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If you *really* want to get ahead, get a bunch of theories . . . and data to test them.[1,2]

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Abstract: *This paper addresses questions of mathematics teachers' professional development. My goal is not to provide "answers," although I have worked for some years to enhance teachers' capacity to create rich learning environments for their students. Rather, my goal is to problematize the issue, to ask: How do we frame questions of professional development in ways that are theoretically grounded? What theories do you need to know, in order do a good job of professional development? In the light of this kind of theoretical framing, I will discuss two related attempts at supporting teachers in their work.*

Keywords: Mathematics teacher professional development; Theories of teaching; Teaching-in-context

Introduction

To motivate the kind of approach I take in this paper, I begin with a metaphor. Suppose we consider the "problem" of air travel. The analogous question one might ask is: What theories do you need to know, in order do a good job of operating an air travel system?

One might start with grand theories. What does it take to get a plane off the ground? Newton's laws aren't bad for a start. If you're building airplanes, it helps to understand

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[1] As readers will recognize, my title is a shameless rip-off of Annette Karmiloff-Smith & Barbel Inhelder's (1975) "If you want to get ahead, get a theory." That's fine for simple problems – but as we'll see, complex problems require more than one theory, even if it's a grand theory.

[2] This paper is based on two presentations. The first, "Creating Support Structures to Help Teachers Engage in Formative Assessments," was given at the conference "Teachers as Stakeholders in Mathematics Education Research," Banff, Canada, December 5 - 10, 2010. The second, which evolved from the first, was entitled "If you *really* want to get ahead, get a bunch of theories . . . and data to test them," and was an invited presentation at the annual meeting of the American Educational Research Association, New Orleans, April 8 - 12, 2011.

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about gravity. But that's just a start. What about air lift? There are issues of wing configuration, for example; Bernoulli's laws apply there. Note that Bernoulli's laws, which are more "local," are at a different level of grain size than Newton's. Both the grand theory and the second, more local theory are essential. Moreover, to manufacture various parts of the plane you need theories of: metal expansion and contraction; jet propulsion (if you're manufacturing a jet), and relevant theoretical frames regarding the construction of every system (e.g., braking, oxygen, radar, etc.) that the plane will use. In addition, and absolutely essential: All of the theoretical ideas MUST be backed by rigorous empirical work, preferably in dialectic with the theoretical. Component systems are refined and tested rigorously before they are used for commercial flights.

All that, of course, is just for the airplane. But what about the human and material contexts within which air travel takes place? Having put together the most advanced and capacious airplane won't do any good if the local airport has a dirt runway and no radar; the technology has to fit the context. Beyond that, there is the question of how one builds a robust infrastructure for dealing with normal and not-so-normal travel-related issues. Recent news photographs of people stranded for days and weeks at airports because of volcanic eruptions or unexpectedly heavy snow make the point that a broad range of systems shape "local" day-to-day operations.

In sum, to make progress on complex, multifaceted issues (and supporting teachers' professional growth is certainly a complex and multifaceted issue!),

> We need lots of theories, at different levels of grain size;
>
> The theories should be refined and tested empirically;
>
> The systems and practices one builds must be context sensitive and adaptable.

This last point implies that there is no "one size fits all" solution to issues such as professional development.

Now, let us turn our attention to the issue of teachers' growth and development. I will argue that to address this issue in a reasonable way, one should be informed by (and contribute to the development of) theory related to:

- The dimensions of competency one would like to see develop;
- *How* things develop;
- A theory of change, and a plan embodying the theory;

All of which must be context- and material resource-sensitive, which means that it is also necessary to have

- A theory of individual learning and growth, and
- A theory of how ideas can be spread or squashed in a social (i.e., district, state) context.

In what follows, I will lay out some of our theoretical ideas about (mathematics) teachers' proficiency and its development, describe some current efforts to promote teachers' professional growth and say why I am worried about them, and frame the enterprise in terms of a larger, (prospective) data gathering effort. My hope is that the kind of effort one can envision based on the consideration of such ideas would be an appropriate way to approach teacher preparation and other heretofore intractable problems.

Theoretical backdrop

Theory, Part 1. Dimensions of teaching proficiency.

If one is engaged in supporting teachers' professional growth, it helps to have a theory of the dimensions in which one hopes teachers will grow. To sum up a chapter in a table, Schoenfeld and Kilpatrick (2008) offer a provisional framework for looking at the dimensions of teacher proficiency.[4] See Table 1.

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[4] Schoenfeld and Kilpatrick's chapter is specifically about dimensions of mathematics teachers' proficiency. However, I believe that the framework is general, in that one could replace "mathematics" with any other content domain, and it would remain valid.

Knowing school (mathematics) in depth and breadth
Knowing students as thinkers
Knowing students as learners
Crafting and managing learning environments
Developing classroom norms and supporting classroom discourse as part of “teaching for understanding”
Building relationships that support learning
Reflecting on one's practice

Table 1. A Provisional Framework for Proficiency in Teaching Mathematics

To presage what lies ahead, I shall not try to address all seven of these dimensions in this paper. What I will argue is that many of the proficiencies characterized in Table 1 are embodied by teachers who attend to student thinking in formative ways – teachers who building on student understandings and find productive ways to address student misunderstandings. Thus, I will argue, a productive focus for professional development is to help teachers engage in formative or diagnostic teaching (grounded in rich content-based understandings, of course).

Theory, Part 2. A theory of teachers’ in-the-moment decision making.

At a more fine-grained level, the consequential question in the classroom is, what “moves” is the teacher going to make, and why? Here I shall do my best to condense a book into a page or so. The key assertion in Schoenfeld (2010a), which builds on Schoenfeld (1998; 2000; 2008), is that *teachers' in-the-moment decision-making is a function of their knowledge/resources, goals, and beliefs/orientations.* The major theoretical-empirical claim in that body of work is that if one knows “enough” about a teacher’s resources, goals, and orientations, one can model his/her actions and explain them on a line-by-line basis. Here I will just suggest these three categories are "make or break" elements of effective teaching.

Little needs to be said about knowledge or resources: the importance of subject matter knowledge, general pedagogical knowledge, and pedagogical content knowledge is well understood. And, it goes without saying that material and social resources are highly consequential (cf. Kozol, 1992).

Thompson's (e.g., 1992) pioneering work on teacher beliefs established their importance; here I just describe one story to indicate they way they play out. Some years ago I sat in on a year-long geometry course in which the teacher took a very procedural approach to the mathematics: "First do this, then do that," etc. One day I asked him if he had ever thought of just giving the students a problem and seeing what they might do with it. "Not these kids," he said; "it would just confuse them. I might do that with my honors students" (and he did). To sum up in brief: he had the knowledge and skills to teach this group of students very differently. But, his beliefs about what the students were capable of doing and what was thus an appropriate pedagogy for them resulted in his choosing an approach that actually deprived them mathematically! (One can see analogies in reading groups, where high-flying readers get to discuss important ideas about the text, while other students are focused by the teacher on decoding.)

Finally, anyone who has seen the distortion of classroom practice because the teacher devoted days if not weeks to drilling students in preparation on low-level but high-stakes multiple choice standardized tests knows how the choice of goals for students can make a *big* difference.

In sum, any serious look at professional development should be concerned with the growth and change of teachers' knowledge/resources, goals, and beliefs/orientations.

Theory, Part 3. A Hypothesized Development Space: Dimensions of Teacher Growth[5]

In different countries, different styles of teaching (for example, depending on well constructed lectures as the primary form of instruction, or depending on having students interact with each other) tend to be valued differently (see, e.g., Stigler and Hiebert, 1999). Thus, the ideals to which teachers aspire will vary; and the trajectories toward those ideals will vary. In this section I am not suggesting a universal trajectory, but, rather, one that is consistent with my values as a researcher and professional developer, and that is consistent with developmental trajectories in the United States and some other nations (see, e.g., Fuller, 1969; Hord, Rutherford, Huling-Austin, & Hall, 1987; Lesh &Sriraman, 2010; Ryan, 1986; Smith, 2000).

From my perspective, a particular form of teaching expertise that is highly valuable and worth aspiring to is the ability to conduct "diagnostic teaching." This kind of teaching, in which teachers make significant use of formative assessment to see what their students understand, and shape their lessons according to what they discover about their students, exemplifies the productive use of pedagogical content knowledge as first described by Shulman (1986, 1987), and is consistent with the kind of teaching described in the U.S. National Council of Teachers of Mathematics' (1991) *Professional Standards for Teaching Mathematics*. In diagnostic teaching (or, teaching with a heavy emphasis on using formative assessments), the teacher has specific mathematical goals. In addition, the teacher recognizes that students have varied understandings of the mathematics under discussion. He or she probes for what the students know and then responds in ways that address errors and misconceptions, and that build on student understanding, to move the students toward the instructional goals.

Diagnostic teaching is a form of instruction to which some teachers in the U.S. aspire. This form of teaching is not well supported by typical teacher preparation programs, or what are called "in-service" or ongoing professional development programs for teachers. In the U.S., one sees typical development toward diagnostic teaching as represented in Figures 1, 2, and 3. Each figure includes three planes of teacher activities: "managing"

[5] This section is modified from Schoenfeld (in press).

the classroom, having students engage in mathematically productive and (one hopes) engaging activities, and engaging in diagnostic teaching. Typically, beginning teachers in the U.S. are still learning to manage classroom activities, and a large amount of their time and attention is devoted to this: see Figure 1.

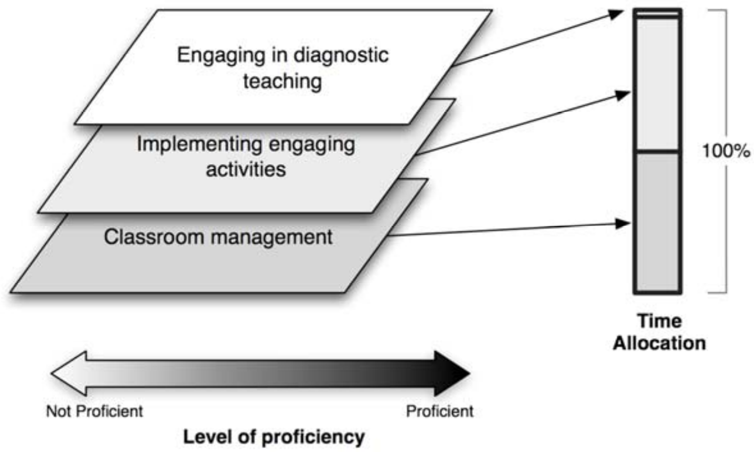

Figure 1. Levels of proficiency and time allocations of a typical beginning teacher. (The degree of shading in the planes represents the level of proficiency, and the arrows point to the percentage of time devoted to each plane of activity.) Reprinted with permission from Schoenfeld, 2010a.

As teachers become more proficient they spend less time on classroom management, both because they are better at it and because students who are actively engaged in doing mathematics do not need to be "managed" as much as those who are not productively engaged. Figure 2 provides the typical profile of an "accomplished" or "proficient" teacher.

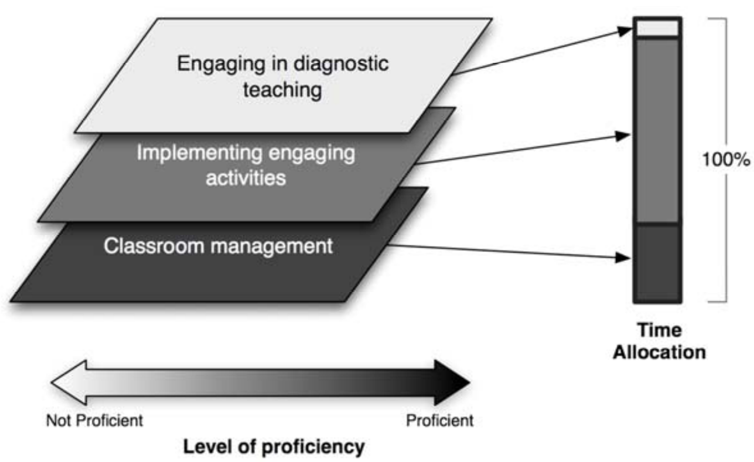

Figure 2. Levels of proficiency and time allocations of a typical accomplished teacher. Reprinted with permission from Schoenfeld, 2010a.

Many teachers – perhaps the majority of experienced teachers in the U.S. – have profiles as represented in Figure 2. A much smaller percentage of teachers engage, to any significant degree, in diagnostic teaching. When it is well done, diagnostic teaching is very responsive to student understandings, and it is likely to be engaging; as a result, classroom management does not require much time and attention, and the students are involved in productive mathematical activities a large percentage of class time. This kind of teaching, when well done, results in a profile such as the one given in Figure 3.

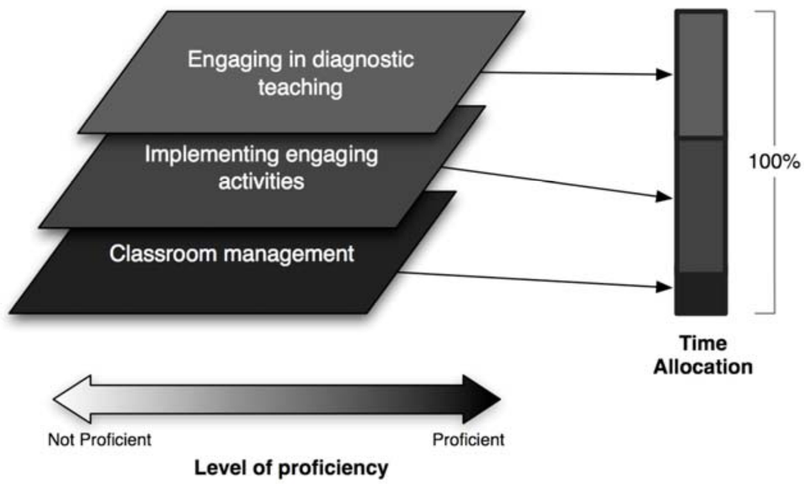

Figure 3. Levels of proficiency and time allocations of a highly accomplished teacher. Reprinted with permission from Schoenfeld, 2010a.

From my perspective a major challenge for professional development is to help teachers develop the resources, goals, and orientations that enable them to function in the ways represented in Figure 3. As indicated above, this is of necessity a slow process: even if a teacher has aspirations to teach in a particular manner, it takes some time to develop the resources (e.g., pedagogical content knowledge) that support teaching in that way. Some beginning attempts to provide teachers with that kind of support are described below in the next section.

The categories identified immediately above are not independent; they are deeply intertwined. For example, beliefs and goals "recruit" resources, in that the knowledge that becomes salient to a teacher is that which is relevant to current beliefs and goals; but resources constrain progress toward goals, in that (for example) a teacher who adopts a new set of goals may not have the resources with which to achieve them. (Cf. Cohen, 1990; Toerner, Roesken, Rolka & Sriraman, 2010). Moreover, the main point of *How we*

Think (Schoenfeld, 2010a) is that knowledge and resources, goals, and beliefs and orientations are all deeply intertwined. One has, thus, the following Serious Theoretical Corollary: *The evolution of professional competency is, of necessity, slow*. (Of course, this comes as no surprise: across the boards, the literature says that the development of expertise in any field takes 5,000-10,000 hours of focused, reflective action.)

In sum (a fact known to everybody in the business): there can be no "magic bullets" in professional development.

Theory, Part 4. Individual growth and change.

There is an obvious but often overlooked fact, which was specifically noted in the expanded edition of *How People Learn*: the conditions that are appropriate for children as learners are also appropriate for adult learners! Thus,

Learning environments for adults (e.g. professional development programs) must be

- Learner Centered
- Knowledge Centered
- Assessment Centered
- Community Centered (NRC, 2003, pp. 36-7)

Theory, Part 5. Institutional Surround and the need for coherence.

Systemic incoherence is the death knell of professional development. Specifically, there is evidence that there can be significant progress within a school district when:

- There is a set of high standards
- Curriculum is aligned with those standards
- Assessment is aligned with those standards
- Professional development is aligned with those standards
- There is enough stability for growth and change to take place.

If any of these are lacking, the chances for progress are significantly diminished. (See, e.g., Schoenfeld, 2006; 2009.)

I note that these comments, which focus largely on the conditions for classroom coherence, are consistent with a broader institutional perspective on the conditions needed for progress in urban school districts. For example, Bryk et al. (2010) identify the following "essential supports" at the systemic level: leadership as the driver for change; parent-community school ties; professional capacity; instructional guidance; and a student-centered learning climate.

Two practical attempts to make a difference.

To put things simply (and putting it this way to remind myself that all of what we do is a matter of values): The central issue of professional development is how to help teachers, over time, develop some of the skills and understandings we value.

As noted above, my focus in this paper, and in my work with teachers, is on formative assessment or "diagnostic teaching." I begin, here with a few words about formative assessment, to distinguish it clearly from summative assessment.

Summative assessments show what students "know and can do" *after* instruction. That's important, but it's too late to help the students learn. In contrast, formative assessments reveal students' current understandings so you can help them improve. There are some important issues to understand about formative assessments:

1. Formative assessment is not summative assessment given frequently! Pacing guides with monthly or biweekly scored exams may keep teachers and students in line, and let both know how well they are doing (in terms of scores at least), but they are not the same as formative assessment – especially because of the next point.

2. Scoring formative assessments rather than, *or in addition to,* giving feedback destroys their utility. Unscored comments help students learn; when papers are also scored, students don't read the comments. (Black & Wiliam, 1998).

3. Learning to attend to student thinking, and build on it, is difficult for teachers to do (at least in mathematics, but I suspect more generally. It's clearly the case in the sciences, given the misconceptions literature).

4. Thus, it is useful to provide teachers with tools to help them make effective use of the information that formative assessments provide.

In consequence, the central question I address in my professional development work is:

How can one support teachers in doing formative assessments, as a mechanism for the teachers' professional growth and *to the benefit of their students?* In what follows I will describe two synergistic but different attempts to do so:

I. The National Research Council's SERP (Strategic Educational Research Partnership) collaboration with San Francisco, and

II. The "Mathematics Assessment Project," or MAP, funded by the Bill and Melinda Gates Foundation.

In the San Francisco SERP partnership, we have collaborated with teachers from San Francisco Unified School district to create instructional support materials ("formative assessment lessons"; see below for a descripion) for grades 6, 7, and 8. For a description of SERP, see Donovan & Pellegrino, 2003, or the link at <http://www.serpinstitute.org/content/index.php>. MAP is a joint project between U.C. Berkeley and the University of Nottingham, in collaboration with the Silicon Valley Mathematics Initiative and Inverness Research Associates. The goal of the MAP project is to create and distribute, free for non-commercial use, a spectrum of summative assessments and formative assessment lessons. Some summative assessments and many of our formative assessment lessons (including the full formative assessment lesson that addresses instructional challenge 2, described below) can be found on the project web site, <http://map.mathshell.org/materials/>. For a discussion of the process of designing such lessons, see Swan (2008). For a comprehensive treatment of related design issues, see Swan (2005).

A major goal of both projects is to help teachers develop the kinds of skills, understandings, habits of mind, supportive beliefs, etc., that will enable them to engage in formative or diagnostic teaching. As suggested by my air travel metaphor, what one needs at minimum in order to address large-scale, socially embedded issues such as professional development is:

- A focal mechanism for helping teachers teach in formative ways, and
- A cultural surround that supports the "take-up" of the focal mechanism.

I address both in turn.

The focal mechanism

A major part of the mechanism at the heart of our professional development work in both projects is what we call the *formative assessment lesson.*

In simplest terms, a formative assessment lesson includes a rich "diagnostic" problem, and things to do when one sees the results of the diagnosis.

What follows are two examples of instructional challenges around which we have built formative assessment lessons and brief descriptions of attributes of the packages themselves. The idea, in brief, is that a formative assessment lesson is designed to provide enough lesson scaffolding to enable teachers who use the materials we design to teach, with moderate success, a formative or "diagnostic" lesson.

Instructional Challenge 1: In their algebra or pre-algebra classes students learn the point-slope formula for a line, and sometimes other formulas such as the two-point formula. However, they often have trouble crafting graphs if the information they are given is not in "standard form." More generally, although students may know that there are various formulas for the equation of a line (the point-slope formula, the two-intercept formula, etc.) they rarely understand that the formulas are specific instantiations of the following general fact: knowing any two pieces of information about a line (e.g., two points on it, a point and the slope of the line, the two intercepts, a point and a rate of change, . . .) will enable you to determine the graph and equation of the line.

The way we approach this challenge is to first give students a "mystery" - a story about a stolen pie[6]. Students are told that passengers on a bus saw the pie on a ledge when they were riding northward, but that the pie was gone when the bus made its return trip. They are given the information about the whereabouts of a number of suspects, for example, where Tom lives, and the fact the Tom left his house for a walk at 4 PM, walking 9 miles north to arrive for dinner at 6:15 PM. See Figure 4.

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[6] The inspiration for this task was an enrichment activity in a Prentice-Hall algebra text book (Smith, 2001).

WHO STOLE THE APPLE PIE?

The Mystery…

Ms. Lee lives 24 miles north of Oakville on the way to Albany. She baked an apple pie and set it out on her windowsill at 3:00 pm to cool. She went to get it at 6:30 pm and found it had been STOLEN!

Tom, Dora, Harry, and Joan, who all live on the road between Oakville and Albany, are the main suspects. Ms. Lee needs you to use the following information to solve the mystery of who stole the pie, and find the thief.

THE BUS DRIVER: A **bus** left Oakville at 4:OO p.m. going nonstop to Albany, 30 miles north. The bus arrived in Albany at 4:45 p.m., waited 15 minutes, and then returned to Oakville at the same rate. The driver saw the apple pie on Ms. Lee's windowsill as she drove north, but it was gone when she passed by on the way south back to Oakville.

TOM Tom lives 17 miles north of Oakville. He left home at 4:00 p.m. and walked 9 miles north to arrive at Curley's Burger Stand for dinner at 6:15 p.m.	**DORA** Dora lives 12 miles north of Oakville. At 3:00 p.m., she left home and rode her bicycle north at 9 miles per hour to the bus stop in Albany.
HARRY Harry left his house, 2 miles north of Oakville, at 3:30 p.m. He drove north for a half an hour at 36 miles per hour. He stopped at the Science Museum for 45 minutes. He then drove to Curley's Burger Stand, 26 miles from Oakville, arriving there at 5:00.	**JOAN** Joan was at her house, across the street from the Science Museum, 20 miles north of Oakville, until 5:00 p.m. She then jogged north for one hour at 6 miles per hour. She stopped at Curley's Burger Stand for a soda and met a friend.

Figure 4. The Apple Pie Problem.

The given information allows one to rule out all but one suspect – but, if one tries to reason one's way through the problem directly, it is very hard to keep track of all of the relevant information. Thus, the first meta-lesson for the students (who are given the mystery the day before the lesson) is that graphing can be useful!

The lesson packet for the teacher includes a complete set of instructional materials (summarized below) and a lesson plan. There are also sections designed to help the teacher make effective use of the materials. There are teacher-to-teacher notes, intended to explain our approach. The notes for this lesson include the following:

- Without the use of graphing, this problem is VERY hard to solve! That's why we give it as a "teaser" the day before. If the students see how hard it is to solve without the graphs, they'll appreciate the use of the graphs when they work through the problem.

- There are a lot of words in this problem. That's deliberate. Our goal is to help students with second language or other linguistic challenges to make sense of the problem situations, rather than simplifying the language into "bite-size" pieces that no longer represent typical spoken or written English.

- The question sets are written so that they will reveal student misunderstandings. Extra questions below are intended to do the same. A series of pointed questions will often get students to see the correct mathematics; or when they explain what they have done correctly, it will be reinforced. Then they will remember it better.

There are notes on expected student difficulties, for example,

- Students are likely to have difficulty seeing that a fixed object, such the pie on Ms. Lee's windowsill or the bus station in Albany, is represented as a horizontal line segment on a distance-versus-time graph.

- The x-axis represents time. Some students may not be able to locate "fractional times" (e.g., 3:30) as points on the axis, and some may be confused because the x-values do not start at 0. Similarly, that some people's trajectories start in the "middle" of the graph may be confusing.

- Understanding that the coordinate pair (time, location) corresponds to a where an object is a some point in time – e.g., answering the question "Where is everyone at 4:15 PM?

- Interpreting rates such as 18 mph, especially when a person is traveling for less than 1 hour.
- Understanding simultaneity – that when two graphs cross, the two things represented are at the same place, at the same time.
- Seeing that steeper lines mean greater speed.

For each of the expected difficulties there are some suggestions regarding questions the teacher might ask the students, in order to help them address those difficulties. In addition, there are samples of student work, which allow the teacher to see what his or her students are likely to produce, and a discussion of how the issues the student work raises might be addressed.

There is, of course, a lesson plan, and a full set of lesson materials. These materials include a "starter" graph (Figure 5), which the teacher uses to get the students going on the task:

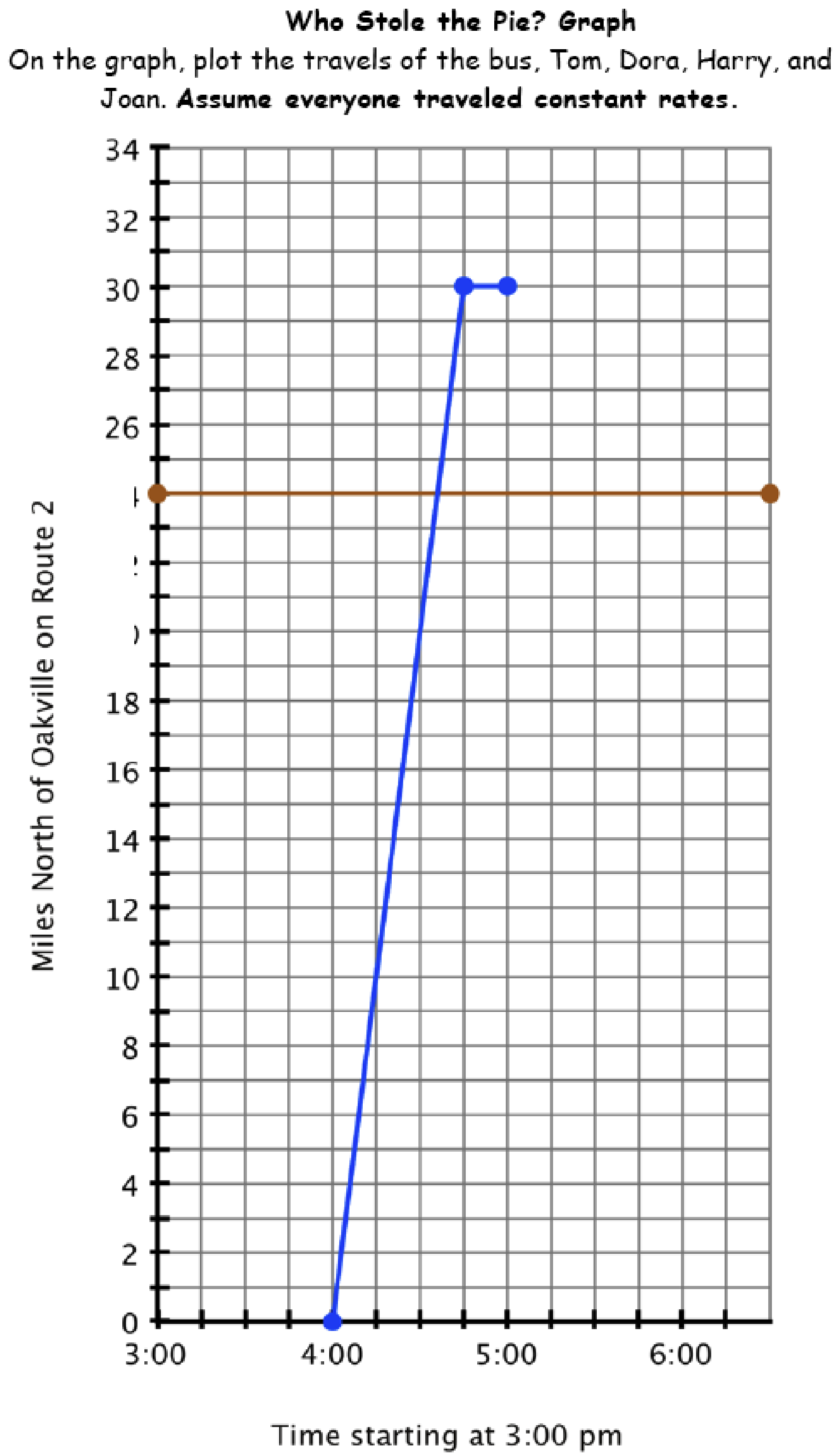

Figure 5: The starter graph.

The students are also given a set of "Getting Started" questions to address:

- Where are Oakville and Albany on the graph?
- What is the meaning of the horizontal line at the point 24 miles north?
- How is the x-axis different from other x-axes you've worked with?
- Where would you find 4:30 on the x-axis?
- Why does the line for the bus have a horizontal section?
- How would you graph the bus's return trip?

Figure 6. Getting started" questions"

These are followed by a series of question sets that the students work over the course of the lesson (which may take two to three days):

Question Set I

Use your graph to answer these questions and solve the mystery. Use extra paper if needed to give complete answers to each of the questions.

- Which of the suspect's graphs did you find easiest to graph? Explain why.
- Which of the suspect's graphs did you find most challenging to graph? Explain why.
- Who stole the apple pie? Write a convincing explanation about how you know who is guilty and who is innocent.

Question Set II

Use your graph from Who Stole the Apple Pie to answer these questions. You need to be able to explain your answers.

- Where are each of the characters in this problem (Tom, Dora, Harry, Joan, the bus, and the apple pie) at 4:15? Explain how you know this.

- Who stops at Curley's Burger Stand? How does the graph help you see when each person arrived at Curley's Burger Stand?

- When did Dora bike past Curley's burger stand? How can you figure this out from the graph?

- Does Harry drive at the same speed from his house to the museum as he drives from the museum to Curley's? Explain how you figured this out.

- Who does Tom see on his walk? When does Tom see the bus pass him?

Question Set III

- Write two questions (similar to those in Question Set II) that can be answered using this graph. Include the answers after your questions.

- You find out that Edward is also a suspect. Here is his "graph" (not shown). Write a story to fit his graph and decide if he could have been the thief.

These questions provide students with an opportunity to grapple with the relevant mathematics and to reflect on their thinking and learning. The questions are discussed in small groups as the class works on them, and then in whole class discussions.

In sum, this lesson tries to support student learning through question-asking, and tries to support the teacher by highlighting issues that are likely to arise and discussing things the teacher can do about them. (The teacher who co-designed this lesson, Shauna Poong, likes to have the questions in written form for her students. This way they all get to address all the questions when they (or their group) is ready, and she can circulate through the classroom working with small groups or individual students.)

The cultural surround for this lesson is discussed following the presentation of the second instructional challenge.

Instructional Challenge 2: We know that students have many graphing misconceptions, e.g., confusing a picture of a story with a graph of the story in a distance-time graph. The following lesson, designed by Malcolm Swan, addresses that issue directly[7].

The lesson sequence begins with a pre-assessment, a task that serves to reveal what the students understand about distance-time graphs and what might be problematic for them. The students work the pre-assessment the day before the full lesson, providing the teacher the opportunity to analyze student responses, observing student strengths and seeing what is problematic. In preparation for the full lesson, the lesson packet provides the teacher with a set of common student issues. One such example is:

Student interprets the graph as a picture

For example: The student assumes that as the graph in “Matching a graph to a story” (Figure 6) goes up and down, Tom's path is going up and down.

Or: The student assumes that a straight line on agraph means that the motion is along a straight path.

Or: The student thinks the negative slope means Tom has taken a detour.

To address such issues, the lesson packet offers a corresponding set of suggested questions and prompts. For the issue above, for example, the packet offers the following:

Suggested questions and prompts.

- If a person walked in a circle around their home, what would the graph look like?
- If a person walked at a steady speed up and down a hill, directly away from home, what would the graph look like?
- In each section of his journey, is Tom's speed steady or is it changing? How do you know?

[7] As noted above, the full lesson can be downloaded from the MAP web site, <http://map.mathshell.org/materials/>. Thus this discussion is more telegraphic than the discussion of challenge 1.

• How can you figure out Tom's speed in each section of the journey?)

The lesson itself begins with a task for individual work and then group discussion. The task, “Matching a graph to a story,” is designed to elicit typical misconceptions, so they are “aired” in classroom discussion. See Figure 6.

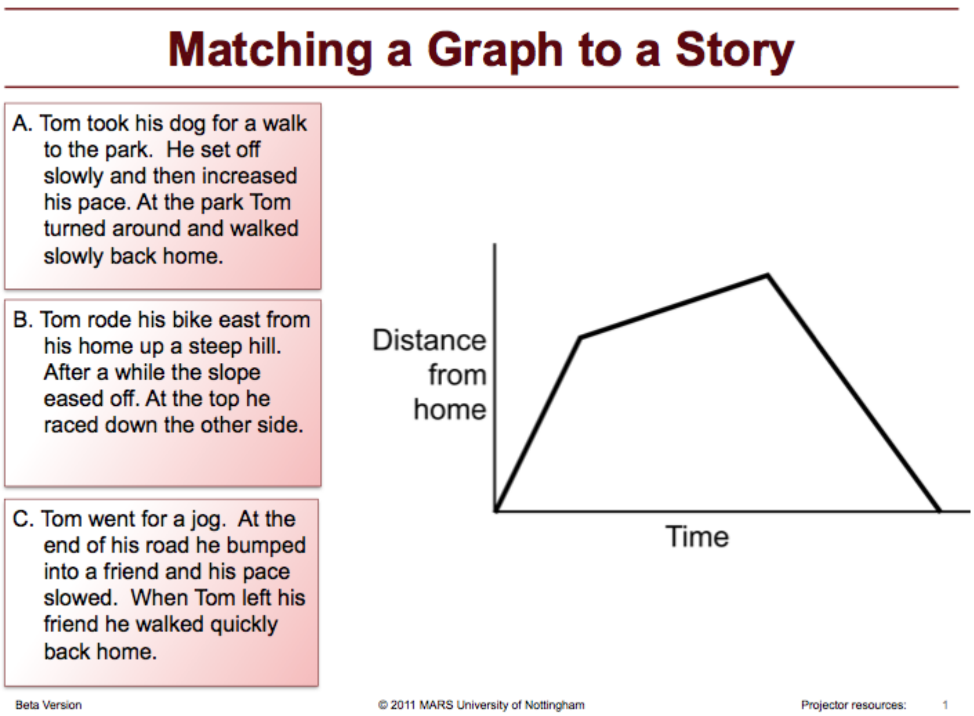

Figure 6.

After the class has discussed this task – but not resolved it – the students are given a first card sorting activity, in which they are asked to match a series of distance-time graphs with a collection of stories – see Figure 7 for some of the graphs, and figure 8 for some of the stories.

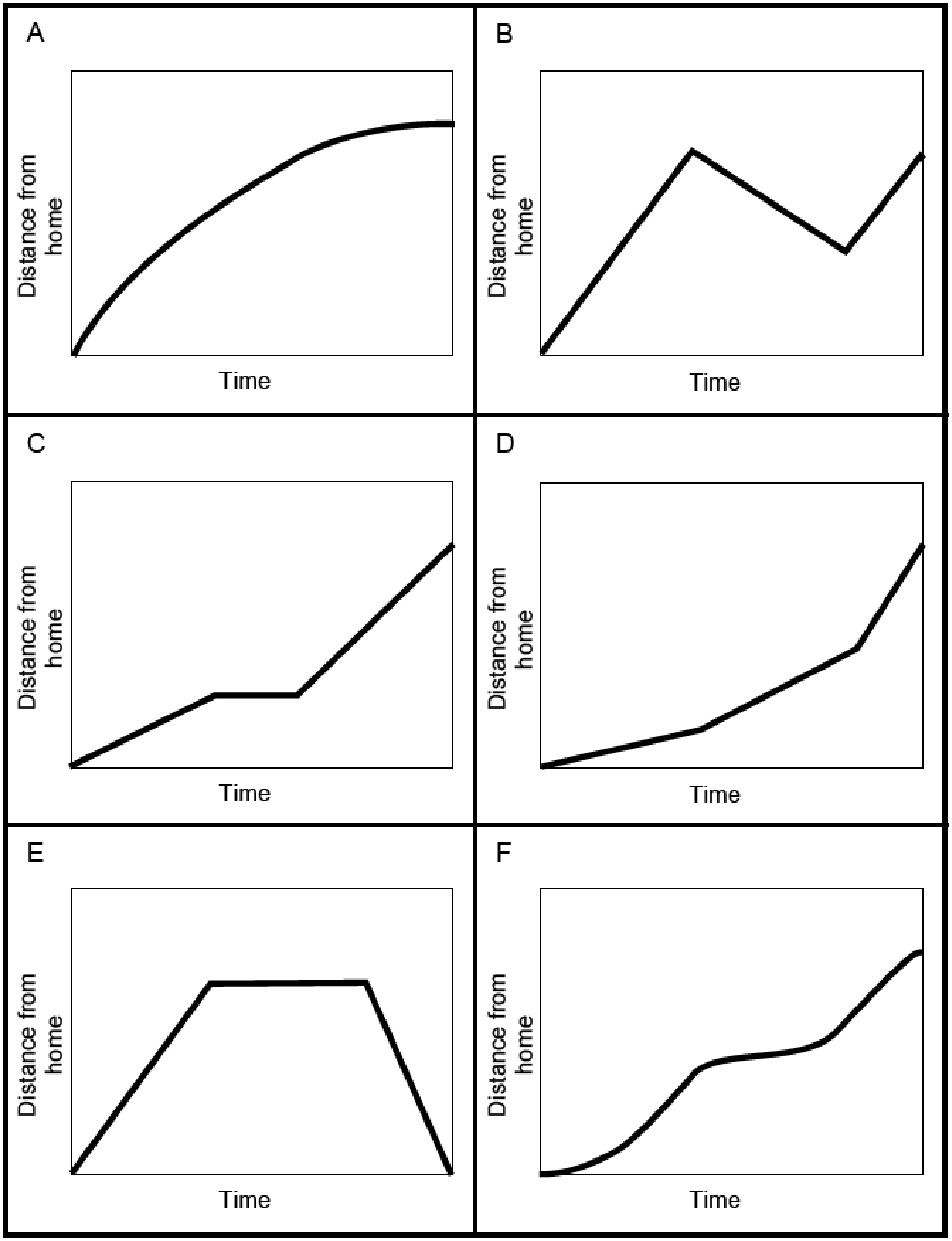

Figure 7. The first six graphs for the card sort (there are 10)

1 Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.	**2** Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.
3 Tom skateboarded from his house, gradually building up speed. He slowed down to avoid some rough ground, but then speeded up again.	**4** Tom walked slowly along the road, stopped to look at his watch, realized he was late, and then started running.
5 Tom left his home for a run, but he was unfit and gradually came to a stop!	**6** Tom walked to the store at the end of his street, bought a newspaper, and then ran all the way back.
7 Tom went out for a walk with some friends. He suddenly realized he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.	**8** This graph is just plain wrong. How can Tom be in two places at once?
9 After the party, Tom walked slowly all the way home.	**10** Make up your own story!

Figure 8. Stories to be matched to the graphs in Figure 7.

The matching activity gives rise to lively debates, as the students realize that two incompatible stories have been matched to the same graph, or that they want to attach the same story to two different graphs. After the students have discussed these issues for a while, the class is given the “mediating” tables of values in Figure 9. By considering how a story would generate a distance-versus-time table, and then graphing their table, the students can iron out many of their difficulties.

P

Time	Distance
0	0
1	40
2	40
3	40
4	20
5	0

Q

Time	Distance
0	0
1	10
2	20
3	40
4	60
5	120

R

Time	Distance
0	0
1	18
2	36
3	54
3	84
5	120

S

Time	Distance
0	0
1	40
2	80
3	60
4	40
5	80

T

Time	Distance
0	0
1	20
2	40
3	40
4	40
5	0

U

Time	Distance
0	0
1	30
2	60
3	0
4	60
5	120

V

Time	Distance
0	0
1	20
2	40
3	40
4	80
5	120

W

Time	Distance
0	0
1	45
2	80
3	105
4	120
5	125

X

Time	Distance
0	120
1	96
2	72
3	48
4	24
5	0

Y

Make this one up!

Time	Distance
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Z

Make this one up!

Time	Distance
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Figure 9. Data tables for the distance-time card sort.

The full lesson package provides a lesson plan and various other supports for the teacher.

The cultural surround.

The previous section describes the formative assessment lessons we have been producing. The question, then, is: How do they fit into a coherent program of professional development? This is where context and a host of social issues arise.

Among the issues one needs to confront in providing a productive professional development context for the use of these lessons are the following:

- How might one make these ideas accessible to individual teachers;
- How one might foster the individual changes in knowledge, habits of mind, beliefs consistent with this approach; and
- How to consider larger social & professional context things that shape perceived opportunities (or lack thereof).

Here, first, is the story with regard to SERP.

We (my students and I, some San Francisco Unified School District personnel, and some others) have worked with 6 teacher "co-developers" over a period of years. We started by focusing on attunement to student thinking, and moved on to develop and teach lessons of the type in the appendices. ("Who Stole the Apple Pie?" is one of our 8th grade lessons.)

Our first attunement technique, aimed at attending to student thinking, was to ask our partner teachers to interview some of their students[8]. We gave them tape recorders and asked them to pick students in whom they were interested, and then interview the student as he or she worked a typical problem from the curriculum.

A seventh grade teacher interviewed a particular student because she felt that the student did not belong in her class. This student's homework had never revealed more than "chicken scratches on the page," providing no evidence that she was following the material. The teacher chose to interview the student in order to get a better sense of what

[8] This discussion is radically condensed from Schoenfeld (2010b).

she knew and did not know. For the interview she chose a problem straight from the curriculum:

> A five-pound box of sugar costs $1.80 and contains 12 cups of sugar. Marella and Mark are making a batch of cookies. The recipe calls for 2 cups of sugar. Determine how much the sugar for the cookies costs.

This task is linguistically complex, especially for second language students. Many students had had difficulty with this problem, and the teacher expected this particular student to struggle. She asked the student to read through the problem and then to think out loud as she worked on its solution.

In the interview the student did some very sensible things – and then produced some incomprehensible chicken scratches on the page. (The result of her dividing 1.80 by 12 was ".13.3 cents.") When the teacher asked the student for an explanation, the student made it clear that to get the cost of one cup of sugar she had to divide $1.80 by 12; then she had to double that number to get the cost of the sugar needed for the recipe. Moreover and unlike many students, she checked the reasonableness of her answer. She knew she had done something wrong because the numerical value she had obtained did not fit the conditions of the problem.

When the teacher brought this tape to the SERP meeting, she was radiant. "I thought the student didn't belong in the class," she said. "All I'd seen were chicken scratches on the page. But now I see she totally gets it conceptually; she just has problems with the algorithms. She definitely belongs in the class, I can do remediation on the algorithms."

The teacher paused, shook her head, and then said, "I had a completely wrong impression of her . . . Oh my god, I'm going to have to interview all my students!"

In short, our discussions focused on student thinking. This focus made a difference, and resulted in our basing some instructional techniques on what we observed in typical student behavior. (For example, students tend to do what one teacher called "number mashing," combining the numbers in a problem statement before they understand the conditions of the problem. We developed classroom techniques to forestall this.)

Now, what I have just described . . . is what I truly believe is a wonderful teacher-researcher community that has stimulated real professional growth for all concerned. However, it may be the world's most expensive per capita professional development program for 6 teachers.

The real question is, How do we get these ideas out to the larger San Francisco teaching community? As indicated, we build formative assessment lessons, which contain a fair amount of support structure.

But that isn't enough, for a number of reasons – the first of which is that what we're trying to do is embody a particular approach to teaching, and we have seen that teachers can take the lesson packages as *scripts*, following the letter of the lesson rather than its spirit. This is not what we intend. We want teachers to be able to work in the spirit of the lessons.

We do have a mechanism to address this issue. The idea is to provide a "surround" for the lessons, which will (if all goes according to plan) be available on the web for all of the district's teachers. Here is a (somewhat idealized) version of how that process works.

First, teacher-researcher dyads (called co-developers) develop the lessons.

Second, a teacher co-developer teaches the lesson in collaboration with the researcher co-developer. We videotape:

- a pre-lesson interview, in which the focus is on what the teacher is trying to learn about the students and what the lesson is supposed to reveal;
- the lesson itself, focusing largely on student work;
- a post-lesson interview, in which the focus is on what the lesson revealed about student thinking and how it might be modified to be more effective.

Third, we create a package of materials that contains the lesson, annotated student work, and teacher comments. This package includes video records (from the taped lessons) that serve as opportunities for focusing on student thinking and how to build on it.

Fourth, the other teacher co-developer at the same grade level goes through the same process with the lesson package. He or she teaches the same lesson and annotates the resources with his/her own comments.

This should, we hope, produce shareable resources grounded in instruction. We have a lovely development plan, tying our packages to the district's curriculum and assessment schedule. If all goes well, by the end of next academic year all of this will be available through the web to all of San Francisco's middle school teachers.

Let us, for the moment, view all this through rose-colored lenses. These are just materials. There are still issues of how you really get things out to a district, in ways that make a difference.

It's time for a brief theoretical interlude:

Theory, Part 6: "Spread"

On the one hand, we know that top-down mandates from a district tend to die rapidly. Teachers who are not enthusiastic about the latest, greatest top-down ideas know how to "duck and cover."

On the other hand, we know that bottom-up activities are hard to maintain, and lose momentum. Innovators tend to burn out. (McLaughlin & Talbert, 2001).

How might one cope with this? Imagine a hybrid, justified in part by social network theory? (see, e.g., Coburn & Russell, 2008). One can envision a scheme where the district introduces the idea of SERP lessons, but the SERP teachers are the experts who provide much of the district-wide live professional development. (Recall, there should be an extensive web support network as well.) In this way, the SERP teachers are given credibility as lesson developers by the district, and the teachers see themselves as being helped by their colleagues rather than as the simple victims of a top-down mandate.

This, as noted above, is the rose-colored version. I'll get to the need for data in a moment.

Now, lets us return to the issue of the cultural surround.

What about districts where we don't have such access or resources? How does one provide a cultural surround, and the grounds for the development of the appropriate pedagogical practices and habits of mind, absent the kinds of support potentially available in the San Francisco Unified School district?

That's the Mathematics Assessment Project. MAP is producing 20 formative assessment lessons per grade, targeted to central concepts in the Common Core State Standards. The Gates foundation will make these lessons available to all, on the web, at no cost! Thus, any school district can have access to these materials. In addition to the 20 lessons at each grade addressing core content via formative assessment, there will be some videotapes for each grade showing teachers how they might work with other teachers at their school to build a teaching community that supports the productive use of such formative materials.

Twenty lessons of one-to-two-day length amount to a substantial part of the curriculum. Our hope is that, having taught 20 formative lessons, even if by partly following a script, teachers will see the value of such an approach and perhaps even begin to develop habits of mind consistent with formative/diagnostic teaching.

Reflections, Worries, and the Need for Data

Can the ideas described in the preceding section work? Will they? That depends on many things, most of which are systemic and beyond our control.

First, consider the SERP process. Will the San Francisco Unified School District do what we think it needs to do? Ultimately, that's a matter of resources and will. Mixed messages or lack of district support can torpedo our effort before it gets off the ground. Will teachers buy in? That remains to be seen. What we have is a set of hypotheses, grounded in theory, about an approach that might have a chance to work.

Second, consider the Gates project lessons. In all honesty, the support for professional development is small: Without district or school support, a few videotapes are not enough to support teachers in building a supportive community. And, what we have right now is an untested hypothesis – that teaching 20 formative lessons will make a difference, as a form of professional development. In the spirit of the Hippocratic oath, *primum non nocere* ("do no harm"), I am confident that at minimum, teachers using those lessons will be teaching powerful mathematics. Is teaching 20 such lessons, perhaps repeating the process for a few years, enough to help teachers develop "formative/diagnostic habits of mind?"

These are *lovely* research questions. And I hope to study them like mad. More importantly, the field should be studying such things like mad. The point is that in educational development and research we often design and implement interventions such as professional development in an atheoretical way, without having:

(i) a theoretical frame (or more) within which the efforts can be contextualized or examined, or

(ii) systematic data analysis to tell us just what is happening, and allow us to do better next time.

To return to my opening metaphor, professional development – like air travel – is multidimensional, and requires multiple theoretical perspectives. Moreover, making progress on this and other such historically intractable problems requires us to be very serious about gathering data and testing our ideas against the data.

Doing this won't be cheap or easy – it calls for resources and ingenuity in constructing design experiments (Cobb, Confrey, diSessa, Lehrer, Schauble, 2003) and in data analysis; it calls for approaching these issues theoretically from multiple perspectives and at multiple levels of grain size. However, the cost of not approaching things in this serious and systematic way is the perpetuation of the status quo, or worse.

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