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Worth the Risk? Modeling Irrational Gambling Behavior

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Abstract: In math class, expected value is often used when deciding whether or not a game is worth playing. A common refrain is that games with negative expected value should be avoided. However, nearly all games of chance have a negative expected value, and a simple expected value analysis fails to explain why these games are so popular. In this article, we consider three psychological factors leading to irrational gambling behavior – the illusion of control, hypersensitivity to reward, and beginner’s luck – and explore how these factors affect an otherwise purely rational model of gambling behavior.

Keywords: expected value, gambling, roulette, luck, utility.

Introduction

Imagine I invite you to play a game. The rules are simple: each round, you have a 53% probability of losing a dollar (to me), and a 47% probability of winning a dollar (from me). We can play this game for as long as you like. Would you play?

If you’re a math teacher, your answer is probably no. After all, this game, besides sounding as dull as can be, has a *negative expected value*: over time, you should expect to lose money and I should expect to gain it. Indeed, given these rules, your expected value for any one round is equal to:

$$0.53 \times (-\$1) + 0.47 \times (+\$1) = -\$0.06.$$

That is, on average, you can expect to lose 6 cents every time we play. Surely, there’s no upside for you, and you’d be better off walking away.

And yet, these probabilities match some of the most common bets in a spin of the roulette wheel. The wheel has 18 red wedges and 18 black wedges, plus 2 green wedges, all of equal size. This means a bet on red (or black) has an $18/38 \approx 47\%$ probability of winning, and a $20/38 \approx 53\%$ probability of losing. But in spite of these numbers, people play roulette in droves. How silly of them; with but a little knowledge of probability, we decry in math class, people would know the risks and could save themselves considerable heartache.

This is the story students often hear when we talk about games of chance. If a game has negative expected value, you shouldn’t play it; if it has positive expected value, you should. As with so many mathematical models, though, the reality is typically much more complicated. Offering games with positive expected value isn’t exactly a viable long-term strategy for casinos, so most games with money on the line have a negative expected value. More importantly, people play games with negative expected value all the time, in spite of the risk involved. The interesting takeaway, especially from the standpoint of a classroom discussion, isn’t: *people shouldn’t do this*. Rather, it’s: *why do people do this?* The answer to the latter question requires us to think more critically about our mathematical model, and try to adapt it more faithfully to our reality.

There are a number of reasons why people gamble, even though rationally they shouldn't. For example, the gambler's fallacy – the (incorrect) idea that if you've lost a number of times in a row, you're "due" for a win – is a common refrain. But there are subtler reasons that get touched upon less often, even though they can influence our decision-making in significant ways. In what follows, we'll touch upon three ideas: *illusion of control*, *hypersensitivity to reward*, and *beginner's luck*, and see how each of them affect our willingness to take on risk.

The Model

Since we'll return to it often, let's take a moment to abstract our model a bit. Instead of a game in which you win \$1 with a 47% probability, and lose \$1 with a 53% probability, we'll talk about a game in which you win an amount a_W with probability p , and lose an amount a_L with probability $1 - p$. For this to make sense, let's assume that a_W and a_L are both nonnegative. As before, your expected value in this game is equal to:

$$a_W p - a_L (1 - p) = (a_W + a_L)p - a_L.$$

If you believe that you shouldn't play a game with negative expected value, then you should only play this game if

$$(a_W + a_L)p - a_L \geq 0,$$

that is, if

$$p \geq \frac{a_L}{a_W + a_L}.$$

In particular, if the stakes are equal to the potential payout (that is, if $a_W = a_L$) you should only play the game if you have at least a 50% chance of winning.

People rarely follow this advice, however. Let's explore some reasons why.

Illusion of Control

The game I described above lacked any sort of context about *how* the winning and losing probabilities are determined. Is it by spinning a roulette wheel? Rolling dice? Playing with cards? Drawing marbles from a bag?

For the mathematically minded, these details may seem irrelevant. After all, once we know the probabilities, everything else is window dressing, at least in terms of our decision about whether or not to play.

But in reality, context is extremely important. This is due, in part, to the *illusion of control*, i.e., the belief that people have much more control over a random outcome than they actually do. When people believe they control over the outcome of a game, they are likely to take greater risks, even if objectively they have no control at all.

There are several ways to elicit the illusion of control in a game of chance. One way is to give the player something to do, or some decision to make, even if they have no control over the outcome of what they're doing. In roulette, players can choose what to bet on: whether the ball will stop on red or black, an even number or an odd number, and so on. In dice games, players roll the dice. Even though the player has no control over how the dice will land, simply giving the player the opportunity to roll can elicit the illusion of control. This manifests itself in subconscious ways: for example, an experiment by

Henslin (1967) found that when players needed to roll low numbers in a dice game, they rolled the dice more softly, and when they needed high numbers, they rolled more forcefully!

Take away the activity granting players an illusion of control, and their confidence will waver as a result. Strickland, Lewicki and Katz (1966) illustrated this point with an experiment: they separated students into two groups, both of which played a dice game. In one group, however, they players bet before they rolled the dice; in the other group, the players bet *after* they rolled the dice (but before they knew the results of the roll). The experiment revealed that people who bet after rolling the dice tended to be more conservative in their wagers than people who bet beforehand.

Players also experience more illusion of control when the game itself is familiar. Burger (1986) found that people with a “high desire for control” will bet more on a card game when playing with a familiar deck of cards (using the standard club, spade, heart and diamond symbols) than when playing with cards that used unfamiliar symbols. Of course, when it comes to a random process, familiarity makes no difference in one’s ability to predict the outcome.

The illusion of control is great for casinos, but not so great for individual players. And the problems that arise from this illusion are compounded by the fact that mathematically, most casino games operate on a relatively small negative expected value. The illusion of control isn’t likely to make you think you should pay \$90 to play a game where you have only a 1% chance of winning \$100. But typically the house edge is quite small, and just a tiny nudge in your perceived probability of winning may make you willing to take a risk that you otherwise wouldn’t.

Put another way, for any given casino game, it likely isn’t true that $p \geq \frac{a_L}{a_W + a_L}$ holds. However, if you believe your probability of winning is actually some value $p^* > p$, it may be true that $p^* \geq \frac{a_L}{a_W + a_L}$, in which case you may play the game in spite of the risk involved. When betting a dollar on black in roulette, for instance, $p \approx 47\%$, and $\frac{a_L}{a_W + a_L} = 50\%$. So if the illusion of control can bump up your confidence in winning by just three or four percentage points, you may opt to put your money on a spin of the wheel. Unfortunately, the *illusion* of control is just that, and while confidence is an asset in many aspects of life, a little humility when gambling is probably wise.

Hypersensitivity to Reward

Most analyses of games of chance are framed in terms of dollars: how much will you make if you win, and how much will you lose if you don’t? But for many people – compulsive gamblers especially – this isn’t necessarily the right perspective. Sometimes, people don’t gamble because they think they can win. Rather, they gamble because it feels *good* to win.

Instead of talking about dollars, then, maybe it makes sense to talk about gambling in terms of something more abstract, like *utility*, which serves as a proxy for measuring happiness. Unlike the illusion of control, which affects the *probabilities* of winning, swapping dollars for utilities affects the *payouts* if we win or lose.

Notationally, not much changes. If you gain a utility u_W by winning and lose a utility u_L by losing, then we should only play a game of chance if $p \geq \frac{u_L}{u_W + u_L}$. But this leaves us with a couple of questions:

1. How do utilities compare to dollar amounts?
2. How do utilities vary among people?

The first question is difficult to quantify. At first, it may seem reasonable that utility should be roughly proportional to dollar amounts. If you win twice as much, you should be twice as happy; winning \$10 should bring you as much happiness as losing \$10 costs you. And so on.

However, this doesn't make for much of an interesting story, since if people's utilities were proportional to amounts won and lost, the ratios $\frac{u_L}{u_W + u_L}$ and $\frac{a_L}{a_W + a_L}$ would be the same, and we'd be no better off trying to explain people's behavior by using utility instead of money.

Nevertheless, it may be true that for *some* people, utility is roughly proportional to actual amounts wagered. Unlike dollar amounts, though, which are objective and uniform, there's likely much more variation among individuals when it comes to utility fluctuations while gambling.

Which brings us to our second question. Utility variation among individuals is of particular interest when it comes to two groups: compulsive gamblers and everyone else. People who have a gambling problem will dig themselves into deeper and deeper financial holes, taking on financial risks well past the point at which any reasonable person would walk away (or so we like to tell ourselves). Illusion of control may play a part in this, but utility plays a role as well.

There are two possible explanations for how a change in utility could lead to riskier gambling behavior: either a person doesn't feel the sting of losing (u_L is relatively low), or the person feels a heightened euphoria after winning (u_W is relatively high). Of these options, the work of Hewig et. al. (2010) has shown that the second is more plausible. By comparing the electrophysiologic responses of problem gamblers to a control group, their experiment showed that problem gamblers are hypersensitive to reward, possibly due to an above-average increase in dopamine when they win.

In other words, the factor u_W may be higher for a problem gambler than for a non-problem gambler. But as u_W increases, the ratio $\frac{u_L}{u_W + u_L}$ decreases, meaning that a problem gambler may be more willing to risk playing a game with relatively poor odds.

For instance, suppose there are two people thinking of playing a game. One person's utility is proportional to the dollar amounts she can win or lose, while the other is hypersensitive to reward. Their utilities may look something like this:

	Person A	Person B (Hypersensitive)
Utility if Win	u_W	$1.5u_W$
Utility if Lose	u_L	u_L

Now suppose the potential payoff in the game equals the potential loss ($a_W = a_L$). Since Person A's utilities are proportional to the dollar amounts, this means that

$$\frac{u_L}{u_W + u_L} = \frac{a_L}{a_W + a_L} = 0.5,$$

so Person A will only play the game if her probability of winning is at least 50%. But for person B, the threshold is lower:

$$\frac{u_L}{1.5u_W + u_L} = \frac{a_L}{1.5a_W + a_L} = 0.4.$$

Returning again to the example we began with, Person A won't play this game if the probability of winning is 47%, but Person B will!

Framed in terms of money, both people have the same expected value for this game: $-0.06a_W$. For Person A, this is proportional to their expected utility from playing the game: $-0.06u_W$. However, Person B's expected utility is equal to $0.175u_W$. Person B expects to gain utility from this game, but lose money. (Of course, utility and money aren't independent, and even someone who is hypersensitive to reward will likely start to feel bad after losing a lot of money in one sitting.)

Combined with the illusion of control, hypersensitivity to reward can be even more dangerous. The illusion of control inflates our perceived probability of success (from p to p^*), while hypersensitivity to reward inflates the value of a win (from a_W to u_W). Both of these factors increase our willingness to take on risk. Taken together, instead of checking the inequality

$$p \geq \frac{a_L}{a_W + a_L},$$

for some people, the relevant inequality is

$$p^* \geq \frac{u_L}{u_W + u_L}.$$

When $p^* > p$ and $\frac{a_L}{a_W + a_L} > \frac{u_L}{u_W + u_L}$, it's quite possible for the second inequality to hold even if the first one doesn't.

Beginner's Luck

We've seen that illusions of control and hypersensitivity to reward can skew our perceptions of games of chance. Research also suggests that some people are more susceptible to these phenomena than others. But before you breath a sigh of relief, convinced of your resistance to these risky behaviors, you should know that, given the right circumstances, none of us seems completely immune.

For example, in one of the more famous papers on the illusion of control, Langer and Roth (1975) found that *beginner's luck* has a measureable affect on people's perceptions of success. One important aspect of the experiment was that the subjects weren't control freaks or problem gamblers; they were simply undergraduates at Yale.

In the experiment, 90 male subjects were asked to predict the outcome of a coin flip thirty times in a row. While the experiment was perceived to be random, subjects were actually put into one of three groups. Members of one group (the *descending* group) were told they guessed correctly very frequently early on, but less frequently as the game progressed. Members of another group (the *ascending* group) were told the opposite: many incorrect guesses early on, but more correct guesses later. A third group was given a randomly generated string of wins and losses to serve as a control.

Descending Group	WWWLWWLLLWWLLWLLWLLLWLLLW
Ascending Group	WLLLWLLWLLWLLWWLLLWWLWWW
Random Group	WLWLLWLWLWLLWLLWLWWLWLWLLWLW

The members of each group were told they won 15 times and lost 15 times; the only difference between groups was the way in which the wins and losses were arranged. Afterwards, each subject was asked a few questions about his performance. Here are some of the questions, along with the average response for each group:

	Descending	Ascending	Random
From 0 to 10, how good do you think you are at predicting outcomes like these?	5.70	4.93	5.10
How many times did you correctly guess the coin toss?	16.7	13.3	14.4
After 100 more rounds, how many more correct guesses do you think you'd make?	54.2	49.1	51.1

People who started off with more wins believed they were better at predicting coin flips than people in the other two groups (the illusion of control rears its head once again). Also, even though people in each group “won” exactly 15 times, people in the descending group thought they had won more than 15, and people in the ascending group thought they had won fewer. Finally, when asked about their probability of future success, people in the descending group had a higher estimate (54.1%) than people in the ascending group (49.1%).

In other words, early success when playing a game can influence feelings of control, even among people who are ostensibly highly rational. A person who wins k rounds out of a total of n rounds in a game of chance should estimate their probability of winning a future round by dividing the number of wins (k) by the number of games played (n):

$$\frac{\#\{\text{wins}\}}{\#\{\text{games played}\}} = \frac{k}{n}.$$

In reality, however it seems that people tend to give more credence to wins that occur earlier on.

We can attempt to model this mathematically as well. Suppose a player weighs the n^{th} round of a game according to some weighting function $w(n)$. If the player wins k rounds i_1, i_2, \dots, i_k out of a possible n , then he will estimate his probability of future success by the ratio

$$\frac{w(i_1) + w(i_2) + \dots + w(i_k)}{w(1) + w(2) + \dots + w(n)}.$$

A perfectly rational person will weigh the importance of each round equally, in which case we can take w to be the constant function 1, and the fraction above reduces to k/n – that is, the number of wins divided by the number of plays. For someone who values early wins more highly, however, w is likely to not be a constant function. Instead, it seems reasonable to insist on the following two properties:

1. w is non-increasing,
2. $w(n) \rightarrow 1$ as $n \rightarrow \infty$.

The first assumption ensures that we really do weigh initial rounds more than (or at least, not less than) later rounds. If the second assumption holds, we become more rational as time progresses; that is, eventually, all rounds are weighed roughly the same.

For instance, suppose we give the n^{th} round a $(100/n)\%$ significance bonus, i.e. we take $w(n) = 1 + 1/n$. This satisfies both of the assumptions above, and compared to the rule $w(n) = 1$, we see that it does indeed favor earlier rounds to later rounds. We can also apply this rule to the ordering of wins and losses in each group listed above in order to estimate the perceived probabilities of future success for each group:

	Descending	Ascending	Random
Weighted probability ($w(n) = 1 + 1/n$)	52.9%	49.6%	50.7%
Self-reported probability (from above)	54.2%	49.1%	51.1%

This relatively simple weight function does a reasonable job predicting the self-reported outcomes, and no doubt a more sophisticated model could do even better. Regardless of the model, the point remains: even for people with no history of gambling or control issues, just a little bit of beginner's luck can be enough to get people to engage in risky gambling behavior. So if you do decide to gamble, maybe the best thing for your wallet is a string of early losses.

Conclusion

Talking about gambling in the classroom can be tricky, but the moral of the story is usually one that everyone can get behind. The math says you shouldn't gamble, because you can expect (in the mathematical sense) to lose. However, this surface-level analysis doesn't do much to explain why people *do* gamble.

One approach to consider is that while the expected value on most gambling events is negative, it's frequently only slightly negative. In this case, it doesn't take much psychological persuasion to convince people that their odds seem better than they objectively are. The illusion of control, hypersensitivity to reward, and beginner's luck are three ideas that can help explain why people take on more risk than they should, and all three can be incorporated into mathematical models that use expected value. However, these ideas are rarely explored in the classroom. But in the end, if we want to discourage students from taking irrational risks with their money, it seems better to try to account for that irrationality rather than to ignore it.

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