The Mathematics Enthusiast

Volume 11
Number 3 *Number 3*

Article 9

12-2014

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Lim Chap Sam

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Lim Chap Sam[1]
School of Educational Studies
Universiti Sains Malaysia

Introduction

"Can you name me a mathematician?" "Einstein??"

"Do you wish to become a mathematician one day?"

If these questions were asked to the public or any school students, the most likely answer to both questions might be a big 'No!'. Why is this so?

The director of the Public Understanding of Mathematics Forum, Gene Kloz (1996) claims that the mathematics profession is the most misunderstood in all of academia. According to him, the public thinks that mathematicians contemplate ancient proofs and work as lonely recluses. Moreover, the most common public image of a mathematician has been furnished by a physicist (example, Einstein) rather than a mathematician. Why is there such a lack of appreciation of mathematicians' work by the public?

Although generally most people agree that mathematics is important in our daily life and useful for many careers, yet many people shy away from doing mathematics. In Malaysia, Lee et al. (1996) found that "the science and non-science student ratios have deteriorated from 31:69 in 1986 to 20:80 in 1993, indicating a drop of 11% in the proportion of students taking science subjects" (p.i). The most common reason as quoted by 58% of 766 Form 4 students and 59% of 489 Form 6 students for not choosing science was 'poor foundation in science and mathematics'. The other common reason was 'no confidence in mathematics" as quoted by 42.8% of Form 4 students and 48.1% of Form 6 students. These findings thus raise concern over the falling enrolment of students in

[1] chpsam@gmail.com

The Mathematics Enthusiast, **ISSN 1551-3440, vol. 11, no. 3**, pp. 586-642

science and mathematics. This is particularly so in the midst of preparing our youth for the era of information technology.

These are some of the issues and problems that are of concern not only to mathematics educationists, but also to school administrators and parents. The related problems include students' mathematics performance in the SPM and STPM examinations which is still far from satisfactory, students' attitudes towards the learning of mathematics which is becoming more and more negative, the decline in the number of student choosing mathematics as a major subject in secondary schools and the small number of students majoring in mathematics in tertiary institutions.

Indeed all of these problems and issues are closely related to one another. The negative attitudes towards mathematics or "mathematics phobia" and the perception that mathematics is a difficult subject certainly does not encourage a student to major mathematics in their tertiary education. Consequently, this shortage of mathematics graduates might result, as it has done in the UK (Hayes, 2002, Cassidy, 1998), in the shortage of mathematics teachers in schools as well as a shortage of mathematicians in Malaysia. As a result, mathematics might have to be taught by teachers who have not majored in mathematics. Ultimately, the problem of teaching and learning of mathematics in schools may become more serious.

Various efforts have been made to improve the performance and attitudes towards mathematics of students. A number of studies have been carried out to find the causes of this negative attitude (for examples, Vanayan, et al., 1997; Elliott, et al., 1999; Gipps & Tunstall, 1998) and to research learning that is more effective and teaching strategies. However, most studies have concentrated on mathematics students who are weak in mathematics.

Perhaps many researchers have forgotten that there exist a small number of students who are so interested in mathematics that they have taken up mathematics as their lifelong career. Who are these people? They include the mathematicians who teach and research in mathematics in universities. Why do they have such an intense interest in mathematics? What has attracted them to mathematical work? What are their experiences in learning mathematics that have motivated them to become mathematicians?

Thus, this study aims to explore how mathematicians come to know mathematics, learn mathematics and research in mathematics. This study has taken a different focus by looking at people who are interested in mathematics such as mathematicians, and not those who are weak at mathematics. It is hoped that findings from this study, with regard to the mathematicians' experience of learning and researching might offer a new model for teaching and learning of mathematics in schools or support an already-existing model. Their struggles and enjoyment in mathematics might provide a source of motivation for school students to be more interested in mathematics. Following this, it is hoped that the findings of this research will provide recommendations and implications that are useful to encourage students in schools and universities to adopt a more positive attitude and interest in learning mathematics.

Research Objectives

More specifically, this study aims:

1. To find out how mathematicians come to know, learn and understand, and think of mathematics.
2. To explore mathematicians' attitudes and beliefs about mathematics and its learning.
3. To explore the mathematicians' learning experiences and their practices
4. To compare Malaysian mathematicians' way of knowing with that of British mathematicians studied by Burton (1999a)

Review of related literature

Review of related literature shows that there are very few studies on mathematicians (except Lim, 1999; Wong, 1996; Burton 1999a). Most people do not have a clear picture about who mathematicians are and how they carry out their practice. In a related research by Lim (1999), when asked about the characteristics of a mathematician, 58% (out of a total of 548 participants in England) chose "intelligence" as the most important characteristic. More than 1/4 of the sample perceived a mathematician as a male who is intelligent, serious, confident and wears glasses. In the same research, a sample of 407 Malaysian students and teachers were asked the same question. Lim found that more than 75% of the Malaysian sample chose "intelligence" and "confidence" as the most important characteristics of a mathematician. More than half of them held the same view of a mathematician as a male who is intelligent, serious, confident, strict and wears glasses. Thus, both samples seem to share a rather similar picture of a mathematician.

Earlier, in 1996, Wong asked a group of Singapore secondary school students (137 boys and 144 girls) to draw a picture of a mathematician. He classified the students' drawings depicting a mathematician into four main categories:

1. Type I: mature and intellectual (69%) who was characterised as a male of mature age, bald, with glasses and mustache, and wearing clothes covered with mathematics symbols.
2. Type II: the strange messy look (10%) who was characterised as a stern looking, weird male with unkempt hair working on mathematics.
3. Type III: feminine qualities (10%) characterised as a female mathematician who looks like a mathematics teacher, but is also kind looking and patient.
4. Type IV: The mechanical icon (10%) such as a calculator. These students perceived mathematicians as having non-human qualities such as being mechanical and unfeeling.

In short, mathematicians were viewed as a group of special human beings, alienated, and seemingly non-human. These research findings (Lim, 1999; and Wong, 1996) may not be generalised due to the small sample size. Nevertheless, they indicate that many people know very little about mathematicians and their practice. Perhaps this view of a mathematician as someone who is mysterious and unpopular will deter many students from specialising in mathematics or becoming mathematicians. Thus, more studies on mathematicians and their work practices need to be done.

Research carried out by Burton (1999a) with 70 mathematicians all over United Kingdom (UK) shows that mathematicians are also emotional and collaborative in their work practices. Her study shows that there is "a substantial cultural shift in mathematics from a discipline dominated by individualism to one where team work is highly valued" (p. 131). The majority of the mathematicians, whom she interviewed, spoke about the importance of collaboration or cooperation in their research as well as publication. This is a surprising result as it contrasts with the social stereotype of mathematicians being aloof, lonely, odd, locked away in an attic room, as portrayed in media and some literature (e.g.

Simon Singh, 1999). Moreover, when talking about ‘knowing ‘ mathematics, these mathematicians tended to give two metaphors: (a) the jigsaw puzzle and (b) the views or the map. These mathematicians’ experiences of coming to know mathematics were also represented by feelings, such as the powerful sense of Aha! In addition, it is this kind of aesthetic feeling and deep involvement in mathematical exploration that holds them in mathematics. However, her data are limited to UK mathematicians only. What are the experiences and practices of mathematicians in Malaysia? Are they similar?

In fact, the school science and mathematics curricula of Malaysia are largely based on the western mathematics model, in particular because of the British colonial rule in the past. Nevertheless, Malaysia and UK are very different in terms of culture, language and religions. The question is, are our local mathematicians' experiences of mathematics learning and doing mathematics similar to those mathematicians studied by Burton (1999a). Do cultural differences make an impact in this aspect?

It is hoped that these research findings will make an important and significant contribution to encouraging young students to major in mathematics. Mathematics is important for the development of our country and mathematicians can contribute significantly in various ways. Understanding how mathematicians do mathematics, more so how they develop a deep interest in mathematics may help us to encourage more positive attitudes among our students. In what ways do mathematicians appreciate the beauty of mathematics and enjoy its elegance [as revealed by some of the mathematician participants in Burton’s study (2001)]? How can this appreciation and enjoyment in mathematics learning be channelled to our young students so that they too could appreciate and enjoy mathematics? With the above goals in mind, this study is deemed both important and necessary.

Methodology

Participants

Twenty five mathematics lecturers from three local universities were interviewed. These universities were chosen as they were among the oldest and have the most established mathematics faculties. Most of the academic staffs in these universities are not only involved in teaching mathematics but also in mathematical research. Thus, they are defined as mathematicians in this study. This definition of a mathematician is similar to what was defined by Burton (1999a) in her study on mathematicians. Therefore, this allows the findings of the two studies to be compared.

The detailed distribution of the participants is provided in Table 1. In total, there are five professors, eight associate professors and 12 lecturers; 16 are males and 9 are females. In terms of ethnic group, 17 are Malays and 8 are Chinese. Each university has about 30-40 academic staff members. Thus, I have interviewed about one quarter of the staff members from each university.

Table 1: Distribution of participants by profession, gender and university

	Professor				Associate Professor				Lecturer				Total
	Male		Female		Male		Female		Male		Female		
University	Ma	Ch	Ma	Ch	Ma	Ch	Ma	Ch	Ma	Ch	Ma	Ch	
M	-	1	-	-	1	2	-	1	1	-	2	1	9
S	1	-	-	-	1	1	-	-	1	1	2	-	7
K	3	-	-	-	1	-	1	-	1	1	2	-	9
Subtotal	4	1	-	-	3	3	1	1	3	2	6	1	25
Subtotal	5		0		6		2		5		7		25
Total	5				8				12				25

Ma=Malay Ch = Chinese

Research design

This study is an interpretative study employing a qualitative method, mainly a single life-history interview, to collect data. The main aspects covered in the interview were:

- experience of learning mathematics in schools;
- experience of learning mathematics in university;
- experience of being supervised as post graduate students;
- experience of supervising students;
- images of mathematics;
- images of mathematicians;
- experience of working as a mathematician – research and publication;
- suggestions for improving mathematics education.

A list of questions was prepared to be used as a guideline and prompt for the interviews (see Appendix A and B). Some of these questions were adapted from Burton (1999a). All questions were written in two languages, Malay and English. This allows the participants to choose whichever language in which they are fluent. The majority of the participants chose to speak in English. Only two of the 25 participants preferred to use Malay to English. In most cases, there was a mixture of languages as this is a typical way of communication for most Malaysians.

The list of interview questions was shown to the participants before the interview started to give them the opportunity to prepare themselves and to allow them to delete any question that they may find sensitive or unsuitable. Each participant’s consent was sought before the conversation was tape-recorded. Of the 25 participants, only one refused to be tape-recorded. In this case, the main details of the conversation were noted during and immediately after the interview. All the interviews were conducted in the offices of the participants or in another private room suggested by the participants. All the interviews were face to face. The duration of the interviews ranged from half an hour to two hours, with an average of about an hour. Each interview was tape-recorded and transcribed for qualitative analysis using computer software, QSR N5 (Richards, 2000).

Findings and Discussion

This section reports and discusses the findings and results of the qualitative analysis of the interview data.

Experience of learning mathematics in schools

Of the 25 participants, nearly half of them (12) displayed great interest in learning mathematics since their primary school. Yet four of them admitted that they were very poor in mathematics when they were young, and the remainder (9) felt their mathematics learning experience at primary school was just routinised. Many of them could not recall, or did not have much memory about, any significant events. The following quotations display some typical answers:

> Q: Can you still remember any moment that you started to be very interested in mathematics?
> A: Interested in maths? I suppose I did well.
> Q: since primary school?
> A: Yes.
>
> (K5, 50's, female associate professor)

> A: I love mathematics since primary school.
>
> (S3, 30's, male lecturer)

> A: to me, it's a normal routine learning process.
>
> (K2, 50's, male professor)

> A: I did not have full experience of mathematics [learning] in primary school. I was not a good student in mathematics at that time.
>
> (S7, 40's, male associate professor)

So, when did the first ‘spark’ of interest in mathematics begin? The ‘spark’ of interest seemed to occur for most of them when they entered upper secondary schools. The three main contributing factors cited by most of them were mathematics teachers, interesting teaching and successful experience of learning.

> Q: Could you please describe your experience of learning mathematics in primary school or secondary school?
> A: I got interested in mathematics during my secondary school.
> Q: Which form?
> A: Since Form Three.
> Q: Any special moment that you can remember?
> A: It is all due to a good teacher, ha, ha. I think a good teacher plays a good and important role.
>
> (S2, 40’s, female lecturer)

A similar view was given by two other mathematicians that,

> I got interested when I was in secondary school. My primary school was not that good. I was just average. But when come to secondary school, I had one very good mathematics teacher.
>
> (M2, 50’s, male associate professor)

> Fortunately, I’ve a good teacher. I think Form 1, Form 2, Form 3, as usual-lah, not very…. When I got to Form 4, I’ve a very good teacher Mr. Ang. I started to really like mathematics especially Add Maths.
>
> (K8, 30’s, female lecturer)

> …jumpa pula Cikgu Chakra ini, cara dia kendali saya masa Form 5 level tu cukup seronok. Tapi malangnya dia ajar kita orang sampai dekat Form 5, and then dia pergi R&G. Kalau dia continue lagi, saya ingat result mungkin lebih baik lagi.
>
> (K6, 50’s, male lecturer)

That some of them can still remember their teachers’ name even after ten or twenty years indicates that mathematics teachers have a considerable impact on the students’ experience of learning mathematics.

Further analysis of the data has shown that mathematics teachers usually play a role as motivator or inspiration:

> Teachers also inspired me. They always considered me to do well in Mathematics. That gave me the encouragement, like a push offer that you can do Maths and you should do Maths.
>
> (M5, 30’s, female lecturer)

The encouragement and motivation of mathematics teachers have pushed many students to work harder. Even though mathematics may be viewed as a difficult subject, some students were willing to take up this challenge as inspired by the teachers. The following conversation illuminates this point:

> S: When did you start to get interested in mathematics?
> J: I think that come very much later, may be Form 5 or 6 when you find there is a necessity to really master mathematics. And of course, I think at that time the teachers also played a role. You see there are not many good teachers in mathematics. Mathematics is regarded as a very difficult subject. Even the teachers themselves think so, what do you expect from the students? *But I think that I do meet some very good teachers in mathematics. I think that really interest me to learn why it is such a difficult subject.*
> (M1, 50's, male associate professor, italic my emphasis)

Besides being motivators, good mathematics teachers also provided innovative teaching and exciting learning experience that captured the interest of their students in mathematics.

> Well, the first part on…, I still remember the first chapter was on continued and discrete, you know, we started with that. And then come to significant figures and things, you know, But most interesting was when he drew out a diagram and asked us to point out which was the angle and so on. You have to go to the board and then show him, so everybody will see you doing that, you see…..
> Then we started to feel that our maths results were much better, you know. So with that I feel I slowly I start to like mathematics….
> (M7, 50's, male lecturer)

Very similar experience of good teaching that led to interest in mathematics was also described by another younger mathematician:

> This tuition teacher was very good and he had his own way to teach Mathematics.
> His method of teaching was very good. Especially when came to memorizing sin, cos. He used our fingers to memorize angles especially 0^o, 30^o & 45^o. All those kind of important angles, we used our fingers to memorize that, but now I can't remember exactly how.
> That was the first moment when I started to realize, wow! What are the wonderful things about Mathematics?
> I think the most important thing is his way of teaching is very good. That is one thing and then I did it very well. Before that, when I was in form 1, 2 & 3, not to say I was not good but nothing that got me interested, I just manage to get through, just pass only. …

> I attended the tuition and I was so interested and I did it very well from then. …The way he taught me, from that I did very well in my Mathematics so I can say that I get the excellence result that was Addition Mathematics.
>
> (S5, 30's, female lecturer)

Thus, one may notice that interesting teaching approaches lead to better interest in mathematics and consequently better results in mathematics. The latter subsequently enhance the students' confidence and thus intensify their interest in mathematics learning. Hence, the vicious cycle of interest leading to better results and better result leading to deeper interest seems to perpetuate.

However, the feeling or experience of success may not necessarily come from good teaching. As one of the mathematicians described his experience:

> When I started Form 1, 40 is the passing mark so I made it 41, 42 to pass. Well, I was still not doing well. I was just managed to pass. It wasn't my favorite subject at all. I like history a lot.
> When I was in Form two the teacher was teaching the simultaneous equation like **X+Y=5** or **2X-3Y=4** something like that. I was ill so I did not go to school on one particular day. So for some reasons. I decided to do well, of course, I couldn't do it, and then I try to think logically and let me try to think rational how to do it. I try to do methodically as in the examples, [I check the answer] in the back [of the textbook] and it's correct. When I did the next question with the same technique it worked too. Than suddenly every thing worked and from that point, my interest increased. I used the same kind of methodical method and my performance shoot-out. I reach 70 [marks] and then in Form 3, it was close to hundred in all tests. I got A1 in my LCE. So, it pushes out without any help from anybody. I am sure the teachers did their works but they were not very inspiring.
>
> (S7, 40's, male associate professor)

Again, one may argue that this type of successful experience perhaps is unique to certain people only. However, the pertinent question should be how to provide each and every student with this type of successful feeling or experience of success in mathematics learning?

Other than mathematics teachers and experiences of success in mathematics, the data also show that the other important influential factors could be home environment, especially the moral support of parents as well as peers influence.

One associate professor described how his father helped to cultivate his interest in mathematics from an early age:

> A: It all started when, you know… when you're young, when my father played a big role in this. Normally, I used to follow my father down in the evening to see a football match or something like that, we ride on a bicycle, my father will paddle the bicycle and I always sitting at the back. He always figures something interesting like a flock of birds fly up, example: "Berapa burung tu," and I used to count up, you know I was little…count up the numbers of birds. "ada 13" okay "13"
> Q: How old were you then?
> A: 5, 6 or 7 years old. You know I always follow him. The next batch of birds, because we were not rich we ride on a bicycle, "ada 17" yang tadi ada berapa ? Ohh 13 so campur, so that helped a lot, you know, we add up he's not properly educated…but he's wise in his own way…
> (M4, 40's, male associate professor)

Likewise, the importance of peer support was recognised by a young female mathematician:

> I think ….okay, good examination results of course inspire me. I had a few friends, they always talked about who got the best in Maths. They always mentioned my name. Teachers also inspired me. They always considered me to do well in Mathematics. That gave me the encouragement, like a push offer that you can do Maths and you should do Maths.
> (M5, 30's, female lecturer)

The above example displays that even though some families may be poor, it is always the conducive environment that promotes mathematics learning that matters. This is also evidenced in literature (for e.g. Rabusicova, 1995) that parents' educational background, parents' occupation and parents' interest in students' performance in schools have great correlation with the students' educational trajectory. Therefore, a family environment that promotes good reading habits from young will help to promote mathematics learning. Two mathematicians whom I interviewed believed that their early exposure to reading might have contributed to their interest in mathematics. The following conversation illustrate this:

> A: … Because I've been a good reader. My parents provided me books to read. While my friends they didn't do that way… in my primary school level.
> Q: What kind of books do you read during primary school?
> A: some storybooks… I've the general knowledge, concerning science and maths.
> (K5, 50's, female associate professor)

> When I was very young … at the primary level I used to visit the USIS library and the British Council a lot. There are a lot of very good

> science books. Actually, my interest started not in mathematics but in science. I used to pester my mother to show me how to go down to USIS (United State Information Service) library near the riverside. I'm local. I took the old Tong Fong bus down to the riverside near Bukit Aman. It's still there.
>
> (M1, 50's, male associate professor)

In addition, the data also reveal an interesting finding. Not all mathematicians whom I interviewed experienced a wonderful and enjoyable mathematics learning during primary schools. In fact, at least three of them had negative experiences because they found themselves weak and disappointing in learning mathematics. Not all of them excelled or were interested in mathematics since young, but they managed to pick up later due to a mathematics teacher or a successful experience of problem solving. The following stories of a senior mathematician exemplify a typical case.

A senior mathematician (S1, male associate professor) relates in the interview that,

> I was very weak in mathematics when I was in primary school because no one taught me mathematics.

When he entered secondary school, he still faced problems in mathematics because,

> When I attended the secondary school, everything changed to English. Language problem again and I don't understand because of language. I don't like mathematics at all. I almost failed my mathematics. I always got 50 or 40. In Chinese school, this was considered fail. But in Form 3 that year I've to sit for the LCE. …. Well, even if you do not know, you have to try your best. So luckily, I passed and I was admitted to Form 4.

He managed to pass his Form 3 examination and was admitted to Form 4. Then his fate changed as he met a very special mathematics teacher:

> Then in Form four I was taught by an American teacher, Peace-corp teachers. ……I think, I was interested in some.. that is what I say, even a weaker student, when something struck him, let him understand certain thing, he can also be interested in mathematics. It's my own experience that when this peace corp teacher taught me some modern mathematics. I think it was something different from the traditional way, we were no more just applying formula.

> Now we have to use our brains to think. And then I'm so proud because I can understand it and I can answer the questions whereas my peers cannot. *Something that made me feel I am so proud about myself. Suddenly I feel oh! I'm good at mathematics. I can do it*.
> [italic my emphasis]

Perhaps this type of story should be told to our students and to our prospective mathematics teachers. It clearly indicates the important role of mathematics teachers. Besides that, it reflects the hard work and successful experience of mathematics learning that can boost the interest and confidence of students towards learning mathematics.

Experience of learning mathematics in university

Of the participants interviewed, all except four did their postgraduate studies overseas. The majority of them went to further studies in mathematics or fields related to mathematics. Most of them went to either United Kingdom or United States. All of them were fully sponsored by their universities as part of an academic staff training programme.

Of the seven Chinese Mathematicians, four of them did not have postgraduate experience overseas. All Malay Mathematicians have studied overseas, sponsored by the Public Service Department (JPA), Ministry of Education or a university staff training programme. The four who did not go overseas did their studies from undergraduate to PhD in the same local university. Some of them studied through tutorships where they gave tutorials while studying for postgraduate courses.

Some of them did struggle to get through their postgraduate studies. Choosing the right field and a suitable university were some of the problems faced. At least two of them did not manage to finish their PhD study overseas.

Therefore, it may be interesting to find out if:

- the experience of studying overseas affect their world view of mathematics?
- the experience of being supervised overseas influence their supervision of students later?

Experience of being supervised as postgraduate students

As mentioned earlier, the majority of the mathematicians in this study went to do their postgraduate studies overseas, to some extent, these experiences seemed to have impact on their world view about mathematics, in particular with regard to the learning and doing research in mathematics. One female mathematician did her master degree locally but then went to Australia for her PhD degree. She compared both her experiences as follows:

> I think it's a correct decision to go to Australia because you are exposed to a wider range of experience there. You can see the research there. They emphasized a lot about research there. The facility, the encouragement, the understanding of what you want to do research. In a way, you won't face as many obstacles as I think one was facing here. I mean here [locally] if you want to find some good journals in the library, you don't have that many journals. Some of the papers that you want might not be available. The length of time you take to take something [there] compare to here, it is much faster there. I think it is just the general environment and the understanding of when you do the research I think those that around you have to understand that you were not to be disturbed and try not to put too much of interruptions.
>
> (M5, 30's, female lecturer)

In comparison, she found the library facilities, and the culture of research were much more conducive in oversea universities than those provided by local universities. She recommended that local universities should strive to provide similar facilities and environment that will encourage this type of research culture.

Another interesting observation was that all the four Malay professors seemed to have a very positive and encouraging learning experience while they were doing postgraduate studies overseas. They managed to collaborate with their supervisors in writing journal articles. One of them proudly presented a book written by his supervisor which gave credit to his work in mathematics. In addition, their supervisors also provided them with many academic opportunities. For example, one of them revealed that he was brought to attend the Annual Mathematics Conference by his British supervisor. He found attending conference like this not only open up another new avenue for learning, but also it was a very helpful way of networking. At the conference, he was introduced to his supervisor's former PhD students, some of them were already established mathematicians. This gave him opportunities to ask for comments and suggestions about his own research. Besides this, he also built up a very close relationship with his American supervisor whom he regarded as his "uncle". He found meeting with his supervisors and other academic staffs during 'coffee time' every morning had provided him great opportunities for academic discussion. Indeed, all these cultural exchanges have influenced him to do the same thing with his students now.

Similarly, another young female mathematician (S5) have learnt up "to be quite independent student in that time because my supervisor let me do my work on my own. I need to see him twice a week. So once I did something, I went to see him and asked for comments, and he will direct me to whatever things that I should do. Yes, regular meetings and I feel the meeting was always very helpful". Consequently, she acknowledged that her experience is going to influence what she would like her future students to be. She has just completed her PhD study a year ago and yet to supervise any students. However, she has laid down her criteria as: "first of all, I need them to have a strong background in everything they want to do, and also working independently. A part from that, of course they will need my consultation".

"To be independent" seemed to be one of the dominant cultural qualities that these mathematicians have learnt from studying overseas. As a result, they would like to impose the same kind of training to their students. One of the professors also acknowledged that, his postgraduate experiences have influenced how he supervises his students now.

> Pengalaman saya masa buat PhD dulu itu mempengaruhi saya. Jadi, misalnya student has to be more independent. Dia kena cari tajuk sendiri. Dia kena cuba develop his own problem. Setengah pelajar mungkin suka dengan cara itu. Tapi saya rasa kebanyakan Malaysian memang tidak suka.
> [My experience while I was doing my PhD did influence me. For example, (I want my) students have to be more independent. He has to find his own topic. He has to develop his own problem. Some students may like this way, but I know most Malaysian (students) do not like this]
>
> (K1, 50's, male professor)

Even though he was aware that some of his students might not appreciate his way of emphasising "independent", he would still enforced it because he believed that it was a good way of training them.

However, not all mathematicians whom I interviewed had positive experiences of being supervised. One of them (M8) described her experience:

> I had a problem because my basic was already in OR… by the time, I wanted to do PhD I was asked to do something else, in numerical analysis because you know. Internal politic or what so ever. From my department, they wanted me to do numerical analysis. I do not know what was the plan at that particular time but because I was about to go already so I felt it was not fair, so I went to... So I got everything ready, I got supervisor, I got the university everything ready for OR. Suddenly I was offered numerical analysis. I got to change university to find somebody who can bridge between numerical analysis and OR. I went to somebody.
>
> When you go to a supervisor who is not statistics. You know what sort of surprise you got. So basically, I was all on my own. That was not a very nice experience because before you go and do your PhD, you already have this idea in your mind, what PhD constitute with. How and what you expect what the supervisor should be, so it turns out that you had a very different experience.
>
> (M8, 40's, female lecturer)

She was ordered to change her area of study from her sponsor at the last minute and she was given a supervisor who was not an expert in her field. Nevertheless, she managed to obtain her PhD degree with much hardship.

> You just present your work, he will comment, because he is not in that field, his comment is more to on the surface. You have to be keen yourself, I have to inform him what I have done until I reach to this stage and this stage.
>
> (M8, 40's, female lecturer)

However, she still appreciated and was grateful to her supervisor for making an effort to invite experts in her area to come and help her:

> One thing good about my supervisor is he gives me all the opportunity. He invites visitors from other countries, just for me. To help me with my research. So that is something that I was thankful.
>
> (M8,40's, female lecturer)

Therefore, have the negative experiences of being supervised during PhD study influence her supervision of her students now? She replied,

> Sometimes people say that you have been treated badly by supervisor. Normally you become a lousy supervisor, but I find it in different way, I thought the lack of supervision from my supervisor which I yearn to have at that particular time, that's why I sometime, I write the programme for my student, believe it or not, I do the defrauding for them, I spend hours in front of the computers with them because

> that's worry me if they cannot do. So, I do not know but I had gone through that extend where I get so involved with my students' work, so time consuming. I'm not the type, if you don't finish this you don't come and see me.
>
> (M8,40's, female lecturer)

It seems that the lack of proper supervision made her understand the needs of her students better, and she was trying to do everything for her students to compensate what she lacked.

Experience of supervising students

As most of the mathematicians in this study seemed to have postgraduate studies overseas, thus indirectly this reflected the lack of postgraduate students studying at local universities. Perhaps it is therefore not surprising to notice that the majority of the participants interviewed did not have much experience in supervising Masters or PhD students. Most of them only supervised the final year project mathematics students. Some senior mathematicians have supervised a handful of master students but relatively very few PhD students in mathematics. One of the professors suggested three possible reasons:

> A:Ya. Dia buat master sini, dia buat PHD oversea. Ini salah satu constraint. Kedua, because of the field. Siapa nak buat PHD in mathematicss? (laughter). As I said Buat undergraduate pun Difficult. So the source is not there. Yang buat undergraduate dalam matematik pun is the third class punya material. Unless they are, there are a few so called late developer, yang itulah ada potential. Memang dia just looking forward to finish it and forget mathematicss. Get something else, MBA tu (laughter).

First, most students prefer to study master degree locally but PhD degree overseas. Second, mathematics is not considered as a popular field among students. Third, there is a lack of high mathematical ability students, even among the mathematics undergraduates. This shortage of postgraduate mathematics students could be a worrying trend as this might lead to a shortage of future mathematicians and mathematics teachers in the country.

In terms of supervision, there seemed to have two styles. One is let the students explore the literature and come out with their own topic or problem to be researched on.

Another style is that the supervisor provides the topics or problems for the students to research. So, what is the best style? One experienced supervisor offered his view:

> A: Well, for me it is a question of whether they should be closely supervised or left to be more independent. It depends on the calibre of the students.
> Q: So far how do you find your 2 PhD students?
> J: Well, they are very hard-working, they are okay. Of course, certain time they need a bit of hand-holding. (laughter).
> (M1, 50's, male associate professor)

According to him, it depends on the ability of the students, whether they are independent or not. More often, their supervision included both 'independent' as well as 'hand-holding'. This was echoed by another senior mathematician.

> A : Yes. Independent in certain way and also need guidancelah! Actually it is a mixture-lah, to be fair.
> (M2, 50's, male associate professor)

Others found supervising students as a mutual learning process. They found they were learning and maturing together with their students. For example,

> I did together with my students because it's a new area. So when he entangles a certain things, I give ideas. And then as I give the ideas, I also go back and find resources and try to solve the problem. So, at least both of us could be, we are maturing together with the topic.
> (M4, 50's, male associate professor)

Apparently this type of training was inherited from his experience of being supervised, as he explained further:

> We [start with] just blank ideas. We give a thought of the idea before we proceed and then I also try to find other resources. Maybe from that sources, I pass it to him. And then when he explains it to you, and then you understand the thing better because your experience to your own supervisor in the last time. You should apply it. And that helps, there should make the system fit and then the way you ask the question, is how they [your supervisor] treat you the last time.
> (M4, 50's, male associate professor)

Again, this shows that the experiences of being supervised overseas did influence their supervision of students later.

Images of mathematics

When the mathematicians were posed with the question, "what is mathematics?" at least six out of the 25 interviewed stressed that:

> Maths is not just number. How you solve the real life problem actually. It is problem solving. Like in OR [operational research] how we solve real life complex.
>
> (K8, 30's, female lecturer)

> Mathematics is a field of study. Not really with numbers. Numbers are only fundamental.
>
> (K3, 50's, male professor)

If mathematics is not just about numbers only, then what should mathematics include?

> To me, mathematics is thinking. A way of thinking.
>
> (K2, 50's, male professor)

> Mathematics is just an art of or is a technique of understanding things [in the world?] in anything. Technique of understanding things... Technique tu apa ? is study-lah like you looking at pattern, looking at law.
>
> (K1, 50's, male professor)

Thus, mathematics seemed to be more than just manipulation of numbers and symbols, but a way of thinking, a way of understanding the world. Although,

> A: … to the layman, mathematics is number. But to me it could be more on formulae, sometimes some abstract thinking, we are trying to model something. Actually it is a system…
> Q: A system is a system of number, and formulae?
> A: Yes. We are using formulae and numbers to represent the real world.
> Actually, I would say, Mathematics is multi-disciplinary. Sorry, it is a language of science. So, in every field of science, we need mathematics.
>
> (K7, 30's, male lecturer)

Therefore, two of them regarded mathematics as 'the king of science' because:

> Ha! It's the king. Because other fields would depend on maths. Whereas, we don't need to depend on others.
>
> (K5, 50's, female associate professor)

But then, a younger mathematician preferred mathematics to be the 'queen of science' as he explained,

> No, 'queen' refers more to mother. Like the mother figure is nurturing the others. While the king is more like power, or the one that governing the others. I think, in a way, it is lucky to use the word queen. Instead of if mathematics is a king, then mathematics is dying now. When you come to a university, in real world, mathematics become secondary in the university, they called it School of computing and mathematics. And some people just call it as School of Computing.
>
> (K7, 30's, male lecturer)

Most of the mathematicians agreed upon the importance of mathematics in real life and in providing the foundation for applied sciences. Perhaps the only difference in images of mathematics between the pure mathematicians and applied mathematicians was that for the pure mathematician,

> Mathematics is something either right or wrong. So everything we say it is right, think it is right under any condition, any situation. In Chinese, we call it as 真理 "Zhen Li" [nobel truth]. This is what we are pursuing in mathematics.
> It is quite different from mechanical engineering. They are constructing a bridge, they are doing something which they have no idea whether this bridge can last... how long it can last. They just do some estimation. If they say that there may be earthquake in that area, they will use more materials, stronger materials.
> But for mathematics, we want precision. We want everything to be exact. That is the difference. So, when we learn mathematics, we are learning it in a different way. In our mind, everything is so idealise, ...Whatever we think, we want precise, we want everything to be absolutely correct, in that sense as possible. So, mathematics is different from applied science. But mathematics is a guideline for applied science. So, you must have mathematics first before you have applied science.
>
> (S1, 50's, male associate professor)

So, a pure mathematician is very much concerned about the precision, the accuracy and the absolute idealistic model, rather than the usefulness or the value of practicality of their findings. Another pure mathematician agreed that,

> …We don't try to be ahead of thoughts. We just want to know the answer, that's all. It is not the intention of trying to be ahead or

> trying to take the challenge. The most important thing is we just work more on the problem.
>
> (M5, 30's, female lecturer)

However, for the applied mathematician, they are also concerned about the empirical aspect. One of them explained that,

> Mathematics is the way of reaching or obtaining knowledge about the physical world, without resolve to experiment. …so that you are not deceived by your senses.
>
> I think there are two ways to obtain knowledge, one is to do experimental work, another one is from mathematics, where you develop a set of axioms then from the set of axiom, you made up your theorem and then develop into a theory. So, for me both are valid.
>
> But I think [pure] mathematicians, they don't really appreciate the experimental work. Having been trained in the civil engineering department, [I believe that] Experimental work is very important. When you do experiment, you get the real feeling for the subject matter. When you do mathematics, what does this mean? It is very theoretical.
>
> For example, for [pure] mathematicians certain terms will represent viscosity. It just a term but for the engineer they know, they feel the viscosity they know once you reduced the viscosity what will happen, they have feeling. They have physical sense which is important to help them. For [pure] mathematicians it is a certain term. You said this thing could take a certain value, for [pure] mathematicians is correct but for engineer that is impossible to reach certain value under certain circumstances. So what I am trying to say is that [pure] mathematicians sometimes they don't really appreciate the empirical work which is important.
>
> (S7, 40's, male associate professor

Another senior professor of pure mathematics also argued that the values of mathematics change as time changes. Historically, in Greek period, the scholars at that time were very engrossed with the motion of heavenly bodies, so they developed many theories concerning trigonometry and geometry. They were concerned with religious values,

> But Modern maths, is no longer religious values, it's of economy value, it's rationalistic value. …Like trigonometry, geometry is certainly [religious value]. Statistics is very recent it is not. Calculus is economy value, it is in the 8th century.

(K1, 50's, male professor)

Due to this change of values in mathematics, some mathematicians were worried that in the pursuit of values that are more utilitarian or more applications we may lose the support of the fundamental theory. Two of them voiced similar comments that,

> Because now the government and the universities, all towards applied which to me it's ridiculous when you don't have the background, but it is important that you have a basic foundation, [then] the applied can come anytime.
>
> (M8, 40's, female lecturer)

> This is something that Malaysians don't understand. What they want is applied, applied they never care about the fundamental design. But just applied, we spend a lot of money and get nothing. But you see, they do not have the support of a theory. They do not know what they will get there.
>
> (S1, 50's, male associate professor)

In brief, mathematicians viewed mathematics as more than just numbers and symbols, but a way of thinking, using symbols and equations to represent the real world problem in a mathematical model and then trying to analyse and solve it mathematically. This finding seemed to echo what Lim (1999) found in her study that among those who reported liking mathematics, in particular, the mathematics teachers and mathematicians, tended to hold a problem solving view. They tended to relate mathematics to solving problems and mental work.

Moreover, a pure mathematician seemed to hold a different image of mathematics from an applied mathematician or a statistician. Similar to what was found by Burton (1999a), applied mathematicians stressed practicality and utilitarian values, while pure mathematicians looked for theoretical foundation. By and large, findings of this study concurred with Burton's (1999a) study that mathematicians' views of mathematics are personal and social cultural related.

Besides viewing mathematics as more than numbers and symbols, there are two more aspects, which emerge and are worth mentioning in these mathematicians' images of mathematics. These two aspects include the notion of difficulty in mathematics and the aesthetic aspect of mathematics. They are discussed as follows:

(i) Mathematics is difficult?

To many laymen on the street, mathematics is considered one of the difficult subjects in schools. But we would expect that the mathematicians would not find mathematics difficult as they liked and excelled in mathematics. To my surprise, at least three of them described mathematics as

> Yes, it can be difficult. (M1, 50's, male associate professor)

> Actually it is difficult. (M2, 50's, male associate professor)

> A:No such thing as mathematics without tears…There is nothing like that. There is nothing like an easy way to ... if you look at those books written like comic or lighter to be read as mathematics, they cannot go too far. All are on elementary, made it interesting, a lot of examples, fun, ok. These are all elementary. When you go on for more advances, you need to be very serious. There is no alternative that you can ...
> Q: Do you think mathematics is difficult then?
> A: I don' think mathematics is difficult. University mathematics is different, because you are talking about structures, but high school mathematics, no, is concerned with everyday life.
> (S1, 50's, male associate professor)

Although most mathematicians did not find mathematics difficult, neither was it easy. One young mathematician felt that,

> A: Actually it is not the easiest. But if you compare with Sejarah [History] and what not, [which] you need a lot of facts to remember, biology, Chemistry. But maths if you can understand how to solve one sort of problem, then you can do anything. Because it is just deal with numbers, for school- [mathematics]. If you know how to deal with that, differentiation, operation, then you can do any type of questions or problems.
> Q: How about higher degree, let's say university maths?
> A: University maths. You need to have a very good basis in that. So in school, you need to know how to differentiate, basic concept...Because in university they don't really teach you those things. I think for school students, they need a lot of exercises, just to familiarise with that problem, I think I learnt like that, ya, a lot of drill and practice, and then after that it becomes easy.
> (K8, 30's, female lecturer)

According to her, a way to conquer the 'difficulty' in mathematics was to work hard, do a lot of "drill and practice"; or learn it step by step as suggested by another mathematician:

> You go virtually in stages. Just like, you want to climb a high mountain. You cannot climb it overnight. One day one hour a few steps, and you do it slowly, then it's ok. You got to do it slowly and it's fun.
>
> (M2, 50's, male associate professor)

Perhaps this notion of 'difficulty' is relative. How one perceives what mathematics should be and how one is confident about it seem to determine the degree of difficulty. One of the mathematicians gave her opinion in the interview:

> Q: Do you find maths difficult?
> A: No. (laughter)
> Q: All the while you find maths easy?
> A: At school level, yes.
> Q: How about at higher level? Let's say the university level?
> A: Not that difficult. I don't know the school students are concerned, who put the idea that mathematics is difficult. Like my students here too, I think they have been like some other people, like their seniors who put the idea in their heads that mathematics is difficult. If these students are able to discover something by themselves, with ...I think it shouldn't be difficult.
>
> (K5, 50's, female associate professor)

Consequently, she suggested that it was important for students to keep an open mind so that this 'self fulfilling prophecy' did not take place:

> A: they should be more open- minded. When you start reading something, some topics in mathematics, you have to read with an open mind, and you have to think that you don't know anything about that, and you are trying to discover new thing and then you just go on with that. We shouldn't be stuck with a fixed mind that it is difficult.
>
> (K5, 50's, female associate professor)

However, the concept of difficulty had a totally different meaning for another male mathematician. He took the difficulty of mathematics as a challenge and was attracted to mathematics because of this.

> Mathematics is regarded as a very difficult subject. Even the teachers themselves think so, what do you expect from the students? But I think that I do meet some very good teachers in mathematics. I think that really interests me to learn why it is such a difficult subject. When you say the subject is hard, I take it as a challenge. Because I

> think it is in my nature. Because of that, I ended up ...or sort of being hooked up in mathematics for quite some time.
>
> (M1, 50's, male associate professor)

It seems that the notion of 'difficulty' is relative. If one can do it or is interested in it, one may not find it difficult. On the other hand, if one cannot do it or is not interested in it, one tends to find it difficult.

Since the notion of 'mathematics is difficult' is such a relative concept, should we then acknowledge its difficulty and transmit this notion to our students? But then we could encourage them to take this difficulty as a challenge. Alternatively, should we conceal or ignore this difficulty, but attempt to make it 'easy and fun' for our students? Is it possible to learn mathematics without tears?

(ii) Mathematics is beautiful

One of the female mathematicians gave her reason for interest in mathematics as,

> 'How I got interested in mathematics? I suppose I found maths is beautiful'.
>
> (K5, 50's, female associate professor)

Her reason was echoed by another male mathematician that,

> Other than that [difficulty], mathematics by itself is also a very nice and beautiful subject. Because if it is just the difficulty alone I think it won't be that appealing
>
> (M1, 50's, male associate professor)

From the data, at least four mathematicians mentioned that they liked mathematics because they appreciated the beauty of mathematics. According to them, the beauty of mathematics lay on "to be in it and work in mathematics, then you know the beauty" (K1, 50's, male professor). One needs to be able to appreciate the beauty by really immersing in it, even when people around you may not understand as fully as you are. One of them described her experience,

> For example like octagonal polynomial, it might be very messy and so on. But then later on… later on... you find that, to you now it is very beautiful, something very beautiful, but people can't understand that it is beautiful. You tend to be able to see that the beauty is there.
>
> (M6, 40's, female lecturer)

However, these descriptions still sounded abstract and implicit. So I asked,

> Q: Can you quote some examples that you view mathematics as something enigmatic, something beautiful?
> A: Well I think you have a lot of symmetry, consistency. You can actually solve problems sometimes just by 'seeing', by intuition alone. It can be very interesting.
>
> (M1, 50's, male associate professor)

> A: I think is the thinking. I mean the solving of problems. The thinking, the problem solving, manipulation of like property, when you reach the answer and it's very rewarding.
>
> (K5, 50's, female associate professor)

Thus, it seems that the beauty of mathematics is closely related to the process of working on mathematical problems, thinking and solving mathematical problems. Once you are able to see or find the solution to it, you will feel it to be so rewarding and beautiful! Again, this aesthetic feeling of mathematics was also revealed by Burton's research participants (2001). The issue is how can we convey this beautiful feeling to our students who are doing mathematics in schools. As argued by Burton (2001) that,

> "People experience aesthetics differently and find values in different aspects of mathematical practice and outcomes, but they all express delight and motivation from the pleasure of touching perceived beauty" (p. 597).

Thus, the more pertinent thing may be providing students with experiences of learning mathematics through explorations and discoveries of mathematical concepts, rather than presenting mathematics as an objective knowledge. However, this may mean a big change in the existing culture of mathematics learning in schools.

Images of learning mathematics

Many of the mathematicians whom I interviewed suggested that drill and practice is one of the best ways to learn mathematics. Instead of just doing the number of sums or questions that were selected by your teachers or lecturers, a male mathematician suggested a student who wanted to excel in mathematics needed to do all types of questions. He/she should attempt all the questions that are given in the textbooks. This is because,

> Mathematics you got to solve the problems over and over again and then you find the satisfaction by knowing, then you gain mastery over the problem. Whatever form the problems come then you find it easy to solve so what I would suggest, you know, the old method of doing mathematics here.

As he explained further that,

> When questions came in, when the chapters when the exercise for when the teacher said do all. So the first question 1 to question 10 are almost similar question but different.. What do you call that? The different variables or numbers there. Then you manipulate. The time when you solve the first problem you look at the formula. The second time you look at the formula. The third time you look at the formula, the fourth time you don't look at the formula anymore because it has stuck in your head already. By the time you reach number 10, you are solving the problem [without] memorizing the formula. Next, when you come to problem 11,12 and it's a real problem question, your mind is investigating more than simple direct problems, but application problems. Then you'll find that you have [reach certain] standard towards the topic. On that thinking, so I always believe that, when students are young, they need to do more exercise.
>
> (M4, 40's, male associate professor)

His suggestion of learning mathematics by a lot of drill and practice was echoed by another female mathematician:

> I think for school students, they need a lot of exercises, just to familiarise with that problem, I think I learnt like that, ya, a lot of drill and practice, and then after that it becomes easy.
>
> (K8, 30's, female lecturer)

This belief of learning mathematics with emphasis on 'drill and practice' is not to be understood as 'rote learning' which is not a true learning and may not bring about understanding. However, as argued by Marton (1997), this type of repetitive learning which is "continuous practice with increasing variation", which can lead to deep understanding or mastery of the skills. The latter has also been identified as one of the features of East Asian culture of learning by Leung (2001). Perhaps this is not a surprise as Malaysia is part of East Asian, and thus it shares similar culture of mathematics learning.

Many of my mathematician participants believed that one only needed pen and paper plus your brain to do mathematics, mathematics is clean and free from the use of doing practical work, such as in Chemistry or other sciences. Mathematics also does not need so much memory work as studying biology. These are some of the reasons these mathematicians gave for choosing to major in mathematics rather than other sciences.

> It is very convenient, you see. [laughter] I don't know why. I don't like the practical work, you see, so I end up here. It is just the mind, just like when you walk, you don't have to bring anything, you see. When you are free, you can 'think' it anytime.
>
> (M2, 50's, male associate professor)

> A: because It require less memory work, that is my main reason.
> Q: You think in mathematics you don't have to memorize?
> A: Just memorize the formula only
>
> (S3, 30's, male lecturer)

Even a female mathematician shared a similar view of mathematics as a clean and less memory work:

> A: Yes . challenging, maths is challenging as well-kan? I don't like doing .. satulah lab… saya tak suka kotor-kotor tangan, lepas itu… Even chemistry , kena hafal itulah… saya tak suka . I jenis yang macam
> [yes, mathematics is challenging. I don't like …one thing is laboratory work. I don't like to dirty my hand. Even chemistry, have to memorise, I didn't like this type of subject…]
>
> A: Masa matriculation kena ambil semua subjek: chemistry, biology… I masih remember ..Kena belah kataklah semua itu . Saya memang tak suka benda-benda macam tu kotor .
> [During matriculation, have to take all subjects: Chemisty, biology… I still remember..have to dissect frogs.. all that. I didn't like these things, they are dirty…]
>
> (K9, 40's, female lecturer)

These Malaysian mathematicians seemed to hold the belief that mathematics is best learned through drill and practice. It does not required much memorization as compared to other sciences such as Biology and Chemistry. This belief is coherent with their images of mathematics (discussed earlier) as a way of thinking and not merely symbols and formulae to be remembered. This finding is important because it shows the difference in images of mathematics and mathematics learning between mathematicians and school students (as found in Lim, 1999; Kalsom & Lim, 2001). Most Malaysian students tended to hold a symbolic view of mathematics and consequently they tended to learn mathematics by memorizing formulae and procedures.

Intuition and insight in mathematics

These terms, 'intuition' and 'insight' were not asked deliberately in the interviews. However, at least five of the mathematicians whom I interviewed brought out the importance of intuition in their conversation.

> There are explicit and implicit information there. So to do proof, it is not just looking at the surface, you have to have the insight of looking at things. The geometrical intuition of it...the algebraic intuition of it...
>
> (K2, 50's, male professor)

> Well I think you have a lot of symmetry, consistency. You can actually solve problems sometimes just by 'seeing', by intuition alone. It can be very interesting.
>
> (M1, 50's, male associate professor)

How do mathematicians gain intuition? Although they did not seem to be able to explain it in explicit terms, they believed that it was something related to experiences:

> A: I think I can see there is something could be done there.
> Q: From your own experience?
> A: Yes, Intuition, in fact. You just feel that there is something there. Even though you might be in the dark, when you really… when you are settle down, you are able to....
> Q: How do you get this kind of intuition?
> A: I just get it.
>
> (M6, 40's, female lecturer)

> A : The intuition is also something like because of your experience. Somehow, someway but you don't know how you see. The only thing is you have that knowledge-based already. You think of something now and all these link to... they all help you to have that concept... So called intuition also have a base somewhere, you see.
> Q : It is not come out from nothinglah?
> A: No, no... It is not like this. Intuition has a base somewhere.
>
> (M2, 50's, male associate professor)

In fact, one of them suggested that an intuitive way of thinking is so important that we should inculcate this in teaching and learning of mathematics in schools:

> A: To solve a problem you have to follow certain steps. I think, for most cases, you can do that if it's very routine. But usually it is not. Basically it shows with the application of insight you can actually get to the answer quickly rather than follow a certain number of steps. I'm not saying that it's no use to have to follow steps. That can be very useful. But that's not the way human being thinks. Nowadays we have what is known as an algorithm... there are certain fixed steps to get a certain answer. Definitely very good for implementation on a computer and for many other tasks. But when it comes to mathematical and perhaps other types of thinking it's not always like that. I think a lot of unconventional or fuzzy modes of thinking come in, . .. like making use of intuition etc.
> Q: Do you think mathematical research needs a lot of intuition too?
> A: Well exactly. Able to see, to visualize ... have to be creative.
> (M1, 50's, male associate professor)

Similar findings were observed by Burton (1999a) in her study to explore how research mathematicians come to know the mathematics they develop. She concludes that,

> 'Intuition, insight, or instinct was seen by most of the seventy mathematicians whom I interviewed as a necessary component for developing knowinng. Yet none of them offered any comments on whether, and how, they themselves has had their intuitions nurtured as part of their learning processes' (p.31).

Perhaps this is something to be pursued or looked into by our teachers and students alike, in view of the importance of intuition and insight in mathematical practices. (for further discussion see 'implications to mathematics education').

Images of mathematician

When the participants were asked "who do you define as mathematicians", they said mathematicians are those:

> 'who are interested, keen on doing mathematics'
> (K2, 50's, male professor)

> 'who work in mathematics and who do research in mathematics
> (K3, 50's, male professor)

> 'who study, who explore and who are expert in the field of mathematics'
> (K5, 50's, female associate professor)

> ‘those with PhD degree, conducting some kind of research in mathematics’
>
> (M1,50’s, male associate professor)

> ‘A Mathematician is someone who is actively working on maths problem, actively trying solve maths problems, trying to come out with something new, or contribute something to mathematics, not just taking what is known and sharing with others. I think some one have to contribute new things, or trying to help to increase knowledge.’
>
> (M5, 30’s, female lecturer).

Consequently, a female pure mathematician (M5) believed that,

> ‘Not necessary someone with a PhD in maths because I won't consider because someone with PhD in mathematics but latter on not really working on maths. He or she can be considered as an academician but not really doing research in mathematics. I will just call the person a teacher maybe, she is just teaching but not a mathematician.’
>
> (M5, 30’s, female lecturer)

But almost all of them agreed that mathematicians must be somebody who can contribute, although “not necessarily 100% new [knowledge/theories], you criticise the definitions, criticise the concepts, that’s the beginning of becoming a mathematician” (K1, aged 50, male professor). With this criteria in mind, most of them disagreed that ‘all mathematics lecturers in universities can be considered as mathematicians’. This is because not all of them do research and contribute to new theorems or new mathematical knowledge. Likewise, mathematics teachers in schools are certainly not mathematicians because they merely transmit mathematical knowledge to their students without making any new contribution in mathematics.

However, at least four of them felt that the condition of making new contributions was too stringent. This was because, “if you want only to say those who discover new theory are mathematicians, then you don’t have any real mathematicians, I think”(K8). Instead, they gave a rather loose and general definition of mathematicians to include mathematics teachers in schools as long as “some of them are because they are writing articles for mathematics, you see. Some of them are very productive also, they are writing books, do research, I mean more on education side perhapslah! In that sense they are still mathematicians.” (M2, 50’s, male associate professor). Thus, some mathematics teachers could be considered as mathematicians because they have contributed in terms of writing books and even those “who can come out with new way of teaching mathematics, okay, they can be [mathematicians]” (K8, 30’s, female lecturer).

One male participant also included engineers as applied mathematicians because “they are user…of mathematics, …some discipline, the pillar of the subject is mathematics. For example, engineers, the pillar is still mathematics” (M4, 40’s, male associate professor)

One of the participants differentiated pure mathematicians from applied mathematicians. According to him, “pure mathematicians are those type of people who are thinking in formula. They are excited about numerical problems, about numeric, just like Fermat’s last theorem. …whereas the applied mathematics is more like using. They are users, they are not really inventors. A pure mathematician is someone who invent some kind of formula to solve some problems”(K7, 30’s, male lecturer). Again, this kind of differentiating pure mathematicians from applied mathematicians was also espoused by the mathematicians participated in Burton’s (1999a) study.

It is interesting to note that the pure mathematicians seemed to have stricter criteria for being a mathematician than the applied mathematicians. The former tended to believe that it was vital for a mathematician to contribute new knowledge or do pioneer work in mathematical research. Interestingly, they also did not consider themselves as mathematicians. The following conversation illustrated this point:

> Q: How do you become a mathematician?
> A: I'm not a mathematician (laughter). I am doing mathematics that's all. We have seen mathematicians. Mathematician ...does not mean who do mathematics or who teach mathematics is mathematician. No. We have seen some real mathematicians. But I do not consider myself as a mathematician.
> Q: In your opinion, what kind of people is a mathematician?
> A: A mathematician, I think, he must have some contributions to mathematics in terms of research, yes. In terms of some pioneer work in mathematics. Of course, certainly we have published paper , but depends on what kind of paper you are publishing. A lot of paper are computer garbage. For the sake of publication, we published paper. But, compared to those real mathematicians whose work influence mathematics, the development of mathematics. These are mathematicians. I mean, when you want to say mathematics , there is a field called group theory. If you want to study that field you must learn the theorem by certain people. Those people are mathematicians.
> (S1, 50’s, pure mathematician, male associate professor)

However, applied mathematicians especially the younger ones seemed to be more reliant and they held a broader definition for mathematicians.

Experience of working as mathematician

To which community do you belong?

As the participants' images of mathematicians were heterogeneous, they also seemed to equate themselves to different communities. When the participants were asked, 'of which community do you claim membership?', they seemed to divide themselves into three broad categories: pure mathematicians, applied mathematicians and statisticians. Perhaps this categorisation is not surprising because it was also practiced by their British counterparts (Burton, 1999a). As discussed earlier, the majority of the Malaysian mathematicians were trained in United Kingdom or United States of America, so the same kind of system could have easily brought back to Malaysia.

Even though one or two of them disagreed with this division, the majority of them found this division necessary as it helped to identify oneself or to promote one's expertise:

> A: I think it is more for convenience you want to classify yourself. I don't think we can...
> Q: So it is not that important?
> A: Yes, but it can be good, like you say, you're in certain area and people know if you want to do then they know what area you're in.
> (M1, 50's, male associate professor)

Even within the broad category of pure mathematics, they grouped themselves into very specific fields like topology, cohomology etc.

> Q:Do you think it is very important that you associate with 1 specific community like topology or Fuzzy mathematics?
> A:Yes. That's important. We also attend the general talk, general seminar. But we don't research in those fields, we concentrate in our field only.
> (K3, 50's, male professor)

Likewise, for the applied mathematicians, they have their own cliques such as operational research (OR), industrial computing or optimization. When they go to conferences, they tend to meet only the members of their own cliques or communities. They have their reasons as illustrated in this conversation:

> Q: Of which mathematical community do you claim membership?
> A: In OR, operational research.
> Q: When you go to a conference, you are sure to go for this?
> A: I wouldn't go for mathematics [conferences], sometimes we do go for mathematics, but we do not get so much feedback as we go to OR. OR is a bridge between management, engineering, computer science and mathematics.
> Because it is inter disciplinary with many subjects, so when we go to OR conference, OR focused, we got all combinations of people from different fields. So the feedback is better compared to if I go to mathematics conference, I wouldn't get this varieties, so normally, I do not go for mathematics conference, I always go to OR conference or computer conference only.
>
> (M8, 40's, female lecturer)

Indirectly, this illustrated that each mathematical community is so specialised that any non-member may feel alienated even if they tried to join in.

The 25 participants whom I interviewed seemed to place themselves into one of the three broad categories without overlapping. Table 2 displays the broad distribution in terms of membership claimed.

Table 2: Distribution by membership and gender

	Pure Mathematician	Applied mathematician	Statistician	Total
Male	7	5	4	16
female	4	2	3	9
Total	11	7	7	25

The participants of this study were not selected deliberately by any criteria, but more by chance and voluntary. I identified them by going through the telephone directory of the staff members, and asking for their voluntary participation. Nevertheless, I did try to stratify them by their academic positions such as professor, associate professor and lecturer. Although the distribution in Table 2 may not be representable of all Malaysian mathematicians in universities, to some extent, it reflected the much higher number of male mathematicians than female mathematicians. It also showed that there were more pure mathematicians than applied mathematician or statisticians.

Mathematicians and Research

In terms of research, I was interested to find out what kinds of research were these mathematicians engaged with. Do pure mathematicians or applied mathematicians have equal chances of being involved in research?

The pure mathematicians whom I interviewed seemed to find themselves lonely and isolated as most of them tended to be very specialised in one or two particular areas. Moreover, their areas of interest or research were so specific that very few people were involved or engaged in the same area. For example, one of them specialised in cohomology and she found only about a handful of people in the world of mathematics research on this same area. Therefore, many pure mathematicians like she, tended to work alone, even though she appreciated the positive effect of collaboration:

> A: …Sometimes it's quite nice-lah, you have some one to exchange and to interact. But, most of the time, a lot of people here are working on their own.
> Q: Is this typical for mathematicians?
> A: I think it's quite typical for pure mathematicians. But I think applied mathematics and Statistics may be less, …unless they are doing the pure or theoretical part of statistics. Most statisticians that I know here work with the people in the Medical faculty, some of them in finance… So I think it is easier for them to find people to talk to.
> [M5, pure mathematician, female, 30's]

From the analysis, it is observed that the number of research projects being carried out by the majority of the mathematicians in this study is still limited. Perhaps this is due to three possible factors. First, there is generally a lack of research grants, especially for pure mathematics. One of the mathematicians complained that,

> No. For pure mathematics, we hardly can get a grant. You want to get the grant, you have to join those in applied mathematics.
> (S1, 50's, male associate professor)

But then some found that doing mathematical research did not need much funding, as one of them experienced that,

> We don't really need a lot of grant. Ha! Ha! paper and pencil and computer. Last time I have a grant, I don't know what to do, I have a lot of money. About RM17, 000. I did not finish the grant.
>
> (S7, 40's, male associate professor)

Second, collaboration among colleagues seemed not to be encouraged due to differences in specialisation as discussed earlier. Last, as one of the mathematicians whom I interviewed who was also the former dean of a mathematics department observed that,

> A: I feel in this university, we have too many.... just a significant number of people who are just, I feel are, complacent.
> Q : Do you think, there is a lack of the collaboration spirit?
> A : No, I think these are people who are very comfortable in their positions. You know, when they have classes, they come a little earlier, otherwise, they don't have..
>
> (S6, 40's, male professor)

Perhaps this kind of 'complacent' attitude of some academic staff may have partly explained the less active participation and involvement in mathematics research.

On the whole, there seemed to be some observable cultural differences in mathematical communities or research atmosphere among the three different universities. University M which is the oldest, and with more established and senior mathematicians tended to engage more in individual research projects. University K with the most number of professors tended to lead big group research projects. Normally these projects were led by a senior professor and a group of collaborators. Lately university K seemed to have the tendency to engage more in educational research, such as ethno mathematics. However, university S seemed to still lack in a spirit of research collaboration among its staff members, as disclosed by its dean in the interviews.

Collaboration or individual work?

As noted earlier that pure mathematicians seemed to find it harder to have collaborative projects with the others than their applied counterparts. Of the 11 pure mathematicians whom I interviewed, six of them did not have or were yet to have any collaborative project with the others. As one of the female pure mathematician expressed that,

> Q: Do you have any experience of like collaborating on a research project or not?
> A: No. Because I am in pure mathematics. So, most of these project, I mean, usually.. the one collaborated is mainly the one on applied mathematics such as statistics.
>
> (K5, 50's, female associate professor)

Likewise, another professor in pure mathematics also agreed that,

> "Normally it is difficult to team up with other people. So most of us do our work on our own."
>
> (K3, , 50's, male professor).

However, this is not to say that these mathematicians do not value collaboration. Many of them did believe that collaboration could be helpful because

> Yes, I learn something from there. I think it is helpful. Just to see how your colleague sometimes work and how they think.
>
> (M5,30's, female lecturer)

> because some thing relate a little bit of everything, you see, a little bit of pure-mathematics, a little bit of applied-mathematics, a little bit of statistics. At least, you team out the people of various fields, so it helps one another.
>
> (S3, 30's, male lecturer)

And yet, only very few of them managed to have collaborative projects with the others. Why was this so? One of the reasons could be that different colleagues, even within one instituition, tended to research on different areas. One mathematician agreed that,

> That's very true because sometimes the area that one mathematician do, we cannot even understand. It takes time for you to understand it. So far I think mathematics is different in the form.. if you are a pure mathematician, the area is so wide and then there are so little of you. In Malaysia itself, pure mathematicians here we have about only 13 or 12 people. The area in pure mathematician is so large but in the end. The master, the PhD students or the lecturers, who has mastery in that particular, he's alone there. How do he transmit his knowledge to other people so that they can understand in this, this, this alone. Like I'm doing this Bayscian statistic. So when I talk about this Bayscian thing, there's none, even in the country so now I heard about 2 or 3 is coming back, and That also they are doing different sets. Some people are doing Bayscion, some people are doing basic time series, and some people are doing some thing else.
>
> (M4, 40's, male associate professor)

Thus, many mathematicians found the differences were so wide that some of them just could not understand each other. They felt that they were speaking different languages. Hence, most of them tended to collaborate mainly with their supervised students. A few of them collaborated with one or two colleagues overseas, including with their former supervisors.

Nevertheless, applied mathematicians did find ways to collaborate in some research projects with their counterparts in other fields, in particular, with engineers, medical researchers and computer scientists. One statistician that I interviewed managed to have collaborative linkage with researchers in Canada and USA, even though he agreed that,

> Q: Most people tend to think that mathematicians used to do work on their own, without much collaboration with others. Do you agree with this?
> A: I would agree to a great extend. That is seriously the norm. Because I think in mathematics you do a lot of thinking. You tend to be alone most of the time.
> Q: Sort of you don't have to collaborate with your colleagues?
> A: Yes to a certain extend yes, you don't have to. But I think, thing has come to a stage where many things are connected. You cannot be in an island to yourself... I think that is where collaboration comes in.
> (M1, 50's, male associate professor)

Perhaps there was a slight reaslisation of this "cultural shift in mathematics from a discipline dominated by individualism to one where team work is highly valued" as pointed out by Burton (1999a, p.131). Nevertheless, the data showed that collaboration or team works among Malaysian mathematicians were relatively less significant as compared to their British counterparts.

In short, the collaboration seemed to be in three categories: (a) in research project (b) with their supervised students and (c) with oversea colleagues. Due to specialisation, most mathematicians seemed not able to collaborate with their colleagues even within the same institution or in the same country.

Justification of new knowledge

When the question of “how do you justify that what you have come to know is new?”, most mathematicians whom I interviewed agreed that the best way to justify is through peer review. But first of all, one needs to make a comprehensive literature review before one starts working on a problem.

> As I told you just now, I didn't look at the books initially when I started my phd. but I went through papers. So papers are very specific, you went to library and pick out Mathematical review and you go under number theory. So you just pick out any paper at random on number theory, and just look at that, so that's where you can be confidence and you will fill safe because you are in that area already. I am sure that I have cover all the review, from A-to Z. and you know that you are picking right papers. The lastest publications and what you need to improve.
>
> (S5, 30’s, female lecturer)

After a thorough review of related literature, one’s work needed to be scrutinized by his/her peers. This was usually done by presenting at conferences or sending for publications in mathematical journals.

Application of mathematical knowledge/theories

It is interesting to notice that some mathematicians, especially the pure mathematicians seemed not to worry about the immediate application of the mathematical knowledge or theories that they found. As they believed that,

> A:My thesis is about generalisation of the Van Kampen Theorem, which we call union theorem.
> Q:What is the application?
> A:in mathematics ? Maybe in many years to come someone might find application in the real world, in the applied science, but now is still in mathematics.
>
> (K3, 50’s, male professor)

> A : Well, I am talking for myself because I have been doing pure Mathematics. for pure Math like me the application is very lack and of course in my area is purely theory so to made people realize the usage of it is quite difficult. You don't know the result will be consider important or interesting. That's one lecturer asking for what I have been doing in my PhD. I told him that, I am doing something very pure and theoretical, so he said he cannot apply the theory to the real world. I said, well, at this moment, I don't see the application yet. So he said, oh, is that so? I am not interested and I am not really sure. The most important things is I like what's I am doing. I have look into

> the quite several Mathematicians work and from there I improve their work. Yes, for myself I am happy with what I am doing. I am very sure that but later on in future may be they may need my theory. I am happy by continuing the works have been done by the Mathematics sometimes ago. Even though some of the lecturers don't really interested with what I am doing because of that purpose, like no application and things like that, who cares? Ha!
>
> (S5, 30's, female lecturer)

They were not worried about the immediate application of what they found as that would be the responsibility of applied mathematicians or others users of mathematics such as engineers. Could this be the result of a conflict of interest between the present world view with its emphasis on materialism or instant application/utilitarianism causing pure mathematicians to live in their own world?

Yet, this lack of immediate application of the mathematicians' research should not be taken to imply that mathematicians do not care about the 'connectivities' (Burton, 2001) of mathematical knowledge with the real world problems. In fact, as one pure mathematician pointed out that,

> Mathematics is applicable, except that… when you have a problem here. We use mathematics to solve your problem. But after we solve the problem, the matematics is still alive. We still go on with the research and the research is so fast that we have reached a stage where we cannot apply the result yet. Mathematics goes faster than the practical world. That's why people think mathematics is useless.
>
> (S1, 50's, male associate professor)

Thus, it is believed that mathematical knowledge is advancing faster that what is needed in the practical world.

Suggestions for improving mathematics education

The mathematicians were asked to give some suggestions for improving the current mathematics education in Malaysia. Their suggestions can be grouped under the following aspects:

a) School textbooks need to be revised

At least three mathematicians whom I interviewed commented that the textbooks used currently in schools need to be revised. One of them voiced his opinion that,

> Kalau kita baca buku bidang lain, ia ada knowledge. Knowledge created by that katalah buka buku Fizik, buku Sejarah, you baca, ambil baca ada knowledge. Tapi kalau kita buka buku Matematik dari Darjah satu sampai Tingkatan 6, it just simbol and teknik. Dia don't

> produce any knowledge. That's my impression about mathematics textbooks.
>
> (K1, 50's, male professor)

According to him, unlike textbooks in other fields, such as physics or history, where one gains knowledge, in mathematics textbooks from Primary one to 6th Form, one sees just number and techniques. One may not gain any knowledge from reading these textbooks. He also observed that many textbooks have been written with the aim of helping students to pass examinations rather than to gain knowledge in mathematics. Thus, he advocated that there is an urgent need to revise the mathematics textbooks in use now.

b) **Utilitarian view** -- Relevance to real life

Some suggested that learning of mathematics should relate to real life application. For example,

> I can give suggestion like try to make the application side, and make them realize the importance, especially those are more relevant to daily life. That part of mathematics should be taught first, so that they can see the relevance of what they are learning and what they need, for example counting their money.
>
> (K7, 30's, male lecturer)

Other echoed that,

> Relate to real life so that they can open their minds about maths, not just calculating, not number only. (K8, 30's, female lecturer)

This suggestion seems not new as many mathematics teachers and educators alike have advocated it. More importantly, finding the right application of mathematics in real life problems and relating it to the topic taught seem to pose a major challenge to our mathematics teachers today.

c) **Make it more fun –** more colorful, using ICT, games and puzzles

Others suggested that mathematics learning should be made more fun and colourful. Indirectly they agreed that mathematics can be dry and boring for many students.

> So far when mathematics is concern, it is always black and white. Now there is a process of how to make it colorful. I mean to start it by using a lot of other medium and then the set of example that we give to the students, it cannot be anymore, it have to make a good blend of sea shell and also some computer animated thing, out of it then the blending of this will make it more interesting. Hands on in computing and what ever it is.
>
> (M4, 40's, male associate professor)

The idea of using multimedia in teaching mathematics was also supported by another mathematician:

> One way is by means of multimedia software because of the animation. Some of the things can be taught by multimedia.
>
> (M2, 50's, male associate professor)

This is because, as he explained that,

> Q : Do you think that the animation and all these things will improve their understanding in mathematics or their interest?
> A : Both their understanding and interest. Because this sort of thinglah can be repeated at any time and as many times as you like, you see. It is not like one to one teaching where teacher may have to explain a few rounds. Then it depends on how creative the software prices is. Eventually, there are good software coming out, to teach all levels not only, even the primary school and so on. I think the potential is tremendous now. Nowaday even the 3D graphic ..... These are used for teaching.
>
> (M2,50's, male associate professor)

Besides the use of multimedia, others suggested adding quizzes and games in the teaching of mathematics.

> Q : What is your suggestion?
> A : may be we should put some... I am not sure it is practical or not. Put some graphic. Like we draw some graph, instead of data. At least we tell them from this formula, it can represented in a form of graph. So that they can visualise it. [More concrete form]. And also some quiz and games, so that they feel that mathematics is quite active, they can can take part actively in mathematics classroom.
>
> (S3, 30's, male lecturer)

> S: Do you think we should promote some of these puzzles in school mathematics ?
> J: Yes. Some of these are very good puzzles that if you try to solve it, it actually helps to develop the base for problem solving.
>
> (M1,50's, male associate professor)

Thus, by introducing mathematical games and puzzles in mathematics classroom, active participation may be promoted as well as developing students' problem solving skills.

d) Mathematics teachers

A few of them suggested the need to upgrade the content knowledge and the teaching approaches of mathematics teachers. As they themselves experienced,

> A : Maybe one of the problems that we are facing today is not having many teachers who are capable in teaching.
> Q : Teaching at all levels?
> A : I mean it is not only teaching alone but also to make the students interested in the subject because I know I feel that is so because I also teach some education students who will eventually go to the schools to teach. I feel that their maths already so poor?
> Q: Their content knowledge?
> A: Yes. They do not have the ability to do mathematics also, sometimes. You expect them to have the ability to something which they are suppose to teach later, but you find that they can't. That is the worry-lah. So how are you going to teach students if youn yourself are not good. So I think one of the problem is getting good teachers-lah!
>
> (M5, 30's, female lecturer)

Some had experienced good teaching during their school time, therefore they highly recommended that mathematics teachers needed to be good at mathematical content before they could make learning of mathematics interesting and meaningful for their students. As they believed mathematics might be a difficult subject for many students, thus they believed that it was important for mathematics teachers to help students to make learning easy. One of them recalled that,

> S: Any suggestions why do you love mathematics so much?
> J; Well, I did or met ...I think a lot of the teachers who are very good in mathematics. In fact, they have shown that in spite of the apparent difficult of that subject, it can become very easy. I think one in particular has make me curious, you know.
>
> (M1,50's, male associate professor)

A similar suggestion was given by another female mathematician (S5),

> They should have a very good teacher. They should be able to attract attention of the students. They should be able to influence the students so that they can see the subjects very interesting.
> I think it is the way how you teach them, and that is the most important thing. As I have experienced that myself. Math teacher out there need to find out how to made the subject more interesting

> because children like apply things, like myself, not just memorize the sin, put it in such way which easy to memorize and interesting. It is easier to say than done.
>
> (S5, 30’s, female lecturer)

Again, these suggestions reflected the significant role of mathematics teachers in mathematics teaching and learning which was very much recognised and appreciated by these mathematicians.

e) Hard work and practice

The majority of the mathematicians in this study viewed mathematics as a difficult but challenging subject; perhaps it is therefore not surprising that they stressed the importance of drill and practice as well as the understanding of mathematical concepts.

> I mean mathematics is like ... exercising-lah, you must do a lot of practice constantly and consistently. No practice, not enough practice is like I mean going into the ring without any preparation. You need a lot of practice. First thing you must know the concepts, master the concepts first. After you give the concepts, some examples, we give some exercises, they don't know how to start. The first thing, we must understand the concept, what is a group? And then understand the examples and then do a lot of practice. And then there is no problem in mastering it.
>
> (K3, 50’s, male professor)

Likewise, another mathematician also agreed that,

> Q: How about to excel? How to promote the student can be better achievement in maths?
> A: I think must be hardworking and understand the theory. And then, you must practice, you can't do anything without practice. If you can't practice much, at least you must read as many questions and start to think how to tackle the problem.
> Q: Is it must be quite flexible in thinking because you said manipulate.
> A: yes, How to play around with the facts given, the hypothesis, you know, but of course they must be foundation to what they do.
>
> (K5, 50’s, female associate professor)

> Mathematics is a doing subject, you need to work on it.
>
> (M10, 50’s, female associate professor)

In brief, to excel to mathematics means to strive a balance between concept understanding and 'drill and practice'. A students needs to master the basic concepts and then a variety of practices to enhance the skills as well as the understanding.

Conclusion and Implications

Summary of findings

This study aims to explore a sample of 25 Malaysian mathematicians' way of knowing. It focuses on the following four aspects:

a) their experiences of learning mathematics in schools and universities
b) their images of mathematics, mathematics learning and mathematicians
c) their experiences of working as mathematicians
d) a comparison between Malaysian and British mathematicians' way of knowing

An analysis of the data shows that not every mathematician whom I interviewed had positive experiences of mathematics learning since young. In fact, at least three out of the 25 mathematicians had negative experiences because they found themselves weak and disappointing in learning mathematics. Not all of them excelled or were interested in mathematics since primary schools, but they managed to pick up, usually at upper secondary. It was often due to either an inspiring mathematics teacher or a successful experience of problem solving. In fact, some mathematicians can still recalled vividly their experiences of mathematics learning in schools. Some can still remember their teachers' name though after ten or twenty years. This finding confirms the significant role of mathematics teachers in a student' experience of mathematics learning as documented in literature (see Lim, 2001; Brown, 1992; Fennema, Peterson, Carpenter, & Lubinski, 1990). Besides mathematics teachers, many of them also acknowledged the important role of family support and peer influence.

The majority of the mathematicians in this study have postgraduate experience overseas, either in United Kingdom, United States of America or Australia. This cross cultural experiences seemed to have influenced their worldview about the culture of learning and research in mathematics. In particular, their experiences of being supervised as 'independent learners' have, to some extent, influence the way they supervise their students now. Many of them tried to adopt or adapt their experiences of being supervised to the supervision of their present students. However, some of them met with 'cultural conflict' as they found many local students are too dependence on supervisors or prefer to be 'spoon feed'.

Mathematics was viewed by the mathematicians in this study as not just manipulation of numbers and symbols, but a way of thinking and a way of understanding the world. Consequently, these mathematicians perceived the beauty of mathematics as closely related to the process of thinking and solving mathematical problems. They seemed to hold the belief that mathematics is best learnt through a lot of drill and practice with a variety of problems. For many of them, school mathematics is perceived as a subject that does not require much memorization as compared to other science subjects such as Biology and Chemistry. In fact, these mathematicians believed that solving mathematical problems needs intuitive thinking and gaining insight through deep involvement. This is an interesting finding because these mathematicians' images of mathematics and mathematics learning are very much different from those espoused by the school students and prospective mathematics teachers (as found by Kalsom and Lim, 2001). Many Malaysian students and prospective mathematics teachers tended to hold an utilitarian view (41%) or a symbolic view (26%) of mathematics. Consequently they tended to learn mathematics by memorizing formulae and procedures. As a result, many students found mathematics learning boring and dislike the subject. Does this imply that there is a need to change our students' images of mathematics and mathematics learning experience in schools?

Although 'intuition' and 'insight' were not asked deliberately in the interviews, at least five of the mathematicians in this study brought out the importance of intuition in mathematicians' work and research. They suggested that an intuitive way of thinking is very important and it should be inculcated in the teaching and learning of mathematics in schools.

In this study, the participants's images of mathematicians were heterogenous. The pure mathematicians seemed to set stricter criteria for being a mathematician than the applied mathematicians. The pure mathematicians tended to believe that it was vital for a mathematician to contribute new knowledge or to do pioneer work in mathematical research. However, the applied mathematicians especially the younger ones seemed to be reliant and held a broader definition of mathematician.

From the interviews, I noticed that the culture of research and publication was still not pervasive among most Malaysian mathematicians whom I interviewed. Perhaps this was due to three possible factors. First, there was generally a lack of research grant, especially for pure mathematics research. Second, due to the wide differences in specialisation, especially the pure mathematicians, tended to research individually or collaborate with their supervised students only. Third, some mathematicians seemed to be so "complacent" with their present positions that there was generally a lack of research culture and collaborative efforts among them. However, the applied mathematicians seemed to acquire research funding much easier than their pure mathematics colleagues. This is because the former could collaborate with their colleagues in applied sciences, such as medicine and technology.

Thus, in comparison with what was reported by Burton (1999a, 1999b and 2001), there seemed to be more similarities than differences among the British and Malaysian mathematicians' way of knowing. These similarities include: (i) mathematicians's images of mathematics and mathematics learning; (ii) the perceived beauty of mathematics and the importance of intuition in solving mathematical problems; (iii) the categorisation of mathematicians into pure, applied and statisticians; and (iv) mathematicians' justification of mathematical knowledge and its connectivities with real life applications. Nonetheless, one considerable difference was the relatively less collaborative project among the Malaysian mathematicians as compared to their British counterparts. This situation applied most to the pure mathematicians of Malaysian who tended to do individual research. However, with the recent increase in research funding and promotion of internationalisation policy for all universities, it is hope that there will also be a shift of this individualistic culture to a more collaborative culture of research and publication for Malaysian mathematicians.

Perhaps these findings of similarities are not surprises as most Malaysian mathematicians had their postgraduate experiences in the United Kingdom, United States of America or Australia. Moreover, Malaysian education system has great resemblance with the British system due to the historical ties. It is very likely that there is a transfer of culture from the west to the East. However, Since 1997, as a result of the economic downturn of Malaysia, the Malaysian Ministry of Education has limit the number of students sending to study overseas. This possibly mean that the next generation of Malaysian mathematicians may not have similar academic background as the present ones. Will the above scenerio change in the next decade then?Or is there a universal culture in mathematicians' way of knowing?

Implications to mathematics education

From the data, it is apparent that there exist differences between mathematics learning in schools and mathematicians' way of knowing. As noted by Burton (2001) that mathematicians "are engaged on a creative endeavour which demands a very different epistemological stance from the one which pervades the teaching and learning of mathematics"(p. 595) in schools. Therefore, if we are to encourage our students to learn mathematics and to be interested in mathematics, then perhaps we should adopt or adapt the model of learning and researching in mathematics of these mathematicians into our school model of mathematics teaching and learning. More specifically, some implications to pedagogical changes in schools are discussed in the following section.

(a) the notion of 'mathematics may be difficult but challenging'

Analysis of the data indicates that although the majority of mathematicians interviewed found mathematics not difficult for them, it is not too easy either. One needs to work hard, to do a lot of drill and practice, to learn it step by step. There is rarely 'mathematics without tears'. According to some of them, the fun and beautiful part of mathematics comes from the experience of successful solving mathematical problems. The fun, easy and game-like mathematics only apply in elementary levels. When one comes to higher level mathematics, one needs to build up strong foundations and to keep an open mind. Thus, the most pertinent part is to take the difficulty as challenges and keep working on it, the reward and satisfaction will follow then.

This finding is not unique as it coincides with a study by Lim (1999) on a sample of British public members about their images of mathematics. She found that among those who espoused liking mathematics, especially mathematics teachers and mathematicians, most of them held a problem solving view of mathematics. They viewed the difficulty of mathematics as a challenge and they felt rewarded and satisfied when they managed to find solutions to mathematical problems.

Therefore, the message to be conveyed to our school students is that the fun part of learning mathematics lies in the process of doing and exploring mathematics. Merely emphasising the procedural and utilitarian importance of mathematics may not be enough to sustain students' interest in mathematics. This implies that students should be encouraged to investigate, analyse and solve mathematical problems and justify the solutions by themselves. Perhaps this can be done through activities such as open problem strategy (Nohda, 2000); investigation projects, daily life problems or mathematical quizes and games. In this way, students will not only learn to apply whatever mathematical concepts and skills that they have learnt, but also build up their self confidence and interest in mathematics.

(b) inculcation of intuition and insight

In view of the importance of intuition and insight in mathematicians' practices, perhaps this is another thing to be pursued or looked into by our teachers and students alike. Yet, Burton (1999b) pointed out that very little mathematics education literature has accounted the nurture of intuition and insight in school mathematics teaching and learning. Why is this so? One possible reason is the expectation that mathematics is infallible (Hersh, 1998). Consequently, students are expected to learn from the basics and to acquire 'absolute' mathematical knowledge. This clearly does not encourage intuitive way of learning.

However, Hersh (1998) argued that every one of us is capable of doing intuitive thinking "because we have mental representation of mathematical objects. We acquired these representations, not mainly by memorizing formulas, but by repeated experiences" (p. 65). For examples, on the elementary level, students can experience manipulating physical objects while on the advanced level; students can experience doing problems and discovering things for themselves. Therefore, one pedagogical implication is that if we aim at promoting intuitive and creative mathematics learning in school mathematics, then we must allow students to explore and discover mathematical solution by themselves. School mathematics learning must be provided with activities that demand students to argue, to control and to justify by themselves. Moreover, the image of mathematical knowledge as infallible may have to be changed.

(c) the significant role of mathematics teachers and teaching approaches

The fact that some mathematicians in this study can still vividly remember their teachers' name and their experiences of mathematics learning in schools implies the significant role of mathematics teachers in a student's life. The story of how two mathematicians who were very weak at mathematics when they were students but they managed to become mathematicians later should be used as inspiring examples for our mathematics students and prospective mathematics teachers in schools. The first one was inspired by his mathematics teacher's teaching approach which was 'something different from the traditional way, we were no more just applying formula. Now we have to use our brains to think" (S1). This new approach has not only changed his image about mathematics but also gain him a lot of self confidence. As he acclaimed that 'And then I'm so proud because I can understand it and I can answer the questions whereas my peers cannot. Something that made me feel I am so proud about myself. Suddenly I feel oh! I'm good at mathematics. I can do it". (S1). This feeling of success was also experienced by the second mathematician and that inspired him to become a mathematician now. This finding reflects the crucial role of mathematics teacher, hard work and successful experience of mathematics learning that can boost the interest and confidence of students towards learning mathematics.

Therefore, how can we, as mathematics teachers, strike to provide our students innovative teaching approaches that can boost up their interest in mathematics? More important, how can we provide students with the successful experience of learning mathematics that will sustain their interest and inspire them to go further?

On the other hand, perhaps this finding could also boost up the morale of mathematics teachers today. In view of the declining in respect to teachers and heavy word load in schools, there seemed to have an increasing number of teachers losing their morale and interest in teaching. Indeed, a teacher who is disappointed with the negative attitudes of his/her pupils, may not strive to work hard and teach well. As a result, his/her stundents may not show much respect towards the teacher and consequently less interest in his/her teaching. This is a vicious cycle of cause and effect. Therefore, findings of this study should be served as a motivator or stimulus for our mathematics teachers in schools that they are still being respected and well remembered by their inspired students.

(d) the enculturation of collaboration and research culture

In comparison with what was reported by Burton (1999a, 1999b and 2001), there seems a lack of collaboration and research culture among the Malaysian mathematicians. Apart from the constraint of specialisation and limited research funding, perhaps the crucial point is that there seems a lack of conducive environment that promote academic discussion, intellectual discourse or exchange of ideas among the local mathematicians in most universities. One plausible solution could be the setting up of the culture of 'coffee break'which provides a common time and space for academic discourse. This was a common practice in many leading western universities, such as Princeton University (as described in the biography of the Nobel prize winner mathematician, John Nash by Nasar, 2001), as well as mentioned by one of the professor (K2) in this study.

In view of the advantage of collaboration in advancement of knowledge and research, as pointed by mathematicians participants of Burton (1999), perhaps it is time for Malaysian mathematicians to strive a shift of individualistic culture to a culture of collaboration.

Suggestion for further research

Participants in this study only included experienced mathematicians, perhaps further research could focus on mathematics postgraduate students. Similar methodology can be used to find out how they are being encultured into the mathematician world. The experience of transition may provide practical hints and cue to upgrade the mathematics learning in schools.

Conclusion

This study is, albeit exploratory, gives a glimpse of how Malaysian mathematicians come to know, learn and think of mathematics. To the best of my knowledge, this study contributes to be the first study about the work practice of Malaysian mathematicians. The findings have provided some pedagogical implications and recommendations as discussed above, which I hope will be helpful in improving the present mathematics learning in schools.

On the whole, this study points to the importance of promoting a problem solving view of mathematics rather than a symbolic view among present mathematics students. This means changing the model of mathematics teaching approach to a more open, explorative and which encourages intuitive thinking and reasoning. Mathematical activities provided by schools should allow students to gain a better insight of mathematics as well as appreciate the beauty of mathematics. Moreover, students could be encouraged to acknowledge the notion of 'difficulty' of mathematics and take it as a challenge rather than as an obstacle to mathematics learning.

Acknowledgement

This study was made possible with the funding from the short term grant provided by Universiti Sains Malaysia, Penang, Malaysia.

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Appendix A: interview questions (English language)

The questions below are meant to give direction if we need it but not to be a requirement of how our conversation must develop.

About you

1. Can we chart the historical trajectory of your becoming a mathematician?
   -- What's your experience of learning mathematics in primary and secondary school?
   -- When did you start to get interested in mathematics?

2. Can you briefly describe your undergraduate or postgraduate experiences of coming to know mathematics?
   -- How do you come to your choice of PhD topic?
   -- Whose/what influences?

3. How would you describe your experiences of research supervision and yourself as a supervisor of research students?
   -- your experience of supervisors
   -- how these experiences influence (or not) your supervision of your students?

4. Of which mathematical community would you claim membership? Is that membership important and in what ways?
   -- If you go to a conference, which one do you normally go to?
   -- At the conference, which community do you associate yourself to?

5. Do you have experience of collaborating on any research projects? Will you describe that experience and say what you have learnt from it or explain why you think collaboration is helpful (or not helpful) to you?

About how you come to know mathematics

1. What do you now believe mathematics is?
   -- how do you explain what is "mathematics"?

2. Who are the mathematicians?
   -- how do you define them?

3. When you are acting as a mathematician, can you explain what you do, what choices you have, what leads you to make one choice rather than another?

   -- Just think about a problem that you are working now, can you explain what you do?

   -- How many problems do you work at the same time? Where do you think about your problems – in the office, the bath, walking…? Are these problems all of the same style?

   -- Where do you find the problems on which you work and what makes them something which engages you?

4. Do you always know when you have come to know something new? How? Have you been justified/unjustified in this confidence?

   -- "I was told that mathematicians often think they have reached the resolution of a problem, but later they find out that they were wrong" What do you think?

   -- so, how do you know "when you know"?

5. Do you know whether a result will be considered important, interesting or rejected by your community?

   --When do you decide to send a paper for review or publication?

   --What is the criteria for publication? What makes a paper publishable?

   --Do you share their criteria? What are they?

About mathematics education

1. What are your suggestions for getting our secondary students to be interested and excel in mathematics ?