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Prospective Elementary Mathematics Teacher Content Knowledge: What Do We Know, What Do We Not Know, and Where Do We Go?

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ABSTRACT: In this Special Issue, the authors reviewed 112 research studies from 1978 to 2012 on prospective elementary teachers' content knowledge in five content areas: whole numbers and operations, fractions, decimals, geometry and measurement, and algebra. Looking across these studies, this final paper identifies the trends and common themes in terms of the counts and types of studies and commonalities among findings. Analyses of the counts show that the number of articles published each year focusing on prospective teacher (PT) content knowledge is increasing. Most articles across the content areas show that PTs tend to rely on procedures rather than concepts. However, the focus of most articles is identifying PTs' misconceptions rather than understanding PTs' conceptions and the development thereof. Both the limitations of the reviews and the directions for future research studies are elaborated.

Keywords: mathematical knowledge for teaching, mathematical content knowledge, preservice teachers, prospective teachers, elementary, teacher education

Introduction

The collection of papers in this Special Issue is the result of a PME-NA Working Group titled "Preservice Elementary School Teachers' Content Knowledge in Mathematics" (Thanheiser et al., 2009; Thanheiser et al., 2010; Thanheiser, Lo, Kastberg, Canda, & Eddy, 2007) that met three times (2007, 2009, and 2010) and continued to collaborate after those years. All of the authors of this volume are mathematics educators teaching content and methods courses to prospective elementary teachers (PTs) and are involved in research related to PTs' content knowledge in various content areas. The goal of the group was to provide a summary of the research (as of 2012) conducted on PTs' mathematical content knowledge needed for teaching and to inform the research community on (a) what we currently know, (b) what we do not know yet, and (c) what we need to know.

The collection of papers in this Special Issue represents a summary of PTs' mathematical content knowledge for teaching mathematics to children up to age 14 (see the *Common Core State Standards* [CCSS], National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010), with emphasis on number and operations (treated in three papers: whole numbers, fractions, and decimals), geometry and measurement, and algebra. For each of these listed areas, an individual paper summarizes the current state of the research literature. The papers provide an insight into areas well researched (e.g., division of fractions) and areas that need more work (e.g., fraction number sense) to round out our understanding of PTs' mathematical content knowledge for teaching.

This final paper of the Special Issue is based on a focused collection of findings spanning across the five content area papers. We acknowledge that it provides a somewhat incomplete perspective on what we know about PTs' content knowledge and development due to the following limitations: (a) the exclusion of mathematics outside the scope of our Working Group, (b) the exclusion of Standards of Mathematical Practice, and (c) the limitations of the methodology of the Working Group (described in the introduction of this Special Issue).

Descriptive Themes of the Summarized Research: Counts and Types

In this Special Issue, we summarized a total of 112 peer-reviewed research articles published in journals reporting on prospective teachers' content knowledge, spanning the years 1978 to 2012. We categorized the research articles into three sections: *A Historical Look* (pre 1998), *A Current Perspective* (1998–2011), and *A View of the Horizon* (2011–2012). We incorporated a review of an additional 18 papers published in PME and PME-NA conference proceedings in the years 2011 and 2012 to allow us to see what is on the horizon; however, those 18 papers are not included in the summary totals we are reporting in this section as they are conference papers and did not appear in peer reviewed journals. Thus, the total numbers reported in this section refer to peer-reviewed research articles from journals.

Number of Research Articles Published Increased Over Time

The number of published research articles across the content areas can be seen in Table 1. Before 1998, we found a total of 38 research articles focusing on PTs' content knowledge; the number increased to 68 in the timespan from 1998 to 2011. The count of published research articles for 2012 suggests a decline in research on mathematical content knowledge of PTs; however, if we include the counts of papers from the proceedings (parenthetical counts in the table),[18] the View of the Horizon promises a possible increase in publications for the next decade. Across two of the three time periods, we note that the content area of fractions has the highest frequency of publications, suggesting perhaps that the challenges faced when PTs are learning fraction content prompts more research attention. When we view the counts by individual years (see Figure 1), we see that 1989 marks an increase in research focused on PTs' content knowledge, followed by a second increase in 2007. It is interesting to reflect on particular events in mathematics education that occurred during and near those particular years, such as Shulman's (1986) introduction of pedagogical content knowledge and the launch of the *Curriculum and Evaluation Standards for School Mathematics* by the National Council of Teachers of Mathematics (1989). These events are followed by the debut issue of the *Journal of Mathematics Teacher Education* (1998), with the initial articulation of mathematical knowledge for teaching (Ball & Bass, 2002; Ball, Hill, & Bass, 2005; Hill, Rowan, & Ball, 2005) setting the stage for the second increase.

[18] We include papers from the conference proceedings here as they may evolve into publications in the coming years.

Table 1

Peer-Reviewed Research Articles Reporting on PTs' Mathematical Content Knowledge for Teaching

	A Historical Look (Pre 1998)	A Current Perspective (1998–2011)	A View of the Horizon (2012)	Total
Whole Number	7	18	1 (2)	26
Fractions	12	17	5 (7)	34
Decimals	9	5	0	14
Geometry & Measurement	9	12	0 (5)	21
Algebra	1	16	0 (4)	17
TOTAL	38	68	6 (18)	112

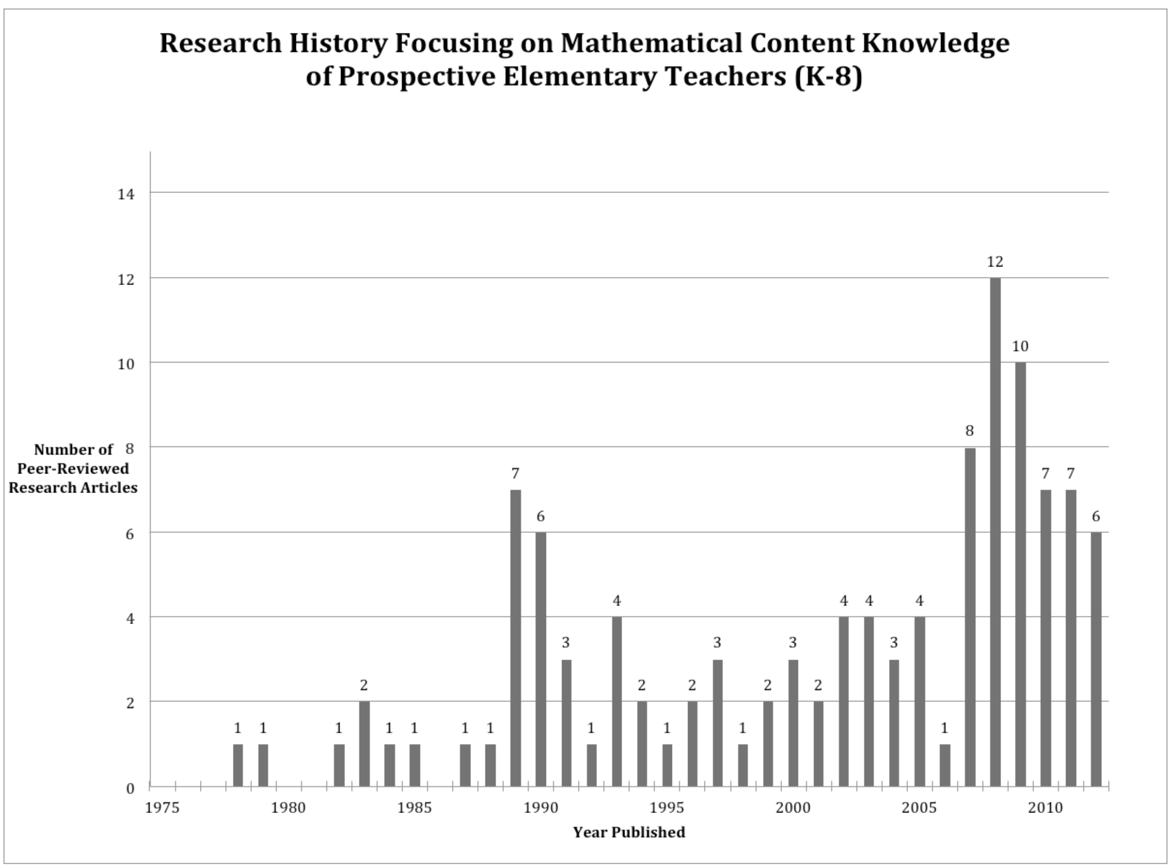

Figure 1: The number of peer-reviewed journal articles from 1978 to 2012.

While the number of research articles increases over time, the relative number of research articles focusing on PTs' mathematical content knowledge for teaching is still small. In addition to examining the counts of studies across topics, we also considered the frequency of the research in different geographical locations.

Number of International Versus U.S. Studies

In our review of 112 articles, 72 presented research conducted in the United States, while 40 were based in other countries (see Table 2). Thus, while most of the reviewed studies were done in the United States, about a third of them are international and show that PTs' mathematical content knowledge is of interest around the world. More than half of the international studies were conducted in four countries: nine studies in Austria, seven in Canada, and five each in Turkey and Taiwan. While we do not attempt to claim that our review examined research in all international journals, the common concerns that arose through the summary work were found to exist across studies conducted in the United States as well as in the other included countries.

Table 2

The Number of Peer-Reviewed Articles Published Focusing on PTs Outside the United States

	Total	International
Whole Number	26	7 (27%)
Fractions	34	13 (38%)
Decimals	14	5 (35%)
Geometry & Measurement	21	9 (42%)
Algebra	17	6 (35%)
TOTAL	112	40 (35%)

Static Studies of Knowledge Versus Motion Studies of Learning

Of the 112 studies surveyed, 104 (93%) focused on assessing PTs at a certain point in time, or several points in time (*static studies of knowledge*), while only eight (7%) of the studies (two in whole number, one in fractions, and five in algebra) focused on closely examining PTs' learning (*motion studies of learning*) (see Table 3). We use the phrase *static studies of knowledge* to describe studies that focus on multiple data captures with assessments on PTs' mathematical understanding at specific moments in time, but do not focus on the development of learning mathematical ideas. The work of Kaasila, Pehkonen, and Hellinen (2010) described by the whole numbers and operations group in this Special Issue presents an example of this type of static study. PTs enrolled in a mathematics methods course in Finland were given an item related to the division of whole numbers that they were asked to solve without using the traditional division algorithm. Responses were analyzed for evidence of and difficulties in conceptual understanding, adaptive reasoning, and procedural fluency. Data were collected only once, with findings presented from the single analysis. This type of static research of mathematical knowledge is useful in order to identify areas of concern for a subsequent, careful examination of PTs' conceptions and the development thereof.

Table 3

Motion Studies of Learning Versus Snapshot Studies of Knowledge for Each Content Area

	Motion Studies of Learning	Static Studies of Knowledge	Total
Whole Number	2 (8%)	24 (92%)	26
Fractions	1 (3%)	33 (97%)	34
Decimals	0	14	14
Geometry & Measurement	0	21	21
Algebra	5 (29%)	12 (71%)	17
TOTAL	8 (7%)	104 (93%)	112

We use the term *motion studies of learning* to describe a careful examination of learning. In such studies it is not enough to report pre data, describe the treatment, report post data, and indicate potential change; with motion studies, a clear description of the learning, the treatments implemented, any developmental change in learning, and an examination of possible correlations of the developmental change to implemented treatments are needed. Examples of such studies could be case studies conducted during an extended period of time assessing how any interventions were related to the learning or a systematic analysis of a sequence of learning segments determining the strength of correlations between treatment and learning. Richardson, Bereson, and Staley (2009) provide an example of a motion study of learning in their study with PTs focusing on algebraic reasoning. Their teaching experiment focused on the processes of teaching PTs how to generalize and justify rules, noting critical moments in the PTs' development of algebraic reasoning during the experiment, finding positive associations between the tasks the instructors developed and the PTs' learning.

We note that Mewborn's (2001) review of research on PTs' mathematical knowledge also found a prevalence of "snapshot studies" (p. 33) and a dearth of "video-taped" (p. 33) studies in the literature up to that point. More than a decade later, we are still making this same observation. Very few studies of PTs' content knowledge have analyzed the processes by which that knowledge develops (see Table 3). While understanding PTs' knowledge is a critical component of our understanding of how PTs learn (since we want to build on the knowledge they bring with them to their preparation programs), we also need to focus on understanding how PTs learn and construct knowledge so we can help them build the mathematical understandings from which they will need to teach.

Descriptive Themes of the Summarized Research: Two Commonalities

In the next sections we highlight two common themes that emerged in the findings across all of the content areas we examined. We highlight a few examples for each theme and refer the reader to the individual summary papers within this Special Issue for a more in-depth reading of the themes.

Most Research Focuses on Deficit Descriptions

One noted theme that was found within the static studies of knowledge across the content areas was the focus on identifying and describing deficits in PTs' content knowledge as opposed to (a) providing a useful characterization of the PTs' conceptions, and (b) identifying knowledge that can serve as a resource in learning. While establishing what PTs know and do not know is essential, and thus deficit studies are useful, we need to move beyond those studies to understand what PTs do know and how learning happens.

For example, the work of Tirosh and Graeber (1989, 1990a, 1990b, 1991) highlighted PTs' misconceptions about multiplication and division. Findings of such "static" work can prompt subsequent research to focus on the how such misconceptions develop in PTs' learning. Yet, still what is needed is a characterization of PTs' conceptions, such as what is presented in Thanheiser (2009, 2010), that details how PTs think about number. Whitacre (2013) argued for the need to view PTs' prior knowledge as a resource in their learning. He offered several examples from his research of ways in which PTs' prior knowledge—including their knowledge of procedures—can be built upon productively. Another example that goes beyond identifying and describing deficits in PTs' content knowledge comes from geometry, where the work of Battista, Wheatley, and Talsma (1982, 1989) examined the importance of spatial visualization in learning geometry.

PTs' Focus on Procedures Rather Than Concepts

Related to a focus on deficits in PTs' content knowledge was the theme related to PTs' procedural understanding. The studies reviewed for this Special Issue highlighted PTs' tendency to focus on procedures rather than concepts across all content areas; several examples are shared below.

When PTs were asked to reason about alternative algorithms or nonstandard strategies when working with whole numbers, Harkness and Thomas (2008) found that only 7 of 71 PTs were able to explain conceptually why the algorithm worked. Fifteen more PTs showed some understanding but gave incomplete explanations. The remaining PTs' arguments relied on comparing the alternative algorithm to the standard multiplication algorithm.

Similar procedural thinking was exhibited in decimal place value understanding when PTs worked with converting 12.34_{five} to base ten (Khoury & Zazkis, 1994; Zazkis & Khoury, 1993) by relating the fractional part of a number to the base in the number in non-standard ways. For example, in 12.34_{five}, some PTs suggested that the 3 was in the 0.5 position and the 4 was in the 0.05 position, reasoning that is aligned with the consistent use of 1 in decimal notation for tenths (0.1) and hundredths (0.01). Other PTs ignored the fractional part of the number, noting that decimals exist only in base ten (Zazkis & Khoury, 1993). The digits after the decimal were unchanged, while the integer part of the number was converted using a conventional strategy.

PTs demonstrated the ability to use algorithms to multiply, divide, and compare fractions, but were unable to explain why these procedures worked (e.g., Ball, 1990; Borko et al., 1992) or to stray from them, even if using number sense would be more appropriate (e.g., Yang, Reys, & Reys, 2008).

Baturo and Nason (1996) found PTs relied on procedural understandings of area and that they could not explain why one must divide by 2 in the area formula for a triangle, as the rule had not been connected to any concrete experiences.

And finally, PTs generally had strong procedural skills in the context of linear functions such as calculating a slope (Nillas, 2010; You & Quinn, 2010). However, results of both studies suggest that many PTs struggled to (a) interpret the slopes of the graphs of linear functions in real-world contexts, and (b) flexibly translate between multiple representations of a function.

Both themes of deficit and procedural understanding arise from the wealth of static studies summarized across content areas, showing what we know in the moment, that PTs struggle when it comes to learning and understanding mathematics. Again, these common themes strongly suggest our need to examine how the PTs can be successful in learning mathematics so we can move beyond these noted limitations of understanding.

Conclusions

Through the collective summary of research from 1978–2012 on the mathematical content knowledge of PTs, we found that:

1. The number of peer-reviewed research articles focusing on elementary PTs' content knowledge that have been published in journals has increased from roughly one study per year in the 1970s and 80s, to six or more per year since 2007, with the trend suggesting this count per year will continue to be maintained and possibly increase.
2. Within the five specific content areas examined (whole numbers and operations, decimals, fractions, geometry and measurement, and algebra), research on fraction content knowledge generally had the highest frequencies of published work (twice out of the three time periods).
3. Similar research on PTs' mathematical content knowledge was conducted in the United States and several other countries with similar findings.
4. Far more static studies of knowledge (104) were reported than motion studies of learning (8).
5. While many individual research findings were summarized, two themes across all five content areas emerged: (1) PTs' reliance on procedural understanding,

and (2) the tendency of the literature to focus on describing deficits in PTs' understandings.

We also realize there are limitations to these findings in that:

1. Reviewed articles are published in mathematics education research journals. There may be other relevant articles published in journals in other fields.
2. Almost all of the articles reviewed were in English. Given the known work in international venues, relevant studies published in other languages unfortunately were missed and not examined.
3. We did not attempt to systematically search for all related research published prior to 1998. There were limited available resources for an exhaustive review of research.

As stated several times in this Special Issue, the goal of our work was to provide a summary of the research (as of 2012) conducted on PTs' mathematical content knowledge needed for teaching and to inform the research community on (a) what we currently know, (b) what we do not know, and (c) what we yet need to know. If we use these three points to frame our summative findings, we see that *we currently know* and have identified many misconceptions that PTs hold across all content areas, and we have some more nuanced descriptions of PTs' conceptions (e.g., Thanheiser, 2009, 2010). Our summaries also suggest *we do not know* enough yet about how PTs learn, showing a lack of nuanced descriptions for PTs' conceptions across most content areas, as well as a dearth of studies on PTs' learning. What we do not know provides a sufficient context for what *we yet need to know*. Thus, we suggest that more research articulate characterizations of the PTs' conceptions and focus on PTs' learning, conducting more of what we have described as *motion studies* in learning that examine the PTs' learning process and describe any associations between what is done in the classroom and developmental changes in learning. We see these types of studies as particularly fertile ground for future research that can help mathematics teacher educators understand how to support PTs' critical development of important content knowledge.

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