3 | 4 | 14 |
| | 2 | 4 | 3 | 4 | 13 | | 2 | 3 | 2 | 2 | 3 | 4 | 16 | | 3 | 2 | 4 | 2 | 3 | 14 |
| | 3 | 4 | 4 | 4 | 15 | | 1 | 3 | 2 | 2 | 3 | 4 | 15 | | 2 | 1 | 4 | 4 | 4 | 15 |
| | 3 | 4 | 4 | 4 | 15 | | 3 | 4 | 4 | 4 | 3 | 4 | 22 | | 3 | 2 | 4 | 3 | 3 | 15 |
| | 5 | 5 | 5 | 3 | 18 | | 1 | 4 | 3 | 5 | 4 | 3 | 20 | | 5 | 3 | 4 | 2 | 3 | 17 |
| | 3 | 3 | 3 | 4 | 13 | | 3 | 4 | 4 | 4 | 4 | 3 | 22 | | 3 | 3 | 4 | 4 | 4 | 18 |
| | **3** | **4** | **4** | **4** | **14.32** | | **2** | **3** | **3** | **2** | **3** | **3** | **15.75** | | **3** | **3** | **4** | **3** | **3** | **15.52** |
| | 1 | 1 | 1 | 1 | 2.4005 | | 1 | 1 | 1 | 1 | 1 | 1 | 3.554 | | 1 | 1 | 1 | 1 | 1 | 1.9684 |
| | | | | | | | | | | | | | | | | | | | | |
| | | pval 0.36 | | | | | | | | | | | pval 0.17 | | | | pval 0.65 | | | |
| | | no sig diff | | | | | | | | | | | no sig diff | | | | no sig diff | | | |

| Growth Mindset | | | | | Persistence | | | | | Interest | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **5** | **6** | **#** | **#** | | **8** | **#** | **#** | **#** | | **#** | **#** | **#** | |
| 3 | 5 | 3 | 2 | 13 | 4 | 3 | 2 | 4 | 13 | 4 | 4 | 3 | 11 |
| 3 | 4 | 2 | 4 | 13 | 4 | 4 | 3 | 3 | 14 | 1 | 4 | 5 | 10 |
| 3 | 2 | 3 | 4 | 12 | 4 | 4 | 2 | 2 | 12 | 1 | 4 | 3 | 8 |
| 2 | 3 | 2 | 2 | 9 | 4 | 2 | 3 | 4 | 13 | 1 | 5 | 2 | 8 |
| 2 | 4 | 3 | 3 | 12 | 4 | 3 | 3 | 4 | 14 | 4 | 3 | 3 | 10 |
| 2 | 4 | 2 | 3 | 11 | 3 | 3 | 2 | 3 | 11 | 3 | 2 | 4 | 9 |
| 2 | 4 | 2 | 1 | 9 | 3 | 2 | 2 | 2 | 9 | 2 | 3 | 3 | 8 |
| 3 | 4 | 2 | 4 | 13 | 4 | 3 | 2 | 3 | 12 | 3 | 2 | 4 | 9 |
| 1 | 5 | 1 | 2 | 9 | 4 | 4 | 1 | 3 | 12 | 5 | 1 | 1 | 7 |
| 2 | 4 | 2 | 2 | 10 | 4 | 3 | 3 | 2 | 12 | 1 | 2 | 3 | 6 |
| 1 | 4 | 2 | 1 | 8 | 4 | 3 | 1 | 4 | 12 | 2 | 5 | 5 | 12 |
| 2 | 4 | 3 | 5 | 14 | 4 | 4 | 3 | 3 | 14 | 4 | 2 | 2 | 8 |
| 1 | 4 | 2 | 2 | 9 | 3 | 4 | 2 | 3 | 12 | 4 | 3 | 2 | 9 |
| 1 | 5 | 2 | 2 | 10 | 2 | 2 | 1 | 3 | 8 | 3 | 2 | 4 | 9 |
| 2 | 4 | 2 | 2 | 10 | 4 | 4 | 2 | 1 | 11 | 1 | 4 | 5 | 10 |
| 3 | 3 | 3 | 4 | 13 | 4 | 4 | 3 | 5 | 16 | 4 | 2 | 5 | 11 |
| 1 | 4 | 2 | 2 | 9 | 3 | 4 | 2 | 4 | 13 | 3 | 2 | 4 | 9 |
| 2 | 4 | 2 | 4 | 12 | 3 | 2 | 2 | 1 | 8 | 2 | 4 | 5 | 11 |
| 4 | 2 | 3 | 3 | 12 | 4 | 3 | 5 | 4 | 16 | 1 | 4 | 5 | 10 |
| 3 | 5 | 1 | 1 | 10 | 2 | 2 | 1 | 1 | 6 | 5 | 1 | 1 | 7 |
| **2** | **4** | **2** | **3** | **10.9** | **4** | **3** | **2** | **3** | **11.9** | **3** | **3** | **3** | **9.1** |
| **1** | **1** | **1** | **1** | **1.758** | **1** | **1** | **1** | **1** | **2.51** | **1** | **1** | **1** | **1.513** |
| | | | | | | | | | | | | | |
| **5** | **6** | **#** | **#** | | **8** | **#** | **#** | **#** | | **#** | **#** | **#** | |
| 3 | 3 | 3 | 3 | 12 | 2 | 3 | 2 | 4 | 11 | 1 | 4 | 4 | 9 |
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| 2 | 3 | 3 | 2 | 10 | 4 | 4 | 2 | 4 | 14 | 1 | 5 | 4 | 10 |
| 2 | 4 | 3 | 2 | 11 | 4 | 4 | 4 | 2 | 14 | 4 | 5 | 3 | 12 |
| 3 | 4 | 3 | 5 | 15 | 5 | 5 | 5 | 4 | 19 | 2 | 2 | 3 | 7 |
| 1 | 5 | 2 | 2 | 10 | 4 | 4 | 2 | 3 | 13 | 5 | 1 | 2 | 8 |
| 1 | 4 | 2 | 2 | 9 | 4 | 4 | 2 | 2 | 12 | 2 | 5 | 3 | 10 |
| 2 | 4 | 2 | 3 | 11 | 4 | 2 | 3 | 3 | 12 | 2 | 4 | 3 | 9 |
| 2 | 5 | 2 | 2 | 11 | 3 | 2 | 1 | 3 | 9 | 3 | 2 | 3 | 8 |
| 2 | 5 | 1 | 1 | 9 | 1 | 2 | 1 | 1 | 5 | 5 | 1 | 1 | 7 |
| 1 | 5 | 2 | 4 | 12 | 2 | 2 | 2 | 1 | 7 | 4 | 2 | 3 | 9 |
| 1 | 4 | 2 | 1 | 8 | 4 | 2 | 2 | 2 | 10 | 3 | 2 | 2 | 7 |
| 2 | 4 | 3 | 1 | 10 | 3 | 2 | 2 | 2 | 9 | 2 | 5 | 5 | 12 |
| 3 | 4 | 3 | 2 | 12 | 4 | 4 | 2 | 4 | 14 | 4 | 2 | 3 | 9 |
| 1 | 5 | 2 | 1 | 9 | 2 | 2 | 2 | 1 | 7 | 5 | 1 | 1 | 7 |
| 3 | 1 | 2 | 3 | 9 | 4 | 4 | 3 | 4 | 15 | 2 | 2 | 4 | 8 |
| 1 | 5 | 3 | 2 | 11 | 2 | 2 | 2 | 3 | 9 | 2 | 3 | 2 | 7 |
| 1 | 4 | 2 | 4 | 11 | 4 | 4 | 2 | 3 | 13 | 2 | 4 | 3 | 9 |
| 3 | 4 | 3 | 4 | 14 | 4 | 4 | 2 | 2 | 12 | 4 | 3 | 3 | 10 |
| 2 | 4 | 3 | 2 | 11 | 4 | 3 | 3 | 4 | 14 | 3 | 4 | 4 | 11 |
| 1 | 4 | 2 | 4 | 11 | 4 | 2 | 3 | 2 | 11 | 3 | 3 | 4 | 10 |
| 2 | 4 | 3 | 2 | 11 | 2 | 2 | 2 | 2 | 8 | 2 | 5 | 4 | 11 |
| **2** | **4** | **2** | **3** | **10.82** | **3** | **3** | **2** | **3** | **11.4** | **3** | **3** | **3** | **9.045** |
| 1 | 1 | 1 | 1 | 1.5851 | 1 | 1 | 1 | 1 | 3.127 | 1 | 1 | 1 | 1.5515 |
| | | | | | | | | | | | | | |
| | pval 0.88 | | | | | pval 0.5458 | | | | | pval 0.9089 | | |
| | no sig diff | | | | | no sig diff | | | | | no sig diff | | |

| Real World | | | | | Verification | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| # | # | # | # | | # | # | # | # | # | # | |
| 4 | 5 | 2 | 4 | 15 | 2 | 2 | 3 | 3 | 3 | 4 | 17 |
| 2 | 2 | 5 | 2 | 11 | 4 | 2 | 2 | 2 | 2 | 2 | 14 |
| 2 | 3 | 4 | 2 | 11 | 3 | 3 | 3 | 2 | 1 | 2 | 14 |
| 3 | 3 | 4 | 3 | 13 | 2 | 2 | 2 | 4 | 3 | 5 | 18 |
| 3 | 5 | 3 | 3 | 14 | 3 | 3 | 3 | 3 | 3 | 4 | 19 |
| 3 | 3 | 3 | 4 | 13 | 2 | 2 | 2 | 4 | 4 | 4 | 18 |
| 2 | 3 | 4 | 3 | 12 | 4 | 3 | 3 | 2 | 3 | 4 | 19 |
| 3 | 3 | 3 | 3 | 12 | 4 | 3 | 3 | 3 | 3 | 4 | 20 |
| 4 | 4 | 4 | 5 | 17 | 2 | 2 | 4 | 4 | 4 | 4 | 20 |
| 2 | 3 | 3 | 3 | 11 | 3 | 3 | 2 | 3 | 3 | 3 | 17 |
| 1 | 3 | 4 | 3 | 11 | 5 | 5 | 1 | 3 | 3 | 5 | 22 |
| 3 | 3 | 5 | 2 | 13 | 3 | 2 | 2 | 1 | 2 | 4 | 14 |
| 2 | 4 | 4 | 3 | 13 | 3 | 2 | 3 | 3 | 3 | 5 | 19 |
| 3 | 4 | 3 | 3 | 13 | 4 | 3 | 5 | 5 | 5 | 4 | 26 |
| 1 | 4 | 2 | 3 | 10 | 5 | 4 | 3 | 2 | 3 | 4 | 21 |
| 4 | 4 | 4 | 4 | 16 | 4 | 1 | 2 | 2 | 4 | 3 | 16 |
| 4 | 2 | 3 | 3 | 12 | 3 | 3 | 3 | 4 | 4 | 4 | 21 |
| 3 | 4 | 4 | 4 | 15 | 4 | 3 | 3 | 3 | 4 | 3 | 20 |
| 2 | 2 | 3 | 4 | 11 | 3 | 3 | 3 | 1 | 1 | 4 | 15 |
| 5 | 5 | 1 | 4 | 15 | 3 | 1 | 3 | 4 | 4 | 4 | 19 |
| **3** | **3** | **3** | **3** | **12.9** | **3** | **3** | **3** | **3** | **3** | **4** | **18.45** |
| **1** | **1** | **1** | **1** | **1.868** | **1** | **1** | **1** | **1** | **1** | **1** | **2.941** |
| | | | | | | | | | | | |
| # | # | # | # | | # | # | # | # | # | # | |
| 2 | 2 | 5 | 3 | 12 | 4 | 2 | 3 | 2 | 3 | 4 | 18 |
| 2 | 2 | 4 | 3 | 11 | 4 | 3 | 3 | 2 | 2 | 2 | 16 |
| 2 | 3 | 4 | 2 | 11 | 2 | 2 | 2 | 2 | 4 | 4 | 16 |
| 2 | 4 | 4 | 4 | 13.5 | 4 | 2 | 4 | 4 | 4 | 5 | 22.5 |
| 3 | 4 | 5 | 4 | 16 | 2 | 1 | 1 | 1 | 1 | 4 | 10 |
| 5 | 4 | 2 | 4 | 15 | 3 | 3 | 4 | 4 | 4 | 4 | 22 |
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| 3 | 5 | 3 | 4 | 15 | 3 | 2 | 3 | 2 | 2 | 4 | 16 |
| 4 | 4 | 1 | 4 | 13 | 1 | 1 | 3 | 5 | 5 | 5 | 20 |
| 4 | 4 | 3 | 4 | 15 | 3 | 2 | 3 | 4 | 5 | 5 | 22 |
| 2 | 2 | 4 | 4 | 12 | 4 | 3 | 4 | 2 | 4 | 3 | 20 |
| 2 | 3 | 3 | 3 | 11 | 5 | 3 | 2 | 1 | 3 | 5 | 19 |
| 3 | 3 | 3 | 2 | 11 | 3 | 3 | 3 | 2 | 4 | 4 | 19 |
| 5 | 5 | 2 | 5 | 17 | 3 | 2 | 3 | 5 | 5 | 3 | 21 |
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| 4 | 4 | 3 | 4 | 15 | 3 | 3 | 4 | 4 | 4 | 4 | 22 |
| 3 | 4 | 3 | 4 | 14 | 3 | 3 | 3 | 3 | 3 | 4 | 19 |
| 3 | 4 | 2 | 3 | 12 | 5 | 5 | 2 | 4 | 4 | 5 | 25 |
| 2 | 4 | 3 | 4 | 13 | 3 | 3 | 4 | 3 | 2 | 4 | 19 |
| **3** | **4** | **3** | **3** | **12.93** | **3** | **3** | **3** | **3** | **3** | **4** | **19.3** |
| 1 | 1 | 1 | 1 | 1.9087 | 1 | 1 | 1 | 1 | 1 | 1 | 3.1285 |
| | | | | | | | | | | | |
| pval 0.9568 | | | | | pval 0.3735 | | | | | | |
| no sig diff | | | | | no sig diff | | | | | | |

MAPS Expert Consensus Data

Period 1:

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Pre** | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | 1 | | | 1 | 1 | 1 | | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | | 1 | 1 | 1 | 1 | | 1 | 1 | | 1 | 21 |
| | | 1 | 1 | | 1 | 1 | | | 1 | 1 | 1 | | | 1 | | | 1 | | 1 | 1 | | | 1 | 1 | 1 | | | | | 14 |
| | | 1 | 1 | | | 1 | | | 1 | | 1 | | | 1 | 1 | | 1 | | 1 | | | 1 | 1 | | 1 | 1 | | 1 | 1 | 15 |
| | | 1 | 1 | 1 | 1 | 1 | | | 1 | 1 | | | | 1 | | | 1 | 1 | 1 | | 1 | 1 | 1 | 1 | | | 1 | | 1 | 17 |
| | 1 | 1 | 1 | | 1 | 1 | | 1 | 1 | | 1 | | | 1 | 1 | 1 | 1 | 1 | 1 | | | 1 | 1 | | | | | | | 16 |
| | 1 | | 1 | 1 | 1 | 1 | 1 | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | | 1 | 1 | 1 | 1 | | 1 | | | 1 | 22 |
| | 1 | | 1 | | 1 | 1 | | 1 | 1 | | 1 | 1 | | 1 | 1 | 1 | 1 | 1 | 1 | | | 1 | 1 | 1 | 1 | | 1 | 1 | 1 | 21 |
| | | | | 1 | 1 | 1 | | | | 1 | 1 | | | 1 | | 1 | 1 | | 1 | | | 1 | | 1 | 1 | | | | 1 | 13 |
| | | | | 1 | | 1 | | | | | 1 | | | | | 1 | | 1 | | | | 1 | | 1 | | | 1 | | | 8 |
| | | 1 | 1 | | | | 1 | | | | 1 | | | | | | | | 1 | | | | | | | 1 | 1 | | | 7 |
| | 1 | 1 | | 1 | 1 | | 1 | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | | | 1 | 1 | 1 | | 1 | 1 | | 1 | 22 |
| | | 1 | | | | 1 | 1 | | | | | | | 1 | 1 | 1 | 1 | | 1 | | | | 1 | 1 | | 1 | 1 | | | 12 |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| **Post** | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| | 1 | | 1 | 1 | 1 | 1 | | | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | | | 1 | 1 | | | 1 | | | | 17 |
| | 1 | 1 | | 1 | 1 | | | | 1 | 1 | 1 | | 1 | | 1 | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | | | 1 | | 1 | | 18 |
| | 1 | 1 | 1 | | 1 | 1 | | | | | | | 1 | 1 | 1 | 1 | 1 | 1 | | 1 | 1 | 1 | 1 | | 1 | | | 1 | | 17 |
| | 1 | 1 | | | 1 | 1 | | | 1 | | 1 | | | | 1 | 1 | 1 | 1 | 1 | | | | | | | | | | | 11 |
| | 1 | 1 | 1 | 1 | | 1 | 1 | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | | 1 | 1 | 1 | 1 | | 1 | | 1 | 1 | | 1 | 1 | 1 | 23 |
| | 1 | 1 | 1 | | | 1 | | | 1 | | 1 | 1 | 1 | 1 | 1 | 1 | | 1 | 1 | | 1 | 1 | 1 | | | | 1 | | 1 | 18 |
| | 1 | 1 | | 1 | 1 | 1 | | | | 1 | | | | | 1 | | | 1 | 1 | | | 1 | 1 | | | 1 | 1 | 1 | | 14 |
| | | 1 | | | 1 | | | | 1 | | 1 | | | 1 | 1 | | | 1 | 1 | | 1 | 1 | 1 | 1 | | 1 | | 1 | 1 | 15 |
| | 1 | 1 | | 1 | 1 | 1 | 1 | | | 1 | 1 | 1 | | 1 | 1 | | 1 | 1 | 1 | 1 | | 1 | | 1 | 1 | 1 | 1 | 1 | | 21 |
| | | | 1 | | | 1 | | | | | 1 | | | | | | | | 1 | | | | | | | | | | | 4 |
| | | 1 | | 1 | 1 | 1 | 1 | | | | 1 | 1 | | 1 | 1 | | | 1 | 1 | 1 | | | | | 1 | | 1 | | 1 | 15 |
| | 1 | 1 | | | 1 | 1 | | | | | 1 | | | 1 | | | 1 | | 1 | | | | | | | | | | | 8 |
| | | 1 | | | 1 | 1 | | | | 1 | | | | 1 | 1 | | | | 1 | | | 1 | 1 | | | | | | | 9 |
| | | | 1 | | 1 | 1 | | 1 | | | 1 | | | | 1 | | | | 1 | | 1 | | 1 | 1 | | | 1 | 1 | | 12 |
| | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | | 1 | | 1 | 25 |

Pre
15.67 mean
4.972 stdev
12 n

Post
15.13 mean
5.524 stdev
15 n

Period 2:

| Pre | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 20 | 21 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 32 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| |  | 1 |  |  | 1 | 1 | 1 |  |  | 1 | 1 |  |  |  |  | 1 | 1 |  | 1 |  | 1 | 1 | 1 |  |  |  |  |  |  | 12 |
| |  | 1 |  |  | 1 | 1 |  |  | 1 | 1 |  |  |  |  | 1 |  |  | 1 | 1 |  | 1 | 1 | 1 |  |  |  |  |  |  | 11 |
| |  | 1 |  | 1 |  | 1 | 1 |  | 1 |  | 1 |  |  |  | 1 |  |  |  | 1 |  |  | 1 |  |  |  | 1 |  | 1 |  | 11 |
| |  | 1 |  | 1 | 1 | 1 |  |  |  |  | 1 |  |  |  |  | 1 |  |  | 1 |  | 1 |  | 1 | 1 |  |  |  | 1 |  | 11 |
| | 1 | 1 |  |  | 1 | 1 | 1 |  | 1 |  |  | 1 |  | 1 |  | 1 | 1 |  | 1 |  |  | 1 | 1 | 1 | 1 |  | 1 |  | 1 | 17 |
| | 1 | 1 |  | 1 | 1 | 1 | 1 |  |  | 1 | 1 |  |  | 1 | 1 |  |  | 1 |  |  |  | 1 |  |  |  |  |  |  |  | 12 |
| |  | 1 |  |  |  | 1 | 1 |  |  |  |  |  |  |  | 1 |  |  | 1 |  |  | 1 |  |  |  |  |  |  | 1 |  | 7 |
| |  |  | 1 |  | 1 |  |  |  |  |  | 1 |  |  |  |  |  |  |  | 1 |  |  |  |  |  |  |  |  |  |  | 4 |
| |  | 1 |  | 1 | 1 | 1 |  |  | 1 | 1 |  |  |  |  | 1 |  |  |  |  | 1 | 1 | 1 |  | 1 |  |  | 1 |  |  | 12 |
| |  | 1 | 1 | 1 | 1 |  | 1 |  | 1 |  | 1 |  |  |  |  |  |  | 1 |  |  |  | 1 |  | 1 |  |  | 1 | 1 |  | 12 |
| |  |  |  | 1 | 1 |  | 1 |  | 1 | 1 | 1 |  |  | 1 | 1 | 1 |  | 1 |  | 1 | 1 | 1 | 1 |  | 1 | 1 |  | 1 |  | 17 |
| |  | 1 |  |  | 1 | 1 |  |  | 1 |  |  |  |  |  | 1 |  | 1 | 1 | 1 |  |  | 1 |  |  |  |  |  |  |  | 9 |
| | 1 | 1 |  | 1 | 1 | 1 |  |  | 1 | 1 |  |  |  | 1 |  |  | 1 |  | 1 |  |  | 1 | 1 |  | 1 |  | 1 |  |  | 14 |
| |  |  | 1 |  |  | 1 | 1 |  |  |  | 1 |  |  |  |  |  |  | 1 |  |  | 1 | 1 | 1 |  | 1 |  |  |  |  | 9 |
| |  | 1 |  | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |  |  |  | 1 | 1 |  | 1 |  |  |  | 1 | 1 |  | 1 |  |  | 1 |  | 15 |
| **Post** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| | 1 | 1 |  |  | 1 | 1 | 1 |  |  |  |  |  |  |  | 1 |  |  | 1 |  | 1 |  | 1 |  |  |  |  |  |  |  | 9 |
| |  | 1 |  | 1 | 1 | 1 |  |  | 1 | 1 | 1 |  |  |  | 1 | 1 |  |  |  | 1 |  | 1 |  | 1 | 1 |  |  |  |  | 13 |
| | 1 |  | 1 | 1 |  |  | 1 |  | 1 |  | 1 |  |  |  | 1 | 1 |  | 1 |  | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 |  | 17 |
| | 1 | 1 |  | 1 | 1 | 1 |  |  | 1 | 1 |  |  |  | 1 |  | 1 | 1 |  | 1 | 1 |  | 1 |  |  |  |  | 1 |  |  | 14 |
| |  | 1 |  | 1 |  | 1 |  |  | 1 |  | 1 |  |  |  |  | 1 |  |  |  |  |  | 1 |  |  |  |  | 1 |  |  | 8 |
| | 1 |  | 1 | 1 | 1 | 1 |  |  |  | 1 | 1 | 1 | 1 |  | 1 |  |  | 1 | 1 | 1 |  | 1 | 1 |  |  |  |  | 1 | 1 | 17 |
| |  | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 |  |  |  |  |  |  | 1 |  | 1 |  |  |  | 1 | 1 |  |  | 1 |  | 1 |  | 13 |
| |  | 1 |  |  | 1 | 1 |  |  |  |  |  |  |  |  | 1 |  |  | 1 |  |  |  | 1 |  |  | 1 | 1 |  |  |  | 8 |
| |  |  | 1 |  | 1 | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 |  |  |  |  |  |  |  | 4 |
| | 1 | 1 |  | 1 | 1 | 1 |  |  | 1 | 1 |  | 1 | 1 |  | 1 |  |  |  | 1 |  |  | 1 | 1 |  |  | 1 | 1 | 1 |  | 16 |
| |  | 1 |  | 1 | 1 | 1 |  | 1 |  | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 |  |  | 1 |  |  |  |  |  |  |  | 14 |
| |  | 1 |  | 1 | 1 |  | 1 |  |  |  | 1 |  |  |  | 1 |  |  | 1 |  |  |  |  |  |  |  |  |  | 1 |  | 8 |
| |  | 1 |  |  | 1 | 1 |  |  |  | 1 | 1 |  | 1 |  | 1 |  |  | 1 | 1 | 1 | 1 | 1 | 1 |  |  |  |  | 1 |  | 14 |
| |  | 1 |  |  | 1 | 1 | 1 |  |  | 1 |  |  | 1 |  | 1 | 1 |  |  | 1 | 1 |  | 1 | 1 |  |  |  |  |  |  | 12 |
| | 1 | 1 |  | 1 | 1 | 1 | 1 |  | 1 |  | 1 | 1 |  |  |  | 1 | 1 |  | 1 |  |  | 1 | 1 | 1 |  |  | 1 |  | 1 | 17 |
| |  | 1 |  | 1 | 1 | 1 | 1 |  |  | 1 | 1 |  |  |  |  |  |  | 1 |  |  |  | 1 | 1 |  | 1 |  |  | 1 |  | 12 |

Pre:

11.53333 mean
3.36386 stdev
15 n

95% Ci -1.9066 to 3.3399
no sig diff pval 0.5806

Post

12.25 mean
3.75 stdev
16 n

Period 6:

| Pr | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | # | 21 | # | # | # | # | # | # | # | # | # |  | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  |  | 1 | 1 |  |  | 1 |  |  | 1 |  | 1 | 1 | 1 |  | 1 |  |  |  | 1 | 1 |  | 1 | 1 |  |  |  |  | 1 |  |  | 13 |
|  | 1 | 1 |  | 1 |  | 1 |  |  |  |  |  |  |  | 1 |  | 1 |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  | 7 |
|  |  | 1 |  |  |  |  | 1 |  |  |  | 1 |  |  |  |  | 1 |  |  |  |  |  | 1 |  |  |  |  | 1 |  |  |  | 6 |
|  |  |  |  |  | 1 |  |  |  | 1 | 1 | 1 |  |  |  |  |  |  |  |  |  | 1 |  |  |  |  |  |  |  | 1 |  | 6 |
|  |  |  | 1 | 1 | 1 | 1 |  |  |  |  | 1 | 1 |  |  | 1 |  |  | 1 | 1 |  |  |  |  |  | 1 |  |  |  |  |  | 10 |
|  | 1 | 1 | 1 | 1 | 1 | 1 |  |  | 1 |  | 1 |  |  |  |  | 1 | 1 |  | 1 |  | 1 | 1 | 1 | 1 | 1 | 1 |  |  |  |  | 17 |
|  |  | 1 |  | 1 | 1 | 1 |  |  | 1 | 1 | 1 |  |  | 1 |  |  |  |  | 1 |  |  | 1 |  |  | 1 | 1 | 1 | 1 |  |  | 14 |
|  | 1 | 1 |  | 1 |  | 1 | 1 |  |  |  |  |  |  |  |  | 1 |  | 1 | 1 |  |  | 1 |  | 1 | 1 |  |  |  |  |  | 11 |
|  |  | 1 |  |  | 1 | 1 |  |  |  |  | 1 | 1 | 1 |  | 1 | 1 |  | 1 | 1 |  |  | 1 | 1 | 1 |  |  |  |  | 1 |  | 14 |
|  |  | 1 |  | 1 | 1 | 1 |  |  | 1 |  |  |  |  | 1 |  |  |  |  | 1 |  | 1 |  |  | 1 |  |  | 1 |  |  |  | 10 |
|  |  | 1 |  |  | 1 | 1 | 1 |  |  |  | 1 |  |  |  |  |  |  |  |  |  | 1 | 1 |  |  |  | 1 |  |  |  |  | 8 |
|  |  | 1 | 1 |  | 1 | 1 | 1 |  | 1 |  | 1 | 1 |  |  |  | 1 |  |  |  |  |  |  |  | 1 |  | 1 |  | 1 | 1 |  | 13 |
|  |  | 1 |  | 1 | 1 | 1 |  |  |  |  | 1 | 1 |  |  | 1 | 1 |  |  | 1 |  | 1 | 1 |  |  |  | 1 |  | 1 | 1 |  | 14 |
|  | 1 | 1 |  | 1 | 1 | 1 |  | 1 |  | 1 |  |  |  | 1 | 1 |  | 1 | 1 | 1 |  |  | 1 |  | 1 | 1 | 1 |  |  |  |  | 16 |
|  |  | 1 |  |  | 1 | 1 |  |  |  |  | 1 |  |  |  | 1 |  |  |  | 1 | 1 |  | 1 |  |  |  |  | 1 |  |  |  | 9 |
|  |  | 1 | 1 |  |  |  |  |  | 1 |  | 1 | 1 | 1 |  | 1 | 1 |  | 1 | 1 |  |  |  | 1 | 1 |  | 1 |  |  |  |  | 13 |
|  |  |  |  |  | 1 | 1 |  |  | 1 |  | 1 |  | 1 | 1 |  |  |  | 1 |  |  |  | 1 |  | 1 |  | 1 |  |  |  |  | 10 |
|  |  | 1 | 1 |  | 1 | 1 |  |  |  | 1 | 1 |  |  |  | 1 |  |  |  | 1 |  |  | 1 | 1 |  | 1 |  | 1 |  |  |  | 12 |
|  |  |  | 1 | 1 |  |  |  |  |  |  |  |  |  |  |  | 1 |  |  |  |  |  |  | 1 |  |  |  |  |  |  |  | 4 |
|  | 1 | 1 |  | 1 |  | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 |  | 24 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **Post** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | 1 | 1 |  |  |  |  | 1 |  |  |  |  |  |  |  | 1 |  |  |  |  |  | 1 |  |  |  | 1 |  |  |  |  | 6 |
|  |  | 1 | 1 |  |  |  |  |  | 1 | 1 | 1 |  |  |  |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 6 |
|  |  | 1 |  |  | 1 |  | 1 |  | 1 |  | 1 |  |  |  |  | 1 |  | 1 |  |  |  | 1 |  |  |  | 1 |  |  |  |  | 9 |
|  |  | 1 |  |  | 1 | 1 | 1 |  |  |  | 1 | 1 |  |  | 1 | 1 |  |  | 1 |  | 1 |  | 1 |  |  | 1 | 1 |  |  |  | 13 |
|  |  | 1 | 1 |  |  | 1 | 1 |  |  |  | 1 |  |  |  | 1 | 1 |  |  |  |  |  |  | 1 | 1 |  |  |  |  |  |  | 9 |
|  |  | 1 |  |  | 1 | 1 |  |  | 1 |  | 1 | 1 | 1 |  | 1 |  |  | 1 | 1 | 1 |  | 1 | 1 | 1 |  |  |  |  | 1 |  | 15 |
|  |  |  | 1 |  | 1 | 1 |  |  | 1 |  | 1 |  | 1 |  | 1 | 1 |  |  | 1 | 1 |  | 1 | 1 |  |  | 1 | 1 |  |  |  | 14 |
|  |  | 1 | 1 | 1 | 1 | 1 |  |  |  | 1 | 1 |  |  |  |  |  |  | 1 |  | 1 |  |  |  |  |  |  |  |  |  |  | 9 |
|  |  | 1 |  |  | 1 | 1 | 1 |  |  | 1 | 1 |  |  |  | 1 | 1 |  | 1 | 1 |  |  | 1 | 1 | 1 |  | 1 |  | 1 |  |  | 15 |
|  | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 |  | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | 26 |
|  | 1 | 1 |  | 1 | 1 | 1 |  | 1 |  | 1 | 1 | 1 | 1 |  | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 |  |  | 1 |  |  |  | 19 |
|  | 1 | 1 | 1 | 1 | 1 | 1 |  |  | 1 | 1 | 1 |  |  |  |  | 1 |  | 1 | 1 |  |  | 1 | 1 | 1 |  | 1 | 1 |  | 1 |  | 18 |
|  |  | 1 |  |  | 1 | 1 | 1 |  |  | 1 | 1 |  |  |  |  | 1 |  |  |  |  | 1 | 1 |  |  |  | 1 | 1 |  |  |  | 11 |
|  | 1 | 1 |  |  |  | 1 |  |  |  |  |  | 1 |  |  |  | 1 |  | 1 | 1 |  |  | 1 |  | 1 | 1 |  |  |  |  |  | 10 |
|  | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 | 1 | 1 |  | 1 | 1 | 1 |  | 1 | 1 | 1 | 1 |  | 25 |
|  |  | 1 | 1 |  |  |  | 1 |  |  |  | 1 |  |  |  |  | 1 |  |  |  |  |  |  |  | 1 |  |  |  |  |  |  | 6 |
|  | 1 | 1 |  | 1 | 1 | 1 |  | 1 |  | 1 |  |  |  | 1 | 1 | 1 | 1 |  | 1 |  |  | 1 |  |  |  |  |  |  | 1 |  | 14 |
|  | 1 | 1 |  | 1 | 1 | 1 |  |  | 1 |  | 1 |  |  |  |  | 1 |  | 1 | 1 |  |  | 1 |  |  |  |  |  |  |  |  | 11 |
|  |  | 1 | 1 | 1 |  | 1 |  |  | 1 |  | 1 | 1 | 1 |  | 1 | 1 |  |  | 1 |  |  | 1 | 1 |  |  |  | 1 |  |  |  | 14 |
|  |  |  |  | 1 | 1 | 1 |  |  |  |  | 1 |  |  |  | 1 |  |  |  | 1 |  |  |  | 1 |  | 1 |  |  |  |  |  | 8 |
|  |  | 1 |  |  | 1 | 1 |  |  |  | 1 | 1 |  |  |  | 1 |  |  | 1 |  | 1 |  |  |  |  |  |  | 1 |  |  |  | 9 |
|  |  |  |  |  | 1 | 1 |  | 1 |  | 1 | 1 |  |  |  | 1 |  |  |  | 1 |  |  | 1 |  |  |  |  | 1 |  |  |  | 9 |

Pre:

11.55 mean
4.443816 stdev
20 n

Post:

| | |
|---|---|
| 12.54545 | mean |
| 5.433292 | stdev |
| 22 | count |

pre-post CI

95%CI: -4.0829 <-0.99545< 2.092

not significant at 0.05 pval 0.5220

MAPS Open Ended Scored Data

| | | | Period 1 | |
|---|---|---|---|---|
| | Pre | Post | Pre | Post |
| Algorithm | 0 | 3 | 0.0 | 20.0 |
| Solve | 1 | 5 | 8.3 | 33.3 |
| Apply | 0 | 1 | 0.0 | 6.7 |
| Together | 1 | 5 | 8.3 | 33.3 |
| Learn | 3 | 3 | 25.0 | 20.0 |
| Homework | 8 | 7 | 66.7 | 46.7 |
| Behave | 4 | 4 | 33.3 | 26.7 |
| Explain/Ask | 0 | 0 | 0.0 | 0.0 |
| | | | | |
| Self | 0 | 4 | 0.0 | 26.7 |
| Proof | 1 | 6 | 8.3 | 40.0 |
| Check | 8 | 5 | 66.7 | 33.3 |
| Others | 5 | 6 | 41.7 | 40.0 |
| Numeric | 0 | 1 | 0.0 | 6.7 |
| Teacher | 0 | 4 | 0.0 | 26.7 |
| *sample size* | 12 | 15 | | |

| | | | Period 2 | |
|---|---|---|---|---|
| | Pre | Post | Pre | Post |
| Algorithm | 1 | 1 | 6.7 | 6.3 |
| Solve | 4 | 7 | 26.7 | 43.8 |
| Apply | 0 | 0 | 0.0 | 0.0 |
| Together | 1 | 5 | 6.7 | 31.3 |
| Learn | 1 | 1 | 6.7 | 6.3 |
| Homework | 6 | 10 | 40.0 | 62.5 |
| Behave | 2 | 4 | 13.3 | 25.0 |
| Explain/Ask | 2 | 6 | 13.3 | 37.5 |
| | | | | |
| Self | 1 | 3 | 6.7 | 18.8 |
| Proof | 2 | 7 | 13.3 | 43.8 |
| Check | 7 | 9 | 46.7 | 56.3 |
| Others | 4 | 3 | 26.7 | 18.8 |
| Numeric | 0 | 0 | 0.0 | 0.0 |
| Teacher | 2 | 2 | 13.3 | 12.5 |
| *sample size* | 15 | 16 | | |

| | | | Period 6 | |
|---|---|---|---|---|
| | Pre | Post | Pre | Post |
| Algorithm | 0 | 1 | 0.0 | 4.5 |
| Solve | 4 | 2 | 20.0 | 9.1 |
| Apply | 1 | 1 | 5.0 | 4.5 |
| Together | 4 | 8 | 20.0 | 36.4 |
| Learn | 0 | 2 | 0.0 | 9.1 |
| Homework | 11 | 16 | 55.0 | 72.7 |
| Behave | 4 | 7 | 20.0 | 31.8 |
| Explain/Ask | 6 | 6 | 30.0 | 27.3 |
| | | | | |
| Self | 3 | 8 | 15.0 | 36.4 |
| Proof | 4 | 5 | 20.0 | 22.7 |
| Check | 8 | 8 | 40.0 | 36.4 |
| Others | 7 | 10 | 35.0 | 45.5 |
| Numeric | 2 | 2 | 10.0 | 9.1 |
| Teacher | 1 | 3 | 5.0 | 13.6 |
| *sample size* | 20 | 22 | | |

# Appendix C:

## Student Work

Proven Statements (each period)

Period 1

PROVEN THEOREMS

1. If opposite vertices are connected in a trapezoid, then triangles formed by bases are similar.
2. If there are two pairs of congruent base angles in a trapezoid, then the legs are congruent.
3. If it's a trapezoid, then angles between bases are supplementary.
4. If a kite is cut by longus angles, then two isosceles triangles are formed.
5. If trapezoids with congruent base angles and midpoints of bases are connected, then two congruent quadrilaterals form.
6. If it's a kite, then it is not a rhombus.
7. If rhombus and all parallelograms are cut by diagonal, then two congruent triangles are formed.
8. If it's a parallelogram, then opposite angles are congruent.
9. If it's a parallelogram, then consecutive angles are supplementary.
10. If it's a rectangle, then opposite sides are congruent.
11. If it's a square, then it is a rhombus.
12. If it's a rhombus, it can be a square.
13. All squares are similar.
14. If it's a square, it's also a rectangle.
15. If it's a parallelogram, then opposite sides are congruent.
16. If it's a kite, the longus angles are congruent.

17. If it's an isosceles triangle, then the altitude hits at the midpoint.
18. If it's a kite, then the sahngus angles are not congruent.
19. If it's a kite, then the diagonal is perpendicular.
20. If the consecutive sides of a parallelogram are congruent, then it's a rhombus.
21. If the trapezoid is isosceles, then the two base angles are congruent.
22. If it's an isosceles trapezoid, then the diagonal is congruent.
23. If it's a square it's also a parallelogram.
24. If opposite sides are congruent, then it's a parallelogram.
25. If it's a quadrilateral, then the angles add up to 360 degrees.
26. If it's a rectangle, then diagonals are congruent.
27. If the parallelogram has one right angle, then they're all right angles.
28. If it's a rhombus, then it's a parallelogram.
29. If it's a kite, then it's not a parallelogram.
30. If there's a diagonal in the kite, then there are two sets of congruent triangles formed.
31. If it's a rectangle, then it's a parallelogram.
32. If the rectangle is cut by two diagonals, then there are two sets of two congruent triangles.
33. If a trapezoid has congruent base angles, then the line formed by connecting the midpoints of the base are perpendicular to the bases themselves.
34. If a diagonal is drawn in a rectangle, then the diagonal is the longest line segment.

PERIOD 2
PROVEN THEOREMS

1. If parallelogram, then consecutive angles are supplementary
2. If parallelogram, then all angles add to 360.
3. If parallelogram, then opposite angles are congruent
4. If parallelogram, then opposite sides are congruent.
5. If parallelogram, then diagonals bisect each other.
6. If parallelogram with a diagonal drawn, then opposite interior angles are congruent
7. If parallelogram with one right angle, then all angles are right.
8. If parallelogram with one set of consecutive sides, then it is a rhombus.

9. If opposite sides are not parallel, then not a parallelogram
10. If opposite angles are not congruent, then it is not a parallelogram

11. If rectangle, then all angles are 90
12. If rectangle, then diagonals are congruent

13. If rhombus with a diagonal, then 2 congruent triangles are formed.

14. If square, then rhombus
15. If square, then diagonals are congruent

16. If square, then diagonals bisect the angles.
17. If square with a diagonal, then 2 congruent triangles are formed.
18. If a square with 2 diagonals, then 4 congruent triangles are formed.

19. If trapezoid, then all angles add to 360.
20. If trapezoid, then angles between parallel lines are supplementary.
21. If isosceles trapezoid, then one line of symmetry
22. If isosceles trapezoid, then base angles are congruent.
23. If diagonals in a trapezoid are congruent, then it is isosceles.

24. If kite, then one line of symmetry
25. If kite, then line of symmetry creates 2 congruent triangles
26. If kite, then one set of opposite angles are congruent and the others are not congruent.
27. If kite, then diagonals are perpendicular to each other.

PERIOD 6

PROVEN THEOREMS

1. If parallelogram, then opposite sides congruent
2. If parallelogram, then opposite exterior angles are congruent
3. If parallelogram, then all interior angles add up to 360.
4. f parallelogram, then adjacent angles add to 180.
5. If parallelogram, then 2 opposite acute angles and 2 opposite obtuse angles exist.
6. If parallelogram, then opposite angles are congruent
7. If parallelogram has 4 congruent sides, then rhombus
8. If parallelogram, then diagonals bisect each other.
9. If parallelogram with one angle 90 degrees, then all angles are 90 degrees.
10. If parallelogram with diagonal drawn across, then 2 congruent triangles are formed

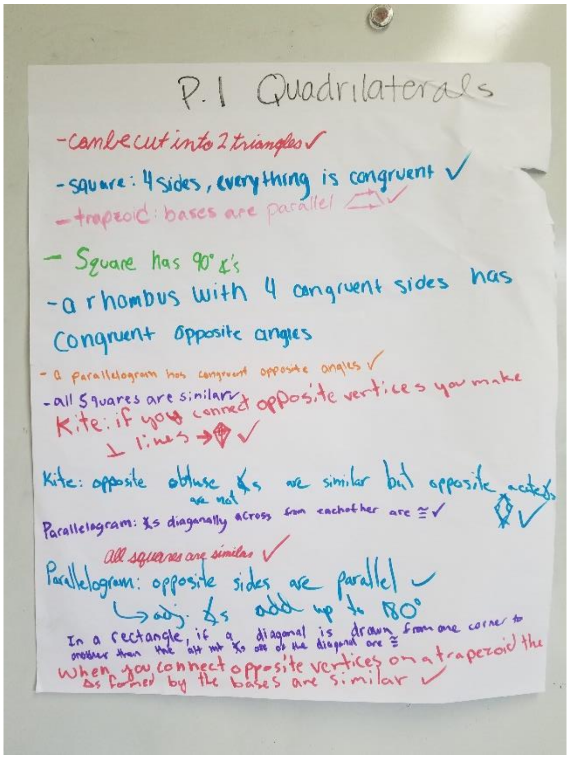
11. If quadrilateral with one set of congruent parallel sides, then it has to be a parallelogram
12. If both pairs of opposite sides are parallel, then it is a parallelogram

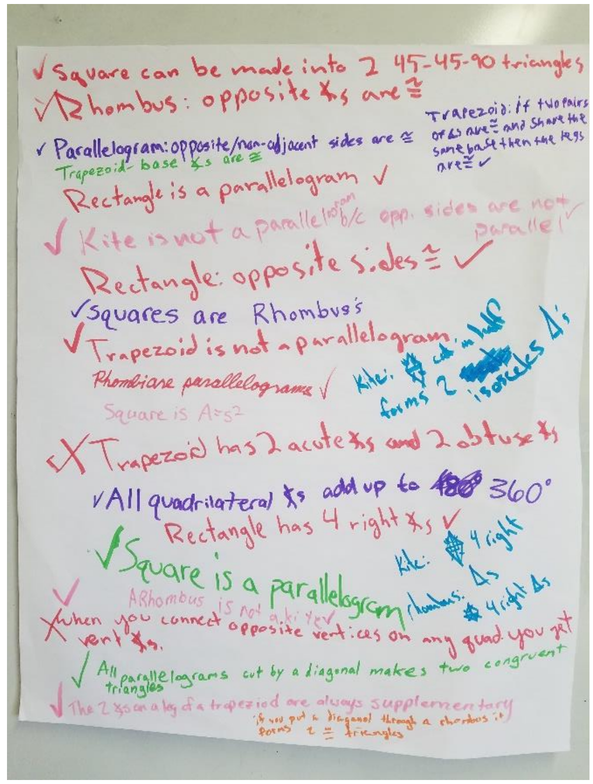
13. If square, then parallelogram
14. If square, then diagonals are perpendicular
15. If square with perpendicular drawn across, then 2 rectangles form
16. If rectangle, then parallelogram

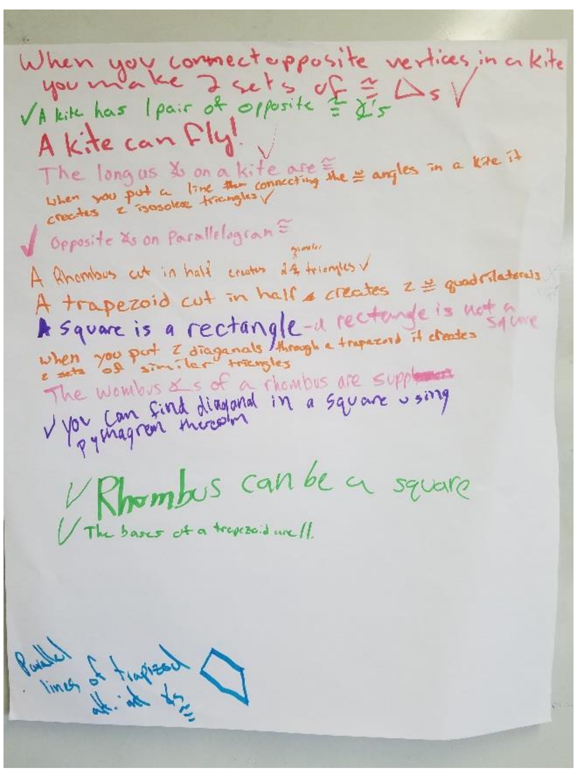
17. If rhombus, then diagonals perpendicular
18. If rhombus with 90 degree angle, then square

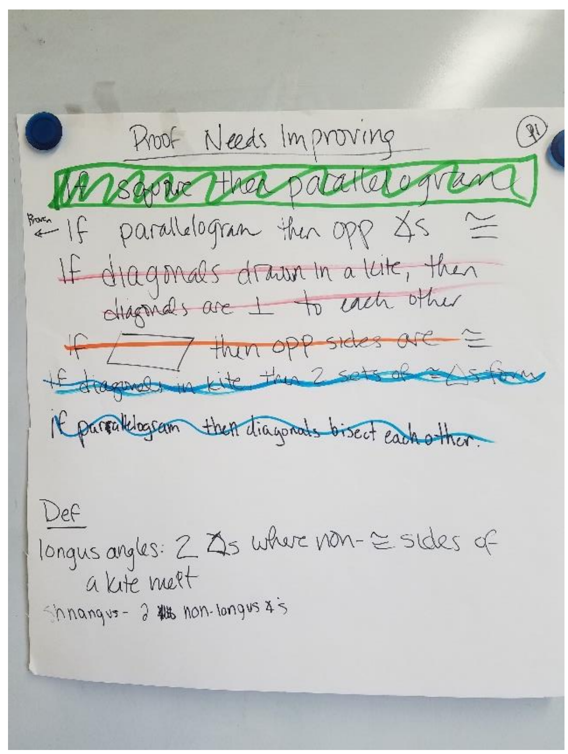
19. If kite, then diagonals are perpendicular.
20. If kite cut by diagonal, then opposite angles are congruent (between the non-congruent sides)
21. If kite cut by diagonal, then the noncongruent angles are bisected.

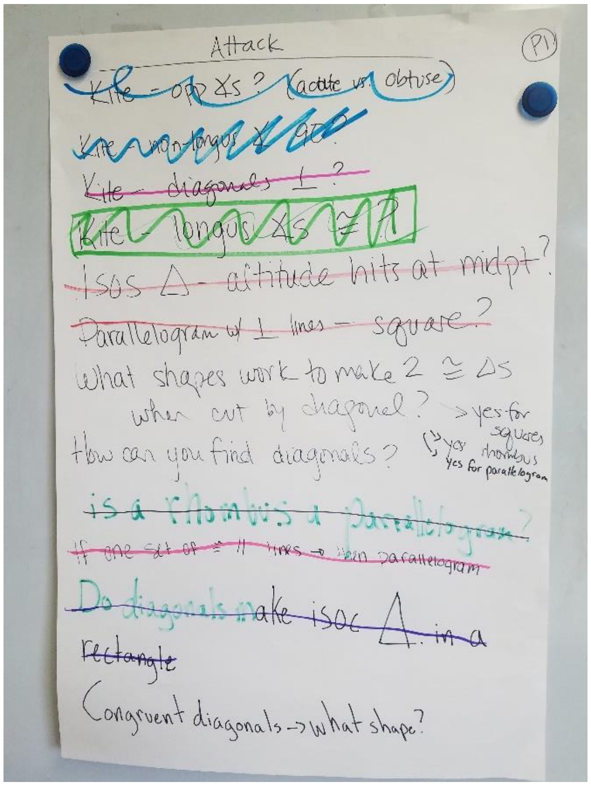
Student Generated Conjectures (Period 1)

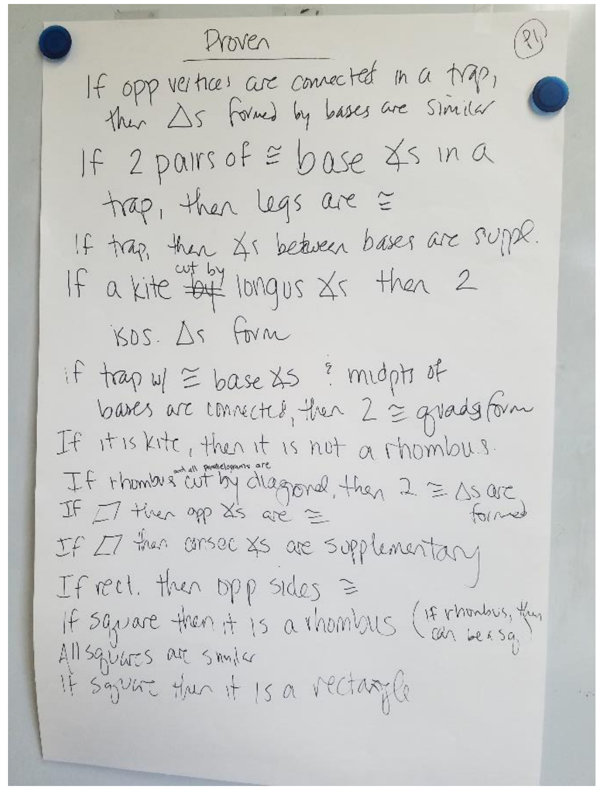


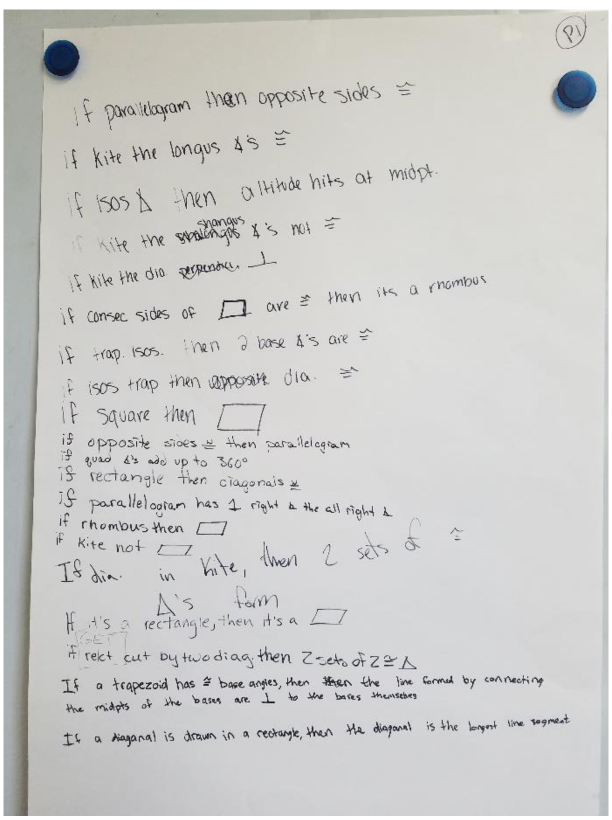



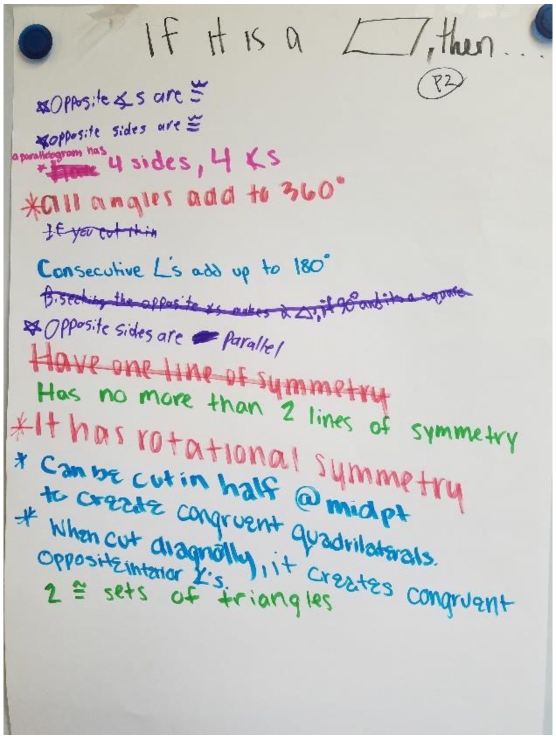



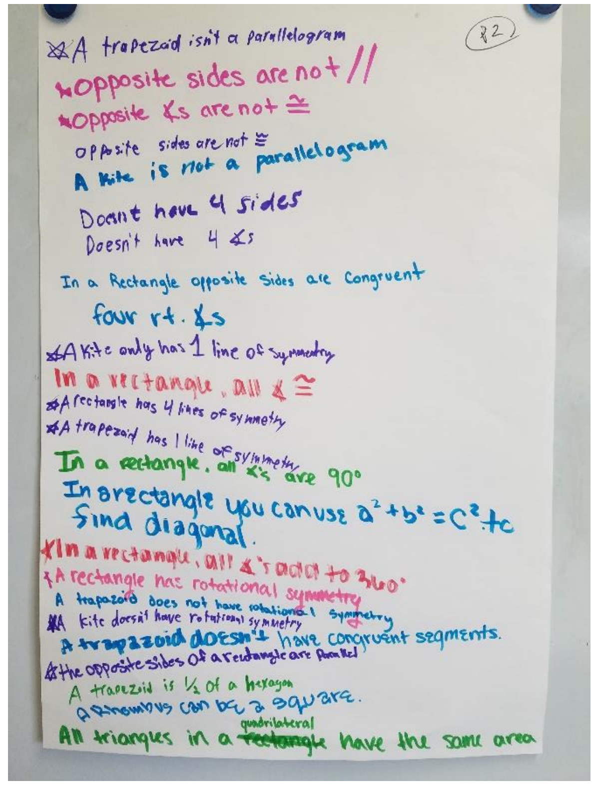

Attack
P1
~~Kite - opp ∡s ? (acute vs obtuse)~~
~~Kite - non-longus ∡ ≅?~~
~~Kite - diagonals ⊥ ?~~
~~Kite - longus ∡s ≅ ?~~
~~isos Δ - altitude hits at midpt?~~
~~Parallelogram w/ ⊥ lines - square?~~
What shapes work to make 2 ≅ Δs when cut by diagonal?
→ yes for squares
yes rhombus
yes for parallelogram
How can you find diagonals?
~~is a rhombus a parallelogram?~~
~~If one set of ≅ ∥ lines → then parallelogram~~
~~Do diagonals make isos Δ in a rectangle~~
Congruent diagonals → what shape?

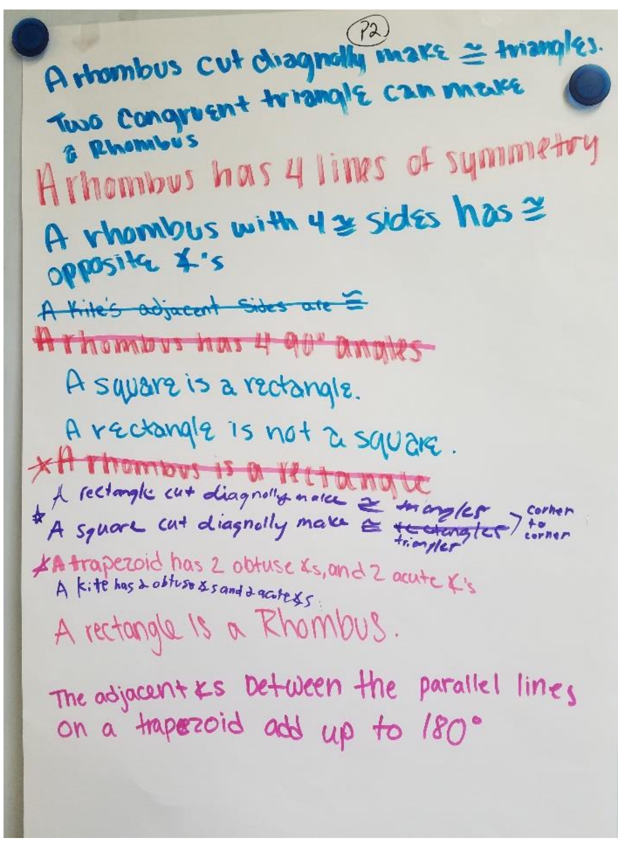

Proven
P1
If opp vertices are connected in a trap, then Δs formed by bases are similar
If 2 pairs of ≅ base ∡s in a trap, then legs are ≅
If trap, then ∡s between bases are suppl.
If a kite cut by longus ∡s then 2 isos. Δs form
if trap w/ ≅ base ∡s & midpts of bases are connected, then 2 ≅ quads form
If it is kite, then it is not a rhombus.
If rhombus and all parallelograms are cut by diagonal, then 2 ≅ Δs are formed
If ▱ then opp ∡s are ≅
If ▱ then consec ∡s are supplementary
If rect. then opp sides ≅
If square then it is a rhombus (If rhombus, then can be a sq.)
All squares are similar
If square then it is a rectangle

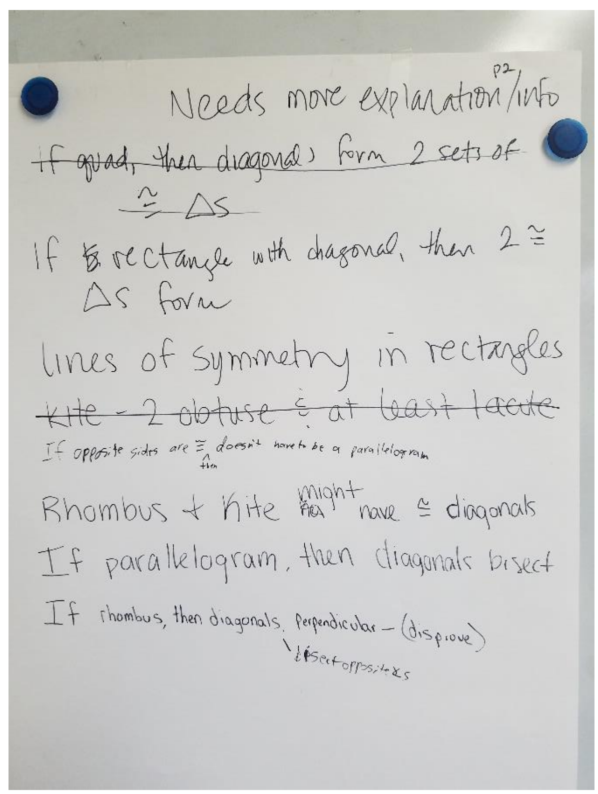

P1
if parallelogram then opposite sides ≅
if kite the longus ∡'s ≅
if isos Δ then altitude hits at midpt.
if kite the shlongus ∡'s not ≅
if kite the dia. perpendic. ⊥
if consec sides of ▱ are ≅ then its a rhombus
if trap. isos. then 2 base ∡'s are ≅
if isos trap then opposite dia. ≅
if square then ▱
if opposite sides ≅ then parallelogram
if quad ∡'s add up to 360°
if rectangle then diagonals ≅
if parallelogram has 1 right ∡ the all right ∡
if rhombus then ▱
if kite not ▱
If dia. in kite, then 2 sets of ≅ Δ's form
If it's a rectangle, then it's a ▱
if rect cut by two diag. then 2 sets of 2 ≅ Δ
If a trapezoid has ≅ base angles, then the line formed by connecting the midpts of the bases are ⊥ to the bases themselves
If a diagonal is drawn in a rectangle, then the diagonal is the longest line segment

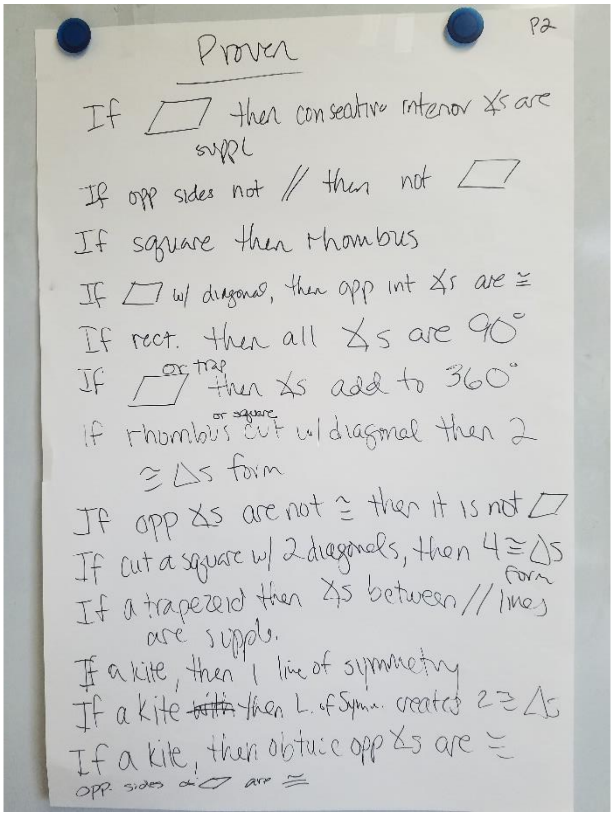
Student Generated Conjectures (Period 2)

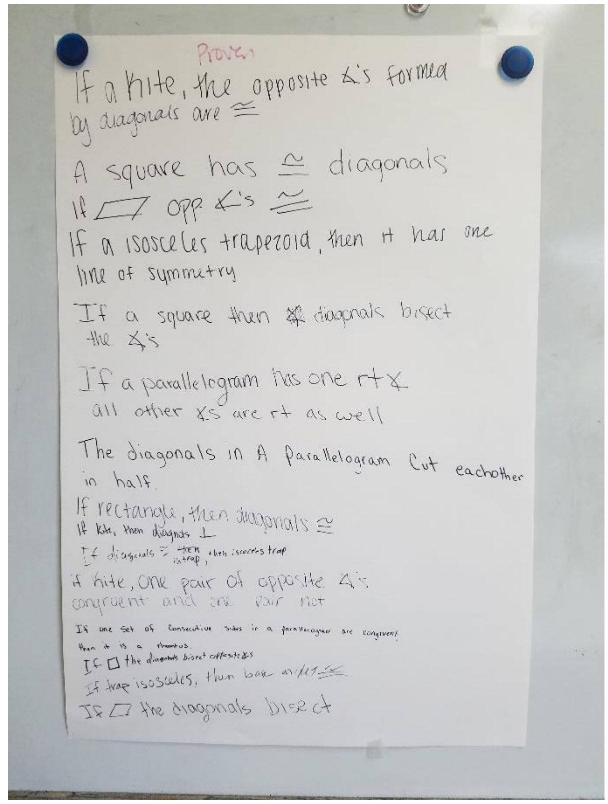


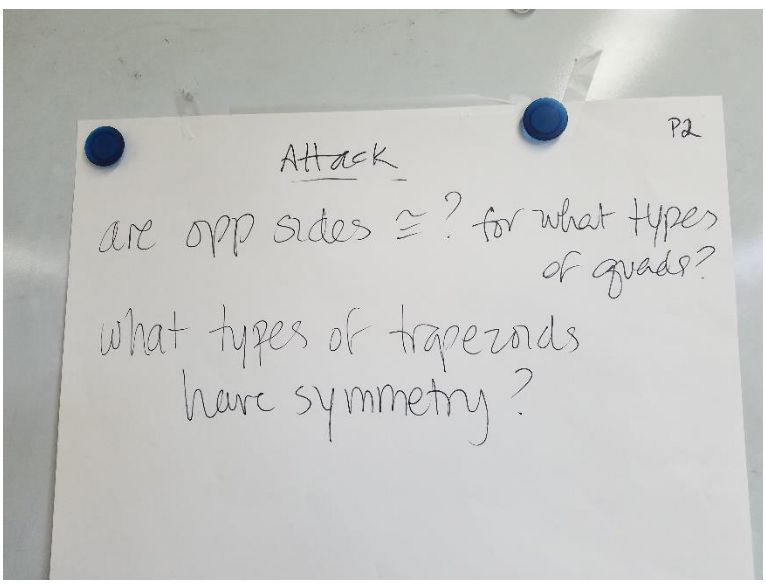



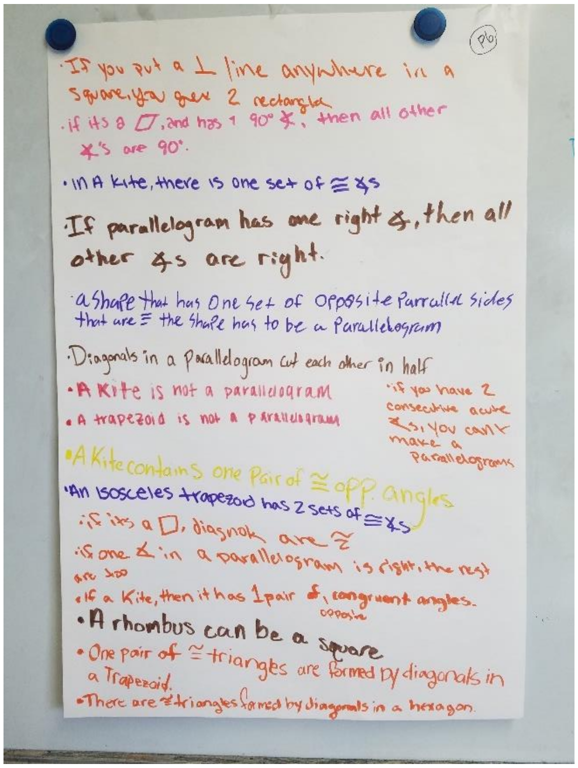



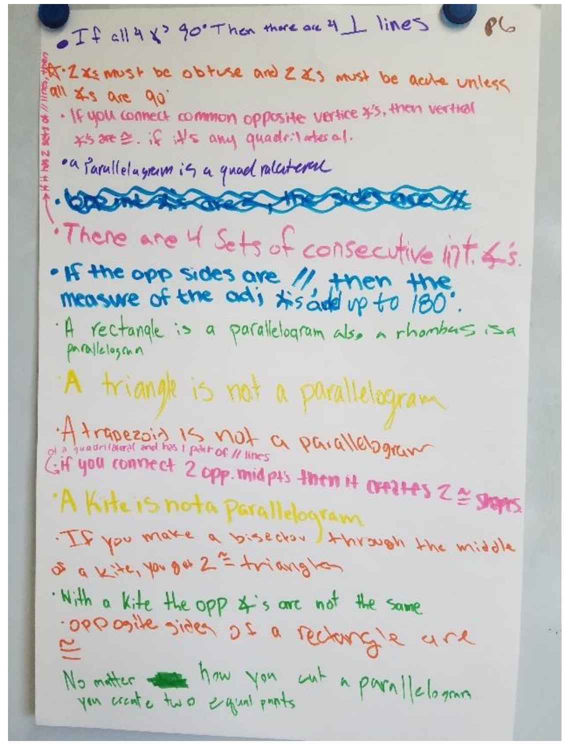

P2
Proven
If ▱ then consecutive interior ∡s are suppl
If opp sides not // then not ▱
If square then rhombus
If ▱ w/ diagonal, then opp int ∡s are ≅
If rect. then all ∡s are 90°
If ▱ or trap then ∡s add to 360°
if rhombus or square cut w/ diagonal then 2 ≅ △s form
If opp ∡s are not ≅ then it is not ▱
If cut a square w/ 2 diagonals, then 4 ≅ △s form
If a trapezoid then ∡s between // lines are suppl.
If a kite, then 1 line of symmetry
If a kite ~~with~~ then L. of Symm. creates 2 ≅ △s
If a kite, then obtuse opp ∡s are ≅
opp. sides of ▱ are ≅

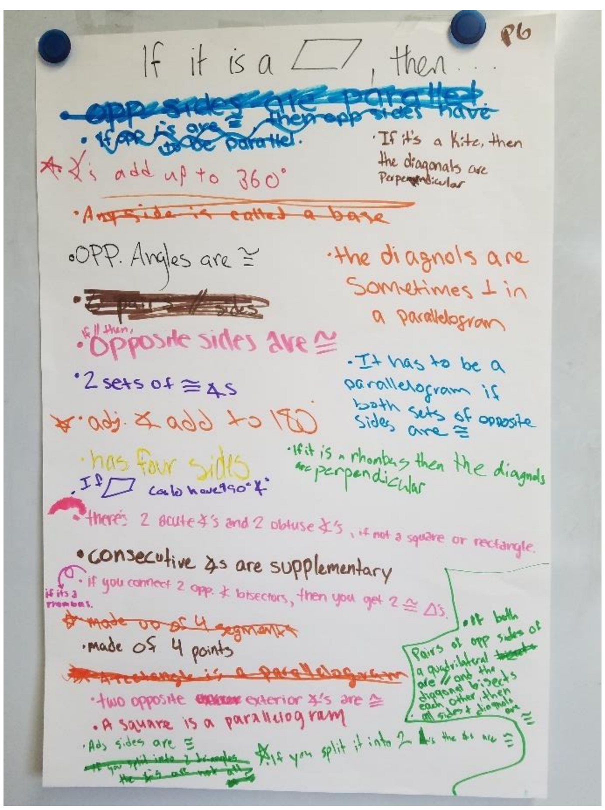

Proven
If a kite, the opposite ∡'s formed by diagonals are ≅
A square has ≅ diagonals
If ▱ opp ∡'s ≅
If a isosceles trapezoid, then it has one line of symmetry
If a square then diagonals bisect the ∡'s
If a parallelogram has one rt ∡ all other ∡s are rt as well
The diagonals in A Parallelogram Cut eachother in half
If rectangle, then diagonals ≅
If kite, then diagonals ⊥
If kite, one pair of opposite ∡'s congruent and one pair not
If □ the diagonals bisect opposite ∡s
If ▱ the diagonals bisect

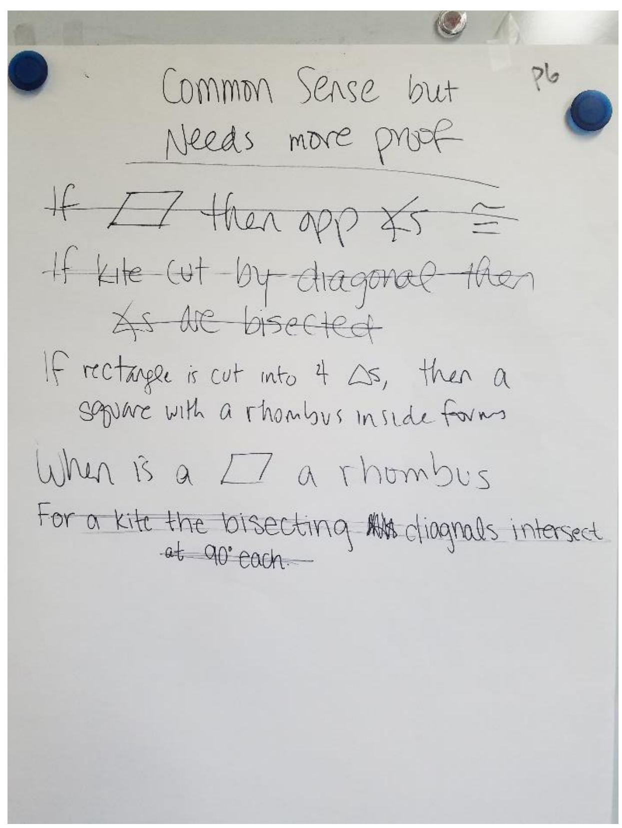

P2
Attack
are opp sides ≅? for what types of quads?
what types of trapezoids have symmetry?

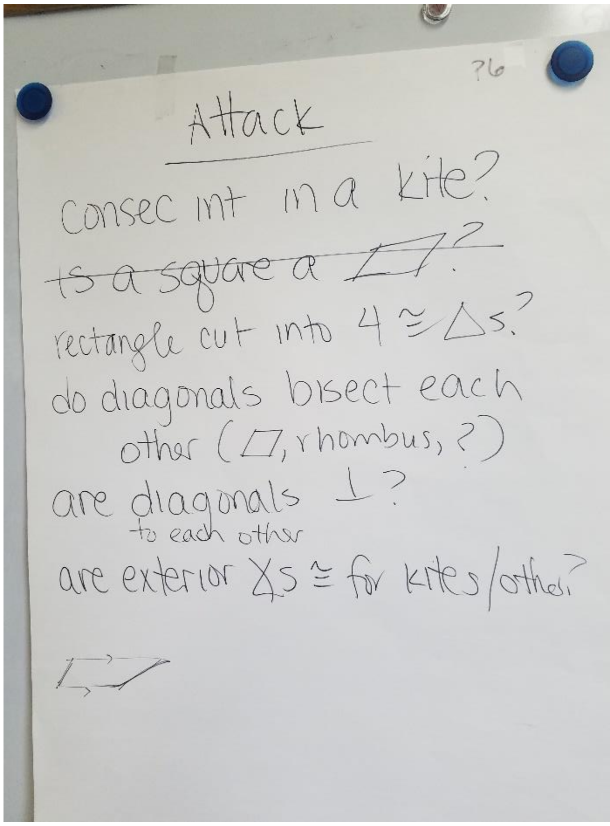
Student Generated Conjectures (Period 6)

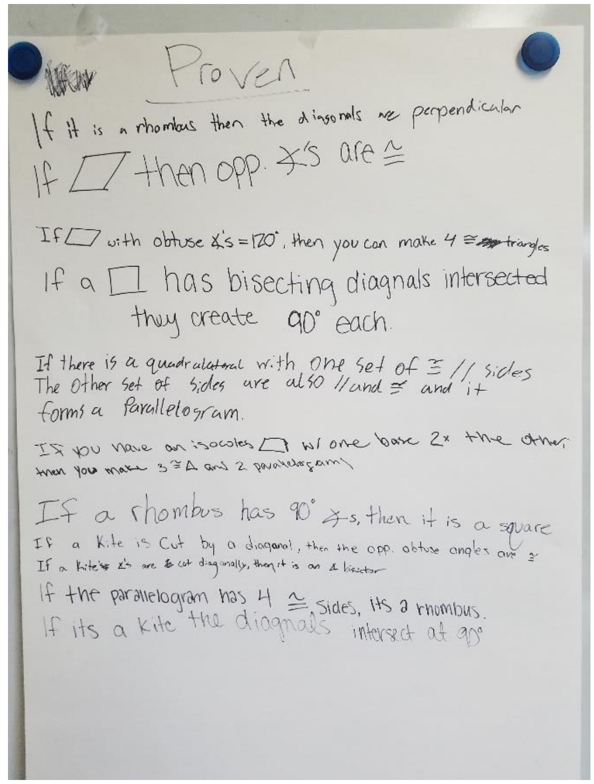


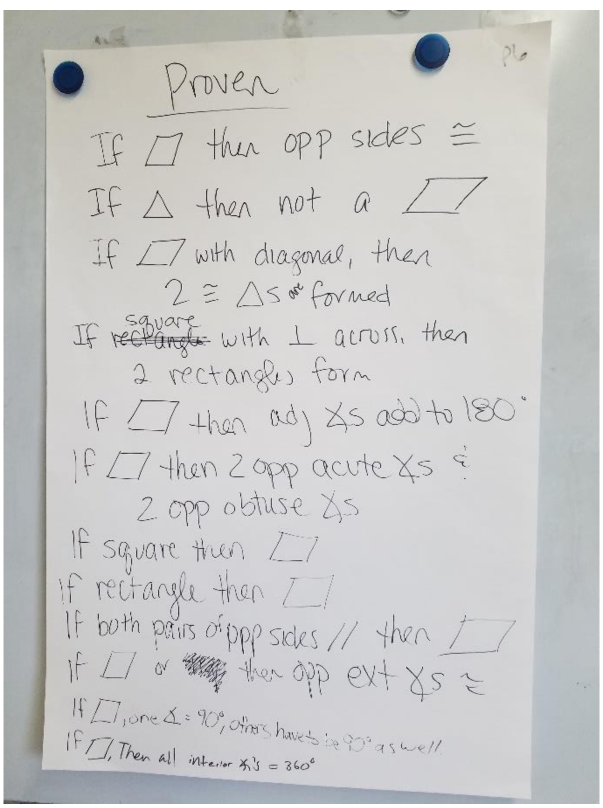




Attack
consec int in a kite?
~~is a square a~~ ▱?
rectangle cut into 4 ≅ △s?
do diagonals bisect each other (▱, rhombus, ?)
are diagonals ⊥? to each other
are exterior ∡s ≅ for kites/others?


Proven
If it is a rhombus then the diagonals are perpendicular
If ▱ then opp. ∡'s are ≅
If ▱ with obtuse ∡'s = 120°, then you can make 4 ≅ triangles
If a □ has bisecting diagnals intersected they create 90° each.
If there is a quadralateral with one set of ≅ // sides The other set of sides are also // and ≅ and it forms a Parallelogram.
If you have an isocoles ⏢ w/ one base 2x the other then you make 3 ≅ △ and 2 paralellograms
If a rhombus has 90° ∡s, then it is a square
If a kite is cut by a diagonal, then the opp. obtuse angles are ≅
If a kite's ∡'s are cut diagonally, then it is an ∡ bisector
If the parallelogram has 4 ≅ sides, its a rhombus.
If its a kite the diagonals intersect at 90°


Proven
If ▱ then opp sides ≅
If △ then not a ▱
If ▱ with diagonal, then 2 ≅ △s are formed
If square ~~rectangle~~ with ⊥ across, then 2 rectangles form
If ▱ then adj ∡s add to 180°
If ▱ then 2 opp acute ∡s & 2 opp obtuse ∡s
If square then ▱
If rectangle then ▱
If both pairs of opp sides // then ▱
If ▱ or then opp ext ∡s ≅
If ▱, one ∡ = 90°, others have to be 90° as well.
If ▱, Then all interior ∡'s = 360°

Quadrilateral Genealogy – Student Sample Work (top: Period 1. Bottom: Period 6)