PURPOSEFULLY PLANNING FOR MATHEMATICS DISCOURSE: A STUDY OF TEACHER LEARNING AND LESSON ENACTMENT

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PURPOSEFULLY PLANNING FOR MATHEMATICS DISCOURSE:
A STUDY OF TEACHER LEARNING AND LESSON ENACTMENT

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Mathematics discourse plays a crucial role in students’ learning of mathematics, yet facilitating productive mathematics discourse is a complex task for teachers. This embedded single-case study sought to explore teacher learning around planning for and facilitating mathematics discourse and capture any changes in the nature of classroom discourse as a result of teacher learning. Teacher learning was facilitated through a professional learning community (PLC). The PLC was designed to honor, build upon, and develop teachers’ expertise in purposefully planning opportunities for student talk during mathematics instruction. It also attended to the teachers’ contexts by asking teachers to incorporate math talk strategies into their adopted non-reform mathematics textbook. The participating teachers self-facilitated the PLC using a protocol designed by the researcher, which blended elements of best practice in professional development for mathematics teachers and strategies for facilitating mathematics discourse. The teachers completed three mini-design cycles in which they selected a math talk strategy, co-planned a lesson, enacted the lesson, reflected on the experience within the PLC, and worked with their colleagues to revise the lesson based on their learning.

In examining teacher learning in the case of the PLC and examining the experience of the embedded cases of each participating teacher’s classroom, several themes emerged: (a) teachers had to re-establish classroom norms and expectations for math talk; (b) students began to drive the mathematics learning; (c) the nature of math talk in the classroom began to change; and (d) teachers had to negotiate tensions with their adopted text in terms of instructional format and pacing. While there was evidence of changing instructional practice around facilitating mathematics discourse, significant change was difficult to realize given the study’s short timeframe. However, the teachers were successful in self-directing their learning and some of them were beginning to take steps to challenge the nature of mathematics teaching presented in their adopted non-reform textbook. Therefore, while this professional development approach shows promise in terms of supporting teachers to self-facilitate instructional change around productive mathematics discourse, additional study for a longer timeframe is recommended.
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Chapter 1

A Study of Mathematics Discourse and Teacher Learning

The teaching and learning of mathematics is a complex endeavor. Central to all learning, including mathematics, is the role of language and social interactions in that process. It is through our use of language that we negotiate meaning with others; that is, we communicate our thinking, consider the thinking of others, and, together, confirm or revise our understanding collectively through shared meaning making, as well as individually (Cai & Lester, 2010; Vygotsky, 1962, 1978; Zwiers & Crawford, 2011). Therefore, “mathematical discourse among students is central to meaningful learning of mathematics” (NCTM, 2014, p. 35).

It is because of this crucial role that discourse plays in learning that there has been an effort over the past two decades among the mathematics education community to increase research about mathematics discourse, as well as systematically promote teaching reforms that incorporate more opportunities for students to engage in mathematics discourse. As Smith and Stein (2011) noted,

In mathematics classrooms, high-quality discussions support student learning of mathematics by helping students learn how to communicate their ideas, making students’ thinking public so it can be guided in mathematically sound directions, and encouraging students to evaluate their own and each others’ mathematical ideas. (p. 1)

Additionally, Franke, Webb, Chan, Ing, Freund, and Battey (2009) found that “student talk can lead to increased student mathematical knowledge and understanding” because
listening to student talk informs both teachers and classmates about a student’s mathematical thinking and “the act of talking itself” leads to deeper understanding when the student must explain and justify (p. 381). Thus, we can see the critical role that mathematics discourse plays in helping students to reason, co-construct meaning, and learn mathematics deeply.

**Statement of the Problem**

Despite efforts to reform mathematics instruction over the past several decades to include opportunities for students to engage in discourse as means of thinking, reasoning, and learning, many educators continue to envision mathematics teaching as “memorizing facts, formulas, and procedures and then practicing those skills over and over again” (NCTM, 2014, p. 9). Hull, Balka, and Miles (2013) described the nature of discourse situated in traditional mathematics teaching, which heavily focuses on procedural skill and fluency, as “the teacher poses the problem, the students work independently on a solution, the teacher asks for an answer, hands are raised, and a student responds” (p. 54). Such interactions, which tend to characterize “transmission styles” of instruction, leave few opportunities for students to explain, justify, or construct their own meaning (Truxaw & DeFranco, 2008, p. 491). As NCTM (1991) noted,

> Because the discourse of the mathematics class reflects messages about what it means to know mathematics, what makes something true or reasonable, and what doing mathematics entails, it is central to both what students learn about mathematics as well as how they learn it. (p. 54)

Furthermore, the adopted mathematics curriculum, especially the textbook, greatly influences what and how students learn (NCTM, 2014), just as it often reflects the culture
and beliefs of a school and/or district in terms of the nature of teaching and learning mathematics. Many textbooks reflect transmission styles of instruction, which rely on direct instruction in whole class settings where teachers tell students procedures for solving mathematics problems and the students practice those procedures with few opportunities to engage in discussion. As Cai and Lester (2010) noted, “Students’ actual opportunities to learn depend on…the kinds of classroom discourse that takes place during problem solving, both between the teacher and students and among students” (p. 3). Therefore, both teacher beliefs about the nature of mathematics learning and the adopted curriculum can create barriers for students to engage in mathematics discourse as a means for meaningful learning.

The question then becomes, if discourse is necessary for mathematics learning, how do we support teachers in moving away from more traditional methods of teaching mathematics and opening opportunities for students to engage in mathematics discourse? Unfortunately, there is no easy answer. Facilitating productive mathematics discourse is very complex; therefore, providing teachers with a set of instructional practices to implement is not enough to necessarily impact student learning (NCTM, 1991; Truxaw & DeFranco, 2008). While there are a variety of research-based best practices in facilitating productive mathematics discourse, there are also a variety of factors that influence the success of implementing these practices, such as the adopted curriculum, teacher beliefs about mathematics teaching and learning and/or student capacity for learning, opportunities for teacher collaboration and coaching support, and the nature of the classroom community, just to name a few. Therefore, any professional development
program designed to support teachers in changing their practice around facilitating productive mathematics discourse will have to account for these many complexities.

**Purpose of the Study**

The purpose of this qualitative study was to explore what impact, if any, a professional development program had on teacher learning about planning for and facilitating mathematics discourse as well as what changes in mathematics discourse, if any, occurred in the classrooms of the participating teachers. It aimed to support teachers as they collaboratively co-planned lessons that increased opportunities for students to engage in mathematics discourse in whole class discussions “within the complexity of the actual classroom setting” (Zawojewski, Chamberlin, Hjalmarsone, & Lewis, 2008, p. 219). That is, the professional development was designed to complement the context of the participating classrooms by building on the adopted mathematics curriculum, which happened to be a more traditional textbook that structured most of the lessons as whole class direct instruction. It was my hope, as the researcher, that by purposefully attending to the participating teachers’ context, especially with regard to the adopted textbook, that teachers would begin to see and create opportunities to open spaces in the curriculum for mathematics discourse. Therefore, this study sought to document teacher learning around planning and facilitating mathematics discourse in the collaborative setting of a professional learning community as well as capture any changes in the nature of the whole class mathematics discussions in the participating teachers’ classrooms.

**Research Design Overview**

This qualitative study sought to explore how teachers purposefully planned for discourse as a part of their mathematics instruction. The professional development was
designed to provide support to teachers working collaboratively in a professional learning community (PLC) to co-plan lessons that incorporated more opportunities for mathematics discourse. These lessons were based on the teachers’ current adopted mathematics curriculum, which complemented their current resources and context. At the same time, the professional development was designed to help teachers learn research-based best practices in facilitating productive mathematics discussions and provide the space for them to try to implement some of these effective instructional practices in their classrooms. Therefore, this study was designed to capture teacher learning, create educational objects that reflect teachers’ learning, and document any impacts teacher learning had on the nature of mathematics discourse in the classroom.

The study used multiple sources of qualitative data, including observations, lesson plan artifacts, co-created documents, and interviews. Data was collected before, during, and after the participating teachers’ work in the professional learning community to document any changes with regard to teacher learning and the classrooms’ mathematics discourse.

**Research questions.** This study was focused on teacher learning about incorporating discourse into mathematics teaching and any changes in the nature of the mathematics classroom discourse as a result of that teacher learning. Therefore, this study was focused on two levels of inquiry: (1) what teachers learned and applied with regard to planning and facilitating mathematics discourse; and (2) how the nature of the discourse community in the mathematics classroom may have changed because of the enactment of those plans. To that end, this study explored two central questions:

- How did teachers learn about planning for and facilitating mathematics discourse as a result of participating in a professional learning community (PLC)?
In what ways did a teacher’s participation in the PLC impact the nature of whole class discussion in the mathematics classroom during the enactment of the co-planned lessons and beyond that enactment?

In using these central questions to frame the research design, this study sought to understand what impact, if any, this professional development in the form of a PLC had on teacher learning, students’ mathematics discourse, and the nature of discourse in the mathematics classroom.

**Definition of Terms**

The following definitions were used in order to clarify key terms in this study and to provide a common language for readers.

**Mathematics discourse.** Mathematics discourse is “the ways of representing, thinking, talking, agreeing and disagreeing” (NCTM, 1991, p. 34) that “happens among students and teacher” (Cazden, 2001, p. 60). The National Council of Teachers of Mathematics (NCTM) describes it as “the purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication” (2014, p. 29). For the purpose of this study, we will narrow the definition of mathematics discourse to verbal exchanges about mathematics that take place during whole class discussions among students and between teacher and students.

**Whole class discussion.** Whole class discussions are one type of structure in which mathematics discourse can take place. During whole class discussion, the teacher and all students are involved in conversation about mathematics. The teacher’s role is to “get students to share their thinking, explain the steps in their reasoning, and build on one another’s contributions” (Chapin, O’Connor, & Anderson, 2013, p. 81). Thus, whole
class discussions “give students the chance to engage in sustained reasoning,” where the teacher and students co-construct shared meaning about mathematics (Chapin et al., 2013, p. 81).

**Delimitations**

This study was delimited to the population of the study, the “participating classrooms” from an elementary school (K–5) in western Montana. Classrooms had to meet the pre-established inclusion criteria, which are detailed in Chapter 3, in order to be considered a participating classroom. Two grade levels were selected with at least two participating classrooms in each grade level. This was not a school-wide study because I acted as the researcher as well as a participant-observer in the professional learning community. Therefore, it was designed to be a small-scale study.

The study was also delimited to a focus on whole class mathematics discussion with an *emphasis* on verbal exchanges. Mathematics discourse is a broad term that includes “the purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication” (NCTM, 2014, p. 29). These exchanges can take place between students, between a teacher and a student, and among students and teacher. Moreover, these exchanges might be situated within partner, small groups, or whole group settings. Additionally, the form of communication can be verbal, visual, writing, or combinations of these forms. Thus, mathematics discourse is broad and varied. So, in order to keep this study manageable and focused, I chose to delimit the scope of mathematics discourse to whole group discussions with an *emphasis* on verbal exchanges. The choice to focus on whole group discussion was driven by the adopted mathematics curriculum used in the selected elementary school, as
the primary setting for instruction in the textbook was whole class. The choice to focus on verbal exchanges was that they were a primary means of communication during a whole class discussion. Also, verbal exchanges were public reflections of mathematics thinking on the part of the students and public reflections of instructional moves on the part of the teacher, which allowed for documentation and analysis. However, all data, including written student solutions or visual representations, were documented during the whole class discussion in order to capture and illuminate the context of the discourse.

Finally, the study was delimited to a review of the literature that defines mathematics discourse research in terms of effective instructional practices. There are different branches of mathematics discourse research, including those that focus on instructional practices teachers can use to facilitate productive discourse, those that focus on issues of power and position, and those that focus on linguistic analysis of the discourse, often drilling-down to word-level analysis (Ryve, 2011). Because this study focused on supporting teachers in planning for and facilitating mathematics discourse within the context of a professional learning community, it made sense to delimit the review of research literature to that with a focus on instructional practices teachers can use to facilitate productive discourse.

Limitations

This study had some limitations that could impact the transferability of the findings. One limitation was the potential for the participants to change their behavior or responses based on what may be perceived as socially desirable (Cozby & Bates, 2015). While every precaution was taken to ensure participants felt comfortable to share their experiences honestly during the interviews and behave typically during videotaped
professional development sessions and classroom observations, participants could have modified their responses or behaviors to meet perceived notions of social acceptance. Another limitation of the study was related to its short timeframe. Best practice in mathematics professional development call for intensive and sustained programs that typically last for more than six months and total between 60 and 100 hours (NCTM, 2014). These recommendations were well beyond the scope that I was able to implement in my limited timeframe of approximately 12 weeks to enact the professional learning community. Therefore, the less intensive and sustained nature of this professional development program could have limited its impact on teacher learning and the nature of discourse in the mathematics classroom. Thus, both of these limitations could have impacted my ability to observe transfer of teacher learning into the classroom.

**Significance of the Study**

The significance of this study existed in its design to attend to the complexity of classroom teaching. Each classroom represented multiple layers of context, including teacher beliefs and experiences, student experiences, the adopted curriculum, and the broader school context to name a few; all of which impact the nature of mathematics learning. Moreover, bringing a group of teachers together and negotiating these myriad complexities in the midst of making instructional change highlights the messy reality of schools. Therefore, the researcher sought to “embrace the complexity and the dynamic nature of the system in which teachers learn and grow” by situating this work within a professional learning community that is teacher-driven and attends to the context of the school and the participating teachers (Zawojewski et al., 2010, p. 236).
Thus, teacher learning was at the heart of this study. It was the goal of the researcher, through the design of the professional development to: (1) honor, leverage, and develop teacher expertise in facilitating mathematics discourse; (2) work within the school and classroom context by using and enhancing the adopted non-reform mathematics curriculum; and (3) create a system of collegial support for making instructional changes around mathematics discourse practices that is both sustainable and transferrable. As a result, this study sought to capture the learning of teachers as they co-constructed meaning around mathematics discourse, collaboratively planned and reflected, and implemented instructional strategies to open up spaces for students to talk about mathematics.

By situating the professional development within the context of the adopted non-reform mathematics curriculum, this study also sought to fill a gap in the literature about teacher learning around mathematics discourse. Most studies about effective instructional practices to facilitate productive mathematics discourse are either add-ons to the curriculum or they are situated within reform-based curricula that already reflect a student-centered pedagogy. Therefore, this study sought to explore and understand how teachers, working within the confines of their adopted curriculum, collaboratively planned for and facilitated mathematics discourse in a professional learning community. It also sought to capture any changes that took place in the teachers’ classrooms with regard to the nature of the mathematics discourse.

Summary

There is a need to increase opportunities for mathematics discourse as a regular part of instruction because of the vital role it plays in students learning mathematics.
deeply and meaningfully. However, there are obstacles that exist in terms of teacher beliefs about the nature of mathematics learning and in terms of how many curricula focus on a more traditional transmission style to teaching mathematics. Additional challenges exist when taking into account the complex nature of facilitating productive mathematics discussions across the varied contexts of classrooms. While there are a variety of research-based best practices for facilitating productive mathematics discourse, there is no perfect combination that guarantees student learning. Therefore, professional development programs for mathematics teachers must find ways in which to acknowledge the complexities of facilitating discourse and the varied contexts of classrooms while also providing the space for teachers to develop their repertoire of effective discourse practices in their classrooms. The review of literature that follows explored these issues and the research about them in greater depth.
Chapter 2

Review of the Literature

In the past two decades, there has been an increased research focus on discourse and student talk in the mathematics classroom. Many teachers tend to use a more traditional “transmission style” of instruction, which leaves little opportunity for students to explain, justify, or construct their own meaning (Truxaw & DeFranco, 2008, p. 491). Reform efforts in mathematics education have pushed for more purposefully planned and facilitated student talk about mathematics. These efforts to increase the use of discourse are based on the belief that “students learn mathematics best when they are given opportunities to speak about mathematics using the language of mathematics” (Cirillo, 2013a).

The focus on discourse in the mathematics classroom traces back to the reform movement of the late 1980s and continues through today with the advent of the Common Core State Standards for Mathematics (CCSSM). The National Council of Teachers of Mathematics (NCTM) has played a pivotal role in setting expectations for mathematics instruction, including a strong emphasis on communication. In both the Curriculum and Evaluation Standards for School Mathematics (1989) and the Principles and Standards for School Mathematics (2000), NCTM detailed the importance of student discourse within the strand of communication. They noted the need for students to have opportunities to engage in “math talk,” because “interacting with classmates helps children construct knowledge, learn other ways to think about ideas, and clarify their own thinking” (NCTM, 1989, p. 26). The Principles and Standards for School Mathematics
(2000) laid out specific standards around communication across the K–12 grade span, which included students “communicating their mathematical thinking coherently and clearly to peers, teachers, and others” (p. 61). At the core of these communication standards is the expectation that students are integral participants in classroom discourse, which encompasses talk around explanation, justification, questioning, and sense making.

More recently, there has been a continued focus on discourse in the mathematics classroom, which is reflected in the CCSSM and Principles to Actions (NCTM, 2014). While the CCSSM do not intend to prescribe particular pedagogical approaches or curriculum, the standards do establish specific “processes and proficiencies” based on the NCTM Process Standards and the work of the National Research Council in Adding It Up that are reflected in the Standards for Mathematical Practice (NGA Center & CCSSO, 2010). The need to communicate or engage in discourse is inherent in many of the Mathematical Practices, especially when students are asked to “construct viable arguments and critique the reasoning of others” and when students are expected to “attend to precision” when sharing their reasoning during discussions (NGA Center & CCSSO, 2010, pp. 6–7). In an effort to bring implementation of the CCSSM to fruition, NCTM identified eight Mathematics Teaching Practices in their publication, Principles to Actions, which reflect research-based best practices that will facilitate deep learning of mathematics. Among these practices is one focused on discourse: “Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments” (NCTM, 2014, p. 29). Key in both the Standards for Mathematical Practices and the
discourse Mathematics Teaching Practice is the expectation that teachers must facilitate productive student talk about mathematics.

**Theoretical Framework: Learning through Discourse**

Classroom discourse lies at the heart of learning. Learning is a “socially mediated process,” where language is a tool that children acquire through their social interactions with others and the internalization of that language results in learning (Schunk, 2012, pp. 251–252). Thus, academic conversations allow students to engage in and develop language as they use it for real purposes through talking, listening, and negotiating meaning (Zwiers & Crawford, 2011). Teaching mathematics through problem solving creates a learning environment that allows students to engage in academic conversations as they “present various solutions to their group or class and learn mathematics through social interactions, meaning negotiation, and reaching shared understanding” (Cai & Lester, 2010, p. 3). Key to learning, regardless of the content area, is the act of negotiating meaning while engaging in discourse, as it is this process in which students must articulate their thinking, consider the thinking of others, and, together, confirm or revise their understanding both collectively and individually.

However, talking or having conversations during school is not enough to result in learning. Rather, students must engage in *productive academic* conversations for learning to occur, which might include actions such as negotiating meaning, deeply engaging with a topic, co-constructing ideas, clarifying thoughts, and supporting opinions (Zwiers & Crawford, 2011). Therefore, the role of the teacher in facilitating these productive academic conversations is key as to whether or not children will learn as a result of engaging in discourse. As Cazden (2001) noted, “classroom discourse happens
among students and teacher” (p. 60). Thus, both students and the teacher have important roles to play as members of a discourse community as they engage in the process of co-constructing meaning through productive academic conversations.

**Vygotsky’s social constructivism.** The role of language and social interactions in learning are central to Vygotsky’s social constructivism perspective and provide a lens through which to understand how students learn when engaging in classroom discourse. Vygotsky (1962) believed that the main function of speech was to communicate, to interact socially; that is, speech acts as the medium by which humans share their thoughts and experiences with others. It is through interactions, or discourse, that meaning is shared and negotiated among learners, both interpersonally and intrapersonally. Therefore, social constructivism is based on the premise that language is central to human development, and hence, discourse is a reflection of this “shared meaning-making process” (Stetsenko & Arievitch, 1997, p. 162). Because of the key role that language plays in learning, classroom discourse should be carefully considered in the context of mathematics learning for these reasons:

(a) mathematics is a specialized form of literacy; (b) spoken language is a primary mode of teaching and learning; (c) the particular context in which language is used plays a role in what is appropriate to say and do; and (d) language is intimately related to culture and identity. (Herbel-Eisenmann, 2009, p. 5)

Stetsenko and Arievitch (1997) described three “grounding” assumptions about social constructivism: (1) the individual is an active participant in making meaning; (2) learning and development is situated within “culturally and historically defined contexts;” and (3) language is a “social mediator” of individual learning (pp. 160–161). Like
constructivism, social constructivism positions the learner as actively involved in making sense of concepts by integrating them into already established schema or modifying the schema to accommodate the new understanding. However, social constructivism conceives this meaning making as “not a natural process but as a social artifact;” that is, learning is a reflection of socially constructed meaning and not something that happens within the individual learner in isolation (Stetsenko & Arievitch, 1997, p. 161). In fact, Vygotsky (1978) explained how learning happens through social interaction:

*An interpersonal process is transformed into an intrapersonal one.* Every function in the child’s cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (*interpsychological*) and then inside the child (*intraspsychological*). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher [mental] functions originate as actual relations between human individuals. (pp. 60–61)

Vygotsky’s explanation reinforces the idea that learning (and language) is first social and second personal. That is why Vygotsky “considered the social environment critical for learning and thought that social interactions form learning experiences” (Schunk, 2012, p. 242). Therefore, classrooms can provide the rich social environment that is necessary for learning, and discourse can serve as opportunities for social interactions in which language is used to co-construct meaning among students and the teacher. In the classroom, mathematics discourse (or language) “functions as a psychological tool when students put their mathematical ideas into words and as a cultural tool for sharing and joint construction of knowledge” (Adler, 1999, p. 55). Likewise, teacher collaboration
within the context of a professional learning community also relies on language as a means for teacher learning. That is, teachers work together to co-construct meaning about their practice. Thus, we can see parallels in terms of how language can be a mediator of learning for both students and teachers.

**Lave and Wenger’s theory of learning.** Lave and Wenger (1991) offered a theory of learning that seeks to understand how participation in *communities of practice* leads to individual and collective knowing within a given context. Thus, situated learning is “a transitory concept, a bridge, between a view according to which cognitive processes (and thus learning) are primary and a view according to which social practice is the primary generative phenomenon, and learning is one of its characteristics” (Lave & Wenger, 1991, p. 34). That is, learning is an outgrowth of social interaction. It is a “process that takes place in a participation framework, not [just] in an individual mind. This means, among other things, that it is mediated by the differences of perspective among the coparticipants” (Hanks, 1991, p.15). Hence, participation in these communities of practice is both “personal and social” and it is an “active process” in which learning happens both individually and collectively through negotiating meaning (Wenger, 1998, p. 55). Thus, the work of Lave and Wenger (1991) and Wenger (1998) reflect Vygotsky’s social constructivism, as learning is situated as a social endeavor. While situated learning is not a theory of education or a pedagogical approach, it is “an analytic viewpoint on learning, a way of understanding learning” that we can use to examine how a community of practice engages in learning through the use of language and the co-construction of meaning (Lave & Wenger, 1991, p. 40).
While Lave and Wenger (1991) purposefully avoided considering classrooms as one of the situated learning contexts they investigated, classrooms do function as communities of practice. Classrooms are situated within multiple contexts, including the broader school community, the backgrounds and perspectives brought by each student, and the teacher’s pedagogical philosophy to name a few. Thus, each classroom functions as a community of practice in which students and the teacher co-participate to negotiate meaning. Discourse provides a space for this co-construction of knowledge because “language is a part of practice, and it is in practice that people learn” (Lave & Wenger, 1991, p. 85). Thus, for students to become participating members of a mathematics discourse community of practice, they must learn how to speak mathematically, both in terms of language and the norms of the classroom (Adler, 1999). Furthermore, mathematics language is a means for learning the practice of doing mathematics, and the teacher serves a valuable role in facilitating those learning experiences during discourse through the questions and structures used to move the discussion along.

Likewise, professional learning communities (PLCs) also function as communities of practice. PLCs are comprised of teachers working collaboratively to grow their practice and positively impact student learning. These professional learning communities are often organized around grade level teams; therefore, PLCs exist within multiple contexts, including the individual experience of each teacher, the dynamics of the grade level team(s), the expectations and/or mandates from administrators to be satisfied within those PLCs, and how that all fits into the school’s culture. Thus, teachers must negotiate these layers of context and, in doing so, they create a shared language of reflective practice and change that is both individual and collective. “Teachers’ learning
and thus their professional development is seen as situated in forms of co-participation in joint activities, for which they have mutually held goals and to which they bring varying levels of expertise” (Potari, Sakonidis, Chatzigoula, & Manaridis, 2010, p. 475).

Therefore, we can see that communities of practice can be a powerful framework through which to make sense of learning for students and teachers.

**What is Mathematics Discourse?**

Before we delve into the many practices that can support productive mathematics discourse, it is necessary to define what we mean by discourse. Ryve (2011) reported from a meta-analysis of 108 journal articles about discourse research in mathematics education found that only about 20% of them explicitly defined “discourse” in their article. For the purposes of this literature review, discourse is being used to describe social interactions, typically, conversations that take place in the mathematics classroom. Thus, discourse can be thought of as a “dialogue between at least two voices where some form of transformation takes place and new meaning is generated” (Truxaw & DeFranco, 2008, p. 491). In *Principles to Actions*, NTCM describes discourse as “the purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication” (2014, p. 29). To further narrow our scope of discourse, this literature review will focus on verbal communication during classroom discussions as the primary form of discourse examined. As Chapin, O’Connor, and Anderson (2013) noted, “it takes students a great deal of practice to become solid and confident mathematical thinkers, and the whole-class discussion talk format provides a space for that practice” (p. 81).
Facilitating Mathematics Discourse

As teachers work to increase mathematics discourse in their classrooms, it is important to acknowledge that just having students talk more is not enough; rather, mathematics discussion must be purposefully crafted in order for it to be productive (Smith & Stein, 2011; Truxaw & DeFranco, 2008). As Smith and Stein (2011) stated, Many mathematics teachers believe that students learn through sharing their ideas, listening to and critiquing the ideas of others, and by having others critique their approaches to problem solving. Classroom discussions in which these activities occur do not materialize out of thin air. (p. 69)

Talking about mathematics is not something that students will naturally know how to do (NCTM, 1989, 2000). Therefore, teachers need to facilitate mathematics discourse, especially in terms of inviting students to share and explain their thinking, as well as asking probing questions to uncover students’ mathematical understanding (NCTM, 1989, 1991). As a word of caution, Truxaw and DeFranco (2008) noted that facilitating effective mathematics discourse is more complex than just giving teachers certain components to use or moves to make; rather, such recommendations must be considered within the context of a teacher’s pedagogical approach as well as the classroom community.

Classroom community. Creating and maintaining a classroom community that is safe, respectful, and encourages student participation in discourse is essential for mathematics learning. Teachers and students should work together to establish a community where it is safe to take risks and all members are expected to engage in sense making (Anthony & Walshaw, 2009; NCTM, 1991, 2000). Involving students in the
process of establishing such a community will support a shift from teacher-led to student-led discussion (Cirillo, 2013a). That process should include developing norms for mathematics discussions, including how to question each other’s ideas and how to respectfully disagree with someone (Cirillo, 2013a).

As part of creating and maintaining a positive mathematics discourse community, one must consider the goals of the community, both in terms of learning and social relationships. Burns (2012/2013) asserted “developing reasoning strategies and understandings should become part of the classroom culture, and at the forefront” (p. 46). Likewise, Middleton and Jansen (2011) advocated for framing the mathematics classroom as a knowledge building community. In a knowledge building community, members work from the belief that ideas can and should be improved; thus, the mathematics conversation does not end once a solution or strategy has been shared (Middleton & Jansen, 2011). Moreover, a knowledge building community grows out of the belief that learning is a social endeavor. “Its members are interdependent…they need to hear from one another” to make sense of and grow the collective understanding of the mathematics (Middleton & Jansen, 2011, p. 164). One potential obstacle to enacting a knowledge building community was uncovered by Bray (2011) when she found that teachers who viewed their students’ learning capacity as limited were reluctant to encourage collaboration among students and tended to use the traditional instructional method of teacher as teller when errors occurred during whole class discussions.

In a positive and supportive mathematics classroom, all members of the community should feel that their contributions to the discourse are valued. All students, regardless of their cultural or linguistic backgrounds, bring a wealth of mathematical
knowledge and perspectives to their classroom communities that should be shared and built upon (Anthony & Walshaw, 2009; Ernst-Slavit & Slavit, 2007). Cribbs and Linder (2013) found in their case study of one fifth-grade teacher that she leveraged student funds of knowledge as a means to build mathematical understanding. This teacher encouraged students to “reflect on their prior knowledge and experiences outside of the classroom in order to make meaning of the mathematics content,” such as experiences with family and community members (Cribbs & Linder, 2013, p. 77). Because the teacher positioned students’ funds of knowledge as “legitimate ways of knowing and learning mathematics in the classroom,” all students had opportunities to engage in the mathematics discourse knowing that “their knowledge and experiences were valued by the teacher and their peers” (Cribbs & Linder, 2013, p. 76).

Furthermore, teachers should actively position students as mathematically competent through their questions and comments (Kazemi & Hintz, 2014). This type of positioning is especially important for supporting the mathematical (and language) development of students who are English language learners (Turner, Dominguez, Maldonado, & Empson, 2013). Such positioning, however, may be challenging. Atweh, Bleicher, and Cooper (1998) found that “classroom context is informed by and in turn reproduces the construction of mathematics in the wider sociocultural context,” which includes how issues such as gender, socioeconomic status, or language use can impact the discourse (p. 63). Thus, teachers must be cognizant of their perceptions about students’ ability to engage in mathematics in order to provide equitable access to rich discourse.

Another important aspect of creating and maintaining a positive classroom community is for teachers to consider how they will respond to student errors or
mathematical ideas that are not yet fully developed. Kazemi and Hintz (2014) suggested establishing the practice of putting ideas, questions, or confusions out for the community of learners to grapple with collectively. This sends a message that challenges and/or mistakes are a normal part of learning (Cirillo, 2013a; Middleton & Jensen, 2011). In fact, discussing wrong solutions is a powerful practice by which students clarify and refine thinking and confront common misconceptions (Anthony & Walshaw, 2009; Cirillo, 2013a; Kazemi & Hintz, 2014). However, Bray (2011) found that teacher beliefs influenced how they handled errors or mistakes during mathematics discussions. Some teachers believed that sharing errors during whole class discussions would embarrass the student who made the error or cause confusion among students with underdeveloped mathematical concepts, but others found it to be an effective means by which to revise thinking and deepen conceptual understanding (Bray, 2011). A key element in supporting students revising their thinking is to ensure the teacher returns to the student with whom the idea originated and attribute that revised thinking to him/her. This practice is especially important for students who are English language learners as it positions them as mathematically competent (Turner et al., 2013).

**Math-talk learning community.** Research around math-talk learning communities comes from the work of Hufferd-Ackles, Fuson, and Sherin (2004, 2014). A math-talk community is defined as “a classroom community in which the teacher and students use discourse to support the mathematical learning of all participants,” where the goal is for participants to “understand and extend” their thinking and the thinking of everyone in the community (Hufferd-Ackles et al., 2014, p. 125). This definition reflects
and reinforces the idea that discourse promotes a shared responsibility for learning mathematics (Cirillo, 2013a, 2013b; NCTM, 2000).

Hufferd-Ackles et al. (2004, 2014) developed and refined the Math-Talk Learning Community (MTLC) Framework in order to identify characteristics of mathematics discourse communities across a continuum from teacher-led to student-led discourse. The MTLC Framework is comprised of four levels (0, 1, 2, 3) that progressively shift authority from teacher to the learning community within five components: teacher role, questioning, explaining mathematical thinking, mathematical representations, and building student responsibility within the community (see Appendix A). Growth in the level on the MTLC Framework was, for the most part, concurrent across the components (Hufferd-Ackles et al., 2004, 2014). Not surprisingly, the longest time of transition was from Level 1 to Level 2, as that change marks the most significant shift in practice from teacher-led to student-led discussion (Hufferd-Ackles et al., 2004). Additionally, Hufferd-Ackles et al. (2004) found that when new mathematics concepts were introduced, there was a brief return to more teacher-led discourse as new vocabulary and other concept-specific language were introduced and modeled.

In addition to describing the nature of mathematics discourse, the MTLC Framework is also a helpful tool for supporting teachers in making instructional change. In fact, Hufferd-Ackles et al. (2004) found that the MTLC Framework was “accessible and doable” for teachers and it could be used as a scaffold for changing instructional practice (p. 113).

**Purposefully planning for discourse.** As noted earlier, productive mathematics discourse does not just *happen*. That is, teachers must purposefully plan how they will
facilitate mathematical discussions to engage students in reasoning, justifying, and sense making. Likewise, “the discourse should not be a goal in itself but rather should be focused on making sense of mathematical ideas and using them effectively in modeling and solving problems” (NCTM, 2000, p. 194). To that end, it is imperative that teachers have a clear goal in mind when planning a math discussion (Kazemi & Hintz, 2014; NCTM, 2014; Smith & Stein, 2011). Different foci require different structures; for instance, structuring a discussion to prove or justify a mathematical idea will look and sound different than how a discussion is structured in which the teacher is trying to elicit a variety of different strategies for a given problem (Chapin et al., 2013; Kazemi & Hintz, 2014; Zwiers & Crawford, 2011). Thus, establishing goals to focus mathematics learning during a discussion is one of the first steps a teacher can take toward effectively facilitating mathematics discourse.

Another key element in purposefully planning for productive mathematics discourse is selecting a quality task as the impetus for discussion. Smith and Stein (2011) emphasized the need for teachers to choose appropriate tasks, in terms of cognitive demand, multiple entry points, and multiple pathways, which is echoed by the work of other researchers (Anthony & Walshaw, 2009; Cai & Lester, 2010; Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013; Middleton & Jansen, 2011; Trocki, Taylor, Starling, Sztajn, & Heck, 2014/2015; Wenrick, Behrend, & Mohs, 2013). Ni, Zhou, Li, and Li (2014) conducted a study of fifth-grade classrooms in China and they found that teachers were more likely to explore their students’ ideas and/or require their students to explain and justify solutions when using tasks that had high cognitive demand and/or allowed for multiple representations. It is, however, important to keep in mind that teacher beliefs
about the capacity of students for learning and/or what it means to *do mathematics* can impact task selection. For example, Muir (2008) found that some elementary teachers selected more procedurally oriented tasks to teaching numeracy concepts because they believed that would get students “ready for high school” (p. 92). Thus, we must be mindful of perceptions such as this because if teachers do not begin with rich tasks, then there will be little opportunity for students to engage with challenging mathematics in a meaningful way.

Moreover, setting up the task is a key aspect in supporting student learning (Jackson et al., 2013). If fact, the manner in which teachers and their students explicitly discuss key contextual features, key mathematical ideas and relationships, and develop common language impacts whether or not and how students engage with the task (Jackson et al., 2013). Findings from Jackson et al. (2013) suggested that purposefully setting up a task may be linked to students’ opportunities to learn during whole class discussions that wrap up the task in a middle school setting. Similarly, Trocki et al. (2014/2015) investigated the use of teacher think-alouds as a means to launch discourse-rich lessons in elementary classrooms. They found that it was challenging for teachers to not do too much of the thinking for students, so the think-alouds had to be carefully pre-planned (Trocki et al., 2014/2015). Yet Trocki et al. (2014/2015) also found that the teacher think-alouds flowed into rich mathematical conversations among the students about conjectures and different solution strategies and/or solutions. So we can see that not only is selecting a quality task necessary for rich discussion, but the manner in which the task is presented also impacts how students participate in mathematics and the ensuing discourse.
Questioning also plays a crucial role in facilitating mathematics discourse. To support rich mathematics discussions, teachers should purposefully pre-plan questions (Cirillo, 2013a; NCTM, 2014). Smith and Stein (2011) recommended using a variety of questions, especially questions that explore and connect mathematics ideas, probe student thinking, and invite students to contribute to the discourse, and they cautioned teachers to avoid asking leading questions.

These recommendations are echoed in NCTM’s *Principles to Actions* (2014) in which two patterns of questioning were explicated: focusing and funneling. When teachers use a focusing pattern of questioning, they try to understand what students are thinking, they press them to explain their ideas clearly and with precision, and they require students to engage with each other’s thinking (NCTM, 2014). Herbel-Eisenmann and Breyfogle (2005) found that focusing question patterns “encourage students to participate, show that students’ thinking is valued, and helps [students] clarify their thinking” (p. 484). Conversely, when teachers use a funneling pattern of questioning, they often use a series of closed questions that lead students to an answer using a procedure that may not be connected to the students’ thinking (Herbel-Eisenmann & Breyfogle, 2005; NCTM, 2014). Therefore, pre-planning questions is a key practice teachers can use to support them in facilitating productive mathematical discourse.

In addition to the initial questions posed to students, teachers must also consider the types of questions they ask as follow-ups to student responses and how that impacts mathematics discourse. Franke et al. (2009) examined elementary teachers’ questioning practices after an initial question-response interaction and found that follow-up questions do not necessarily result in students elaborating or explaining their thinking further. If
students had incomplete or incorrect explanations, teachers who asked a “probing sequence of specific questions” supported those students most effectively in modifying their explanations (Franke et al., 2009, p. 390). Additionally, Franke et al. (2009) found that single questions were rarely sufficient to undercover student thinking. Thus, while asking follow-up questions are key to helping students understand mathematics more deeply and contribute to the classroom discourse, a series of probing questions appears to be most effective in supporting students in articulating their mathematical thinking.

While teachers cannot purposefully plan specific teachable moments that might arise during mathematics learning, they can plan to notice and act upon opportunities to build on student thinking. Key to recognizing and capitalizing on a teachable moment is the teacher’s ability to listen to students. As Burns (2012/2013) noted, “One of the challenges of teaching is to listen to how students reason, rather than listening for responses we expect to hear” (p. 43). Muir (2008) proposed capturing teachable moments as a teacher action that contributes to effective teaching of numeracy. More recently, Leatham, Peterson, Stockero, and Van Zoest (2015) offered a framework for identifying a Mathematically Significant Pedagogical Opportunity to Build on Student Thinking (MOST). MOSTs are characterized by three components: (1) student mathematical thinking (can be inferred and has a related mathematical point); (2) mathematically significant (accessible and central goal for learning); and (3) pedagogical opportunity (intellectual need and right timing) (Leatham et al., 2015, p. 103). The MOST Analytic Framework can be used to identify teachable mathematics moments during classroom discourse in a variety of settings, including whole class discussions, and can help teachers pursue a line of questioning or investigation that values and builds upon
student mathematical thinking. At the same time, Lobato, Hohensee, and Rhodehamel (2013) cautioned that “teachers play an important role in directing students’ attention toward or (unintentionally) away from what is centrally important for students to notice for a given topic” (p. 845). Thus, teachers must be cognizant of how they rename or highlight the mathematics functions as discourse moves and may well influence what students notice and how they reason about the mathematics (Lobato et al., 2013).

**Teacher discourse moves.** Mathematics education researchers have identified some specific teacher moves that are effective in facilitating rich mathematics discussion. Chapin et al. (2013), Cirillo, Steele, Otten, Herbel-Eisenmann, McAneny, and Riser (2014), and Smith and Stein (2011) identified specific discourse moves that teachers can use to engage and extend mathematics discussions: waiting, revoicing, asking students to revoice, creating opportunities to engage with another’s reasoning, and inviting student participation.

Waiting is about providing students with time to process and think during a discussion (Chapin et al., 2013; Cirillo et al., 2014; Smith & Stein, 2011). In a study about wait time in middle school mathematics and language arts classrooms, Tobin (1986) found that increased wait time led to fewer instances of the teacher interrupting a student and increased length of student responses. Rowe (as cited in Cai & Lester, 2010) found that teachers wait less than one second between asking a question to which they receive no immediate answer and intervening again, which impeded students’ opportunities to engage in classroom discussions. Providing wait time after a question is posed is critical, but it is equally important to provide wait time after a response is made during the discussion (Cirillo et al., 2014).
Revoicing and asking students to revoice, which are two different teacher discourse moves, take place when the teacher or a student restates or rephrases another student’s contribution to the discussion (Anthony & Walshaw, 2009; Chapin et al., 2013; Cirillo et al., 2014; Smith & Stein, 2011; Zwiers & Crawford, 2011). The purpose of revoicing, whether by teacher or student, is to confirm understanding or mark a significant contribution to the discussion (Cirillo et al., 2014; Smith & Stein, 2011). Revoicing is a central strategy for supporting students who are English language learners (Turner et al., 2013). That being said, teachers must guard against revoicing too often, as it can take away the ownership of ideas from the student (Smith & Stein, 2011). Krusi (2009) found through her action research in her middle school classroom that as she limited her revoicing and increased pauses after student responses, more of her students took on the responsibility of revoicing their peer’s comments and student participation in discussions increased. Thus, while revoicing can be an effective discourse-promoting strategy, it must be used carefully and purposefully.

Another important teacher discourse move is creating opportunities to engage with another’s reasoning (Cirillo et al., 2014; Smith & Stein, 2011). These opportunities might include comparing student strategies, trying a specific student’s strategy, agreeing or disagreeing with a solution, or adding to or revising another student’s strategy (Chapin et al., 2013; Cirillo et al., 2014; Smith & Stein, 2011; Zwiers & Crawford, 2011). Barlow and McCrory (2011) suggested purposefully facilitating mathematical disagreements as a means to prompt students to engage in reasoning and discourse. They noted “the discourse that surrounds the disagreement allows students to organize their thoughts, formulate arguments, consider other students’ positions, and communicate their positions
to their classmates” (Barlow & McCrory, 2011, p. 531). Yet, Ni et al. (2014) found that some Chinese fifth-grade teachers did not press students to make connections among strategies when multiple solution methods were shared. The researchers hypothesized that “if the teacher’s intention was to encourage students to talk more, then the teachers therefore were less evaluative of students’ responses” (Ni et al., 2014, p. 37). Thus, this possible tension between a teacher’s need to support students’ social interactions and the need to support students’ analysis of mathematics may represent a professional development need for some teachers in learning how to balance support for both purposes, especially given that helping students make connections among representations and strategies is key for building deep mathematical understanding.

Related to the discourse move of creating opportunities to engage with another’s reasoning is an additional move that Cirillo et al. (2014) emphasized: probing student thinking. When probing student thinking, the teacher is following up to a student’s strategy or solution or question or comment by asking questions to prompt the student to consider his/her own reasoning, explain further, or share his/her thinking with others (Cirillo et al., 2014). Chapin et al. (2013) described the process of probing student thinking as a “press for reasoning” (p. 21). Wenrick et al. (2013) encouraged teachers to make probing student thinking a classroom norm, regardless of whether or not the answer was right or wrong; that is, students should expect to be asked about their mathematical thinking and reasoning whenever they contribute to the classroom discourse.

Inviting student participation is yet another key teacher discourse move. Inviting student participation encompasses both soliciting multiple ideas or solution strategies and involving multiple students in the discussion (Cirillo et al., 2014; Smith & Stein, 2011).
Rigelman (2011) found that structuring student interactions by providing for “private think time” first and followed with “pair shares” throughout whole class discussions invites all students to actively participate in the meaning making process (p. 194). Middleton and Jansen (2011) asserted that teachers need to open up possibilities for how students participate in mathematics discussions. They offer a variety of ways in which students can contribute to the classroom discourse, including asking questions, offering alternate solutions, making connections among representations or strategies, explaining or justifying, making conjectures, and sharing partially developed ideas (Middleton & Jansen, 2011, p. 170).

Closely related to the discourse move of inviting student participation is the work of Schleppenbach, Perry, Miller, Sims, and Fang (2007), which focused on extended discourse. Extended discourse “represent moments in the classroom dialogue in which a student has already provided a correct answer and is still required to discuss, explain, or justify that answer” (Schleppenbach et al., 2007, p. 381). In a sense, extending the discourse invites further student participation. Interestingly, when Schleppenbach et al. (2007) compared the use of extended discourse in U.S. and Chinese classrooms, they found that Chinese classrooms engaged in more extended discourse than U.S. classrooms and for longer periods of time. Perhaps implementing the teacher discourse moves suggested by Chapin et al. (2013), Cirillo et al. (2014), and Smith and Stein (2011) would support U.S. teachers in facilitating more rich mathematics discussions.

have been set and an appropriate task has been selected, then teachers can use Smith and Stein’s five practices—anticipating, monitoring, selecting, sequencing, and connecting—to engage students in rich mathematical discussions (2011, pp. 8–11): First, teachers must anticipate what strategies students might use for a given task, which is usually accomplished by solving the problem in as many ways as possible. Additionally, teachers should identify any potential misconceptions that might arise. Second, teachers should monitor students while they are working by watching and listening, as well as probing student thinking with questions. Teachers should also make note of which strategies students are using to solve the task. Third, teachers must purposefully select which student strategies will be shared during the whole class discussion. Fourth, teachers must also determine the sequence in which those strategies will be shared. Finally, teachers support students in making connections among solutions and the big mathematical ideas in the problem. As we can see, Smith and Stein (2011) offered teachers specific moves to employ that will support their students in engaging in productive mathematics discourse.

**Supporting student engagement in discourse.** As noted previously, talking about mathematics is not something that students will naturally know how to do (NCTM, 1989, 2000). Therefore, teachers need to put structures in place that support student engagement in mathematics discourse. Teachers and their students should work together to create shared norms for mathematics discussions (Kazemi & Hintz, 2014; NCTM, 1991). For instance, these norms might include the idea that “getting the right answer is not enough;” rather, students are expected to explain how they know (Wenrick et al., 2013, p. 358). Or, the norms might address the seating arrangement of students, as how
students are physically oriented (toward the teacher or toward each other) might invite or discourage participation in discourse (Cazden, 2001).

Norms should also include the way in which members of the math community initiate responses and follow up to others’ responses. Cazden (2001) referred to these types of norms as “speaker rights and listener responsibilities,” where teachers must consider how students enter the discourse (e.g., called on by teacher, self-select/volunteer by raising hand or just joining conversation, require all to participate, use of a “talking stick”) as well as how to support students in actively listening to their peers (pp. 82–91). Shindelar (2009) addressed expectations for listener responsibilities in her action research in her middle school classroom by explaining to students that it would be their responsibility to listen, respond through agreement/disagreement or questions, and help each other make sense of the mathematics (p. 171). On the other hand, Brooks and Dixon (2013) addressed speaker rights when they advocated for eliminating the practice of students raising their hands during mathematics discussions. While the transition from teacher-led to student-led discussions presented some challenges, Brooks implemented the following expectations in her second-grade classroom to facilitate this change: (a) addresses each other by name; (b) you must talk about your classmate’s response before presenting their own ideas; (c) take turns—only one person speaks at a time; and (d) speak loud enough and clear enough for everyone to hear you (Brooks & Dixon, 2013, p. 88). Regardless of what norms are developed for a given mathematics classroom, it is necessary that norms for discourse are created and maintained.

Closely related to the speaker rights and listener responsibilities noted by Cazden (2001) is the work of Chapin et al. (2013) that asserted teachers should have two major
goals when setting up norms for mathematics discourse: (1) developing expectations for “respectful discourse;” and (2) putting structures in place to ensure “equitable participation” (p. 69). Part of developing expectations for respectful discourse involves setting ground rules for cordial exchange, such as not denigrating people’s ideas, and delineating students’ rights and obligations as participants in the discourse community. Chapin et al. (2013) also suggested that students brainstorm potential dilemmas that might arise, like someone interrupting a student explanation or someone taking credit for another’s idea, and discuss how to prevent these situations or correct them if they happen (p. 70).

In terms of putting structures in place to ensure equitable participation, Chapin et al. (2013) offered the following strategies for creating opportunities for all students to engage in mathematics discourse: (a) give students “time to think” through the use of wait time and partner sharing; (b) give students “time to practice” by alerting them to the fact you will ask them to share their ideas with the whole class; (c) randomly select speakers to share on behalf of partners or groups; (d) “encourage students to self-monitor their participation,” especially for students who constantly raise their hands—do they need to contribute to every question or can they “sit this one out;” (e) use a talking chips protocol where each student must contribute to the discussion; and (f) ask that students who are reluctant to participate do “one out of three” each time—make a comment, respond when called on, or ask a question (pp. 74–77). Chapin et al. (2013) also cautioned teachers against using random calling techniques, like pulling popsicle sticks, during mathematical discussions as it may interfere with the coherence of the discourse.
This attention to norms of equitable participation in the work of Chapin et al. (2013) is related to larger issues of equity in the mathematics classroom that must be considered within the instructional context of discourse. As Ernst-Slavit and Slavit (2007) noted, “putting communication in the center of instructional reform places potential inequities on students whose languages and cultures are different from those of the school” (p. 21). How teachers honor, value, and integrate varied languages and diverse cultural backgrounds and experiences will determine the level of a student’s participation in mathematics discourse. Adler (1999) found that secondary teachers of multilingual students expressed a tension between providing opportunities for students to express their thinking in mathematical language and the teacher scaffolding or revoicing the precise mathematics language.

If [the teacher] focused on language for too long, she would inadvertently obscure the mathematics under consideration. If she left too much implicit, she would run the risk of losing or alienating those who most needed opportunity for access to educated discourse. (Adler, 1999, p. 62)

Thus, the teacher is challenged to find the right balance for supporting student participation in mathematics discourse. NCTM (1991) acknowledged the challenging and critical role that teachers play in promoting equitable participation among students when they noted “teachers must…be perceptive and skillful in analyzing the culture of the classroom, looking out for patterns of inequality, dominance, and low expectations that are primary causes of nonparticipation by many students” (p. 34). Thus, ensuring equitable participation requires that teachers view the exchanges during mathematics
discourse through an equity lens and examine how their instructional moves might impact the ability for all students to have access to the mathematics being discussed.

When considering issues of equity and access, Meyer, Rose, and Gordon (2014) advocate that teachers utilize the Universal Design for Learning (UDL) guidelines as they facilitate student talk about mathematics. UDL is about proactively planning instruction to open access to all learners; that is, teachers should expect there to be variability among learners and, as such, should plan instruction that offers flexibility in meeting the varied needs of all students by providing students with multiple means of engagement, representation, and action and expression (Meyer et al., 2014). Moreover, UDL takes both cognition and emotion into account when considering how to open opportunities for learning, just as it also recognizes that learner capacity shifts based on context (Meyer et al., 2014). Interestingly, many of the means for supporting student engagement in mathematics discourse already discussed, such as building a community of learners, honoring students’ funds of knowledge, creating tasks with choice and multiple entry points, and supporting the use of multiple representations and strategies, reflect these UDL principles. But that is not enough. Teachers should systematically consider additional ways in which the UDL principles can be applied when they plan for productive mathematics discourse in order to support access to learning for all students.

To support all students’ participation in math talk, teachers need to establish and model expectations for what it looks like and sounds like for students to explain and justify, ask questions of peers, and respectfully disagree with another person’s reasoning (Anthony & Walshaw, 2009; Cirillo, 2013a; Kazemi & Hintz, 2014; Middleton & Jansen, 2011; NCTM, 1991, 2000; Zwiers & Crawford, 2011). Some supports that teachers
might put in place to help students engage in these discourse practices include providing sentence starters, like “I agree/disagree with ___ because,” “How is your way different from ____,” “What did you mean by ____,” or “I want to revise my thinking” (Kazemi & Hintz, 2014, p. 4). When students have a picture of what it means to do math talk, then they can enact it.

In order to fully participate in a mathematics discourse community, students need certain dispositions and skills, like being willing to share their ideas, listen to each other, explain and justify their thinking, and see other students’ thinking as a means for learning (Cirillo, 2013b). For young students in the primary grades, it can be very helpful to have them work in pairs and small groups because it allows them to develop their skills in organizing and articulating their mathematical thinking (NCTM, 2000). For students who are English language learners, it is beneficial to allow them to communicate in their native language whenever possible (Ernst-Slavit & Slavit, 2007; Turner et al., 2013). It is also helpful to broaden the means for communicating by encouraging use of multiple representations as well as nonverbals, like gestures, drawings, and physical models (Turner et al., 2013). To foster confidence and competency among students, teachers should validate student thinking by drawing attention to the contributions made by students, which is especially powerful for students who are English language learners (Kazemi & Hintz, 2014; Turner et al., 2014).

Even when planning a whole class mathematics discussion, it is helpful to allow students to interact in smaller groups initially. Several researchers, including Anthony and Walshaw (2009) and Cazden (2001), advocated for mixed ability levels/heterogeneous grouping when forming small groups. A flow of instruction from
small group to whole group is well described in Brown and Hirst (2007) where they shared a structure for facilitating whole class discussion in elementary classrooms they termed Collective Argumentation. In Collective Argumentation, students initially solve a problem individually using a strategy of their choice, then they compare their strategy with members of their small group, and finally small groups share their findings in a whole class discussion of the problem (Brown & Hirst, 2007). This structure (individual work, followed by small group, followed by whole group) works well for problem solving and gives students multiple opportunities to talk about the mathematics prior to the whole class discussion. Moreover, as Cribbs and Linder (2013) noted, small group interactions allow students to utilize peer funds of knowledge when co-constructing mathematical meanings. Structures like Collective Argumentation represent the types of supports that students need to engage in mathematics discourse.

Another way in which teachers can support engagement in discourse is the nature of the tasks presented. As previously noted, the selection of a quality task can impact the subsequent discussion about it. In order to open up the task and allow for multiple entry points, Wenrick et al. (2013) suggested giving students different number choices within word problems to allow students to choose the complexity of number that is right for them. The ability to discuss these tasks is not hindered because the mathematics of the task is the same across the different sets of numbers. Holbert and Barlow (2012/2013) explored mathematical tasks that invited participation from students who were reluctant problem solvers. These students who are reluctant problem solvers were slow to engage and often relied on group members or the teacher to do the thinking and reasoning for them (Holbert & Barlow, 2012/2013). Holbert and Barlow (2012/2013) found that their
students who were reluctant problem solvers were more willing to engage in mathematics “tasks without words,” which are tasks that more visual and hands-on that do not require reading text, like a traditional word problem (p. 312). The authors hypothesize that these tasks may be more appealing because they do not require reading and/or the visual or hands-on nature of the tasks may seem more puzzle-like thereby appealing to children’s curiosity (Holbert & Barlow, 2012/2013).

Another important aspect in supporting students’ engagement in mathematics discourse is how the mathematical ideas discussed are recorded and preserved for future reference, consideration, and revision. If the classroom is functioning as a knowledge building community, then ideas are modified and improved collectively over time (Middleton & Jensen, 2011). Having a means to capture this running record of evolving mathematical knowledge is powerful and necessary for learning. Technology offers many options to help keep track of the community’s mathematical ideas, such as taking pictures of manipulative or whiteboard work, image/screen captures from document cameras or SMART Boards, videos of presentations, and students recording their work with verbal explanations on applications such as Show Me or Explain Everything (Middleton & Jansen, 2011). Additionally, mathematical thinking can be captured by more traditional means such as written work, anchor/flip charts, or math journals. Ritchhart, Church, and Morrison (2011) argued that documentation should “serve to advance learning, not merely capture it” (p. 38). That is, the content of the discussion (e.g., conjectures, connections, conclusions, and reflections) about the mathematics must be preserved in addition to the mathematics itself. Regardless of the medium, it is
valuable to preserve students’ mathematical thinking during classroom discussions in order to reference it, revisit it, and revise in the future.

**Benefits of Discourse**

When teachers increase opportunities for students to engage in rich mathematics discourse, both students and teachers benefit. First, students who participate in discourse engage in “authentic mathematics” as they reason, problem solve, justify, and make sense of their own thinking (Schleppenbach et al., 2007, p. 382). Additionally, discourse allows students to explore mathematical ideas from multiple perspectives, which helps them clarify their ideas, refine their thinking, and make connections (Cai & Lester, 2010; NCTM, 2000; Schleppenbach et al., 2007; Zwiers & Crawford, 2011). Discussions also provide feedback, both for students about their thinking and for teachers about how to guide instruction (Anthony & Walshaw, 2009; Cirillo, 2013b; Franke et al., 2009; Zwiers & Crawford, 2011).

Another joint benefit is that discourse promotes a shared responsibility for learning mathematics. Atweh et al. (1998) characterized the mathematics classroom as “a place where the teacher and students socially construct an interactive environment with the primary goal of promoting learning” (p. 64). It is in such an environment that members of the classroom community create shared meanings and understandings about mathematics. As students progress through grade levels, they should take increasingly greater responsibility for participation during whole class discussions (Carrillo, 2013a; NCTM, 2000). Thus, purposeful mathematics discussions can be an opportunity to shift the authority from the teacher to a community of learners (Cirillo, 2013b; Hufferd-Ackles et al., 2004, 2014).
Teacher Professional Development

Given the many benefits of discourse and the many effective practices teachers can implement to facilitate productive mathematics discourse, it makes sense to include improving classroom discourse as a central component of mathematics professional development. However, as Ni et al. (2014) noted, “the inherent nature of classroom discourse is greatly determined by the behaviors of individual teachers and students” and is, therefore, a more challenging aspect of pedagogy in which to affect change (p. 3). This challenge reminds us that teaching is complex and, therefore, professional development programs need to offer experiences for teachers to examine the complexities of their practice (Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010). Therefore, it is necessary to consider how best practices in mathematics professional development can support teachers in changing their practice around facilitating productive mathematics discourse.

Attending to context. It is important for professional development programs to complement existing school and/or district goals to provide both context and motivation for teachers to change their practice (Loucks-Horsley et al., 2010). To that end, it is critical for professional developers to be in the classrooms to observe teacher practice and match the teacher’s needs to the content of the professional development program. Otherwise, teachers may view professional development as an add-on not related to their current situation (Loucks-Horsley et al., 2010) or something “being done to them” (NCTM, 2014, p. 101). Feelings such as this will make it difficult to cultivate buy-in among the teachers for engaging in the challenging work of making instructional change.
One way in which professional developers can complement the existing contexts of teachers is to consider how to incorporate the current adopted mathematics curriculum in the professional development program. Many teachers rely on and/or are required to use the adopted mathematics curriculum to guide instruction. But teachers must keep in mind that “published curriculum materials must be written for a generic classroom, so teachers have an important role in adapting and using the materials to meet the strengths, needs, and experiences of specific children” (Drake, Land, Bartell, Aguirre, Foote, McDuffie, & Turner, 2015, p. 348). Herein lies the opportunity for professional developers. Drake et al. (2015) suggested three strategies for opening spaces in curriculum for richer mathematics teaching: (1) rearrange lesson components by moving problem solving to the beginning of the lesson and eliminating sections of the lesson that focus on “telling” or “showing” how to solve; (2) adapting tasks by offering choice of number and encouraging students to represent and solve problems in multiple ways; and (3) adjust the contexts of textbook problems to be more connected to real-world problems, home and community situations, and shared experiences (pp. 349–352). Minor adjustments, such as these, honor the reality that teachers use their adopted curriculum, but also facilitate creating opportunities to improve practice around engaging students in problem solving and discourse that should lead to more positive student learning outcomes.

**Collaborative structures.** Professional development programs must provide opportunities for teachers to collaborate. In fact, “teachers’ co-planning of lessons provides one of the greatest opportunities for making a positive difference on student learning” (NCTM, 2014, p. 104). At the same time, “intentional and structured
reflection” is equally necessary to affect instructional change (NCTM, 2014, p. 105).

However, collaboration without structure and a concerted effort to focus on mathematics pedagogy and student learning will most likely fail to facilitate professional learning. Two effective forms of teacher collaboration that provide the space, structure, and focus to co-plan and reflect are professional learning communities and lesson study.

Professional learning communities (PLCs) are one collaborative structure that can be a support for teachers to make instructional change. While PLCs can take many different forms, essentially they are a group of teachers who meet regularly and over time to focus on improving some aspect of teaching and learning. NCTM (2014) identified several characteristics of effective mathematics PLCs: (a) examine and make decisions about math content and practices that students will learn; (b) develop and use common assessments to gather data about student learning; (c) use data to drive instruction; (d) set immediate and long-term instructional goals; (e) implement research-based best practices; (f) develop action plans based on student learning outcomes; and (g) provide a space for continual learning, around mathematics content and pedagogy (pp. 103–104).

While it is important to describe what a PLC it, it is equally important to describe what it is not. DuFour and Reeves (2016) criticize the misuse of PLCs, which they term “PLC lite,” in which faculty meetings or grade level meetings are just renamed as PLCs. Instead, PLCs must have an actionable element that focuses on improving teacher practice and, ultimately, student learning (DuFour & Reeves, 2016). While DuFour and Reeves (2016) emphasized action in the form of creating common assessments and/or planning interventions, effective PLCs can also focus on instructional planning and purposeful use of strategies within curriculum (Little, 2006; Slavit & McDuffie, 2013).
That is, PLCs can be a space for teachers to try out research-based best practices around mathematics instruction in order to improve their adopted curriculum and the learning experience of their students (Blank, 2013). Inherent in all of these endeavors is collaboration, but collaboration that is structured and focused on improving teaching practice and student learning (Blank, 2013; DuFour & Reeves, 2016; Little, 2006).

The collaborative nature of lesson study can also be an effective means for supporting teachers in changing their practice. In lesson study, teachers collaboratively design, implement, and reflect on mathematics lessons. Robinson and Leikin (2012) detailed how lesson study supported one elementary teacher in changing her mathematics instruction. Based on analysis of two lessons, they found that, over time, the lesson study process helped this teacher to enhance the mathematics discourse in her classroom by allowing more time for students to talk and engage with their strategies and solutions, by asking more open-ended questions that pushed students to reason, explain, and justify, and by selecting tasks that provided an appropriate level of challenge for all students (Robinson & Leikin, 2012).

Regardless of the type of collaborative structure employed, lesson study or PLCs, teachers need opportunities to collaborate with their colleagues and collectively work to change their instructional practice. Teachers engaging in both PLCs and lesson study become members of a community of practice, in which they co-construct meaning about the teaching of mathematics based on their shared experience of collaboratively co-planning and reflecting on mathematics instruction (Lave & Wenger, 1991; Wenger, 1998). Furthermore, these communities of practice can support professional development
efforts by providing collegial support for implementing and sustaining instructional change.

**On-going support.** As teachers try to implement their learning in their classrooms, it is crucial that they have on-going support (Loucks-Horsley et al., 2010; NCTM 2014). Structuring professional development to be job-embedded can facilitate a means for on-going support and address the obstacle of finding or making time for professional learning and growth. Such job-embedded professional development can include instructional coaching, model lessons, co-teaching, co-planning, or other collaborative structures like PLCs or lesson study. Instructional coaching is an especially effective way in which to support change in teacher practice. “Teachers who receive coaching are more likely to implement new teaching strategies and effective coaching can have a positive impact on student learning” (NCTM, 2014, p. 106).

Bay-Williams, McGatha, Kobett, and Wray (2014) established the Leading for Mathematical Proficiency (LMP) Framework as a structure for coaching mathematics instructional change within the context of the CCSSM. The LMP Framework links the Standards for Mathematical Practice, shifts in classroom practice, and teaching skills as a means to “create opportunities for students to become mathematically proficient,” which “requires teaching practices that focus on helping students acquire these practices while simultaneously working on creating deep understanding of the content” (Bay-Williams et al., 2014, p. 3). The LMP Framework specifically addresses the teaching practice of facilitating classroom discourse. Bay-Williams et al. (2014) noted,

Discourse involves various teacher actions, including: asking challenging questions, listening to students’ responses to gauge their learning; encouraging
students to listen and respond to their peers, requiring students to explain their thinking, encouraging students to use multiple representations, and allowing students to engage in productive struggle. (p. 78)

These myriad teacher actions reflect the complexity of facilitating mathematics discourse, as well as the complexity of supporting teacher learning in this area. The LMP Framework offers coaching suggestions to help teachers shift their classroom practices toward a classroom that focuses on UDL principles to make mathematics accessible to all, cultivates a positive and supporting learning community, attributes mathematical authority to sound reasoning by students, and focuses discussion on explanation and understanding (Bay-Williams et al., 2014, p. 81). Thus, the LMP Framework could be a powerful resource for instructional coaches and professional developers who are supporting teachers as they work to change their practice around facilitating mathematics discourse.

**Summary**

There are many structures and strategies that teachers can use to promote and facilitate productive mathematics discourse. While there is a definite focus at both the elementary and secondary levels to identify promising practices to increase opportunities for students to engage in rich mathematics discussion, many of these practices are not situated within the reality of classroom contexts in terms of adopted curricula, especially when the curricula are not reflective of reform mathematics philosophy. This issue reflects the complexity of facilitating effective mathematics discourse that Truxaw and DeFranco (2008) noted. Therefore, it would be beneficial to the field of mathematics
education to examine how to support teachers to use these discourse-promoting practices within the confines of an adopted non-reform mathematics curriculum.

Developing and implementing effective professional development for teachers around facilitating mathematics discourse is challenging work. As noted by NCTM (1991), “Because teaching mathematics well is a complex endeavor, it cannot be reduced to a recipe for helping students learn” (p. 22). However, there are several research-based practices in professional development that will support teacher change and student learning, such as situating the professional development within the context of the school’s or district’s goals, providing structured and focused opportunities for collaboration, and providing job-embedded, on-going support. It is necessary for professional developers to strike a balance between utilizing these professional development best practices and customizing the professional development to meet individual teachers’ and/or schools’ needs. The next chapter explores the methodology of this study, which utilizes a professional development treatment that seeks to blend effective instructional practices around mathematics discourse with best practice in professional development with providing flexibility to meet varied classroom contexts.
Chapter 3
Study Methodology

This qualitative study sought to explore teacher learning about planning for and facilitating mathematics discourse and to capture any changes in mathematics discourse in teachers’ classrooms. This chapter will elaborate on the methods that were used to investigate this phenomenon. The study was designed using an embedded single-case study approach to attend to the case of teacher learning as well as the cases of mathematics discourse in classrooms (Yin, 2014). In order to facilitate teacher learning, a protocol for a professional learning community was developed. Data were gathered and analyzed throughout the study, themes were identified, and an interpretation of teacher efforts to facilitate productive mathematics discussions was developed.

Central Questions

The purpose of this qualitative study was to explore teacher learning about planning for and facilitating mathematics discourse in the context of a professional learning community as well as document any changes in mathematics discourse that occurred in the classrooms of the participating teachers. Therefore, the two central questions were:

• How did teachers learn about planning for and facilitating mathematics discourse as a result of participating in a professional learning community (PLC)?

• In what ways did a teacher’s participation in the PLC impact the nature of whole class discussion in the mathematics classroom during the enactment of the co-planned lessons and beyond that enactment?
These central questions reflected the multiple tiers of interaction inherent in this study, which included the teachers in their classrooms, the students, the teachers as members of a professional learning community, and the researcher.

**Research Design**

This qualitative study sought to explore teacher learning about planning for and facilitating mathematics discourse and capture any changes in the nature of mathematics discourse in the teacher’s classrooms. After obtaining approval from my committee to move forward with my planned study design, I developed the Purposefully Planning for Mathematics Discourse (PPMD) professional development program. Then I obtained approval from the Institutional Review Board (IRB) to ensure the proper safeguards were in place for the participants. Next, I identified possible schools to host the study. After purposefully selecting the school site and obtaining permission from the principal to conduct the study, the participating classrooms were identified based on the established inclusion criteria and the participating teachers were recruited. Then, pre-data were collected around current discourse practices during whole class mathematics discussions. Next, the PPMD professional development program was implemented. Data collection continued through the implementation of the PPMD professional development program. Post-data were collected to check for any sustained changes in mathematics discourse beyond which time the PPMD professional development program concluded. After all data were collected, the data were analyzed for themes and an interpretation of the case of the PLC and the embedded cases of each participating teacher’s classroom was developed.
**Embedded single-case study.** When developing the design of this study, I knew that I wanted to capture teacher learning about increasing opportunities for mathematics discourse as well as any impact that learning might have on the nature of the classroom discourse. Therefore, it made sense to consider the case of the professional learning community in which teacher learning was taking place, given it was a bound system (Creswell, 2013). At the same time, I knew that the participating teachers would likely have different experiences as they enacted the lessons within their individual classroom contexts. It was for these reasons that an embedded single-case study design was used in this qualitative study.

Embedded single-case studies are an effective research approach when dealing with multiple units of analysis that may interact in complex ways (Yin, 2014). The case of the professional learning community addressed Central Question 1, which was concerned with how teachers learn about planning for and facilitating mathematics discourse. The *case* was bound by place and time as the study took place during the 2015–2016 school year in the participating classrooms and within the professional learning community though which the PPMD professional development was facilitated (Creswell, 2013). But as noted previously, embedded cases are required when there are multiple units of analysis involved (Yin, 2014). Within the broader case of the PLC, each participating teacher’s classroom also represented a case, or bound system, in which these discourse strategies would be implemented. Examining these embedded cases sought to address Central Question 2, which was concerned with how the nature of mathematics discourse might change in each classroom as a result of teacher learning. Thus, there were two units of analysis: (1) the case of the PLC and (2) the embedded cases of each
participating classroom (see Figure 1). Additionally, the embedded single-case design allowed for examining the complexities of interacting units of analysis (Yin, 2014). As the teachers were learning how to plan for and facilitate mathematics discourse, they were trying out those strategies in their classrooms. Likewise, their experiences in their classrooms enacting those strategies shaped their subsequent experiences in the PLC. The embedded single-case study approach provided a means for making sense of these interactions among multiple units of analysis.

![Visual design for an embedded single-case study](image)

**Figure 1.** Visual design for an embedded single-case study

**Creating the PPMD professional development program.** Because this study was about teacher learning around planning for and facilitating mathematics discourse, it was necessary to develop a protocol to focus the work of the professional learning community. Thus, the Purposefully Planning for Mathematics Discourse (PPMD) professional development program was created. It was developed by the researcher using research-based best practices to support teachers in facilitating productive mathematics discourse within the confines of their adopted curriculum. The professional development
was job-embedded, as it utilized the school’s existing PLC structures. The PPMD professional development program consisted of three mini-design cycles, in which the teachers worked within the PLC to co-plan lessons that purposefully planned for mathematics discourse, enacted those lessons, reflected on the teaching episode, and made revisions to the lesson plan. The following details the design of the PPMD professional development program (see Appendix B for the complete program).

**Connection to the theoretical framework.** The PPMD professional development program was designed based upon the theoretical framework of social constructivism (Vygotsky, 1962, 1978) and the concept of communities of practice (Lave & Wenger, 1991) were used. Just as discourse opens opportunities for students to interact with others as they co-construct meaning about mathematics, the PPMD professional development program provided those same opportunities for teachers as they planned, revised, and reflected together, thereby co-constructing knowledge about teaching mathematics. Thus, situating the professional development program within the collaborative structure of a PLC provided a space for teachers to form communities of practice in which discourse served as the means by which pedagogical knowledge was constructed individually and collectively.

**Research-base.** Three strands of research were attended to when developing the Purposefully Planning for Mathematics Discourse (PPMD) professional development program: (1) best practices in professional development for mathematics teachers; (2) effective instructional practices for supporting productive mathematics discourse in the classroom; and (3) multitiered design study for professional development.
Best practice in professional development for teachers of mathematics. With regard to best practice in mathematics professional development, the elements of context and collaboration were used when designing the PPMD professional development program. Connecting professional development to the context of teachers was necessary to motivate instructional change (Loucks-Horsley et al., 2010). The PPMD professional development program attended to context by involving teachers in determining which discourse strategies they wanted to incorporate into their existing mathematics curriculum. Another element necessary for effective professional development was providing opportunities for teachers to collaborate, both in terms of planning and structured reflection (NCTM, 2014). Hence, the PPMD professional development program utilized professional learning communities as the space for such collaboration. Because PLCs were an existing collaborative structure at the participating school, using them also complemented and connected to the school’s context.

For any professional development program, it is necessary to provide on-going support in order for teachers to implement what they have learned and make sustained instructional changes (Loucks-Horsley et al., 2010; NCTM 2014). Therefore, the PLC served a key role in providing this on-going support, as teachers were collaborating and supporting one another through the learning process. The PPMD professional development program was developed so that the teacher members rotated facilitating the PLC themselves. This design honored the expertise that each teacher brought to the learning community and it provided for scalability as outside facilitators and/or content or pedagogical experts were not required. Therefore, once teachers were familiar with mini-design cycle process, they could sustain their work beyond the timeline of the study as
they had an established, but flexible structure that would allow them to continue to improve their practice in facilitating productive mathematics discourse or focus on other areas of mathematics instruction.

*Best practices in facilitating mathematics discourse with students.* While the mini-design cycle process could be used with any instructional focus, the review of literature showed that providing opportunities for students to engage in mathematics discourse positively impacts their learning (Cai & Lester, 2010; Cirillo, 2013b; Franke et al., 2009; NCTM, 2000; Schleppenbach et al., 2007; Zwiers & Crawford, 2011). Therefore, the PPMD professional development program was designed to focus on this high-leverage instructional practice in mathematics teaching. There are a plethora of effective instructional practices for supporting productive mathematics discourse in the classroom. Key to successfully implementing these strategies is creating a classroom community that is safe for students to share ideas, explain and justify solution strategies, and engage in respectful disagreement (Anthony & Walshaw, 2009; Cirillo, 2013a; NCTM, 1991, 2000). Beyond creating a mathematics discourse community, there are many different considerations for teachers in terms of facilitating productive mathematics discourse, including: selecting tasks, the types of questions asked, building upon student reasoning, and teacher discourse moves (e.g., using waiting time or revoicing). This list is far from exhaustive, which demonstrates the complexity of facilitating mathematics discourse. Thus, there was no specific set of practices or teacher moves that will ensure productive mathematics discussions will happen (NCTM, 1991; Truxaw & DeFranco, 2008). It is for this reason that the PPMD professional development program was designed to reflect best practices in effective instructional practices for supporting
productive mathematics discourse in the classroom, but also allowed for flexibility and choice to meet the needs of the participating teachers and their students.

*Multitiered design study for professional development.* Given the complexity of facilitating productive mathematics discourse, professional development to support teachers in improving their practice in this area is an equally complex undertaking. The philosophy of multitiered professional development design study (Zawojewski et al., 2008), which was used as a basis to develop the professional learning community protocol, is “to embrace the complexity and the dynamic nature of the system in which teachers learn and grow” (p. 236) and to “carrying out research about learning within the complexity of the actual classroom setting” (p. 219). This approach matched with best practice in professional development for mathematics teachers by situating professional development within the local context of the school and/or classroom, which helps to produces buy-in and supports established goals for teacher and student learning (Loucks-Horsley et al., 2010). Moreover, it honors and builds from the experiences, strengths, and needs that teachers bring to the table, which allows for flexibility. At the same time, it also provided a collaborate environment in which teachers learn and growth together around a focused topic, in this case facilitating productive mathematics discourse.

In the PPMD professional development program, this collaborative environment was the professional learning community (PLC). The teachers in the PLC conducted multiple mini-design cycles (Zawojewski et al., 2008), in which they will co-plan an annotated lesson plan that modifies their current adopted mathematics curriculum to include opportunities for mathematics discourse using agreed upon instructional strategies. After enacting the lesson plan, teachers will meet again in the PLC to engage
in structured reflection and revise the lesson plan based on their learning. This mini-design cycle will take place multiple times throughout the professional development.

Additionally, the multitiered professional development design supports teachers in designing and implement educational products to improve their practice. As Zawojewski et al. (2008) explained,

teachers are engaged in the development of artifacts that reveal aspects of their own thinking; teachers are engaged in testing and revising the artifact; and teachers are asked to describe and document the guiding principles they have used while revising the artifact. (p. 221)

In this study, the artifacts developed by teachers were annotated lesson plans that modify the adopted curriculum by infusing more deliberate uses of mathematics discourse as a way of knowing and learning. Hiebert and Morris (2012) described annotated lesson plans as containing two types of knowledge: “what to do and why/how to do it that way” (p. 95). As part of their work in the professional learning community, the participating teachers will also co-create other artifacts that reflect their understanding of and beliefs about mathematics discourse, including creating goals for the PLC, a definition of mathematics discourse, and a Venn diagram of the roles of students and the teacher during mathematics discourse. Thus, these artifacts allow the researcher another means by which to document teacher learning and growth.

Validating the PPMD professional development program. In addition to being based on research, the PPMD professional development program was also based on elements of professional development that I have used with elementary teachers in the past. Professional learning communities have been shown to be an effective means of
improving teaching and learning over time, especially when they contain an actionable element, which could include instructional planning and purposeful use of strategies within a curriculum, like in the PPMD professional development program (DuFour & Reeves, 2016; Little, 2006; Slavit & McDuffie, 2013). Moreover, PLCs are flexible enough to attend to teachers’ needs and context. I have found providing teacher choice within a professional learning community to be a powerful motivator for instructional change, which is why I wanted to incorporate that element within the PPMD professional development program. Additionally, most of the PLCs I have worked with previously were teacher-facilitated using prepared meeting protocols. Structuring a PLC to be teacher-facilitated provides a mechanism to build leadership capacity among teachers. It also allows for ease of implementation as outside facilitators are not required. It is for these reasons that I designed the PLC to be teacher-facilitated. Therefore, the crucial design elements in the PPMD professional development program, namely a teacher-facilitated PLC with elements of choice built into a structured protocol, have been field tested through many years of my previous professional development work with elementary teachers. Finally, experts in the field of mathematics education reviewed the PPMD professional development program prior to its implementation.

Participants in the Study

The participants in this study were purposefully selected from an elementary school (K–5) in western Montana. The study participants were the teacher and students from a participating classroom. In order for the classroom to participate, two inclusion criteria had to be met. First, the classroom teacher had to be willing to participate in and implement the discourse structures agreed upon in the PLC, as well as consent to
participating in the study. Secondly, at least two classrooms from a grade level had to be willing to participate because grade level collaboration in a professional learning community was an essential component of the PPMD professional development program. Based on these inclusion criteria, at least four classrooms were selected, in order to keep the number of members in the PLC at a level conducive to functionality. While optimal sizes of PLCs vary among researchers, PLCs need to be small enough for all members to have a voice, but not so small that new ideas and innovations fail to be considered (Goodwin, 2014). Therefore, the size of the PLC was limited to 5-7 members, including the teachers from the participating classrooms and the researcher who was an active member in the PLC in the role of participant-observer.

**Data Collection Procedures**

To explore the central questions, multiple types of data were collected. In case studies, it was recommended that the researcher gather multiple types of data from multiple sources (Yin, 2014). To that end, the data sources included classroom observations, observations of the PPMD professional development sessions, lesson plan artifacts, co-constructed documents from the PLC, and interviews with participating teachers. Table 1 provides an overview of the study’s timeline, activities, and data collection.

First, each participant classroom was observed before, during, and after implementation of the PPMD professional development program. During the study, classrooms were observed when co-planned lessons developed as part of the mini-design cycles from the PPMD professional development program were enacted. During all classroom observations, an observation protocol (see Appendix C) was used to
standardize and focus note taking, especially to make note of who was speaking and pertinent artifacts of thinking (e.g. drawings, use of fingers) (Hancock & Algozzine, 2011; Yin, 2014). The observations were also audiotaped using the Voice Record Pro application in order to transcribe the discussion. A research assistant reviewed 25% of the participant classroom audiotapes, which were randomly selected, to ensure accuracy in the transcription. This research assistant was selected based on the following criteria: (a) she successfully completed the University of Montana Online Research Ethics course (Section One - Ethical Issues in Research: A Framework, Section Two - Interpersonal Responsibility, and Section Six - Human Participation in Research); and (b) she was available and willing to commit to the required data analysis for the duration of the study. After each observation, field notes were made to capture the context of the observation as well as record my thoughts and reactions (Yin, 2014).

Additionally, I captured data about teacher learning throughout the PPMD professional development program by videotaping the Math Talk PLC sessions, gathering lesson plan artifacts, and archiving co-constructed documents. I was as a participant-observer during the data collection, because I was actively participating as a member of the PLC (Yin, 2014). First, all of the professional development sessions as they took place during the Math Talk PLC meetings were videotaped using the video function on an iPad and transcribed in order to capture the planning, reflection, and revision process. The Math Talk PLC meetings were also audio recorded using the Voice Record Pro application, as a form of back up data. A research assistant reviewed 25% of the professional development sessions, which were randomly selected, to ensure accuracy in the transcription. Secondly, the annotated lesson plans were used as data. Annotated
lesson plans contained both what was taught and why teachers made those instructional choices (Hiebert & Morris, 2012). Copies of both the original co-planned version of the lesson plan and the revised version of the lesson plan were gathered from each of the three mini-design cycle to capture the evolution of these educational products. Co-constructed documents, including a definition of math talk, goals for the Math Talk PLC, and a Venn diagram of the roles of teacher and students during math talk were maintained in a Google folder.

Finally, semi-structured interviews were conducted with the participating teachers. Hancock and Algozzine (2011) noted that semi-structured interviews were quite appropriate for case studies because they provide a blend of pre-planned questions to address the central questions, but also allow for spontaneous follow up and more open sharing on the part of the interviewees. During the interviews, the experiences of the classroom teachers were captured using an interview protocol (see Appendix D) in order to consistently implement the interview procedure, as well as to ensure informed consent (Creswell, 2013). Each interview was audio-recorded using the Voice Record Pro application and transcribed, and notes were taken on the interview protocol. I also engaged in member checking during the interview when I sought clarification from the interviewees that their thoughts and/or feelings have been captured accurately. After each interview, field notes were made to capture the context of the interview as well as record my thoughts and reactions (Yin, 2014). Interviews with the participating classroom teachers took place before, during, and after implementation of the PPMD professional development program.
Table 1
Overview of Study Timeline, Activities, and Data Collection

<table>
<thead>
<tr>
<th>Week(s)</th>
<th>Activities</th>
<th>Data Collection</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–3</td>
<td>Recruiting and Obtaining Consent</td>
<td>Pre-Data: 2 Classroom Observations &amp; Teacher Interview</td>
</tr>
<tr>
<td>4–6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Implement PD Program*

7     PPMD Session 1              PD Session Observation
8     PPMD Session 2              PD Session Observation

*Mini-Design Cycle 1*

9     PPMD Session 3              PD Session Observation & Initial Lesson Plan
10    Enact Co-Planned Lesson     Classroom Observation
11    PPMD Session 4              PD Session Observation & Revised Lesson Plan

*Mini-Design Cycle 2*

12    PPMD Session 5              PD Session Observation & Initial Lesson Plan
13    Enact Co-Planned Lesson     Classroom Observation & Teacher Interview
14    PPMD Session 6              PD Session Observation & Revised Lesson Plan

*Mini-Design Cycle 3*

15    PPMD Session 7              PD Session Observation & Initial Lesson Plan
16    Enact Co-Planned Lesson     Classroom Observation
17    PPMD Session 8              PD Session Observation & Revised Lesson Plan
18    PPMD Session 9              PD Session Observation & Teacher Interview

*End PD Program*

21    Post-Data: 1 Classroom Observation & Teacher Interview
24    Post-Data: 1 Classroom Observation

**Role of the researcher.** It was necessary to clarify my role as researcher in this study. First, I acted as the researcher and, as such, I was responsible for data collection, data analysis, and interpretation of the cases. Secondly, I wrote the PPMD professional development program. I have engaged in creating and facilitating mathematics
professional development for elementary teachers for the past eight years. For three years of my career in public schools, I worked as a mathematics curriculum and instructional specialist, during which time I led more than 1000 hours of mathematics professional development with elementary and middle school teachers. Many of the programs included professional learning communities as an element of job-embedded professional development. Therefore, I have experience developing and facilitating PLCs as well as training teachers to be PLC facilitators. Also during that same time, I developed a 45-hour mathematics content course for Kindergarten and First Grade teachers, which was followed by 15 hours of professional learning community work that I designed around looking at student work. I also developed professional development for teachers of grades K-2 and 3-5 and for special education teachers of grades K-6 around the Common Core State Standards for Mathematics. As a result of this work, I have extensive experience in planning and facilitating mathematics professional development, especially at the elementary school level.

Although I wrote the PPMD professional development program, my intent was to observe what happened when it was implemented in terms of (a) what learning, if any, happened among the participating teachers with regard to planning for and facilitating mathematics discourse; and (b) how, or if, this learning transferred into the classroom with regard to how students engaged in mathematics discourse. Thus, the study was exploratory in nature because data was being collected for the purpose of documenting what happened during the PPMD professional learning community meetings and what happened during whole class mathematics discussions in the participating classrooms.
Additionally, I acted as a participant-observer in the PPMD professional learning community, in which I took an active role as a member of the learning community. Participant-observers have an opportunity to experience the perspective of someone inside the case, which can provide additional insights into the phenomenon (Yin, 2014). At the same time, Yin (2014) warned that when acting as a participant-observer, the researcher must guard against exerting influence or manipulating the situation in a way that will change the nature of the phenomenon. That is why the PPMD professional development program was designed so that the teacher members facilitate the Math Talk PLC themselves. I did not plan to facilitate any of the PPMD professional development sessions; instead, each of the participating teachers will take turns being the facilitator for a session, rotating the leadership role. Additionally, I actively worked to not assume the role of expert during the PLC meetings; instead, each member in the PLC assumed an equal role in terms of bringing her individual expertise to the group as well as building our collective expertise around planning for and facilitating mathematics discourse. This design honored the expertise that each teacher brought to the learning community and it provided for scalability as outside facilitators and/or content or pedagogical experts were not required. Moreover, it built leadership capacity among the participating teachers, which could benefit the entire school community. In an effort to provide for transparency with regard to my role as a participant-observer, the PPMD professional development sessions were videotaped and transcribed. A research assistant was also used to monitor the accuracy of my data collection, and I utilized member checking as a means to assure my interpretations were accurate to add to the trustworthiness of the data.
**Trustworthiness of the data.** It was necessary to plan for accuracy and verification during the data collection and the data analysis processes to ensure trustworthiness of the data. I agreed with Eisner (as cited in Creswell, 2013) that researchers should take steps to develop credibility and confidence in our data collection, our interpretation of the data, and the conclusions that we draw from it. Trustworthiness of the data provides credibility and confidence through an emphasis on accuracy, verification, and transferability (Creswell, 2013).

**Accuracy.** Accuracy during data collection was ensured using several different strategies. First, an observation protocol and an interview protocol were used to make sure both the observations and interviews were conducted in a similar manner among all participants and participating classrooms (Creswell, 2013; Hancock & Algozzine, 2011; Yin, 2014). Secondly, the professional development sessions were videotaped and transcribed, and the classroom observations and interviews were audiotaped and transcribed to accurately capture the participants’ discussion, actions, and thoughts and/or feelings during the observations and interviews. Notes were taken as a means of recording fidelity with the observation and interview protocols, and field notes were taken to capture contexts, nonverbal behaviors, or possible anomalies that may influence the accuracy of the data collected (Yin, 2014). For all audio recordings, the Voice Record Pro application was used, which featured a function to slow down playback, thus allowing the researcher to more accurately transcribe with ease. Finally, a research assistant reviewed 25% of the videos and audiotapes, which were randomly selected, to ensure accuracy in transcription for the classroom and professional development observations and interviews.
Verification. To ensure the accuracy of the data collection and analysis, several verification strategies were used (Creswell, 2013). First, researcher bias was clarified by using field notes as a means to record my thoughts and feelings in order to check that my bias is not influencing my observations, interviews, interactions during the PPMD professional development sessions, or interpretations. Additionally, a research assistant was utilized to ensure accuracy of transcription of the professional development observation videos and classroom observation and interview audiotapes. As another means of verification, member checking, was conducted during the interview; that is, I clarified participant responses as they occurred in the interview to ensure the participants’ thoughts and/or feelings are captured accurately. Additionally, Yin (2014) recommended collecting multiple types of data from multiple sources in order to verify that emerging themes and interpretations are supported through the various strands of evidence. Creswell (2013) recommended using at least two verification strategies in a qualitative study, which this study utilized to strengthen the trustworthiness of the data.

Data Analysis Procedures

This study used a qualitative approach to data analysis. I chose to use this approach for several reasons, including: (a) data was collected in the natural setting of a school; (b) multiple sources of data were collected; (c) the study sought to understand the meaning the participants were taking from the PPMD professional development; and (d) the study tried to provide a holistic account of the complexities surrounding facilitating mathematics discourse in terms of planning and in-the-moment instruction (Creswell, 2014). The embedded single-case study design guided the data analysis, as Yin (2014) cautioned that it is necessary to attend equally to both levels of the case when conducting
analysis and interpretation. Broadly, the data analysis process entailed coding data for emergent themes, identifying relationships among the themes, and developing an interpretation of the case.

**Unit of analysis.** Within a social constructivism perspective, the unit of analysis is “human activity,” that is, the interaction of individuals and the environment (Stetsenko & Arievitch, 1997, p. 162). For the first central question (How did teachers learn about planning for and facilitating mathematics discourse as a result of participating in a professional learning community (PLC)?), the unit of analysis was the PLC that the structure through which the PPMD professional development was facilitated. For the second central question (In what ways did a teacher’s participation in the PLC impact the nature of whole class discussion in the mathematics classroom during the enactment of the co-planned lessons and beyond that enactment?), the unit of analysis was the mathematics classroom, which encompassed the social enterprise of learning as students and their teacher co-constructed meaning by engaging in discourse. Because there were multiple units of analysis, an embedded single-case study design guided the data analysis (Yin, 2014). Thus, the data analysis attended to the PLC and the individual teachers’ classrooms that made up the learning community as well as the interactions between the case and the embedded cases when developing an interpretation (see Figure 1).

**Identifying themes and developing an interpretation.** Once the data were collected in this embedded single-case study, the data analysis determined results and, eventually, allowed the researcher to draw conclusions to what impact, if any, the professional learning community had on teacher learning about planning for and facilitating mathematics discourse as well as what changes in mathematics discourse, if
any, occurred in the classrooms of the participating teachers. When engaging in data analysis, Creswell (2013) emphasized the importance of making detailed descriptions of the case, as well as attending to its setting. Hancock and Algozzine (2011) also recommended researchers engage in continuous examination and interpretation of the data throughout the collection process.

Multiple sources of data were examined, including transcripts of the classroom observations, the professional development sessions, and the participant interviews, field notes, the annotated lesson plans, and the co-constructed documents about math talk, for any patterns or trends (Creswell, 2013; Yin, 2014). Through this process, a means for coding and organizing the data was developed (Creswell, 2013; Hancock & Algozzine, 2011; Yin, 2014). The researcher did an initial read through the transcribed data, making notes of passages that were of particular interest. On subsequent reads, the researcher identified initial broad categories and coded the data as such. Then those passages were organized by category and the researcher looked for connections among the data.

Then those initial categories and connections were used to develop themes within the cases, using the multiple sources of data to verify the credibility of those conclusions as well as comparing the themes with relevant theories and/or the literature (Creswell, 2013; Hancock & Algozzine, 2011). Additionally, a research assistant was used to establish reliability with regard to coding the data. The coding would be considered reliable if the research assistant agrees with 80% or more of the researcher’s coding from 25% of the coded data, which was randomly selected, based on the criteria established by Miles and Huberman (as cited in Creswell, 2013, 2014). Once those themes were
confirmed and verified, an interpretation of the case was developed. These study findings will be discussed in the next chapter.

**Summary**

The research approach described outlined a method of inquiry that sought to explore teacher learning about planning for and facilitating mathematics discourse within the context of a professional learning community and to capture any changes in mathematics discourse that occurred in the classrooms of the participating teachers. An embedded single-case study was used to examine the broader case of the PLC and the embedded cases in each participating teacher’s classroom. Within this qualitative study, multiple data sources were collected, including classroom and professional development session observations, lesson plans and other co-constructed artifacts, and interviews with participant teachers. Moreover, the study ensured accuracy and trustworthiness of the data and used multiple means of verification in order to maintain credibility and confidence in the data collection, findings, and conclusions. The next chapter shares the findings from the study.
Chapter 4

Study Results

This qualitative study sought to capture teacher learning about infusing more mathematics discourse into instruction within the collaborative structure of a professional learning community as well as to capture any changes in the nature of mathematics discourse in the classrooms of the participating teachers. Therefore, multiple sources of data were examined, including observations of the PLC and classroom lessons, interviews, field notes, and other artifacts such as lesson plans and co-constructed documents about mathematics discourse. During the analysis of the case of the PLC and the embedded cases of the classrooms, themes were developed and verified. From that, an interpretation of this embedded single-case study was developed. This chapter will share the enactment of the methodology and the study results.

Study Site and Participants

After developing my methodology and the PPMD professional development program, the first step to enact the methodology was to select and gain access to the school that would host the study, and then recruit the teachers to participate in the study. The following describes my process of gaining access and recruiting the participants. It also describes the context of the school and its mathematics curriculum and provides a brief biographical overview of each participant teacher.

Gaining access. I had an on-going relationship with the school that I selected as the study site. I had a positive working relationship with the school’s principal, and I had worked in various capacities with three of the four participating teachers prior to
beginning the study as well as several other teachers in multiple grade levels. It was based on these established relationships that I purposefully selected the study site, knowing that I would receive the necessary administrative support to conduct the study and knowing that there were many teachers among multiple grade levels who were open to and interested in growing as mathematics teachers.

**School context.** The school purposefully selected as the study site was an elementary school (K-5) in western Montana. During the 2015-2016 school year, the school had approximately 290 students. The student body was comprised of primarily White/Non-Hispanic students with about 15% of the students who were Native American. The school also served a lower socio-economic student population with 75% of its students receiving free and reduced lunch. A variety of programming was provided to support student success both in terms of academics and behavior, including the Multi-Tiered System of Support/Response to Intervention approach (Montana Office of Public Instruction, 2015). Thus, there were often several other adults in any given classroom including para-educators and behavior specialists. The school had a warm and welcoming feel upon entering that permeated throughout the site.

**Instructional context.** The school selected for the study used a non-reform mathematics textbook as part of its adopted curriculum. While this textbook series has been in use in the school district for at least the past 8 years, teachers were using an updated Common Core version during the 2015-2016 school year, which aligned with Montana’s Common Core Standards for Mathematics. The textbook series, especially in the primary grades, promoted drawing and counting-based strategies as a means to make sense of operations. Whole group instruction was the primary structure promoted in the
textbook. The teachers did have autonomy with regard to how they used the adopted textbook, but most of them used it as is, occasionally omitting a section they believed was less relevant. The only supplemental instructional materials the teachers used were games and computer-based math websites when they had a blended learning day. On these blended learning days, the students would rotate through four centers: seatwork, computer, game, and teacher. The frequency of blended learning days varied, but the first grade team typically engaged in blended learning days at least twice a week while the third grade team typically engaged in blended learning once a week.

Participants. The participating teachers were purposefully selected from the study site. Initially, I made contact with the third grade team, as I had worked with one of the teachers on two different previous projects and I knew she had a strong interest in mathematics teaching. I set up a short face-to-face meeting with the team to review the purpose and details of the study using the Informed Consent form approved by the IRB. Both the two third-grade teachers volunteered to participate. Next, I contacted the Kindergarten team, but only one teacher was interested in participating, which failed to meet my pre-established inclusion criteria. Then I reached out to the first grade team and, after the face-to-face meeting, all three first grade teachers initially volunteered to participate. However, one of the first grade teachers withdrew from participating in early January due to being overcommitted with district committee work, which resulted in continued scheduling conflicts with our study meeting dates. Thus, there were four participating teachers who completed the study in full. The following provides a brief background of each of the participating teachers. Pseudonyms were assigned to maintain confidentiality.
Ms. Mitchell. This was the first year Ms. Mitchell was teaching first grade and mathematics. She began her teaching career as a junior high language arts teacher. After taking a break for several years, she worked in family resource centers and eventually took a reading specialist position for two years. During the 2015-2016 school year, Ms. Mitchell was doing a job-share for first grade. Because she taught in the afternoons and that was when math was taught, Ms. Mitchell found herself teaching math for the first time. In many regards, she was going through the same adjustments that new teachers face in terms of becoming familiar with a new grade level and a new curriculum. Even though Ms. Mitchell found “teaching math a little challenging,” she enjoyed the newer approaches to teaching math, as illustrated when she shared, “It makes so much sense to me to have multiple strategies for solving problems; that we’re not just memorizing facts and spitting them back out.” One of Ms. Mitchell’s personal goals, which arose in multiple conversations, was her desire to become more familiar with the adopted math curriculum. I had not met or worked with Ms. Mitchell prior to her participation in this study.

Ms. Green. Ms. Green is a veteran teacher, who has spent more than thirty years teaching primary grades. During the 2015-2016 school year, she was teaching first grade. Given her many years of experience, Ms. Green has seen instructional approaches in mathematics ebb and flow, often cycling back into best practice, such as teaching in whole group settings versus teaching in small groups with centers. Ms. Green strongly believes in considering what is developmentally appropriate when planning mathematics instruction; several times she discussed the role of conservation of number as being foundational for counting and operating. She expressed an interested in becoming more
efficient with using and integrating technology in her instruction. I had only begun
working with Ms. Green for about two months prior to the beginning of the study. She
was hosting one of my pre-service teachers as part of the field experience.

Ms. Jansen. Ms. Jansen has been teaching third grade for the past nine years,
including during the 2015-2016 school year in which this study took place. Prior to that,
she taught a few years in other primary grades as well as working as a Title 1 academic
specialist. Ms. Jansen recently completed her Masters degree with an emphasis in
mathematics education. Not surprisingly, Ms. Jansen really enjoys teaching math. She
strives to make math fun for the students by incorporating different forms of media and
infusing humor; she believes that element of fun is key to student engagement. Ms.
Jansen “loves the way that [the] program teaches math because it gives so many
strategies.” She strongly believes in allowing students to select a strategy that makes
sense to them. This year, Ms. Jansen’s class has many students with challenging
behaviors, so management, especially when students are interacting with each other, has
been an on-going focus of her efforts. I have worked with Ms. Jansen on two previous
professional development projects, both of which focused on mathematics professional
development; therefore, I had a long-term working relationship with her.

Ms. Davis. This was Ms. Davis’s second year teaching third grade. Prior to that,
she taught first grade for two years. Ms. Davis works closely with Ms. Jansen in terms of
planning math instruction and she continues to become more familiar with the adopted
text. A theme that resonated throughout my interactions with Ms. Davis was her belief
that building perseverance in her students is key to their success in mathematics learning.
She noted, “I mean the tendency to want to give up is there and I feel like if there’s a way
they can just stick with it and try different things, then I think that they can get where they need to be.” Like the other third grade class, Ms. Davis also has many students with challenging behaviors. She usually has two or three adults in her classroom during math instruction to provide academic and behavioral supports to specific students. While I had not worked directly with Ms. Davis before this study, I had met her on several occasions when she was finishing up a planning meeting with Ms. Jansen.

The Experience of the Math Talk PLC

This study was designed as an embedded single-case study. That design was used because there were multiple units of analysis and those units interacted with each other in complex ways (Yin, 2014). The case of the professional learning community examined how the participating teachers learned about planning for and facilitating mathematics discourse. Each individual teacher’s classroom was an embedded case in which the teacher’s learning was applied and enacted within the unique context of her students and her beliefs about the teaching and learning of mathematics. The following examines the case of the professional learning community that was facilitated through the PPMD professional development program.

After recruiting the participants for the study and collecting the pre-data, we began our Math Talk PLC. During our very first meeting, the participants began using the term math talk instead of mathematics discourse, so what had been termed our Mathematics Discourse PLC quickly became our Math Talk PLC. We continued to use the term math talk in lieu of mathematics discourse throughout our subsequent work together. Additionally, what was referred to as the PPMD professional development
program in Chapter 3 is now being referred to as the Math Talk PLC in the remainder of this study report to honor the co-constructed language of the participants.

At our initial Math Talk PLC meeting, we established our norms. All of the participants were familiar with and had used norms before in professional development settings. We agreed upon the following norms: (a) Start and end on time; (b) Be flexible about and understanding of our lives beyond our work; (c) “Math Talk” will be our friendly reminder phrase to get our conversation back on track; and (d) Take bio breaks as needed. Our second norm, “Be flexible about and understanding of our lives beyond our work” became essential to the workings of our PLC, as everyone had very busy schedules both in terms of responsibilities at work and at home. The acknowledgement of those responsibilities honored everyone’s contexts and helped to build a strong bond among the participants.

When establishing a professional learning community, it is important to attend to building relationships. Even though all of the participants were colleagues and each of the pairs of teachers worked closely within their grade level teams, it was necessary to extend and situate that collegiality within the specific context of the Math Talk PLC. Hence, each meeting would begin with an opportunity to share something that had been going on recently. While I had initially planned for all of this sharing to be around mathematics instruction, it became clear that sharing about our lives outside of teaching was necessary to deepen our bond and to get to know each other in this new context. Therefore, I modified several of the Math Talk PLC meetings to incorporate sharing about our experiences outside of work (e.g. Thanksgiving Break, Winter Break) (see Appendix E for the changes made during the study). These opportunities to share the
more personal sides of us strengthened our bond and positively contributed to the cohesiveness and trust in our Math Talk PLC. Likewise, there were several times when tangential conversations would arise during the meetings. Again, it was important to allow these brief discussions to happen as a means to maintain and solidify those relationships. Interestingly, we never had to use our phrase, “Math Talk,” to get us back on topic. These asides served a valuable purpose in that they acknowledged the many issues and concerns, beyond just our work in the Math Talk PLC, which shaped the participants’ contexts, both individually and collectively.

Creating a shared understanding of math talk. One of our tasks in the Math Talk PLC was to develop a shared understanding of math talk. There were several activities designed to facilitate this conversation, including developing a working definition of math talk, considering the roles of students and teachers during math talk, sharing our own experiences with math talk, and anchoring our work by reading some research briefs about best practices in planning for and facilitating math talk. Our co-constructed definition of math talk evolved over the course of our work together. During the first Math Talk PLC meeting, we began to develop our definition of math talk. Initially, it was not completely formed; rather, it was list of key words and phrases: shared conversation, conceptual knowledge, safe learning environment, teacher as facilitator, critical thinking, reasoning and explaining, partnerships, give and take, artifacts of thinking (e.g., words, pictures, manipulatives), and shared understanding. This definition in progress was posted in our Google folder, where participants could access it and edit if they wanted. However, no one made edits outside of our formal PLC meetings. We revisited the working definition from time to time during our meetings, but
no significant changes were made. It was not until the last Math Talk PLC meeting that we were able to formulate our final definition of math talk. We likened it to creating an *elevator speech* to answer the question: What is math talk?

Math talk is an inclusive, shared conversation where members of the classroom community persevere, think, reason, and explain their mathematical understanding using words, pictures, and objects.

Many of the initial ideas from our first session were incorporated into the math talk definition, but the ideas of inclusiveness, perseverance, and a sense of community as opposed to delineating teachers and students were additions. These additions reflected threads of conversations that took place in both the PLC meetings and in teacher interviews. For instance, Ms. Davis noted during one of our PLC meetings, “That’s something that I have. I feel like I have been able to include everybody [in math talk].” She was the strongest advocate for incorporating the concepts of inclusion and perseverance as part of our definitions, to which the rest of the participants agreed.

Another activity that built our shared understanding of math talk was determining the roles that students and teachers play during mathematics discourse. During our first Math Talk PLC meeting, the participants brainstormed within their grade level partnerships and then each group shared. There was a lot of overlap between the first and third grade teams. From that, we created a Venn diagram of the roles of students and teachers during math talk. Again, this chart was posted in the Google folder where participants could access and edit it if they wanted. Like the definition of Math Talk, no one made changes outside of the formal PLC meetings nor did they indicate other changes when we revisited it during our subsequent meetings. In our eighth Math Talk
PLC meeting, we formally re-examined the Venn diagram and only a single change from the original was made; we moved “Be okay with feeling uncomfortable” from the teacher’s role to a role shared by teachers and students. Figure 2 displays the final version of the co-constructed Venn diagram.

<table>
<thead>
<tr>
<th>Students</th>
<th>Both</th>
<th>Teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understand mathematics</td>
<td>Listening</td>
<td>Understand thinking</td>
</tr>
<tr>
<td>Use materials appropriately</td>
<td>Respectfully questioning</td>
<td>Provide wait time</td>
</tr>
<tr>
<td>Take responsibility for learning</td>
<td>Supporting each other’s learning</td>
<td>Students talk more and teacher talks less</td>
</tr>
<tr>
<td>Stay on topic with responses and</td>
<td>Make things fun</td>
<td>Look for/present complex problems</td>
</tr>
<tr>
<td>questions</td>
<td>Restate what others say</td>
<td>Provide the “why” we are learning this</td>
</tr>
<tr>
<td></td>
<td>Be okay with feeling uncomfortable</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 2.** Roles of students and teachers during math talk

Ultimately, the participants brought a lot of prior knowledge about math talk to the Math Talk PLC. From early on, they were identifying key elements in effective mathematics discourse, as noted in the literature, when working to create a definition of math talk and to identify the roles of students and teachers during math talk. At the same time, they did not report using and applying what they know about math talk on a regular basis during their mathematics instruction. For instance, Ms. Jansen commented, “This is stuff I’ve learned so many times through DMI [Developing Mathematical Ideas professional development], through being on the Math Committee, but do I put them into practice? For some reason, no.” So the challenge in the Math Talk PLC would be for the participants to build on this prior knowledge and find a way to motivate enactment of
math talk consistently in their instruction. Fortunately, the Math Talk PLC was designed to prompt enactment through the mini-design cycle process.

**Mini-design cycle process.** The Math Talk PLC was developed using multitudes of design study for professional development, which is an approach to educational research about learning that honors the “complexity of the actual classroom setting” (Zawojewski et al., 2008, p. 219). To that end, the PPMD professional development program was designed to allow the participant teachers to select the instructional strategies they wanted to try out with their students once they had a shared understanding of math talk and some background information about best practices to promote math talk. The participants completed three mini-design cycles in which they (a) gathered research and resources about their selected instructional strategy to share with the members of the PLC; (b) co-planned a lesson with their grade level teammate in which they purposefully incorporated the selected instructional strategy into the adopted mathematics textbook lesson; (c) enacted the lesson with their class; (d) shared and reflected upon how the lesson went when being enacted during the next Math Talk PLC meeting; and (e) revised the lesson with their grade level teammate based upon their experience and reflection. As they moved through each subsequent mini-design cycle, the participants continued to use the previously selected instructional strategies, but highlighted the use of the new strategy when planning and enacting the lesson.

**Mini-design cycle 1.** In Mini-Design Cycle 1, the participants decided to focus their efforts on providing more wait time/think time. When sharing the resources they found, it was surprising for the participants to see that much of the research on wait time was conducted several decades ago (see Appendix F). As Ms. Mitchell noted,
The article I found I thought was fascinating because it was from 1986, which was when I graduated high school, and that was so long ago and it’s exactly…exactly what we’ve been talking about—teachers need to offer more wait time…I mean it was so current, all of the material in it felt like it was so current. So it just struck me like, “What the hell is wrong with us that we’re not there yet?” It’s been so long.

Most of the participants acknowledged that they know providing wait time/think time in any subject is best practice, but that it can be challenging to enact that consistently.

Several ideas for providing wait time/think time were shared, including talk in your hand, the board flip, and thinking caps; these strategies were examples of ways in which the participants have tried to give students time to think during math instruction. The *talk in your hand* strategy asks students to whisper their response into their cupped hands. Thus, it is meant to allow the student to rehearse his/her response and to prevent the student from blurting out the response. The *board flip* is a helpful strategy if students are creating written solutions. Students write or draw their solution on a personal whiteboard, and they put their marker down and look up when they are ready to share their thinking. This process allows the teacher to monitor who is done and who needs more time to think. When everyone is ready to share, the students all flip their boards to show their solutions, which allows the teacher to gather formative assessment data and the students to see each other’s solutions. At that point, the teacher can select students to share their thinking. The *thinking caps* strategy is used to promote quiet thinking time. The students pretend to put their thinking cap on their heads and hold it there while everyone has time to think. The physical action of putting on the thinking cap helps the
students focus on the act of thinking about the mathematics and it keeps those who are ready before others from raising their hands, which can cause students who need more thinking time to feel rushed. The participants agreed to use any combination of these wait time/think time strategies that they felt would work best with their students.

Even though the participants came to a shared understanding of these wait time/think time strategies, their enactment differed. As they co-planned, the first and third grade teams took different approaches about how to enact these wait time/think time strategies. The first grade team selected a problem in which to enact each of the three strategies; whereas the third grade team was more general in indicating use of wait time/think time strategies, but leaving that up to Ms. Jansen and Ms. Davis to individually select which of the three strategies they thought would be best for their classes. Table 2 provides examples of how these wait time/think time strategies were enacted in some of the participant’s classrooms.

As the participants were debriefing their experiences during the Math Talk PLC, they noted difficulties in eliciting rich math talk as a result of providing wait time/think time. For instance, Ms. Mitchell noted,

There was very little math talk but I think it was because the concept was so complex for them, like they just didn't get it. We did do the board flip, but they didn’t have a whole lot to say about their strategies.

The first grade lesson was introducing the strategy of making a ten to solve word problems. The third grade lesson was about measurements of weight and mass. Similarly, Ms. Davis commented that as the complexity of the lesson increased from
identifying appropriate measures of weight to applying concepts of measurement, the richness of the discussion was limited:

When we switched to applying it to story problems...we were at a stand still.
They couldn’t remember what operation to use and, so it felt like it was running along smoothly and then it got too easy, and then I tried...to push them a little bit and then I just lost them.

At this point, the researcher shared that math talk can be impacted by the complexity of the mathematics concept:

If you remember back to that Math-Talk Learning Community framework (Hufferd-Ackles et al., 2004), part of the research that was done around that actually found that to be a typical phenomenon—that you might have higher levels of math talk going on in your classroom but then when you introduced a brand new concept, it would shift back down to a little more teacher-directed and then once the kids...get used to that new concept and start kind of taking it on their own, then it shifts back up to the...more student-led, so I think it sounds like you just experienced something that’s a really normal thing to have happen when you’re introducing a new concept.

To support this possible interpretation of experience, a few of the teachers then went on to share how they were having success with the wait time/think time strategies when they introduced them prior to the enacted lesson. For instance, Ms. Mitchell noted,

Talking into your hand and doing this and those [board flip and thinking caps], those went really well and one other thing I noticed...I wrote the decade numbers on the board and said, “What do you guys notice about these?” And those kinds
of questions lend themselves to all kinds of talk among them. I mean even down to “I noticed that you used green and blue markers.”

When the participants revised their lessons, they looked for opportunities to implement these wait time/think time strategies earlier in the school year and/or to implement them more systematically with specific modeling of expectations for engaging in each strategy as well as what both appropriate and inappropriate math talk looks like and sounds like.

Table 2
Classroom discourse examples from mini-design cycle 1: Wait time/think time

Ms. Green’s Class
7 tigers play games. 5 more tigers join them. How many tigers are playing games now?

T: Please solve it any way you want to. When you’re done, please put your marker on your board. Students get 3:10 to work. Markers should be down now. So we’re not gonna blurt. What you’re gonna do is you’re gonna board flip. So flip your board. When you board flip, you have to actually show us. As we look around, we a lot of different ways that people solved it. Let’s start with [S1]. What did you do to solve yours?

S1: I used dots.

T: You used dots. Did you use x’s down below that?

S1: Yes

T: She used kind of what we do when we have our problems on the math page. She used one symbol for one of the numbers and she used another symbol for the other one. She came up with 12. Is that one technique that you can use to do it? Did it work?

S1: Yes.

T: Okay, I saw something totally different. [S2] what did you do?

S2: I drawed the tigers and circled and wrote the number and drew an arrow to them.

T: She did what we talked about yesterday, the actual physical drawing of them. Did you come up with the answer?

S2: Yes.
Ms. Davis’s Class
T: Think of a situation in which you or someone you know needed to weigh something. What is measured when we weigh something? Let’s put our thinking caps on. Provides 10 seconds of wait time. [S1], what is measured when we weigh something?

S1: [No response]

T: Do you need some help? [S2]?

S2: If you’re at the grocery store and you get apples, you could weigh apples and if it was like 16 ounces it would equal 1 pound.

T: You said two things—ounces and pounds. Now there’s a new one you’re gonna learn. So [S2]’s right—16 ounces equals 1 pound. There’s another weight we’re going to learn, too. Can anyone think of it? I’m thinking that [S3] might know what it is? [S3] what is it?

S3: Mass

T: Mass, we are weighing mass? What’s another form of weight besides pounds and ounces?

S3: My brain is blank.

Conversation continues as students try to figure out T is looking for “grams.”

Mini-design cycle 2. In Mini-Design Cycle 2, the participants focused on using revoicing as a strategy to promote more math talk. Revoicing is a strategy in which the teacher or another student restates or rephrases what someone has said; it is typically used to confirm what has been said or to mark the significance of a comment (Chapin et al., 2013; Cirillo et al., 2014; Smith & Stein, 2011). When the participants shared their research during the Math Talk PLC, they provided both web-based resources and video clips of revoicing being enacted (see Appendix F). As the participants were talking through the revoicing strategy, the idea of a no opt-out arose. That is, if a student is asked to revoice and they cannot, for whatever reason, the person who made the original statement will repeat it and that same student will be given another opportunity to revoice. The participants agreed to incorporate this element of revoicing as they believed
it would encourage students to attend to the discussion and require them to make sense of what their classmates are saying about the mathematics.

As both grade level teams were planning how to incorporate revoicing into their mathematics lessons, they decided to use a sentence stem to support students in revoicing during the whole class discussion. The phrases they planned to use were, “I heard ____ saying ____” or “I think ____ is saying ____.” During the observations, I noticed the third grade teachers had this stem written on the board. Table 3 includes some examples from the classroom as the revoicing strategy was enacted during the lesson.

As the teachers reflected on their experiences implementing the revoicing strategy, they felt that the revoicing strategy within whole group instruction went smoothly. As an example, Ms. Jansen said, “I thought the revoicing piece went really well. They were able to really explain what the other people were thinking.” Furthermore, some of the participants noted that revoicing seemed to increase engagement and active listening, like when Ms. Jansen described how it went in her classroom, “I think the revoicing has caused my kids to pay closer attention cause I do the no opt-out, so if they aren’t listening, we go to somebody else and then come back to them.” Ms. Green also used the no-opt out and she would repeat a word problem again so a student could revoice it. While the first grade teachers had to facilitate that process, the third grade teachers could put the onus of asking for a repeat on the students. Ms. Davis said, “I told them ahead of time that if they needed to repeat whatever was being revoiced that they could do that. Cause some of my kids need that multiple times, so there was no shame.” Overall, the participants found success in implementing the revoicing strategy with their students.
Both the first and third grade teams purposefully planned to use the revoicing strategy during whole class discussions when conducting their initial lesson planning. Likewise, both grade levels also planned to include sharing among partners. When revising their lesson plans, the participants added the revoicing strategy within the partner talk. That is, when they paired students to restate the problem in their own words and/or share their solution strategies, they would structure their talk so that Partner A would share and Partner B would revoice Partner A’s statement and then the roles would reverse. The participants believed that this structure would help the partner’s attend to each other’s thinking more closely if they knew they would have to revoice.

Table 3
Classroom discourse examples from mini-design cycle 2: Revoicing

**Ms. Mitchell’s Class**

T: Okay, now show me a signal if you have an idea about these equations. [S1], what’s your idea?

S1: The 50 and the 40 and the 90 all have zeros at the end.

T: Tell us what [S1] just said [S2].

S2: I don’t know what she said.

T: So you ask [S1] what she said.

S2: What did you say?

S1: 50 and 40 and 90 all have zeros at the end.

T: [S2] nice and loud, what did [S1] just say?

S2: I forget really quick.

T: [S1], tell [S2] one more time what you said.

S1: 50 and 40 and 90 all have zeros at the end of them.

S2: 50 and 40 all have zero at the end.

T: Right on.
Ms. Jansen’s Class

Hirva left home at 9:45 a.m. and returned home at 11:20 a.m. She spent 55 minutes at the gym and the rest of the time at the library. How much time did Hirva spend at the library?

T: Where do you think we should start? Should we do a timeline or use our clocks? [S1] what do you think we should do? How should we start?

S1: We should use a number line.

T: He said we should use a number line, so everybody please write a number line. [S2], what did [S1] say?

S2: I think [S1] is saying we write a number line.

T: That’s what we’re gonna do. And what should we do now, [S3]?

S3: Mark 9:45 a.m.

T: [S4], what did he say we should do?

S4: We do something with 9:45.

T: [S3] said we should put 9:45 on there. The question is how much time did Hirva spend at the library. What do you think our next step is?

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Mini-design cycle 3. In Mini-Design Cycle 3, the participants incorporated turn and talks into their mathematics instruction. Turn and talks, sometimes referred to as think-pair-shares, involve partnering students for a brief conversation based on a specific prompt or question for students to discuss. When the participants shared their research about turn and talks, there was discussion about the need to be clear about expectations and to model what an appropriate and productive turn and talk looks like and sounds like. Ms. Mitchell shared some videos that featured the fishbowl method as a means to teach these expectations. During a fishbowl, the students sit in a circle and a pair of students sit in the middle and do a turn and talk while their classmates observe them. When the turn and talk ends, the teacher and students debrief the experience noting what went well and what could be adjusted based on the expectations for the turn and talk. Thus, considering
how to help students understand the expectations of the turn and talk was a focus of the participants when they planned their lessons. When enacting the lessons, Ms. Jansen did use the fishbowl technique to set expectations for the turn and talk, and she provided feedback to the students after their first few turn and talks. Similarly, Ms. Mitchell had students practice the turn and talk by sharing what they did over the weekend as means to set expectations. Both Ms. Green and Ms. Davis had pre-taught the turn and talk as a strategy prior to enacting it in the observed lesson for this design cycle. Table 4 shares some examples from the classrooms during the lesson in which the turn and talk strategy was enacted.

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Classroom discourse examples from mini-design cycle 3: Turn and talk</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ms. Mitchell’s Class</strong></td>
<td><strong>Rosa reads 8 stories. Tim reads 5 stories. How many stories do they read in all?</strong></td>
</tr>
<tr>
<td></td>
<td>Some students have whiteboards and others have Unifix cubes to solve.</td>
</tr>
<tr>
<td></td>
<td>T: I want you to turn and talk to your partner. Your partner is going to be the person sitting across from you. Remember, eyes, ears, and take turns talking. Go ahead and tell them how you solved the problem. <em>Turn and talk lasts 1:20 min.</em> When you are finished, turn your chair so it looks at me. You have nothing in your hands. I should see your eyeballs. [S1], how did you choose to solve this problem?</td>
</tr>
<tr>
<td></td>
<td>S1: Use cubes. I put them together when I was counting them and I saw it was 13.</td>
</tr>
<tr>
<td></td>
<td>T: What you told me when I was at your desk is that you started with one of those numbers. Which number did you start with?</td>
</tr>
<tr>
<td></td>
<td>S1: 8</td>
</tr>
<tr>
<td></td>
<td>T: Started with 8 and then counted…</td>
</tr>
<tr>
<td></td>
<td>S1: 5</td>
</tr>
<tr>
<td></td>
<td>T: Then counted 5 more. And what answer did you get?</td>
</tr>
<tr>
<td></td>
<td>S1: 13</td>
</tr>
<tr>
<td></td>
<td>T: 13 in all. If you also got 13 in all, would you show [S1] that you got 13 in all. <em>Students use agree hand signal</em></td>
</tr>
</tbody>
</table>
Ms. Davis’s Class

Round 989 to nearest 100.

T: Same shoulder partner, but the other person is going to talk. **Turn and talk**—go! **Provides 1:30 minutes of time to talk.** [S1], looking at this number, what do we round it to? Do we round down or round up? We’re rounding to hundreds.

S1: Up

T: So what do you think the number would be?

S1: [No response]

T: Do you need some help? [S2], what would you say?

S2: I would say that it was 1,000 because it was past the 5.

T: And [S1], I would like for you to repeat what [S2] just said. So I need you to say, “I think [S2] just said…Do you need [S2] to repeat it? [S2] would you repeat it?

S2: I think it’s a thousand because it was over 5 so it’s closer to 1,000.

S1: I think it’s a 1,000.

T: I think [S2] is saying it is closer to 1,000.

S1: I think [S2] said it is closer to 1000.

T: Good—because it’s over the 5.

When planning their original lesson, the participants focused on clearly setting expectations when using the turn and talk. The ease with which the turn and talk was enacted varied across the participating classrooms, but all the teachers reviewed the procedures for the turn and talks using one of the posters shared in the Math Talk PLC at the beginning of the lesson and some of them referred to it again as needed during the lesson. Moreover, the teachers provided clear expectations for who would be partners, including elbow partners (adjacent) and eyeball partners (across). When revising their lesson plans, the first grade teachers added a turn and talk in which both students would revoice the word problem to ensure they had an opportunity to make sense of the problem.
before solving it. Then they would engage in another turn and talk to share their strategy with a partner. The third grade team decided they would like to model turn and talks earlier in the school year. When revising the rounding lesson, they noted that because the students had solid background knowledge about rounding, they would jump ahead in the lesson to the activities that required more reasoning and, therefore, would have more rich mathematics for students to discuss in their turn and talks. Overall, the turn and talk strategy was successful in its implementation.

**Themes across the mini-design cycles.** Throughout the three mini-design cycles, the Math Talk PLC participants engaged in continuous reflection both in the PLC setting and in interviews with the researcher. During those conversations, several themes emerged about the participating teachers’ learning process and experiences in the professional learning community. In what follows, the themes of connecting research and practice, intentional planning, collegiality and support, and allowing time for change are explored in greater depth.

*Connecting research and practice.* The mini-design cycles in the Math Talk PLC were developed to be grounded in theory and action (Loucks-Horsley et al., 2010). That is, the participants would engage in research about a math talk strategy in order to develop a shared understanding of that strategy before enacting it in their classrooms. Several of the participants discussed the role of research as part of the mini-design cycle process. Ms. Jansen noted, “I really enjoyed the fact that we had to do some research first. It brought it all home—Gosh, this is research-based and it does increase how well kids do.” Ms. Green discussed how just doing research is not enough to make lasting change when she said,
So I think on the one hand, if you don’t have the research and you’re just copying someone, you don’t have anything to keep you going, and I think the other one, if…you do all the research, but you really don’t have anyone to watch or you don’t really know what you’re doing, you just read about it…you’re going, “Okay, I’ll try it,” you might not follow through either.

Ms. Green’s comment underscored the need for a balance between theory and action. Action without a research-base may not translate into effective change, just as research without an opportunity to enact it and receive support and feedback may not translate into lasting change. Therefore, the mini-design cycle process sought to find that balance by including research to ground the co-planning, enacting, reflecting, and revising. Ms. Jansen focused on that element of feedback when she said, “I appreciate that…we did research, and then we planned, and then we came back and we got to share. I think that sharing piece—Well, I think this time you could try this—that was hugely beneficial.”

Thus, the mini-design cycles facilitated teacher learning in which research informed practice.

*Intentional planning.* As the participants worked through the mini-design cycles in the Math Talk PLC, many of them noted how their math planning process changed in terms of being more intentional about looking for opportunities to incorporate math talk. For instance, Ms. Jansen said, “I know [Ms. Davis] and I, as a team, are being a lot more thoughtful about our planning.” Likewise, Ms. Mitchell commented how the mini-design cycle process supported intentional planning by considering, “Here’s where I’m going to have them revoice. Here’s where I’m gonna have them turn and talk or whatever.” In fact, Ms. Mitchell went on to say, “I want to just continue to really be aware of them [the
math talk strategies] and build them in intentionally. I think that has been super helpful and that is something that I definitely want to continue to work on.” Ms. Jansen also discussed the evolution of her math lesson planning as a result of being part of the Math Talk PLC:

I think I had gotten into the routine of, “Okay I just have my printing and I have my PowerPoint—I’m ready to go.” And I didn’t really even think about instructional methods and that’s huge, that’s if not over half the battle—getting them engaged and learning. So I think just the focused intentional planning has been helpful and then the revisiting—that’s the piece I really appreciate is coming back and saying, “Well instead maybe we should have done revoicing here instead of whatever.” I think that’s the helpful piece.

In addition to addressing the need to purposefully plan for math talk, Ms. Jansen also touched upon the value of structured reflection and engaging in the revision process as part of the broader lesson planning process.

Collegiality and support. One of the purposeful design elements of the Math Talk PLC was to create a system of collegial support for the participants to make instructional change around planning for and facilitating math talk. Professional learning communities are an ideal setting to create these systems of support and provide structure for co-planning and reflection (Blank, 2013; NCTM, 2014). In fact, Ms. Davis explained how the collaborative process was necessary for her successful learning:

I don’t think it would be helpful for just me to do it [by myself] because I feel like…I need to be able to run the ideas off my teaching partner and the other grade levels—that was great to hear their ideas.
Similarly, Ms. Jansen discussed how the Math Talk PLC facilitated creating a shared understanding that she and Ms. Davis can “incorporate into our teaching, so when we plan, I can say, ‘Oh this would be a great place to incorporate revoicing.’ So I appreciate that we’ve had the chance to get together and do that as a team.” Because most teachers typically work within their grade level, the Math Talk PLC was also specifically designed to facilitate cross-grade level collaboration. When considering that structure, Ms. Jansen noted, “[I]t’s nice to have the camaraderie of the different grade levels, to see what first grade is doing as compared to third.” Likewise, that cross-grade level structure widens the sphere of support for the participating teachers. Even when the Math Talk PLC comes to a close, Ms. Mitchell noted, “I’ll continue to have the support of the colleagues I actually work with.” That is, the participants have on-going support within their grade levels after the professional learning community formally disbands at the end of the study.

Allowing time for change. Once the participants began engaging in the process of changing their instruction to create more opportunities for math talk, they discussed the need for time to practice these new strategies. After completing the three mini-design cycles, Ms. Jansen said, “We don’t need any more resources. We have enough of those. I think just that time piece—it’s important.” Making instructional changes does take time. Some of the participants found themselves trying to reconcile the need to allow themselves time to make that change and the pressure to cover all of the content in their mathematics textbook. As Ms. Green noted, “The challenge has been: Breathe. Do it right.” She went on to say, “[I] think people just have to be patient and realize you don’t have to do all of the changes all at once.” In fact, Ms. Davis brought up the fact that
implementing these math talk strategies over a longer time period was different than some professional development she had done in the past “where it’s like two consecutive days and a month later, I’ve forgotten everything. So, I think because it was over a couple months, that I was able to apply it, and apply different strategies at a different time.” Given the longer timeframe to try these math talk strategies out supported the teachers in realizing instructional change that could last beyond the Math Talk PLC. At the same time, the participants had to balance this learning curve with the demands to keep moving through their mathematics curriculum, especially the third grade teachers whose students had to be ready to take the Smarter Balanced Assessment in April.

Moving beyond the Math Talk PLC. As our work in the Math Talk PLC drew to a close, it was time for the participants to consider how they might sustain the work of purposefully planning for and facilitating math talk during instruction. Grade level collaboration, including co-planning, would continue between Ms. Mitchell and Ms. Green in first grade and Ms. Jansen and Ms. Davis in third grade. However, the Math Talk PLC was designed to have participants create an action plan as a means to ensure they would continue to incorporate the instructional strategies to promote math talk into their lesson plans and practice facilitating them with their students (See Figure 3 for Math Talk Action Plan).

When the conversation about the action plan began, the participants immediately began discussing how they wanted to begin using math talk with their students next year. Ms. Davis shared her idea for systematically teaching the math talk strategies to her students:
I’m looking to next year and…I was thinking I would implement this the way that I did the Daily Five, so maybe the first week is we are just practicing revoicing. The entire week we are practicing revoicing. The next week revoicing and then turn and talk, so kind of like the way we did where we had a new strategy, but I’d like to start the year off with all of those strategies, just working a few days for each one.

Moreover, the participants indicated that they wanted to infuse these math talk strategies into more subject areas, so that they become *talk strategies* for all aspects of learning, as seen in the following exchange at our final Math Talk PLC meeting:

Ms. Davis: Yeah, start those strategies before we even start math…

Researcher: Talk.

Ms. Davis: Instead of math talk, it will just be called talk.

Researcher: Or student talk, right? Or academic talk, maybe?

Ms. Mitchell: You might start with social interaction talk.

Ms. Davis: Getting to know you talk.

Based on this exchange, the participants decided that the would begin using these talk strategies as part of building their classroom community and developing social skills among their students at the beginning of the school year. From there, they would apply it within their mathematics instruction as well as instruction in other subject areas.

After the lively discussion about how to begin math talk next year, the researcher guided the conversation toward creating an action plan for the remainder of the school year. There was a lot of interest, especially by Ms. Davis, to create anchor charts about the math talk strategies. The belief was that while the anchor charts would support the
students when using the particular math talk strategies, more importantly, it would serve as a visual reminder to the teacher to purposefully plan for math talk and it would support the teacher in enacting math talk strategies in the moment during instruction. At the same time, some of the participants expressed that they might want to employ a different type of visual reminder, perhaps something electronic that they could project on the InterWrite Board. In the end, the participants agreed to develop some type of visual reminder, but the format was flexible to customize each teacher’s situation and preference. In addition to the visual reminder of the math talk strategies, the participants also agreed to intentionally plan some of the math talk strategies into their lessons at least once a week. This way, they could engage in continued practice with the math talk strategies, but keep it manageable to where they could feel continued success. Also, using the at least language left space to use math talk more often if time permitted.

<table>
<thead>
<tr>
<th>For This Year....</th>
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<tbody>
<tr>
<td>• Purposefully add in Math Talk to at least one lesson a week</td>
</tr>
<tr>
<td>• Create and Use an Anchor Chart or Posters (physical or electronic) for Math Talk Strategies</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>For Next Year....</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start talk strategies at the beginning of the year</td>
</tr>
<tr>
<td>• Social skills</td>
</tr>
<tr>
<td>• Math and other subjects</td>
</tr>
<tr>
<td>• Introduce and practice strategy each week</td>
</tr>
<tr>
<td>Will meet after Spring Break to plan this out</td>
</tr>
</tbody>
</table>

**Figure 3.** Math talk action plan

Two post-observations were conducted at three weeks and six weeks past the end of the Math Talk PLC, and a post-interview was conducted in order to track how the
participants were implementing the action plan. All of the teachers reported they were still actively using the math talk strategies of wait time/think time, revoicing, and turn and talk, which I also observed during their post-observations. The Math Talk Action Plan called for the teachers to purposefully incorporate the math talk strategies into at least one lesson a week. Most of the teachers reported in their interviews that they were meeting this goal. For instance, Ms. Mitchell said, “Nearly all of my lesson plans that I’m doing in math, I build in some sort of math talk strategy that we’ve talked about.” Ms. Green also stated she was doing math talk more than once a week. Ms. Davis noted, I feel like I do that on every lesson, so when I am looking ahead to the lesson of the day, I just consciously look at, okay, where can I fit this in there? And then, and I kind of, I guess, mentally rehearse it before I instruct it. Ms. Jansen reported that meeting the goal was challenging because she had a student teacher. Therefore, she was not teaching math consistently as she had turned most of the mathematics instruction over to her student teacher by March. Likewise, her student teacher had not been part of the Math Talk PLC and was, therefore, not as familiar or comfortable with the math talk strategies. With regard to the second part of the Math Talk Action Plan, Ms. Davis was the only teacher who made an anchor chart about the math talk strategies and she referred to them consistently while teaching. Overall, the post interviews and post-observations did show that the Math Talk PLC participants were continuing to apply what they learned about planning for and facilitating math talk.

Even though the participant teachers and the researcher all had shared experiences as a result of their time together in the Math Talk PLC, each individual also experienced the Math Talk PLC through her own lens. Within the more inclusive case of the Math
Talk PLC, Ms. Mitchell, Ms. Green, Ms. Jansen, and Ms. Davis had unique experiences within each of their classrooms as they enacted the math talk strategies. What follows is an examination of each embedded case.

The Experience of Ms. Mitchell and her Class

Ms. Mitchell’s journey throughout the study was best characterized as discovering who she was as a mathematics teacher. This was Ms. Mitchell’s first year of teaching first grade and teaching mathematics. She was focused on wanting to increase her “familiarity with the curriculum and the language” of mathematics teaching. Given her newness to the subject, Ms. Mitchell usually asked to debrief her lesson observations. She noted that she appreciated getting feedback and to “have a chance to really process [the lesson] with [the researcher] has been really nice.” At the same time, Ms. Mitchell did acknowledge that having me in her classroom observing her created some anxiety. She said, “I’m always nervous to be observed.” I tried to be cognizant of that when observing and especially when we debriefed the lesson. During our debrief, we began by talking about what went well and then I let Ms. Mitchell lead the conversation about possible adjustments or refinements. Most of our discussions centered on Ms. Mitchell’s frustration with whole group instruction and how that limited her ability to meet the varying needs of her students.

As Ms. Mitchell began to implement the math talk strategies from the Math Talk PLC into her classroom, she found the process a little overwhelming. But the focus on specific math talk strategies as well as the support from her colleagues has helped her understand how to facilitate math talk among her students. She noted,
I mean I think I knew that allowing kids time to talk about math was super positive but in terms of like how to do that management-wise wasn’t clear to me. And I, unlike [another teacher] and some other people in the group, I hadn’t had any of the coursework on teaching math at all, so you know it was very abstract to me—like, “Yes, we all need to chat,” but how? So yeah, I think that’s been really great to just have specific strategies that I already knew, that really were already in my toolkit, just needed to take them out and polish them and use them.

Several times Ms. Mitchell noted that using talk strategies had been an integral part of her teaching practice in the past, especially during language arts instruction. Thus, the Math Talk PLC provided the space for her to re-envision those talk strategies in the context of mathematics instruction. At the end of the Math Talk PLC, Ms. Mitchell discussed some of her favorite math talk strategies:

I love “Give me the signal when you’re ready with an answer.” I love the talk into your hand for the whole group management. That is just so handy; it makes them feel validated and for the smaller group, I love the turn and talk and then coupling that with revoicing. Those are probably my favorites.

In both of my post-observations, I saw Ms. Mitchell consistently using these strategies as well as giving students think time before a response.

When reflecting on her approach to teaching mathematics, Ms. Mitchell noted, “I think the best approach for me and the kids that I’m teaching is to use small groups and to use lots of hands-on.” Over the course of the study, this emphasis on small group instruction resonated in Ms. Mitchell’s comments, like when she said, “Small group lessons are just my thing.” Ms. Mitchell and I discussed the logistics of shifting to using
the blended learning structure regularly instead of the heavy reliance on whole group instruction found in the adopted textbook. In addition to being able to differentiate instruction, Ms. Mitchell also discussed how the students are able to engage in more math talk on blended learning days when she stated, “During those lessons, they’re talking constantly about math because they’re doing games together where they’re talking about ‘I hid 6 bears, so there must be 4 out here.’ At the teacher table during the blended learning, they’re talking amongst themselves.” By late January, Ms. Mitchell had made the decision to use blended learning as the primary structure for her math instruction.

During one of my post-observations, I was able to see Ms. Mitchell enact her small group teaching approach in mathematics using the blended learning model. The lesson was about reading pictographs and making comparison statements about them using words like greater than and less than. Ms. Mitchell had a single page that displayed three different graphs, which she used in various ways with the different groups she saw. She grouped the students by their skill level for this concept. The following is an excerpt from one of the small groups in which Ms. Mitchell uses some math talk strategies to get the students to discuss a graph that shows how many eggs the chickens, Clucker, Daisy, and Vanilla, laid:

Ms. Mitchell: As soon as you have something that you notice, would you give me this signal [thumbs up on chest]. Then I’ll know we’re ready as soon as you’re ready to share. Provides 25 seconds of think time. [Student 1], what did you notice about this chart?

Student 1: They have names.

Ms. Mitchell: They have names. Who has names?
Student 1: The chickens.

Ms. Mitchell: And whoever made this chart labeled it, didn’t they? They used the chicken’s names as labels. We’ve been using mostly colors for our labels, right? In this case they used names. [Student 2], what did you notice about this?

Student 2: 4 and 7 are less than 9.

Ms. Mitchell: Wow! Where in the world are you getting those numbers?

Student 2: I can’t.

Ms. Mitchell: Show us please. Can you write the number next to the information on the chart? See the blanks over here, those blanks are meant for numbers. Would you guys fill in those numbers? I see what you noticed. So you noticed that, which chicken?

Student 2: Clucker…

Ms. Mitchell: Clucker laid…

Student 2: 4 eggs.

Ms. Mitchell: [Student 2] noticed that Clucker laid 4 eggs. [Student 3], can you say something about Vanilla?

Student 3: Vanilla laid 9 eggs.

Ms. Mitchell: Vanilla laid 9 eggs. And [Student 1], what about Daisy?

Student 1: Daisy. 7.

Ms. Mitchell: Can you say it in a sentence?

Student 1: Daisy laid 7 eggs.
In this excerpt, Ms. Mitchell utilized the math talk strategies of think time and revoicing. First, she provided students think time and then asked them to use a signal to indicate they were ready with their response. This allowed her to monitor who still needed more time to think and it eliminated the possibility of students feeling rushed because their peers had their hands raised. Ms. Mitchell also used revoicing to support students in forming complete thoughts about the graph. This group of students needed that extra language support. The subsequent groups that met with Ms. Mitchell had varying skill levels, but did not need as much language support as this small group. Ms. Mitchell continued to use the think time and revoicing strategies, but had more students involved in the revoicing.

By shifting to small group instruction, Ms. Mitchell was able to better meet the different needs in her classroom. While she was meeting with a small group, students were engaged in three other centers: (1) seatwork completing a *remembering* worksheet; (2) computer game focused on fact fluency; and (3) playing a game in which they sorted *Unifix* cubes, created a graph, and made comparison statements working with a partner. During all of these activities, students were conversing about their math work, especially at the game center. When reflecting on what she had learned as a result of participating in the Math Talk PLC, Ms. Mitchell said,

I think that I’ve learned I’m not a whole group teacher; that I’m a small group teacher and…that’s totally fine. That I can actually get all of the content that I want to get across in the way that feels most comfortable for me.

Not only did this shift to small group instruction mark a departure from the adopted math textbook, it also represented Ms. Mitchell’s discovery that small group teaching with
centers is a structure for mathematics teaching that matches her philosophy and more readily allows her to meet the needs of her students.

**The Experience of Ms. Green and her Class**

Ms. Green’s journey during the Math Talk PLC was one of refining her practice. Given her many years of teaching experience, Ms. Green has developed her skills as a mathematics teacher in the primary grades. She believes that instruction must take into account a student’s developmental growth, and it must be useful to the students:

I think what my basic belief is that…whatever skill you’re going to teach, you introduce the skill with the idea that everybody is gonna understand it. And I think you have to introduce that same skill with the idea of how does it apply to real life. I mean, how in the world could this be useful?

Therefore, Ms. Green usually grounds problems in context. She also talks with her students about the need to be confident and accurate as they will use mathematics in the real world:

You do math every single day of your life. And I want children to know that all the stuff that you actually have to do the rest of your life, you might not be doing trigonometry, but you will be doing addition and subtraction the rest of your life. If you are gonna depend on your cell phone or if you’re gonna depend on your calculator, boy don’t do that…You know, get some tricks down so you can actually do it in your head or, you know, however you’re going to do it. Get so you can actually trust yourself.

Therefore, Ms. Green approaches mathematics teaching from a problem solving perspective and tries to make it engaging and relevant for her students.
Problem solving played a central role in Ms. Green’s mathematics instruction, even before beginning the Math Talk PLC. This is not surprising given what she explained she enjoys most about teaching math:

I like to see the kids—how they figure things out. I like to see just their thought processes and how they, like I said, how they figure things out. And that there’s more than one way to figure things out.

A typical math lesson in Ms. Green’s classroom, based on my observations and other times I had dropped by her room, was focused on students sharing how they solved problems. The students would sit on the carpet in a circle, Ms. Green would pose a problem, the students would solve the problem using manipulatives, their fingers, and/or whiteboards, and then they would share their solution and strategy. She works to support them in being “able to explain their rationale—their thinking processes is kind of entertaining sometimes with the little guys ‘cause they’re so logical and they’re so—it’s, you know, 1 + 1 is 2—literally—I just know it.” An instructional strategy she often uses is asking her students to imagine they are explaining their thinking to a kindergartener.

Ms. Green reported occasionally using partner talk when she taught math, but she did not consistently incorporate math talk into her instruction other than asking students to explain their strategies for solving problems.

As she began to purposefully plan for and facilitate math talk, Ms. Green continued to have students engage in problem solving and sharing their thinking on a regular basis. However, she also incorporated more formal math talk strategies, like the board flip, talk in your hand/whisper response, the agree signal, and turn and talks. Because she has a very small class during math instruction, Ms. Green regularly changes
up partnership so that students have a chance to turn and talk with multiple classmates during a lesson. The one strategy that Ms. Green rarely uses is revoicing, except in the context of having a student repeat/revoice the word problem she poses.

Ms. Green was especially concerned about maintaining a positive and respectful classroom environment because two students from the Extended Resource classroom joined her class during math. The two students were fully included in class during math; they sat in the circle, they were paired with other students, and they were called on to share their solutions strategies. Sometimes the para-educator who accompanied the students would support their explanation when sharing with the whole class. Ms. Green was watchful of the interactions among the students and she was quick to praise students for working respectfully with all students. Moreover, Ms. Green worked to create an environment in which students felt safe to share, even if they might not get the right answer. She shared,

They have to feel comfortable enough to know that they can get it wrong, you’re not going to cut off their head or something like that. And it’s perfectly fine. And your peers will not laugh at you, if you say 1 + 1 is 3 or something like that. They have to understand that it’s…you learn by even making mistakes—that’s perfectly fine. I think they have to understand that they have to be willing to take that step forward and being able to talk out there.

Based on my observations, students did appear to feel comfortable to share, and when students had an error in thinking their classmates were kind and respectful. Consider the following exchange when solving the problem 30 + 40:
Ms. Green: [Student 1], how’d you solve yours?

Student 1: I pounded the smaller the number.

Ms. Green: Show me what you did.

Student 1: I pounded the smaller number. 30, 31, 32, 33, 34.

Ms. Green: I think I know where you got stuck. Does anyone else have a suggestion about where [Student 1] might have gotten stuck?

Student 2: I think you counted by ones.

We can see from this exchange that the second student respectfully identified what she thought was the first student’s error. Ms. Green had Student 1 build the problem with Base Ten blocks and then he was able to count on using tens, not ones. Ms. Green also shared an anecdote about how her students inducted a new student into the norms of their math learning community:

They taught her [the new student] this [the agree symbol]… She was looking at them like this [confused] and then they taught her how to do it because she didn’t know how to do it [physically forming her hand]. She was kind of like [forming it wrong]…No, no, put your pinkie out, put your thumb out, and you put your thumb toward your chest, like this if you agree with her. And all of a sudden she was going, “This is cool.”

As this example and the others illustrate, Ms. Green’s work to ensure a positive, safe learning environment during mathematics instruction was successful as reflected in the behaviors of her students.

In reflecting on her participation in the Math Talk PLC, Ms. Green believed that both she and her students were engaging in deeper understanding as a result of using the
math talk strategies. Ms. Green noted that because the students were talking more about mathematics, she had greater insight into their thinking:

It’s increased my understanding for their understanding, if that makes sense. So I think what it does is it makes me have a better comprehension of just how deep their understanding is. Cause I mean it’s one thing to take a little quiz and pass a quiz and all of a sudden you’re like, “Okay, they’ve got it, but to what depth?” So that’s where I’m seeing the improvement significantly.

Likewise, students also gained greater insight into their own understanding of mathematics when they engaged in talk. Ms. Green shared an anecdote about one of her students who was quite advanced in his mathematics knowledge, but struggled to articulate his thinking. She explained how using the math talk strategies helped him:

And so getting him to actually verbalize what he is thinking and he has gotten much better. He is much better than he was when we started and so I think…having him be able to break it down into steps for someone who doesn’t understand it is also a huge step. Cause all of a sudden, then he finally understands, “This is what my brain is doing.”

Additionally, Ms. Green concluded that the greater emphasis on math talk and explanation has also changed the students’ perceptions of what it means to do mathematics:

I feel like the kids have had to do a little more thinking because they’ve had to explain what they’ve done and it has been very interesting to watch. Before, I think a lot of times it was just the answer, right or wrong—that was just it. And so children thought, “Okay, I’ve got the answer right,” so they understood it, but
now they’ve had to explain their thinking and I think it’s actually made them take it to a new depth. I think that’d probably be the best way to explain it. I’m hoping what’s happened is it’s given them a deeper understanding of what they’ve been doing.

The students in Ms. Green’s classroom are learning that in mathematics they must go beyond the right answer just as they must go beyond “I did in my head” or “I just knew it” as an explanation of their thinking. Thus, Ms. Green’s journey provided a space for her to refine her problem solving approach to teaching mathematics by purposefully planning for and facilitating math talk to support deeper student understanding.

The Experience of Ms. Jansen and her Class

Ms. Jansen persevered through her Math Talk PLC journey; it had some bumps along the way, but she persisted and the road is smoother now. Ms. Jansen has been teaching third grade for nine years, and she characterized this year’s class as her most challenging in terms of behaviors. Based on my observations, she has many students who struggle to self-regulate in terms of attention and impulsivity, which results in students often being engaged in something other than the lesson. Thus, classroom management, especially during mathematics, has been an on-going focus of Ms. Jansen’s efforts. Consequently, Ms. Jansen faced some challenges when trying to implement the math talk strategies with her students.

Ms. Jansen is passionate about mathematics learning. She strongly believes that students should solve problems in ways that make sense to them, so she provides opportunities for students to share their solution strategies with the class. Ms. Jansen explained,
I do think that a child, as long as they, you know, have a way of solving that works every time, I think they should be able to use whatever works in their little mind…And I definitely think sharing those ideas is important because it might click with somebody else in a better way than how I teach it.

Having worked with Ms. Jensen on previous professional development projects, I know she has a high-energy approach to mathematics instruction, where she uses a lot of humor, movement, and connections to the real world to engage her students in learning. Ms. Jensen explained, “I try to make it [math instruction] fun, I think. I think the kids have fun—sometimes too much fun and then they get out of control, so it’s hard to reel them back in after that.” Herein lies the challenge—balancing her energy with the energy of her students and making that work during mathematics instruction when the students are talking and interacting with each other on a more regular basis.

As Ms. Jansen worked to implement the agreed upon strategies from the Math Talk PLC, she became more explicit about her expectations for student behavior during the enactment of those strategies. She said, “I need to better model what I expect out of them.” To that end, Ms. Jansen also engaged in more modeling to show what the math talk strategy looked like and sounded like. For instance, when teaching the revoicing strategy, she modeled what revoicing would sound like in the context of the lesson by playing the part of two students: Ms. Jansen 1 and Ms. Jansen 2. Similarly, Ms. Jansen was very systematic in how she introduced the turn and talk strategy. First, she used the turn and talk poster shared during the Math Talk PLC to review the procedures for the strategy. Secondly, she used the fishbowl technique to demonstrate for students what the
turn and talk strategy should look like and sound like. After setting those expectations, the students engaged in their first turn and talk:

Ms. Jansen: When I say turn and talk, you are going to discuss what you know about estimation. Maybe you don’t know what that word means, maybe you do. I’m just gonna give you a few minutes. Turn and talk—remember these things as you turn and talk [references Turn and Talk poster]. Go. Remember, I know you’re done when you face me.

_Gives 45 seconds to talk._

Ms. Jansen: It looked liked most of you were waiting your turn. It wasn’t too loud, but we probably could’ve brought down the noise a little bit, but don’t forget that piece about taking turns and looking forward. So [Student 1], what do you and your partner know about estimation?

Student 1: It’s when you guess…It’s like a guess. It’s a guess. Also, it’s like a hypothesis, which is the same thing.

Ms. Jansen: You said a guess or a hypothesis. [Student 2], what is estimation? What do you know about it?

Student 2: Estimation when you ask…is when you make a guess about something.

Ms. Jansen: [Student 3], what is estimation?

Student 3: Estimation is like a guess…like guessing how long something is or how tall something is.
Ms. Jansen: So an estimation is a guess, but it’s not just a guess—it’s a good guess.

In this exchange, Ms. Jansen provided feedback to the students about how well they met the expectations for the turn and talk. She had them continue to practice the turn and talk several more times throughout the lesson. In reflecting on her journey to plan for and facilitate math talk, Ms. Jansen shared:

So this has been my toughest class. Anytime I let them kind of, you know, do the math talk it’s so hard to reel them in. So I think that’s a struggle that I’ve had. It just takes so long, so it’s almost tempting not to do it because it takes so long, but I know it’s beneficial so I have to talk myself through that, “It’s so beneficial, you’ve gotta let them do it.” So, that’s been a struggle.

Investing the time to set up the math talk strategy from the beginning was key to helping Ms. Jansen implement more talk during her math instruction. Likewise, investing the time to reinforce expectations and model the strategy again, if needed, also contributed to more successful implementation of the math talk strategies. Ms. Jansen mentioned a few times where she had to “backtrack” and revisit those expectations, but doing so helped.

While it has been challenging to help her students engage in math talk, Ms. Jansen persisted and, by the end of the Math Talk PLC, she was finding more success in implementing the math talk strategies. For instance, she shared:

I love the turn and talk now. Whereas before, it didn’t work out so well. But I kind of refined it and I really love the turn and talk. When I have been teaching, it seems like they’re more engaged because, you know, there [are] more opportunities to share ideas.
When asked what changes she made to the turn and talk, Ms. Jansen explained, “I assigned roles and I made it a little bit more exciting because I switch how I pair them up, so they never know what they’re going to get.” She also commented that consistently reviewing expectations and procedures helped students engage in productive math talk:

I think they’re a lot better than when we started. They know their routines; they know they have to…well a few of them still don’t talk as much as they should, but I have a lot more participation in the appropriate way than I did before.

Ms. Jansen also saw changes in her students’ ability to communicate about their mathematical thinking:

I think there ability to speak about their thinking has become better. They’re able to voice their thoughts better, which is important. And the reason I know that is because at the end of some of their worksheets their called, “Going Deeper or Deeper Thinking” and they’re better able to, you know, verbalize or write down what their thinking is to solve that problem.

Thus, Ms. Jansen’s persistence in finding ways to make the math talk strategies work for her class of students paid off. While navigating the management aspects of math talk did present some challenges for her, Ms. Jansen persevered. By continuing to review and reinforce her expectations for the math talk strategies, Ms. Jansen helped her students come to understand how to engage in math talk productively.

The Experience of Ms. Davis and her Class

Ms. Davis had one of the most transformative experiences of the participants. Over the course of the study, as Ms. Davis applied the strategies from the Math Talk PLC, she changed the nature of her mathematics instructional practice from primarily
teacher-led to much more student-led. When self-assessing how much her students talk during a mathematics lesson, Ms. Davis reported students engaging in math talk about 10-15% of the time before beginning the Math Talk PLC. She estimated students were engaged in math talk 60-70% of the time during a typical lesson by the end of the study. My observations in her classroom support this dramatic change. So what brought about such a shift in practice during Ms. Davis’s journey?

Ms. Davis has been teaching for four years, and this was her second year teaching third grade. She enjoyed helping her students be successful learners, but she found it challenging at times. Ms. Davis said, “I like those light bulb moments when you see them make the connection; when they are confused and you’ve talked through those strategies for days and all of a sudden they…get it.” Key to making those connection was her belief that students need to persevere. She noted, “They need to stick it out and stick with it. That it’s [math] going to be a challenge, but once you’ve overcome that challenge…you’ll see your end goals met.” At the same time, when interviewed about what she found challenging about getting her students to talk about math, Ms. Davis said,

That they get off task. That the level of knowledge is…I might have one kid that really gets it and then another kids that’s struggling… I guess differentiation. And then I think it goes back to perseverance. I think they would rather just shut down and say they don’t know what they’re doing instead of engaging in that conversation.

Thus, Ms. Davis saw the potential in her students to be successful math learners, but she was struggling with how to make that a reality given behavior and academic challenges. At the end of the first interview, Ms. Davis shared, “Well, I guess I would just like to
have my math lessons run a little more smoothly and I guess I’d like to feel confident while I’m teaching.”

Ms. Davis’s participation in the Math Talk PLC helped to facilitate the change in her mathematics instruction. In reflecting on the mini-design cycle process, Ms. Davis explained that the initial research phase could be overwhelming,

but once we’re able to apply it [the math talk strategy], then so we have all these ideas coming together and it looks hard in the beginning, but then when we problem solve it and come up with a plan for how to incorporate it into the classroom, it ended up being successful.

Ms. Davis shared that she liked being able to focus on one math talk strategy at a time and integrate it into her teaching. When reflecting on how things went during the first mini-design cycle about wait time/think time, she explained,

The think time I felt like I was still adjusting…to the math discourse and letting go of being the lead speaker. But I feel like once I trusted them and felt like that they could do it, then each lesson…each subsequent lesson has been easier.

This idea of coming to trust her students and being confident that they were on-task and engaged helped Ms. Davis move her practice toward more student-led instruction. Along the way, she acknowledged the challenges of moving away from the traditional teacher as teller role and the feelings of comfortableness that she encountered. For instance, she said, “I think one challenge for me is volume. It’s hard for me to not want to be the only voice.” She shared a similar sentiment when she discussed changing the physical arrangement of her classroom:
It took a lot for me to move desks together. At the beginning of the year I had them all in rows and I felt comfortable that way. I felt comfortable standing in front but I don't know that the engagement was very high, so putting them together and allowing myself to feel a little uncomfortable when it got a little loud but knowing great conversations were going on was a good thing.

Interestingly, I had noticed Ms. Davis changed her desks from an array into tables in December, but I did not think to ask why she had done so. She reported that she needed to move them into groups to facilitate their math talk more easily. Her desks are still grouped in tables as of my final post-observation in April.

A key tool that helped Ms. Davis regularly implement math talk strategies into her instruction was her use of anchor charts. During an interview in January, she noted,

The one thing that I’m concerned with and I’ll just have to fix this on my own is remembering all of the tools that I’ve learned and remembering to use them in my lessons. And I was thinking I’d probably do an anchor chart as a refresher.

As we began the final mini-design cycle that focused on the turn and talk strategy, Ms. Davis found a turn and talk chart on Pinterest and shared that as one of the resources during our Math Talk PLC. Before enacting her lesson, she reproduced the turn and talk chart in a larger scale, which she referred to regularly as she enacted the lesson. When I conducted her first post-observation, I noticed Ms. Davis had reproduced another anchor chart, based on the work of Chapin, O’Connor, and Anderson (2013), which displayed the math talk moves of revoicing, repeating, reasoning, adding on, waiting, and turn and talk. Ms. Davis was still using the strategies of think time, revoicing, and turn and talk, but she was also asking students to add on to either their own or others’ responses as
means to extend the math talk. She noted, “If one student has trouble explaining their reasoning, I’ll have another student add on.” Here is an excerpt from that post-observation of a geometry lesson about properties of quadrilaterals:

Ms. Davis: Now, I don’t want you to write—I want you to discuss with your group. So whoever [presented] last time, I’m going to have somebody else present to the class this time. I want you all to discuss this: Write what you know about opposite sides of a parallelogram. So you’re going to discuss that as a group and then somebody is going to report to the class. Go.

_Gave 1:20 seconds to discuss._

Ms. Davis: Class, class. I’m hearing conversations that have nothing to do with parallelograms. Okay, who would like to report from this table what a parallelogram is and I might have somebody repeat it back. Or not what a parallelogram is, but what you know about the opposite sides of a parallelogram.

Student 1: Parallelogram is a shape with two sets of parallel lines.

Ms. Davis: So what does that mean?

Student 1: So it means like if…like a triangle…no, not a triangle…rectangle cause it has two long lines and two short lines—those are parallel. But in a rectangle you have to have four right angles in it.

Ms. Davis: We’re not talking about rectangles right now, but you’ve got it on the parallelogram. [Student 2], what did [Student 1] say that made
a parallelogram when she was talking about a rectangle?

Student 2: I couldn’t really hear it.

Ms. Davis: [Student 1] could you talk about that again and then [Student 2] will repeat it again.

Student 1: A parallelogram has two sets of parallel lines, so it has to…like in a rectangle it has two long lines and two short lines. The two short lines are one and the two long lines are one. [When she says “one” she gestures with her hands parallel]

Ms. Davis: Go ahead and repeat that, [Student 2].

Student 2: I think what [Student 1] is saying a rectangle has two sides, but it…has two longer and two shorter and that’s why is parallel.

Ms. Davis: Does anybody have something they can add to it? [Student 3]?

Student 3: It always has…a quadrilateral always has lines that don’t cross over.

Ms. Davis: Not a quadrilateral, but a parallelogram, the lines never cross. Yes, the two sets of parallel lines never cross. So now what I’m going to have you draw three different parallelograms. One person will in your group will draw one, preferably someone who not talked yet. Then you’ll pass your board to another person and you’ll talk about what makes the parallelogram that your drawing a parallelogram.

Students begin working again in their small groups.
As this excerpt shows, Ms. Davis is still incorporating a variety of the math talk strategies into her instruction, namely the turn and talk, revoicing, and adding on. Additionally, she seems to be more comfortable with using some of them, like adding on *in the moment* during instruction. Therefore, these anchor charts Ms. Davis created serve as a support by providing a visual reminder to help her continue to implement math talk strategies.

As Ms. Davis reflected on her journey of planning for and facilitating math talk, she described how her instruction changed:

I’ve been able to release my instruction to them, so they’re able to instruct their classmates and I feel like there’s a reciprocal relationship going on. I feel like they listen to their classmates and their classmates listen to them, instead of checking out when I call on one student, they’re listening to what each person has to say.

Ms. Davis saw her role as that of a facilitator and she envisioned her instruction as more student-led. Moreover, she was especially pleased with the inclusive nature of her classroom community:

I like that I have every voice heard, and that I would say hasn’t always been the case. Sometimes I would go for safety and know that I could get the right answer, but then I wouldn’t feel like the whole class got it, so I feel like I’ve come up with some different strategies to make sure everybody has their input in the lesson.

Those math talk strategies do allow for including all members of the mathematics learning community. Once Ms. Davis opened up spaces for all student voices to be heard and contribute to their shared mathematical understanding, she was confident that her students could take the lead in their own learning and she was there to support them.
The Experience of the Researcher

My experience as a researcher and participant-observer in the Math Talk PLC was both rewarding and challenging. I enjoyed being able to observe in the participating teachers’ classrooms and to be a part of the Math Talk PLC. I think my role as an observer was a benefit because the other teachers did not have an opportunity to engage in peer observation, so I was able to share and describe things that I had noticed the teacher doing as she enacted the lesson during the PLC meetings. At the same time, I did encounter some challenges, especially with regard to taking on the role of instructional coach and being pulled into the facilitation process of the Math Talk PLC.

Even though instructional coach was not a role that I had planned to take during this study, I did provide instructional coaching and/or other supports to the participants throughout the study at their request. For instance, Ms. Mitchell asked that I debrief each observed lesson with her. During these debriefs, I was careful to focus my feedback around topics other than math talk, so as not to influence or preempt sharing that would take place during the next Math Talk PLC meeting. Because this was Ms. Mitchell’s first year teaching mathematics, it was understandable that she would want to receive feedback. Our conversations coalesced around ways in which she could use more small group instruction and how to bolster the effectiveness of her center activities. To that end, I shared several resources for centers with Ms. Mitchell. As another example, Ms. Jansen and Ms. Davis both asked me on different occasions to discuss various topics with them, such as classroom management during mathematics instruction and scope and sequence for teaching multiplication.
There were also times in which I offered instructional support during the Math Talk PLC meetings. For instance, when the grade level teams were revising their lessons, issues other than how to incorporate the math talk strategies often arose. When revising their lesson on elapsed time, Ms. Jansen and Ms. Davis were discussing how to scaffold the problems and asked if they should remove the context; that is, make it a *naked number* problem instead of a word problem. I shared with them how context can be a helpful support and that, instead, they might consider solving a few one-step elapsed time word problems as a transition into the two-step elapsed time word problems in the textbook. Additionally, there were times during the Math Talk PLC meetings when something related to discourse would come up and I had a resource to share, such as some agree/disagree statements or a list of questions to use during math instruction. Thus, I made those resources available through our Google folder. Given what the participants knew about my background and experience, it was not surprising that they sought out my feedback and/or asked about resources I could share.

Another challenge that I encountered, in terms of my role as participant-observer, were the sometimes-blurred lines around facilitation. When I planned the study, it was not my intention to facilitate any of the Math Talk PLC meetings. However, I ended up having to facilitate the first meeting because Ms. Jansen was unable to attend due to a last minute family emergency and no one else was prepared or felt comfortable enough to jump in for the initial meeting. I also became the *de facto* facilitator for our final PLC meeting as I offered to take notes as we finalized some of our co-created documents, like our definition of math talk and our Venn diagram of student and teacher roles during mathematics discourse, and I conducted the group interview. Similarly, there were many
times when one of the participants was facilitating, but they would ask me to clarify a direction or something that was noted in the facilitator’s guide. And, sometimes, I would have to interject in the meeting in my role of data collector to explain and/or remind the participants that I needed to take pictures of their lesson planning. Thus, there were these reminders, no matter how much I tried to downplay them, that I was the researcher as well as a participant in the Math Talk PLC.

In examining my role as a researcher and a participant-observer in the Math Talk PLC, I found it challenging to negotiate that dual role. While I was certainly not trying to disregard my breadth of experience as a professional development facilitator and instructional coach, sometimes I felt that I was walking a tightrope in trying to interact in an authentic way during the Math Talk PLC and not influence what happened. At the same time, I do not know if I would have gained as much insight about the teachers’ learning process had I not been in the thick of the conversation and able to follow up with comments made in the moment during the Math Talk PLC. Likewise, I am not sure if the participants would have been as open during the interviews and as comfortable during the classroom observations had they not gotten to know me as a member of the PLC. Therefore, despite the challenge of negotiating this dual role, I believe it was worth it given the richness of experience I had being immersed in the participants’ journeys.

Common Themes and Patterns of Interaction Across Experiences

Throughout the data analysis, I was constantly reviewing the various sources of data for emerging themes and patterns. From this continuous process, several themes and patterns of interactions emerged across the experiences of the participants within the case
of the Math Talk PLC as well as within the embedded cases of the individual participants and their classes.

**Establishing and maintaining a community of mathematics learners.** A variety of issues arose throughout the study around the process of establishing and maintaining a community of mathematics learners. As the study began in November, each participating teacher and her students had already established a classroom community in terms of setting rules and expectations and cultivating positive relationships among all students and the teacher. Yet, as the participating teachers began their journeys to implement more math talk among their students, they found the need to re-examine their norms and expectations during mathematics instruction specifically.

First and foremost, the participants worked to make sure students understood the expectations for math talk. Because the teachers were introducing new math talk strategies during each mini-design cycle, it was necessary for them to explicitly teach the strategy and the procedures for using it. For instance, when teaching students how to turn and talk, most of the participants referred to a poster that was shared during the Math Talk PLC. Here is an excerpt from Ms. Mitchell’s classroom where she is setting those expectations and procedures:

Ms. Mitchell: When I turn and talk, it means that I’m going to turn to a neighbor and I’m going to talk. Here are rules. #1: I look at my partner. I do what?

Students: [Choral response] I look at my partner.

Ms. Mitchell: The second rule is I listen to my partner. What’s the rule?

Students: [Choral response] I listen to my partner.
Ms. Mitchell: I’m ready to speak when it’s my turn. I’m ready to what?

Students: [Choral response] Speak when it’s my turn.

Ms. Mitchell: [Whispering] I speak so only my partner can hear me. I do what?

Say it quietly with me.

Students: [All whispering] I speak only so my partner can hear me.

Ms. Mitchell: Then I’m done sharing with my partner, when we’ve both had a turn to talk, I turn and face the front. So do that with your body real quick. Turn your body so you’re facing me. So that your body’s straight on. If you were doing this right now, I’d know you’re ready to share.

Then Ms. Mitchell had the students practice a turn and talk about what they did the previous weekend. Being explicit about expectations was a focus of Ms. Jansen’s throughout the study, especially given that she had a challenging class in terms of behavior. She had to re-teach the expectations for the different math talk strategies several times throughout the study. When reflecting on that work, Ms. Jansen noted,

I’ve found that teaching my expectations…I mean I know this and we do it at the beginning of the year, but really explicitly teaching, “This is what this looks like. This shouldn’t sound like this...” is really important to facilitate math talk, so I’ve done a lot more of that and backtracked and it’s been much more successful.

Once those expectations for productive math talk were established, the participating teachers had to consistently reinforce them and hold students accountable to those expectations. Ms. Davis shared that she “would just circulate around and make sure that all 10 of those conversations were applicable…Otherwise I’d call them out, ‘Oh, that
doesn’t sound like math talk.” This helped Ms. Davis feel confident that the students were engaging in math talk, including staying on topic and interacting appropriately, and “that things were happening and that they were learning.” Therefore, implementing the math talk strategies of wait time/think time, revoicing, and turn and talk brought about a need to re-establish expectations for interactions during mathematics instruction.

Cultivating a respectful and supportive environment during mathematics instruction was another aspect of the classroom community that the participants had to attend to when implementing more math talk. While the participants had certainly established norms for their classroom community, once students were being asked to engage in more sharing of their thinking, it was necessary to reinforce respect and kindness during mathematics instruction. Both first grade teachers were especially focused on bolstering this feeling of community as they each had a few students that were not consistently kind in their interactions with others. Therefore, the teachers made a conscious effort to emphasize there are many different ways to solve problems and it is okay if your answer is incorrect the first time. Ms. Green commented, part way through the study, that her students had been more supportive of each other and that they were “far nicer to each other with the wrong answer. So for example, even if it’s 1 + 1 and some kid says, ‘Six’ or something like that, they are far more respectful of when the wrong answer crops up.” When I observed in the classrooms, I noticed, for the most part, the students interacting in kind, positive ways with their peers. If there was a student who was not being respectful, the teacher would step in and help them adjust their behavior. Ms. Jansen summarized the need to have a strong classroom community when she said, “It made me realize that becoming a community that shares ideas is very
important and it helps others learn.” Thus, cultivating and maintaining that positive classroom community was key to mathematics learning.

**Shift toward student-driven instruction.** All of the participants, from the beginning, shared student-centered beliefs about the nature of learning in general and of mathematics learning specifically. These beliefs were especially prominent in terms of allowing students to select strategies that made sense to them when solving and explaining mathematics problems. Despite these beliefs, their instruction, prior to beginning the Math Talk PLC was mostly teacher-directed, with the exception of eliciting multiple strategies for solving problems. This teacher-directed focus was most likely a consequence of the participant teachers’ use of the adopted mathematics textbook, which utilized a significant amount of whole class direct instruction. Yet, as the participant teachers began to look for opportunities to incorporate more math talk within the adopted curriculum, they began to make adjustments to the lessons that opened up spaces for student thinking and allow for students to begin to drive the mathematics instruction.

As students engaged in more math talk, the participants came to understand that their students were capable of problem solving without direct instruction about specific strategies. Ms. Green discussed this realization and how it impacted her instructional approach when she commented,

I think the thing that has been going the best is having the kids discuss their thought processes. Before it was just a case of I would tell them this is how you’re gonna do it and then we would do it, and I didn’t really pay attention to what was…what was their process—it was my process. “You will do it this way,
that’s how we’ll do it.” And so I think it’s actually been very nice to see them coming up with different ways to solve it.

Having multiple students explain their different solutions strategies was characteristic of Ms. Green’s classroom. What shifted as we moved through the study was that Ms. Green showed her solution strategies less and focused more on asking questions to make sense of the student’s process. Here is an example from Mini-Design Cycle 2, which took place in January:

Ms. Green: Here’s my question, did this problem [6 + 2] help you on this problem [60 + 20]?

Students: [Many respond chorally] Yes.

Ms. Green: [S1], how did that help you? How did this problem [6 + 2] help you on this problem [60 + 20]?

Student 1: The answer is like the same with this problem [6 + 2], but we just added a zero to it.

Ms. Green: Okay. So like I’m a kindergarten student, okay. I’m a 5 year old student. So if I think 6 + 2 is 8, then this [60 + 20] must be 8. Is that true?

Student 1: No, it’s 80.

Ms. Green: But how would you explain the difference to me so I would understand it? Remember, I’m 5.

Student 1: I would…I would draw some ten sticks. [Drawing on whiteboard]

Student 2: But what it they don’t know ten sticks yet?

Ms. Green: So what have I got there?
Student 1: I’d count by ten, I’d be like 10, 20, 30, 40, 50, 60. And then I’d add two more [ten sticks] and then I you would…and then I would say, “60, 70, 80.”

Ms. Green: And would it be up here [6 + 2]?

Student 1: 6 plus 2 would be 8.

Ms. Green: But how would this [6 + 2] look? How would it be different?

Student 3: It’s just singles.

Ms. Green: But I mean how would this look [6 + 2] compared to the ten sticks?

Student 1: You would do dots.

In this exchange, the student’s choice of the strategy to draw ten sticks drives the instruction. Ms. Green follows that line of thinking and uses questions to help the student explore how it would look different if the problems was ones, not tens.

The move toward letting student thinking drive instruction required a release of authority on the teacher’s part. Ms. Green discussed how retaining that authority could be motivated by trying to manage challenging behaviors among students when she said, “So you figure if I just bring them all into control, then I’ll be able to make sure everybody’s doing exactly what I want them to be doing—which is not math.” Thus, the participants acknowledged that sharing responsibility is necessary for mathematics learning. To that end, Ms. Davis described her role as changing to be more of a “guide on the side” and “instead of teaching to,” she feels that the students “[are] learning from each other and not just from me.” A key component in being able to shift this authority
from herself to her students was premised upon seeing and believing in her students’
mathematical abilities and understanding as a result of their math talk. She said,
I would say that I am able to trust that they can problem solve things without
being told how to problem solve. That they can come up with the solutions to
problems either as a group or…mostly as a group and then do independent work,
too, and come up with the solutions themselves.

As a result of this trust, Ms. Davis describes her instruction as changing from the
beginning of the year to the end of the Math Talk PLC, when she stated, “I think that I
really felt it had to be teacher-led and I really now feel like it’s student-led and teacher-
guided. Because I do see success in their daily math conversations.” When observing
Ms. Davis, I saw this change as well. During the pre-data collection and the early part of
the Math Talk PLC, Ms. Davis would typically draw a picture or model of the problem
being presented to students to get them started and then have them finish it their own
using that strategy. By the end of the Math Talk PLC, Ms. Davis was presenting
problems and then turning them over to her students to engage in the problem solving
process, usually in pairs or small groups.

The participants noticed that a positive consequence of more student-driven
instruction was increased confidence among their students as mathematics learners. Ms.
Jansen commented that implementing the math talk strategies helped her “see that if you
just give kids time and a chance to think and reflect and voice their ideas, that they can
become better math students.” As a result of that, she is “seeing some kids start to open
up and feel a little bit more empowered.” Ms. Mitchell echoed this notion, and she noted
that a shift to accepting all strategies and building on them was benefitting her students,
especially those who were struggling with mathematics learning. She explained, “I feel like that this validates all the different approaches that they’re taking to solving the problem. That has helped my intensive kiddos feel like, ‘Oh yeah, I can do it cause I can use my fingers.’” Ms. Mitchell shared an anecdote from her lesson about the relationship between adding with ones and adding with tens:

I had a couple of kids who stood up to express themselves at their desks, so I think they’re feeling a lot of confidence in their math abilities and in their math thinking. I think having them articulate it, even when they can’t quite, is valuable cause it makes a connection for them between numbers and language, which I don’t think is always present in math teaching, and just that metacognition that they’re doing. You know, and on a first grade level, but it was obvious that they’re thinking about their own thinking when my one student said, “Oh, this is really hard.” I thought, yeah, it is. Awesome.

Thus, using math talk as a means to shift to more student-driven instruction benefits students by validating their mathematical thinking, increasing their self-confidence, and helping them to engage in metacognition.

**Changes in mathematics discourse.** Throughout the study, the participant teachers reported and the researcher observed some changes in the nature of the mathematics discourse in the classrooms. These changes reflected several different elements, including (1) extending responses; (2) engaging with the reasoning of others; and (3) transfer of math talk to other subjects. It is important to note that given the Math Talk PLC began mid-year and it only lasted a little more than three months, many of these changes in mathematics discourse were in the beginning stages of development.
**Extending responses.** Prior to beginning the Math Talk PLC, participants were already using some strategies to get students to talk about mathematics. Several of the teachers mentioned using partner talk, but not on a regular basis. All of the teachers mentioned posing questions to students during their whole group instruction and having students explain their strategies for solving problems. During my pre-observations, I noticed a questioning pattern that is typical of traditional mathematics teaching where a (usually lower level) question is posed, a student is called on either because their hand was raised or their name was pulled from the jar of popsicle sticks, the student answers, and the teacher moves on (Hull, et al., 2013). As a result, the student’s response was often only one or two words in length. It must be noted that I did observe some responses that were more extended, but shorter responses during mathematics instruction were typical of all the participating classrooms.

Over the course of the Math Talk PLC, as the teachers were implementing the selected math talk strategies during their lessons, students’ responses did become more extended, both in terms of length of response and in terms of the mathematical thinking represented in the response. It is important to note that the mathematics discourse ebbed and flowed in terms of length and mathematical depth of response throughout the study. It seemed that the opportunities to talk were somewhat dependent upon the content of the mathematics lesson; that is, if the mathematics in the lesson was particularly complex and/or the lesson was introducing a new concept, the students’ responses were limited. Table 5 displays samples of discourse from some of the participating classrooms that illustrates the changing nature of the responses over the course of the study.
### Ms. Mitchell’s Class: BEFORE PLC
*Teacher displays $3 + _ = 9$ on board.*

**T:** What’s the total? [S1], what’s the total?

[S1]: 9

[S2]: I have something.

**T:** What is it?

[S2]: You need six more.

**T:** Oh, goodness sake…3 plus blank equals nine. 9’s the total. Use one of the strategies we just did. [S3], come show us.

[S3]: 6.

**T:** How did you do that?

[S3]: I used my fingers.

**T:** You used your fingers. [S3], when you used your fingers, did you start with the total or did you start with the partner?

[S3]: 3

**T:** You started with the partner and counted up. Did anybody start with total 9 and subtract 3? [S4], is that what you did? Show us with your fingers.

Start with 9 and subtract…And she put down 3…1, 2. 3.

[S4]: *Shows 9 fingers and puts 3 down—did not talk.*

**T:** She ends up with the other partner, which is 6. Awesome.

### Ms. Mitchell’s Class: DURING/AFTER PLC
*Rosa reads 8 stories. Tim reads 5 stories. How many stories do they read in all?*

**T:** [S1], how did you choose to solve this problem?

[S1]: Used cubes. I put them together and I counted them and I saw it was 13.

**T:** What you told me when I was your desk, what you were telling [S2] and me is that you started with one of those numbers. Which of those number did you start with?

[S1]: 8.

**T:** Stared with 8 and then counted…

[S1]: 5.

**T:** Counted 5 more. And what answer did you get?

[S1]: 13.

**T:** 13 in all. If you also got 13 in all, would you also show [S1] you also got 13 in all. *Students give agree signal.*

**T:** How did you do it, [S2]?

[S2]: *Silent*

**T:** I’ll come back to you because I’d like you to share—you used a good strategy. [S3], how did you do it?

[S3]: I just knew the partner.

**T:** Would you say that again?

[S3]: I just knew the partner.

**T:** [S2] it sounded like your strategy was the same as [S3]’s. If you just knew the partners would you show [S3] that’s what you did. *Many students give agree signal.*

**T:** Anyone have a different way of doing it?
S4: First I colored in 8 x’s, then I colored in 5 triangles, then I counted 8 up. 8, 9, 10, 11, 12, 13.

T: Can you hold up your board and show us? And so you also counted on from 8?

S4: Well, it’s just on my whiteboard.

T: What I’m wondering, [S4], is did you start from 8 here or did you notice the ten? What did you decide to do? Either way is great, I’m just wondering what you did.

S4: Well first I drawed 8 and then I drawed 5.

T: So you counted on from the 8 then. One more. [S5], what did you do to solve the problem?

S5: I used the number line.

T: Would you show us? Can you hold up your board and show us? The rest of us should be listening to [S5] when he tells us. So listening respectfully to [S5] please. Tell us what you did.

S5: I used the number line and it was 13.

T: So did you start at 8 and count on to 13? Awesome. So these are great strategies!

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Ms. Jansen’s Class: BEFORE PLC

Students are making number paths for their six multiplication facts.

T: Please circle one group of six. If you have one group of six, that is what [S1]?

S1: 6

T: So write six off to the side. So please in the middle of your paper, would you write the multiplication for one group of six. One group of six equals six. You’re going to write a lot of these facts, so write it kind of small please. Starting with 7, would you count on six more groups. [S2] where did you stop at?

S2: 12

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Ms. Jansen’s Class: DURING/AFTER PLC

Students are sharing their ideas from a turn and talk about a grouping of angles. They have decide if they are right angles or not.

T: Which of these are right angles?

S1: W.

T: Can you come and prove it to me? How could you prove it? You’re gonna do thumbs up if you agree, thumbs down if no, so she’s gonna try and prove it to you. You’ll have to tell her thumbs up if it is a right angle and thumbs down if no. So how could you prove to us that W is a right angle?

S1: W isn’t small like an acutie and also it doesn’t stop going, it’s keeps on going and going and going.
T: If you agree, thumbs up.

Students give thumbs up.

T: Oh, so you are saying two groups of six is 12?

S2: Yes

T: I like it. So please write 12 off to the side of that group and then write your facts: $2 \times 6 = 12$. I’m glad you noticed something, store it in your head and we’ll talk about it in a minute. Now starting at 13, please count on six more groups, that’s 3 groups—or six more boxes. That’s another group of six; three groups of six. [S3] where did you stop?

S3: 24

T: Ummm, do you agree [S4]? Where did you stop?

S4: 18

T: Oh, okay. Do you agree with that?

Some students say, “Yeah”

T: [S2] any help to proving that’s a right angle?

S1: He didn’t talk with me.

T: [S3], I think you figured out how to prove it. Would go and show us how you would prove that W is a right angle. But before that, do you agree with her? Thumbs up-yes? Thumbs up-no?

S3: If you put it [a piece of paper] right there on the corner, it just goes straight down. Student fits the corner of a piece of paper perfectly in the corner of the angle.

T: So it goes straight across the top like a piece of paper and straight down the side like a piece of paper. Thumbs up if you agree that she proved it to you?

Students give thumbs up.

T: Okay. Did you have any others? Any other right angles? Which of these angles are smaller than a right angle? You’re the reporter…Which ones?

S4: X and y

T: Could you prove to me that they’re smaller than a right angle? They’re the acute.

S4: Because it doesn’t fit right in with the paper and the corner.

T: Now what do you mean it doesn’t fit right in?

S4: Cause you couldn’t make a square in here and you couldn't...

T: Would you show me again? Would you put your paper up there and show me what you mean—it doesn't fit right in?

S4: Cause it goes straight across the paper instead of slanting down. Student lines up one side of paper with top ray in angle, but the angle’s other ray cuts across the paper.

While all children are capable of engaging in math talk, there can be some challenges when beginning this work with very young children. The first grade teachers, Ms. Mitchell and Ms. Green, both commented at the beginning of the study about the
difficulties of supporting young children to explain their thinking and expand on their responses. For instance, Ms. Mitchell noted, “Some of them because their skills aren’t particularly advanced or their vocabulary, their math vocabulary, isn’t really advanced, they’ll say things like, “Well I got 6.” and kind of stop there.” To combat the common first grade response of “I just knew it” or “I had it memorized,” Ms. Green tries to support elaboration by saying, “Pretend I’m a kindergarten student. I need you to explain it to me like I was a kindergarten student.” Over the course of the study, the first graders became markedly more articulate when verbalizing their thinking. Both Ms. Mitchell and Ms. Green attributed a lot of these more extended responses to developmental growth in the first graders in terms of their expanding vocabularies. At the same time, both teachers were very deliberate in the scaffolding of language they were using to promote rich math talk during instruction. For instance, Ms. Green noted, “You have to be the one who says, ‘I noticed you counting on your fingers.’ Because if you think they’re actually going to come right out and say, Oh, I’d use the counting on method,’ that’s not going to happen.” Toward the end of the study, however, there were some first graders who were beginning to articulate their solution strategies using precise mathematics vocabulary, such as explaining they used the counting on method.

**Engaging with the reasoning of others.** Prior to beginning our work in the Math Talk PLC, other than occasional partner talk, there were not many opportunities for students to engage with each other’s reasoning. The typical pattern of interaction was that one student answered one question from the teacher and the teacher moved on to the next question. A shift in interaction began when the revoicing strategy was implemented during Mini-Design Cycle 2. While revoicing often constituted repeating what a
previous student had said, sometimes the students would revoice it in their own words. Likewise, some participants also began to implement agree/disagree prompts using a thumbs up/down signal, which required students to consider what their classmate had just stated. Additionally, Ms. Davis began using the *add on* talk move (Chapin, et al., 2013), where she prompted students to elaborate on either their own response or the response of another. Table 6 contains examples of students engaging with the reasoning of their classmates.

**Table 6**  
Examples of engaging with others’ reasoning

**Ms. Green’s Class**

9 alligators are in the river. 7 more jump in. How many alligators are in the river now?

Students solve the problem their own way. Then, they are paired up and share their strategy with a partner.

T: Did you find a way with your partner that was different?

*Many students say, “Yeah.”*

S1: Cause she did squares and I did dots. She [S2] did 6 on the bottom and wrote 15, but I was right because it was 16. Because she wrote 6 squares and when I counted the bottom.

*S1 and S2 were partnered to share their strategies.*

T: [To S2] Did you just not draw enough squares?

S2: Yeah.

T: But who discovered that? Did you discover that or did [S1]?

S2: [S1]

T: But did that work out okay? Did he laugh at you or was he very nice?

S2: Nice.

T: Thank you, [S1], I like to hear that, too.
Ms. Davis’s Class
Rounding 71 to the nearest hundred.

Students discussed this problem with assigned partners and were supposed to be ready to share their thinking.

T: Class, class. [S1], what did you and [S2] come up?

S1: 100?

T: And why did you come up with 100, bud?

S1: Because 100 is like…if you add up…71 and [S2] said it makes it 100 because 100 is the bigger number and I’m like 71 is the bigger number.

T: And 71 is bigger than what?

S1: 7

T: And do you have something to add to that [S2]? What would you add to that?

S2: It’s not in the 40s or 30s or 20s or 10s or 1s.

T: It’s higher than what number?

S2: 50.

T: It’s higher than 50. Good. So 100.

As you can see from the examples, these student interactions represent the beginning stages of engaging with each other’s reasoning, especially given that the teacher must prompt the interaction by asking the students to revoice, add on, or agree/disagree. While there are a few instances in Ms. Davis’s classroom where students are initiating that interaction themselves, that is the only classroom in which I observed this more student-led engagement with the thinking of others.

Transfer of math talk to other subjects. During the Math Talk PLC meetings and during the interviews, the participants all mentioned how they and their students were transferring the math talk strategies to other subjects. Ms. Green noticed that, in general,
the students were more willing to engage in discourse when she said, “They are trying to have more discussions to see more possibilities. And it doesn’t have to be about math, it can be about anything. They’re more willing to have a discussion.” She also commented that the students were using the agree symbol “not just on math talk; they do this on everything.” In addition to the students carrying over the math talk strategies to other subjects, the participating teachers stated they were also incorporating those strategies into their teaching of other subjects. Ms. Mitchell noted, “That has actually extended throughout my teaching. I’ve been much more cognizant of allowing kids think time and we use those strategies all of the time.” Additionally, some of the teachers mentioned the specific subjects in which they are incorporating those strategies for discourse. For example, Ms. Jansen stated, “I’ve even carried it over into reading. So, yeah, it’s been a practice that’s helped in all areas.” Ms. Mitchell has also discussed similar carryover when she said,

I feel like lately, even when we’re talking about science or doing a read aloud or whatever, we’re doing a lot more signaling, a lot more—“Oh, tell your hand or your partner” so, yeah, it’s just becoming part of my repertoire and it’s exciting!

The ease of transferring the math talk strategies to other subjects was also evident when the participants were creating their Math Talk Action Plan (see Figure 3). They decided to introduce these strategies as a means for increasing academic talk in math and other subjects at the beginning of the next school year.

**Tensions with the adopted mathematics curriculum.** All of the participants liked various aspects of the adopted mathematics textbook and felt, for the most part, it fit well enough with their philosophies of and approaches to teaching mathematics. As Ms.
Green put it, “It’s better than some. It’s not as good as other ones.” The participants liked the emphasis on multiple strategies, but were concerned, especially at the third grade level, with the heavy reading demands given so many word problems for their students who were struggling readers. All of the participants discussed how they made minor adjustments in their math lesson planning processes, which might include omitting activities not central to the lesson’s focus and/or stretching the lesson across multiple days. Even though the participating teachers had the autonomy to make changes to the adopted mathematics textbook, most of them described following the lessons as they were written other than the minor adjustments already noted. Over the course of our time together in the Math Talk PLC, during the participant interviews, and in other impromptu conversations, tension with how to best use the adopted mathematics curriculum arose. This tension took one of two forms: (1) concerns about the pacing of the lessons; and (2) a desire for more flexibility and variation in instructional format.

**Concerns about pacing.** The issue of pacing, especially in terms of how many days the adopted mathematics textbooks suggests for a lesson versus the number of actual days it takes to enact all of it, was a continuing concern among all participants. Ms. Green noted, “If you follow their scope and sequence and everything that they have in there, you’d be lucky if you get anything else [other subjects] in.” Ms. Davis expressed that she believes the textbook “moves too fast for the level that our kids are at.” Ms. Jansen echoed that sentiment when she stated, “Many times I feel like…the kids need extra practice and they’re moving on to something else, so I guess the pacing is pretty difficult.” The lessons, as written in the adopted textbook, are quite extensive in terms of how many activities the students are expected to complete in the designated time. For
example, in the third grade lesson about measurement from Design Cycle 1, the lesson was written to take one day of instruction. In approximately 65 minutes, students were expected to explore U.S. customary measures of weight, metric measures of mass, and estimate measures of weight/mass by completing 40+ workbook problems. Needless to say, the issue of needing more days to complete the lessons than recommended by the adopted textbook was a continuous concern.

Another aspect of concern with regard to pacing was the length of the lessons, especially for the first grade students. For instance, in the first grade lesson about word problems with various unknowns from Design Cycle 3, students were supposed to solve and discuss about ten word problems for approximately one hour in a whole group setting. As Ms. Mitchell reflected on this lesson in particular, she noted, “My lesson as far as adjustments was way too long, and I actually cut it in half even from what I had planned to do…But even that was too long.” While asking first grade students to engage in mathematics learning for one hour is not unreasonable, using the adopted textbook’s lessons in their original form led to management issues for Ms. Mitchell because her students could not attend that long.

Inherent in this sense of feeling like the textbook is moving too fast and that the teachers are not keeping up with the pacing is the perception that taking time to slow down to allow students to talk about mathematics will only compound being behind. Ms. Mitchell addressed this feeling when she said, “I’m always kind of watching the clock and worried about sort of losing instructional time, even though I understand that conversation on math is instructional time.” Yet, as this concern with pacing resurfaced in our discussions in the Math Talk PLC, the participants began to challenge the
prescribed pacing of the textbook. For instance, Ms. Davis asserted, “I think that would be a good norm—to be okay with not following the lesson, the pace of the lesson, being okay with slowing down.” By January, Ms. Davis went further in looking to next year and how she would approach teaching multiplication when she said,

I think I’m just gonna slow it down and let the lessons guide themselves, instead of like, okay we finished Class Activity A, B, C and not really checking for understanding because I feel like I have to get through the lesson and I feel they’re getting more out of a lesson if we can go slower and they can talk to each other.

Likewise, Ms. Green noted that our conversations during the Math Talk PLC have “made me stop and go, ‘Wait. Okay, I need to take the time.’ This isn’t just about getting the unit done. This isn’t just about getting—this is more about getting the kids to understand.” While their concerns about pacing were not completely resolved by the end the study, the participants were beginning to reevaluate how they viewed the use of time during mathematics instruction and were aiming for quality as opposed to quantity.

**Desire for more flexibility in instructional format.** The adopted mathematics textbook used whole group instruction as its main format for delivering content. There was also an individual work section in which students completed worksheets that (1) reinforced that lesson’s content, and (2) reviewed previous content. Occasionally there were activities in the lesson in which students would work in pairs. The textbooks series also provided differentiated tasks for each lesson for students who were above grade level, on grade level, and below grade level; these tasks could be completed individually
during seatwork, with a partner, or the teacher could use them during small group instruction. None of the participants used these differentiated tasks on a regular basis.

Throughout the study, several participants commented that they would like to change the format of their instruction; that is, to move away from so much whole group instruction. For instance, Ms. Davis commented, “I think if there was a way to change, I guess, the way the math is done, so maybe group work instead of just whole group all the time. Maybe me doing a short whole group lesson and then small groups.” Interestingly, the participants are allowed to make such changes to their curriculum, as the district does not have strict scope and sequences for each grade level defined. And, to some extent, the participants were making such changes when they employed blended learning.

Blended learning days, as described by the participants, consisted of students rotating through four centers: seatwork, computer, game, and teacher. Thus, the participants were employing small group instruction, but the frequency of these blended learning days varied. Despite the use of blended learning, however, the participants still perceived their ability to make significant changes to the format of lessons to be limited.

Grappling with whole group instruction as the primary format for teaching was of particular concern for the first grade team, especially Ms. Mitchell. Over the course of our work together, though, she began to push against the whole group instructional format in the textbook. In January, Ms. Green shared a plan that she and Ms. Mitchell were contemplating in terms of moving away from so much whole group instruction. “[Ms. Mitchell] and I have been talking about…just not doing the textbook, instead finding out what they have to do and then…planning out all the lessons and doing small group, which we’ve done as far as our rotation goes.” Ms. Mitchell did go forward with
using the blended learning format as the primary structure for her teaching. The following is an exchange during our final Math Talk PLC meeting after Ms. Mitchell has explained that she prefers small group instruction to whole group instruction:

Ms. Davis: What does that look like?

Ms. Mitchell: So we’ve been doing blended learning. So they’re doing the game, tech…

Ms. Davis: Every day?

Ms. Mitchell: I’ve been doing it almost every day.


Ms. Mitchell: It just works so much better for me.

Ms. Davis: How much time do you have?

Ms. Mitchell: We have from 12:35-1:45. So it’s a good chunk. Yeah. And I mean we’ll see, they’re gonna do an assessment here pretty quick so we’ll see if this is what I ought to be doing, we’ll see…

Researcher: But you feel better about it?

Ms. Mitchell: I feel way better about it. And they, like, I have been doing blended learning almost every day this last week and every day they cheer when I say, “It’s blended learning today.”

Ms. Mitchell’s move opened up the possibility for the other participants to envision changing the adopted textbook’s emphasis on whole group instruction. Whether or not that move prompts them to make such changes has yet to be seen.
Summary

This embedded single-case study sought to capture teacher learning around planning for and facilitating mathematics discourse within the context of a professional learning community and to capture changes in discourse in the mathematics classrooms. The case of the Math Talk PLC and the embedded cases of each participating teacher’s classroom tell collective and individual stories of teacher learning—learning that was teacher-directed and self-facilitated. Through those stories, common themes and patterns of interaction emerged, including the need for the participants to re-establish and maintain their classroom communities within the context of mathematics learning, the shift to allowing student thinking to drive more mathematics instruction, the changing nature of the mathematics discourse in the participating classrooms, and how the teachers negotiated tensions with their adopted mathematics textbook and sometimes pushed back. These stories reveal the complexity of facilitating productive math talk and the complexity of teacher learning. In Chapter 5, the take aways from the experience of the Math Talk PLC and the experience of enacting these math talk strategies in elementary classrooms are examined.
Chapter 5
Discussion and Conclusions

The case of the Math Talk PLC and the embedded cases of each participating teacher’s classroom revealed the collective and individual experiences of teacher learning around planning for and facilitating mathematics discourse. Additionally, this embedded single-case study examined the changing nature of mathematics discourse as a result of the transfer of teacher learning from the Math Talk PLC to the classroom. The narrative shared common themes and patterns of interactions across the case, which reflect the complexity of teacher learning and facilitating productive math talk. In this chapter, the overall findings are linked to those in the literature. Finally, conclusions, recommendations based on the study findings, and implications for future research are discussed.

Overall Findings

When considering the overall findings of this qualitative embedded single-case study, it was important to return to the central questions, which served to guide and focus the researcher throughout her work. The two central questions in this study were:

- How did teachers learn about planning for and facilitating mathematics discourse as a result of participating in a professional learning community (PLC)?
- In what ways did a teacher’s participation in the PLC impact the nature of whole class discussion in the mathematics classroom during the enactment of the co-planned lessons and beyond that enactment?
Central question 1. The case of the Math Talk PLC sought to answer the first central question: How did teachers learn about planning for and facilitating mathematics discourse as a result of participating in a professional learning community (PLC)? Cultivating positive relationships among the participants was key to creating a safe space for teacher learning throughout the work of the Math Talk PLC. The participants began their work by co-constructing a shared understanding of math talk, which was grounded in research as well as their own background knowledge and experiences as mathematics teachers. From there, the participants engaged in the mini-design cycle process (Zawojewski et al., 2008) in which they selected a math talk strategy, researched it, co-planned a lesson using that strategy within their adopted textbook, enacted the lesson, reflected on it during the Math Talk PLC, and revised their lesson based on their learning.

In reflecting on that mini-design cycle process, the participants identified several factors that facilitated their learning and supported them in enacting change in their mathematics instruction. First, the teachers found it very helpful to connect research and practice by grounding the enactment of the math talk strategies in literature and resources they sought out and shared in the Math Talk PLC. Secondly, the teachers appreciated the opportunities to intentionally plan for math talk within their adopted textbook. They noted that shared understanding of the math talk strategies among their teammates facilitated the planning process, and they enjoyed the opportunity to engage in the structured reflection and revision process. Moreover, the participants mentioned how they enjoyed working with the other grade level team and hearing perspectives and ideas from colleagues they do not have a chance to work with regularly. Additionally, they appreciated the on-going support they would have as they continued to work with their
grade level partners beyond the Math Talk PLC. Finally, the teachers felt that having the opportunity to implement the math talk strategies over a period of time made it more manageable in terms of making instructional change. Table 7 summarizes the findings that answer Central Question 1.

**Table 7**  
Study Findings that answer Central Question 1

<table>
<thead>
<tr>
<th><strong>Central Question 1</strong></th>
<th><strong>Finding 1:</strong> Cultivating relationships within the PLC</th>
<th><strong>Finding 2:</strong> Supports in the mini-design cycle process</th>
</tr>
</thead>
<tbody>
<tr>
<td>How did teachers learn about planning for and facilitating mathematics discourse as a result of participating in a professional learning community (PLC)?</td>
<td>Safe space for teacher learning</td>
<td>Connecting research and practice</td>
</tr>
<tr>
<td></td>
<td>Co-constructing shared meanings</td>
<td>Intentionally planning for math talk</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Support from colleagues</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Allowed time for change</td>
</tr>
</tbody>
</table>

**Central question 2.** Each participant experienced the Math Talk PLC through her own lens and enacted the math talk strategies within the individual context of her classroom. Thus, the embedded cases of the participating teachers’ classrooms sought to answer the second central question: In what ways did a teacher’s participation in the PLC impact the nature of whole class discussion in the mathematics classroom during the enactment of the co-planned lessons and beyond that enactment? There was variation in how much mathematics instruction changed across the four participants. However, there were some changes in patterns of interaction across the classrooms. First, the teachers had to re-establish norms and procedures within their mathematics classroom communities specific to the math talk strategies being implemented. It was necessary for
the teachers to explicitly teach the students how to enact the math talk strategy as well as model what that strategy looked like and sounded like. Additionally, as the teachers and their students were using these math talk strategies, there were some changes in the nature of the classroom discourse: (1) student responses extended in terms of length and the mathematical thinking represented in the response; (2) students were beginning to engage with the reasoning of others; and (3) the math talk strategies were carrying over to other subject areas. Finally, the participating teachers began to let student thinking drive the mathematics instruction more often, which shifted the instruction in some classrooms to a more student-led model. Again, there was variation in how much the nature of the mathematics discourse changed across the classrooms, but all of the teachers were purposefully creating opportunities for math talk. Therefore, students were talking more about mathematics. Table 8 summarizes the findings that answer Central Question 2.

Table 8
Study Findings that address Central Question 2

**Central Question 2**
In what ways did a teacher’s participation in the PLC impact the nature of whole class discussion in the mathematics classroom during the enactment of the co-planned lessons and beyond that enactment?

**Finding 1: Mathematics community**
Re-established norms specific to math talk:
- Explicitly taught math talk strategy
- Modeled what math talk strategy looks like/sounds like

**Finding 2: Changes in discourse**
Extended responses (length & mathematical thinking)
Engaging with the reasoning of other
Transfer of math talk strategies to other subjects

**Finding 3: Student-led instruction**
Teachers beginning to let student thinking drive the course of the mathematics lesson
Connecting the Findings to the Literature

As a means of lending credibility to the findings, it was important to consider them in relation to research about mathematics discourse and teacher learning that was discussed in the review of the literature in Chapter 2. One theme that emerged in the findings was the role that building a strong sense of community plays in creating an environment in which learners, both teachers and students, feel comfortable to share their thinking. The literature showed that establishing and maintaining such a community of mathematics learners is key for productive discourse (Anthony & Walshaw, 2009; NCTM, 1991, 2000). Within that umbrella of community, the literature also indicated the need for the teacher to establish and model expectations for what math talk looks like and sounds like, which was a major focus of the work the participants did as they implemented math talk strategies into their instruction (Anthony & Walshaw, 2009; Cirillo, 2013a; Kazemi & Hintz, 2014; Middleton & Jansen, 2011; NCTM, 1991, 2000; Zwiers & Crawford, 2011).

Additionally, the work in the Math Talk PLC and the resulting changes in classroom mathematics discourse were grounded in research. The math talk strategies (i.e., wait time/think time, revoicing, and turn and talk) that were selected by the participants for the three mini-design cycles were identified as effective instructional practices to facilitate productive mathematics discourse (Cai & Lester, 2010; Chapin et al., 2013; Cirillo et al., 2014; Smith & Stein, 2011). Likewise, the changes in mathematics discourse that came about as a result of implementing these strategies, namely extended responses and beginning to engage with the reasoning of others, were identified in the literature as possible benefits of mathematics discourse (Chapin et al.,
Moreover, some of the study participants began to shift their instruction to a more student-led model as they were allowing student thinking and responses to drive the course of the lesson. Cirillo (2013b) and Hufferd-Ackles et al. (2004, 2014) both found that purposeful math talk could create an opportunity to shift the authority from the teacher to a community of learners. The variation among the participants in terms of how much the nature of discourse changes in the classroom was also supported in the literature given that facilitating productive mathematics discourse is a complex endeavor that is dependent upon a multitude of contexts (Truxaw & DeFranco, 2008).

Conclusions

In reflecting on the findings of this study, there are several conclusions that can be drawn. Table 9 provides a summary of the conclusions drawn from the study’s findings.

Table 9
Summary of Conclusions from Study Findings

Conclusion 1:
Parallels between teacher learning and student learning
• Learning through discourse
• Need to establish and maintain a community of learners
• Math Talk PLC and classroom instruction were learner-driven
• Universal Design for Learning (UDL) principles were utilized

Conclusion 2:
Teachers can successfully self-facilitate and self-direct their learning in a PLC
• Prior experience with professional learning communities
• Structured facilitator guides
• Opportunities for choice

Conclusion 3:
Benefits of situating PLC within the context of a non-reform mathematics curriculum
• Created space for teachers to begin to critique and modify textbook
• Fill gap in literature
First, there were many parallels between how to best facilitate teacher learning and student learning. For instance, both teachers and students learned through discourse, which reflects the theoretical framework of this study. Language mediates our learning as we co-construct shared meanings of our experience (Schunk, 2012; Vygotsky, 1962, 1978). Just as the teachers co-constructed a shared understanding about math talk and the math talk strategies they enacted, the students in the participating classrooms co-constructed meaning about mathematics such as operations, geometry, and measurement concepts. Both teachers and students learned within communities of practice, the Math Talk PLC and the classroom respectively (Lave & Wenger, 1991; Wenger, 1998).

This sense of community and the need to establish and cultivate that community is another parallel between teacher and student learning. The participants developed norms to guide their interactions in the Math Talk PLC, and the PLC facilitator guides were designed to assist the participants in developing their relationships as colleagues. Likewise, the teachers cultivated relationships among their students to ensure a positive and safe environment where students would feel comfortable and valued as they shared their thinking. As the teachers began implementing the math talk strategies, they found the need to explicitly teach the procedures for how students should engage in those strategies and to model what the strategies looked like and sounded like so the learning community had a shared understanding of how to do math talk.

Yet another parallel between teacher and student learning was how the Math Talk PLC and the classroom instruction was learner-led. In the case of the Math Talk PLC, the teachers drove their learning in several ways, including building their shared understanding of math talk and the math talk strategies, selecting the math talk strategy to
enact, and revising the lessons based on their experience enacting them. As the teachers implemented the math talk strategies, they began to take their cues from the students’ responses and allow the lesson to be guided by the students’ thinking. While the extent of this shift to more student-led mathematics instruction varied across the four participants, the students’ math talk was beginning to drive instruction.

The final parallel between teacher and student learning was the use of principles of Universal Design for Learning (UDL) (Meyer et al., 2014). UDL is about proactively planning instruction to open access to all learners, and as such, teachers should expect there to be variability among learners and should plan instruction that offers learners multiple means of engagement, representation, and action and expression (Meyer et al., 2014). In terms of the Math Talk PLC, choice was deliberately built into the professional development to allow teachers to pursue math talk instructional strategies that were of interest to them and that would best meet the needs of their students. In terms of the students in the mathematics classroom, the teachers intentionally planned to give them choice of which strategy to use and how to represent and express their thinking when sharing it with their classmates. Even though there was an emphasis on verbal discussion, students were able to share their thinking using pictures and manipulatives, including physical objects and their fingers, as well as use hand signals to indicate their agreement/disagreement. Thus, UDL principles were applied in both teacher and student learning as means to support access to learning for all learners.

On another note, this study showed that teachers can self-direct and self-facilitate their learning about planning for and facilitating math talk through a professional learning community. The PLC was the ideal setting in which to facilitate the professional
development because it was an established collaborative structure at the study site. Thus, teachers had prior knowledge of the workings of PLCs and a few of them had acted as facilitators prior to this study. What made the Math Talk PLC particularly successful in terms of teacher self-facilitation were the structured facilitator guides that were provided (see Appendix B). These guides had clear timelines and activities to ensure that actionable element, which is critical to effective PLCs, was present (DuFour & Reeves, 2016). Likewise, choice was built into the Math Talk PLC, which allowed teachers to drive their learning and better attend to their context. While the Math Talk PLC is a viable model for facilitating teacher learning, it may be challenging to make space for this process within a school’s current professional learning context, especially depending upon what kind of mandates are required at the school, district, and state levels.

Finally, situating the Math Talk PLC within the context of the adopted non-reform mathematics curriculum opened up spaces for the teachers to begin to challenge the nature of mathematics teaching as presented in a traditional textbook. It also sought to fill a gap in the literature about teacher learning around mathematics discourse because most studies about effective instructional practices to facilitate productive mathematics discourse are either add-ons to the curriculum or they are situated within reform-based curricula that already reflect a student-centered pedagogy. NCTM (2014) noted the adopted textbook often drives the nature of mathematics instruction. Likewise, the adopted textbooks sends messages about what it means to do mathematics, which often reflect the more broadly held beliefs about the nature of mathematics teaching and learning within a school’s or district’s culture. Therefore, it was important to explore efforts to change mathematics instruction within the context of a traditional curriculum.
Throughout the Math Talk PLC, the participating teachers shared some frustrations about the adopted textbook. Namely, they had concerns that the pacing of the text was too fast and they expressed a desire to teach in structures other than whole group. As they incorporated more math talk into their lessons and student thinking began to drive more of the instruction, they had to navigate increasing tensions with the textbook, as it was even more difficult to keep up with the pacing given the time it takes for math talk. At the same time, the teachers were seeing benefits from incorporating the math talk and they were successfully facilitating partner and small group structures. This experience created the space necessary for some of the teachers to not only begin to pushback against following the textbook mostly as is, but it also provided the opportunity to reflect on and challenge the broader school culture’s beliefs about what it means to teach and learn mathematics in this grade level, in this school, in this district. A few of the participants challenged the text and those beliefs more than others, like Ms. Mitchell who changed her math instruction to mostly blended learning (i.e., small group instruction with centers). Regardless of the magnitude of change, the Math Talk PLC helped teachers begin to critique and, for some, modify their adopted mathematics textbook as well as assert their beliefs about the teaching and learning of mathematics.

**Recommendations**

Based on the findings of this study, there are several recommendations to consider that have implications for professional development for elementary teachers around planning for and facilitating mathematics discourse. First, when designing mathematics professional development, it is recommended to use the adopted mathematics curriculum, even if it is a non-reform textbook. While working within the context of a traditional
mathematics textbook can present some challenges, it has two advantages: (1) the professional development will complement the work teachers are already doing as opposed to being perceived as an add on or one more thing to do; and (2) as teachers’ practice begins to change, they will have to negotiate tensions with the textbook and what it means to teach and learn mathematics within their school’s and district’s culture, which may create a space for them to begin to critique and modify their curriculum and challenge those institutional beliefs. It is important to keep in mind that when working within the context of a traditional textbook, instructional change may come more slowly depending on how tightly the teachers adhere to the text, but it will come as result of their choice to challenge the adopted mathematics curriculum and the beliefs about the nature of teaching and learning mathematics that the curriculum represents.

It is also recommended to select high leverage instructional practices when planning mathematics professional development. In the case of this study, a focus on mathematics discourse was selected because of the extensive research-base that showed it to be an effective instructional practice for improving the mathematics learning of students. Moreover, there were myriad ways in which teachers could plan for and facilitate math talk among their students, so it made space for teacher choice. Thus, the participants could consider which math talk strategies might best fit with their context (e.g. adopted textbook, teaching philosophy, or student needs). This element of choice was crucial for allowing the teachers to drive their own learning as they selected math talk strategies from research-based best practices in mathematics teaching. Therefore, their work in the professional learning community was grounded in theory, but the participants had control of how that would be translated into action.
To that end, another recommendation when developing self-facilitated mathematics professional development for teachers is to provide a structured protocol for the professional learning community. That is, facilitator guides should be developed providing enough detail to support the teachers in the self-facilitation process. Likewise, those facilitator guides also need to reflect an actionable element within the PLC that focuses on improving teacher practice and, ultimately, student learning (DuFour & Reeves, 2016). In the case of the Math Talk PLC, this was the mini-design cycle process (Zawojewski et al., 2008). While the process of researching, co-planning, enacting, reflecting, and revising was structured, the what of that process was left up to the choice of the participants. Any structure for facilitating teacher learning can be used, but the necessary component in professional development for mathematics teachers is that such a structure exists to guide the work of the PLC.

Finally, it is recommended that multiple grade levels are combined when forming professional learning communities as means for mathematics professional development. Within existing PLC structures at many schools, elementary teachers typically work with their grade level team members. While such a structure can increase the applicability of their work to their specific grade level needs, it can limit varied perspectives, especially when consider how mathematics teaching and learning can differ across grades. Therefore, pairing more than one grade level in the PLC can broaden the conversation by bringing different perspectives, experiences, and expertise to the table. Moreover, blending multiple grade level perspectives within a PLC can contribute to the richness of the conversation and develop new relationships among colleagues.
Limitations of the Study

This study had some limitations that impacted the transferability of the findings. One potential impact on the study’s results was my role as a participant-observer during the Math Talk PLC. While this role was necessary for me to capture the process of teacher learning around planning for and facilitating mathematics discourse for the purpose of this study, the PPMD professional development program was not designed to have an outside expert facilitating the professional learning community. Therefore, even though I tried to position myself not as an expert and tried to be conscious of my interactions and limit my influence, the additional supports that I provided, such as sharing resources and providing feedback on the lessons, could have impacted some of the outcomes of the study. That being said, I believe that the benefit of the supports that I provided to the study participants outweighed any potential impact on the study’s findings, which is why I chose to take a more active role in the study at times. However, given that I did facilitate the PLC at times when I had not intended to, further research on the Math Talk PLC is recommended in order to see if similar findings occur when only participating teachers facilitate the PLC.

The most significant limitation of the study was related to its short timeframe. Best practice in mathematics professional development calls for intensive and sustained programs that typically last for more than six months and total between 60 and 100 hours (NCTM, 2014). The Math Talk PLC was implemented over three and a half months for a total of approximately 20 hours, including the PLC meetings, time to research and lesson plan, and the actual enactment of the lessons. Therefore, the less intensive and sustained nature of this professional development program could have limited its impact on teacher
learning and the extent of the application of that learning, as making sustained
instructional change takes time. Likewise, the transfer of teacher learning into the
classroom in terms of how the nature of discourse in the mathematics classroom changed
could have also been limited. I did observe some changes in the discourse, like student
responses during instruction did extend over the course of the study in terms of length
and more mathematical thinking. At the same time, students were just beginning to
engage with the reasoning of others, so this discourse practice was not yet fully
developed, which could have been a consequence of the limited time in which teachers
had to implement the math talk strategies. Thus, the study’s abbreviated timeframe may
have limited the impact the PPMD professional development program had on teacher
learning and the transfer of that learning into classroom practice.

**Implications for Future Research**

There are several opportunities for future research using the PPMD professional
development program. First, it recommended that the Math Talk PLC be implemented
with only teachers facilitating the professional learning community as originally
intended. Because of the unplanned instances in which I took on the role of facilitator,
there were times an outside expert was guiding the PLC which was not a planned part of
the study and, thus, additional study of this professional development program is
warranted. Additionally, I have made revisions to the PLC protocol that includes
extending the Math Talk PLC throughout an entire school year (see Appendix G). This
longer, more intense timeframe better matches the recommendations for best practice in
mathematics professional development. Likewise, the revisions also provide guidance
for implementing multiple Math Talk PLCs to facilitate the professional development
across the school. Thus, implementing this revised version of the PPMD professional development program would provide opportunities to research its impact on teacher learning and the nature of classroom discourse over a longer timeframe and, possibly, as a school wide model. Finally, the mini-design cycle structure is flexible enough that the PPMD professional development program could be modified to focus on other areas of mathematics instructional practice, such as questioning or modifying mathematics tasks. Therefore, its flexibility opens many opportunities for future research supporting teachers to make changes in their mathematics instructional practice.

**Summary**

As a result of this qualitative inquiry, I have gained a deeper understanding of the process of teacher learning around purposefully planning for and facilitating mathematics discourse in elementary classrooms. This study showed that teachers can self-facilitate and self-direct their learning within the collaborative setting of a professional learning community. Moreover, this study showed that purposefully incorporating research-based math talk strategies within a non-reform mathematics curriculum can facilitate change in the nature of mathematics discourse in elementary classrooms. While teacher learning and facilitating productive mathematics discourse are complex endeavors, professional learning communities provide a collaborative space in which to develop and grow teacher expertise in increasing opportunities for students to engage in math talk.
References


www.amazon.com


Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2014). Describing levels and components of a math-talk learning community. In E. A. Silver & P. A. Kenney (Eds.), *More lessons learned from research: Useful and usable research related*


*Teaching Children Mathematics, 18*(3), 190–197.

*International Journal of Science and Mathematics Education, 10*(1), 139–161. 
Retrieved from  

http://www.jstor.org/stable/10.5951/jresematheduc.42.2.0167

doi:10.1037/0022-0663.99.2.380


Appendix A:

Math-Talk Learning Community (MTLC) Framework
Table 11.1
Levels of math-talk learning community components

<table>
<thead>
<tr>
<th>Teacher role</th>
<th>Questioning</th>
<th>Explaining mathematical thinking</th>
<th>Mathematical representations</th>
<th>Building student responsibility within the community</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Level 0</strong></td>
<td>Teacher is at the front of the room and dominates conversation.</td>
<td>Teacher questions focus on correctness. Students provide short answer-focused responses. Teacher may tell answers.</td>
<td>Representations are missing or teacher shows them to students.</td>
<td>Culture supports students keeping ideas to themselves or just providing answers when asked.</td>
</tr>
<tr>
<td><strong>Level 1</strong></td>
<td>Teacher encourages sharing of math ideas and directs speaker to talk to the class, not to the teacher only.</td>
<td>Teacher questions begin to focus on student thinking and less on answers. Only teacher asks questions.</td>
<td>Students learn to create math drawings to depict their mathematical thinking.</td>
<td>Students feel their ideas are accepted by the classroom community. They begin to listen to each other supportively and to restate in their own words what another student said.</td>
</tr>
<tr>
<td><strong>Level 2</strong></td>
<td>Teacher facilitates conversation between students, and encourages students to ask questions of one another.</td>
<td>Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in an explanation. Students provide brief descriptions of their thinking in response to teacher probing.</td>
<td>Students label their math drawings so others are able to follow their mathematical thinking.</td>
<td>Students believe they are math learners and that their ideas and the ideas of classmates are important. They listen actively so that they can contribute significantly.</td>
</tr>
<tr>
<td><strong>Level 3</strong></td>
<td>Students carry conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others.</td>
<td>Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask “why” and call for justification. Teacher questions may still guide discourse.</td>
<td>Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.</td>
<td>Students follow and help shape the descriptions of others’ math thinking through math drawings and may suggest edits in others’ math drawings.</td>
</tr>
</tbody>
</table>

Appendix B:

Purposefully Planning for Mathematics Discourse (PPMD)

Professional Development Program

Original Before Study
Planning and implementing effective professional development for teachers of mathematics is as complex an endeavor as planning and implementing effective mathematics instruction for students. Key to success in this endeavor is the ability of the professional development program to be grounded in research-based best practices for both teacher learning and student learning, as well as be flexible enough to meet the varied needs of the participating teachers. As Loucks-Horsley, Stiles, Mundry, Love, and Hewson (2010) explained, “Skilled professional developers have one foot planted firmly in theory (knowledge and beliefs and vision) and the other in action (the local context, data about students, issues faced, and planning and doing professional development)” (p. 25). To that end, a purposeful effort was made to balance between theory and action when designing the Purposefully Planning for Mathematics Discourse (PPMD) professional development program.

**Purpose**

The purpose of the PPMD professional development program is to support teachers in their efforts to increase opportunities for students to engage in mathematics discourse as a regular part of instruction. From a teacher learning perspective, participating teachers will collaborate to create and revise annotated lessons that create space for student talk about mathematics as well as develop their skills in facilitating productive mathematics discussions. From a student learning perspective, students will have increased opportunities to discuss mathematics and engage in shared meaning making.
Research-Base

Three strands of research were attended to when developing the PPMD professional development program: (1) multitiered design study for professional development; (2) best practices in professional development for mathematics teachers; and (3) effective instructional practices for supporting productive mathematics discourse in the classroom.

The PPMD professional development program is a multitiered professional development design study. The philosophy of multitiered professional development design study is “to embrace the complexity and the dynamic nature of the system in which teachers learn and grow” by situating teachers as co-developers of educational objects (Zawojewski, Chamberlin, Hjalmarson, & Lewis, 2010, p. 236). This approach to professional development honors and builds from the experiences, strengths, and needs that teachers bring to the table, which allows for flexibility. At the same time, this approach to professional development also provides a collaborate environment in which teachers learn and growth together around a focused topic, in this case facilitating productive mathematics discourse. In the PPMD professional development program, this collaborative environment will take place in a professional learning community (PLC). The teachers in the PLC will conduct multiple mini-design cycles, in which they will create an annotated lesson plan that modifies their current adopted mathematics curriculum to include mathematics discourse. After enacting the lesson plan, teachers will meet again in the PLC to reflect and revise the lesson plan based on their learning. This mini-design cycle will take place multiple times throughout the professional development. Thus, the multitiered professional development design study supports
teachers in designing and implement educational products to improve their practice, and it allows researchers to document teacher learning and growth, which can contribute to the broader field of mathematics education research.

With regard to best practice in mathematics professional development, the elements of context and collaboration were used when designing the PPMD professional development program. Connecting professional development to the context of teachers, in terms of school-wide goals and individual teacher needs, is necessary to motivate instructional change (Loucks-Horsley et al., 2010). The PPMD professional development program will attend to context by involving teachers in determining which discourse strategies they want to incorporate into their existing mathematics curriculum. Another element necessary for effective professional development is providing opportunities for teachers to collaborate, both in terms of planning and reflecting (NCTM, 2014). Hence, the PPMD professional development program will utilize professional learning communities as the space for such collaboration. Because PLCs are an existing collaborative structure at the participating school, using them also complements the school’s context.

For any professional development program, it is necessary to provide on-going support in order for teachers to implement what they have learned and make sustained instructional changes (Loucks-Horsley et al., 2010; NCTM 2014). Within the PPMD professional development program, the PLC serves a key role in providing this on-going support, as teachers are collaborating and supporting one another through the learning process. The PPMD professional development program was developed so that the teacher members rotate facilitating the PLC themselves. This design honors the expertise
that each teacher brings to the learning community and it provides for scalability as outside facilitators and/or content or pedagogical experts are not required. Therefore, once teachers are familiar with mini-design cycle process, they could easily sustain their work beyond the timeline of the PPMD professional development program as they will have an established, but flexible structure that would allow them to continue to improve their practice in facilitating productive mathematics discourse or focus on other areas of mathematics instruction.

There are a plethora of effective instructional practices for supporting productive mathematics discourse in the classroom. Key to successfully implementing these strategies is creating a classroom community that is safe for students to share ideas, explain and justify solution strategies, and engage in respectful disagreement (Anthony & Walshaw, 2009; Cirillo, 2013a; NCTM, 1991, 2000). Beyond creating a mathematics discourse community, there are many different considerations for teachers in terms of facilitating productive mathematics discourse, including: selecting tasks, the types of questions asked, building upon student reasoning, and teacher discourse moves (e.g., using waiting time or revoicing). This list is far from exhaustive, which demonstrates the complexity of facilitating mathematics discourse. Thus, there is no specific set of practices or teacher moves that will ensure productive mathematics discussions will happen (NCTM, 1991; Truxaw & DeFranco, 2008). It is for this reason that the PPMD professional development program is designed to reflect best practices in effective instructional practices for supporting productive mathematics discourse in the classroom and to allow for flexibility to meet the individual needs of a given teacher and his/her students.
Theoretical Framework

When designing the PPMD professional development module, the theoretical framework of social constructivism (Vygotsky, 1962, 1978) and the concept of communities of practice (Lave & Wenger, 1991) were used. Just as discourse opens opportunities for students to interact with others as they co-construct meaning about mathematics, the PPMD professional development program will provide those same opportunities for teachers as they plan, revise, and reflect together, thereby co-constructing knowledge about teaching mathematics. Thus, situating the professional development program within the collaborative structure of a PLC provides a space for teachers to form communities of practice in which discourse serves as the means by which pedagogical knowledge is constructed individually and collectively.

Goals

There are multiple levels of goals for the PPMD professional development program:

- Support teacher learning in effectively facilitating productive mathematics discourse as part of regular instruction;
- Provide a space for teacher collaboration to co-construct knowledge about teaching mathematics individually and collectively;
- Create annotated lesson plans that build upon the currently adopted mathematics curriculum by opening opportunities for mathematics discourse;
- Provide access to productive mathematical discussions for all students, which allow them to engage in shared meaning making;
• Shift responsibility for mathematics discourse from teacher-led discussion to more student-led discussions;

• Document teacher growth in the area of facilitating mathematics discourse; and

• Contribute to the field of mathematics education research as a result of implementing the PPMD professional development program.

**Overview of PPMD Module**

The PPMD professional development will utilize a professional learning community (PLC) as the collaborative structure. The PLC will consist of teachers from multiple grade levels, but focus on the shared goal of collaboratively planning for mathematics discourse building from lessons in the adopted textbook. The PLC will meet for nine sessions that are approximately one-hour long. The first and second sessions will orient the teachers to the goals of the PPMD professional development program, establish norms, develop a shared understanding of mathematics discourse, build background in mathematics discourse research, and identify focus areas. The subsequent six sessions will consist of three mini-design cycles, in which teachers co-plan a lesson, enact it, and come together to debrief and revise the lesson. The members of the PLC will decide the focus of each mini-design cycle within the area of facilitating productive mathematics discourse. The final session will summarize teacher learning and create a plan for moving forward.
Session 1: Establishing our PLC & What is Mathematics Discourse?

Materials Preparation

- Chart paper or access to an InterWrite Board to capture discussions
- Scratch paper or post-its

Opening (10 minutes)

- Welcome to everyone! Share your favorite part of teaching math.
- Share Overview of the Professional Development Program
  - The goal of this professional development program is to support us in planning for and facilitating mathematics discourse in our classrooms. We will work as a professional learning community to collaborate and learn together. Everyone bring valuable insights, experiences, and knowledge to this PLC. There is no “expert,” instead we are all here to share our individual expertise and develop a shared expertise around mathematics discourse. Although Andria is the researcher on this project, her role in this PLC is that of a member, equal to everyone in this learning community.
  - We will begin by establishing a shared understanding of mathematics discourse and exploring some research about best practices in planning for and facilitating math talk among students during instruction. We will go through three mini-design cycles, in which we will select a practice to incorporate in a lesson, try it out with our students, and then come back together to share our learning and make revisions to the lesson. We will
collaborate and reflect on our learning as work through professional development together.

- Establish norms for PLC
  - What behaviors and/or expectations do we need to make our PLC function effectively?
  - Create an agreed upon chart of norms (limit to 3–5 total)

**Defining Mathematics Discourse (10 minutes)**

- Think-Write-Share: What is mathematics discourse?
- Discuss to reach consensus definition

**Roles of Student and Teacher during Mathematics Discourse (15 minutes)**

- Working grade level teams, create a Venn diagram to answer the following questions:
  - What roles should teachers take during mathematics discourse?
  - What roles should students take during mathematics discourse?

- Examine each others’ charts:
  - Look for similarities and differences
  - Clarify ideas as needed

- Create a consensus Venn diagram of the roles teacher and students take during mathematics discourse

**Characteristics of Productive Mathematics Discussions (15 minutes)**

- Working grade level teams, create chart to answer the following questions:
  - What does a productive mathematics discussion look like?
  - What does a productive mathematics discussion sound like?
o  What does a productive mathematics discussion feel like?

• Examine each others’ charts:
  o  Look for similarities and differences
  o  Clarify ideas as needed

• Create a consensus three-column chart that shows what a productive mathematics discussion looks like, sounds like, and feels like.

Closing (10 minutes)

• Think-Pair-Share: What do you want to learn as a result of participating in this PLC?
  o  Capture the group’s learning goals on a chart (limit to 2–3 total)

• Leadership: Remember, we will rotate the leadership role among all of us, so who will facilitate next time?

• Homework: Read the two Discussion Research Briefs on NCTM’s website:
  Benefits of Discussion and Strategies for Discussion

Session 2: Selecting a Focus

Materials Preparation

• Post Goals, Norms, Definition of Mathematics Discourse, Venn Diagram of Roles, and Characteristics of Productive Mathematics Discourse Chart

• Chart paper or access to an InterWrite Board

• Copies of the Math-Talk Learning Community Framework (Hufferd-Ackles, Fuson, & Sherin, 2014)
• Copies of “Three Strategies for Opening Curriculum Spaces” TCM article (Drake et al., 2015)

Opening (5 minutes)

• Welcome to everyone! Please share a positive math experience from this past week.
• Begin by reviewing the goals and norms you created in the first session.
• Share the definition of mathematics discourse, the Venn diagram of roles, and the characteristics of productive mathematics discourse you created. Does anyone have suggestions for change? If so, modify accordingly.

Examine the Math-Talk Learning Community (MTLC) Framework (20 minutes)

• Compare the MTLC to our conceptualization of productive mathematics discussions.
  o Read the MTLC Framework; feel free to make notes on it
  o How is the MTLC Framework similar to the documents we’ve produced? How is it different?
  o Do we want to make any deletions, additions, or modifications?
• What elements of the MTLC Framework do we see reflected in our math textbook?
• Take a few minutes to rate yourself on the continuum of the MTLC Framework.
  o This rating can remain personal, but if there is something you want to share, feel free.
Discussion of Research Briefs (20 minutes)

- Facilitate a discussion about the two Research Briefs everyone read for this week. If some members are reluctant to share, try to invite them into the conversation. Ask someone to capture the key points of the discussion, either on a board or chart paper.

- Here are some possible questions for the discussion, but you are not limited to these:
  - What are your reactions to the research briefs we read?
  - What do you agree/disagree with? What surprised you?
  - Which of the practices mentioned in the briefs would complement our math textbook?
  - Which practices are you most interested in trying out? Why?

Determining a Focus (10 minutes)

- Next week, we will be planning a lesson that focuses on student discourse. We will use our textbook as a basis and look for opportunities for student talk and identify instructional moves we can make to facilitate productive discourse. But, there are many elements within facilitating mathematics discourse, so we need to narrow our focus…

- Which instructional practice do we want to try first? Why? (Must reach consensus)

- Once focus is agreed upon, each member is responsible for bringing an idea, information, or research about that practice to share with the PLC next week…
Closing (5 minutes)

- Think-Pair-Share: What is your “take-away” from today’s discussion? Why?
- Leadership: Remember, we will rotate the leadership role among all of us, so who will facilitate next time?
- Homework: Read “Three Strategies for Opening Curriculum Spaces” article; Bring your idea, information, or research about our agreed upon focus

Sessions 3-4: Mini-Design Cycle 1

Session 3

Materials Preparation

- Post Goals, Norms, Definition of Mathematics Discourse, Venn Diagram of Roles, and Characteristics of Productive Mathematics Discourse Chart
- Chart paper or access to an InterWrite Board
- Bring Math Textbooks (hardcopy or electronic)

Opening (5 minutes)

- Welcome to everyone! Please share a positive math experience from this past week.
- Begin by reviewing the goals and norms you created in the first session.
- Share the definition of mathematics discourse, the Venn diagram of roles, and the characteristics of productive mathematics discourse you created. Does anyone have suggestions for change? If so, modify accordingly.
Mathematics Discourse Practice of Focus (15 minutes)

- Ask someone to capture the key points of the discussion, either on a board or chart paper.
- What was your most important take-away from the Drake article?
- Each member shares his/her idea, information, or research about the selected focus.

Co-Planning the Lesson (35 minutes)

- Work with your grade level partner(s) to identify a lesson you will be teaching in the next week that you want modify by incorporating more mathematics discourse.
- Think about the Drake article and the ideas shared in our discussion as you look for opportunities to infuse student talk into the lesson…
- You will need to produce a written lesson plan, in a format of your choice, that (1) tells when you will use student talk and how you will facilitate that; and (2) why you made those choices. It is recommended that the “what” of the lesson and the “why” of the lesson are distinguished in some way (font, color, etc.).
- Time permitting, have each grade level share an overview of their plan and rationale and the other members provide feedback.

Closing (5 minutes):

- Think-Pair-Share: What supports do you need to carry out your lesson plan?
- Leadership: Remember, we will rotate the leadership role among all of us, so who will facilitate next time?
• Homework: Enact your lesson before our next meeting and be sure to schedule your observation with Andria.

Session 4

Materials Preparation

• Post Goals, Norms, Definition of Mathematics Discourse, Venn Diagram of Roles, and Characteristics of Productive Mathematics Discourse Chart
• Chart paper or access to an InterWrite Board
• Bring Math Textbooks and Annotated Lesson (hardcopy or electronic)

Opening (5 minutes)

• Welcome to everyone! Please share a positive math experience from this past week.
• Begin by reviewing the goals and norms you created in the first session.
• Share the definition of mathematics discourse, the Venn diagram of roles, and the characteristics of productive mathematics discourse you created. Does anyone have suggestions for change? If so, modify accordingly.

Reflecting on the Mathematics Discourse Lesson (15 minutes)

• Grade-level groups share a brief reminder/overview of lesson.
• Facilitate a discussion about how the lessons went when enacted. If some members are reluctant to share, try to invite them into the conversation. Ask someone to capture the key points of the discussion, either on a board or chart paper.
• Here are some possible questions for the discussion, but you are not limited to these:
  o What went well? Why do you think that is?
  o What was challenging? Why do you think that is?
  o What adjustments would you make in the future? Why?

Revise the Lesson Plan (25 minutes)

• Work with your grade level partner(s) to modify the lesson based on your experience and our discussion.

• Your revisions need to be captured in a written lesson plan, in a format of your choice, that (1) shows what changes you made in terms of telling when you will use student talk and how you will facilitate that; and (2) why you made those changes. It is recommended that the “what” of the lesson and the “why” of the lesson are distinguished in some way (font, color, etc.) and that the Revised version of the lesson is distinguished in some way from the Original version of the lesson.

• Time permitting, have each grade level share an overview of their changes and rationale and the other members provide feedback.

Determining a Focus (10 minutes)

• Next week, we will be planning another lesson that focuses on student discourse. We can continue to use the focus from this design-cycle or we can choose a focus…

• Which instructional practice do we want to try next? Why? (Must reach consensus)
• Once focus is agreed upon, each member is responsible for bringing an idea, information, or research about that practice to share with the PLC next week…

Closing (5 minutes)

• Think-Pair-Share: What is your “take-away” from our first mini-design cycle? Why?
• Leadership: Remember, we will rotate the leadership role among all of us, so who will facilitate next time?
• Homework: Bring your idea, information, or research about our agreed upon focus

Sessions 5-6 Mini-Design Cycle 2

Session 5

Materials Preparation

• Post Goals, Norms, Definition of Mathematics Discourse, Venn Diagram of Roles, and Characteristics of Productive Mathematics Discourse Chart
• Chart paper or access to an InterWrite Board
• Bring Math Textbooks (hardcopy or electronic)

Opening (5 minutes)

• Welcome to everyone! Please share a positive math experience from this past week.
• Begin by reviewing the goals and norms you created in the first session.
• Share the definition of mathematics discourse, the Venn diagram of roles, and the characteristics of productive mathematics discourse you created. Does anyone have suggestions for change? If so, modify accordingly.

Mathematics Discourse Practice of Focus (10 minutes)

• Ask someone to capture the key points of the discussion, either on a board or chart paper.

• Each member shares his/her idea, information, or research about the selected focus.

Co-Planning the Lesson (40 minutes)

• Work with your grade level partner(s) to identify a lesson you will be teaching in the next week that you want modify by incorporating more mathematics discourse.

• Think about the Drake article and the ideas shared in our discussion as you look for opportunities to infuse student talk into the lesson…

• You will need to produce a written lesson plan, in a format of your choice, that (1) tells when you will use student talk and how you will facilitate that; and (2) why you made those choices. It is recommended that the “what” of the lesson and the “why” of the lesson are distinguished in some way (font, color, etc.).

• Time permitting, have each grade level share an overview of their plan and rationale and the other members provide feedback.

Closing (5 minutes):

• Think-Pair-Share: What supports do you need to carry out your lesson plan?
• Leadership: Remember, we will rotate the leadership role among all of us, so who will facilitate next time?

• Homework: Enact your lesson before our next meeting and be sure to schedule your observation with Andria.

**Session 6**

Materials Preparation

• Post Goals, Norms, Definition of Mathematics Discourse, Venn Diagram of Roles, and Characteristics of Productive Mathematics Discourse Chart

• Chart paper or access to an InterWrite Board

• Bring Math Textbooks and Annotated Lesson (hardcopy or electronic)

Opening (5 minutes)

• Welcome to everyone! Please share a positive math experience from this past week.

• Begin by reviewing the goals and norms you created in the first session.

• Share the definition of mathematics discourse, the Venn diagram of roles, and the characteristics of productive mathematics discourse you created. Does anyone have suggestions for change? If so, modify accordingly.

Reflecting on the Mathematics Discourse Lesson (15 minutes)

• Grade-level groups share a brief reminder/overview of lesson.

• Facilitate a discussion about how the lessons went when enacted. If some members are reluctant to share, try to invite them into the conversation. Ask
someone to capture the key points of the discussion, either on a board or chart paper.

• Here are some possible questions for the discussion, but you are not limited to these:
  
  o What went well? Why do you think that is?
  
  o What was challenging? Why do you think that is?
  
  o What adjustments would you make in the future? Why?

Revise the Lesson Plan (25 minutes)

• Work with your grade level partner(s) to modify the lesson based on your experience and our discussion.

• Your revisions need to be captured in a written lesson plan, in a format of your choice, that (1) shows what changes you made in terms of telling when you will use student talk and how you will facilitate that; and (2) why you made those changes. It is recommended that the “what” of the lesson and the “why” of the lesson are distinguished in some way (font, color, etc.) and that the Revised version of the lesson is distinguished in some way from the Original version of the lesson.

• Time permitting, have each grade level share an overview of their changes and rationale and the other members provide feedback.

Determining a Focus (10 minutes)

• Next week, we will be planning another lesson that focuses on student discourse. We can continue to use the focus from this design-cycle or we can choose a focus…
• Which instructional practice do we want to try next? Why? (Must reach consensus)

• Once focus is agreed upon, each member is responsible for bringing an idea, information, or research about that practice to share with the PLC next week…

Closing (5 minutes)

• Think-Pair-Share: What is your “take-away” from our second mini-design cycle? Why?

• Leadership: Remember, we will rotate the leadership role among all of us, so who will facilitate next time?

• Homework: Bring your idea, information, or research about our agreed upon focus

Sessions 7-8: Mini-Design Cycle 3

Session 7

Materials Preparation

• Post Goals, Norms, Definition of Mathematics Discourse, Venn Diagram of Roles, and Characteristics of Productive Mathematics Discourse Chart

• Chart paper or access to an InterWrite Board

• Bring Math Textbooks (hardcopy or electronic)

Opening (5 minutes)

• Welcome to everyone! Please share a positive math experience from this past week.

• Begin by reviewing the goals and norms you created in the first session.
• Share the definition of mathematics discourse, the Venn diagram of roles, and the characteristics of productive mathematics discourse you created. Does anyone have suggestions for change? If so, modify accordingly.

Mathematics Discourse Practice of Focus (10 minutes)
• Ask someone to capture the key points of the discussion, either on a board or chart paper.
• Each member shares his/her idea, information, or research about the selected focus.

Co-Planning the Lesson (40 minutes)
• Work with your grade level partner(s) to identify a lesson you will be teaching in the next week that you want modify by incorporating more mathematics discourse.
• Think about the Drake article and the ideas shared in our discussion as you look for opportunities to infuse student talk into the lesson…
• You will need to produce a written lesson plan, in a format of your choice, that (1) tells when you will use student talk and how you will facilitate that; and (2) why you made those choices. It is recommended that the “what” of the lesson and the “why” of the lesson are distinguished in some way (font, color, etc.).
• Time permitting, have each grade level share an overview of their plan and rationale and the other members provide feedback.

Closing (5 minutes):
• Think-Pair-Share: What supports do you need to carry out your lesson plan?
• Leadership: Remember, we will rotate the leadership role among all of us, so who will facilitate next time?

• Homework: Enact your lesson before our next meeting and be sure to schedule your observation with Andria.

Session 8
Materials Preparation
• Post Goals, Norms, Definition of Mathematics Discourse, Venn Diagram of Roles, and Characteristics of Productive Mathematics Discourse Chart
• Chart paper or access to an InterWrite Board
• Bring Math Textbooks and Annotated Lesson (hardcopy or electronic)

Opening (5 minutes)
• Welcome to everyone! Please share a positive math experience from this past week.
• Begin by reviewing the goals and norms you created in the first session.
• Share the definition of mathematics discourse, the Venn diagram of roles, and the characteristics of productive mathematics discourse you created. Does anyone have suggestions for change? If so, modify accordingly.

Reflecting on the Mathematics Discourse Lesson (15 minutes)
• Grade-level groups share a brief reminder/overview of lesson.
• Facilitate a discussion about how the lessons went when enacted. If some members are reluctant to share, try to invite them into the conversation. Ask
someone to capture the key points of the discussion, either on a board or chart paper.

- Here are some possible questions for the discussion, but you are not limited to these:
  - What went well? Why do you think that is?
  - What was challenging? Why do you think that is?
  - What adjustments would you make in the future? Why?

Revise the Lesson Plan (25 minutes)

- Work with your grade level partner(s) to modify the lesson based on your experience and our discussion.

- Your revisions need to be captured in a written lesson plan, in a format of your choice, that (1) shows what changes you made in terms of telling when you will use student talk and how you will facilitate that; and (2) why you made those changes. It is recommended that the “what” of the lesson and the “why” of the lesson are distinguished in some way (font, color, etc.) and that the Revised version of the lesson is distinguished in some way from the Original version of the lesson.

- Time permitting, have each grade level share an overview of their changes and rationale and the other members provide feedback.

Determining a Focus (10 minutes)

- Next week, we will be planning another lesson that focuses on student discourse. We can continue to use the focus from this design-cycle or we can choose a focus…
• Which instructional practice do we want to try next? Why? (Must reach consensus)

• Once focus is agreed upon, each member is responsible for bringing an idea, information, or research about that practice to share with the PLC next week…

Closing (5 minutes)

• Think-Pair-Share: What is your “take-away” from our last mini-design cycle? Why?

• Leadership: Remember, we will rotate the leadership role among all of us, so who will facilitate next time?

• Homework: None

Session 9: Reflection & Moving Forward

Materials Preparation

• Post Goals, Norms, Definition of Mathematics Discourse, Venn Diagram of Roles, and Characteristics of Productive Mathematics Discourse Chart

• Chart paper or access to an InterWrite Board

• Copies of the Math-Talk Learning Community Framework (Hufferd-Ackles, Fuson, & Sherin, 2014)

Opening (5 minutes)

• Welcome to everyone! Please share a positive math experience from this past week.

• Begin by reviewing the goals and norms you created in the first session.
• Share the definition of mathematics discourse, the Venn diagram of roles, and the characteristics of productive mathematics discourse you created. Does anyone have suggestions for change? If so, modify accordingly.

Reflect on your Learning (30 minutes)

• Facilitate a discussion about the take-aways from participating in this professional development. If some members are reluctant to share, try to invite them into the conversation. Ask someone to capture the key points of the discussion, either on a board or chart paper.

• Here are some possible questions for the discussion, but you are not limited to these:
  o What have you learned as a result of participating in this PLC?
  o How has your instructional practice been impacted?
  o How have your students been impacted?
  o What were some of the challenges you faced during this process? Were you able to overcome them? Why or why not?
  o What were some of the successes you had during this process? What structures or supports enabled you to have those successes?
  o Has your conception of planning for and facilitating mathematics discourse changed? If so, how? If not, why do you think that is?

• Reflect on goals set for PLC—Did we meet them? Why or why not?

• Revisit the Math-Talk Learning Community (MTLC) Framework
  o Review the MTLC Framework
- Take a few minutes to rate yourself on the continuum of the MTLC Framework.
- Consider the following questions for personal self-reflection—the intent is not that each member share, although they are welcomed to if they want…
  - How does it compare to where you were prior to engaging in this professional development process?
  - What areas might you want to continue to attend to moving forward?

Moving Forward (20 minutes)

- Have a discussion about “next steps.” Ask someone to capture the key points of the discussion, either on a board or chart paper.
- Here are some possible questions for the discussion, but you are not limited to these:
  - What are some aspects of planning for and/or facilitating mathematics discourse that you still want to work on? Why? How might you accomplish that?
  - How could you apply what we’ve learned in this PLC to your daily classroom practice?
  - What continuing supports do you need to be successful in continuing to increase opportunities for students to engage in mathematics discourse as a regular part of instruction? Where could you find/get those supports?
• After the discussion, determine individual and/or collective next steps. Make these steps manageable and actionable…

Closing (5 minutes)

• Final Thoughts: As our formal professional learning community draws to a close, we’d like to give everyone an opportunity to share their final thoughts about this experience…
Appendix C:

Classroom Observation Protocol
# Classroom Observation Form

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<thead>
<tr>
<th>Participant Code:</th>
<th>Grade Level:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date:</td>
<td>Time:</td>
</tr>
<tr>
<td>Lesson Topic:</td>
<td></td>
</tr>
<tr>
<td>Classroom Context:</td>
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</table>

<table>
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<tr>
<th>Time</th>
<th>Descriptive Data</th>
<th>Observer Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix D:

Interview Protocol
Opening Statements:

Thank you for agreeing to take time from your busy schedule to participate in this interview. There are a few things that I would like to make sure you understand before we get started:

• I will be asking you some questions and writing notes as we proceed.

• With your permission, I will audiotape this interview. The audiotape will be destroyed as soon as I transcribe it. Do I have your permission to audiotape today?

• All information from this interview will be confidential. That is, you will not be identified by name, location, or place of employment in this study or in any report from this study.

• You may stop this interview at anytime without any negative consequences.

• You may hear the term discourse used in the interview; I am referring to the different ways you and your students communicate during learning.

Please be assured that there are no correct answers to the questions that I will be asking. The intent of this interview is to gather your thoughts, feelings, and experiences, not to make judgments on your responses.
Possible Interview Questions

Before PPMD Treatment:

• Tell me about your teaching background.
• What do you like about teaching math? What do you find challenging?
• What are your strengths as a math teacher? What is an area you’d like to grow in as a math teacher?
• What is your philosophy for teaching math?
• How well does your adopted textbook fit with your philosophy?
• Tell me about your process for planning math lessons.
• Do you adjust the lessons in your textbook? If so, how?
• In a typical lesson, how often do you have students talk about math?
• What are some strategies or instructional moves you make to get students to talk about math?
• What are some challenges you face when having students talk about math?

During PPMD Treatment:

• What are your reactions to participating in the PLC so far/now that we’re ending?
• What have enjoyed about the process? What have you found challenging?
• Describe your experience going through the first/second/third mini-design cycle.
• Describe your feelings going through the first/second/third mini-design cycle.
• Is this process impacting your beliefs about teaching math? If so, how?
• Is this process impacting your students? If so, how?
• What has been going well as you’ve been teaching math?
• What challenges have you faced trying to incorporate more math discourse?
• What supports do you need as you continue to incorporate more math discourse?

After PPMD Treatment:

• What do you like about teaching math? What do you find challenging?
• What are your strengths as a math teacher? What is an area you’d like to grow in as a math teacher?
• What has been going well as you’ve been teaching math?
• In reflecting back on the PPMD, what did enjoy you about the experience? What did you find challenging about it?
• Did participating in the PPMD change your beliefs about teaching math? If so, how?
• Did participating in the PPMD change how you plan your math lessons? If so, how?
• Did participating in the PPMD change how you teach math? If so, how?
• Did participating in the PPMD impact your students? If so, how?
• Tell me about how you are doing with the action plan you made.
• What challenges have you faced in continuing to plan and facilitate math discourse?
• What supports do you still need?
• Would you recommend that other teachers participate in the PPMD? Why or why not?
• Would you consider using the mini-design cycle process to help you make other instructional changes in math? Why or why not?
### Interview Form

<table>
<thead>
<tr>
<th>Participant Code:</th>
<th>Date:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1) Text of question…</strong></td>
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<tr>
<td>Observer Notes</td>
<td>Descriptive Data</td>
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</tbody>
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Appendix E:

Purposefully Planning for Mathematics Discourse (PPMD)

Professional Development Program

Changes Made During Study
Session 1: Establishing our PLC & What is Mathematics Discourse?

Materials Preparation

- Access to Math Discourse PLC in Google Drive Folder
- Scratch paper or post-its

Opening (10 minutes)

- Welcome to everyone! Share your favorite part of teaching math.
- Share Overview of the Professional Development Program
  - The goal of this professional development program is to support us in planning for and facilitating mathematics discourse in our classrooms. We will work as a professional learning community to collaborate and learn together. Everyone bring valuable insights, experiences, and knowledge to this PLC. There is no “expert,” instead we are all here to share our individual expertise and develop a shared expertise around mathematics discourse. Although Andria is the researcher on this project, her role in this PLC is that of a member, equal to everyone in this learning community.
  - We will begin by establishing a shared understanding of mathematics discourse and exploring some research about best practices in planning for and facilitating math talk among students during instruction. We will go through three mini-design cycles, in which we will select a practice to incorporate in a lesson, try it out with our students, and then come back together to share our learning and make revisions to the lesson. We will
collaborate and reflect on our learning as we work through this professional development together.

- Establish norms for PLC
  - What behaviors and/or expectations do we need to make our PLC function effectively?
  - Create an agreed upon chart of norms (limit to 3–5 total)

Defining Mathematics Discourse (10 minutes)
- Think-Write-Share: What is mathematics discourse?
- Discuss to reach consensus definition

Roles of Student and Teacher during Mathematics Discourse (15 minutes)
- Working grade level teams, create a Venn diagram to answer the following questions:
  - What roles should teachers take during mathematics discourse?
  - What roles should students take during mathematics discourse?
- Examine each others’ charts:
  - Look for similarities and differences
  - Clarify ideas as needed
- Create a consensus Venn diagram of the roles teacher and students take during mathematics discourse

Closing (10 minutes)
- Think-Pair-Share: What do you want to learn as a result of participating in this PLC?
Please bring these to our next meeting or make note of them in our Google Folder prior to our next meeting

- Leadership: Remember, we will rotate the leadership role among all of us, so who will facilitate next time?
- Homework: Read the two Discussion Research Briefs on NCTM’s website:
  
  *Benefits of Discussion* and *Strategies for Discussion*

**Session 2: Selecting a Focus**

**Materials Preparation**

- Access to Math Talk PLC in Google Drive Folder: Norms, PLC Goals, Definition of Mathematics Talk, and Venn Diagram of Roles
- Copies of the Math-Talk Learning Community Framework (Hufferd-Ackles, Fuson, & Sherin, 2014)

**Opening (5 minutes)**

- Welcome to everyone! Please share a positive math experience from this past week.
- We’ll begin by reviewing the norms we created in the first session. Do we need to make any adjustments?
- Let’s look at the PLC Goals ideas we generated. We need to narrow this list down to 2-3 total goals.

**Examine the Math-Talk Learning Community (MTLC) Framework (20 minutes)**

- We will compare the MTLC to our understanding of mathematics discourse.
  
  o Read the MTLC Framework; feel free to make notes on it.
How is the MTLC Framework similar to the documents we’ve produced? How is it different?

Do we want to make any deletions, additions, or modifications based on what’s included in the MTLC Framework to our definition of Math Discourse or the roles of students and teachers?

- Let’s take a few minutes to rate ourselves on the continuum of the MTLC Framework.
  
  This rating can remain personal, but if there is something we want to share, feel free.

Discussion of Research Briefs (20 minutes)

- Facilitate a discussion about the two Research Briefs everyone read for this week.
  
  If some members are reluctant to share, try to invite them into the conversation.
  
  Ask someone to capture the key points of the discussion in a Google doc.

- Here are some possible questions for the discussion, but we are not limited to these:
  
  - What are your reactions to the research briefs we read?
  - What do you agree/disagree with? What surprised you?
  - Which of the practices mentioned in the briefs would complement our math textbook?
  - Which practices are you most interested in trying out? Why?

Determining a Focus (10 minutes)

- Next week, we will be planning a lesson that focuses on student talk during math.
  
  We will use our textbook as a basis and look for opportunities for student talk and
identify instructional moves we can make to facilitate productive discourse. But, there are many elements within facilitating math talk, so we need to narrow our focus…

• Which instructional practice do we want to try first? Why? (Must reach consensus)

• Once our focus is agreed upon, each of us is responsible for bringing an idea, information, or research about that practice to share with the PLC next week…

Closing (5 minutes)

• Think-Pair-Share: What is your “take-away” from today’s discussion? Why?

• For Next Time: Bring your idea, information, or research about our agreed upon focus

Sessions 3-4: Mini-Design Cycle 1

Session 3

Materials Preparation

• Access to Math Talk PLC in Google Drive Folder: PLC Goals, Definition of Math Talk, and Venn Diagram of Roles

• Bring Math Textbooks (hardcopy or electronic)

Opening (5 minutes)

• Welcome to everyone! Please share a highlight from your long weekend.

• Review of Norms.

• Quick check-in on our PLC Goals and Definition of Mathematic Talk--Does anyone have suggestions for change? If so, modify accordingly.
Math Talk Practice of Focus: Wait Time/Think Time (15 minutes)

- Each of us will share our idea, information, or research about wait time/think time.
- Ask someone to capture the key points of the discussion to be added to our Google folder.

Co-Planning the Lesson (35 minutes)

- Work with our grade level partner(s) to identify a lesson we will be teaching in the next week that we want modify by incorporating more math talk.
- Think about the ideas we shared in our discussion as we look for opportunities to infuse student talk into the lesson and use our agreed upon instructional practice of wait time/think time strategies.
- We will need to produce a written lesson plan, in a format of our choice, that (1) tells when we will use student talk and how we will facilitate that; and (2) why we made those choices. It is recommended that the “what” of the lesson and the “why” of the lesson are distinguished in some way (font, color, etc.).
- Time permitting, each grade level will share an overview of their plan and rationale and the rest of us will provide feedback.

Closing (5 minutes):

- Think-Pair-Share: What supports do you need to carry out your lesson plan?
- Homework: Enact your lesson before our next meeting and be prepared to share the strengths and areas for revision.
Session 4

Materials Preparation

• Access to Math Talk PLC in Google Drive Folder: PLC Goals, Definition of Math Talk, and Venn Diagram of Roles

• Bring Math Textbooks and Annotated Lesson (hardcopy or electronic)

Opening (10 minutes)

• Welcome to everyone! Please share something fun or relaxing you have planned for Winter Break.

• Review of Norms.

• Quick check-in on our PLC Goals and Definition of Math Talk --Does anyone have suggestions for change? If so, modify accordingly.

Reflecting on the Math Talk Lesson with Wait Time/Think Time (20 minutes)

• Grade-level groups share a brief overview of lesson.

• Facilitate a discussion about how the lessons went when tried out. If some members are reluctant to share, try to invite them into the conversation. Ask someone to capture the key points of the discussion to be added to our Google folder.

• Here are some possible questions for the discussion, but you are not limited to these:
  
  o What went well? Why do you think that is?

  o What was challenging? Why do you think that is?

  o What adjustments would you make in the future? Why?
Revise the Lesson Plan (15 minutes)

• Work with your grade level partner(s) to modify the lesson based on your experience and our discussion.

• Your revisions need to be captured in a written lesson plan, in a format of your choice, that (1) shows what changes you made in terms of telling when you will use student talk and how you will facilitate that; and (2) why you made those changes. It is recommended that the “what” of the lesson and the “why” of the lesson are distinguished in some way (font, color, etc.) and that the Revised version of the lesson is distinguished in some way from the Original version of the lesson. Please be sure to send Andria a picture or electronic copy of the revised lesson plan.

• Time permitting, have each grade level share an overview of their changes and rationale and the other members provide feedback.

Determining a Focus (10 minutes)

• Next week, we will be planning another lesson that focuses on student discourse. We will continue to use the focus from this design-cycle (wait time/think time) and we will add another element/practice…

• Which instructional practice do we want to try next? Why? (Must reach consensus)

• Once focus is agreed upon, each member is responsible for bringing an idea, information, or research about that practice to share during our next PLC…
Closing (5 minutes)

- Think-Pair-Share: What is your “take-away” from our first mini-design cycle? Why?
- Logistics: Scheduling our next meeting after Winter Break
- Homework: Bring your idea, information, or research about our agreed upon focus

Sessions 5-6 Mini-Design Cycle 2

Session 5

Materials Preparation

- Access to Math Talk PLC in Google Drive Folder: PLC Goals, Definition of Math Talk, and Venn Diagram of Roles
- Bring Math Textbooks (hardcopy or electronic)

Opening (7 minutes)

- Welcome to everyone! Please share a highlight from your break.
- Review of Norms.

Mathematics Discourse Practice of Focus (20 minutes)

- Each of us will share our idea, information, or research about revoicing.
- Ask someone to capture the key points of the discussion to be added to our Google folder.

Co-Planning the Lesson (30 minutes)

- Work with our grade level partner(s) to identify a lesson we will be teaching in the next week that we want modify by incorporating more math talk.
• Think about the ideas we shared in our discussion as we look for opportunities to infuse student talk into the lesson and use revoicing as well as continue to use wait time/think time…

• We will need to produce a written lesson plan, in a format of our choice, that (1) tells when we will use student talk and how we will facilitate that; and (2) why we made those choices. It is recommended that the “what” of the lesson and the “why” of the lesson are distinguished in some way (font, color, etc.).

• Time permitting, each grade level will share an overview of their plan and rationale and the rest of us will provide feedback.

Closing (3 minutes):

• Think-Pair-Share: What supports do you need to carry out your lesson plan?

• Homework: Enact your lesson before our next meeting and be prepared to share the strengths and areas for revision.

Session 6

Materials Preparation

• Access to Math Talk PLC in Google Drive Folder: PLC Goals, Definition of Math Talk, and Venn Diagram of Roles

• Bring Math Textbooks and Lesson Plan (hardcopy or electronic)

Opening (10 minutes)

• Welcome to everyone! Please share a positive math teaching experience from the past week.

• Review of Norms.
Reflecting on the Mathematics Discourse Lesson (20 minutes)

- Grade-level groups share a brief overview of lesson.
- Facilitate a discussion about how the lessons went when tried out. If some members are reluctant to share, try to invite them into the conversation. Ask someone to capture the key points of the discussion to be added to our Google folder.
- Here are some possible questions for the discussion, but you are not limited to these:
  - What went well? Why do you think that is?
  - What was challenging? Why do you think that is?
  - What adjustments would you make in the future? Why?

Revise the Lesson Plan (15 minutes)

- Work with your grade level partner(s) to modify the lesson based on your experience and our discussion.
- Your revisions need to be captured in a written lesson plan, in a format of your choice, that (1) shows what changes you made in terms of telling when you will use student talk and how you will facilitate that; and (2) why you made those changes. It is recommended that the “what” of the lesson and the “why” of the lesson are distinguished in some way (font, color, etc.) and that the Revised version of the lesson is distinguished in some way from the Original version of the lesson. Please be sure to send Andria a picture or electronic copy of the revised lesson plan.
• Time permitting, have each grade level share an overview of their changes and rationale and the other members provide feedback.

Determining a Focus (10 minutes)

• Next week, we will be planning another lesson that focuses on student discourse. We will continue to use the focus from our previous design-cycles, wait time/think time and revoicing, and we will add another element/practice…

• Which instructional practice do we want to try next? Why? (Must reach consensus)

• Once focus is agreed upon, each of us is responsible for bringing an idea, information, or research about that practice to share during our next meeting…

Closing (5 minutes)

• Think-Pair-Share: What is your “take-away” from our second mini-design cycle? Why?

• Logistics: Scheduling our final three meetings, including location(s) and facilitators

• Homework: Bring your idea, information, or research about our agreed upon focus

Sessions 7-8: Mini-Design Cycle 3

Session 7

Materials Preparation

• Access to Math Talk PLC in Google Drive Folder: PLC Goals, Definition of Math Talk, and Venn Diagram of Roles
• Bring Math Textbooks (hardcopy or electronic)

Opening (7 minutes)

• Welcome to everyone! Please share a brief update on how continuing to include math talk into your instruction is going…

• Review of Norms.

Math Talk Practice of Focus (20 minutes)

• Each of us will share our idea, information, or research about Turn and Talk.

• Ask someone to capture the key points of the discussion to be added to our Google folder.

Co-Planning the Lesson (30 minutes)

• Work with our grade level partner(s) to identify a lesson we will be teaching in the next week that we want modify by incorporating more math talk.

• Think about the ideas we shared in our discussion as we look for opportunities to infuse student talk into the lesson and use Turn and Talk as well as continue to use wait time/think time and revoicing…

• We will need to produce a written lesson plan, in a format of our choice, that (1) tells when we will use student talk and how we will facilitate that; and (2) why we made those choices. It is recommended that the “what” of the lesson and the “why” of the lesson are distinguished in some way (font, color, etc.).

• Time permitting, each grade level will share an overview of their plan and rationale and the rest of us will provide feedback.
Closing (3 minutes):

- Think-Pair-Share: What supports do you need to carry out your lesson plan?
- Homework: Enact your lesson before our next meeting and be prepared to share the strengths and areas for revision.

**Sessions 7-8: Mini-Design Cycle 3**

**Session 8**

Materials Preparation

- Access to Math Talk PLC in Google Drive Folder: PLC Goals, Definition of Math Talk, and Venn Diagram of Roles
- Bring Math Textbooks and Lesson Plan (hardcopy or electronic)
- Copies of the Math-Talk Learning Community Framework (Hufferd-Ackles, Fuson, & Sherin, 2014)

Opening (5 minutes)

- Welcome to everyone! What are your plans for the upcoming long weekend?
- Review of Norms.

Reflecting on the Mathematics Discourse Lesson (15 minutes)

- Grade-level groups share a brief overview of lesson.
- Facilitate a discussion about how the lessons went when the Turn and Talk strategy was tried out. If some members are reluctant to share, try to invite them into the conversation. Ask someone to capture the key points of the discussion to be added to our Google folder.
- Here are some possible questions for the discussion, but you are not limited to these:
  - What went well? Why do you think that is?
  - What was challenging? Why do you think that is?
  - What adjustments would you make in the future? Why?

Revise the Lesson Plan (15 minutes)

- Work with your grade level partner(s) to modify the lesson based on your experience and our discussion.

- Your revisions need to be captured in a written lesson plan, in a format of your choice, that (1) shows what changes you made in terms of telling when you will use student talk and how you will facilitate that; and (2) why you made those changes. It is recommended that the “what” of the lesson and the “why” of the lesson are distinguished in some way (font, color, etc.) and that the Revised version of the lesson is distinguished in some way from the Original version of the lesson. Please be sure to send Andria a picture or electronic copy of the revised lesson plan.

- Time permitting, have each grade level share an overview of their changes and rationale and the other members provide feedback.

Revisit Venn Diagram of Roles (10 minutes)

- Review the Venn Diagram of Teacher and Student Roles in Discourse (in Google folder)

- Facilitate a brief discussion and make modifications to the document as needed:
What aspects of the roles we created have we affirmed through our work together?

What changes do we want to make? Why?

Revisit the Math-Talk Learning Community (MTLC) Framework (10 minutes)

- Review the MTLC Framework
- Take a few minutes to rate yourself on the continuum of the MTLC Framework.
- Consider the following questions for personal self-reflection—the intent is not that each member share, although they are welcomed to if they want…
  - How does it compare to where you were prior to engaging in this professional development process?
  - What areas might you want to continue to attend to moving forward?

Closing (5 minutes)

- Think-Pair-Share: What is your “take-away” from our final mini-design cycle? Why?
- Logistics: Scheduling our final meeting—when and where?
- Homework: Bring your ideas and materials for our Math Talk anchor charts

Session 9: Reflection & Moving Forward

Materials Preparation

- Access to Math Talk PLC in Google Drive Folder: PLC Goals, Definition of Math Talk, and Venn Diagram of Roles

Opening (10 minutes)

- Welcome to everyone! How have things been going since we last met?
- Review of Norms.
- Definition of Math Talk—review current form and make final revisions

Reflect on your Learning (20 minutes)

- Facilitate a discussion about the take-aways from participating in this professional development. If some members are reluctant to share, try to invite them into the conversation. Ask someone to capture the key points of the discussion, either on a board or chart paper.

- Here are some possible questions for the discussion, but you are not limited to these:
  - What have you learned as a result of participating in this PLC?
  - How has your instructional practice been impacted?
  - How have your students been impacted?
  - What were some of the challenges you faced during this process? Were you able to overcome them? Why or why not?
  - What were some of the successes you had during this process? What structures or supports enabled you to have those successes?
  - Has your process of planning for and facilitating math talk changed? If so, how? If not, why do you think that is?
  - What changes or improvements would you suggest if others were to use this protocol for the Math Talk PLC?

- Review of PLC Goals—Did we meet them? Why or why not?
Moving Forward (20 minutes)

• Have a discussion about “next steps.” Ask someone to capture the key points of the discussion, either on a board or chart paper.

• Here are some possible questions for the discussion, but you are not limited to these:
  o What are some aspects of planning for and/or facilitating mathematics discourse that you still want to work on? Why? How might you accomplish that?
  o How could you apply what we’ve learned in this PLC to your daily classroom practice?
  o What continuing supports do you need to be successful in continuing to increase opportunities for students to engage in mathematics discourse as a regular part of instruction? Where could you find/get those supports?

• After the discussion, determine individual and/or collective next steps. Make these steps manageable and actionable…

• Time to work on Math Talk Anchor Charts

Closing (5 minutes)

• Final Thoughts: As our formal professional learning community draws to a close, we’d like to give everyone an opportunity to share their final thoughts about this experience…
Appendix F:

Research and Resources Shared by Participants

During the Mini-Design Cycles
Research and Resources Shared by Participants

Mini-Design Cycle 1: Wait Time/Think Time


Mini-Design Cycle 2: Revoicing


Mini-Design Cycle 3: Turn and Talk


TeachLikeThis. (2013). How to do a turn and talk [YouTube video]. Retrieved from https://www.youtube.com/watch?v=2zSUCH0vHY

Appendix G:

Purposefully Planning for Mathematics Discourse (PPMD)

Professional Development Program

Final Version After Study
Planning and implementing effective professional development for teachers of mathematics is as complex an endeavor as planning and implementing effective mathematics instruction for students. Key to success in this endeavor is the ability of the professional development program to be grounded in research-based best practices for both teacher learning and student learning, as well as be flexible enough to meet the varied needs of the participating teachers. As Loucks-Horsley, Stiles, Mundry, Love, and Hewson (2010) explained, “Skilled professional developers have one foot planted firmly in theory (knowledge and beliefs and vision) and the other in action (the local context, data about students, issues faced, and planning and doing professional development)” (p. 25). To that end, a purposeful effort was made to balance between theory and action when designing the Purposefully Planning for Mathematics Discourse (PPMD) professional development program.

**Purpose**

The purpose of the PPMD professional development program is to support teachers in their efforts to increase opportunities for students to engage in mathematics discourse as a regular part of instruction. From a teacher learning perspective, participating teachers will collaborate to create and revise annotated lessons that create space for student talk about mathematics as well as develop their skills in facilitating productive mathematics discussions. From a student learning perspective, students will have increased opportunities to discuss mathematics and engage in shared meaning making.
Research-Base

Three strands of research were attended to when developing the PPMD professional development program: (1) multitiered design study for professional development; (2) best practices in professional development for mathematics teachers; and (3) effective instructional practices for supporting productive mathematics discourse in the classroom.

The PPMD professional development program is a multitiered professional development design study. The philosophy of multitiered professional development design study is “to embrace the complexity and the dynamic nature of the system in which teachers learn and grow” by situating teachers as co-developers of educational objects (Zawojewski, Chamberlin, Hjalmarsrud, & Lewis, 2010, p. 236) This approach to professional development honors and builds from the experiences, strengths, and needs that teachers bring to the table, which allows for flexibility. At the same time, this approach to professional development also provides a collaborate environment in which teachers learn and growth together around a focused topic, in this case facilitating productive mathematics discourse. In the PPMD professional development program, this collaborative environment will take place in a professional learning community (PLC). The teachers in the PLC will conduct multiple mini-design cycles, in which they will create an annotated lesson plan that modifies their current adopted mathematics curriculum to include mathematics discourse. After enacting the lesson plan, teachers will meet again in the PLC to reflect and revise the lesson plan based on their learning. This mini-design cycle will take place multiple times throughout the professional development. Thus, the multitiered professional development design study supports
teachers in designing and implement educational products to improve their practice, and it allows researchers to document teacher learning and growth, which can contribute to the broader field of mathematics education research.

With regard to best practice in mathematics professional development, the elements of context and collaboration were used when designing the PPMD professional development program. Connecting professional development to the context of teachers, in terms of school-wide goals and individual teacher needs, is necessary to motivate instructional change (Loucks-Horsley et al., 2010). The PPMD professional development program will attend to context by involving teachers in determining which discourse strategies they want to incorporate into their existing mathematics curriculum. Another element necessary for effective professional development is providing opportunities for teachers to collaborate, both in terms of planning and reflecting (NCTM, 2014). Hence, the PPMD professional development program will utilize professional learning communities as the space for such collaboration. Because PLCs are an existing collaborative structure at the participating school, using them also complements the school’s context.

For any professional development program, it is necessary to provide on-going support in order for teachers to implement what they have learned and make sustained instructional changes (Loucks-Horsley et al., 2010; NCTM 2014). Within the PPMD professional development program, the PLC serves an key role in providing this on-going support, as teachers are collaborating and supporting one another through the learning process. The PPMD professional development program was developed so that the teacher members rotate facilitating the PLC themselves. This design honors the expertise
that each teacher brings to the learning community and it provides for scalability as outside facilitators and/or content or pedagogical experts are not required. Therefore, once teachers are familiar with mini-design cycle process, they could easily sustain their work beyond the timeline of the PPMD professional development program as they will have an established, but flexible structure that would allow them to continue to improve their practice in facilitating productive mathematics discourse or focus on other areas of mathematics instruction.

There are a plethora of effective instructional practices for supporting productive mathematics discourse in the classroom. Key to successfully implementing these strategies is creating a classroom community that is safe for students to share ideas, explain and justify solution strategies, and engage in respectful disagreement (Anthony & Walshaw, 2009; Cirillo, 2013a; NCTM, 1991, 2000). Beyond creating a mathematics discourse community, there are many different considerations for teachers in terms of facilitating productive mathematics discourse, including: selecting tasks, the types of questions asked, building upon student reasoning, and teacher discourse moves (e.g., using waiting time or revoicing). This list is far from exhaustive, which demonstrates the complexity of facilitating mathematics discourse. Thus, there is no specific set of practices or teacher moves that will ensure productive mathematics discussions will happen (NCTM, 1991; Truxaw & DeFranco, 2008). It is for this reason that the PPMD professional development program is designed to reflect best practices in effective instructional practices for supporting productive mathematics discourse in the classroom and to allow for flexibility to meet the individual needs of a given teacher and his/her students.
Theoretical Framework

When designing the PPMD professional development module, the theoretical framework of social constructivism (Vygotsky, 1962, 1978) and the concept of communities of practice (Lave & Wenger, 1991) were used. Just as discourse opens opportunities for students to interact with others as they co-construct meaning about mathematics, the PPMD professional development program will provide those same opportunities for teachers as they plan, revise, and reflect together, thereby co-constructing knowledge about teaching mathematics. Thus, situating the professional development program within the collaborative structure of a PLC provides a space for teachers to form communities of practice in which discourse serves as the means by which pedagogical knowledge is constructed individually and collectively.

Goals

There are multiple levels of goals for the PPMD professional development program:

- Support teacher learning in effectively facilitating productive mathematics discourse as part of regular instruction;
- Provide a space for teacher collaboration to co-construct knowledge about teaching mathematics individually and collectively;
- Create annotated lesson plans that build upon the currently adopted mathematics curriculum by opening opportunities for mathematics discourse;
- Provide access to productive mathematical discussions for all students, which allow them to engage in shared meaning making;
• Shift responsibility for mathematics discourse from teacher-led discussion to more student-led discussions;
• Document teacher growth in the area of facilitating mathematics discourse; and
• Contribute to the field of mathematics education research as a result of implementing the PPMD professional development program.

Overview of PPMD Module

The PPMD professional development will utilize a professional learning community (PLC) as the collaborative structure. The PLC will consist of teachers from multiple grade levels, but focus on the shared goal of collaboratively planning for mathematics discourse building from lessons in the adopted textbook. The PLC will meet for sessions that are approximately one-hour long. The initial sessions will orient the teachers to the goals of the PPMD professional development program, establish norms, develop a shared understanding of mathematics discourse, build background in mathematics discourse research, and identify focus areas. The subsequent sessions will consist of multiple mini-design cycles, in which teachers co-plan a lesson, enact it, and come together to debrief and revise the lesson. The members of the PLC will decide the focus of each mini-design cycle within the area of facilitating productive mathematics discourse. The final sessions will summarize teacher learning and create a plan for moving forward.
Forming the PLCs

Cross-grade level collaboration is a key design element in the PPMD professional development program. Therefore, it is suggested that two to three grade levels combine based on the size of the faculty. Ideally, there should be at least 4 people in the PLC to ensure diversity in perspective, but no more than 7 or 8 members in a PLC, otherwise it is difficult for all voices to be heard. Some possible pairings of grade levels based on an elementary school that serves grades K-5 include: a) K-2 and 3-5; or b) K-1, 2-3, and 4-5.

Suggested Timeline

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<th>PPMD Sessions</th>
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<tr>
<td>September</td>
<td>Opening Module</td>
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<tr>
<td>October</td>
<td>Mini-Design Cycle 1</td>
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<tr>
<td>November/December</td>
<td>Mini-Design Cycle 2</td>
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<td>Mini-Design Cycle 3</td>
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<tr>
<td>April</td>
<td>Mini Design Cycle 6</td>
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<tr>
<td>May/June</td>
<td>Closing Module</td>
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### Opening Module

**Overview of Process**

The opening module is designed to take one month. During this time, you will be establishing the workings of the Math Discourse PLC, grounding your work in some research about math discourse, and building shared understanding of math talk.

So the timeline for opening module would look like this:

<table>
<thead>
<tr>
<th>Month</th>
<th>Meeting</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Week</td>
<td>Yes</td>
<td>Establish Norms; Explore Math Discourse</td>
</tr>
<tr>
<td>Second Week</td>
<td>Yes</td>
<td>Dig Deeper into Math Discourse</td>
</tr>
<tr>
<td>Third Week</td>
<td>Yes</td>
<td>Math Talk Strategies; Select First Strategy</td>
</tr>
<tr>
<td>Fourth Week</td>
<td>No</td>
<td>Find Research/Resource for First Strategy</td>
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</tbody>
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**First Meeting in Opening Module**

Materials Preparation

- Create Math Discourse PLC Folder in Google Drive
- Scratch paper or post-its

Opening (20 minutes)

- Welcome to everyone! Share your favorite part of teaching math.
- Share Overview of the Professional Development Program
  - The goal of this professional development program is to support us in planning for and facilitating mathematics discourse in our classrooms. We will work as a professional learning community to collaborate and learn together. Everyone bring valuable insights, experiences, and knowledge
to this PLC. There is no “expert,” instead we are all here to share our individual expertise and develop a shared expertise around mathematics discourse.

- We will begin by establishing a shared understanding of mathematics discourse and exploring some research about best practices in planning for and facilitating math talk among students during instruction. We will go through multiple mini-design cycles, in which we will select a math talk strategy to incorporate in a lesson, try it out with our students, and then come back together to share our learning and make revisions to the lesson. We will collaborate and reflect on our learning as we work through this professional development together.

- Establish norms for PLC
  - What behaviors and/or expectations do we need to make our PLC function effectively?
  - Create an agreed upon chart of norms (limit to 3–5 total) in the Google folder

Beginning to Define Mathematics Discourse (15 minutes)

- Think-Write-Share: What is mathematics discourse?
- Discuss to reach consensus on a “working” definition—it will be revisited throughout the year

Characteristics of Productive Mathematics Discussions (20 minutes)

- Working grade level teams, create chart to answer the following questions:
  - What does a productive mathematics discussion look like?
What does a productive mathematics discussion sound like?

What does a productive mathematics discussion feel like?

- Examine each others’ charts:
  - Look for similarities and differences
  - Clarify ideas as needed

- Create a consensus three-column chart that shows what a productive mathematics discussion looks like, sounds like, and feels like in the Google Folder.

Closing (5 minutes)

- Think-Pair-Share: What is your take away from our first meeting together?
- Leadership: Remember, we will rotate the leadership role among all of us, so who will facilitate next time?
- Homework: Read the Benefits of Discussion Research Brief from NCTM.

**Second Meeting in Opening Module**

Materials Preparation

- Access to Math Discourse PLC in Google Drive Folder: Definition of Mathematics Discourse and Productive Math Discussions
- Scratch paper or post-its

Opening (10 minutes)

- Welcome to everyone!
- Review of Norms.
- Share something positive that happened this week.
• Quick review of Definition of Math Discourse and Productive Math Discussions—What changes do we need to make?

Developing PLC Goals (10 minutes)

• Think-Write-Share: What do you want to get out of participating in the Math Discourse PLC?

• Chart people’s ideas and narrow down to no more than 3-5 goals

Roles of Student and Teacher during Mathematics Discourse (20 minutes)

• Working grade level teams, create a Venn diagram to answer the following questions:
  o What roles should teachers take during mathematics discourse?
  o What roles should students take during mathematics discourse?

• Examine each others’ charts:
  o Look for similarities and differences
  o Clarify ideas as needed

• Create a consensus Venn diagram of the roles teacher and students take during mathematics discourse

Discussion of Research Brief (15 minutes)

• Facilitate a discussion about the Benefits of Discussion Research Brief everyone read for this week. If some members are reluctant to share, try to invite them into the conversation. Ask someone to capture the key points of the discussion in a Google doc.
• Here are some possible questions for the discussion, but we are not limited to these:
  o What are your reactions to the research brief we read?
  o What do you agree/disagree with? What surprised you?
  o What type of strategies might facilitate productive discussion?
  o How does our math textbook support productive discussion?

Closing (5 minutes)
• Think-Pair-Share: What is your take away from our meeting today?
• Leadership: Remember, we will rotate the leadership role among all of us, so who will facilitate next time?
• Homework: Read the Strategies for Discussion Research Briefs from NCTM

Third Meeting in Opening Module

Materials Preparation
• Access to Math Discourse PLC in Google Drive Folder: PLC Goals, Definition of Mathematics Discourse, Venn Diagram of Roles, Productive Math Discussions
• Copies of the Math-Talk Learning Community Framework (Hufferd-Ackles, Fuson, & Sherin, 2014)

Opening (10 minutes)
• Review of Norms.
• Share something positive that happened while teaching math this week.
• Quick review of Definition of Mathematics Discourse, Venn Diagram of Roles, Productive Math Discussions—What changes do we need to make?
Examine the Math-Talk Learning Community (MTLC) Framework (20 minutes)

- We will compare the MTLC to our understanding of mathematics discourse.
  
  o Read the MTLC Framework; feel free to make notes on it.
  
  o How is the MTLC Framework similar to the documents we’ve produced?
  
    How is it different?
  
  o Do we want to make any deletions, additions, or modifications based on what’s included in the MTLC Framework to our definition of Math Discourse or the roles of students and teachers?

- Let’s take a few minutes to rate ourselves on the continuum of the MTLC Framework.
  
  o This rating can remain personal, but if there is something we want to share, feel free

Discussion of Research Brief (15 minutes)

- Facilitate a discussion about the Strategies for Discussion Research Brief everyone read for this week. If some members are reluctant to share, try to invite them into the conversation. Ask someone to capture the key points of the discussion in a Google doc.

- Here are some possible questions for the discussion, but we are not limited to these:
  
    o What are your reactions to the research brief we read?
  
    o What do you agree/disagree with? What surprised you?
  
    o Which of the practices mentioned in the brief would complement our math textbook?
Which practices are you most interested in trying out? Why?

Determining a Focus (10 minutes)

- In two weeks we will begin our first mini-design cycle, so we will be planning a lesson that focuses on facilitating discourse by using a math talk strategy. We will use our textbook as a basis and look for opportunities for student talk where we can implement that math talk strategy.
- Which math talk strategy do we want to try first? Why? (Must reach consensus)
- After deciding, each of us is responsible for bringing an idea, information, or research about that practice to share with the PLC at our next meeting...

Closing (5 minutes)

- Think-Pair-Share: In reflecting on the beginning of our Math Talk PLC, what is an “aha” you’ve had? What is a concern or question you have?
- Leadership: Remember, we will rotate the leadership role among all of us, so who will facilitate next time?
- Homework: Bring your idea, information, or research about our first agreed upon math talk strategy.
Mini-Design Cycles

Overview of Process

The mini-design cycle process is designed to take one month. In the first meeting of the cycle, you will share the research and/or resources you found for your selected math talk strategy and then, working in your grade level teams, you co-plan for an upcoming lesson(s) where you incorporate that math talk strategy. You take a week off from meeting to allow everyone to teach their lesson(s). In the second meeting in the cycle, you come back together to reflect on the experience and work with your grade level teams to make revisions to the lesson(s). You also decide on the next math talk strategy you want to try, so you do not meet the following week to allow you time to gather research and/or resources about that strategy.

So the timeline for the mini-design cycle would look like this:

<table>
<thead>
<tr>
<th>Month</th>
<th>Meeting</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Week</td>
<td>Yes</td>
<td>Share Research/Resources &amp; Co-Plan Lesson(s)</td>
</tr>
<tr>
<td>Second Week</td>
<td>No</td>
<td>Teach the Lesson(s)</td>
</tr>
<tr>
<td>Third Week</td>
<td>Yes</td>
<td>Reflect &amp; Revise; Select Next Strategy</td>
</tr>
<tr>
<td>Fourth Week</td>
<td>No</td>
<td>Find Research/Resource for Next Strategy</td>
</tr>
</tbody>
</table>

You repeat this mini-design cycle process multiple times; each time adding a new math talk strategy that you want to try. The idea is to build your repertoire of math talk strategies over time by continuing to use those that you have previously tried, where appropriate, and by adding a new strategy each cycle.
First Meeting in Mini-Design Cycle

Materials Preparation

- Access to Math Discourse PLC in Google Drive Folder: PLC Goals, Definition of Mathematics Discourse, Venn Diagram of Roles, Productive Math Discussions
- Bring Math Textbook Teacher Manual (hardcopy or electronic)

Opening (10 minutes)

- Welcome to everyone!
- Review of Norms.
- Sharing (Facilitator should select a topic to share about—this should rotate among school related and non-school related topics—e.g., What was a success you had while teaching math this week? What are plans for the long weekend?)
- Quick check-in on our PLC Goals—How are we doing?

Mathematics Discourse Practice of Focus (15 minutes)

- Each of us will share our idea, information, or research about the selected focus.
- Ask someone to capture the key points of the discussion to be added to our Google folder.

Co-Planning the Lesson (30 minutes)

- We will work with our grade level partner(s).
- We will identify a lesson or lessons we will be teaching in the next week that we want modify.
- We will think about the ideas we shared in our discussion as we look for opportunities to purposefully plan for student using the select math talk strategy.
• We will need to produce a written lesson plan, in a format of our choice, that (1) tells when we will use the math talk strategy; and (2) how we will facilitate that.

• Each grade level will share an overview of their plan and why they made those instructional choices. The rest of us will provide feedback.

Closing (5 minutes):
• Think-Pair-Share: What supports do you need to carry out your lesson plan? How will you get those supports?
• Leadership: Remember, we will rotate the leadership role among all of us, so who will facilitate next time?
• Homework: Teach your lesson(s) before our next meeting and be prepared to share the strengths and areas for revision.

Second Meeting in Mini-Design Cycle

Materials Preparation
• Access to Math Discourse PLC in Google Drive Folder: PLC Goals, Definition of Mathematics Discourse, Venn Diagram of Roles, Productive Math Discussions
• Bring Math Textbook Teacher Manual AND Lesson(s) (hardcopy or electronic)

Opening (10 minutes)
• Welcome to everyone!
• Review of Norms.
• Sharing (Facilitator should select a topic to share about—this should rotate among school related and non-school related topics—e.g., What was a success you had while teaching math this week? What are plans for the long weekend?)
Quick review of (a) Definition of Math Discourse; (b) Venn Diagram of Roles; or (c) Productive Math Discussions—What changes do we need to make?

(Facilitator should choose just one of the documents—during the next mini-design cycle, select a different document to review)

Reflecting on the Mathematics Discourse Lesson (15 minutes)

- Grade-level groups share a brief overview of lesson.
- Facilitate a discussion about how the lessons went when tried out. If some members are reluctant to share, try to invite them into the conversation. Ask someone to capture the key points of the discussion to be added to our Google folder.
- Here are some possible questions for the discussion, but you are not limited to these:
  - What went well? Why do you think that is?
  - What was challenging? Why do you think that is? How might you overcome that challenge?
  - What adjustments would you make in the future? Why?

Revise the Lesson Plan (15 minutes)

- We will work in our grade levels to revise the lesson based on your experience and our discussion.
- Our revisions need to be captured in the written lesson plan, in a format of your choice—we need to think about it like what information or notes would we want to leave ourselves so that when we teach this next year, we remember the changes we wanted to make…
• Each grade level will share an overview of their changes and why they made those instructional choices. The rest of us will provide feedback.

Determining a Focus (15 minutes)

• When we start our next mini-design cycle, we will be planning another lesson that uses a new math talk strategy.

• Which math talk strategy do we want to try next? Why? (Must reach consensus)

• Once focus is agreed upon, each member is responsible for bringing an idea, information, or research about that practice to share during our next PLC…

Closing (5 minutes)

• Think-Pair-Share: What is your “take-away” from this mini-design cycle? Why?

• Leadership: Remember, we will rotate the leadership role among all of us, so who will facilitate next time?

• Homework: Bring your idea, information, or research about our agreed upon math talk strategy for our next meeting.
**Closing Module**

*Overview of Process*

The closing module is designed to take two weeks—you can meet for two consecutive weeks or put a one-week break in between. During this time, you will be reflecting on the work of the Math Discourse PLC and creating an action plan.

*First Meeting of Closing Module*

Materials Preparation

- Access to Math Discourse PLC in Google Drive Folder: PLC Goals, Definition of Mathematics Discourse, Venn Diagram of Roles, Productive Math Discussions
- Copies of the Math-Talk Learning Community Framework (Hufferd-Ackles, Fuson, & Sherin, 2014)

Opening (5 minutes)

- Welcome to everyone!
- Review of Norms.
- Sharing (Facilitator should select a topic to share about—this should rotate among school related and non-school related topics—e.g., What was a success you had while teaching math this week? What are plans for the long weekend?)

Revisit Venn Diagram of Roles (15 minutes)

- Review the Venn Diagram of Teacher and Student Roles in Discourse (in Google folder)
- Facilitate a brief discussion and make modifications to the document as needed:
Revisit Productive Math Discussions (15 minutes)

- Review the What Productive Math Discussions Look/Sound/Feel Like chart (in Google folder)
- Facilitate a brief discussion and make modifications to the document as needed:
  - What elements of productive math discussions have we confirmed through our work together?
  - What changes do we want to make? Why?

Revisit the Math-Talk Learning Community (MTLC) Framework (15 minutes)

- Review the MTLC Framework
- Take a few minutes to rate yourself on the continuum of the MTLC Framework.
- Consider the following questions for **personal** self-reflection—the intent is not that each member share, although they are welcomed to if they want…
  - How does it compare to where you were prior to engaging in this professional development process?
  - What areas might you want to continue to attend to moving forward?

Closing (5 minutes)

- Think-Pair-Share: What was the most exciting change you experienced?
- Leadership: Remember, we will rotate the leadership role among all of us, so who will facilitate next time?
Second Meeting of Closing Module

Materials Preparation

- Access to Math Discourse PLC in Google Drive Folder: PLC Goals, Definition of Mathematics Discourse, Venn Diagram of Roles, Productive Math Discussions

Opening (15 minutes)

- Welcome to everyone!
- Review of Norms.
- Sharing (*Facilitator should select a topic to share about—this should rotate among school related and non-school related topics—e.g., What was a success you had while teaching math this week? What are plans for the long weekend?*)
- Review our Definition of Mathematics Discourse—What changes do we need to make based on our work together?

Reflect on your Learning (15 minutes)

- Facilitate a discussion about the take-aways from participating in this professional development. If some members are reluctant to share, try to invite them into the conversation. Ask someone to capture the key points of the discussion, either on a board or chart paper.
- Here are some possible questions for the discussion, but you are not limited to these:
  - What have you learned as a result of participating in this PLC?
  - How has your instructional practice been impacted?
  - How have your students been impacted?
What were some of the challenges you faced during this process? Were you able to overcome them? Why or why not?

What were some of the successes you had during this process? What structures or supports enabled you to have those successes?

Has your process of planning for and facilitating mathematics discourse changed? If so, how? If not, why do you think that is?

What changes or improvements would you suggest if others were to use this protocol for the Math Talk PLC?

• Review of PLC Goals—Did we meet them? Why or why not?

Moving Forward (20 minutes)

• Have a discussion about “next steps.” Ask someone to capture the key points of the discussion, either on a board or chart paper.

• Here are some possible questions for the discussion, but you are not limited to these:

  o What are some aspects of planning for and/or facilitating mathematics discourse that you still want to work on? Why? How might you accomplish that?

  o How could you apply what we’ve learned in this PLC to your daily classroom practice?

  o What continuing supports do you need to be successful in continuing to increase opportunities for students to engage in mathematics discourse as a regular part of instruction? Where could you find/get those supports?
• After the discussion, determine individual and/or collective next steps and create an Action Plan. Make these steps manageable and actionable…And post them in the Google Folder!

Closing (10 minutes)

• Final Thoughts: As our formal professional learning community draws to a close, we’d like to give everyone an opportunity to share their final thoughts about this experience.