[The Mathematics Enthusiast](https://scholarworks.umt.edu/tme)

Volume 11
Number 3 *Number 3*

Article 11

12-2014

# Math as a tool of anti-semitism

Jay Egenhoff

# Math as a Tool of Anti-Semitism

Jay Egenhoff
University of Montana

**ABSTRACT:** At Moscow State University's Department of Mathematics during the 1970's and 1980's, there was rampant discrimination against Jewish and other unwanted students. The professors at the math department made a strong effort to keep Jewish students out of the department. They designed "killer" or "coffin" problems and Jewish students had to answer them during an oral exam. These problems have simple solutions, but require a clever strategy to solve them. This paper explores some of the context of this episode and provides several problems with detailed solutions.

**Keywords:** Education, Coffin Problems, Entrance Exam, Soviet, Discrimination

*The Mathematics Enthusiast*, **ISSN 1551-3440, vol. 11, no. 3**, pp. 649-664

## A Brief History of Anti-Semitism in The Soviet Union

Antisemitism has a long history in Russia. Tensions over immigrating Jewish populations date back to the 11th and 12th centuries when Jews expelled from Western Europe settled in the area that is today's Ukraine (Saul, 1999). Institutionalized discrimination dates back to the early czars of Imperial Russia, when the state encouraged anti-Semitic policies during a time of Eastern Orthodox zeal, attempting to impose the Russian national identity across the empire (Rambaud, 1898). In this time period, there were several brutal pogroms in what is now Ukraine.

The state discrimination continued with more restrictive policies. This had the effect of radicalizing Jewish populations who joined the ranks of the revolutionaries. There was a brief period after the Bolshevik Revolution, when the situation seemed to have improved. Discriminatory laws were redacted and revolutionaries aimed for a society of equality. Lenin himself campaigned to try and discourage antisemitism (Vershik, 1994). These progressive ideals laid down by the leaders of the revolution have survived, and the current Russian government maintains that discrimination based on ethnicity is illegal. But just as racism remains here in America half a century after the civil rights movement, antisemitism is alive and well today in the former soviet republics.

While I studied abroad in Kyrgyzstan and traveled through former Soviet Republics and Russia, the topic of Jews came up with regularity in conversations with people across generations and ethnicities. People often ask me if I am Jewish both at home and abroad. People say that I just "look Jewish" whatever that means. The difference is that in the former Soviet Republics the people inquiring would not try to hide their relief when I informed them that I am in fact not Jewish. Several different people I met in my travels have voiced their mistrust of Jews in general. The stereotype that I heard most often was that Jews are too clever, too smart, and control exclusive organizations. It is a kind of irony then, that the bigots in the math department of Moscow State University employed these clever and sneaky math problems to exclude the unwanted Jewish students from their math department.

## Entrance Exam Procedure

The general procedure for admission to a Soviet university consisted of a written and oral test. The written portion of the entrance exam consisted of a few simple problems to test computational accuracy and one or two more challenging questions in order to test mathematical knowledge. Shen(1994) notes that only perfect papers were counted and allowed to advance and there is evidence of either discrimination or incompetence on the part of the examiners at the written level. For example, the answer to one particular question was "$x = 1$ or $x = 2$", a student wrote "$x = 1; 2$ and that answer was marked wrong (Kanevskii, 1980). If a student passed the written exam, they then had to pass the oral exam.

The concept of an oral exam is unfamiliar here in America, but it is a mainstay in the Russian education system. Students walk into the room and take a piece of paper from a pile at the front of the classroom. This piece of paper has two questions on it and is called the *bilyet* or ticket. The students are given some time to prepare their answers with only paper and pencil. When a student has an answer, they raise their had and an examiner comes by to check the solutions. Then the examiner asks one follow up question, evaluates the solutions, and dismisses the student(Frenkel, 2013).

These exams, however, were different for a Jewish student. The oral exam could last as long as five and a half hours in one case (Kanevskii, 1980). Students were given follow up problems one after another until they failed one of them, at which point there were given a failing grade (Kanevskii, 1980). Sometimes they were dismissed on a minute technicality. There is an account of one student being asked by the examiner "What is the definition of a circle?" The student's answer was "It is the set of points in a plane, equidistant from a fixed point" The student was informed that this was an incorrect answer, the correct answer is "the set of *all* points in a plane, equidistant from a fixed point" and the student failed the test (Saul, 1999).

Now this is only one example, but there are many stories of similar instances of ridiculous, often times pedantic reasons for dismissal. These stories began to accumulate, and it became blatantly obvious that an effort was being made to make it difficult for some students in particular. Predominately it was Jewish students who received this inhumane treatment.

The discrimination was not just a form of prejudice on the part of the examiners, it was well known in the university. Frenkel(2013) recalls trying to schedule the exam and being advised not to waste his time by a secretary. His mathematics credentials were impressive for a boy of 16 years, and yet before he even entered the examination room, he was encountering obstacles purely on the basis of his Jewish heritage. However, Frenkel calculated that he had nothing to lose by attempting the entrance exam, and so ignored the secretary's warnings.

Once in the examination room, Frenkel took his ticket and set to solving the problems. Upon completion he raised his hand, but was ignored by the examiners. After the examiners attended to several other students, Frenkel finally asked one directly why they were not reviewing his solutions. The examiner answered that he was not allowed to talk to Frenkel. Eventually, two older professors entered and began to cross-examine Frenkel's solutions. Frenkel says that other the examiners had been pleasant and supportive, but these two professors were aggressive and pedantic.

They looked for the smallest mistakes in Frenkel's solutions. They demanded precise definitions of everything along the way, from the definition of the mentioned above, to the definition of a line. This lasted for an hour and a half.

After this harsh treatment, they administered the follow up question. Frenkel does not relate the exact problem in his book, but he mentions that the solution required the Strum Principle which is not studied in high school, however, Frenkel knew of this through his extracurricular study of mathematics. When the examiner saw that Frenkel was approaching a solution to the problem, he was interrupted and given another problem which was harder still. After a while, Frenkel worked out a strategy for solving the problem. Once it became apparent that he could solve the problem, the examiners interrupted with yet another killer problem. Four hours into this hostile exam, Frenkel surrendered to the inevitable and withdrew his application.

This particular example of the type of hostile atmosphere a Jewish student faced during entrance exams is not exceptional. *"It should be noted that there is absolutely no controversy about whether this discrimination actually took place." (Vardi, 2000)* Khovanova(2011) echoes this sentiment in the introduction to her paper. The work of Kanevskii(1980), Vershik(1994), Shen(1994), and Saul(1999) all corroborate that claim. Antisemitism at the math department "was accepted as a fact of life"(Vardi, 2000) which is supported by Frenkel's account of being dissuaded by the secretary.

The only controversy I was able to find was a letter in response to an article by Kolata(1978), which appeared in *Science*, entitled: *Anti-Semitism Alleged in Soviet Mathematics* . The article mentions some of the cases of antisemitism connected with oral exams, but the main focus is a man named Pontrygin and his connection to antisemitic plots. In the article, Pontrygin is mentioned several times as implementing antisemitic policies, and in particular, working to deny an exit visa to a Jewish Soviet Mathematician named Gregory Margoulis who had won a Fields metal.

Pontryagin replies in a letter to *Science* in 1979. He denies, point by point, his alleged involvement in antisemitic policies and denies that he has the power to influence such policies. Ponyargin(1979) also denies that he himself is antisemitic. What speaks louder, though, is that he does not deny the existence of antisemitism in the Soviet system, just his own involvement. He does not even touch on the allegations of discrimination as part of the oral exams.

In his article, Vershik(1994) calls out the people who actively participated in this discrimination as well as those who witnessed it but did nothing. It is important to note that this took place during a time of fear and control in the Soviet Union, so it is understandable for concerned witnesses to adhere to a doctrine of caution. That is not to say that nothing was done. Several students who were allowed to attend Moscow State University knew of the discrimination. They organized to bring some of the classes to the Jewish students in the form of lecture notes. This became known as "The Jewish University." The idea was admirable, but most students who were denied access to the Moscow State Math Department decided to attend The Institute for Petrochemical and Natural Gas Industry or *Kerosinka* which offered higher mathematics classes.(Saul, 1999). These solutions to the problem do not satisfy Vershik. He believes that the story of injustice must be told and the perpetrators reproached so that we learn to recognize and prevent this kind of discrimination in the future.

## Selected Problems

On top of the strategies mentioned above, the examiners also chose carefully designed problems to give to the undesirable students. These questions were generally chosen to be difficult, yet appear solvable. Over the intervening years since the height of the discrimination at Moscow State University, several people have worked to compile and solve a list of the "killer' or "coffin" problems.

In the following section are a number of "killer problems" which I have selected from the works of Khovanova and Radul (2011) and Vardi (2000). Between the two papers there are 44 enumerated problems with complete solutions. The solutions presented here have been abridged in order to make the paper more approachable. The purpose of these examples is to demonstrate the difficulty of these problems so that the reader can appreciate the unreasonable nature of these problems. For more complete and technical solutions, refer to the works of Khovanova and Radul (2011) and Vardi (2000).

Notice the elementary solutions of these problems. This is by design, if an administrator, or outside party were to inquire about the fairness of the questions, the examiner could point to the simple answer as if to say: “The answer is 3/5, how hard could the question be?” This false logic is nonetheless convincing to someone who is not comfortable with mathematics and does not want to admit to their ignorance. These questions are difficult, and even someone with a solid foundation in mathematics would need some strokes of insight in order to solve them. These are not reasonable questions for an oral exam, and keep in mind that many times that examiners continued to give the students questions until they got one wrong (Khovanova, 2011).

**Question 1 (Khovanova, 2011)** What is larger, $\log_2 3$ or $\log_3 5$?

**Notes** It can be difficult to comprehend logarithms of different bases and this inequality is by no means apparent. So Khovanova and Radul came up with a strategy to rephrase these logarithms in simpler terms by comparing each in turn to $3/2$. This is by no means an obvious or intuitive move.

**Solution** Consider the base of the original logarithm to the power of the original logarithm and that quantity squared.

$$(2^{\log_2 3})^2 = 3^2 = 9 > 8 = 2^3 = (2^{3/2})^2$$

and

$$(3^{\log_3 5})^2 = 5^2 = 25 < 27 = 3^3 = (3^{3/2})^2$$

Then notice that:

$$(2^{\log_2 3})^2 > (2^{3/2})^2$$

and

$$(3^{\log_3 5})^2 < (3^{3/2})^2$$

Implies:

$$\log_2 3 > 3/2$$

and

$$\log_3 5 < 3/2$$

Therefore: $\log_2 3 > \log_3 5$

**Question 2 (Khovanova, 2011)** Solve the following inequality for all positive $x$

$$x(8\sqrt{1-x} + \sqrt{1+x}) \leq 11\sqrt{1+x} - 16\sqrt{1-x}$$

**Notes** This question relies on a substitution which is perhaps more straight forward, but to reach the solution requires several stages in which one could easily make a mistake.

**Solution** First, notice that $x \leq 1$, otherwise the square roots become undefined. Now, multiply both sides of the inequality by $\frac{\sqrt{1+x}}{\sqrt{1+x}}$

$$x(8\sqrt{1-x} + \sqrt{1+x}) \leq 11\sqrt{1+x} - 16\sqrt{1-x}$$
$$x(8\tfrac{\sqrt{1-x}}{\sqrt{1+x}} + 1) \leq 11 - 16\tfrac{\sqrt{1-x}}{\sqrt{1+x}}$$

Then define:

$$y = \tfrac{\sqrt{1-x}}{\sqrt{1+x}} \text{ and } x = \tfrac{1-y^2}{1+y^2}$$

Notice that for our values of $x$, then $0 \leq y \leq 1$. This yields the following:

$$\tfrac{1-y^2}{1+y^2}(8y+1) \leq 11 - 16y$$
$$(1-y^2)(8y+1) \leq (1+y^2)(11-16y)$$
$$-8y^3 - y^2 + 8y + 1 \leq -16y^3 + 11y^2 - 16y + 11$$
$$-8y^3 + 12y^2 - 24y + 10 \leq 0$$
$$(2y-1)(-4y^2 + 4y - 10) \leq 0$$

$(-4y^2 + 4y - 10)$ is always negative for our values of $y$, so the inequality simplifies to:

$$(2y - 1) \leq 0$$

By the earlier definition $y = \frac{\sqrt{1-x}}{\sqrt{1+x}}$, the left hand side of the inequality can be rewritten as:

$$\frac{1-(1/2)^2}{1+(1/2)^2} = \frac{3}{5}$$

And the final answer is simply: $\frac{3}{5} \leq x \leq 1$

**Question 3 (Khovanova, 2011)** Prove that $\sin 10°$ is irrational.

**Notes** This problem involves some standard trigonometry, but the difficulty lies in recognizing that $\sin(10° + 20°) = \sin 30° = 1/2$ and working backwards from there.

**Solution** Employ the angle sum and double angle formulae for sine and cosine.

$$1/2 = \sin(10° + 20°)$$
$$1/2 = \sin 10° \cos 20° + \sin 20° \cos 10°$$

$$1/2 = \sin 10°(1 - \sin^2 10°) + (2 \sin 10° \cos 10°) \cos 10°$$
$$1/2 = \sin 10° - \sin^3 10° + 2 \sin 10° \cos^2 10°$$
$$1/2 = \sin 10° - \sin^3 10° + 2 \sin 10°(1 - \sin^2 10°)$$
$$1/2 = 3 \sin 10° - 4 \sin^3 10°$$
$$0 = 8 \sin^3 10° - 6 \sin 10° + 1$$

Then substitute $x = 2 \sin 10°$ and reduce to:

$$x^3 - 3x + 1 = 0$$

All rational roots must be integers that divide the constant term which is 1 in this case. Since neither 1 nor -1 are solutions to our polynomial, $x$ must be irrational and therefore $2 \sin 10°$ must be irrational.

**Question 4 (Vardi, 2000)** Show that $(1/\sin^2 x) \leq (1/x^2) + 1 - 4/\pi^2$ for $0 < x < \pi/2$

**Notes** This problem has tricky substitution, but the difficulty lies in demonstrating that the inequality is in the strict domain of $0 < x < \pi/2$.

**Solution** First rewrite the inequality as:

$$\frac{1}{x^2} - \frac{1}{\sin^2 x} + 1 - \frac{4}{\pi^2} \geq 0 \text{ for } 0 < x < \pi/2$$
$$\text{and then: } \frac{x^2 - \sin^2 x}{x^2 \sin^2 x} + 1 - \frac{4}{\pi^2} \geq 0 \text{ for } 0 < x < \pi/2$$

Then begin by showing that:

$$\lim_{x\to\ 0}(\frac{x^2-\sin^2 x}{x^2\sin^2 x}) = \frac{1}{3}$$

Observe that the function approaches an indeterminate form:

$$\frac{x^2-\sin^2 x}{x^2\sin^2 x} \to \frac{0}{0}$$

Now employ L'Hopital's rule until the limit exists.

$$\frac{2x-\sin 2x}{2x\sin^2 x + x^2\sin 2x} \to \frac{0}{0}$$

,

$$\frac{1-\cos 2x}{\sin^2 x + 2x\sin 2x + x^2\cos^2 x} \to \frac{0}{0}$$

,

$$\frac{2\sin 2x}{3\sin 2x + 6x\cos 2x + 2x^2\sin 2x} \to \frac{0}{0}$$

,

$$\frac{2\cos 2x}{6\cos 2x - 8x\sin 2x - x^2\cos 2x} \to \frac{1}{3}$$

Since $1/3 + 1 + 4/\pi^2 \leq 0$, this demonstrates that the inequality holds for the lower bound. Now to demonstrate the upper bound has a strict inequality, substitute an $a = 1 - 4/\pi^2$ which yields:

$$\frac{\sin x}{\sqrt{1-a\sin^2 x}} \geq x$$

Note that $x = \pi/2$ yields a solution, but to show that the inequality holds for values strictly less than $\pi/2$, one must take the second derivative of:

$$f(x) = \frac{\sin x}{\sqrt{1-a\sin^2 x}}$$

Which is found to be:

$$f'(x) = \frac{\cos x}{(1-a\sin^2 x)^{3/2}}, \text{ and } f''(x) = \frac{(a-1+2a\cos^2 x)sinx}{(1-a\sin^2 x)^{5/2}}$$

Because $a > 1/3$, it follows that $f''(x) > 0$ for $0 < x < x_0$, where $x_0$ is the unique solution to $f''(x_0) = 0$. So $f''(x)$ is concave up on $0 < x < x_0$ and that $f(x) > x$ on the same interval. Since $f''(\pi/2) = -(\pi/2)^3$, it shows that $f''(x)$ is concave down on $x_0 < x < \pi/2$. Since $f(x_0) > x_0$ and $f(\pi/2) = \pi/2$ the fact that it is concave down at the point $x = \pi/2$ implies $f(x) > x$ for $x_0 < x < \pi/2$. This shows the strict inequality of the interval.

**Question 5 (Vardi, 2000)** Solve the system of equations: $y(x+y)^2 = 9$, and $y(x^3-y^3) = 7$

**Notes** This problem is difficult because it requires some non-intuitive substitution, followed by some tricky algebra. Near the end of the solution is a trap where students must work with an eighth decree polynomial, which can leave students bogged down in calculation. Though, in the end, the only information needed from this polynomial are the signs of its coefficients.

**Solution** Let $x = ty$ which yields:

$$y^3(t+1)^2 = 9, \text{ and } y^4(t^3-1) = 7$$

Take the first equation to the fourth power, the second equation to the third power and then divide. This results in:

$$\frac{(t+1)^8}{(t^3-1)^3} = \frac{9^4}{7^3}$$

Which reduces to a polynomial:

$$f(t) = 9^4(t^3-1) - 7^3(t+1)^8$$

Any real positive root $t_0$ of this polynomial will lead to solutions $x_0$, and $y_0$ for the original system of equations. Where $x_0 = t_0 y_0$ and

$$y_0 = (\frac{9^4}{(t+1)^8})^{1/12}$$

It is fairly clear that 2 is a root of $f(t)$, now one must demonstrate that $f(t)$ has no other positive real roots. This can be done by expanding $f(t)$ and performing long division.

$$\frac{f(t)}{t-2} = 6561t^8 + 12779t^7 + 22814t^5 + 13474t^4 + 2938t^3 + 6351t^2 + 3098t + 3452$$

Notice that all coefficients are positive, so there are no positive real roots beside $t = 2$. So the final answer is found by employing the definitions above.

$$y = 1 \text{ and } x = 2$$

**Question 6 (Vardi, 2000)** Solve the equation: $f(x) = x^4 - 14x^3 + 66x^2 - 115x + 66.25 = 0$

**Notes** This question also requires some tricky substitution, the ability to factor a quartic polynomial, a small system of equations, and imaginary numbers. On top of that, one must maintain computational accuracy and precision throughout this long exerciser.

**Solution** Let $x = y/2$ which reduces $f(x) = 0$ to $g(y) = 0$ where:

$$g(y) = y^4 - 28y^3 + 264y^2 - 920y + 1060$$

Similarly let $y = z + 7$ so that $g(y) = 0$ reduces to $h(z) = 0$ where:

$$h(z) = z^4 - 30z^2 + 32z + 353$$

Now factor this quartic using the constants $a, b, c, d$ as follows:

$$z^4 - 30z^2 + 32z + 353 = (z^2 + a\sqrt{d}z + b + c\sqrt{d})(z^2 - a\sqrt{d}z + b - c\sqrt{d})$$

By isolating the $z^n$ terms one gets the following equalities:

$$\text{(I) } 2b - a^2 = -30, \text{ (II) } -2acd = 32, \text{ (III) } b^2 - c^2 d = 353$$

From (II), one can say that $d$ must be one of -2, 2, or -1, because any other factors of 32 contain a square root, and we know that $d$ does not contain a square root by virtue of our factorization. Consider each possible value of $d$. First, if $d = -2$, then (III) gives $b = \pm 15$ and $c = \pm 8$. But then (I) implies that $a$ is divisible by 15, which contradicts (II). Second, if $d = 2$, then (III) gives $b = \pm 19$ and $c = \pm 2$. But then $a = \pm 4$ which contradicts (I). Finally, if $d = -1$, then (III) gives $b = \pm 17$ and $c = \pm 18$, and (II) shows that $a = \pm 2$. After trying all of the possible combinations of the signs for these three numbers, one eventually finds that $a = -2, b = -17$,and $c = -8$ satisfies (I), (II), and (III). Now introduce these facts to the factorization:

$$z^4 - 30z^2 + 32z + 353 = (z^2 - 2\sqrt{-1}z - 17 - 8\sqrt{-1})(z^2 + 2\sqrt{-1}z - 17 + 8\sqrt{-1})$$

Consider $i = \sqrt{-1}$ so that:

$$z^4 - 30z^2 + 32z + 353 = (z^2 - 2iz - 17 - 8i)(z^2 + 2iz - 17 + 8i)$$

Applying the quadratic formula yields:

$$z = i \pm 2\sqrt{4+2i} \text{ and } z = -i \pm 2\sqrt{4-2i}$$

For the left and right factors respectively. In order to get the final answer, one must go though the backwards substitutions of $y = z + 7$ and then $x = y/2$ which yields the roots of $f(x)$, which are:

$$x = \tfrac{7+i}{2} + \sqrt{4+2i},\ \tfrac{7+i}{2} - \sqrt{4+2i},\ \tfrac{7-i}{2} + \sqrt{4+2i},\ \tfrac{7-i}{2} - \sqrt{4+2i}$$

**Question 7 (Dodys, 2003)** Four circles on a plane are such that each one is tangent to the three others. The centers of three of them lie on a line. The distance from the center of the fourth one to this line is $x$. Find $x$, if the radius of the fourth circle is $r$.

The first step is to draw an accuate picture of the situation:

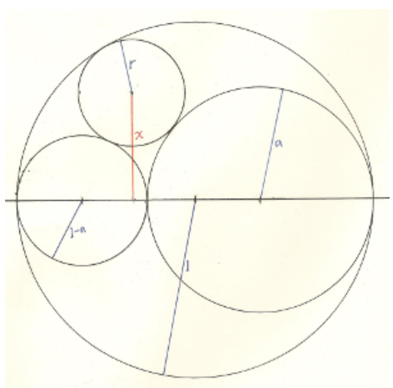


Figure 1: Here the radii are in blue and the desired length $x$ is in red.

**Notes** The solution requires knowledge of Soddy's Formula. This formula was first described by Descartes, but was popularized by Sir Fredrick Soddy in the form of a poem published in Nature, 1936 (Lagarious, 2002). The formula relates the radii of four tangent circles as follows:

$$\left(\frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} + \frac{1}{r_4^2}\right) = \frac{1}{2}\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}\right)^2$$

The solution also relies on Heron's formula for the semi-perimeter. Again, this is not an obvious move and would would have required some inspiration on the student's part. Yet the final answer looks quite simple.

**Solution** Assume the radius of the large circle is 1, and those of the two smaller circles are a and $(1-a)$, while the remaining circle has radius $r$. By Soddy's formula:

$$\frac{1}{a^2} + \frac{1}{(1-a)^2} + \frac{1}{r^2} + 1 = \frac{1}{2}\left(\frac{1}{a} + \frac{1}{(1-a)} + \frac{1}{r} - 1\right)^2$$

It should be noted here that the radius of the largest circle is considered to be negative one, because it is concave to the other three interior circles. Then substitute: $z = a(1-a)$, And the first equality can be rewritten as:

$$\left(\frac{1-2z}{z^2} + \frac{1}{r^2} + 1\right) = \left(\frac{1}{z} + \frac{1}{r} - 1\right)^2$$

This can be solved to give: $z = \frac{r}{1+r}$. Now, consider the triangle whose vertices are the centers of the circles, excluding the center of the outermost circle with a radius of one. The sides of the triangle are $1, a+r, (1-a+r)$, and the height to the side with a length of one is the $x$ we are seeking. The area of this triangle is one half base times height or $x\frac{1}{2} = \frac{x}{2}$.

Heron's formula states that the area of a triangle with side lengths $p$ and edges $a, b, c$, is equal to $\sqrt{p(p-a)(p-b)(p-c)}$. The semiperimeter $p$, is the sum of the side lengths divided by two, so our triangle has a semi-perimiter of $\frac{1+(a+r)+(1-a+r)}{2}$, or $1+r$. It follows that:

$$A = \sqrt{(1+r)r(1-a)a}$$

Since $a(1-a) = z = \frac{r}{1+r}$,

$$A = \sqrt{r^2} = r$$

From earlier it was shown that $A = x/2$ so $x = 2r$.

**Question 8 (Khovanova, 2011)** Is it possible to put an equilateral triangle onto a square grid so that all the vertices of the triangle correspond to vertices of the grid?

**Notes** This solution is not as tricky or difficult as some others, but the solution relies on considering the parity of a number and tracing this parity through some calculations. This strategy is not generally emphasized in high school math.

**Solution** Set one of the triangles vertices at the point $(0, 0)$. Consider the other two vertices to be $(a, b)$ and $(c, d)$. It can be assumed that at least one of the numbers is odd, because otherwise the triangle could be reduced. Let $a$ be odd. The square of the length from the origin to $(a, b)$ is $a^2 + b^2$. Consider two separate cases.

Fist consider $b$ to be odd, then the square of the edge length takes on the form $(2n+1)^2 + (2m+1)^2$ which reduces to the form: $4k+2$. Since this triangle is equilateral, $a^2 + b^2 = c^2 + d^2$. This shows that both $c$ and $d$ must be odd. But the square of the length of the third side equals $(a-c)^2 + (b-d)^2$ which is divisible by 4. So this side does not equal the other of the form $4k+2$ and the triangle is not equilateral.

Next, consider $b$ to be even, then the square of the edge length takes on the form $(2n+1)^2 + (2m)^2$ which reduces to the form: $4k+1$. As before, $a^2 + b^2 = c^2 + d^2$.

This shows that $c$ and $d$ must be one even and one odd. But the square of the length of the third side equals $(a-c)^2+(b-d)^2$ which is even. This fact contradicts with the fact that the square of the first side length is of the form $4k+1$. So this case also fails.

**Question 9 (Vardi, 2000)** Can a cube be inscribed in a cone so that 7 vertices of the cube lie on the cone?

**Notes** This problem requires a good three dimensional imagination and then firm grasp of conic sections. Then, even if a student can convince themselves of the solution, the general proof of the situation requires some algebraic juggling. It is very easy to make simple mistakes along the way.

**Solution** A cube has 8 vertices. So if a cube could be inscribed in a cone with 7 vertices on a cone, that means that there would be a face $ABCD$ so that each corner touch the cone and an opposite face $EFGH$ so that at least three of the vertices touched the cone. The face $ABCD$ lives on a plane which cuts the cone into a conic section, either a hyperbola, parabola, ellipse or two intersecting lines. An ellipse is the only option which can circumscribe a square at all four vertices. Call this ellipse $E_1$. The opposite face $EFGH$ is parallel to $ABCD$, therefore its conic section is also an ellipse. Call this ellipse $E_2$.

It is defined that face $ABCD$ touches $E_1$ at four points and that the sides of $ABCD$ are parallel to the major and minor axes of $E_1$. Since $ABCD$ and $EFGH$ are parallel and $E_1$ and $E_2$ are parallel, it is implied that the edges of face $EFGH$ are parallel to the major and minor axes of $E_2$. This means that $E_2$ can intersect the vertices of $EFGH$ at 0, 2 or 4 points. If 4 is chosen, it implies that the ellipses are equal, which is impossible inside of a cone. The only other possibility is that $ABCD$ is not parallel to the major and minor access, which also cannot be.

This can be proven without loss of generality as follows: Consider an ellipse $x^2+\frac{y^2}{a^2}=1, a>0$, and a line $y=mx+b, m\neq 0$ If these intersect at $(x,y)$, then by solving both the line and the ellipse for $y^2$ and setting the expression to zero, one finds: $(a^2+m^2)x^2+2mbx+b^2-a^2=0$. This is the in the form of a quadratic, so the quadratic formula can be employed:

$$\frac{-2mb\pm\sqrt{(2mb)^2-4(a^2+m^2)(b^2-a^2)}}{2(a^2+b^2)}$$

And simplified,

$$\frac{-mb\pm\sqrt{(mb)^2-(a^2+m^2)(b^2-a^2)}}{(a^2+m^2)}$$

$$\frac{-mb \pm \sqrt{m^2b^2 - (a^2b^2 + m^2b^2 - a^4 - a^2m^2)}}{(a^2 + m^2)}$$

$$\frac{-mb \pm a\sqrt{a^2 + m^2 - b^2}}{(a^2 + m^2)}$$

This gives the $x$ values of the two points of intersection. A similar method can be employed to find the $y$ values at the two points of intersection. So the points of intersection are:

$$I = \left( \frac{-mb - a\sqrt{a^2 + m^2 - b^2}}{a^2 + m^2}, \frac{ba^2 - am\sqrt{a^2 + m^2 - b^2}}{a^2 + m^2} \right)$$

$$J = \left( \frac{-mb + a\sqrt{a^2 + m^2 - b^2}}{a^2 + m^2}, \frac{ba^2 + am\sqrt{a^2 + m^2 - b^2}}{a^2 + m^2} \right)$$

And using the distance formula, the length of the segment $IJ$ is found to be:

$$\frac{2a\sqrt{(1 + m^2)(a^2 + m^2 - b^2)}}{a^2 + m^2}$$

Investigating this result, it can be seen that the only way to get two equal chords is by using the lines $y = mx + b$ and $y = mx - b$. The slope $m$ must remain the same because the opposite edges of $ABCD$ are parallel. The sign of $a$ cannot be changed without changing the sign of the length of $IJ$. So this leaves $b$, whose sign can change without changing the length of $IJ$. The resulting two intersection points are:

$$K = \left( \frac{mb - a\sqrt{a^2 + m^2 - b^2}}{a^2 + m^2}, \frac{-ba^2 - am\sqrt{a^2 + m^2 - b^2}}{a^2 + m^2} \right)$$

$$L = \left( \frac{mb + a\sqrt{a^2 + m^2 - b^2}}{a^2 + m^2}, \frac{-ba^2 + am\sqrt{a^2 + m^2 - b^2}}{a^2 + m^2} \right)$$

If these 4 points lie on a square, then the slope $LJ$ must be $-1/m$ (the opposite reciprocal of the slope of $KL$). Since

$$L - J = \left( \frac{2mb}{a^2 + m^2}, \frac{-2ba^2}{a^2 + m^2} \right)$$

the slope is $-a^2/m$. This implies that $a = 1$, which implies that the ellipse is a circle. This contradicts the assumption that the ellipse circumscribes the square asymmetrically, and the situation is proved to be impossible.

## Conclusion

*"Mathematical audiences (not only in the West) will find it interesting to learn some details and solve the little problems that a [high] school graduate was supposed to solve in a few minutes"*(Vershik, 1994)

The Questions above are just a small sample of many challenging questions. Each one just as unreasonable to ask a high school graduate in the setting of an oral exam. Beyond the difficulty of the content, the true story is the intent of the examiners.

This particular story of discrimination has a happy albeit anti-climactic ending. Shen(1994) cites the 1988 policy of Perestroika as the impetus for reform at MGU. Before Perestroika, serious complaints against the math department could be deemed “anti-Soviet activity” and effectively silenced. Kanevskii and Senderov, two of people most active in trying to shed light on the discrimination at MGU, were arrested for such anti-Soviet agitation. After Perestroika, the issue was brought to the table and discussed openly. This led to the one student being allowed to retake the entrance exam, and general reform in the examination practices. Blatant discrimination by the entrance examiners ended and the controversy with it.

Implicit in this story of bigoted examiners oppressing Jewish students, are the side actors who witnessed the injustice and did little to nothing to stop it. Shen admits to being one of these actors guilty of complacency. In his article he recognizes that his opinions and actions at the time were "largely from cowardice." Although he did take moderate action, it is implied that he wishes he had done more. This chapter in history should serve as a warning and a reminder for each of us to speak out against discrimination.

## References


[1] . Dodys, A. Sahai, 2003, *Solution to Problem 21*, www.tanyakhovanova.com/Coffins/sol21

[2] E. Frenkel, (2013), *"Love and Math, The Heart of Hidden Reality"*, Basic Books, pp.28-39.

[3] C. J. H. Hayes, P. T. Moon, J. W. Wayland, *World History*,The Macmillan Company, New York, 1932 pp. 721

[4] V.I. Lenin, (1919), *«О погромной травле евреев» "On Anti-Jewish Pogroms"*, speech.

[5] B.I. Kanevskii, V.A. Senderov, (1980), *Intellectual Genocide*, Moscow, Samizdat.

[6] G. B. Kolata, 1978, *Anti-Semitism Alleged in Soviet Mathematics*, Science, New Series, Vol. 202, No. 4373, pp. 1167-1170

[7] T. Khovanova, A. Radul, (2011), *Jewish Problems*, arXiv:1110.1556v1.

[8] L. S. Pontryagin, 1979, *"Soviet Anti-Semitism: Reply by Pontryagin"*, Science, New Series, Vol. 205, No. 4411, pp. 1083-1084

[9] A. Rambaud, E. Saltus, (1898),*Russia*, vol. 2, Peter Felelon Collier, New York, pp. 76-79.

[10] M. Saul, (1999), *Kerosinka: An Episode in the History of Soviet Mathematics*, Notices of the AMS, vol. 11.

[11] A. Shen, (1994), *Entrance Examinations to the Mekh-mat*, Mathematical Intelligencer, vol. 16, pp. 6-10

[12] I. Vardi, (2000), *Mekh-Mat Entrance Examinations Problems*, www.ihes.fr/ilan/mekh-mat.ps.

[13] A. Vershik, (1994), *Admission to the Mathematics Faculty in Russia in the 1970's and 1980's*, Mathematical Intelligencer, vol. 16, pp.4-5

[14] J. C. Lagarias, C. L. Mallows, A. R. Wilks, (2002), *Beyond the Descartes Circle Theorem*, The American Mathematical Monthly 109 (4): 338–361.