

12-2014

Reasoning-and-Proving Within Ireland's Reform-Oriented National Syllabi

Jon Davis

Follow this and additional works at: <https://scholarworks.umt.edu/tme>



Part of the [Mathematics Commons](#)

Let us know how access to this document benefits you.

Recommended Citation

Davis, Jon (2014) "Reasoning-and-Proving Within Ireland's Reform-Oriented National Syllabi," *The Mathematics Enthusiast*: Vol. 11 : No. 3 , Article 12.

Available at: <https://scholarworks.umt.edu/tme/vol11/iss3/12>

This Article is brought to you for free and open access by ScholarWorks at University of Montana. It has been accepted for inclusion in The Mathematics Enthusiast by an authorized editor of ScholarWorks at University of Montana. For more information, please contact scholarworks@mso.umt.edu.

Reasoning-and-Proving Within Ireland's Reform-Oriented National Syllabi

Jon D. Davis¹
Western Michigan University

Abstract

As educational systems around the world attempt to reform their mathematics programs to increase students' opportunities to engage in processes central to the practice of mathematics such as proof, it is important to understand how this mathematical act is portrayed in national curriculum documents that drive that change. This study examined the presence of reasoning-and-proving (RP) in Ireland's national reform-oriented secondary syllabi for junior cycle (ages 12-15) and senior cycle (ages 15-18) students. The analyses reveal that there were no differences among *direct* and *indirect* RP learning outcomes within each syllabus, but statistically significant differences did exist across syllabi in these categories. Students were provided with statistically different opportunities to engage in pattern identification, conjecture formulation, and argument construction in both syllabi. There were significantly fewer opportunities to engage in conjecture formulation for junior cycle students and significantly more opportunities to construct arguments for senior cycle students. There were no instances of proof as falsification across both syllabi, but students were given similar opportunities to experience proof as explanation, verification, and generation of new knowledge. Across both syllabi there were statistically significantly more RP learning outcomes that were divorced from content than those that were connected to content. The results as well as the implications of these results for the design of national curriculum documents are discussed.

Keywords: pattern, conjecture, proof, standards, reform

Introduction

Mathematicians have argued that proof is the material with which mathematical structures are constructed (Schoenfeld, 2009). Proof is also becoming instantiated as an important component through which one learns school mathematics (Common Core State Standards Initiative [CCSSI], 2010; Epp, 1998; Hanna, 2000; Martin et al., 2009; National Council of Teachers of Mathematics [NCTM], 2000). Due to the acknowledgement of proof as

¹ jon.davis@wmich.edu

important in the practice and learning of mathematics researchers are beginning to analyze this practice (Hanna & de Bruyn, 1999) or practices related to it such as reasoning-and-proving (RP) (Davis, Smith, Roy, & Bilgic, 2013; Stylianides, 2009) in textbooks. While analyses of standards at the state level in the United States for reasoning (Kim & Kasmer, 2006) or for conjecturing and proving (Porter, McMaken, Hwang, and Yang, 2011) have been conducted, we know little about the standards in other countries with regard to proof or its related actions of pattern identification or conjecture formulation. National standards play an important role in shaping classroom practices in the United States (Cogan, Schmidt, & Houang, 2013), Ireland (National Council for Curriculum and Assessment [NCCA], 2012), and other countries (Eurydice, 2011). The curriculum documents at the center of this study are two national syllabi designed to describe the learning expectations for students ages 12-18 studying mathematics in Ireland. These frameworks have been recently developed to drive a nation-wide reform of the Irish secondary mathematics system. This study introduces readers to a framework and methodology for examining RP in national curriculum documents and addresses the dearth of research of this type by enumerating the nature of RP within these two documents. The analysis of these documents for RP expands our knowledge of the nature of this important process in national curriculum documents and adds to our understanding of the potential effectiveness of RP in this reform. More broadly, this paper makes suggestions for how RP can be more interwoven into curriculum frameworks.

Background

Centrality of Proof-Related Constructs in Mathematics and Mathematics Education

Mathematicians have pointed out that the act of constructing proofs is essential to the practice of mathematics (Ross, 1998) or as Schoenfeld (2009) has stated, “If problem-solving

is the 'heart of mathematics', then proof is its soul (p. xii). National curriculum documents in the United States emphasize the centrality of proof in the learning of mathematics.

Specifically, the *Principles and Standards for School Mathematics* (PSSM) (NCTM, 2000), which has driven reform in the United States for over a decade breaks down the instruction of mathematics into five content areas and five processes, one of which is reasoning and proof. The Common Core State Standards for Mathematics (CCSSM) (CCSSI, 2010) begins their document with eight standards for mathematical practice, which they argue should be present as students engage in the learning of mathematics. The third standard advocates for students' construction of mathematical arguments or proof as well as the critiquing of arguments constructed by others. Other countries have also emphasized the importance of proof in the instruction of mathematics. For example, each of the syllabi documents produced by the NCCA in Ireland break mathematics content down into five different mathematics content strands. At the end of each of these content strands is a topic with the title: synthesis and problem solving skills. This topic includes the identification of patterns, formulation of conjectures, and explanation/justification of assertions. These three actions comprise the related processes of reasoning-and-proving as defined by Stylianides (2009).

Ireland's Secondary Educational System

The secondary educational system in Ireland consists of three components. The first component is called the junior cycle and lasts three years. At the end of the junior cycle mathematics students are required to complete an examination in one of three different levels of difficulty. The lowest level is foundation. The next highest level is ordinary and the most difficult level is higher. After completion of the junior cycle many schools provide students with an optional transition year for students. After the optional transition year students begin

the two-year senior cycle. At the completion of the senior cycle, students can opt to take one of three different levels of mathematics examinations: foundation, ordinary, or higher.

Mathematics at both the junior and senior levels consists of five different strands: Strand 1 – Statistics and Probability; Strand 2 – Geometry and Trigonometry; Strand 3 – Number; Strand 4 – Algebra; and Strand 5 – Functions.

The NCCA developed the syllabus for Junior Cycle students and the syllabus for Senior Cycle students describing the content and methods of the Project Maths reform. Both the syllabus at the Junior Cycle and Senior Cycle level refer to individual standards using the words *learning outcomes* and this terminology is used to refer to them throughout the paper. The reform of the secondary mathematics program in Ireland, Project Maths, began with 24 pilot schools in September 2008. These schools not only implemented the Project Maths curriculum, but also helped to revise the syllabi. Project Maths was designed to address high failure rates in mathematics for ordinary level students, low participation rates in the higher level Leaving Certificate program, a lack of conceptual understanding, and the difficulty students encountered in trying to use mathematical concepts in real-world contexts (Department of Education and Skills, 2010). In September 2010 the program was gradually implemented across Ireland beginning with the statistics/probability content strand. The last content strand to be implemented will be functions and will occur at the junior and senior levels in September 2012.

In the *Junior Certificate Mathematics Syllabus: 2015 Examination (JC Syllabus)* (NCCA, n.d.), students at the foundation level are expected to understand the same learning outcomes as ordinary level students. Higher level students are expected to understand all of the learning outcomes listed for ordinary and foundation level students as well as additional learning

outcomes identified in bold throughout the syllabus. In the *Leaving Certificate Mathematics Syllabus: Foundation, Ordinary & Higher Level: 2014 Examination* (hereafter referred to as the LC Syllabus) (NCCA, n.d.), learning outcomes are listed separately for foundation, ordinary, and higher level students. Foundation level students are expected to learn only those outcomes listed within this level. Ordinary level students are expected learn the outcomes for foundation and ordinary level. Higher level students are expected to learn the outcomes for foundation and ordinary as well as those listed within the higher level.

Consequently, higher level students are expected to learn more content than foundation and ordinary level students. For example, within the LC Syllabus, students at the foundation level are expected to learn how to complete three geometric constructions. Students at the ordinary level are expected to learn the three geometric constructions at the foundation level as well as three more. Students at the higher level must learn the six constructions at the foundation and ordinary levels as well as sixteen more constructions. There are also differences across the levels in terms of the complexity of the learning outcomes that students are expected to learn. For instance, within the LC Syllabus students at the foundation level are expected to be able to apply the theorem of Pythagoras. Ordinary level students are expected to solve problems involving sine and cosine rules in two dimensions in addition to using the theorem of Pythagoras. Higher level students are asked to use trigonometry to solve problems in three dimensions in addition to the earlier described learning outcomes at the foundation and ordinary levels. After completing the LC Syllabus, 22.1% (11,131) of students opted to take the higher level examination, 67.2% (37,506) of students opted to take the ordinary level examination, and 12.4% (6,249) of students opted to take the foundation level examination (Reilly, 2012).

Importance of Syllabi Documents in Irish Mathematics Classrooms

The State Examination Commission in Ireland creates and administers the examinations at both the junior certificate and leaving certificate levels. Students who elect to take the foundation level Junior Certificate Examination complete one assessment while students at the ordinary and higher levels each take two assessments. Students at the foundation, ordinary, and higher levels will each take two Leaving Certificate examinations. The junior certificate examination has all of the characteristics of a high stakes examination as students report an increase in homework demands during the third year of junior cycle focusing on the junior certificate examination, one-quarter of students enroll in private tutoring outside of school to prepare them for this exam, and students' performance on the junior certificate examination influences the levels of courses (e.g., ordinary vs. higher) in which they enroll during the senior cycle (Smyth, 2009). At the end of the senior cycle, students receive points based upon the score and level of the leaving certificate test that they take. These points are used to determine students' eligibility to enroll in different university programs. Universities in Ireland publish points associated with academic programs. These values represent the minimum number of points needed on leaving certificate exams in order to apply to these programs. In high demand programs students who achieve the minimum number of points may not be accepted into the program. Consequently, the leaving certificate examinations also hold high stakes for students.

Both the junior certificate and leaving certificate examinations are based upon content as delineated within syllabi developed by the NCCA. While professional development is being conducted in Ireland to help teachers understand the content and teaching approach

associated with Project Maths and textbooks now exist which purport to contain Project Maths content, the high stakes exams that students take at the junior and senior cycles are based upon syllabi documents produced by the NCCA. Accordingly, this study examines these documents for the presence of RP as they have an important influence on the nature of instruction in secondary mathematics classrooms in Ireland.

Proof-Related Constructs in Mathematics Textbooks

A variety of studies have examined what I describe as proof-related constructs. These constructs include the following: development or discussion of arguments that mathematicians would consider valid proofs (Davis et al., 2013; Stacey & Vincent, 2009; Stylianides, 2005, 2009; Thompson, Senk, & Johnson, 2012); identification of patterns and development of conjectures (Davis et al., 2013; Stylianides, 2009); modes of reasoning (Stacey & Vincent, 2009); and proof-related reasoning consisting of making and testing conjectures, developing and evaluating deductive type arguments, locating counterexamples, correcting mistakes in arguments, creating specific and general arguments (Thompson et al.). Studies conducted on secondary mathematics textbooks in different countries suggest that students' opportunities to engage in proof related constructs are limited. For instance, Stylianides (2009) found that only 5% of 4578 tasks appearing in a secondary mathematics program for students ages 11-14 in the United States asked students to construct valid arguments. Similar to homework exercises, students are provided with infrequent opportunities to read about valid mathematical arguments. By way of example, Thompson et al. assumed that mathematical properties needed to be justified and found that less than half of these mathematics building blocks appearing within the topics of exponents, logarithms and polynomials in 22 different high school mathematics texts were justified with valid

proofs.

Reasoning and Proof in U.S. State and National Standards

Kim and Kasmer (2006) examined reasoning in 35 state curriculum frameworks in the United States from kindergarten through eighth grade. They found that 22 states contained a reasoning section or specific statements that reasoning should appear across all content strands. They also found inconsistencies in messages addressing reasoning within state frameworks. For example, they found that reasoning appeared infrequently at the primary level and was not consistent across different mathematics content strands. Some curriculum frameworks contained reasoning in a general sense that was separated from specific mathematics content. They also found that state frameworks contained inappropriate examples. That is, examples designed to represent reasoning focused on mathematical procedures. Some state curriculum frameworks lacked alignment between benchmarks or what students were expected to know at a certain grade level and their corresponding performance indicators. Oftentimes one of these curriculum components contained reasoning while the other did not.

Kim and Kasmer (2006) also examined the prevalence of different words associated with reasoning in the state curriculum frameworks. The word “prediction” was found in many state frameworks but was most prevalent in data analysis and probability. “Generalization” appeared most frequently in the algebra content strand. “Verification” appeared primarily in two mathematics content strands: Geometry and Number and Operations. More of these states reserved “verification” to the upper elementary grades. The word “conjecture” appeared in a little over half of the 35 states and primarily at grades 5-8. This action was primarily concentrated within Geometry and Data Analysis/Probability content strands. The

words “develop arguments” appeared in less than half of the state curriculum frameworks and predominantly in the Data Analysis and Probability strand.

The U.S. has traditionally been a decentralized curricular system with a variety of curricular frameworks at the state level and textbooks selected by entities that consist of several K-12 schools or individual schools (Dossey, Halvorsen & McCrone, 2008). However, this may now change with the advent of the Common Core State Standards in English language arts and mathematics, which has been adopted by 45 states in the U.S. Porter et al. (2011) examined the alignment among 27 state frameworks, *Principles and Standards for School Mathematics* (PSSM) (National Council of Teachers of Mathematics [NCTM], 2000), and the Common Core State Standards for School Mathematics (CCSSM) (Common Core State Standards Initiative [CCSSI], 2010). This study connects with the research conducted here since one of the categories of cognitive demand is labeled “Conjecture, generalize, prove.” They found that 7.78% of the learning outcomes across the 27 state frameworks and 5.96% of the standards appearing in the Common Core State Standards for Mathematics involved conjecturing, generalizing, or proving. These percentages seem low given the centrality of these practices to mathematics.

Research Questions

In summary, previous research suggests that students are provided with infrequent opportunities to engage in tasks or read text involving proof-related constructs within school mathematics textbooks in the United States as well as other countries. This finding is echoed in state curriculum frameworks and in the current Common Core State Standards for Mathematics. In curriculum frameworks in the U.S., RP appears in a variety of different guises such as prediction, verification, generalization, etc. The majority of state curriculum

frameworks contain either a specific reasoning section or general statements that reasoning should appear throughout the documents. However, these state frameworks also contained the following negative features with regard to RP: differential attention to reasoning across mathematics content strands, inconsistent messages being sent to teachers in different components of the frameworks, and the separation of reasoning from content. The study described in this paper builds on these previous studies with regard to national curriculum documents by using similar methodology, but with a slightly different framework. A total of four research questions guided this study. First, are there statistically significant differences in the frequency of RP learning outcomes by different content strands within each syllabus or across syllabi by student learning level? Second, are there statistically significant differences in the frequencies of mathematical ideas categorized as pattern identification, conjecture formulation, or argument construction by student learning level within each syllabus? Third, are there statistically significant differences in the purposes of proof by learning level within each syllabus? Fourth, are there statistically significant differences in the frequency of content and non-content related RP by student learning level within each syllabus?

Framework

As Stylianides (2005) has pointed out, while different researchers have defined reasoning in a variety of ways, these definitions contain a common thread, proof. Indeed, a recent interpretation of a U.S. national standards document (Martin et al., 2009) defines reasoning as encompassing “proof in which conclusions are logically deduced from assumptions and definitions” (p. 4). Moreover, reasoning can consist of different levels of formality (NCTM, 2000, 2009) and be connected to different mathematics content areas such as algebra (Walkington, Petrosino, & Sherman, 2013) or mathematical ideas such as proportion

(Jitendra, Star, Dupuis, & Rodriguez, 2013). Stylianides (2005) defined the term reasoning-and-proving to consist of four potentially interconnected actions: pattern identification; conjecture formulation; developing non-proof arguments; and creating proofs. Support for Stylianides' decision to connect pattern identification and conjecture development to reasoning and proof come from national standards documents produced by NCTM (1989, 2000). The hyphens within this terminology denote two meanings. First, they signify that these actions can be integrated with one another. Second, they suggest that reasoning is connected to the development of proofs as opposed to other types of reasoning as described above. Stylianides developed an analytic framework for analyzing reasoning-and-proving opportunities in school mathematics textbooks. Several features associated with the learning outcomes appearing in standards documents suggest the need for changes to Stylianides' analytic framework.

First, learning outcomes associated with standards documents come in differing levels of specificity (McCallum, 2012). As a result, learning outcomes require interpretation on the part of users and the analyses presented in this paper indicate *potential* RP processes. In addition, this feature has been taken into account in the framework through the creation of *direct* and *indirect* RP processes. *Direct* RP processes are defined as actions involving pattern identification, conjecture formulation, and/or argument construction as indicated within learning outcomes by the appearance of words tightly connected to these processes (e.g., *pattern*) coupled with a context that an expert would recognize as indicating evidence of these processes within a mathematical community of practice. *Direct* RP processes were used for identifying one or more of the RP categories within fine grain or narrowly specified standards document learning outcomes. For example, in both the JC Syllabus and the LC Syllabus a learning

outcome states that students should be able to formulate conjectures. Due to the close connection of this learning outcome to the RP framework through the word *conjecture* as well as the fact that this phrase appears within mathematics content standards suggests the presence of *direct* RP processes.

Indirect RP processes are defined as actions involving pattern identification, conjecture formulation, and/or argument construction as indicated within learning outcomes by the appearance of words loosely connected to one or more of these processes (e.g., *investigate*) coupled with a context that an expert would recognize as indicating evidence of these processes within a mathematical community of practice. Consider the following learning outcome appearing in the LC Syllabus: investigate theorems **7, 8, 11, 12, 13, 16, 17, 18, 20, 21** and **corollary 6** (NCCA, n.d., p. 22). Readers of this document could interpret the word “investigate” in several different ways. That is, the investigation could consist of a tightly scripted set of steps that students are asked complete that does not include pattern identification, conjecture formulation, or argument construction. However, others could interpret “investigate” to include one or more of these RP processes. Thus learning outcomes that contained words such as investigate as well as a context as described above were coded as involving *indirect* RP processes. Additionally, the lack of specificity of some learning outcomes required the creation of methods for determining the frequencies associated with different components of the RP framework as described later within the methodology section.

Second, as standards documents will not contain statements asking students to construct non-proof arguments this component of the Stylianides’ framework was removed for this study. Third, as standards documents do not typically contain specific examples of student problems, in contrast with school mathematics textbooks, it was not possible to

discern plausible from definite patterns or generic examples from demonstrations. In the case of the former plausible and definite patterns were collapsed into pattern identification and in the case of the latter generic examples and demonstrated were collapsed into argument construction. This lack of specific mathematical problems also necessitated the removal of pattern purposes and conjecture purposes from Stylianides' framework. It was posited that words appearing in learning outcomes associated with the development of arguments could be used to determine the purposes associated with a proof. The analytic framework used in this study is shown in Figure 1.

Students may engage in pattern identification, conjecture formulation, and argument construction separately or in conjunction with one another as noted by the dashed arrows in Figure 1. For instance, learning outcomes may expect students to identify a pattern without constructing a conjecture or developing an argument. Other learning outcomes may expect students to engage in two (e.g., identification of a pattern followed by the construction of a conjecture) or all three of the framework components. If only one of the three components of the framework appeared within the syllabi documents, this was still considered an instance of reasoning-and-proving.

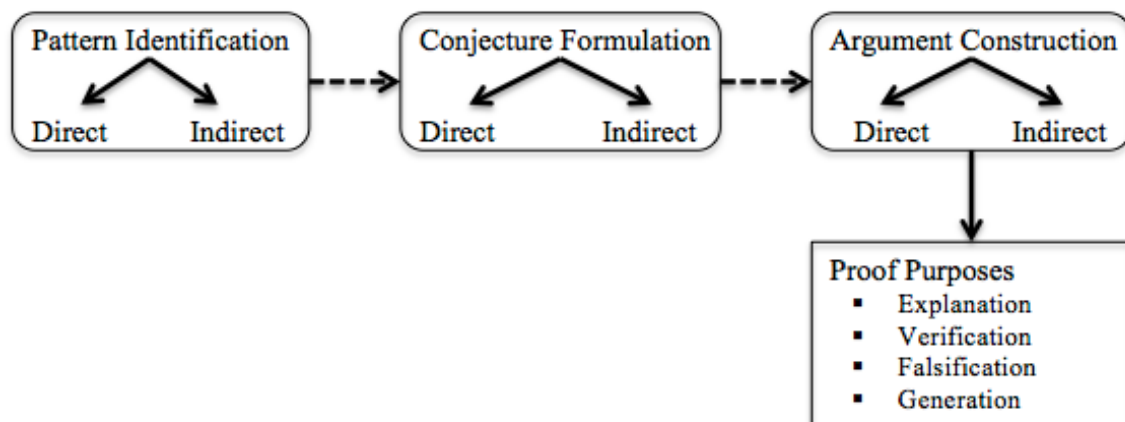


Figure 1. Framework for analyzing reasoning-and-proving in syllabi documents.

Identification of patterns was defined as the act of locating a key feature or key features in a set of data existing in a variety of different forms that one has not encountered before and for which a procedure has not been previously introduced. Conjecturing consists of the development of a reasoned hypothesis extending beyond a particular set of data existing in different representational forms and expressed with uncertainty as to its validity. Argument construction involved the creation of valid proofs, which consist of a set of accepted statements, modes of argumentation, and modes of argument representation (Stylianides, 2007).

Learning outcomes that were coded as argument construction were later categorized in terms of the purposes that these proofs served. In analyzing the work of De Villiers (1990, 1999) and others (e.g., Hanna, 1990), Stylianides (2005) described four different purposes of proof that can be coded in curriculum materials: explanation; verification; falsification; and generation of new knowledge. Explanation denotes why a particular assertion is valid. Verification establishes the truth of a particular assertion. Falsification shows that a particular assertion is false. Generation of new knowledge occurs when a proof develops knowledge that was not previously known by a particular group of individuals.

Methodology

Units of Analysis

Electronic copies of the JC Syllabus and the LC Syllabus were examined for instances of RP. These documents describe the Project Maths reform at the junior certificate and leaving certificate levels, respectively. The four research questions described above necessitated two phases of analysis. The first phase involved enumerating the number of learning outcomes

categorized as containing direct and indirect RP processes. The second phase involved identifying and categorizing learning outcomes as involving pattern identification, conjecture formulation, or argument construction. Electronic versions of the JC Syllabus and the LC Syllabus were the sources used for both phases of the analysis. Each document contains five different mathematics content strands with the learning outcomes in each strand appearing in a matrix format as seen in Figure 2.

Strand 1: Statistics and Probability

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
1.1 Counting	<ul style="list-style-type: none"> – list outcomes of an experiment – apply the fundamental principle of counting 	<ul style="list-style-type: none"> – count the arrangements of n distinct objects ($n!$) – count the number of ways of arranging r objects from n distinct objects 	<ul style="list-style-type: none"> – count the number of ways of selecting r objects from n distinct objects

Figure 2. Excerpt from the LC Syllabus (NCCA, n.d., p. 17).

The first column represents a general mathematics content area with the subsequent columns representing learning outcomes associated with the general mathematics content area differentiated by student learning level. A unit of analysis needed to be defined in order to enumerate the RP learning outcomes appearing in both documents. Since both documents contained the same structure shown in Figure 2, the phrase following a dash was defined as a learning outcome and hence became the unit of analysis for the first phase of the study.

The second phase of the study involved distinguishing among different components of the RP framework (e.g., pattern identification) embedded within a specific learning outcome. Analyses of both the JC Syllabus and the LC Syllabus suggested that one or more mathematical ideas could appear within what was defined as a learning outcome. A mathematical idea was

identified as the set of words separated by commas, plural forms, or by conjunctions such as *and*. For instance, consider the following learning outcome appearing in the LC Syllabus: prove theorems 11, 12, 13 concerning ratios (NCCA, n.d., p. 22). While this was considered to be one learning outcome it was composed of three related mathematical ideas involving argument construction.

Coding

Kim and Kasmer (2006) employed a methodology whereby words associated with reasoning (e.g., *predict*) were located and used as evidence that students were expected to engage in reasoning. In this study, a similar process was used to locate learning outcomes that indicated the *potential* for students to engage in RP. Table 1 lists the words that suggested but did not determine direct and indirect RP processes. Recall that the context within which these words appeared also needed to be evaluated to finally categorize learning outcomes as either direct or indirect RP processes.

Table 1

Words Linking Potential Direct and Indirect RP Processes and RP Framework Components

Word	RP Framework
Direct	
<i>Pattern</i>	Pattern Identification
<i>Conjecture, Guess, Hypothesis, Predict</i>	Conjecture Formulation
<i>Explain, Argument, Prove, Proving, Proof, Justify, Show,</i> <i>Generalize, Generate Rules, Derive, Disprove,</i> <i>Counterexample</i>	Argument Construction
Indirect	
<i>Describe, Interpret</i>	Pattern Identification
<i>Evaluate, Verify, Analyze</i>	Argument Construction
<i>Explore</i>	Pattern Identification, Conjecture Formulation
<i>Investigate</i>	Pattern Identification, Conjecture Formulation, Argument Construction
<i>Draw Conclusions</i>	Argument Construction

As can be seen from Table 1, *direct* RP processes consisted of words that either appeared in the RP framework (*pattern, conjecture, argument*) or were closely connected to these components (e.g., *predict*). Table 1 also lists the words associated with *indirect* RP

processes. These words were less tightly connected to the three main RP categories and hence could be interpreted in a variety of different ways by teachers and students. For instance, the word *investigate* is defined in the following manner: to examine, study, or inquire into systematically; search or examine into the particulars of; examine in detail (dictionary.reference.com).

The words *search* or *examine* contain the potential for components of the RP framework such as looking at a set of data for a pattern or patterns to exist. Thus the appearance of words within the syllabi documents that suggested processes similar to the identification of patterns, formulation of conjectures, and development of arguments were also used as potential evidence of components of the RP framework.

The word *explore* was used to potentially indicate pattern detection and conjecture formulation, but not argument as the word did not necessarily denote the location and solidification of mathematical ideas. Similar to direct instances of RP, the context in which words denoting indirect RP were used was taken into consideration to determine if the learning outcome was indeed connected to RP. Consider the following learning outcome from the JC Syllabus: explore the properties of points, lines and line segments including the equation of a line (NCCA, n.d., p. 20). This instance of the word *explore* in this example would account for pattern identification and conjecture formulation. Moreover, since *explore* is used with respect to four different mathematical ideas (properties of points, lines, line segments, and the equation of a line) four instances of pattern detection and four instances of conjecture formulation would be enumerated for this single learning outcome. However, no argument construction instances would be coded here as *explore* was not considered to encompass this component of the RP framework.

Given the definition of the word *investigate* included above it held the potential to involve the identification of patterns and construction of conjectures. In addition, it was assumed that students engaged in an investigation would locate a mathematical idea. That is, there would be an endpoint at which the investigation would be completed. This suggested that students involved in an investigation would also be asked to construct a valid argument showing that the mathematical idea they located and conjectured actually existed.

The words *draw conclusions* held the potential to indicate argument development, but not pattern identification or conjecture formulation. The location of other words in the syllabi documents potentially indicated the presence of pattern identification and argument construction. The words *interpret* and *describe* were used to potentially indicate identification of patterns. The following words were used as potential evidence of the construction of valid arguments: *evaluate*, *verify*, *analyze*, and *develop*.

Both syllabi were examined for presence of the words appearing in Table 1. Once a word appearing in the table was identified, the rest of the learning outcome associated with this word was considered the context associated with this word. The word was potentially connected with one or more RP categories as indicated in Table 1. The context was examined to determine if there was agreement between it and the RP category definitions associated with that word. The learning outcome was categorized as RP-based if there was no aspect of the context that disagreed with the RP category definitions and the context could be interpreted as involving one or more of the RP categories as determined by the main coder. This process is illustrated in the following example. Students at the leaving certificate level are asked to: “use the following terms related to logic and deductive reasoning: theorem, proof, axiom, corollary, converse, implies” (p. 22). The presence of the word *proof* here

suggests the potential for a *direct* RP process and subsequently an RP-based learning outcome, yet the context involving the words *use the following terms* suggests that students are not required to develop a proof.

Determining frequency of occurrences. Recall that in the first phase of the study the information after a dash in the learning outcomes indicated one instance. In the second phase of the study each learning outcome was broken down into mathematical ideas that were examined for one or more of the three RP categories. In some cases, the mathematical ideas were specifically listed within the learning outcome, resulting in a straightforward determination of the number of occurrences of that particular idea. For instance, in the JC Syllabus the following learning outcome appears: explore the properties of points, lines and line segments including the equation of the line (NCCA, n.d., p. 20). The word *explore* suggests the presence of *indirect* RP processes, but the language here illustrates that four different mathematical ideas are involved: points, lines, line segments, and the equation of the line. Other learning outcomes used plural forms. For example, the following learning outcome appears in the LC Syllabus: generate rules/formulae from those patterns (NCCA, n.d., p. 25). In this example, plural forms (*rules/formulae*) are used and since the exact number was not described in the syllabus it was counted as two instances of argument construction. Whenever plural forms were used in RP-based learning outcomes, these were counted as two mathematical ideas.

Identifying proof purposes. Table 2 shows how words associated with argument construction in analyses of the syllabi documents were connected to the four proof purpose categories described in the framework. The word *explain* indicated the construction of an argument, the purpose of which was coded as explanation. One definition of the word

analyze is as follows: *To examine carefully and in detail so as to identify causes, key factors, possible results, etc.* (www.dictionary.com). This suggests that an analysis leads to a better understanding of a mathematical idea, which helps to explain why something is the case. Consequently, the word *analyze* was linked to an explanation proof purpose. Words that were more closely associated with the development of a valid argument (e.g., *prove*) were coded as verification as these words were often used in association with some mathematical idea such as in the following learning outcome from the LC Syllabus document: “prove that $\sqrt{2}$ is not rational” (NCCA, n.d., p. 25). Since the statement assumes that $\sqrt{2}$ is not rational, the development of an argument would verify that this is indeed the case and hence would constitute a verification proof purpose.

The presence of words such as *counterexample* or *disprove* suggested that students were expected to show that some specific idea was not true in general leading to a falsification proof purpose. Likewise, if students were asked to determine the validity of some mathematical idea that was not true in general with words such as *determine if _____ is true* this was considered to be a falsification proof purpose. The proof purpose of generation of new knowledge was linked to the following words: *generalize*, *generate rules*, *derive*, *investigate*, and *draw conclusions*. The word, *investigate*, was considered to be involved in the generation of new knowledge as this word suggests that students working with a mathematical idea that they had not previously examined. In some cases there were generic descriptions of RP learning outcomes. For example, the learning outcome involving the words *justify conclusions* within the synthesis and problem-solving skills section in each mathematics content strand involved the development of an argument, but could not be coded for a purpose. Such instances were simply coded as unclear. In Stylianides’ work, each

argument could potentially be coded within multiple proof purpose categories, however, in this study each RP learning outcome was only placed into one category.

Table 2

Words Used to Identify Proof Purposes

Words Indicating Argument Construction	Proof Purpose
<i>Explain, Analyze</i>	Explanation
<i>Argument, Prove, Proving, Proof, Justify, Show, Evaluate, Verify</i>	Verification
<i>Counterexample, Disprove, Determine if _____ is true</i>	Falsification
<i>Generalize, Generate Rules, Derive, Investigate, Draw Conclusions, Develop</i>	Generation

Inter-rater Reliability

The author was the primary coder of both syllabi documents. However, in order to determine the reliability of the framework and the coding system another individual possessing experience with the RP framework read through the framework and methodology descriptions and coded two content strands from the JC Syllabus and two strands from the LC Syllabus for RP-based learning outcomes. This individual coded the Probability and Statistics strand and the Number strand within the JC Syllabus as these two strands contain a range of RP-based learning outcomes. The inter-rater reliability using unweighted Cohen's Kappa for this coding was 0.9276. Landis and Koch (1977) consider these values to represent almost perfect agreement. The Geometry and Trigonometry strand and the Algebra strand were coded within the LC Syllabus. These strands were chosen to provide information about

the reliability of coding RP-based learning outcomes within different mathematics content areas. The inter-rater reliability using unweighted Cohen's Kappa for this strand was 0.7682. This lower value when compared to the JC Syllabus was due to the second coder identifying words associated with the framework without also attending to the context of the learning outcome within which the word was embedded. For instance, the second coder identified the word *interpret* in the algebra content strand to indicate the presence of RP, however, the context of the learning outcome is related to interpreting the results of solving equations considered as functions. Such an action does not indicate the pattern identification as it is described in the framework. While this is less than the inter-rater reliability for the strands coded within the JC Syllabus Landis and Koch still consider this value to denote substantial agreement.

Analysis

For the first phase of the study, learning outcomes coded as *direct* and *indirect* RP processes were enumerated. The total number of learning outcomes appearing in each mathematics content strand was then used with the aforementioned numbers to calculate the number of non-RP learning outcomes. The relationship between content strand and *direct/indirect/non-RP* learning outcomes by student learning level within each syllabi was examined using Pearson's Chi Square and Fisher's Exact Test (Field, 2009).

In the analysis associated with the second phase of the study, *direct* and *indirect* processes were collapsed together as they both involved RP. The percentage of RP learning outcomes that contained pattern identification, conjecture formulation, and argument construction were calculated for each mathematics content strand across both syllabi in the following manner. First, the number of RP-based learning outcomes within each strand at

each student level was enumerated. For example, in the Statistics/Probability strand of the JC Syllabus there were a total of seven RP-based learning outcomes. Second, the number of learning outcomes associated with each RP category was counted. Using the example of the Statistics/Probability strand of the JC Syllabus, three of the seven RP-based learning outcomes involved pattern identification resulting in $3/7 * 100$ or 42.9%. The total number of RP-based learning outcomes providing students with opportunities to identify patterns, formulate conjectures, and construct arguments were enumerated within each learning level across both syllabi. Because mathematical ideas can be coded as one or more of the three RP categories (pattern identification, conjecture formulation, or argument construction) a Cochran Q test, which takes interdependence across categories into account (Conover, 1999) was used to examine if the distribution of mathematical ideas across these three categories within a student learning level and syllabus were statistically significantly different from one another. It was not possible to conduct Chi Square tests on the relationship between student learning level and RP-based learning outcomes within either the JC Syllabus or LC syllabus as these learning outcomes were not independent because upper level learning outcomes subsumed learning outcomes at lower levels, but also included new learning outcomes specific to that level.

Connectedness of RP learning outcomes to content. Each of the learning outcomes that had been coded as involving RP in the process described earlier were further examined to determine if they were content related or not. For instance, the following learning outcome from the Number strand of the JC Syllabus was considered to be content related: “investigate the nets of rectangular solids” (NCCA, n.d., p. 24). An RP learning outcome was judged to be unrelated to content if it did not mention any mathematical content or ideas as seen in the

following RP learning outcome from the Statistics/Probability strand of the LC Syllabus: “decide to what extent conclusions can be generalised [sic]” (NCCA, n.d., p. 18). The percentage of learning outcomes by level and strand that were content-related and not related to content were calculated and compared across strands and syllabi. Fisher’s Exact Test was used to determine if there were differences in the distribution of content and non-content related RP learning outcomes by strand for different student levels.

An $\alpha = 0.05$ level of significance was used for the Pearson’s Chi Square, Fisher Exact, and omnibus Cochran’s Q test. There were a total of three different RP levels resulting in three different comparisons for contrasts between two different RP levels. Contrasts using Cochran’s Q test were examined using an $\alpha = 0.0167$ level of significance. This value comes from a Bonferroni correction to reduce type I error as there are three different comparisons to be made and $.05/3 = .0167$ (Field, 2009).

Results

Indirect, Direct, and Non-RP Learning Outcomes

The relationship between direct/indirect/non-RP learning outcomes and content strand for foundation/ordinary level in the JC Syllabus were not statistically significant, $\chi^2(8) = 8.012, p = .424$. Similar results were found at the higher level in the JC Syllabus between content strand and direct/indirect/non-RP learning outcomes, $\chi^2(8) = 9.575, p = .279$. In the LC Syllabus the relationship between direct/indirect/non-RP learning outcomes and content strands for foundation level, ($\chi^2[8] = 8.875, p = .275$), ordinary level, ($\chi^2[8] = 5.801, p = .667$), and higher level, ($\chi^2[8] = 11.113, p = .164$) were statistically nonsignificant. That is, the distribution of direct, indirect, and non-RP learning outcomes by student learning level was not statistically dissimilar across mathematics content strands for

either the JC or LC Syllabus.

The frequency and percentage of learning outcomes that were categorized as involving *direct*, *indirect*, and non-RP across the foundation/ordinary and higher level for the JC Syllabus and for the LC Syllabus are shown in Table 3 and 4, respectively. In order to make comparisons in these categories across syllabi the learning outcomes at the foundation and ordinary levels in the LC Syllabus were combined. There was a statistically significant relationship between learning outcomes categorized as *direct/indirect/non-RP* and foundation/ordinary level students within the JC Syllabus and the LC Syllabus, $\chi^2(2) = 14.796, p = .001$. A similar situation existed between these categories and higher level students in the JC and LC Syllabi, $\chi^2(2) = 20.637, p < .001$.

Table 3

Direct, Indirect, and Total Learning Outcomes in the JC Syllabus by Level

Foundation/Ordinary			Higher		
T ^a	D ^b	I ^c	T	D	I
141	21	29	172	22	36
	(14.9%)	(20.6%)		(12.8%)	(20.9%)

^a T represents the total number of learning outcomes in this strand and level.

^b D represents the total number of direct RP-based learning outcomes in this strand and level.

^c I represents the total number of indirect RP-based learning outcomes in this strand and level.

Table 4

Direct and Indirect RP Learning Outcomes in the LC Syllabus by Strand and Level

Foundation			Ordinary			Higher		
T ^a	D ^b	I ^c	T	D	I	T	D	I
97	19	9	183	21	13	240	29	15
	(19.6%)	(9.3%)		(11.5%)	(7.1%)		(12.1%)	(6.3%)

^a T represents the total number of learning outcomes in this strand and level.

^b D represents the total number of direct RP-based learning outcomes in this strand and level.

^c I represents the total number of indirect RP-based learning outcomes in this strand and level.

Mathematical Ideas Categorized as Patterns, Conjectures, and Arguments

Table 5 shows the breakdown in the three RP categories within the JC Syllabus when direct and indirect RP-based learning outcomes are combined. The differences across these three categories in the JC Syllabus for foundation/ordinary level students were statistically significant, $Q(2) = 24.163, p < .001$. There were statistically significant differences between pattern and conjecture, $Q(1) = 15.000, p < .001$, and between conjecture and argument, $Q(1) = 19.282, p < .001$. There were no statistically significant differences between pattern and argument, $Q(1) = 5.628, p = .018$. The differences across these three categories in the JC Syllabus for higher level students were statistically significant, $Q(2) = 29.163, p < .001$. There were statistically significant differences between pattern and conjecture, $Q(1) = 15.000, p < .001$, between conjecture and argument, $Q(1) = 22.277, p < .001$, and between pattern and argument $Q(1) = 8.000, p = .005$.

Table 5

Mathematical Ideas Categorized as RP by Student Level within the JC Syllabus

Foundation/Ordinary			Higher			Totals ^d
P ^a	C ^b	A ^c	P	C	A	
61	46	83	72	57	105	239
(50.0%)	(34.0%)	(70.0%)	(50.0%)	(34.5%)	(70.7%)	(100.0%)

^a P represents pattern identification.

^b C represents conjecture formulation.

^c A represents the construction of valid arguments.

^d Only the RP categories for the higher level have been added for this column as it contains all

Appendix A shows the breakdown in the three RP categories within the LC Syllabus when direct and indirect RP-based learning outcomes are combined. The differences across these three categories in the LC Syllabus for foundation level students were statistically significant, $Q(2) = 29.280, p < .001$. The differences between pattern and conjecture were not statistically significantly different from one another, $Q(1) = 2.000, p = .157$. However, there were statistically significant differences between conjecture and argument, $Q(1) = 15.000, p < .001$, and between pattern and argument, $Q(1) = 13.520, p < .001$.

The differences across these three categories in the LC Syllabus for ordinary level students were statistically significant $Q(2) = 24.571, p < .001$. The differences between pattern and conjecture were not statistically significantly different from one another, $Q(1) = 4.000, p = .046$. However, there were statistically significant differences between conjecture

and argument, $Q(1) = 15.077, p < .001$, and between pattern and argument, $Q(1) = 10.286, p = .001$.

The differences across these three categories in the LC Syllabus for higher level students were statistically significant $Q(2) = 54.320, p < .001$. The differences between pattern and conjecture were not statistically significantly different from one another, $Q(1) = 4.000, p = .046$. However, there were statistically significant differences between conjecture and argument, $Q(1) = 31.113, p < .001$ and between pattern and argument $Q(1) = 24.653, p = .001$.

Argument Purposes

The most obvious pattern in the area of argument purposes is the omission of proof as falsification across both the JC and LC Syllabi as well as across different student learning levels. The differences for Foundation/Ordinary learning levels in proof purposes between the JC Syllabus and the LC Syllabus were statistically nonsignificant, $\chi^2(2) = 1.022, p = .600$. A similar finding appeared in proof purposes at the higher level across both syllabi, $\chi^2(2) = .327, p = .849$.

Connectedness of RP Learning Outcomes to Content

Tables 6 and 7 show the breakdown of content- and non-content related RP learning outcomes by strand within the JC Syllabus and LC Syllabus, respectively. The distribution of content and non-content RP-based learning outcomes by strand was statistically significant for foundation/ordinary level students as Fisher's Exact test had a value of 13.615, $p = .005$. These differences were also statistically significant for higher level students as the value for Fisher's Exact test was 16.844, $p = .001$. These differences also appeared at the foundation level (14.884, $p = .001$), ordinary (13.638, $p = .004$), and higher level within the LC Syllabus (14.679, $p =$

.003).

Across both syllabi the majority of RP learning outcomes were divorced from specific mathematics content. The only strand within the JC Syllabus where this didn't occur was in number. The ratio of non-content-related to content-related RP learning outcomes decreased as one moved from lower levels in both syllabi. For instance, in the JC Syllabus at the foundation/ordinary level this ratio was 2.2:1, while at the higher level this ratio had dropped to 1.6:1.

Table 6

Frequency of Content and Non-Content Related RP Learning Outcomes by Content Strand and Level in JC Syllabus

Strand	Foundation/Ordinary		Higher	
	C ^a	NC ^b	C	NC
Statistics/Probability	1	6	2	7
Geometry/Trigonometry	1	4	2	4
Number	8	7	12	7
Algebra	6	13	6	13
Functions	0	5	0	5
Total	16	35	22	36

^a Content-related RP learning outcome.

^b Non-content-related RP learning outcome.

Table 7

Frequency of Content and Non-Content Related RP Learning Outcomes by Content Strand and Level in LC Syllabus

Strand	Foundation		Ordinary		Higher	
	C ^a	NC ^b	C	NC	C	NC
Statistics/Probability	1	5	2	5	2	6
Geometry/Trigonometry	2	4	2	5	4	5
Number	2	6	4	6	6	10
Algebra	0	4	0	4	1	4
Functions	0	4	1	4	2	4
Total	5	23	9	24	15	29

^a Content-related RP learning outcome.

^b Non-content-related RP learning outcome

Discussion

As Hiebert (2003) has pointed out, mathematics standards are value judgments that are a composite of society-based values, best educational practices, research, and the visions of what professionals would like students to learn. While research cannot choose standards, this study represents an effort to investigate one country's mathematics standards using an analytic framework generated for research into curriculum that is grounded in how mathematicians engage in the practice of mathematics and is aligned with descriptions of reasoning related to proof in school mathematics (NCTM, 2000). The placement of the Synthesis and Problem Solving skills section within each of the content strands in both the Irish JC and LC syllabus suggests that the authors of these documents believe that RP is essential for students' learning of

mathematics. Consequently, this study was designed to examine the nature of RP within the learning outcomes in these syllabi.

The two syllabi analyzed in this study are the main drivers of reform in Ireland's centralized educational system as they set the learning outcomes from which students' high stakes assessments at the Junior Certificate and Leaving Certificate levels are created. The designers of both syllabi did a good job of providing students with equitable opportunities to engage in *direct* and *indirect* RP learning outcomes as there were no statistically significant differences in these categories among different mathematics content strands in both syllabi. However, there were differences in the learning outcomes within these categories between the two syllabi. Thus, while each syllabus appeared to exhibit internal consistency in learning outcomes across these three categories there was less consistency across syllabi.

When *direct* and *indirect* learning outcomes were combined and mathematical ideas were categorized as pattern identification, conjecture formulation, and argument construction statistically significant differences appeared within each syllabus by student learning level. In the JC Syllabus at the foundation/ordinary level, there were statistically significantly more conjecture opportunities than pattern identification or argument construction. In the JC Syllabus at the higher level students were given different opportunities to engage in all three categories. In the LC Syllabus students at all learning levels were given more opportunities to develop arguments than identify patterns or make conjectures. The falsification purpose of proof did not appear in either syllabus. In addition, there were no differences across syllabi by student learning level within argument proof purposes. In each syllabus by student learning level, there was a statistically higher prevalence of non-content than content related RP learning outcomes.

In terms of the research community, this study developed and advanced the use of indirect

and direct reasoning-and-proving forms for the analysis of national curriculum documents. In addition, it supplied researchers with a set of keywords to be used to suggest the potential for indirect and direct RP forms. The lack of actual tasks appearing in curriculum documents necessitated an adapted RP framework based upon the work of Stylianides (2009). If national curriculum architects in other countries value RP, the framework presented in this paper can be used as a tool to construct documents integrating these processes into student learning outcomes. The results of this study provides researchers as well as educational stakeholders in other countries with a baseline set of data from which similar analyses of other national curriculum documents can be compared. Moreover, the location of RP elements within national curriculum documents can be followed up with the identification of these elements within textbooks, classroom lessons as well as the assessed curriculum to determine the alignment of these components vis-à-vis RP.

Vocabulary

Both direct and indirect instances of the framework were considered to be valid forms of RP in this study. However, indirect RP learning outcomes are more open to interpretation by readers and as the bandwidth of that interpretation increases there is a greater chance that interpretations by users of the curriculum documents may differ from those of the authors. For instance, within the number content strand the LC Syllabus expects ordinary and higher level students to “investigate the operations of addition, multiplication, subtraction and division with complex numbers C in rectangular form for $a + ib$ ” (NCCA, n.d., p. 25). In this study, the verb *investigate* was coded as involving pattern identification, conjecture formulation, and argument construction. In an activity book for ordinary level leaving certificate students in Ireland (Keating, Mulvany, & O’Laughlin, 2012), students are asked to calculate $(2 + 5i) + (3 + 4i)$ and later calculate $(3 + 4i)$

+ $(2 + 5i)$). Students are then asked to fill in the blank in the following sentence, "This illustrates that addition is a _____ operation on the set of complex numbers" (p. 62). Thus the textbook authors' interpretations of the word *investigation* appearing in the syllabus in this instance focus on pattern identification only and the requirement that students make an assertion that is based only on two examples may promote an empirical proof scheme (Harel & Sowder, 1998).

Vocabulary issues also arose in the section titled Synthesis and Problem-Solving Skills. That is, this section contained identification of patterns, development of conjectures, and the justification of conclusions yet from the title it is not obvious that this section pertains to reasoning-and-proving. As a result, national curriculum frameworks should carefully define mathematical processes such as *synthesis*, *investigate*, *analyze*, *synthesis*, etc. so that teachers, curriculum developers, and others interpret such words in similar ways that are aligned with the perspective of mathematics that policy statements are intended to promote. Another tact for national curriculum developers is to use direct RP forms to reduce the chances of misinterpretation if they wish to provide students with opportunities to engage in these mathematical processes.

Presenting RP Apart from Content

The appearance of RP in non-content-related learning outcomes in this study was similar to what Kim and Kasmer (2006) found with regard to reasoning in state curriculum frameworks in the United States. The decision of policy architects to embed mathematical processes such as RP apart from content may not lead to an increase in RP in mathematics classrooms for three reasons. First, teachers may choose not to read non-content-related RP learning outcomes thereby failing to implement them in the classroom because they are in pursuit of content that

students need to learn and that could be assessed on high stakes assessment. Second, for teachers who may have little experience learning about mathematical ideas through pattern identification, conjecture formulation, and argument construction, it may be difficult to decide how mathematical ideas that they may have learned in less meaningful ways could be reimagined to incorporate these processes when they are not directly connected to content in the syllabi documents. Third, as Bieda (2010) has noted, incorporating opportunities for students to engage in RP opportunities during classroom lessons takes time. If teachers feel rushed to prepare students for high stakes examinations they may feel that they do not have the time for such activities as they appear to be an addendum to the syllabus by their presence in locations other than where content is listed.

Writers of national curriculum documents could seek to bridge the chasm between content and RP in two ways. First, they could weave the presence of RP as a central act of mathematics into individual learning outcomes. Take for example the learning outcome related to the fundamental principle of counting within the JC Syllabus. Currently, this learning outcome is stated in the following way: “apply the fundamental principle of counting” (NCCA, n.d., p. 15). As written, this learning outcome may focus teachers’ work on providing students with practice using this mathematical idea to solve problems and less emphasis may be placed on understanding why this principle is valid. This learning outcome could be rewritten in the following way to increase the possibility that teachers would more tightly integrate RP within student learning opportunities related to it: “develop and apply the fundamental principle of counting.” The word *develop* could be defined up front to involve the identification of patterns, development of conjectures, and/or construction of arguments.

Second, a characteristic common to national standards documents is the listing of particular

learning outcomes or objectives. The words used to label this component could be altered to make RP a more central component of the process of learning mathematical ideas. For example, the column headings in the tables listing learning outcomes in the JC and LC syllabi are written as follows: “students should be able to” (p. 15). These headings could be changed to better emphasize the centrality of RP in learning outcomes through the alteration of these column headings to incorporate the following processes: pattern identification, conjecture formulation, and/or argument construction. Learning outcomes appearing in syllabi documents would then list mathematical ideas such as the fundamental principle of counting.

Location of Mathematical Processes

The Common Core State Standards for School Mathematics (CCSSM) (CCSSI, 2010), a set of national standards in the United States, contain a section titled, Standards for Mathematical Practice. These standards include a variety of mathematical processes some of which connect to RP such as: Construct viable arguments and critique the reasoning of others. This section appears at the beginning of the document apart from where content objectives are located. Both the Irish JC Syllabus and the LC Syllabus include a section titled Synthesis and Problem-Solving Skills containing components of the RP framework used here, but it appears at the end of each content strand. In both cases, mathematical processes that curriculum writers believe are central to the act of engaging in mathematics, appear apart from content objectives. This organization choice may cause teachers to underplay the role of RP in engaging in and learning mathematics (Cobb & Jackson, 2011).

Proof Purposes

The falsification purpose of proof was missing across all levels within both syllabi. Accordingly, students may not have an opportunity to learn about the fundamental role that

counterexamples play in showing the falseness of an assertion. The lack of falsification proof purposes in the Irish National Syllabi was also found in a set of U.S. reform-oriented mathematics textbooks for students ages 11-14 by Stylianides (2009). Policy statements as embedded within national syllabi should not only describe objectives in terms of specific mathematical ideas that students need to learn, but should also explicitly promote the development of counterexamples connected to content as specific learning outcomes. For example, students could be asked to show that matrix multiplication is not commutative.

Conclusion

Centralized educational systems can be thought of as an interconnected web of different components. National curriculum documents occupy the central position within this web and are connected to other components within this system via radials. Thus in understanding these systems, it is important to begin with the national curriculum documents that hold this system together. In a similar vein, mathematics can be considered a web, the center of which is held together via reasoning-and-proving. Components of the two national curriculum documents examined here as well as national standards in other countries (NCTM, 2000) value RP as a vehicle by which school students learn mathematics. This study represents an initial foray into the analysis of national curriculum documents through a research-based analytic framework designed to examine RP in curricula. This study provides methodological contributions to future national curriculum analyses through the development of indirect and direct RP categories and the creation of a set of keywords suggesting the potential for each of these processes. While the analyses described in this study focus on Irish national syllabi, the results suggest ways in which RP can be made more central within national curriculum frameworks in general. These suggestions include the careful definition of terminology, the connection of RP to mathematical

content, and the careful attendance to the different purposes that proof can play in school mathematics.

References

- Bieda, K. N. (2010). Enacting proof-related tasks in middle school mathematics: Challenges and opportunities. *Journal for Research in Mathematics Education*, 41, 351-382.
- Cobb, P., & Jackson, K. (2011). Assessing the quality of the Common Core State Standards for Mathematics. *Educational Researcher*, 40, 183-185.
- Cogan, L., Schmidt, W., & Houang, R. (January, 2013). *Implementing the Common Core State Standards for Mathematics: What we know about teachers of mathematics in 41 states: Working paper #33*. East Lansing, MI: The Education Policy Center at Michigan State University.
- Common Core State Standards Initiative. (2010). *Common Core State Standards for mathematics*. Retrieved from http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf
- Conover, W. (1999). *Practical nonparametric statistics* (3rd ed.). New York: Wiley.
- Davis, J. D., Smith, D. O., Roy, A. R., & Bilgic, Y. K. (2013). Reasoning-and-proving in algebra: The case of two reform-oriented U.S. textbooks. *International Journal of Educational Research*, 64, 92-106.
- Department of Education and Skills. (2010). *Report of the Project Maths Implementation Support Group*. Accessed on May 10, 2013 at <http://www.education.ie/en/Publications/Policy-Reports/Report-of-the-Project-Maths-Implementation-Group.pdf>
- De Villiers, M. (1990). The role and function of proof in mathematics. *Pythagoras*, 24, 17-24.

- De Villiers, M. (1999). The role and function of proof. In M. De Villiers (Ed.), *Rethinking proof with the Geometer's Sketchpad* (pp. 3-10). Key Curriculum Press.
- Dossey, J., Halvorsen, K., & McCrone, S. (2008). *Mathematics education in the United States 2008: A capsule summary fact book*. Reston, VA: National Council of Teachers of Mathematics.
- Epp, S. S. (1998). A unified framework for proof and disproof. *Mathematics Teacher*, 91, 708-713.
- Eurydice. (2011). *Mathematics education in Europe: Common challenges and national policies*. Education, Audiovisual and Culture Executive Agency. Accessed on May 16, 2013 from http://eacea.ec.europa.eu/education/eurydice/documents/thematic_reports/132en.pdf
- Field, A. (2009). *Discovering statistics using SPSS* (3rd Ed.). London: Sage Publications.
- Hanna, G. (1990). Some pedagogical aspects of proof. *Interchange*, 21(1), 6-13.
- Hanna, G. (2000). A critical examination of three factors in the decline of proof. *Interchange*, 31, 21-33.
- Hanna, G., & de Bruyn, Y. (1999). Opportunity to learn proof in Ontario grade twelve mathematics texts. *Ontario Mathematics Gazette*, 37(4), 23-29.
- Harel, G., & Sowder, L. (1998). Students' proof schemes: Results from exploratory studies. In A. Schoenfeld, J. Kaput, & E. Dubinsky (Eds.), *Research in collegiate mathematics education, III* (pp. 234-283). Washington, DC: Mathematical Association of America.
- Hiebert, J. (2003). What research says about the NCTM Standards. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *Research companion to Principles and Standards for School Mathematics* (pp. 5-23). Reston, VA: National Council of Teachers of Mathematics.

- Jitendra, A. K., Star, J. R., Dupuis, D. N., & Rodriguez, M. C. (2013). Effectiveness of schema-based instruction for improving seventh-grade students' proportional reasoning: A randomized experiment. *Journal of Research on Educational Effectiveness, 6*(2), 114-136.
- Johnson, G. J., Thompson, D. R., & Senk, S. L. (2010). Proof-related reasoning in high school textbooks. *Mathematics Teacher, 103*(6), 410-418.
- Keating, M., Mulvany, D., & O'Loughlin, J. (2012). *Active maths 3 activity book: Leaving certificate ordinary level*. Dublin: Folens.
- Kim, O.K., & Kasmer, L. (2006). Analysis of emphasis on reasoning in state mathematics curriculum standards. In B. J. Reys (Eds.), *The intended mathematics curriculum as represented in state-level curriculum standards: Consensus or confusion?* (pp. 113-133). Charlotte, NC: Information Age Publishing.
- Landis, J. R., & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *Biometrics, 33*, 159-174.
- Martin, G., Carter, J., Forster, S., Howe, R., Kader, G., Kepner, H., et al. (2009). *Focus in high school mathematics: Reasoning and sense making*. Reston, VA: National Council of Teachers of Mathematics.
- McCallum, W. (2012, July). *The Common Core State Standards in Mathematics*. Paper presented at the 12th Meeting of the International Congress of Mathematics Education, Seoul, South Korea. Retrieved from http://math.arizona.edu/~wmc/Research/2012_07_12_ICME_McCallum.pdf
- National Council for Curriculum and Assessment. (n.d.). *Leaving certificate mathematics syllabus: Foundation, ordinary & higher Level: 2014 examination*. Author.
- National Council for Curriculum and Assessment. (n.d.). *Junior certificate mathematics syllabus: 2015 examination*. Author.

- National Council for Curriculum and Assessment. (2012). *Project maths: Reviewing the project in the initial group of 24 schools – report on school visits*. Author.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- Nordström, K., & Löfwall, C. (2005). *Proof in Swedish upper secondary school mathematics textbooks – the issue of transparency*. Proceedings of the 4th Congress of the European Society for Research in Mathematics Education (CERME4), pp. 448-457. Spain: ERME.
- Porter, A., McMaken, J., Hwang, J., & Yang, R. (2011). Common core standards: The new US intended curriculum. *Educational Researcher*, 40(3), 103-116.
- Reilly, G. (August 15, 2012). Almost 11,000 leaving cert students to benefit from bonus Maths points. Accessed on May 10, 2013 at <http://www.thejournal.ie/leaving-cert-maths-bonus-points-cao-559255-Aug2012/>
- Ross, K. (1998). Doing and proving: The place of algorithms and proof in school mathematics. *American Mathematical Monthly*, 3, 252-255.
- Schoenfeld, A. H. (2009). Series editor's foreword: The soul of mathematics. In D. A. Stylianou, M. L. Blanton, & E. J. Knuth (Eds.), *Teaching and learning proof across the grades: A K-16 perspective* (pp. xii-xvi). New York, NY: Routledge.
- Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20-26.
- Smyth, E. (2009). Junior cycle education: Insights from a longitudinal study of students. *ESRI Research Bulletin*, 4(1), 1-5.
- Stacey, K., & Vincent, J. (2009). Modes of reasoning in explanations in Australian eighth-grade mathematics textbooks. *Educational Studies in Mathematics*, 72, 271-288.

- Steen, L. (1999). Twenty questions about mathematical reasoning. In L. Stiff (Ed.), *Developing mathematical reasoning in grades K-12: 1999 Yearbook* (pp. 270-285). Reston, VA: National Council of Teachers of Mathematics.
- Stylianides, A. J. (2007). Proof and proving in school mathematics. *Journal for Research in Mathematics Education*, 38, 289-321.
- Stylianides, G. J. (2005). *Investigating students' opportunities to develop proficiency in reasoning and proving: A curricular perspective*. Unpublished dissertation, University of Michigan.
- Stylianides, G. J. (2009). Reasoning-and-proving in school mathematics textbooks. *Mathematical Thinking and Learning*, 11(4), 258-288.
- Thompson, D. R., Senk, S. L., Johnson, G. J. (2012). Opportunities to learn reasoning and proof in high school mathematics textbooks. *Journal for Research in Mathematics Education*, 43, 253-295.
- Walkington, C., Petrosino, A., & Sherman, M. (2013). Supporting algebraic reasoning through personalized story scenarios: How situational understanding mediates performance. *Mathematical Thinking and Learning*, 15(2), 89-120.

Appendix A

Mathematical Ideas Categorized as RP by Student Level within the LC Syllabus

Foundation			Ordinary			Higher			Totals ^d
P ^a	C ^b	A ^c	P	C	A	P	C	A	
19	17	45	30	26	54	34	30	77	152
(32.1%)	(28.6%)	(78.6%)	(41.2%)	(35.3%)	(76.5%)	(36.4%)	(31.8%)	(81.8%)	(100%)

^a P denotes pattern detection.

^b C denotes conjecture formulation.

^c A denotes argument construction.

^d Only the RP categories for the higher level have been added for this column as it contains all learning outcomes at the foundation and ordinary levels.

