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Peter Rowlett

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Developing Strategic and Mathematical Thinking via Game Play: Programming to Investigate a Risky Strategy for Quarto

Peter Rowlett

Nottingham Trent University, U.K.

Abstract: The Maths Arcade is an extracurricular club for undergraduate students to play and analyse strategy board games, aimed at building a mathematical community of staff and students as well as improving strategic and mathematical thinking. This educational initiative, used at several universities in the U.K., will be described. Quarto is an impartial game played at the Maths Arcade, in that there is one set of common pieces used by both players, and one where stalemates are a common outcome. While some students play without apparent direction until a winning opportunity appears, others adopt a more risky strategy of building the board towards a winning position, which could allow either player to win. Whether building towards a win is a sensible strategy, when the other player could equally well benefit, is a topic of debate at the Maths Arcade. Intending to suggest a possible student project, this article will describe a method to represent Quarto as an array of binary numbers, making the game suitable for programming in Python. Then, one strategy is programmed to play at random unless a winning move becomes available, while another is programmed to work towards a winning position. These are calibrated by playing against a completely random strategy and against themselves, then they are played against each other. The more risky strategy is found to win over the more naive player in around two thirds of one million games. Some limitations and possible areas of development are discussed.

Keywords: mathematical thinking, strategy, risk, games, programming.

Introduction

The Maths Arcade is an extra-curricular activity to involve students and staff in playing and analysing strategy games. A game played at the Maths Arcade, Quarto, is described, along with playing strategies. One strategy is identified as more risky, because it involves working to move the board to a position where either player could win. Intending to suggest a possible student project, Quarto is represented in a way suitable for programming and this is used to play the risky strategy against a less risky behaviour.

An attempt is made to give outline information such as might be useful to a student embarking on project work in this area (and, indeed, investigations have not gone much further). In this way, it is hoped that this article might form the basis of such project work.

The Maths Arcade

The Maths Arcade was initiated by Bradshaw (2011) as a weekly, extra-curricular drop-in session where students and staff play a variety of strategy games and puzzles. This aimed particularly to support students who are new to university by providing an environment where they could interact with other students and staff as part of a mathematical community. Having observed some students reluctant to attend ‘help sessions’ due to a perception that these are for weaker students, Bradshaw designed the Maths Arcade to both support weaker students and stretch more confident learners. As such, the Maths Arcade is an opportunity to develop mathematical thinking and problem-solving skills in a situation

where prior mathematical knowledge is not particularly relied upon. Different approaches to running the Maths Arcade at various universities in the U.K. are explored by Bradshaw and Rowlett (2012).

At Nottingham Trent University, a description of setting up the Maths Arcade is given by Rowlett and Webster (2013). Students' experience of the Maths Arcade begins as a 'getting to know each other' induction activity for mathematics undergraduates in the first week of term. This continues as a weekly drop-in session in term time during the three years of the degree. Games might be simply played, as a fun activity in a mathematics-themed social support environment, while some students start to think about strategies. Questions which arise in game play include: What is the best strategy? Is there an advantage to a particular position? Is there a benefit to going first? As well as a useful induction activity and an extra-curricular curiosity, at Nottingham Trent University students undertake a substantial individual project in the final year of their degree, and some have chosen to study game theory through attempted analysis of a Maths Arcade game.

Figure 1. A Quarto board and some pieces.

Quarto

Game play

Quarto is an impartial game of perfect information played at the Maths Arcade. This means there is one set of common pieces used by both players, and each player is perfectly informed of all the events that have previously occurred in the game at all times.

Quarto is played on a 4×4 board. Each game piece has four attributes each taking one of two values. Specifically, it is: white or black; short or tall; round or square; and, having a dimpled top or flat. Each combination is used, so the game uses $2^4 = 16$ pieces. This means the number of pieces matches the number of spaces on the board. A Quarto board is shown in Figure 1.

Game play is that players take turns to choose an unplayed piece for their opponent to place in an unused space on the board. Thus, there are two stages to each turn, the same for both players: 1) play the piece handed over by the opponent; 2) hand a piece to the opponent.

The aim is to be the player who places the fourth piece in a line (row, column or diagonal) which all match in any one attribute (i.e., four that are square, or four that are dimpled, etc.). A stalemate is possible, in which case the board is filled with no winning lines and the game is a draw.

Strategies

A reasonable strategy for a new player is outlined below. Call this strategy ‘naive’.

1. Play the piece handed over by the opponent:
 - (a) play a winning position if handed a winning piece;
 - (b) otherwise, play randomly.
2. Hand a piece to the opponent:
 - (a) avoid handing over a winning piece for your opponent to play;
 - (b) otherwise, choose randomly.

As a more mature strategy, some attendees at the Maths Arcade try to build lines of like pieces. That is, pieces which match in at least one attribute. This strategy is outlined below. Call this strategy ‘risky’.

1. Play the piece handed over by the opponent:
 - (a) play a winning position if handed a winning piece;
 - (b) otherwise, play to build a line of like pieces if possible;
 - (c) otherwise, play randomly.
2. Hand a piece to the opponent:
 - (a) avoid handing over a winning piece for your opponent to play;
 - (b) otherwise, choose randomly.

The risky strategy attempts to increase the options for winning moves, thus attempting to direct the board to a winning position. This is more risky because building lines that could win advantages both players. However, even though it is an advantage to both, players playing this strategy appear to win more often than those following the naive strategy – excluding human error. (Students at the Maths Arcade are encouraged to ‘rewind’ silly mistakes and re-play, in order to learn and study strategy.) Since stalemates are common among experienced players, whether this strategy holds an advantage is a topic of debate at the Maths Arcade.

For the risky player, if a winning position cannot be taken then an attempt is made to match like pieces. One strategy for doing this is outlined below.

1. Search the board for a line where the piece to be played has at least one attribute value in common with those already on that line.
2. If such a position is found, check that playing there would not create a situation where there exists a no-win scenario for that attribute. That is, a situation where one line of three pieces match in one value of an attribute and, simultaneously, another line of three pieces match in the other value for that attribute. If one line would win with a black piece and another line would win with white, this results in a loss since every piece is either black or white.

	1101	1111	
	0101	0011	0010
	0000		

Figure 2. A Quarto game in progress, represented by binary strings.

The choice made here is to search as described until the first opportunity to play a like piece arises, then play it.

For testing purposes, a third strategy is proposed: random play. A player simply chooses an available board position at random for each piece handed over, and chooses a piece to hand over at random from the remaining pieces. This is extremely poor strategy and will result in the player not necessarily winning even when handed a winning piece, and handing over a winning piece even when non-winning alternatives are available.

Programming Quarto

Quarto can be represented using four-character binary strings. First, label each attribute value either 0 or 1: white (0) or black (1); short (0) or tall (1); round (0) or square (1); and, dimpled (0) or flat-topped (1). Using a positional system in this order, attributes for a piece can be represented. For example, 0100 would be white, tall, round and dimpled; 1001 would be black, short, round and flat-topped.

Then a winning condition is met if four pieces in a line share at least one digit in common. For example, 0100 and 1001 are both round, so both have the third digit 0.

A board position can be represented by a 4×4 grid of such binary strings (an example is shown in figure 2). In figure 2, if the next player plays piece 0001 into the 3rd row, 1st column position, then a line of four will be created which all match in the leading digit, 0. Such a grid could be stored in an array or list.

If a game system keeps track of the state of the current board and the pieces available to play, then players can query these as necessary, since each player in Quarto operates from a position of perfect information. Then the random player can simply select at random from the available pieces or spaces (depending on the phase of the game), while the naive and risky players can search through the board and the unplayed pieces as necessary to meet their strategic objectives.

For example, both the naive and risky players operate on the principle of playing in a winning board position if handed a winning piece. As a brute force approach, they could check each line which contains three pieces against the piece to be played. If a set of four pieces is thus discovered which matches in one digit, then the piece is played and the game won. If no such set is found, the piece may be played according to the next move in the strategy.

Playing to simply avoid handing over a winning piece would operate on a similar principle of matching each available piece against lines of three on the board until a piece is found that does not match any line, if this is possible.

Method

Three players (random, naive and risky) programmed in Python were run in a series of trials.

Trials took the form of 10^6 games of Quarto with player 1 (P1) playing against player 2 (P2). This was arranged so that P1 and P2 take turns to go first, to negate any first player advantage or other imbalance.

A large number of games were run because all strategies have a random element, in choice of piece to hand over and in where to play. 10^6 was chosen as a suitably large number that would compute in a reasonable amount of time (such that the whole process took hours, not days, to complete; though certainly the program could be made more efficient).

First, to check the system, three trials were run in which both players took the same strategy. As these games are symmetric (same strategy, equal times playing first), we expect roughly equal numbers of wins for each player.

Then, to confirm that the naive and risky strategies are sufficiently better than chance, trials were run with each against a random player. As the random player will not necessarily win even if handed a winning piece, and may choose to hand over a winning piece when alternatives are available, we expect the more elaborate strategies to win much more often.

Finally, the risky strategy is played against the naive. As the risky strategy is building lines for both players to take advantage of, and otherwise both adopt similar behaviour, we might expect roughly equal results.

Results

The results from the three trials with both players taking the same strategy are given in Table 1. In each case, players won roughly the same number of games, with a small number of stalemates.

P1	P2	P1 wins	P2 wins	Stalemates
random	random	490668 (49.07%)	489600 (48.96%)	19732 (1.97%)
naive	naive	495707 (49.57%)	496734 (49.67%)	7559 (0.76%)
risky	risky	498650 (49.87%)	499778 (49.98%)	1572 (0.16%)

Table 1. Results from each strategy played against itself 10^6 times.

The results from the two trials with P2 taking a random strategy and P1 taking naive and risky strategies are shown in Table 2. Both more elaborate strategies win substantially more games than the random player, though the risky player wins more often and loses less often than the naive player did, resulting in fewer stalemates.

P1	P2	P1 wins	P2 wins	Stalemates
naive	random	974407 (97.44%)	23872 (2.39%)	1721 (0.17%)
risky	random	993154 (99.32%)	6356 (0.64%)	490 (0.05%)

Table 2. Results from running risky and naive strategies against random play, each pair 10^6 times.

The results from playing the risky strategy against the naive, shown in Table 3, are that the risky player won about twice as often as the naive.

P1	P2	P1 wins	P2 wins	Stalemates
risky	naive	678113 (67.81%)	317313 (31.73%)	4574 (0.46%)

Table 3. Results from running risky strategy against naive strategy 10^6 times.

Discussion

The Maths Arcade is an extra-curricular activity used at Nottingham Trent University and several other U.K. universities to involve students and staff in playing and analysing strategy games. A program was written in Python to simulate play of Quarto, a game used at the Maths Arcade. Inspired by game play observed at the Maths Arcade, two strategies were programmed and tested against each other. The first played naively, winning if possible but otherwise proceeding randomly. The second actively tried to move the board into a winning position. As Quarto is an impartial game of perfect information, the second strategy invites either player to win, so was described as more risky. When these strategies were tried against each other 10^6 times, the more risky player won around twice as many games as did the naive player.

It is not possible to conclude that the more risky approach to Quarto is more likely to result in a win, since the result could have been affected by how well the strategies were programmed. Certainly, the strategy employed to match like pieces was not very sophisticated and could be improved. The choice here was to search until the first opportunity to play a like piece arises, then play it.

This strategy is sensitive to the order in which the board is searched and may cause the player to block a better option to match a weaker one. For example, in Figure 3 the player creates a column of two like pieces despite the fact that doing so blocks a row of three like pieces. If it had played in another position in the same column, it would have retained both options.

			1110
0000	0001	0010	<u>1100</u>

Figure 3. The risky algorithm as programmed may play the underlined piece as indicated in order to make a column of two with 1110, even though it blocks a row of three matching in two attributes 00**.

The strategy outlined here does not search ahead for which pieces may be handed to the opponent, and this will cause losses. For example, it is a strategic error to make a line of three black pieces when every piece left to be played is also black.

The strategy outlined also does not take into account other lines of three on the board, and this will cause losses too. For example, it is a strategic error to make a line of three black pieces when there is already a line of three square pieces on the board and every piece left to be played is either black or square.

Other considerations have impact on the strategy, and there are alternative strategies that could be analysed. For example, one player at the Nottingham Trent University Maths Arcade tends to play to block potential winning lines, reducing the number of possibilities rather than working to increase them, with some successes.

Another interesting avenue of investigation is the number of stalemates that arise. We might expect that, as players get better, stalemates may become more common (in parallel with Tic Tac Toe), and this is observed at the Maths Arcade, but this is not apparent from the data here. For example, it is interesting that the naive player against random finds more stalemate positions than risky against random. And, indeed, that random playing against random finds more stalemates than the other strategies playing themselves. The cause may be that the risky player is building opportunities to win, but not necessarily being very careful about whether the win may occur on the opponent's move. A more sophisticated pair of players might well generate more draws.

A student wishing to undertake a project in this area would be well advised to investigate relevant areas of combinatorial game theory. Various claims are made in this article that should be identified and verified. Where strategy decisions have been described, these should be re-considered. Work might begin by developing a game system capable of keeping track of the board and available pieces, then running two players with random play, before attempting to program more sophisticated strategies. Such a game system should be designed in a way so that different game strategies can easily be assigned to each player. It should also have routines so that a single game can be displayed in progress graphically or multiple games can be played with only a summary of wins, losses and draws at the end displayed. The former option would be to observe game play as expected, while the latter is for simulation work.

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